RISING WAGE INEQUALITY, RATE OF RETURN ON INVESTMENT IN EDUCATION, AND COST OF EDUCATION

Popovic, Milenko

Montenegro Business School, University Mediteran, Podgorica

25 June 2007

RISING WAGE INEQUALITY, RATE OF RETURN ON INVESTMENT IN EDUCATION, AND COST OF EDUCATION

RASTUĆE NEJEDNAKOSTI U PLATAMA, STOPA POVRATA NA INVESTICIJE U OBRAZOVANJE I TROSKOVI OBRAZOVANJA

MILENKO POPOVIC, Montenegro Business School
Podgorica, Montenegro, cis.um@cg.yu

Abstract: *One of the most interesting facts about the growth of developed nations, especially of the US growth, in the last three decades is significant growth of the ratio of the wage of skilled labor to that of unskilled labor. At the same time, existing evidence seems to suggest that the ratio of the rate of return on investment in skilled labor to that of unskilled labor has stayed pretty stable. This contradicting trend in movement of two ratios is formally easy to explain. Being aware of the fact that all possible measures of the rate of return in education confront, in one way or another, differences in wages of different educational levels with the cost of reaching the concerned level of education, we can with certainty conclude that, in order to keep the rate of return ratio unchanged, the increase of wage ratio should be accompanied with adequate increase in the ratio of the cost of reaching a skilled level of education to the cost of reaching an unskilled level. This is something that follows from identity and as such cannot be questioned. The real question here refers to a possible source of relative increase in the cost of reaching skilled level of education. Possibilities are here enormous and every developed country presents a different story. The purpose of this article is to shed a light on one of the sources of education cost growth which is common to all developed countries and which can explain the greatest part of education cost ratio increase in all developed countries. In what follows we will show that the increase in the cost of education ratio is mostly due, first, to the fact that technological progress in industry of education is negligible, second, to the fact that "products" of industry of education are nontradeables, and third, to the increase of wage ratio itself.*
Key Words: *Inequality, Growth, Capital of Education, Costs of Education*

Apstrakt: *Jedna od najinteresantnijih činjenica u razvoju razvijenih zemalja, posebno o razvoju SAD, u poslednje tri dekade je značajan rast odnosa zarada visoko obrazovanih prema zaradama manje obrazovanih radnika. U isto vreme postojeće činjenice sugerišu da je odnos stopa prinosa na investicije u visoko obrazovanje i stopa prinosa u niže nivoe obrazovanja bio konstantan. Ovaj kontradiktoran trend u kretanju dva odnosa je formalno lako objasniti. Znajući da sve moguće mere prinosa na investicije u obrazovanje na jedan ili drugi način sučeljavaju razlike u platama raznih nivoa obrazovanja sa troškovima dosezanja datog nivoa obrazovanja, možemo sa sigurnošću tvrditi da, pri rastućim razlikama u platama, pomenuta konstantnost odnosa stopa prinosa na ulaganja u obrazovanje mora biti praćena odgovarajućim prirastom odnosa troškova dosezanja visokog obrazovanja i troškova dosezanja nižih nivoa obrazovanja. To je nešto što sledi iz identiteta i kao takvo ne može biti dovedeno u pitanje. Pravo pitanje ovde se odnosi na moguće izvore relativnog rasta troškova dosezanja visokog nivoa obrazovanja. Mogućnosti su ovde mnogobrojne i svaka razvijena zemlja predstavlja priču za sebe. Namera ovog članka je da osvetli jedan od izvora rasta troškova obrazovanja koji je zajednički svim razvijenim zemljama i koji može objasniti najveći deo rasta relativnog odnosa troškova obrazovanja u svim razvijenim zemljama. U redovima koji slede pokazaćemo da je rast relativnog odnosa troškova obrazovanja uglavnom posledica, prvo, beznačajnog nivoa tehnološkog progresa u delatnosti obrazovanja, drugo, činjenice da «proizvodi» delatnosti obrazovanja nisu predmet međunarodne trgovine, i treće, posledica rasta samih raspona zarada.*
Ključne reči: *Nejednakost, rast, kapital obrazovanja, troškovi obrazovanja.*

JEL Classification: O11, O40, D31, J31;
Original scientific paper; Recived: June 25, 2007

1. Introduction

One of the most interesting facts about the economic growth of developed countries in the last three decades is a gap in the growth rate of wages of those with higher level of education and those with less education. This phenomenon is clearly illustrated in Figure 1 below. Data refer to the US but similar paths have been evidenced for other developed nations as well. As we can see, in the US this process started to develop at the beginning of seventies for males and at the beginning of eighties for females. As a result, the ratio of the earning of college educated workers (college educated and postgraduate) to those of non-college graduate workers (all other groups) increased dramatically in respected periods. The mere fact that this wage ratio increase was so prolonged is by itself puzzling. It is even more puzzling when we confront this fact with the data for educational composition of labor force given in Figure 2. As we can see, in the period 1964-2002, the US experienced significant and stable increase in share of those with higher level of education (postgraduate, college graduate and some college) and decrease in share of those with lower level of education (high school graduate and high school dropouts). Other developed countries experienced the same movement. So far, economists have come up with three explanations for this phenomenon, which is sometimes called a wage premium increase puzzle: deunionization, trade liberalization, and skill-biased technological change explanation.

Deunionization explanation is based on assumption that wage compression is positively correlated with degree of unionization. This explanation, however, cannot be accepted simply because of the timing of two processes. In the US the process of deunionisation started in the fifties, while the process of wage premium increase started at the beginning of seventies. In the UK, on the other hand, wage premium increase started in the mid-seventies, while the union density had been increasing until 1980.

Trade liberalization explanation is inspired by Heckscher-Ohlin theory. An ongoing process of trade liberalization is, on the one hand, supposed to increase demand for skilled labor in developed countries where skilled labor is cheap relative to developing countries. On the other hand, liberalization is also supposed do decrease demand for unskilled labor in developed countries which is relatively more expensive than in developing countries. This explanation is, however, not supported by evidence (See Aghion, P. 2001). First, it is very doubtful that trade liberalization can have such big impact on movement of wage premium in such big economies like the US, where trade with non-OECD countries represents no more than 2% of GDP. Second, this explanation would imply a decrease in prices of less skill-intensive goods relative to prices of more skill-intensive goods in developed economies. Empirical researches in the US and Europe have not found evidences to support such price movement. Third implication of this explanation is that labor in developed countries should be reallocated from low-skill to high-skill industries. Evidences of this are not significant as well. Finally, this theory would predict decrease in the ratio of skilled to unskilled employment in skill intensive industries of developed industries. This has not happened as well.

Skill-biased technical change (SBTC) explanation seems to be most widely accepted among economists now. A number of empirical investigations have shown a significant role of SBTC in explaining wage premium increase[1]. According to this explanation, acceleration of SBTC that started by early seventies has caused an enormous increase in the ratio of marginal productivity of skilled labor (post-graduate and college graduate) to marginal productivity of unskilled labor (high school dropouts, high school graduate). As a consequence, demand for skilled labor has increased dramatically. This process has been, indeed, accompanied with, previously illustrated, large increase in supply of skilled labor. However, this increase in supply has not been strong enough to match the increase in demand and as a consequence developed countries have experienced strong increase in the wage ratio.

[1] A number of empirical researches have shown superiority of SBTC compared to trade liberalization explanation. See for example: Bound and Johnson (1992), Lawrence and Slaughter (1993), Berman et al. (1994, 1998), Tyers and Yang (1997, 2000), Winchester and Greenaway (2005).

Krussell et al. (1997) provided the first convincing evidence in support for this hypothesis. They constructed a kind of CES aggregate production function in which physical capital (equipment) is more substitutable to unskilled labor than to skilled labor. Using this production function approach, they developed wage premium growth accounting framework in which the movement of ratio of skilled labor wages to that of unskilled can be decomposed in three components. The first component presents *relative quantity effect.* It is determined by the movement of the ratio of unskilled (some college, high school graduate, high school dropouts) to skilled (postgraduate and college graduate) labor inputs. Decrease of this ratio, and it is what developed nations have experienced in the last decades, should have negative effect on respected wage ratio, exactly opposite to what has happened in reality. The second component presents *capital-skill complementarity effect,* which is determined by the movement of the ratio of capital equipment to skilled labor input. The increase of this ratio, under assumed characteristics of production function, should lead to the increase in the respected wage ratio. Finally, the third component is *relative efficiency effect* and it is determined by the movement of the ratio of skilled labor efficiency to that of unskilled. In the empirical part of the research the authors have not tried to measure relative efficiency effect. Instead, they focused on the first two effects and they showed that capital-skill complementarity effect is able not only to compensate for relative quantity effect, but also to explain most of the wage premium increase. They showed that the observed acceleration in the decline of the relative price of production equipment goods since mid-seventies could account for most of the variation in the college premium over the past twenty-five years.

In the light of the above discussed facts, it is interesting to see what happened in the same period with the movement of the ratio of the rate of return on investment in skilled to the rate of return in unskilled labor force. Empirical evidences are scarce here, but they seem to support the conclusion that this ratio was pretty stable in the respected period. Indeed, Psacharopoulos and Patrinos (2002) in their survey of worldwide researches on the rate of return found evidence of an increasing rate of return in university levels of education. However, their conclusion is, first, derived on the basis of data for all countries surveyed, regardless of the level of development. Second, it refers to the movement of rate of return in university education and not to the trend in the respected rate of returns ratio. Only two developed countries for which this ratio can be calculated on the basis of presented results are Canada and France. In the case of Canada the ratio of the rate of return of skilled to the rate of return of unskilled labor was 2,75 for males and 1,8 for females in 1980. Five years later this ratio was 0,78 for males and 1.01 for females. A similar situation is with France: in 1969 the rate of return ratio was 1,62 for males and 0,9 for females, while in 1976 it was 1,35 for males and 0,78 for females. In all presented calculations we have unexpected downward sloping trend of the rate of return ratio. However, data refer to a very short period and only for two countries. The results, therefore, cannot be regarded as significant.

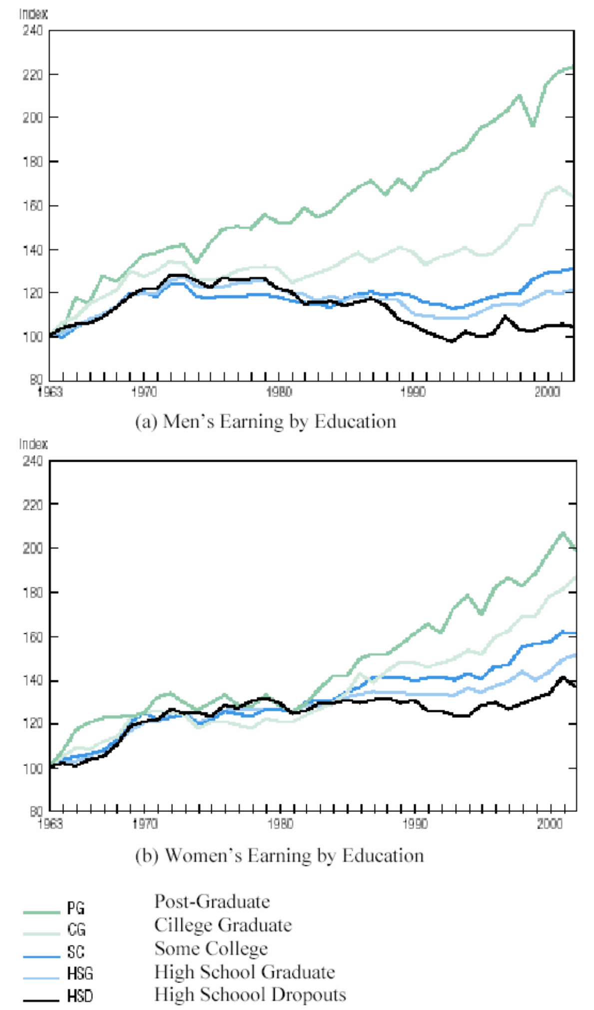
Table 1: Ratio of IRR of college completion to high school completion

Ratio of IRR	1960	1970	1980	1990
White (Nonparametric - Tuition and taxes)	0.7	0.6	0.4	0.4
Black (Nonparametric - tuition and taxes)	0.4	0.4	0.4	0.4
White (Mincer based)	1.0	1.0	1.0	1.0
Black (Mincer based)	1.0	1.0	1.0	1.0

More significant and convincing are results of a recent research for the US economy done by Heckman et al. (2005). They provided numerous econometric estimates of returns on education in the US for years 1940, 1950, 1960, 1970, 1980, and 1990. All those results can be, for our purpose, summarized by results given in their tables 5 and 6, where estimates of internal rate of returns (IRR) for white and black males are given for college and high school completion. On the basis of this result we calculated ratio of internal rate of return of college completion to internal rate of return of high school completion given in the following table.

The results are interesting indeed. For Mincer based estimate of IRR we have a constant rate of return ratio in all years considered, for both white and black males. When allowances for tuition fees, not covered by Mincer approach, and for taxes are taken into account by using a nonparametric approach, we have the constancy of the rate of return ratio for black males and a decreasing trend for white males.

Figure 1: Index of Mean of Real Wages of Full-Time Full-Year Worker:
Ages 22-65 by Education Group
Index, 1963=100

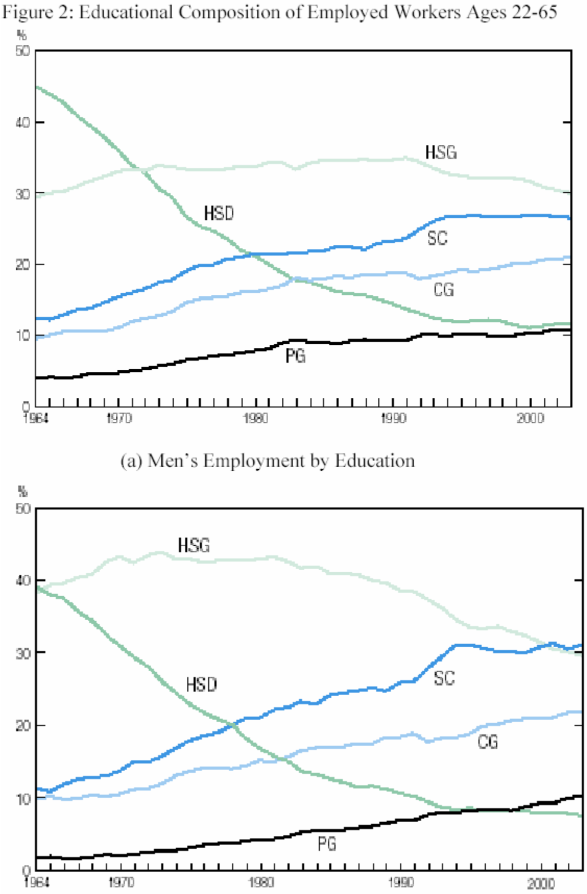


Source: Taken from Eckstein Zvi, Nagupal Eva (2004)

The contradicting trends that we have just described in the movement of wage ratio and the rate of return ratio obviously present a new puzzle itself. A formal solution to this puzzle is not so complicated as it might seem at the beginning. As we know all possible measures of the rate of return in education (IRR, Mincer coefficient, Marginal productivity of ED capital, and other) confront, in one way or another, educational wage premiums with the cost of reaching respected level of education. Having that in mind, we can with certainty conclude that, *in order to keep the rate of return ratio unchanged,* the *increase of wage ratio should be accompanied with adequate increase in ratio of cost of reaching skilled to the cost of reaching unskilled level of education*. This is something that follows from identity and as such cannot be questioned. The real question here refers to a possible source of relative increase in the cost of reaching skilled level of schooling. Possibilities are here enormous and every developed country presents a different story, especially having in mind various educational reforms in different countries. The purpose of this article is to shed a light on one of the sources of a cost of education growth which is common to all developed countries and which can explain the greatest part of the cost of education ratio increase in all developed countries. In what follows we will show that the increase of the cost of education ratio is mostly due, first, to the fact that technological progress in industry of education is negligible, and second, to the increase of wage ratio itself.

In our analysis we will adopt a production function with heterogeneous capital of education approach. More specifically, we will introduce ED capital of skilled and ED capital of unskilled labor. As a measure of the efficiency of ED capital we will, therefore, use marginal productivity of ED capital. It differs from ordinary IRR and Mincer coefficient measure, but it naturally should have similar trend as these alternative measures. In the next chapter different production functions with heterogeneous ED capital will be discussed. To show what their underlying assumptions are, all of them will be derived from general production function with heterogeneous ED capital. In the third chapter the wage ratio and the rate of return in education ratio will be calculated for every specific function. We will see that not all of functions that have been used in growth accounting analysis are able to allow for wage ratio increase. In the forth chapter, a framework for the cost of education growth accounting will be developed. It will be used to prove the thesis that constancy of the rate of return ratio follows from nonexistence of technological progress in the industry of education, and from the wage ratio increase itself. The main conclusions and implications are given in the final chapter.

Figure 2: Educational Composition of Employed Workers Ages 22-65



(a) Men's Employment by Education

(b) Women's Employment by Education

Source: Taken from Eckstein Zvi, Nagupal Eva (2004)

2. Models with Skilled and Unskilled Human Capital

1. In order to develop a model with the capital of education we will first start with a general production function of the following form

$$Q_t = F(K_t, E_{ut}, E_{st}, t) = F(K_t, H_{ut}l_{ut}, H_{st}l_{st}, t) \quad (1)$$

Here, as usual, K_t stands for capital and t presents time. On the other hand, E_{ut} presents quantity of unskilled human / educational capital owned by those who have, let say only 12 or fewer years of schooling, while E_{st} presents quantity of skilled educational capital owned by those with more than 12 years of schooling.[2] Formally, $E_{ut} = H_{ut}l_{ut}$ where l_{ut} presents quantity of educational capital (cost of education) per unskilled worker, while H_{ut} stands for a number of workers with 12 or fewer years of schooling. Similarly, $E_{st} = H_{st}l_{st}$ where l_{st} presents quantity of educational capital (cost of education) per skilled worker, while H_{st} stands for a number of workers with more than 12 years of schooling.

By differentiating expression (1) and dividing with $\boldsymbol{Q_t}$, we are getting the rate of production growth decomposed in the following way

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + \left(\frac{F_{Eu}E_{ut}}{Q_t}\right)\frac{\dot{E}_u}{E_u} + \left(\frac{F_{Es}E_{st}}{Q_t}\right)\frac{\dot{E}_s}{E_s} = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + f_{ut}\frac{\dot{E}_u}{E_u} + f_{st}\frac{\dot{E}_s}{E_s} \quad (2)$$

It is obvious that the first element, $\dot{A}/A$, presents contribution of global factor productivity to the growth rate of GDP, the second element, $a_t(\dot{K}/K)$, measures contribution of capital accumulation, the third part, $f_{ut}(\dot{E}_u/E_u)$, measures contribution of unskilled human capital, while the last element, $f_{st}(\dot{E}_s/E_s)$, expresses contribution of skilled human capital to the rate of growth. A coefficient a_t stands for elasticity of production with respect to physical capital, while coefficients $f_{ut} = (F_{Eut}\, E_{ut}/Q_t)$ and $f_{st} = (F_{Est}\, E_{st}/Q_t)$ present elasticity of production with respect to unskilled and skilled human capital respectively. As usual, $F_{Eut} = \partial Q/\partial E_u$ stands for marginal productivity of unskilled educational capital. Similarly, $F_{Est} = \partial Q/\partial E_s$ presents marginal productivity of skilled human capital. Having in mind usual analysis of educational contribution to the economic growth based on different kind of workers, we can state that $F_{Eut} = \frac{\partial Q}{\partial E_u} = \frac{F_{Hut}}{l_{ut}}$ and $F_{Est} = \frac{\partial Q}{\partial E_s} = \frac{F_{Hst}}{l_{st}}$ and this is even intuitively understandable[3]. It is important to notice that, although F_{Hst} should be larger than F_{Hut}, the same does not necessarily applies for F_{Est} and F_{Eut} because later two symbols measure marginal productivity (rate of return) of money invested in particular level of education and not marginal productivity of hours of work of that level of education. Notice also that $dF_{Hst}=F_{Hst}-F_{Hut}=w_{st}-w_{ut}$ presents wage premium of skilled labor.

1.1. Let us now go back to expression (2) and see what can happen if we take particular assumptions about behavior of its parameters. If we, first, assume that elasticity of substitution between any two kinds of factors of production is equal to one and independent of quantity of the third factor, it will allow us to simplify initial production function. The consequence of this assumption is the constancy of factors elasticity

---

[2] As far as *measurement of human and educational capital* is regarded two different approaches have been proposed so far in economic literature. First one, which will be followed in this article, is *cost-based approach* (Shultz, T. W. 1960, 1961a, 1961b, 1962, Kendrick, J. W. 1976, and Eisner, R. 1985, 1988). Second is *income-based approach* (Jorgenson, D. W. and Fraumeni, B. M. 1989, 1992a, 1992b, Fraumeni, B. M., 2000, Fraumeni, M. B., Reinsdorf, M. B., Robinson, B. B., Williams, M. P., 2004a, 2004b). Simply speaking, capital of education and human capital are here calculated as present value of stream of benefits (increased earning) generated by investment in particular kind of education or human capital in general. For details see Popovic, M., (2005).

[3] For detailed consideration on this issue see Popovic, M. (2006).

of production, that is the constancy of elasticity of production with respect to capital, $a_t = a$, and the constancy of elasticity of production with respect to any kind of human capital, $f_{ut} = f_u$ and $f_{st} = f_s$.

Substituting those new values in expression (2) we get the following decomposition of the rate of growth

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a\frac{\dot{K}}{K} + f_u\frac{\dot{E}_u}{E_u} + f_s\frac{\dot{E}_s}{E_s}$$

Now solving this new differential equation we obtain

$$Q_t = A_t K_t^a E_{ut}^{f_u} E_{st}^{f_s} \tag{3}$$

Both kinds of human capital, as well as physical capital, are here aggregated using geometric index. Or, to put it in other words, we are here totally in realm of CD production function. This model is formally similar to the one used by Mankiw at al (1992): in both, human capital of skilled labor and physical capital are combined using CD production function. An important difference is in the fact that so called "raw" labor, which is presented in Mankiw at al (1992), does not exist as a separate factor of production in this expression.

1.2. On the other hand, if we after certain manipulation divide numerator and denominator of the third and forth part of (2) with F_K we are getting the rate of growth decomposed in the following way

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + \frac{F_{Eut}E_{ut} + F_{Est}E_{st}}{Q_t}\left[\left(\frac{F_{Eut}E_{ut}}{F_{Eut}E_{ut} + F_{Est}E_{st}}\right)\frac{\dot{E}_u}{E_u} + \left(\frac{F_{Est}E_{st}}{F_{Eut}E_{ut} + F_{Est}E_{st}}\right)\frac{\dot{E}_s}{E_s}\right] =$$

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + f_t\left[\left(\frac{z_{ut}E_{ut}}{z_{ut}E_{ut} + z_{st}E_{st}}\right)\frac{\dot{E}_u}{E_u} + \left(\frac{z_{st}E_{st}}{z_{ut}E_{ut} + z_{st}E_{st}}\right)\frac{\dot{E}_s}{E_s}\right]$$

The meaning of f_t is quite obvious here: it is a share of both kind of human capital in GDP. On the other hand $z_{ut}=F_{Eut}/F_{Kt}$ and $z_{st}=F_{Est}/F_{Kt}$.

If we now assume that all relevant parameters are constant, that is $a_t = a$, $(F_{Eut}/F_{Kt}) = z_{ut} = z_u$, $(F_{Est}/F_{Kt}) = z_{st} = z_s$, and $f_t = f$, substitute this value in previous expression and solve in this way obtained differential equation, we get

$$Q_t = A_t K_t^a \left(z_u E_{ut} + z_s E_{st}\right)^f \tag{4}$$

Human capital is here obviously aggregated using arithmetic index. In this way aggregated human capital is combined with physical capital using geometric index that is like in CD production function.

This production function is, in its nature, very close, if not identical, to the one proposed by Jones (Jones, C.I. 1996, 2004; Hall and Jones 1999), which is based on Mincerian tradition. This expression also resembles the one proposed by Mankiw et al (1992). An important difference is in the fact that in Mankiw et al (1992) the expression ED capital is expressed as an ordinary sum of different kinds of ED capital, while it is expressed here as weighted sum of different kinds of ED capital, weights being defined as the ratios of marginal productivity of different kinds of ED capital and marginal productivity of physical capital. Apart from that, so called "raw" labor is here captured by unskilled and skilled human capital, while in Mankiw et al (1992) it stands separately as a part of geometric index.

1.3. If we, however, in the similar manner divide a numerator and denominator of the second and third part of (2) with marginal productivity of capital we get the following decomposition

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+\frac{F_{Kt}K_t+F_{Eut}E_{ut}}{Q_t}\left(\left(\frac{F_{Kt}K_t}{F_{Kt}K_t+F_{Eut}E_{ut}}\right)\frac{\dot{K}}{K}+\left(\frac{F_{Eut}E_{ut}}{F_{Kt}K_t+F_{Eut}E_{ut}}\right)\frac{\dot{E}_u}{E_u}\right)+\left(\frac{F_{Est}E_{st}}{Q_t}\right)\frac{\dot{E}_s}{E_s}=$$

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+(a_t+f_{ut})\left(\left(\frac{K_t}{K_t+z_{ut}E_{ut}}\right)\frac{\dot{K}}{K}+\left(\frac{z_{ut}E_{ut}}{K_t+z_{ut}E_{ut}}\right)\frac{\dot{E}_u}{E_u}\right)+f_{st}\frac{\dot{E}_s}{E_s}$$

The meaning of (a_t+f_{ut}) is obvious: it is a share of unskilled human capital and physical capital in GDP. On the other hand, as before, $z_{ut}=F_{Eut}/F_{Kt}$ while $z_{kt}=F_{Kt}/F_{Kt}=1$.

If we now again assume that all relevant parameters are constant, that is $(a_t+f_{ut})=(a+f_u)$, $(F_{Eut}/F_{Kt})=z_{ut}=z_u$, and $f_{st}=f_s$, substitute this value in the previous expression and solve in this way obtained differential equation, we get

$$Q_t=A_t(K_t+z_uE_{ut})^{(a+f_u)}E_{st}^{f_s} \quad (5)$$

Obviously, this expression is very similar to the one proposed long ago by Griliches (1969) on capital skill complementarity. An important difference is that we here use unskilled human capital while Griliches used a number of unskilled workers.

1.4. If we now in the similar manner divide a numerator and denominator of the second and forth part of (2) with marginal productivity of capital we get the following differential equation

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+\frac{F_{Kt}K_t+F_{Est}E_{st}}{Q_t}\left(\left(\frac{F_{Kt}K_t}{F_{Kt}K_t+F_{Est}E_{st}}\right)\frac{\dot{K}}{K}+\left(\frac{F_{Est}E_{st}}{F_{Kt}K_t+F_{Est}E_{st}}\right)\frac{\dot{E}_s}{E_s}\right)+\left(\frac{F_{Eut}E_{ut}}{Q_t}\right)\frac{\dot{E}_u}{E_u}=$$

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+(a_t+f_{st})\left(\left(\frac{K_t}{K_t+z_{st}E_{st}}\right)\frac{\dot{K}}{K}+\left(\frac{z_{st}E_{st}}{K_t+z_{st}E_{st}}\right)\frac{\dot{E}_s}{E_s}\right)+f_{ut}\frac{\dot{E}_u}{E_u}$$

The meaning of (a_t+f_{st}) is obvious: it is a share of skilled human capital and physical capital in GDP. On the other hand, as before, $z_{st}=F_{Est}/F_{Kt}$ while $z_{kt}=F_{Kt}/F_{Kt}=1$.

If we now again assume that all relevant parameters are constant, that is $(a_t+f_{st})=(a+f_s)$, $(F_{Est}/F_{Kt})=z_{st}=z_s$, and $f_{ut}=f_u$, substitute this value in the previous expression, and solve in this way obtained differential equation, we get

$$Q_t=A_tE_{ut}^{f_u}(K_t+z_sE_{st})^{(a+f_s)} \quad (6)$$

Obviously, this concept of production function and particularly this concept of aggregate capital is similar to the one proposed by Mankiw (1995). A general shape of production function (6) is, having in mind the meaning of $(a+f_s)$ given previously, exactly the same as the one given by Mankiw. An unskilled part of labor input and aggregate capital are, however, measured in a bit different way. An unskilled part of labor input is, first, here measured in money terms (human capital) and not in hours of work, and, second it is here measured by human capital of unskilled labor force and not as unskilled part of the whole labor force. As far as aggregate capital is regarded, it is here, like in Mankiws function, given as a sum of conventional and human / ED capital. However, ED capital is now measured in a different way: it is here multiplied by the ratio of marginal productivity of skilled capital and marginal productivity of physical capital.

1.5. Next, if we divide with F_K a numerator and denominator of all parts of (2) we are getting the following decomposition of the rate of growth

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+(a_t+f_{ut}+f_{st})\left(\left(\frac{K_t}{K_t+z_{ut}E_{ut}+z_{st}E_{st}}\right)\frac{\dot{K}}{K}+\left(\frac{z_{ut}E_{ut}}{K+z_{ut}E_{ut}+z_{st}E_{st}}\right)\frac{\dot{E}_u}{E_u}+\left(\frac{z_{st}E_{st}}{K_t+z_{ut}E_{ut}+z_{st}E_{st}}\right)\frac{\dot{E}_s}{E_s}\right)$$

where $(a_t+f_{ut}+f_{st})=1$ stands for share of all factors in GDP. As before, $z_{st}=F_{Est}/F_{Kt}$ while $z_{ut}=F_{Eut}/F_{Kt}$.

If we now again assume that all relevant parameters are constant, that is $(F_{Est} / F_{Kt}) = z_{st} = z_s$, and, $(F_{Eut} / F_{Kt}) = z_{ut} = z_u$ substitute this value in the previous expression and solve in this way obtained differential equation, we get

$$Q_t = A_t\left(K_t + z_u E_{ut} + z_s E_{st}\right) = A_t C_t \qquad (7)$$

This is obviously a linear production function with all kinds of inputs. Formally, it is similar to the one used long ago by Abramovicz (1956) in one of the first sources of growth analysis. This similarity is, however, only formal and they differ very much in their meaning. In fact, in its nature it is closest to so called "AK" function: what we have in bracket is in fact a sum of all kind of capital specified in this function.

1.6. Now, if in equation (2) we assume constant partial elasticity of substitution between physical and two kinds of human capital, then we can get a model basically the same as the one proposed by Krusell at all (1997)[4]

$$Q_t = A_t\left(\mu E_{ut}^{\sigma} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda)E_{st}^{\rho}\right)^{\frac{\sigma}{\rho}}\right)^{\frac{1}{\sigma}} \qquad (8)$$

1.7. Finally, playing further with different assumptions, especially with assumption that marginal productivity of unskilled human capital is constant, we can derive a model that is only formally very similar to the one used by Galor and Weil (1993).

$$Q_t = A_t\left(\lambda K^{p} + (1-\lambda)E_{st}^{\rho}\right)^{\frac{1}{\rho}} + \mu E_{ut} \qquad (9)$$

First, Galor and Weil (1993) use hours of work, while we here use human capital of skilled and unskilled labor. Second, unskilled input covers here only unskilled workers, while Galor and Weil (1993) refer to unskilled input provided by whole labor force.

2. In order to see how skilled and unskilled labor can contribute to economic growth we can start from somewhat different general production function. This production function also uses two kinds of human capital, but they are now defined in a different way. First is unskilled part of human capital that is owned by all members of labor force, that is by all members who have, let say 12 and fewer than 12 years of education. The second one is a skilled part of labor force, the one reached solely at higher level of schooling and owned only by those who reached this higher level of education (by those who have more than 12 years of education). This general production function will have the following form

$$Q_t = F(K_t, G_{ut}, G_{st}, t) = F(K_t, H_t g_{ut}, H_s g_{st}, t) \qquad (10)$$

Here, as usual, K_t stands for conventional, physical capital, while G_{ut} and G_{st} represents quantity of educational capital reached solely at the years of schooling necessary to reach unskilled and skilled level of education respectively. Since both unskilled (H_{ut}) and skilled (H_{st}) workers passed unskilled level of education it follows that $G_{ut} = (H_{ut} + H_{st})g_{ut} = H_t g_{ut}$ where g_{ut} presents quantity of human / educational capital (cost of education) per worker reached at unskilled level of education. On the other hand, since only skilled workers reached skilled level of education it follows that $G_{st} = H_{st} g_{st}$ where g_{st} presents quantity of educational capital (cost of education) per worker reached at skilled level of education. Notice, here, that g_{st} can be expressed as per capita costs of reaching that additional level of education, *s***,** from the previous one, *u*. Therefore, it should be equal to $g_{st} = l_{st} - l_{ut}$, where l_{st} presents a cumulative cost per capita of reaching a skilled level of education.

[4] Our expression differs from Krussell at al only by the, empirically but not theoretically relevant, fact that Krusssel at al use two kind of physical capital: construction and equipment.

Now, by differentiating the previous expression and dividing it with $\boldsymbol{Q_t}$ we are getting the rate of growth of production decomposed in the following way

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+\left(\frac{F_{Gut}G_{ut}}{Q_t}\right)\frac{\dot{G}_u}{G_u}+\left(\frac{F_{Gst}G_{st}}{Q_t}\right)\frac{\dot{G}_s}{G_s}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+q_{ut}\frac{\dot{G}_u}{G_u}+q_{st}\frac{\dot{G}_s}{G_s} \qquad (11)$$

As usual, $\dot{A}/A$ presents a contribution of global factor productivity to the growth rate of GDP, $a_t(\dot{K}/K)$ measures a contribution of capital accumulation, $q_{ut}(\dot{G}_u/G_u)$ presents a contribution of unskilled human capital, while the last element, $q_{st}(\dot{G}_s/G_s)$ expresses a contribution of skilled part of educational capital to the rate of growth. Coefficients a_t present elasticity of production with respect to physical capital, while $q_{ut} = (F_{Gut}\, G_{ut} / Q_t)$ and $q_{st} = (F_{Gst}\, G_{st} / Q_t)$ stand for elasticity of production with respect to unskilled and skilled level of educational capital respectively. Accordingly, $F_{Gut} = \partial Q/\partial G_u$ and $F_{Gst} = \partial Q/\partial G_s$ stand for marginal productivity of unskilled and skilled level of educational capital respectively. Having in mind a usual analysis of the contribution of educationally heterogeneous labor to economic growth it can be shown that $F_{Gut}=\frac{\partial Q}{\partial G_u}=\frac{F_{Hu}}{g_{ut}}=\frac{F_{Hu}}{g_{ut}}$ and $F_{Gst}=\frac{\partial Q}{\partial G_s}=\frac{F_{Hs}-F_{Hu}}{l_{st}-l_{ut}}=\frac{dF_{Hs}}{g_{st}}$, which is also intuitively understandable.[5] In this relations F_{Hst} and F_{Hut} present marginal productivity of hours of work of skilled and unskilled worker respectively, while $dF_{Hs}= F_{Hst}-F_{Hut}=w_s-w_u$ stands for wage premium. Notice that, while F_{Hst} should be larger than F_{Hut}, the same does not hold for F_{Gut} and F_{Gst} because the latter two measures express productivity of money units invested in respected levels of education. It can easily happen that productivity of money invested in unskilled education be larger than productivity of money invested in skilled level of education.

2.1. Now that we developed this somewhat different approach to human capital we can in the similar way derive a whole new set of an additional specification of production function with human capital. If we make certain assumptions, similar to those used to derive expressions from the previous paragraphs about the movement of respected parameters and solve this differential equation we can get the following set of production functions. First, if we assume the constancy of a physical capital share ($a_t=a$), an unskilled human capital share ($g_{ut}=q_u$), and a skilled human capital share ($q_{st}=q_s$) in GDP, then we can get the following CD production function

$$Q_t = A_t G_{ut}^{q_u} G_{st}^{q_s} K_t^{a} \qquad (12)$$

In this function all forms of capital, and in this case all inputs, are combined like in CD production function.

This expression is much more similar to the one used by Mankiw at al (1992) than the previously mentioned expressions (3) and (4): in this model the whole "raw" labor is captured by $G_{ut}^{q_u}$ part of expression.

2.2. If we now divide a numerator and denominator of the last two parts of expression (11) with marginal productivity of capital (F_K), after a certain manipulation we get

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+\frac{F_{Gut}G_{ut}+F_{Gst}G_{st}}{Q_t}\left[\left(\frac{F_{Gut}G_{ut}}{F_{Gut}G_{ut}+F_{Gst}G_{st}}\right)\frac{\dot{G}_u}{G_u}+\left(\frac{F_{Gst}G_{st}}{F_{Gut}G_{ut}+F_{Gst}G_{st}}\right)\frac{\dot{G}_s}{G_s}\right]=$$

$$\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+(q_{ut}+q_{st})\left[\left(\frac{x_{ut}G_{ut}}{x_{ut}G_{ut}+x_{st}G_{st}}\right)\frac{\dot{G}_u}{G_u}+\left(\frac{x_{st}G_{st}}{x_{ut}G_{ut}+x_{st}G_{st}}\right)\frac{\dot{G}_s}{G_s}\right]$$

The meaning of $(q_{ut}+q_{st})$ is quite obvious here: it is a share of both kinds of human capital in GDP. On the other hand, $x_{ut}=F_{Gut}/F_{Kt}$ and $x_{st}=F_{Gst}/F_{Kt}$.

[5] For detailed analysis in more general case with more than two kind of ED capital see Popovic, M. (2005).

If we now assume that all relevant parameters are constant, that is $a_t = a$, $(F_{Gut} / F_{Kt}) = x_{ut} = x_u$, $(F_{Gst} / F_{Kt}) = x_{st} = x_s$, and $q_{ut} + q_{st} = q_u + q_s$, substitute this value in the previous expression and solve in this way obtained differential equation, we get

$$Q_t = A_t K_t^a \left(x_u G_{ut} + x_s G_{st}\right)^{q_u+q_s} \qquad (13)$$

Human capital is here obviously aggregated using arithmetic index. In this way aggregated human capital is, as before, combined with physical capital like in CD production function. Its prediction of human capital contribution is the same as in the previously mentioned Jones model.

2.3. If we, however, in the similar way as above divide a numerator and denominator of the second and third part of expression (11) with marginal productivity of physical capital and then assume the constancy of relevant parameters, that is $\frac{F_{Gut}}{F_{Kt}} = x_{ut} = x_u$, $\frac{F_{Kt}}{F_{Kt}} = 1$, $q_{st}=q_s$, and $(a_t + q_{ut}) = (a + q_u)$, then by solving this differential equation we get the following Griliches wise function

$$Q_t = A_t [x_u G_{ut} + K_t]^{(a+q_u)} G_{st}^{q_s} \qquad (14)$$

2.4. If we next divide a numerator and denominator of the second and forth part of expression (11) with marginal productivity of physical capital and assume the constancy of a physical and skilled human capital share in GDP ($h_t=a_t+q_{st}=a+q_s=h$), the constancy of an unskilled human capital share in GDP ($q_{ut}=q_u$), as well as the constancy of the ratio of marginal productivity of skilled human capital to marginal productivity of physical capital ($\frac{F_{Gst}}{F_{Kt}} = x_{st} = x_s$), then by solving in this basis the obtained differential equation we can get the following production function

$$Q_t = A_t G_{ut}^{q_u} \left[K_t + x_s G_{st}\right]^{a+q_s} = A_t G_{ut}^{q_u} \left[K_t + x_s G_{st}\right]^h \qquad (15)$$

We can say that this expression explains the growth process in almost identical way as the one used by Mankiw (1995). "Raw" labor is here entirely captured, although in money units, in the second part of expression, $G_{ut}^{q_u}$.

2.5. Again in a similar way, if we now divide a numerator and denominator of all parts of expression (11) with marginal productivity of physical capital, and assume that $\frac{F_{Gut}}{F_{Kt}} = x_{ut} = x_u$, $\frac{F_{Gst}}{F_{Kt}} = x_{st} = x_s$, then, knowing that $a_t+q_{ut}+b_{ut}=1$, we get the following linear production function

$$Q_t = A_t \left[x_u G_{ut} + K_t + x_s G_{st}\right] = A_t C_t \qquad (16)$$

Again, as before, this linear production function can be regarded as a sort of "AK" model.

2.6. Next, if in equation (11) we assume a constant partial elasticity of substitution between different factors of production, then we can get a model formally, but only formally, similar to the one proposed by Krusell at al (1997)

$$Q_t = A_t \left(\mu G_{ut}^{\sigma} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda) G_{st}^{\rho}\right)^{\frac{\sigma}{\rho}} \right)^{\frac{1}{\sigma}} \qquad (17)$$

2.7. Finally, playing further with different assumptions we can, similarly as before, derive the following function which resembles the one used by Galor and Weil (1993)

$$Q_t = A_t \left(\lambda K^{p} + (1-\lambda) G_{st}^{\rho}\right)^{\frac{1}{\rho}} + \mu G_{ut} \qquad (18)$$

This model is obviously closer to that used by Galor and Weil than the previously mentioned expression (9): The last part of this expression captures an effect of unskilled labor of all labor force, not just of unskilled labor.

3. Growth Accounting for Wage Ratio and Rate of ED Capital Return Ratio

What follows is an analysis of wage ratio and the rate of return in human capital ratio (Table 2) for different models of human capital contribution presented in the previous chapter. In the first column versions of models for two types of human capital are given. The second column presents a corresponding wage ratio and the rate of return ratio. The wage ratio is defined as the ratio of marginal productivity of hours of work of skilled labor to that of unskilled labor, $\varpi = F_{Hst} / F_{Hut}$. On the other hand the rate of return ratio is defined as the ratio of marginal productivity of skilled human capital to that of unskilled capital, $\varpi^R = F_{Est} / F_{Eut}$ or $\varpi^R = F_{Gst} / F_{Gut}$.

Before we start our analysis of the results given in Table 1 it is important to remind ourselves that, as far as the theory is regarded, there is no reason whatsoever to expect long run movement of the rate of return on ED capital ratio. Although social rates of return in different levels of ED investment do not need to be equal (externalities, non-monetary benefits, and option values), there is no reason to believe that the above rate of return ratio would have any long run trend and movement. We would expect those ratios to remain constant. Therefore, at the theoretical level one would, for known reasons, expect that a long run rate of return ratio stays constant. Even more, someone would expect this ratio at a private level to be equal to one if we are able to calculate and add in a proper manner private non-monetary and option values of investment in education. In the introduction of this paper we presented some important evidence that prove these theoretical expectations. If this is so, then the wage ratio growth rate should be compensated with differences between the rates of growth of human capital per capita in two groups of workers. Since we measure human capital per capita here by the cost of reaching a certain level of education, it also means that the wage ratio growth rate should be compensated with differences between growth rates of an average cost of postsecondary education and an average cost of education of those with fewer than 12 years of schooling.

Now, let's go back to our Table 2. First, the wage ratio of expression (3) does not allow either for capital skill complementarity effect or for efficiency effect, to use Krusell at al (1997) terminology. Only a relative quantity effect is present here. However, if a skilled part of labor force increases faster than unskilled part, and it is what we have experienced in the last 3 decades, then this model predicts a decrease of wage premium. Since this prediction sharply contradicts to what we have in reality, we can say that this model cannot be used to explain the movement of wage ratio in the last 3 decades. On the top of that, in order to keep the rate of return ratio constant it requires per capita ED capital of an unskilled part of labor force to grow faster than that of a skilled part of labor force, which is, as we will see later, also a very dubious implication. Absolutely the same conclusions apply for expression (12).

Second, the wage ratio of expression (4) allows only for an efficiency effect: the wage ratio can increase only as a result of faster increase of per capita ED capital of a skilled part of labor force relative to that of an unskilled part. The relative quantity and capital complementarity effects are not present here. On the other hand, the rate of return is here constant by definition. Basically the same conclusions apply for expression (13), as well as for linear production functions (or "AK" models) in expressions (7) and (16).

Third, Griliches wise expression (5) allows for a capital complementarity effect as well as for a relative quantity effect. Since the relative quantity effect should be negative (because of faster growth of the skilled part of labor force than that of the unskilled part), the wage ratio here can increase only if capital complementarity effect, growth of $\left(K_t / z_u l_{ut} H_{st}\right)$, is stronger than relative quantity effect, growth of $\left(H_{ut} / H_{st}\right)$. On the other hand, stronger influence of capital complementarity effect is possible only if physical capital grows much faster than the skilled part of labor force. In that circumstance, the rate of return ratio can remain constant only if wage ratio increase is compensated by faster growth of per capita ED capital of skilled labor than that of unskilled labor. Similar reasoning applies for expression (14).

Forth, expressions (6) and (15) have quite a peculiar form and behavior. Take for example Mankiws' celebrated model given in expression (15). The relative quantity effect has expected direction: relative increase of H_{st} / H_{ut} ratio, which characterized last 3 decades of growth, leads to a decrease of wage ratio. Note, however, that there is no capital complementarity effect that might compensate for this. Instead, what we have is K_t / H_{ut} ratio, which also increased in the last 3 decades, and which therefore also should, according to the model, have negative influence on the wage ratio. Only the efficiency effect, g_{st}, has a positive influence on the wage ratio. It is hard to believe, however, that this effect can be so strong to explain the increase of wage ratio and in that way makes this model appropriate for this kind of analysis. The same applies for expression (6).

Fifth, expressions (8) and (17) show all 3 effects and they all behave in expected way. In expression (8), which presents somewhat adopted Krusell at al (1997) model, capital skill complementarity effect is given with $(K_t / E_{st})^{\rho}$, relative quantity effect is given with $(H_{ut} / H_{st})^{(1-\sigma)}$, and efficiency effect is captured with $(l_{st} / l_{ut})^{\sigma}$. A crucial assumption in this nested CES aggregate production function is that physical capital is more substitutable to unskilled labor than to skilled labor. Since elasticity of substitution between capital (or skilled labor) and unskilled labor is given by $\frac{1}{1-\sigma}$, while elasticity of substitution between capital and skilled labor is given by $\frac{1}{1-\rho}$, this means that σ should be larger than ρ, $\sigma > \rho$. The capital complementarity effect, $(K_t / E_{st})^{\rho}$, when confronted with data from the last 3 decades, shows a positive influence on the wage ratio. This influence is somewhat offset owing to the influence of relative quantity effect, $(H_{ut} / H_{st})^{(1-\sigma)}$. Krusell at al (1997) in their research explained a great portion of wage premium increase by these two factors. As we can see, this expression allows also for efficiency effect, which may give additional explanation for wage ratio movement. This effect, however, has not been tested so far. This expression also assumes that, in order to keep the rate of return ratio unchanged, the increase of wage ratio should be compensated by an equal magnitude increase of (l_{st} / l_{ut}) ratio. In other words, the ratio of per capita ED capital (cost of education) of skilled to that of unskilled labor force should increase at the same rate as the wage ratio. A similar consideration applies for expression (17).

Finally, expressions (9) and (18) predict even stronger increase of wage ratio. For example, Galor and Weil (1993) model, given in expression (18), contains only capital complementarity effect, $(K_t / g_{st} H_{st})^{\rho}$, and efficiency effect, (g_{st} / g_{ut}), both of which should have a positive effect. There is no relative quantity effect here, which by its nature would have a negative impact on wage ratio movement. Note, however, that, in order to keep the rate of return ratio constant, this model predicts that the rate of growth of $(1/ g_{st})$ should be equal to the rate of growth of (K_t / H_{st}). A similar consideration applies for expression (9).

Table 2: *Wage Premiums and ED Capital* Rate *of* Return *Premiums in Different Models*

	Expressions	$\varpi = F_{Hst} / F_{Hut}$ $\varpi^R = F_{Est} / F_{Eut}$ or $\varpi^R = F_{Gst} / F_{Gut}$
3	$Q_t = A_t K_t^a E_{ut}^{f_u} E_{st}^{f_s}$	$\varpi = (f_s / f_u)(H_{ut} / H_{st})$ $\varpi^R = (f_s / f_u)(H_{ut} / H_{st})(l_{ut} / l_{st}) = (f_s / f_u)(H_{ut} / H_{st}) \frac{1}{(l_{st} / l_{ut})}$
4	$Q_t = A_t K_t^a \left(z_u E_{ut} + z_s E_{st}\right)^f$ Denison, Jones	$\varpi = (z_s / z_u)(l_{st} / l_{ut})$ $\varpi^R = (z_s / z_u)$
5	$Q_t = A_t (K_t + z_u E_{ut})^{(a+f_u)} E_{st}^{f_s}$ Griliches	$\varpi = \left(\frac{f_s}{a+f_u}\right)\left(\frac{K_t}{z_u l_{ut} H_{st}} + \frac{H_{ut}}{H_{st}}\right)$ $\varpi^R = \left(\frac{f_s}{a+f_u}\right)\left(\frac{l_{ut}}{l_{st}}\right)\left(\frac{K_t}{z_u l_{ut} H_{st}} + \frac{H_{ut}}{H_{st}}\right)$ $= \left(\frac{f_s}{a+f_u}\right)\left(\frac{K_t}{z_u l_{ut} H_{st}} + \frac{H_{ut}}{H_{st}}\right)\frac{1}{\left(\frac{l_{st}}{l_{ut}}\right)}$
6	$Q_t = A_t E_{ut}^{f_u} (K_t + z_s E_{st})^{(a+f_s)}$	$\varpi = \left(\frac{a+f_s}{f_u}\right) z_s l_{st} \left(\frac{H_{ut}}{K_t + z_s E_{st}}\right)$ $\varpi^R = \left(\frac{a+f_s}{f_u}\right) z_s l_{ut} \left(\frac{H_{ut}}{K_t + z_s E_{st}}\right)$
7	$Q_t = A_t\left(K_t + z_u E_{ut} + z_s E_{st}\right)$ Linear / "AK" model	$\varpi = (z_s / z_u)(l_{st} / l_{ut})$ $\varpi^R = (z_s / z_u)$
8	$Q_t = A_t\left(\mu E_{ut}^{\sigma} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda)E_{st}^{\rho}\right)^{\frac{\sigma}{\rho}}\right)^{\frac{1}{\sigma}}$ Krusell at al	$\varpi = \left(\frac{(1-\mu)(1-\lambda))}{\mu}\right)\left[\lambda\left(\frac{K_t}{E_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}}\left(\frac{H_{ut}}{H_{st}}\right)^{1-\sigma}\left(\frac{l_{st}}{l_{ut}}\right)^{\sigma}$ $\varpi^R = \left(\frac{(1-\mu)(1-\lambda))}{\mu}\right)\left[\lambda\left(\frac{K_t}{E_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}}\left(\frac{H_{ut}}{H_{st}}\right)^{1-\sigma}\left(\frac{l_{st}}{l_{ut}}\right)^{\sigma}\frac{1}{\left(\frac{l_{st}}{l_{ut}}\right)}$
9	$Q_t = A_t\left(\lambda K^{p} + (1-\lambda)E_{st}^{\rho}\right)^{\frac{1}{\rho}} + \mu E_{ut}$	$\varpi = \left(\frac{(1-\lambda)}{\mu}\right)\left[\lambda\left(\frac{K_t}{E_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{1-\rho}{\rho}}\left(\frac{l_{st}}{l_{ut}}\right)$ $\varpi^R = \left(\frac{(1-\lambda)}{\mu}\right)\left[\lambda\left(\frac{K_t}{E_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{1-\rho}{\rho}}$
12	$Q_t = A_t G_{ut}^{q_u} G_{st}^{q_s} K_t^a$ Mankiw at al	$\varpi = 1 + (q_s / q_u)\{(H_{ut} / H_{st}) + 1\}$ $\varpi^R = (q_s / q_u)\{(H_{ut} / H_{st}) + 1\}(g_{ut} / g_{st})$
13	$Q_t = A_t K_t^a \left(x_u G_{ut} + x_s G_{st}\right)^{q_u+q_s}$ Jones	$\varpi = 1 + (x_s / x_u)(g_{st} / g_{ut})$ $\varpi^R = (x_s / x_u)$
14	$Q_t = A_t [x_u G_{ut} + K_t]^{(a+q_u)} G_{st}^{q_s}$	$\varpi = 1 + \left(\frac{q_s}{a+q_u}\right)\left(\frac{K_t}{x_u g_{ut} H_{st}} + \frac{H_{ut}}{H_{st}} + 1\right)$ $\varpi^R = \left(\frac{q_s}{a+q_u}\right)\left(\frac{g_{ut}}{g_{st}}\right)\left(\frac{K_t}{x_u g_{ut} H_{st}} + \frac{H_{ut}}{H_{st}} + 1\right)$
15	$Q_t = A_t G_{ut}^{q_u} \left[K_t + x_s G_{st}\right]^{a+q_s}$ Mankiw	$\varpi = \left(\frac{a+q_s}{q_u}\right) x_s g_{st} \left(\frac{H_{ut}}{K_t + x_s G_{st}}\right) + 1 = \left(\frac{a+q_s}{q_u}\right) x_s g_{st} \left(\frac{1}{\frac{K_t}{H_{ut}} + x_s \frac{g_{st} H_{st}}{H_{ut}}}\right) + 1$

		$\varpi^R = \left(\dfrac{a+q_s}{q_u}\right) x_s g_{ut} \left(\dfrac{H_{ut}}{K_t + x_s G_{st}}\right)$
16	$Q_t = A_t\left[x_u G_{ut} + K_t + x_s G_{st}\right]$ Linear / "AK" model	$\varpi = 1 + (x_s / x_u)(g_{st} / g_{ut})$ $\varpi^R = (x_s / x_u)$
17	$Q_t = A_t\left(\mu G_{ut}^{\sigma} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda) G_{st}^{\rho}\right)^{\frac{\sigma}{\rho}}\right)^{\frac{1}{\sigma}}$	$\varpi = \left(\dfrac{(1-\mu)(1-\lambda))}{\mu}\right)\left[\lambda\left(\dfrac{K_t}{g_{st}H_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}} \left(\dfrac{H_{ut}}{H_{st}} + 1\right)^{1-\sigma} \left(\dfrac{g_{st}}{g_{ut}}\right)^{\sigma} + 1$ $\varpi^R = \left(\dfrac{(1-\mu)(1-\lambda))}{\mu}\right)\left[\lambda\left(\dfrac{K_t}{g_{st}H_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}} \left(\dfrac{H_{ut}}{H_{st}}\right)^{1-\sigma} \left(\dfrac{g_{st}}{g_{ut}}\right)^{\sigma} \dfrac{1}{\left(\dfrac{g_{st}}{g_{ut}}\right)}$
18	$Q_t = A_t\left(\lambda K^{p} + (1-\lambda) G_{st}^{\rho}\right)^{\frac{1}{\rho}} + \mu G_{ut}$ Galor and Weil	$\varpi = 1 + \left(\dfrac{(1-\lambda)}{\mu}\right)\left[\lambda\left(\dfrac{K_t}{g_{st}H_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{1-\rho}{\rho}} \left(\dfrac{g_{st}}{g_{ut}}\right)$ $\varpi^R = \left(\dfrac{(1-\lambda)}{\mu}\right)\left[\lambda\left(\dfrac{K_t}{g_{st}H_{st}}\right)^{\rho} + (1-\lambda)\right]^{\frac{1-\rho}{\rho}}$

4. Cost of Education Growth Accounting

1.1. To get further insight in the above-mentioned problem let us now consider in more details the structure of ED capital. Since we use a cost approach in measuring ED capital we should analyze the cost of education structure. The capital of education of a certain person will be presented in *constant money prices* of all cost necessary to provide the level of education reached and "owned" by the respected person.

The total capital of education will, therefore, be equal to a sum of ED capital of all persons employed in a certain economy. Formally

$$E_t = l_t L_t = \sum_{1}^{20} L_{it} l_{it} = \sum_{1}^{20} L_{it} \sum_{1}^{i} g_{it} \tag{19}$$

where E_t stands for total ED capital engaged in economy, L_t a number of employees, l_t an average ED capital per capita, L_{it} a number of employees with *i*-th years of schooling, l_{it} total (cumulative) a cost of reaching *i*-th years of schooling given in current prices (net of inflation), and g_{it} a cost of reaching additional *i*-th years of schooling. Note that here $l_{it} = \sum_{1}^{i} g_{it}$. Note also that we assumed a maximum number of years of schooling to be 20:8 years for elementary school, 4 years for secondary education, 4 years for university degree, 2 years for MA degree and additional 2 years for Ph D.

Total ED capital of *unskilled labor force*, those with 12 years of schooling and less, will similarly be given by

$$E_{ut} = l_{ut} L_{ut} = \sum_{1}^{12} L_{it} l_{it} = \sum_{1}^{12} L_{it} \sum_{1}^{i} g_{it} \tag{20}$$

where E_{ut} stands for total ED capital of unskilled labor, $L_{ut} = \sum_{1}^{12} L_{it}$ a total number of unskilled workers, and l_{ut} average ED capital of unskilled workers.

On the other hand, total ED capital of *skilled labor force* will be given by

$$E_{st} = l_{st} L_{st} = \sum_{13}^{20} L_{it} l_{it} = \sum_{13}^{20} L_{it} \sum_{1}^{i} g_{it} \tag{21}$$

where E_{st} stands for total ED capital of skilled labor force, $L_{st} = \sum_{13}^{20} L_{it}$ a total number of skilled workers, and l_{st} average ED capital of skilled workers.

1.2. Now, from the above relations it follows that *average ED capital* will be equal to

$$l_t = \sum_{1}^{20} (L_{it} / L_t) l_{it} = \sum_{1}^{20} (L_{it} / L_t) \sum_{1}^{i} g_{it} \tag{22}$$

Average ED capital of *unskilled labor force* will similarly be given by

$$l_{ut} = \sum_{1}^{12} (L_{it} / L_{ut}) l_{it} = \sum_{1}^{12} (L_{it} / L_{ut}) \sum_{1}^{i} g_{it} \tag{23}$$

while average ED capital of *skilled labor* will be presented as

$$l_{st} = \sum_{13}^{20} (L_{it} / L_{st}) l_{it} = \sum_{13}^{20} (L_{it} / L_{st}) \sum_{1}^{i} g_{it} \tag{24}$$

4.3. It can be proved that *corresponding rates of growth of the above given average ED capital* can be presented and decomposed in the following way. For average ED capital of total labor force we will have

$$\frac{\dot{l}}{l} = \sum_{1}^{20} \frac{l_i}{l} \Delta\left(\frac{L_i}{L}\right) + \sum_{1}^{20} \frac{L_{it} l_{it}}{L_t l_t} \left(\frac{\dot{l}_i}{l_i}\right) \tag{25}$$

For the rate of growth of average ED capital of the unskilled part of labor force we get

$$\frac{\dot{l}_u}{l_u} = \sum_{1}^{12} \frac{l_i}{l_u} \Delta\left(\frac{L_i}{L_u}\right) + \sum_{1}^{12} \frac{L_{it} l_{it}}{L_{ut} l_{ut}} \left(\frac{\dot{l}_i}{l_i}\right) \tag{26}$$

Finally for the skilled part of labor force we get

$$\frac{\dot{l}_s}{l_s} = \sum_{13}^{20} \frac{l_i}{l_s} \Delta\left(\frac{L_i}{L_s}\right) + \sum_{13}^{20} \frac{L_{it} l_{it}}{L_{st} l_{st}} \left(\frac{\dot{l}_i}{l_i}\right) \tag{27}$$

It is important to note that in all 3 cases the first part of expressions presents an influence of structural change on per capita capital of education. It presents the influence of increase (or decrease) of a share of those with higher level of education in a corresponding number of employees. The second part, on the other hand, reflects an influence of increase of costs of reaching a certain level of education. Note that this increase is net of inflation. Obviously the first part of these equations measure a real increase of labor productive power caused by the increase of real per capita capital of education; it presents the influence of real improvement of labor force caused by education process; it is a real efficiency effect. The second part, on the other hand, reflects a fact that by the passage of time the real cost of reaching a certain level of education increases as well. But we will turn back to this issue later.

2.1. Another way to present *total ED capital* is to sum up all cost of reaching particular levels (years) of education of all labor force. Formally

$$G_t = g_t L_t = E_t = l_t L_t = \sum_{1}^{20} g_{it} \sum_{i}^{20} L_{it} = \sum_{1}^{20} g_{it} R_{it} \tag{28}$$

where $R_{it} = \sum_{i}^{20} L_{it}$ presents a number of those who reached at least *i*-th level (year) of education, while g_{it} presents a cost of education at that particular additional year (level) of schooling.

Similarly, total ED capital of *unskilled part of whole labor force*, using this notation, can be given by

$$G_{ut} = g_{ut} L_t = \sum_{1}^{12} g_{it} \sum_{i}^{20} L_{it} = \sum_{1}^{12} g_{it} R_{it} \tag{29}$$

Finally, it can be shown that total ED capital of *skilled part of whole labor force* is given with

$$G_{st} = g_{st} L_{st} = \sum_{13}^{20} g_{it} \sum_{i}^{20} L_{it} = \sum_{13}^{20} g_{it} \hat{R}_{it} \tag{30}$$

where $\hat{R}_{it} = \sum_{i}^{20} L_{it}$ presents a number of workers with at least *i*-th years of schooling for *i>12*.

2.2. In this case *average ED capital per employee* will be given with

$$g_t = l_t = \sum_{1}^{20} g_{it} (R_{it} / L_t) \tag{31}$$

Average ED capital of *unskilled part of labor force* can in this case be presented as

$$g_{ut} = \sum_{1}^{12} g_{it} \sum_{i}^{20} (L_{it} / L_t) = \sum_{1}^{12} g_{it} (R_{it} / L_t) \tag{32}$$

Similarly, *average ED capital of skilled part of labor* can be given by

$$g_{st} = \sum_{13}^{20} g_{it} \sum_{i}^{20} (L_{it} / L_{st}) = \sum_{13}^{20} g_{it} (\hat{R}_{it} / L_{st}) \tag{33}$$

2.3. Similarly as in the previous case, it can be proved that corresponding *rates of growth* of the above given average ED capital can be decomposed in the following way. For average ED capital of total labor force we will have

$$\frac{\dot{g}}{g} = \sum_{1}^{20} \frac{g_{it}}{g_t} \Delta\left(\frac{R_i}{L}\right) + \sum_{1}^{20} \frac{g_{it} R_{it}}{L_t g_t} \left(\frac{\dot{g}_i}{g_i}\right) = \sum_{1}^{20} \frac{g_{it}}{l_t} \Delta\left(\frac{R_i}{L}\right) + \sum_{1}^{20} \frac{g_{it} R_{it}}{L_t l_t} \left(\frac{\dot{g}_i}{g_i}\right) \tag{34}$$

For the rate of growth of average ED capital of the unskilled part of labor force we get

$$\frac{\dot{g}_u}{g_u} = \sum_{1}^{12} \frac{g_{it}}{g_{ut}} \Delta\left(\frac{R_i}{L}\right) + \sum_{1}^{12} \frac{g_{it} R_{it}}{g_{ut} L_t} \left(\frac{\dot{g}_i}{g_i}\right) \tag{35}$$

Finally for the skilled part of labor force we get

$$\frac{\dot{g}_s}{g_s} = \sum_{13}^{20} \frac{g_{it}}{g_{st}} \Delta\left(\frac{\hat{R}_i}{L_s}\right) + \sum_{13}^{20} \frac{g_{it} \hat{R}_{it}}{g_{st} L_{st}} \left(\frac{\dot{g}_i}{g_i}\right) \tag{36}$$

3.1. Having in mind expressions (26) and (27) we can present differences in rates of growth of ED capital per capita (cost of education) of skilled and unskilled labor in the following way

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_u}{l_u} = \left(\sum_{13}^{20} \frac{l_i}{l_s} \Delta\left(\frac{L_i}{L_s}\right) - \sum_{1}^{12} \frac{l_i}{l_u} \Delta\left(\frac{L_i}{L_u}\right) \right) + \left(\sum_{13}^{20} \frac{L_{it} l_{it}}{L_{st} l_{st}} \left(\frac{\dot{l}_i}{l_i}\right) - \sum_{1}^{12} \frac{L_{it} l_{it}}{L_{ut} l_{ut}} \left(\frac{\dot{l}_i}{l_i}\right) \right) \tag{37}$$

Note that expressions (35) and (36) can also be used for similar decomposition. Obviously, a difference in rates of growth of two kinds of ED capital per capita is here broken down in two parts. The first part, given in the first bracket, presents a difference between two structural effects, an effect of structural change of skilled and a structural change of unskilled part of labor force. In developing nations it is natural to expect both kinds of structural effects to be relatively large and of similar size. In developed nations we can expect an effect of structural change in the skilled part of labor force to be greater than that of unskilled part of labor force. So, for developed nations, we can expect value of expression in the first bracket to be positive. However, the magnitude of this part of expression is very small compared to the magnitude of the second part of this expression. The second part is here given in the second bracket of expression (37). This part, obviously, presents a difference in an average rate of growth of costs of education of skilled and unskilled labor.

3.2. To understand the meaning and possible magnitude of the second part of expression (37) we should further decompose the expression $l_{it} = \sum_{1}^{i} g_{it}$. In other words we should analyze the structure of per capita cost of reaching *i*-th level of education. As we can see from the above expression, it is equal to the sum of all costs made at all previous years of schooling, g_{it}. Costs of education at a particular year of schooling, g_{it}, are on the other hand equal to the sum of direct cost of education, which makes approximately 30% of the total cost, and an opportunity cost of education, which are approximately about 70% of the total cost. Both kinds of cost can be divided on private and social (total) cost. For the time being we will assume that private and social cost of schooling are equal.

Direct cost of education refers to all cost related to operation of school system: teachers and / or professors salaries, administration costs, materials, the cost of capital engaged and so on. The most important and dominant part of this costs are teachers' and professors' salaries. On the other hand all other direct costs are strongly correlated with costs of professors and teachers salaries. For that reason we will assume that other parts of direct costs are proportional to teachers' salaries and that this proportion is constant over time. We will also assume that the ratio of teachers to students is also constant over time. Having above in mind a direct cost per student for the respective year (level) of schooling, d_{it}, can be presented as

$$d_{it} = (1+\psi_{it})\frac{T_{it}}{S_{it}} w_{it}^{p} = (1+\psi_i)t_i w_{it}^{p} \tag{38}$$

In this expression w_{it}^{p} stands for average professors' / teachers' salaries at *i*-th year (level) of schooling in current year given in a constant price, that is net of inflation. The parameter t_i is constant that represents teachers (T_{it}) to students (S_{it}) ratio, while constant ψ_i presents ratio of average other direct costs to average teachers cost at that year of schooling. Under assumption that there is no difference between private and social direct cost we can say that direct cost should be equal to tuition fees paid by students for the respected year of schooling.

Opportunity costs of education for the given year (level) of schooling, on the other hand, are equal to earning lost by students because of their engagement in education instead of working for a wage. So, opportunity costs of a student engaged at *i*-th year of schooling should be equal to possible wages of those with *(i-1)* year of schooling, $w_{(i-1)t}$. Formally

$$o_{it} = w_{(i-1)t} \tag{39}$$

Now we can define an average cost of a particular year of schooling as

$$g_{it} = d_{it} + o_{it} = (1+\psi_i)t_i w_{it}^{p} + w_{(i-1)t} \tag{40}$$

Consequently, per capita cost of reaching a particular *i*-th year (level) of education will be equal to cumulative of all costs necessary to reach that level of education. Formally

$$l_{it} = \sum_{1}^{i} g_{it} = \sum_{1}^{i}\left((1+\psi_i)t_i w_{it}^{p} + w_{(i-1)t}\right) \tag{41}$$

3.3. Now, the rate of growth of average costs at particular year of schooling (g_{it}) are, following expression (40), given with

$$\frac{\dot{g}_i}{g_i} = \Phi_{di}\frac{\dot{w}_i^p}{w_i^p} + \Phi_{oi}\frac{\dot{w}_{(i-1)}}{w_{(i-1)}}$$

Consequently, the rate of growth of per capita cost of reaching a particular *i*-th year of schooling (l_{it}) will be equal to

$$\frac{\dot{l}_i}{l_i} = \sum_1^i \frac{g_{it}}{\sum_1^i g_{it}}\left(\frac{\dot{g}_i}{g_i}\right) = \sum_1^i \frac{g_{it}}{l_{it}}\left(\frac{\dot{g}_i}{g_i}\right) = \sum_1^i \frac{g_{it}}{l_{it}}\left(\Phi_{di}\frac{\dot{w}_i^p}{w_i^p} + \Phi_{oi}\frac{\dot{w}_{(i-1)}}{w_{(i-1)}}\right) \qquad (42)$$

where $\Phi_{di} = d_{it} / g_{it}$ and $\Phi_{oi} = o_{it} / g_{it}$, which are, for the sake of simplicity, assumed to be constant.

Substituting now this value in the second part of equation (37) we get the following important decomposition of the difference between rates of growth of skilled and unskilled labor per capita capital of education

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_u}{l_u} = \left(\sum_{13}^{20}\frac{l_i}{l_s}\Delta\left(\frac{L_i}{L_s}\right) - \sum_1^{12}\frac{l_i}{l_u}\Delta\left(\frac{L_i}{L_u}\right)\right) +$$

$$\left(\sum_{13}^{20}\frac{L_{it}l_{it}}{L_{st}l_{st}}\left(\sum_1^i \frac{g_{it}}{l_{it}}\left(\Phi_{di}\frac{\dot{w}_i^p}{w_i^p} + \Phi_{oi}\frac{\dot{w}_{(i-1)}}{w_{(i-1)}}\right)\right) - \sum_1^{12}\frac{L_{it}l_{it}}{L_{ut}l_{ut}}\left(\sum_1^i \frac{g_{it}}{l_{it}}\left(\Phi_{di}\frac{\dot{w}_i^p}{w_i^p} + \Phi_{oi}\frac{\dot{w}_{(i-1)}}{w_{(i-1)}}\right)\right)\right) \qquad (43)$$

We already know from previous considerations that the first part of this decomposition is of relatively small magnitude. The second part of this expression is dominant in its magnitude and will be subject to our further consideration.

4.1. Let us first take a closer look at expressions (41) and (42). They both tell us something very important and very interesting about the production function of educational industry. In other industries, due to technological progress, an average cost of production decreases by the passage of time. This decrease is followed and compensated by the increase of average wages. In fact, overall rate of technological progress on the long run, as we know, can be approximated by the rate of growth of average wages. On the other hand, average costs of education do not decrease. On the contrary, by passage of time, average costs of education (per capita cost of reaching particular level of education, l_{it}) increase very dramatically. This is clearly shown by expressions (41) and (42). The reason for this lies in the fact that the real increase of teachers' salaries and students' lost wages is not compensated by effects of adequate cost saving innovations in the industry of education.

On the one hand, industry of education, so far, has not been experiencing any significant innovation that might contribute to saving of any part of its costs. A teacher to student ratio (t_i) does not decrease over time. On the contrary, social development is usually followed by decrease of class size and consequent increase of this ratio. Other direct costs also does not decrease: on the contrary, in many developed countries there are evidences that a share of this part of cost, ψ_i, increases over time. Most importantly, there is also no evidence of any innovations that might save time of students learning and in that way contribute to saving of opportunity costs, o_{it}, as most important part of cost of education. Simply speaking, the industry of education has always been very labor intensive, or more precisely, it has always been very teachers intensive and very students intensive.

On the other hand, as far as teachers' salaries, w_{it}^p, and students' lost earning, $w_{(i-1)t}$, are regarded, the industry of education is here "price taker". The education sector has to compete with other market players for these resources. The level of both, teachers' salaries and students' lost earnings, are therefore determined by the level of salaries and wages in the rest of economy, which is itself determined by the level of productivity in overall economy. Since wages and salaries increase with technological progress and the increase of productivity, it means that both, teachers' salaries and students' lost earning, should also increase at the adequate rate. Again, we are here talking about the increase of wages and salaries that is net of inflation.

4.2. It is quite obvious, following expression (40) for g_{it}, that both elements of educational costs should be higher for higher levels of education: teachers' / professors' salaries (w_{it}^{p}) will be higher for a higher level of education simply because lecturers at a higher level of education themselves must be more educated; students' lost earnings ($w_{(i-1)t}$) are for the same reason also higher for a higher level of education.

More importantly, having in mind the evidence on wage ratio increase and following expression $\frac{\dot{g}_i}{g_i} = \Phi_{di}\frac{\dot{w}_{it}^{p}}{w_{it}^{p}} + \Phi_{oi}\frac{\dot{w}_{(i-1)t}}{w_{(i-1)t}}$, we can conclude that higher levels of education will also have higher rates of growth for both elements of ED costs: the rate of growth of teachers' wages as well as of students' lost earning will be higher at a higher level of education than at a lower level of education. Consequently, following previously derived expression (42), $\frac{\dot{l}_i}{l_i} = \sum_{1}^{i}\frac{g_{it}}{l_{it}}\left(\Phi_{di}\frac{\dot{w}_{i}^{p}}{w_{i}^{p}} + \Phi_{oi}\frac{\dot{w}_{(i-1)}}{w_{(i-1)}}\right)$, we conclude that the rate of growth of a unit cost of reaching a certain level of education should be higher for a higher level than for a lower level of education. Again, all this is a direct consequence of wage ratio increase caused by capital skill complementarity and skill biased technological progress in general.

Finally and most importantly, on the basis of the above considerations we can conclude that the second part of expression (43) should be positive and very large. In other words, due to evidenced wage ratio increase, average costs of reaching an average education of skilled labor will grow much faster than that of the unskilled part of labor force. Looking at a whole equation (43), we can conclude that faster increase of ED capital per capita of the skilled part of labor force than that of unskilled part ($\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_u}{l_u}$) can only partially be explained by a difference in real increase of average education (the first part of expression, $\left(\sum_{13}^{20}\frac{l_i}{l_s}\Delta\left(\frac{L_i}{L_s}\right) - \sum_{1}^{12}\frac{l_i}{l_u}\Delta\left(\frac{L_i}{L_u}\right)\right)$). A faster increase is in this case dominantly explained by differences in rates of growth of a cost of reaching a different level of education (the second part of expression (43)). Note, however, that this applies only when there are evidences of the wage ratio increase: If there is no wage ratio increase, the second part of expression becomes equal to zero, and whole differences in corresponding rates of growth of average capital of education have to be explained by real differences in their rate of growth of average education.

5. Concluding Remarks

1. We can now return to the considerations given in the previous section of our paper. Let us take a look at our Table 2. We will focus on the expression (8), which is, as we know, most general and for that reason able to explain the wage ratio movement in the most appropriate way. As we saw, when the wage ratio increases, then the ratio (l_{st}/l_{ut}) should also increase. We can therefore conclude, first, that the efficiency effect, $(l_{st}/l_{ut})^{\sigma}$, should also increase, and in that way may additionally explain an unexplained part of the wage ratio increase. Second, and most important, when we look at the expression for corresponding rate of return ratio, we see that, for the same reason, the wage ratio increase is compensated with the increase of (l_{st}/l_{ut}) ratio. Consequently, the corresponding rate of return ratio increase will be very weak. It might easily happen that the rate of return ratio stays even constant. The empirical evidence that we presented in the introduction shows exactly this situation.

One of the most important consequences of all this is that, owing to the negligible increase of the rate of return ratio, the increase of skilled labor supply may be much smaller than optimal. The process of cyclical adjustment may be, in other words, less dynamic and prolonged. When some great technological innovation, like IT in the seventies or electricity at the turn of the last century, put in force the capital skill complementarity effect, an immediate consequence is a wage ratio increase.[6] An increased supply of skilled labor that follows is supposed after a certain period of time, which may be very long indeed, to reduce wage ratios and put them back again at a new stable position. However, investors in education, as all other investors, make their decisions following rates of return on their investment. More precisely they follow their private rates of return. They do not follow solely wages premium: wages are an only part of the private rate of return formula, which is crucial for their decision. Unfortunately, owing to the wage ratio increase itself the rate of return ratio grows very weakly (or not at all), causing the whole process of cyclical adjustment to be prolonged. The economic policy response to this should, therefore, be focused on looking for the ways to increase investors' rates of return on investment in higher levels of education. One way to do it is to make discrepancy between private and social rate of return in education wider by reducing students' share in financing the cost of education.

2. So far we have seen that the above described constancy (or slow trend) of the rate of return ratio is due to nonexistence of technological progress in the industry of education and to the wage ratio increase itself. To say the full truth, we now must add that all this is possible because "products" of industry of education are nontradeables. If they were tradeables, the lack of technological progress would lead, like in the case of textile industry, to reduction and eventually to disappearance of this industry in developed countries and their reallocation in developing countries. Developed countries would then be able to provide respected products and services on the world market for much smaller prices. This is exactly what happened with textile and many other industries.

Indeed, it is possible to imagine some developed country's university, let say MIT, to go in some less developed country, let say India, and establish there its university units for the education of students from the US. In that case it would be possible to reduce a direct cost of schooling by hiring best Indian lecturers for much smaller salaries. However, direct costs are only around 30% of the total cost of education. Dominant parts of the cost of education are opportunity costs, which are equal to lost earning of enrolled students. Since opportunity costs of enrolled US students are naturally equal to their lost earning in US (not in India), there would not be saving in this cost. Total costs per student would not change significantly, and there would not be possibility for this kind of industry reallocation and for this flow of international trade.

What we have in reality is quite opposite. Best students from developing countries are attending best universities in the US, the UK, and other developed countries. Not all of them return in their country of origins. The best of them usually stay in developed countries. Note that their costs of schooling are much smaller than those of students in developed countries. Although their tuition fees are equal to those of domestic students (sometimes they are even higher), their opportunity costs are much smaller than those of developed country's students: their opportunity costs are equal to lost earning in their country of origins and not to lost earning in the US, the UK, or other developed country.

3. Finally, it is interesting to discuss shortly a possible influence of IT revolution on the cost of education saving. IT revolution has already changed educational practice significantly and it is expected to change it in the future in even more dramatic way. It is sometimes claimed that the usage of Internet, CD and

[6] It is important to note that technological progress has not always been skill biased. According to Goldin and Katz (1996) innovations in nineteen centuries that shifted production from artisanal shops to factories (1830 to 1880) and latter to assembly lines (early 1900) reduced demand for skilled labor and increased demand for unskilled labor. Skill biased technological progress appeared by shift from factories to continuous and batch process (1890 and beyond).

other computer-based delivery of educational content can reduce the above discussed teacher to student ratio and in that way compensate for teachers' salary increase. So far, however, we have not witnessed an important influence of IT revolution on the cost of education. First, substitution of teachers with "computers" can have an influence only on direct costs of education, which, as we know, present only around 30% of all costs of education. It has no influence on opportunity costs whatsoever. Second, there are pedagogical constraints on the above-mentioned substitution. There are empirical evidences that distance education system has much larger dropout ratio than ordinary teacher intensive education. The only area where some cost of education saving can be claimed is at the level of MA and PhD education. This is mainly due to the fact that at that level of education dropout ratio is much smaller owing to the fact that older students are able to work independently.

References

Abramovitz, M. (1956) "Resources and Output Trends in the United States since 1870", *American Economic Review,* p. 5-24.

Aghion, P. (2001) "Shumpeterian Growth Theory and the Dynamics of Income Inequality", *Walras Bowley Lecture,* Econometric Society in Madison, Wisconsin.

Berman, E., Bound, J., Griliches Z. (1994) "Changes in the Demand for Skilled Labour within US Manufacturing: Evidence from the Annual Survey of Manufactures", *Quarterly Journal of Economics*, 109(2), pp. 367-397.

Berman, E., Bound, J., Machin S. (1998) "Implications of Skill Biased Technological Changes: International Evidence", *Quarterly Journal of Economics*, 113(4), pp. 1245-1280.

Bound, J., Johanson, G. (1992) "Changes in the Structure of Wages in the 1980: An Evaluation of Alternative Explanations", *American Economic Review,* 82(3), pp. 371-392.

Denison, E. F. (1962) *Sources of Growth in the United States and Alternatives Before Us,* Supplement Paper 13, New York, Committee for Economic Development.

Denison E. F. (1967) *Why Growth Rates Differ*. Washington, Brooking Institution.

Denison, E. F. (1974) *Accounting for United States Economic Growth: 1929-1969,* Washington, Brooking Institution.

Denison E. F. (1985) *Trends in American Economic Growth 1929-82*. Washington, D. C., Brooking Institution.

Eckstein, Z., Nagypal, E. (2004) "The Evolution of U.S. Earning Inequality", *Federal Reserve Bank of Minneapolis Quarterly Review,* Vol. 28, No. 2, pp. 10-29.

Eisner, R. (1985). "The total incomes system of accounts." *Survey of Current Business,* 65 (1), 24-48.

Eisner, R. (1988). "Extended accounts for national income and product." *Journal of Economic Literature,* 26 (4), 1611-1684.

Fraumeni, M. B. (2000)."The Output of the Education Sector as Determined by Education's Effect on Lifetime Income" *Brookings Program on Output and Productivity Measurement in the Service Sector: Workshop on Measuring the output of the education sector*, April 7, 2000, Brookings Institution, Washington, D. C.

Fraumeni, M. B., Reinsdorf, M. B., Robinson, B. B., Williams, M. P. (2004a) "Real Output Measures for the Education Function of Government: A First Look at Primary & Secondary Education" *Public Services Performance Workshop,* National Institute of Economic and Social Research, London, United Kingdom.

Fraumeni, M. B., Reinsdorf, M. B., Robinson, B. B., Williams, M. P. (2004b) "Price and Real Output Measures for the Education Function of Government: Exploratory Estimates for Primary & Secondary Education" *Conference on Research in Income and Wealth: Conference on Price Index Concepts and Measurement,* Vancouver, BC, Canada.

Galor, O., Weil, N. W. (1993). "The Gender Gap, Fertility and Growth", *NBER Working Papers Series,* Working Paper No 4550, p. 30.

Goldin, C., Katz, L. F. (1996) "The Origins of Technology-Skill complementarity", *NBER Working Papers,* No. 5657, Cambridge, MA, p. 29.

Griliches, Z. (1960). "Measuring Inputs in Agriculture" *Journal of Farm Economics XLII:* 1411-1433.

Griliches, Z. (1963a). "Production Function, Technical Changes, and All That" Netherland School of Economics, Econometric Institute, *Report 6328,* Rotterdam.

Griliches, Z. (1969), "Capital-Skill Complementarity", *Review of Economics and Statistics,* 51(4): 465-468.

Griliches, Z. (1996). "Education, Human Capital, and Growth: A Personal Perspective" *NBER Working Paper Series*, National Birou of Economic Research, Working Paper 5426.

Hall, R. E. and Jones, C. I. (1999) "Why do some countries produce so much more output per workers than other." *The Quarterly Journal of Economics,* 114(1): 83-116.

Heckman, J. J., Lochner, L. J., Todd, P. E. "Earning Functions, Rate of Return, and Treatment Effects: The Mincer Equation and Beyond", *NBER Working Paper Series,* Working Paper 11544, Cambridge, MA

Jones, C. I. (1996) "Human Capital, Ideas, and Economic Growth." *VII Villa Mondragone International Economic Seminar on Finance, Research, Education, and Growth,* Rome, June 25-27, 1996.

Jones, C. I. (2004) "Growth and Ideas." *NBER Working Paper Series,* Working Paper No 10767*, NBER.*

Jorgenson, D. W. and Fraumeni, B. M. (1989). "The accumulation of human and nonhuman capital, 1948-1984." In R. E. Lipsey and H. S. Tice (Eds.), *The Measurement of Savings, Investment and Wealth* (pp. 227-282). Chicago, I.L.: The University of Chicago Press.

Jorgenson, D. W. and Fraumeni, B. M. (1992a). "The output of the education sector." In Z. Griliches (Ed.), *Output Measurement in the Services Sector* (pp. 303-338). Chicago, I.L.: The University of Chicago Press.

Jorgenson, D. W. and Fraumeni, B. M. (1992b). "Investment in education and U.S. Economic Growth" *Scandinavian Journal of Economics* 94: S51-70.

Jorgenson D. W., and Griliches Z. (1967) "The Explanation of Productivity Change." *Review of Economic Studies* 34(3): 249-80.

Kendrick J. W. (1956) "Productivity Trends: Capital and Labour." *Review of Economics and Statistics* 38: 248-57.

Kendrick, J. W. (1976). *The Formation and Stocks of Total Capital.* New York, N.Y.: Columbia University Press for NBER.

Kendrick, J. W. (1979). "Productivity Trends and Recent Slowdown: Historical Perspectives, Casual factors and Policy Options" in Felner, W. (ed) *Contemporary Economic Problems,* p 22-69, Washington: American Enterprise Institute.

Kendrick, J. W. (1980). "Survey of the Factors Contributing to the Decline in US Productivity Growth" in *The Decline in Productivity Growth,* Conference Series, No 22, Boston: Federal Reserve Bank of Boston.

Krusell, P., Ohanian, L. E., Rios-Rull, J. V., Violente, G. L. (1997) "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis", *Federal Reserve Bank of Minneapolis, Research Department Staff Report*, No 236, p. 67.

Lawrence, R. Z., Slaughter, M. J. (1993) "International Trade and American Wages in 1980s: Giant Sucking Sound or Small Hiccup", *Brooking Papers: Microeconomics* 2, pp.161-226.

Madison A. (1972). "Explaining Economic Growth" *Banca Nazionale del Lavoro Quarterly review,* p 211-263.

Madison A. (1987) "Growth and Slowdown in advanced Capitalist Economies: Techniques of Quantitative Assesment." *Journal of Econmic Litherature,* 25, pp 649-98.

Mankiw, N. G., Romer, D., and Weil D. N. (1992) "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics* 107(2): 407-37.

Mankiw. N. G. (1995) "The Growth of Nations." in *Brookings Papers on Economic Activity,* 1: 275-327.

Nelson, R. R. and Phelps, E. S. (1966). "Investment in Humans Technological Diffusion, and Economic Growth." The American Economic Review 56 (1/2, Mar): 69-75.

Popovic, M, (2006) "Capital Augmenting and Labor Augmenting Approach in Measuring Contribution of Human Capital and Education to Economic Growth", *Montenegrin Journal of Economics,* Vol. 2, No.4, pp. 71-109. (www.mnje.com)

Psacharopoulos G. (1972) "Measuring. the Marginal Contribution of Education to Economic Growth." *Economic Development and Cultural Change,* No 4.

Psacharopoulos G. (1985) "Return to Education: A Further international Update and Implications." *Journal of Human Resources* 200(4): 583-604.

Psacharopoulos G., Patrinos, H. A. (2002) "Return to Education: A Further Update" *Policy Research Working Paper,* The World Bank, p.28.

Shultz, T. W. (1960) "Capital Formation by Education" *Journal of Political Economy*, 68: 571-583.

Shultz, T. W. (1961a) "Investment in human capital." *American Economic Review*, 51 (1), 1-17.

Shultz, T. W. (1961b) "Investment in human capital." *American Economic Review*, 51 (5), 1035-1039.

Shultz, T. W. (1962) "Reflections on Investment in Man" *Journal of Political Economy* 70: S1-S8.

Stevens, P., Weale, M. (2003) "Education and Economic Growth" Paper prepared for G. and J. Johnes (ed) *International Handbook on the Economics of Education,* Edward Elgar.

Tyers, R., Yang, Y. (1997) "Trade with Asia and Skill Upgrading: Effects on Factor Markets in the Older Industrial Countries", *Weltwirtschaftliches Archiv 133(3), pp383-418.*

Tyers, R., Yang, Y. (2000) "Capital-Skill Complementarity and Wage outcomes Following Technical Change in a Global Market", *Oxford Review of Economic Policy* 16(3), pp. 23-41.

Winchester, N., Greenaway, D. (2005) "Rising Wage Inequality and Capital-Skill Complementarity", *Economics Discussion Papers,* No. 0528, University of Otago.