The Role of the Private Sector under Insecure Property Rights

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Abstract

It is well known that the so-called voracity effect can be observed in an economy with common and private capital. Voracious behavior is regarded as one of the excess uses of the commons. In this paper, we explore a new interpretation of what causes voracious and investigate the effects of voracious behavior on the economy. For this purpose, we introduce a new direction of capital flow. A government mandates that all groups invest their private capital in the common sector to mitigate the effects of excess use of the commons. We show that, while there is no standard voracity effect, an increase in the contribution of the private sector into the common sector causes more voracious behavior and thus slows economic growth. This implies that policies designed to preserve the commons can lead to a harmful effect on the economy.

Keywords: differential game, Markov perfect equilibrium, voracity effect.

JEL Classification: C73, O10, O40

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1 Introduction

Interest in studying the relationship between the growth rate of an economy and economic institutions has been increasing. Insecure property rights is one of the most interesting fields of study noted among economists. Developing countries generally have a weak property rights system, and it is thought that such a system can function like a set of shackles that cripples economic progress. Some developing countries share a common capital that everyone can access, which is not secured property. The common-pool problem is widely used to analyze such economies.

The excess use of common resources is clearly an interesting phenomenon. In an economy with common capital, each agent freely extracts resources without taking the protection of those resources into account. As a result, the growth rate of the economy is lower than that in an economy with secure property rights. This is called the tragedy of the commons. Excess use of the commons is also a cause of the voracity effect. First studied by Tornell and Velasco (1992), Lane and Tornell (1996), and Tornell and Lane (1999), the voracity effect was defined by them as a positive technology shock in the common sector that leads to an increase in appropriation and thus slows economic growth. However, hereafter we suggest that this is not the only cause of the voracity effect and explore another possible cause of voracious behavior.

Tornell and Velasco (1992) and Tornell and Lane (1999) have also been concerned with the role of private capital. In the model they present, they introduced private capital into the economy with multiple interest groups and the common sector. Each group appropriates a resource from the common sector and can use it not only for its consumption but also for investment to accumulate private capital stock. Private capital is secured property not accessible by the other groups, but it is less productive. In other words, groups in the economy have respective private sectors and accumulate their own capital. Tornell and Velasco (1992) and Tornell and Lane (1999) have shown that under some circumstances, the introduction of secure but less productive capital stock increases the growth rate of the common sector. They have also demonstrated that the voracity effect occurs.

In their model, there is only one direction of capital flow from the common sector to the private sector. As a result, they do not consider the other direction of capital flow, namely, from the private sector to the common sector. In practice, consideration of the interaction between both sectors is important. It is plausible that a portion of private assets is used in the common sector. Schneider (1998) showed empirically that a fraction of the earnings in the informal sector are immediately spent in the private sector.[1] The private sector has been interpreted as an informal or shadow sector in a country.[2] Schneider (1998) shows that the private sector has a positive effect on economic growth. Loayza (1996) uses an endogenous growth model to show that an increase in the size of the private sector negatively affects growth. They also find this result to be observable empirically by using data from Latin America. The role of the private sector in economic growth is, therefore, ambiguous. In our study, we, therefore, introduce capital flow from the private sector to the common sector and study how the voracious behaviours of various interest groups change.

The aims of the present paper are to explore another possible cause of voracious behavior and to investigate the effects of voracious behavior on the economy. We extend the Tornell and Velasco (1992) model by introducing a flow of capital from the private sector to the common sector; a fraction of each interest group's private capital stock is invested in the common sector. In this situation, the obtained results are as follows. First, we show that the balanced growth rates are independent of the technology level in the common sector. This implies that there is no standard voracity effect in the sense that Tornell and Lane (1999) define. We also show that, when each group values the opponents' private capital, their capital has a positive effect on a group's equilibrium consumption strategy. Finally, we show that an increase in the contribution rate leads to an increase in appropriation, and hence the balanced growth becomes slow. The paper predicts that the contribution from the private sector to the common sector has a negative effect on economic growth.

Other lines of literature on the dynamic common-pool problem are as follows. Mino (2006) and Itaya and Mino (2007) introduced labor into the economy without the private sector and considered variable labor-leisure choices by changing the linear production function to an increasing-returns production function. They found that the effects of a rise in productivity and the number of interest groups would be significantly different from the results obtained in the basic framework. Strulik (2011) reconsidered the voracity effect under conditions when basic needs mattered in consumption. It was shown that interest groups are, ceteris paribus, more likely to generate the voracity effect due to more appropriation when an economy is in decline and sufficiently close to stagnation. Tornell (1997) and Lindner and Strulik (2008) used

[1] It is necessary to be careful about the term, the private sector. Since Schneider deals with data on not only developing countries but also developed countries, he regards the national sector as the private sector. In our paper, however, we focus on developing countries without secure property rights, and we represent the national sector as the common sector and the informal sector as the private sector.

[2] In what follows, we represent the sector in which assets are secured as the private sector and the sector in which assets are not secured as the common sector.

trigger strategy equilibria in economic growth models with common access to capital to analyze the features of endogenous property rights. Long and Sorger (2006) extended the Tornell and Velasco (1992) model by adding the following three features. First, extracting the common property asset involves a private appropriation cost. Second, each group derives utility from wealth as well as from consumption. Finally, each group can be heterogeneous. They showed that an increase in appropriation cost and an increase in the degree of heterogeneity of these costs under different appropriation cost across interest groups lower the growth rate of the common capital stock.

There are four remaining sections of the present paper. In Section 2, the model, a solution concept, and each group's maximization problem are described. Section 3 goes on to characterize the balanced growth equilibrium. In section 4, the balanced growth comparative statics will be numerically analyzed. Lastly, section 5 discusses some conclusions.

2 The Model

Our framework builds on the models of Tornell and Velasco (1992) and Tornell and Lane (1999). We consider a continuous time model. There is a developing economy organized by multiple interest groups. The number of multiple interest groups is $n \geq 2$. We suppose that each group is homogeneous in the sense that each group has the same preference, and the subjective rate of discount and the technology level of the private sector are common among all groups. Within each group, there is a set of people who cooperate with other people belonging to the same group. They do not cooperate with those who do not belong to the same group, and they cannot move and belong to other groups. The reason may be that each group has different beliefs or belongs to different ethnic, religious, or occupational categories, so it has no incentive to cooperate with other groups. We can, therefore, interpret a group as the representative agent.

Since each group has the same preference, it has the same utility function. The utility function is assumed to be CRRA. The discounted sum of the utility is, therefore, represented as follows.

$$\int_0^\infty \frac{c_i(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \quad \theta > 0, \;\; \theta \neq 1, \;\; i = 1, 2, \cdots, n \tag{1}$$

where $c_i(t)$ is group i's consumption at instantaneous time t, θ is the inverse of the intertemporal elasticity of substitution in consumption, and ρ is the subjective rate of time preference.

2.1 Secure and Insecure Property Rights

In the economy, there are two capital stocks: the common capital and the private capital. The common capital stock is generally regarded as insecure property right assets; e.g., big, clean fisheries; underground oil; or forests. In the existing literature, private capital has been interpreted in a number of ways, including small, private, and stagnant lakes and bank accounts in foreign developed countries that cannot be deprived by other groups. The common capital stock is assumed to allow each group to have a larger marginal profit than the private-access capital does. In the case of fisheries, common fisheries are large and highly nutritious. The marginal productivity of fish in common fisheries is, therefore, larger than that in small, private, and stagnant fisheries. In the case of bank accounts, the interest rate in foreign developed countries is lower than that of the developing country.

Each group decides how much common capital is appropriated, consumed and invested in order to accumulate its own private capital. Taking the opponents' behavior into account, each group can appropriate any share it desires from the common capital stock. The resource appropriated by a group is used for the consumption of the group or investment in private capital.

We consider, however, the interaction between the common sector and the private sector. For this purpose, we assume that for each group, a portion of its private capital stock must be used for production in the common sector. Since a government or the society in the economy knows that excess use of the resource occurs, it requires all groups to invest in the common sector in order to avoid that phenomenon. An alternative interpretation can be considered. In the fishery case, some fish are moved to the bountiful fisheries – the common sector– because the private fisheries are stagnant. This implies that there exists a positive spillover into the common sector.

In the common sector, an output is produced from the aggregate capital, which is composed of the common capital stock and the sum of a part of group i's private capital stock. We assume that the production function is additively separable for analytical simplicity. The common-access capital stock, therefore, evolves according to the following differential equation:

$$\dot{K}(t) = A\left[K(t) + \sum_{i=1}^{n} u_i h_i(t)\right] - \sum_{i=1}^{n} d_i(t), \tag{2}$$

where $K(t) \in \mathbb{R}_+$ is the common capital stock, A is the productivity of the common sector,

$u_i \in (0,1)$ is the rate of the private sector contribution to the common sector, and $d_i(t) \in \mathbb{R}_+$ is the amount appropriated by interest group i. The aggregate capital stock is represented as $K(t) + \sum_{i=1}^{n} u_i h_i(t)$.

As for the private sector, the resource extracted by each interest group can be either consumed or invested in its private and secure capital, but a fraction of the private capital is used for investment in the common sector. The private capital stock of group i, therefore, evolves according to the following differential equation:

$$\dot{h}_i(t) = B(1-u_i)h_i(t) + d_i(t) - c_i(t), \ i = 1, 2, \cdots, n, \tag{3}$$

where $h_i(t) \in \mathbb{R}_+$ is group i's private capital stock, B is the technology level of the private sector, and $c_i(t) \in \mathbb{R}_+$ is group i's consumption. It is plausible that the technology level of the private sector is common because of the assumption of symmetric groups.

Note that we assume that the government sets the rate, u_i, before each group i solves its problem. This means that u_i is assumed to be an exogenous and constant parameter. In addition, since we focus on homogeneous interest groups, the contribution rate is assumed to be common to all interest groups.

Assumption 1. *The marginal product of the common sector is larger than that of the private sector;* $A > B$. *The contribution rate is common to all groups;* $u_i = u$ *for all* i.

2.2 The Solution Concept: Markov Perfect Equilibrium

We focus on a symmetric Markov perfect equilibrium (henceforth, MPE) of the noncooperative game. In the present model, each group i has two Markovian strategies: the consumption strategy ψ_i and the appropriation strategy ϕ_i. These strategies are functions $\psi_i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ and $\phi_i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$, respectively. This means that group i chooses its consumption and appropriation according to the feedback rules $c_i(t) = \psi_i(K(t), h(t))$ and $d_i(t) = \phi_i(K(t), h(t))$. Let us define h as an n-dimensional vector; that is $h = (h_1, h_2, \cdots, h_n)$. Strategies ψ_i and ϕ_i are called symmetric if for all i and $j(\neq i)$ the relations $\psi_i = \psi_j$ and $\phi_i = \phi_j$ hold. Therefore the definition of MPE is as follows.

Definition 1. *The Markov strategies constitute MPE if and only if each group* i*'s problem maximizing* (1) *subject to* (2)–(3)*, any given initial stock* K_0 *and* h_0*, and* $c_j^*(t) = \psi_j^*(K(t), h(t))$

and the opponents' strategies $d_j^*(t) = \phi_j^*(K(t), h(t))$ *for all* $j(\neq i)$ *have an optimal solution which satisfies equilibrium strategies* $c_i^*(t) = \psi_i^*(K(t), h(t))$ *and* $d_i^*(t) = \phi_i^*(K(t), h(t))$.

From the above discussion, one can understand the information structure defined in the present paper. The government and each interest group can observe not only the common-access capital stock but also all the private capital stocks due to the introduction of the contribution ratio u. Therefore, both strategies in our model depend on the common-access capital stock and all private-access capital stocks.

2.3 The Hamilton-Jacobi-Bellman Equation: Group i's Problem

Each group chooses the optimal levels of consumption and appropriation at each instant time t to maximize (1) subject to (2), (3), the opponents' strategies, and the initial levels of capital. Our model is, thus, a differential game among n interest groups where the control variables are c and d, and the state variables are the common capital stock K and the private capital stock h. Since we consider only a symmetric group case, we focus on one group, group i, in the discussion below.

An MPE is generally derived through the dynamic programming technique and must satisfy the Hamilton-Jacobi-Bellman (HJB) equation. The HJB equation of group i is as follows: for all $t \geq 0$ and $i = 1, 2, \cdots, n$,

$$\rho V_i(K, h) = \max_{c_i, d_i} \left\{ \frac{c_i^{1-\theta}}{1-\theta} + \frac{\partial V_i}{\partial K} \cdot \left(A \left[K + u \sum_{i=1}^{n} h_i \right] - d_i - \sum_{j \neq i} \phi_j \right) \right.$$
$$\left. + \frac{\partial V_i}{\partial h_i} \cdot (B(1-u)h_i + d_i - c_i) + \sum_{j \neq i} \frac{\partial V_i}{\partial h_j} \cdot (B(1-u)h_j + \phi_j - \psi_j) \right\}. \quad (4)$$

Furthermore, the value function V_i must satisfy the following boundary condition:

$$\lim_{t \to \infty} V_i(K, h) \exp(-\rho t) = 0. \quad (5)$$

Differentiating the HJB equation with respect to c_i and d_i yields optimal conditions, for all i,

$$c_i^{-\theta} = \frac{\partial V_i}{\partial h_i}, \quad (6)$$

$$\frac{\partial V_i}{\partial h_i} = \frac{\partial V_i}{\partial K}. \tag{7}$$

Equations (6) and (7) constitute a set of MPE solutions. Note that due to the assumption of the utility function, the maximization problem satisfies the second-order conditions as well.

The Markov strategies simultaneously satisfy (6) and (7). Substituting these conditions into the HJB equation and using the envelope theorem, we obtain the following equations.

$$\begin{aligned}
\rho\frac{\partial V_i}{\partial K} =& \frac{\partial^2 V_i}{\partial K^2}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\sum_{j\neq i}\phi_j^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(A-\sum_{j\neq i}\frac{\partial \phi_j^*}{\partial K}\right)\\
&+\frac{\partial^2 V_i}{\partial K\partial h_i}\cdot\left(B(1-u)h_i+\phi_i^*-\psi_i^*\right)+\sum_{j\neq i}\frac{\partial V_i}{\partial h_j}\cdot\left(\frac{\partial \phi_j^*}{\partial K}-\frac{\partial \psi_j^*}{\partial K}\right)\\
&+\sum_{j\neq i}\frac{\partial^2 V_i}{\partial K\partial h_j}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right),
\end{aligned}\tag{8}$$

$$\begin{aligned}
\rho\frac{\partial V_i}{\partial h_i} =& \frac{\partial^2 V_i}{\partial h_i\partial K}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\sum_{j\neq i}\phi_j^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(Au-\sum_{j\neq i}\frac{\partial \phi_j^*}{\partial h_i}\right)\\
&+\frac{\partial^2 V_i}{\partial h_i^2}\cdot\left(B(1-u)h_i+\phi_i^*-\psi_i^*\right)+\frac{\partial V_i}{\partial h_i}\cdot B(1-u)\\
&+\sum_{j\neq i}\frac{\partial^2 V_i}{\partial h_i\partial h_j}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right)+\sum_{j\neq i}\frac{\partial V_i}{\partial h_j}\cdot\left(\frac{\partial \phi_j^*}{\partial h_i}-\frac{\partial \psi_j^*}{\partial h_i}\right),
\end{aligned}\tag{9}$$

and

$$\begin{aligned}
\rho\frac{\partial V_i}{\partial h_j} =& \frac{\partial^2 V_i}{\partial h_j\partial K}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\phi_j^*-\sum_{k\neq i,j}\phi_k^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(Au-\frac{\partial \phi_j^*}{\partial h_j}-\sum_{k\neq i,j}\frac{\partial \phi_k^*}{\partial h_j}\right)\\
&+\frac{\partial^2 V_i}{\partial h_j\partial h_i}\cdot\left(B(1-u)h_i+\phi_i^*-\psi_i^*\right)+\frac{\partial^2 V_i}{\partial h_j^2}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right)\\
&+\frac{\partial V_i}{\partial h_j}\cdot\left(B(1-u)+\frac{\partial \phi_j^*}{\partial h_j}-\frac{\partial \psi_j^*}{\partial h_j}\right)+\sum_{k\neq i,j}\frac{\partial V_i}{\partial h_k}\cdot\left(\frac{\partial \phi_k^*}{\partial h_j}-\frac{\partial \psi_k^*}{\partial h_j}\right)\\
&+\sum_{k\neq i,j}\frac{\partial^2 V_i}{\partial h_j\partial h_k}\cdot\left(B(1-u)h_k+\phi_k^*-\psi_k^*\right).
\end{aligned}\tag{10}$$

The functions with an asterisk represent the optimal strategies in the model. In the following analysis, we focus on the symmetric MPE and show that the growth rates of c_i, d_i, and h_i, for all i and K grow at a positive constant. Before proceeding to the balanced growth analysis, we refer to a restriction of strategy space for consumption and appropriation.

3 A Special Case of the Model

As a benchmark case, we first consider the case where the value function is independent of the opponents' private capital stocks. This is the same situation analyzed in Tornell and Velasco (1992) and Tornell and Lane (1999). In the next section, we analyze a more general case where each group values the opponents' private capital. We conjecture the following value function

$$V_i(K, h) = \frac{\xi(K + \alpha h_i)^{1-\theta}}{1-\theta}, \tag{11}$$

where ξ and α are constant, and β is a $n-1$ dimensional constant vector. As for the consumption strategy $\psi(K, h)$ and the appropriation strategy $\phi(K, h)$, we assume that they are linear strategies; that is $\psi_i(K, h) = a' + aK + eh_i + bZ_i$ and $\phi_i(K, h) = \gamma K + \delta h_i + \delta Z_i$, where a', a, e, b, γ, and δ are unknown constant. The consumption strategy is a standard linear strategy. For notational simplicity, we define the aggregate private capital of the opponents' group, $\sum_{j \neq i} h_j$, as Z_i. Since we focus on the symmetric MPE, it is assumed to be the equal coefficients b and δ among all the opponents' private capital h_j for all $j(\neq i)$.

Substituting these into (8) − (10), we obtain the following lemma.

Lemma 1. *The optimal parameters are obtained as follows.*

$$a' = b = 0, \;\; a = e = \xi^{-\frac{1}{\theta}} = \left(\frac{\theta - 1}{\theta}\right) B(1-u) + \frac{\rho}{\theta},$$
$$\alpha = 1, \;\; \gamma = \frac{A - B(1-u)}{n-1}, \;\; \textit{and} \;\; \delta = \frac{Au}{n-1}.$$

Proof. See Appendix A. □

We can easily confirm that γ and δ are always positive. Although the sign of a is ambiguous, a must be positive because this is the coefficient of consumption strategy. Therefore, we impose the assumption guaranteeing that this is positive below.[3]

Tornell and Velasco (1992) and Tornell and Lane (1999) considered the situation in which there is no contribution of private capital stocks to the common sector; that is, $u = 0$. In

[3]See Assumption 2.

this situation, the appropriation strategy is independent of the private capital stocks and thus $\delta = 0$. In contrast, we consider the case where u is positive and a fraction of private capital stocks contribute to the production of common capital. Thus, the appropriation strategy is affected by the stock of the private capital.

3.1 Balanced Growth Rates

In this subsection, we derive the balanced growth rates of common capital, private capital, consumption, and appropriation. Before proceeding to deriving them, we impose the following assumption.

Assumption 2. *We assume that the following relations hold:*

$$\frac{A}{n-1} - B(1-u)\left(\frac{n}{n-1} - \frac{1}{\theta}\right) < \frac{\rho}{\theta} < \frac{B(1-u)}{\theta}, \quad \text{and } nB(1-u) > A. \tag{12}$$

Note that under this assumption we can confirm that the coefficients of the consumption strategy is positive because the following relations are satisfied.

$$\left(\frac{1-\theta}{\theta}\right) B(1-u) < \frac{A}{n-1} - B(1-u)\left(\frac{n}{n-1} - \frac{1}{\theta}\right).$$

Let us derive the complete dynamics of the model. In the case of symmetric equilibrium, the amount of group i's private capital stock is equal to that of all the other groups j ($\neq i$) so that $h_i = h_j$ and thus the dynamic system of h_j is identical to that of h_i. This implies that the $n-1$ state equations of private capital are redundant. The complete dynamics are, therefore, represented in two state equations, $\dot{K}$ and h_i. Dividing equations (2) and (3) by social capital K and private capital h_i, respectively, yields:

$$\frac{\dot{K}}{K} = \frac{nB(1-u) - A}{n-1} - \frac{nAu}{n-1}\frac{h_i}{K}, \tag{13}$$

$$\frac{\dot{h}_i}{h_i} = \left[\frac{A - B(1-u)}{n-1} - \left(\frac{\theta-1}{\theta}\right) B(1-u) - \frac{\rho}{\theta}\right]\frac{K}{h_i} + \frac{nAu}{n-1} + \frac{1}{\theta}(B(1-u) - \rho). \tag{14}$$

For the growth rate of the common capital to be positive, the first term must be positive; that is $nB(1-u) > A$. This is guaranteed by Assumption 2. Furthermore, on the balanced growth path, growth rates of K and h_i must be constant and thus the ratio of K to h_i must be constant. This requires that both growth rates are same at the steady state level.

Lemma 2. *The balanced growth rates of the common capital and the private capital are*

$$\frac{\dot{K}}{K} = \frac{\dot{h}_i}{h_i} = \frac{1}{\theta}(B(1-u) - \rho). \tag{15}$$

Proof. For notational simplicity, we set $\chi \equiv \frac{h_i}{K}$. On the balanced growth path, the growth rate of social capital is same as that of private capital so that (13) and (14) are a coincident

$$\frac{nB(1-u) - A}{n-1} - \frac{nAu}{n-1}\chi = \left[\frac{A - B(1-u)}{n-1} - \left(\frac{\theta - 1}{\theta}\right) B(1-u) - \frac{\rho}{\theta}\right]\frac{1}{\chi} + \frac{nAu}{n-1} + \frac{1}{\theta}(B(1-u) - \rho).$$

Arranging this with respect to χ, we obtain

$$(\chi + 1)\left(\frac{NBu}{N-1}\chi + D\right) = 0,$$

where we define $D \equiv \frac{1}{\theta}(B(1-u) - \rho) - \frac{nB(1-u)-A}{n-1}$, which is negative under Assumption 2. Since χ must be positive, the solution χ is

$$\chi = -\frac{n-1}{nAu}D = \frac{n-1}{nAu}\left[\frac{nB(1-u) - A}{n-1} - \frac{1}{\theta}(B(1-u) - \rho)\right] > 0. \tag{16}$$

Substituting this into equations (13) and (14), we confirm that the steady state growth rates are $\frac{\dot{K}}{K} = \frac{\dot{h}_i}{h_i} = \frac{1}{\theta}(B(1-u) - \rho)$. □

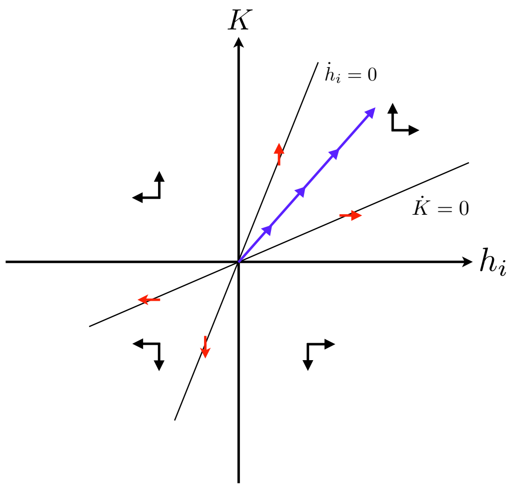


Figure 1: The phase diagram

In the present model, there is no transition dynamics like that found in the AK model. The phase diagram is, therefore, illustrated in Figure 1.[4]

Using Lemma 2, we obtain the following proposition.

Proposition 1. *Under Assumptions 1 and 2, the strategy profile* $\{(\phi_i, \psi_i)\}_{i=1}^n$ *defined by* $\psi_i(K, h) = a' + aK + eh_i + bZ_i$ *and* $\phi_i(K, h) = \gamma K + \delta h_i + \delta Z_i$ *forms a symmetric MPE. The balanced growth rates are*

$$\frac{\dot{K}}{K} = \frac{\dot{h}_i}{h_i} = \frac{\dot{d}_i}{d_i} = \frac{\dot{c}_i}{c_i} = \frac{1}{\theta}(B(1-u) - \rho). \tag{17}$$

Proof. For the first part, we know that, in the equilibrium, parameters are given by Lemma 1. These constitute an MPE.

Before we prove the boundary condition to be satisfied, we derive the growth rates of consumption and appropriation. Differentiating the consumption with respect to time and dividing this by consumption itself, we get

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{K} + \dot{h}}{K + h_i} = \frac{\left(B(1-u) - \frac{A - B(1-u)}{N-1} + D\right)K + \frac{1}{\theta}(B(1-u) - \rho)h_i}{K + h_i} = \frac{1}{\theta}(B(1-u) - \rho).$$

Similarly, we derive the growth rate of appropriation. From Lemma 2,

$$\frac{\dot{d}_i}{d_i} = \frac{\frac{A-B(1-u)}{n-1}\dot{K} + \frac{nAu}{N-1}\dot{h}_i}{\frac{A-B(1-u)}{n-1}K + \frac{nAu}{n-1}h_i} = \frac{\frac{A-B(1-u)}{n-1} + \frac{nAu}{N-1}\frac{h_i}{K}}{\frac{A-B(1-u)}{n-1} + \frac{nAu}{n-1}\frac{h_i}{K}}\frac{\dot{K}}{K} = \frac{1}{\theta}(B(1-u) - \rho).$$

Finally, we check the boundary condition. Substituting the consumption strategy, which is $c_i = a(K + h_i)$ into the value function and using the growth rate of consumption yield

$$V_i(K, h) = \frac{\xi(K_0 + h_{i0})^{1-\theta}}{1-\theta} \exp\left[\left(\frac{1-\theta}{\theta}\right)(B(1-u) - \rho)t\right].$$

The boundary condition is, therefore,

$$\lim_{t\to\infty} V_i(K, h)\exp(-\rho t) = \frac{\xi(K_0 + h_{i0})^{1-\theta}}{1-\theta} \lim_{t\to\infty} \exp\left\{\left[\left(\frac{1-\theta}{\theta}\right)B(1-u) - \frac{\rho}{\theta}\right]t\right\}.$$

Since this converges to zero under Assumption 2, the boundary condition is satisfied. □

[4] Under Assumption 2, we can confirm that the slope of $\dot{h}_i = 0$ line is steeper than that of $\dot{K} = 0$ line.

3.2 The voracity effect

In this subsection, we consider the voracity effect. The voracity effect is one of the most interesting results in the literature. It can be defined as the phenomenon in which countries with multiple interest groups respond to a positive technology shock in the common sector by increasing the appropriation rate, and thus the growth rates become slow. In the existing literature (e.g., Tornell and Velasco (1992), Tornell and Lane (1999), and Long and Sorger (2006)), under some circumstances, the voracity effect is observed.

Proposition 1 states that the balanced growth rate is independent of the productivity of the common sector, A. This implies that, in the situation where there is a fraction of the private capital stock is used for investment in the common sector, there is no effect of a positive technology shock in the common sector on the growth rate as Tornell and Lane (1999) define. However, we can confirm that the contribution rate plays the same role as a technology shock in the common sector.

Proposition 2. *An increase in the contribution ratio,u, leads to an increase in the appropriation rate and a decrease in the balanced growth rate.*

Proof. The appropriation strategy is $d_i = \frac{A-B(1-u)}{n-1}K + \frac{Au}{n-1}h_i + \frac{Au}{n-1}Z_i$. The derivatives of this with respect to u is $\frac{\partial d_i}{\partial u} = \frac{BK+Ah_i+AZ_i}{n-1} > 0$. Similarly, differentiating (22) yields

$$\frac{\partial\left(\frac{\dot{K}}{K}\right)}{\partial u} = -\frac{B}{\theta} < 0.$$

□

The rate is determined by the government in this economy and is an exogenous variable for each group. When u increases, a group is forced to invest its private capital in the common sector. At the same time, however, the remaining $n-1$ groups also are forced to invest their private capital, this is regarded as a positive externality for the group. The externality dominates the impact of an increase in u on the group and, hence, causes it to further extract the resource. This leads to the reduction in the balanced growth rates. This is another channel of the voracity effect.

3.3 The Comparison of Common Capital Growth Rates

We compare the balanced growth rate of the common capital obtained above with that in Tornel and Velasco (1992) and Tornell and Lane (1999). In their model, the growth rate of the common capital is $\bar{g} = \frac{nB-A}{n-1}$. We subtract (22) from g and then obtain

$$\frac{nB - A}{n-1} - \frac{1}{\theta}(B(1-u) - \rho) > \frac{nB(1-u) - A}{n-1} - \frac{1}{\theta}(B(1-u) - \rho) > 0.$$

The second inequality holds due to Assumption 2. This result is summarized below.

Proposition 3. *The growth rate of the common capital under* $u = 0$ *is higher than that under* $u \neq 0$.

In the case where the value function is independent of the opponents' private capital stocks, each group does not value the opponents' private capital stocks. The contribution of the private capital to the common sector increases not only the common capital but also the amount of appropriation because of δZ_i. The latter effect exceeds the former effect and thus the growth rate of the common sector declines.

4 A General Case

In this section, we relax the assumption in the previous section and consider the case where the value function depends on the opponents' private capital stocks. Thus we conjecture the value function as follows.

$$V_i(K, h) = \frac{\xi(K + \alpha h_i + \beta Z_i)^{1-\theta}}{1-\theta}, \tag{18}$$

where ξ, α, and β are unknown constants. Note that although ξ and α are usually positive, β can be either positive, negative, or zero, depending on the model. The consumption strategy is the same as in the previous section, i.e. $\psi_i(K, h) = a' + aK + eh_i + bZ_i$. As for the appropriation strategy, we assume that it depends on the aggregate capital in the common sector, following the existing literature, i.e. $\phi_i(K, h) = \gamma\,[K + uh_i + uZ_i]$.[5] Parameters a', a, b, γ, and e are unknown constants.

Substituting these strategies and (18) into equations (8) – (10), we obtain the candidates for the optimal parameters.

[5]There is another unknown parameter, β, here. If we use the previous appropriation strategy, we cannot identify all the parameters. Thus, we have to eliminate one unknown parameter.

Lemma 3. *The candidates for the optimal parameters are obtained as follows.*

$$
\begin{aligned}
a &= e = \xi^{-\frac{1}{\theta}} = \frac{uB(1-u)}{\beta[(n-1)\beta+1-un]},\\
\beta &= \frac{y \pm \sqrt{y^2+4(n-1)[\rho+(1-u)(\theta-1)B]\theta uB(1-u)}}{2(n-1)[\rho+(1-u)(\theta-1)B]},\\
\gamma &= \frac{A[(n-1)\beta+1-un]-B(1-u)[(n-1)\beta+1]}{(1-\beta)(n-1)[(n-1)\beta+1-un]},\\
a' &= 0, \quad \alpha = 1, \quad \text{and } b = a\beta,
\end{aligned}
$$

where $y \equiv (un-1)\rho + (1-u)[n(1+u)+1](\theta-1)B$*.*

Proof. See Appendix B. □

The lemma states that there are two candidates for β; one is positive and the other is negative. The parameters a and γ must be positive in the present model. For a to be positive, the term $(n-1)\beta+1-un$ must be positive (negative) if β is positive (negative). The sign of γ is not immediately confirmed. In the next subsection, we show that one of two candidates is ruled out by considering the dynamic system of the model. After this, we will confirm the positivity of a and γ.

Before proceeding to the next subsection, we impose the following assumption.

Assumption 3. *We assume that* $\rho > B(1-u)(1-\theta)$*.*

In addition, we derive the growth rate of consumption and the balanced growth ratio of private capital stock to common capital stock, $\chi \equiv \frac{h_i}{K}$.

Lemma 4. *The growth rate of consumption is*

$$g = \frac{\dot{c}_i}{c_i} = \frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}. \tag{19}$$

Proof. See Appendix C. □

The lemma states that the growth rate of group i's consumption is constant over time. For the growth rate of consumption to be positive, it is necessary that $\beta > u$ if β is positive and $\beta(n-1)+$ is negative if β is negative. Conversely, if $\beta > u$, $\beta(n-1)+1-un > u(n-1)+1-un = 1-u > 0$, and if $\beta(n-1)+1 < 0$, the equation $\beta(n-1)+1-un$ is negative. Therefore, these conditions guarantee that the parameter a is positive.

4.1 Dynamic System and Stability

We derive the complete dynamics of the model as follows. As explained in the previous section, in the case of symmetric equilibrium, we can represent the dynamic system in terms of the following two equations composed by the common capital, K, and group i's private capital, h_i:

$$\dot{K} = (A - n\gamma)K + n(A - n\gamma)uh_i, \tag{20}$$

$$\dot{h}_i = (\gamma - a)K + \{B(1-u) + n\gamma u - a[(n-1)\beta + 1]\}h_i. \tag{21}$$

For the growth rate of the common capital to be positive, the term $A - n\gamma$ must be positive.[6] As for (21), the coefficient of K is positive if β is positive and negative if β is negative.[7] In addition, the coefficient of h_i is positive in spite of the sign of β.[8] Therefore, we can illustrate the phase diagrams of the model as follows.

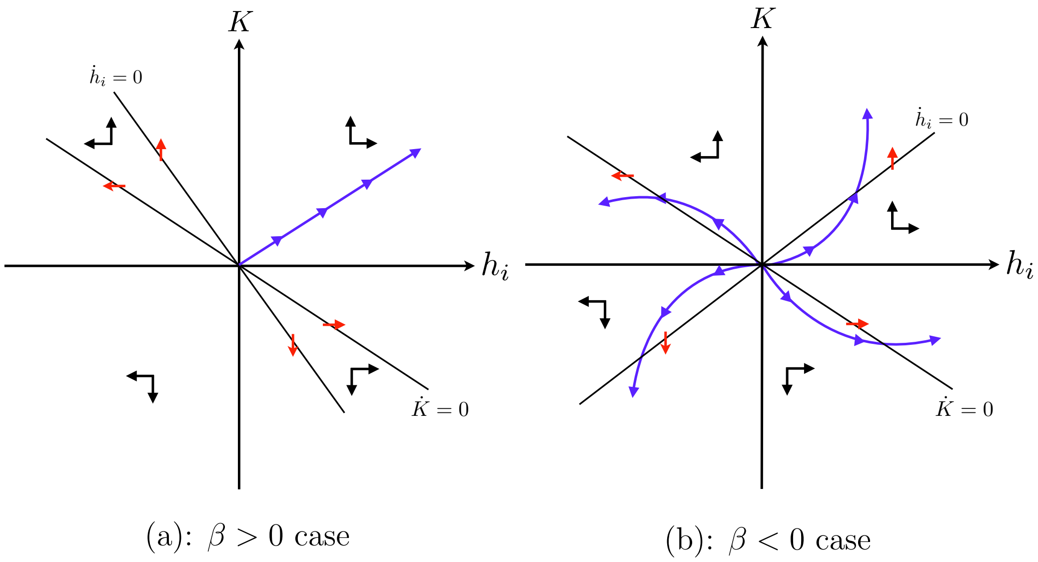


Figure 2: The phase diagrams

In the case where β is positive, we can easily confirm that the slope of $\dot{h}_i = 0$ line is steeper than that of $\dot{K} = 0$ line. The dynamic system is unstable and there is no transition dynamics

[6] It will be numerically confirmed in Section 4.2.

[7] The proof is given in Appendix D.

[8] In the case where β is negative, it is clear because $\beta(n-1)+1 < 0$. In the case where β is positive, from Lemma 3, we can confirm $B(1-u) + n\gamma u - a[(n-1)\beta + 1] = \frac{B(1-u)(\beta - u)}{\beta} + n(\gamma - a)u > 0$.

as seen in the previous section . In contrast, in the case where β is negative, the dynamic system does not ensure the positivity of the state variables over time. Therefore, the negative β is not optimal.

In what follows, we show that the growth rates of all the other variables correspond to that of consumption and characterize the balanced growth path. As seen in the previous section, we define the balanced growth ratio of private capital stock to common capital stock as $\chi \equiv h_i/K$. We obtain the following proposition.

Proposition 4. *Under Assumptions 1 and 3, the strategy profile* $\{(\phi_i, \psi_i)\}_{i=1}^n$ *defined by* $\phi_i(K, h)$ *and* $\psi_i(K, h)$ *forms a symmetric MPE. In the equilibrium, the optimal strategies are*

$$\psi_i^* = aK + ah_i + bZ_i \ \textit{and} \ \phi_i^* = \gamma\left[K + uh_i + uZ_i\right],$$

where the optimal parameters are

$$\begin{aligned}
a &= e = \xi^{-\frac{1}{\theta}} = \frac{uB(1-u)}{\beta[(n-1)\beta + 1 - un]}, \\
\beta &= \frac{y + \sqrt{y^2 + 4(n-1)[\rho + (1-u)(\theta-1)B]\theta uB(1-u)}}{2(n-1)[\rho + (1-u)(\theta-1)B]}, \\
\gamma &= \frac{A[(n-1)\beta + 1 - un] - B(1-u)[(n-1)\beta + 1]}{(1-\beta)(n-1)[(n-1)\beta + 1 - un]}, \\
a' &= 0, \ \ \alpha = 1, \ \ \textit{and} \ b = a\beta,
\end{aligned}$$

where $y \equiv (un-1)\rho + (1-u)[n(1+u)+1](\theta-1)B$. *The balanced growth rates are*

$$g = \frac{\dot{c}_i}{c_i} = \frac{\dot{d}_i}{d_i} = \frac{\dot{K}}{K} = \frac{\dot{h}_i}{h_i} = \frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}, \tag{22}$$

and the ratio of private capital stock to common capital stock is

$$\chi = \frac{g - (A - n\gamma)}{nu(A - n\gamma)}.$$

Proof. See Appendix E. □

Note that the marginal productivity in the common sector and that in the private sector are constant due to the assumption of a linear technology, so that balanced growth is achieved without transitional dynamics in the economy. The growth rate of the common capital is equivalent to the growth rate of the private capital. All the variables grow at the same positive and constant rate regardless of the initial level of common-private capital ratio (see Figure 2(a)). Note also that the balanced growth rate is not dependent on the marginal productivity of the common sector because β is also independent of A. This implies that there is no standard voracity effect as asserted by Tornell and Lane (1999).

Furthermore, χ must be positive because it is the ratio of private capital to common capital. Positive $A - n\gamma$ is required for keeping the accumulation of the common capital over time. The ratio χ has a finite positive value unless $A - n\gamma$ is close to zero or exceeds the growth rate. If $A - n\gamma$ is negative, χ is also negative. Therefore, positive χ means that $A - n\gamma$ is positive and is smaller than the growth rate.

At the end of this section, we derive another proposition.

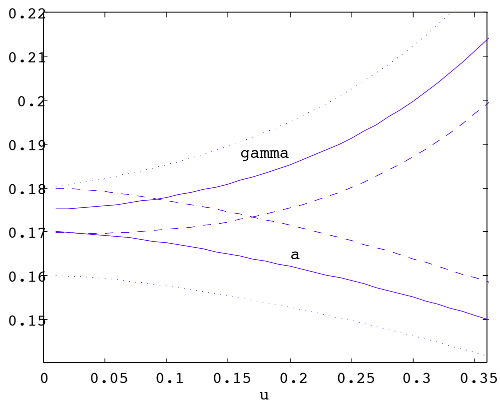
Proposition 5. *The opponents' private capital stock has a positive effect on a player's consumption.*

Proof. When the value function depends on the opponents' private capital stocks, optimal a and β are positive. Differentiating the consumption strategy with respect to h_j yields $\frac{\partial \psi_i^*}{\partial h_j} = a\beta > 0$. □

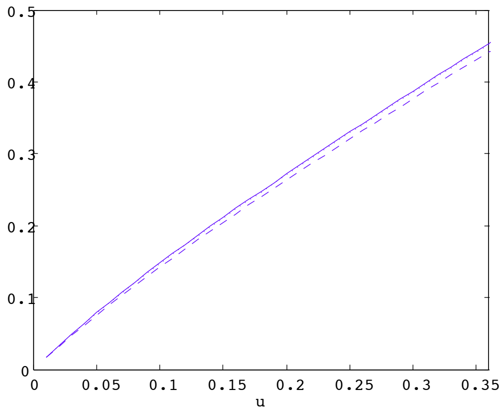
Two features are worth noting. First, Tenryu (2013) considers the case in which $u = 0$ and derives that the consumption strategy is negatively affected by the opponents' private capital. In this situation, once groups extract the resource, it cannot be returned to the common sector, and the more of the resource a group extracts, the less of it the other groups can obtain. Furthermore, the marginal product of the common sector is assumed to be larger than that of the private sector. These lead to the negative effect. Second, there is a crucial difference between the present paper and Tenryu (2013). We consider the interaction between the common sector and the private sector by introducing u; i.e., a fraction of the private capital is used to produce output in the common sector. A group is not only forced to contribute its own capital but also the other groups are forced to. This situation is equivalent to the positive externality in the common sector. As a consequence, the proposition is derived.

4.2 A Numerical Example

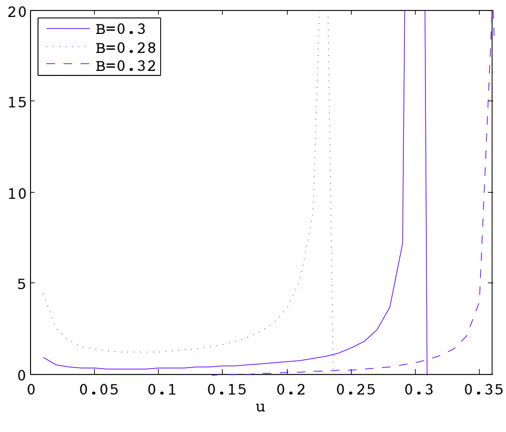
In this section, we consider the effect of the contribution ratio, u, on the parameters, a, β, γ, g, and χ. We will explore how these parameters change as the ratio increases. However, due to the complicities associated with investigating this analytically we do so numerically. We first need to assert values to the structural parameters of the model. In the numerical analysis below, we use the following values as the baseline: $\theta = 2$, $\rho = 0.04$, $A = 1.0$, $N = 5$, and $B = 0.3$. The elasticity of intertemporal substitution, the discount rate, and the technology level of the common sector are followed by the values in Mulligan and Sala-i-Martin (1992). The number of interest groups is equal to that in Lindner and Strulik (2004) and Strulik (2011). We set the technology level of the private sector to 0.3 in order to characterize the balanced growth comparative statics well. Figure 3 shows the effect on the major parameters and variables of the model.



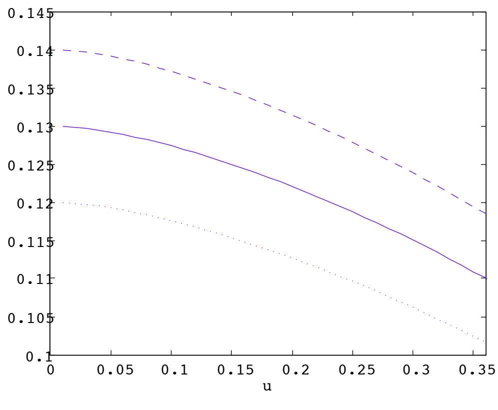
(a) a and γ



(b) β



(c) The ratio of h_i to K



(d) Growth rate

Figure 3: The effects of the contribution rate on major parameters, private-common capital ratio, and growth rate

Figure 3(a) shows that the appropriation parameter, γ, rises and one of consumption parameter, a, declines as the contribution rate increases, u. As explained in the previous section, when u increases, group i is forced to invest its private capital in the common sector whereas the other groups are also forced to invest their private capital. Since the latter effect dominates group i' effect, this is regarded as a windfall for group i and it further extracts the resource. The figure also shows that the parameter restriction $\gamma - a > 0$ is satisfied. Figure 3(b) traces out the effects of u on a coefficient of the value function, (18). We can confirm that β is an increasing function of and larger than u. A rise in β causes the marginal value of the opponents' private capital to increase and those of the common and group i's private capital to fall.[9] Thus, a is a decreasing function of u.

Figure 3(c) shows that χ has a U-shape, i.e., χ is decreasing with respect to u when u is relatively low, and χ is increasing when u is relatively high. The last result is interpreted as follows. When u is relatively low, the marginal increase of appropriation is dominated by that of u. On the other hand, when u is relatively high, the marginal increase of appropriation dominates that of u. Therefore, there exists a point where both effects are set off. In addition, when u is relatively large, the positivity of χ is not satisfied. When B is 0.3, for example, χ is negative over the region where u is 0.31. This means that an increase in u leads to so much appropriation that the economy cannot be sustainable. On the other hand, when B is 0.32, γ is smaller than a for a relatively small u.

Figure 3(d) shows that the balanced growth rate declines as u increases. As u increases, the appropriation parameter is increasing and thus the balanced growth rate is decreasing. This is the same phenomenon and another channel of the voracity effect as discussed in Section 3.2. Similarly, the growth rate of common capital will be smaller than that in Tornell and Velasco (1992). To see this, we consider the case in which B is 0.32.[10] The growth rate of the common capital in their model is $\bar{g} = \frac{nB-A}{n-1} = 0.15$. This is always higher than that in our model, which is the same result obtained in Proposition 3. Therefore, in the general case, each agent values the opponents' private capital stock; that is their private capital has a positive effect on its consumption strategy. This causes each group to extract the resource still further, thus prompting the growth rate to decline.

[9]See Proposition 4.

[10]In case where B is 0.28 or 0.3, the conditions required in Assumption 2 under is not satisfied under $u = 0$.

5 Conclusion

We analyzed a developing economy with multiple interest groups. There are the common sector without secure property rights and the private sectors with secure property rights. A government requires each group to invest a fraction of its own private capital in the common sector in order to protect the commons. In this situation, we explore another cause of voracious behavior and investigate the effects of voracious behavior on the economy. First, we show that the balanced growth rates are independent of the technology level in the common sector. This implies that there is no standard voracity effect in the sense that Tornell and Lane (1999) define. We also show that, when each group values the opponents' private capital, their capital has a positive effect on a group's equilibrium consumption strategy. Finally, we show that an increase in the contribution rate leads to an increase in appropriation, and hence the balanced growth becomes slow. The paper predicts that the contribution of the private sector to the common sector has a negative effect on economic growth and that the policy for preservation of the commons leads to the harmful effect on the economy.

Our model has some limitations and directions of possible extensions. First, we assumed that the contribution rate is exogenously chosen by a government for analytical simplicity. It is possible that the government or another agent chooses the contribution rate endogenously. Second, since we assumed homogeneous interest groups, we could not analyze what happens when there are heterogeneous interest groups. Introducing some kinds of asymmetry into the model would be an important issue. Third, we assumed simplified production, i.e., linear technology. We can consider other types of production and utility functions. For example, it is interesting to use the production with externality, as Mino (2006) and Itaya and Mino (2007) did, and to add appropriation costs and wealth effects to the utility function, as Long and Sorger (2006) did. Finally, we have treated only the linear Markov strategies. Characterizing equilibrium under other Markov strategies, including non-linear Markov strategies, would be important.

Appendix A. Proof of Lemma 1

In the case where the value function is independent of the opponents' private capital stocks, (11), we obtain

$$\frac{\partial V_i}{\partial h_j} = \frac{\partial^2 V_i}{\partial h_j \partial K} = \frac{\partial^2 V_i}{\partial h_j \partial h_i} = \frac{\partial^2 V_i}{\partial h_j^2} = 0.$$

Substituting these and appropriation strategies into (10) yields $\frac{\partial V}{\partial K}(Au - (n-1)\delta) = 0$. Since $\frac{\partial V}{\partial K} \neq 0$, for the equation to be satisfied, $Au - (N-1)u\delta$ must be zero and thus $\delta = \frac{Au}{n-1}$. The optimal condition (7) requires that $\alpha = 1$ and that the conjectured value function must hold

$$\frac{\partial^2 V}{\partial K^2} = \frac{\partial^2 V}{\partial h_i \partial K} = \frac{\partial^2 V}{\partial h_i^2} = -\theta\xi(K + \alpha h_i)^{-\theta-1}.$$

These conditions mean that (8) is equivalent to (9), which leads to $\gamma = \frac{A-B(1-u)}{n-1}$.

Next, from the optimal condition (6), the value function (11), and the consumption strategy, we confirm that

$$(a' + aK + eh_i + bZ_i)^{-\theta} = c_i^{-\theta} = \frac{\partial V_i}{\partial K} = \left(\xi^{-\frac{1}{\theta}} K + \xi^{-\frac{1}{\theta}} h_i\right)^{-\theta}.$$

For the condition to be satisfied, $a' = b = 0$ and $a = e = \xi^{-\frac{1}{\theta}}$ are required.

Since we focus on symmetric equilibrium, h_i is equivalent to h_j for $j \neq i$ at equilibrium. Using this and the results obtained above, we can arrange (8) as follows.

$$[\rho - B(1-u)]\frac{\partial V_i}{\partial K} = \frac{\partial^2 V_i}{\partial K^2}[B(1-u)(K+h_i) - c_i] \iff \rho - B(1-u) = -\theta B(1-u) + \theta\xi^{-\frac{1}{\theta}}.$$

This leads to

$$a = e = \xi^{-\frac{1}{\theta}} = \left(\frac{\theta - 1}{\theta}\right) B(1-u) + \frac{\rho}{\theta}.$$

Appendix B. Proof of Lemma 3

First, we confirm that, in the case of (18), the optimal condition (7) requires $\alpha = 1$ and thus the following relations are obtained.

$$\frac{\partial V_i}{\partial h_j} = \beta\frac{\partial V_i}{\partial K}, \qquad \frac{\partial^2 V_i}{\partial K^2} = \frac{\partial^2 V_i}{\partial h_i \partial K} = \frac{\partial^2 V_i}{\partial h_i^2},$$

$$\frac{\partial^2 V_i}{\partial h_j \partial K} = \frac{\partial^2 V_i}{\partial h_j \partial h_i} = \beta \frac{\partial^2 V_i}{\partial K^2}, \text{ and } \frac{\partial^2 V_i}{\partial h_j^2} = \beta^2 \frac{\partial^2 V_i}{\partial K^2}.$$

Next, substituting these and the strategies into equations (8) − (10), we obtain

$$\frac{\partial^2 V_i}{\partial K^2} \cdot F(K,h) = \{\rho - A + (1-\beta)(n-1)\gamma + a\beta(n-1)\} \frac{\partial V_i}{\partial K}, \tag{23}$$
$$\frac{\partial^2 V_i}{\partial K^2} \cdot F(K,h) = \{\rho - u[A - (1-\beta)(n-1)\gamma] - B(1-u) + a\beta^2(n-1)\} \frac{\partial V_i}{\partial K},$$

and

$$\beta \frac{\partial^2 V_i}{\partial K^2} \cdot F(K,h) = \{\beta\rho - u[A - (1-\beta)(n-1)\gamma] - \beta B(1-u) + a\beta[1+\beta(n-2)]\} \frac{\partial V_i}{\partial K},$$

where the function $F(K,h)$ represents

$$\begin{aligned} F(K,h) = & \{A - (1-\beta)(n-1)\gamma - a[1+\beta(n-1)]\}K \\ & + \{u[A - (1-\beta)(n-1)\gamma] + B(1-u) - a[1+\beta^2(n-1)]\}h_i \\ & + \{u[A - (1-\beta)(n-1)\gamma] + \beta B(1-u) - a\beta[2+\beta(n-2)]\}Z_i. \end{aligned}$$

We can summarize the three equations as follows:

$$(1-\beta)(n-1)(1-u)\gamma = (A-B)(1-u) - a\beta(n-1)(1-\beta), \tag{24}$$
$$(1-\beta)(n-1)(\beta-u)\gamma = A(\beta-u) - \beta[B(1-u) - a(1-\beta)]. \tag{25}$$

The unknown parameters, a, β, and γ, must satisfy both of the above equations simultaneously. First, if $\beta = 1$, the above conditions require that the contribution rate u must be a unity because of the assumption $A > B$. This contradicts the assumption $u \in (0,1)$, and thus this is not an equilibrium. Second, we consider the possibility that β is zero. Substituting $\beta = 0$ into (24) and (25), we get two equations, $(n-1)\gamma = A - B$ and $(n-1)\gamma = A$. For the two equations to be satisfied simultaneously, B must be zero, which contradicts the positivity of B. Therefore, $\beta = 0$ is not an equilibrium. Finally, we consider the case $\beta \neq 0, 1$. Substituting (24) into (25), we obtain $a\beta^2(n-1) - ua\beta(n-1) + a\beta(1-u) - uB(1-u) = 0$. We solve this for a,

$$a = \frac{uB(1-u)}{\beta[(n-1)\beta + 1 - un]}.$$

Substituting it into (24), we obtain the appropriation rate γ:

$$\begin{aligned}\gamma &= \frac{A-B}{(1-\beta)(n-1)} - \frac{uB}{(n-1)\beta+1-un} \\ &= \frac{A[(n-1)\beta+1-un] - B(1-u)[(n-1)\beta+1]}{(1-\beta)(n-1)[(n-1)\beta+1-un]}.\end{aligned} \tag{26}$$

Next, from the optimal condition (6) and (18), and the consumption strategy, we confirm that

$$(a' + aK + eh_i + bZ_i)^{-\theta} = c_i^{-\theta} = \frac{\partial V_i}{\partial K} = \left(\xi^{-\frac{1}{\theta}}K + \xi^{-\frac{1}{\theta}}h_i + \xi^{-\frac{1}{\theta}}\beta Z_i\right)^{-\theta},$$

which leads to $a' = 0$, $a = e = \xi^{-\frac{1}{\theta}}$, and $b = a\beta$.

Finally, since we focus on symmetric equilibrium, h_i is equivalent to h_j for $j \neq i$ at equilibrium. We substitute the above results into (23), and after some manipulation, we obtain the following equation:

$$\begin{aligned}&\left[\rho - B + \frac{(n-1)uB(\beta-u)}{(n-1)\beta+1-un}\right]\xi(K+h_i+\beta Z_i)^{-\theta} \\ &\qquad = -\theta\xi(K+h_i+\beta Z_i)^{-1-\theta}\left[\frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[(n-1)\beta+1-un]}\right](K+h_i+\beta Z_i).\end{aligned}$$

It is rewritten as $(n-1)[\rho+(1-u)(\theta-1)B]\beta^2 - y\beta - \theta uB(1-u) = 0$, where $y \equiv (un-1)\rho + (1-u)[n(1+u)+1](\theta-1)B$. Solving the quadratic equation for β,

$$\beta = \frac{y \pm \sqrt{y^2 + 4(n-1)[\rho+(1-u)(\theta-1)B]\theta uB(1-u)}}{2(n-1)[\rho+(1-u)(\theta-1)B]}.$$

This implies that if the quadratic equation has two different real roots, one is negative and the other is positive.

Appendix C. Proof of Lemma 4

Let us derive the growth rate of consumption. The consumption of group i is represented by $c_i = \psi_i^* = a(K + h_i + \beta Z_i)$. Differentiating this with respect to t and dividing it by c_i yields

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{K} + \dot{h}_i + \beta\dot{Z}_i}{K + h_i + \beta Z_i}.$$

The state dynamics of the model are represented as follows.

$$\begin{aligned}
\dot{K} &= (A - n\gamma)K + (Au - n\gamma u)h_i + (Au - n\gamma u)Z_i, \\
\dot{h}_i &= (\gamma - a)K + (B(1-u) + \gamma u - a)h_i + (\gamma u - a\beta)Z_i, \\
\dot{h}_j &= (\gamma - a)K + (\gamma u - a\beta)h_i + (B(1-u) + \gamma u - a)h_j + (\gamma u - a\beta)\sum_{k\neq i,j} h_k.
\end{aligned}$$

Substituting these into the numerator, we obtain

$$\begin{aligned}
\dot{K} + \dot{h}_i + \beta \dot{Z}_i &= [A - \gamma(n-1) - a + \beta(n-1)(\gamma - a)]K \\
&\quad + [Au - \gamma u(n-1) + B(1-u) - a\beta(n-1)(\gamma u - a\beta)]h_i \\
&\quad + [Au - \gamma u(n-1) - a\beta + \beta(B(1-u) + \gamma u - a) + \beta(n-2)(\gamma u - a\beta)]Z_i, \\
&= \frac{B(1-u)(\beta - u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}(K + h_i + \beta Z_i).
\end{aligned}$$

Therefore, we obtain the following growth rate of consumption,

$$\frac{\dot{c}_i}{c_i} = \frac{B(1-u)(\beta - u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}.$$

Appendix D. The sign of $\gamma - a$

According to Lemma 3, we can represent $\gamma - a$ as follows:

$$\gamma - a = \frac{A\beta[(n-1)\beta + 1 - un] - B(1-u)[nu + (\beta - u)(1 + (n-1)\beta)]}{(n-1)(1-\beta)\beta[(n-1)\beta + 1 - un]}. \tag{27}$$

Similarly, we can represent $A - n\gamma$ as follows:

$$A - n\gamma = \frac{[\beta(n-1)+1]\{nB(1-u) - A[\beta(n-1)+1-un]\}}{(1-\beta)(n-1)[\beta(n-1)+1-un]}.$$

We subtract $A - n\gamma$ from the growth rate of consumption, g:

$$g - (A - n\gamma) = \frac{\begin{array}{l}[(n-1)\beta+1]\{A\beta[(n-1)\beta+1-un] \\ \qquad -B(1-u)[nu + (\beta-u)(1+(n-1)\beta)]\}\end{array}}{(n-1)(1-\beta)\beta[(n-1)\beta+1-un]}. \tag{28}$$

Substituting (28) into (27), we obtain

$$\gamma - a = \frac{g - (A - n\gamma)}{(n-1)\beta + 1}. \tag{29}$$

If β is positive, the denominator in the right-hand side is positive and if β is negative, it must be negative because of the need to assure positive growth of consumption.

Furthermore, on the balanced growth path of the model, the growth rate of consumption and common capital must coincide; that is from (19) and (20),

$$g = \frac{\dot{K}}{K} = (A - n\gamma) + nu(A - n\gamma)\frac{h_i}{K},$$

and therefore we obtain the ratio of private capital to common capital:

$$\chi = \frac{h_i}{K} = \frac{g - (A - n\gamma)}{nu(A - n\gamma)}. \tag{30}$$

Since χ must be positive, the numerator $g - (A - n\gamma)$ must be also positive. Therefore, from (29),if β is positive, $\gamma - a$ is positive and if β is negative, $\gamma - a$ is negative.

Appendix E. Proof of Proposition 4

First, we derive the symmetric MPE strategies. As discussed in 4.1, β is positive. Substituting β into the parameters obtained in Lemma 3 yields the MPE parameters. Therefore, the optimal strategies are $\psi_i^* = aK + ah_i + bZ_i$ and $\phi_i^* = \gamma\,[K + uh_i + uZ_i]$.

Next, the ratio of private capital stock to common capital stock is (30) obtained in Appendix D. To derive the common and private capital growth rates, we divide (20) and (21) by K and h_i, respectively.

$$\frac{\dot{K}}{K} = (A - n\gamma) + nu(A - n\gamma)\frac{h_i}{K}, \tag{31}$$

$$\frac{\dot{h}_i}{h_i} = (\gamma - a)\frac{K}{h_i} + B(1-u) + n\gamma u - a[(n-1)\beta + 1]. \tag{32}$$

Substituting (30) into (31), we obtain

$$\frac{\dot{K}}{K} = \frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}.$$

Also, substituting (30) into (32), we obtain

$$\begin{aligned}\frac{\dot{h}_i}{h_i} &= \frac{1}{(n-1)\beta+1}\left[\{g-(A-n\gamma)\}\frac{K}{h_i} - nu(A-n\gamma)\right] + g \\ &= \frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}.\end{aligned}$$

Third, we derive the growth rate of appropriation. The appropriation of group i is represented by $d_i = \gamma[K + uh_i + uZ_i]$. As in the discussion above, since we focus on the symmetric MPE, $h_i = h_j$ for all $j \neq i$. Therefore, differentiating this with respect to t yields

$$\frac{\dot{d}_i}{d_i} = \frac{\dot{K} + un\dot{h}_i}{K + unh_i}.$$

On the balanced growth path, the growth rate of the common capital is equivalent to that of the private capital, $\dot{K}/K = \dot{h}_i/h_i$, and thus

$$\frac{\dot{d}_i}{d_i} = \frac{\frac{\dot{K}}{K}\left(1 + nu\frac{h_i}{K}\right)}{1 + nu\frac{h_i}{K}} = \frac{\dot{K}}{K} = \frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[\beta(n-1)+1-un]}.$$

Finally, we check the boundary condition. Note that since the value function $V_i(K,h)$ has the properties $V_i(0,0) = 0$ and strict concavity, holding the boundary condition (5) guarantees that the transversality conditions are satisfied. In the same manner as the previous section, for the boundary condition to be satisfied,

$$\frac{\xi(K_0 + h_{i0} + \beta Z_{i0})^{1-\theta}}{1-\theta} \lim_{t\to\infty} \exp\left(\left[\frac{Bu(1-u)(1-\theta)(\beta-1)}{\beta[(n-1)\beta+1-un]} - \{\rho + (\theta-1)(1-u)B\}t\right]\right).$$

must converge to zero. If $\theta > 1$, it is easy to verify that this is satisfied. Let us consider the case where $0 < \theta < 1$. According to (26) in Appendix B, β must be smaller than one to assure positive appropriation, which means that the first term is negative. Under Assumption 3, the equation $\rho + (\theta-1)(1-u)B$ is positive so that the second term is negative. Therefore, the boundary condition is satisfied.

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