Sraffa’s price equations in light of Garegnani and Pasinetti - The ‘core’ of surplus theories and the ‘natural’ relations of an economic system

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The ‘core’ of surplus theories and the ‘natural’ relations of an economic system*

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“Quelli che ti spiegano le tu idee senza farte le capire ...”
(E. Jannacci, “Quelli che ... ”, 1975, Italian pop song)

Abstract. Within the rich literature that has flowed from Sraffa’s framework of Production of Commodities by means of Commodities a prominent position is occupied by the research programmes carried out independently by two authoritative exponents of this school: Pierangelo Garegnani and Luigi Pasinetti. Certain specific features of their approaches might lead one to perceive them as alternative to one another. Yet, when analysed through a constructive perspective, one discovers not only a common origin and methodology, but also strict complementarity in analysing the main characteristics of industrial systems.

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1. Introduction

The ‘modern classical political economy’ is a field of research that has flowed directly from the analytical framework proposed by Sraffa (1960) and from the works of those scholars that gravitated around Cambridge (UK) between the Sixties and the Eighties.

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This group of scholars is addressed by several labels such as Sraffians, modern classicals, Neo-Ricardians, post-Keynesians (although this latter adjective is less appropriate\(^1\)), etc. Many surveys have outlined multiple aspects of these research programmes: for example, Harcourt (1972) reconstructed the first steps of the capital criticism, Pasinetti (2007) gave a picture of the main topics debated in the light of the personal relations among the original members of the Cambridge school. The present paper aims to observe, in a comparative way, how Sraffa’s research programme has been carried out by two of the more authoritative exponents of this school: Pierangelo Garegnani and Luigi Pasinetti. It is undeniable that the common origin of these authors is found in classical political economy and in particular in Sraffa’s approach. The divarication of their investigations in two different directions is also clearly apparent: the delimitation of the logical structure of classical political economy for Garegnani, and the analysis of structural change for Pasinetti. The former describes how classical analysis can be separated in two ‘stages’ of analysis, with a particular focus on the determination of the prices of commodities and their relations with the distributive variables. To achieve these goals, Sraffa and Garegnani emphasize the necessity to consider the quantities produced \textit{as given} in this stage of analysis. Moreover, they refer this analysis to a situation characterized by a \textit{uniform} rate of profit. Garegnani justifies this uniformity as a result of a ‘gravitation’ process induced by the phenomenon of capital mobility in searching the highest return; hence he explicitly refers the entire analysis to a \textit{capitalist} system. Pasinetti, on the other hand, considers a multisectoral system where population, technology and final consumption evolve over time in a sectorally-differentiated way. Consequently, the quantities produced \textit{change} as time goes by; moreover, he refers his analysis to a \textit{pre-institutional} stage, where an entire structure of \textit{differentiated} sectoral rates of profits is determined to meet the sectoral necessities of accumulation of productive capacity in relation to the evolution of the final demand of each commodity.

What will be argued in this work is that these research projects are not incompatible to one another; they are in fact complementary. We will see how the

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\(^1\) A detailed analysis of these classifications is beyond the purpose of this essay. Simply recall that the term ‘post-Keynesian’ is normally referred to the group of scholars directly connected with Keynes, like Richard Kahn, Nicholas Kaldor, Michael Kalecki and Joan Robison. Luigi Pasinetti belongs, in some ways, both to post-Keynesian and to Sraffians: he represents a sort of ‘bridge’ between the groups.
structural change model is built upon, and is an extension of, the ‘core’ equations of the surplus approach: in particular, in order to determine a structure of rates of profits connected to the evolution of final demand of commodities, Pasinetti needs to avoid any biunique (mechanical) co-determination—typical of neoclassical analysis—between prices and distributive variables, on the one side, and quantities on the other. This will be possible simply because quantities are the object of a separate determination in the stage of analysis where the relation between prices and distributive variables are studied. This requirement is so stringent that when Pasinetti formulates the laws of evolution of the parameters of the system, he follows a way which is parallel (though not equal) to that followed by Sraffa and Garegnani.

As is usual with new path breaking works, Sraffa’s framework has been the object of several, sometime antithetic, interpretations and uses. The conclusion that will be reached here points in the opposite direction: the interpretative frameworks delineated by our authors of the bare ‘core’ of Sraffa’s price equations outline the skeleton of a unified and coherent framework suitable to analyse the basic workings of capitalist economies as well as the conditions which need to be satisfied in order to achieve specific goals concerning employment and the satisfaction of (changing) final wants.

In this work I do not mean to interpret the authors nor their approaches. Rather, I will present, in a constructive way, what I have learned from their approaches, and what I consider a common and solid basis for grounding an economic investigation of our contemporary systems. Hence, I will be more inclined to uncovering connections and indicating their possible integration rather than emphasizing the differences among the approaches, which I do not deny exist.

2. The logical framework of classical surplus theories

For expositional reasons it is convenient to start from the reconstruction of the logical framework of classical surplus theories provided by Pierangelo Garegnani (see, in particular, Garegnani, 1984 and 2007).

The main focus of Classical Political Economy was the determination of the size of social surplus and of its distribution. The peculiar feature of these theories is the view
that the shares of the product other than wages are determined residually. This means that once the replacements of the means of production employed and wages (predetermined on the basis of an institutional mechanism) are deducted from the social product, what remains, the residuum, goes to profits and rents. In other terms profits and rents arise because wages do not absorb the entire net product. This view clearly outlines a society where capitalists and land-owners have a prominent position in the distributive process. This principle can be expressed by the following equation:

$$\Pi + R = (X - A) - W, \quad (1)$$

where \(\Pi\) are profits, \(R\) are rents, \(X\) is the social product, \(A\) is the replacement of the means of production and \(W\) are wages. There is a fundamental logical requirement in order to provide relation (1) a theoretical meaning, i.e. to interpret it as an equation and not just as an accounting identity: all the magnitudes on the r.h.s. of (1) must be considered as given in the stage of analysis where we study the determination of the magnitude of the shares other than wages. In other terms, \(X, A\) and \(W\) must be ‘intermediate data’, by using a term quite recently introduced by Garegnani (2007).

It is well known that the satisfaction of this logical requirement represented an Achille’s heel for Smith’s surplus theory of profits, that Ricardo perceived very clearly, but was unable to solve satisfactorily. The difficulties arise from the fact that \(X, A\) and \(W\) are values of aggregates having different compositions. The determination of the prices of the commodities entering them should thus be prior to the determination of the shares other than wages, but the determination of ‘natural’ prices for Smith requires the knowledge of wages, profits and rents. Sraffa’s (1960) framework provides a solution to these difficulties, and Garegnani (1984) has defined in details the logical requirements of this determination. It is useful to recall here briefly the main steps. Consider a system where commodities are produced by themselves and labour, all capital is circulating and there is no joint production; abstract from rents for the sake of simplicity. Free competition ensures a tendency of prices to cover wages and gross profits. The prices of commodities must thus satisfy the following equations

$$p^T\hat{x} = q^TA + w\ell^T, \quad (2)$$

where \(p\) is the price vector, \(\hat{x}\) is the diagonal matrix of gross output of the various industries, \(A = [a_{ci}]\) is a square matrix where \(a_{ci}\) is the quantity of commodity \(c\) annually
employed by industry \( i \), where \( c, i = 1, 2, \ldots, C = I \) (processes are represented on the rows), \( q \) is the vector of the gross rental prices (they include depreciation) of the various commodities used as capital goods, \( w \) is the wage rate and \( \ell \) is the vector of the annual quantities of labour employed in each industry. This formulation, which recalls Smith’s notion of natural prices\(^2\), is still incomplete, as it overlooks the elementary fact that it is impossible to fix all distributive variables, \( q \) and \( w \), independently one another.\(^3\) This inconsistency, typical of an adding up theory of prices for which the claim of each class can be accommodated by a suitable variation of prices, is eliminated once it is recognized that in normal conditions gross rental prices of capital goods, \( q \), are linked to the prices of production of these goods, \( p \), by the relation\(^4\)

\[
q = (1 + \pi)p, \quad (3)
\]

where \( \pi \) is the uniform rate of return or of profit. After replacing equation (3) into equation (2) we obtain the usual formulation of the price system:

\[
p^T x = (1 + \pi)p^T A + w^T \ell. \quad (4)
\]

It should be recognized that price equations (2) plus equations (3) or, which is equivalent, system (4), is common to both classical and long-period neoclassical approaches.\(^5\) What really differentiates them are the forces which regulate income distribution: the relative scarcity of factors in the neoclassical approach—expressed by the supply and demand curves which co-determine the prices of commodities and the distributive variables—or social and historical (i.e. institutional) factors in the classical approach.

\(^2\) Smith (1776, chp. vii) writes: “[w]hen the price of any commodity is neither more nor less than what is sufficient to pay the rent of the land, the wages of the labour, and the profits of the stock employed in raising, preparing, and bringing it to market, according to their natural rates, the commodity is then sold for what may be called its natural price.”

\(^3\) Smith (1776, chp. VII) writes “The natural price itself varies with the natural rate of each of its component parts, of wages, profits, and rents”. On this, see Garegnani (1984, § 9) and Sraffa (1951, p. xxxv).

\(^4\) Equations (3) holds for the case where all commodities are circulating capital goods. In the case of fixed capital (with a constant depreciation rate) they should be replaced by \( q = (\hat{\delta} + \pi I)p \), where \( \hat{\delta} = [\hat{\delta}_i] \) is the diagonal matrix of the depreciation rates of the various commodities used as capital goods.

\(^5\) Equation system (2) correspond to the price equations of capital goods of Walras’ system, while conditions (3) are the conditions of uniformity of the rates of return on the supply prices of capital goods; the same conditions, written in the form (4), are the Sraffa price system.
It is well known that the neoclassical determination of income distribution is affected by the logical difficulties connected with the notion of ‘quantity of capital’, both in the demand and in the supply side. In order to avoid to prejudice the validity of the whole logical construction Sraffa, following the classical tradition, decided to consider the quantities produced, $x$, the quantities of commodities employed as means of production, $A$, and the quantities of labour $\ell$ as given when he writes the price equations (4). In the Preface of his book he warns:

[...] anyone accustomed to think in terms of the equilibrium of demand and supply may be inclined, on reading these pages, to suppose that the argument rests on a tacit assumption of constant returns in all industries. If such a supposition is found helpful, there is no harm in the reader’s adopting it as a temporary working hypothesis. In fact, however, no such assumption is made. No changes in output and [...] no changes in the proportions in which different means of production are used by an industry are considered, so that no question arises as to the variation or the constancy of returns. The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportions of ‘factors’ (Sraffa, 1960, p. v).

In this way, once a numéraire has been chosen, it is possible to deduce from the price equations (4) the relation between the rate of profit and the wage rate. As known, when the Standard commodity is chosen as numéraire this relation takes the simple form

$$\pi = R(1 - w), \quad \text{or} \quad w = 1 - \frac{\pi}{R},$$

(5)

where $R = (1 - \lambda^*)/\lambda^*$ is the maximum rate of profit and $\lambda^*$ is the dominant eigenvalue of $A\hat{x}^{-1}$. Formula (5) depicts clearly the trade-off between profits and wages, which was impossible to be caught by looking at the price equations (2) only; in other terms it displays very clearly the residual character of one distributive variable with respect to the other. With another numéraire, the relation between $\pi$ and $w$ takes a more complicated form, but it also shows the trade-off between profits and wages. Relation (5) (or the analogous relation between $\pi$ and $w$ entailed by the numéraire chosen) shows that that income distribution must be determined outside the price equations, that is, outside the sphere of production.

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6 We are referring to the impossibility to ensure the inverse monotonicity of the demand of capital with respect to the rate of profit highlighted by the rewitching debate and the impossibility to obtain a scalar expression of the quantity of capital independent of the rate of profit.
While the attitude of marginalists has been that of searching this determination in the ‘factors market’, the attitude of classical economists (both ‘old’ and modern) has been that of searching this determination in the ‘institutional sphere’. This point has not always been clear in the literature about Sraffa’s work. One recurrent point was the idea that Sraffa’s price equations represented just one side of the economic relation, the supply side, and that they needed a set of demand equations to close the model: for example Samuelson writes:

[m]y fundamental point, let it now be clear, was that Piero Sraffa sought to have but one leg to stand on. Competitive prices, everyone now knows, must stand squarely on the two legs of (1) tastes, desires, needs and distribution of endowments (in short, on consumer-demand factors), and (2) technology and production costs. At one time or another, Adam Smith (very briefly), David Ricardo, and Frank Knight (briefly), have tried to concentrate on subcases of reality where competitive prices (price ratios, and goods prices relative to factor prices) can be determined autonomously in terms of technology and costs alone: the one-leg case. What is consistent throughout the lifeline of Piero Sraffa – in 1925, 1926, between 1926 and 1930, in 1951 and 1960 – is the attempt to emphasise the singular cases in which the theory of value happens to be dependent only on technology and costs independently of the composition of demand (Samuelson, 1991, p. 570).

Also Joan Robinson writes:

We are concerned with equilibrium prices and a rate of profit uniform throughout the economy, but we are given only half of an equilibrium system to stand on. We need a fence to prevent us plunging off into the abyss (Robinson, 1961, p. 54).

In order to set-up Sraffa’s theoretical framework within the realm of classical political economy, and to outline the main characteristics of this approach it is useful to adopt the device proposed by Pierangelo Garegnani, who enucleated the ‘core’ of this theory, which consists in a subset of relations (equations) that, given the value of some economic magnitudes which are provisionally considered as independent variables (also called ‘intermediate data’), determines the remaining variables as dependent or endogenous variables.

In ‘old’ classical economists (Smith, Ricardo and Marx) the intermediate data are:

(i) the social product, \( \mathbf{x} \),
(ii) the real wage rate, \( \mathbf{w}^T = [w_1, \ldots, w_C] \), i.e. a bundle of commodities,
(iii) the technology of the system, \( \mathbf{A} \);

the relations belonging to the ‘core’ determine the following dependent variables:
(a) labour employment, \( E^* = \ell^T x \),

(b) the shares other than wages; in our case where rents are not considered, profits, whose rate is given by\(^7\) \( \Pi = (1 - \lambda_M) / \lambda_M \),

(c) the price system, \( p^T = p^{*T} \),

where \( \lambda_M \) and \( p^{*T} \) are the dominant eigenvalue and the corresponding eigenvector of the socio-technical matrix \( A + w\ell^T \).

The attitude of ‘modern’ classical economists, since Sraffa (1960, § 44), is that of considering the rate of profit as the independent distributive variable. Hence in this case the intermediate data are

(I) the social product, \( x \),

(II) the rate of profit, \( \pi \),

(III) the technology of the system, \( A \);

the relations belonging to the ‘core’ determine the following dependent variables:

(A) labour employment, \( E^* = \ell^T x \),

(B) the wage rate, \( w^* = 1 - \pi / R \),

(C) the price system, \( p^{*T} = w^*[I - (1 + \pi)A]^{-1} \).

This distinction makes clear that the relations of the ‘core’ of the system can be adequately expressed by ‘necessary quantitative relations’ (Garegnani, 2007, p. 186), i.e. by equations and formal relations. On the contrary, the magnitudes which are taken as given in the core, i.e. the intermediate data, are determined by forces that are less susceptible to be represented by means of formal relations.

Obviously, no one in the surplus approach denies that there are influences and feedbacks among the various intermediate data and between the dependent variables on the one hand and the intermediate data on the other hand. But these influences have not the same level of generality and unambiguousness: they may change significantly according to the institutional circumstances and may go in both directions, partially compensating one another. These relations are thus better studied outside the ‘core’ of

\(^7\) In this case the price system becomes \( p^T = (1 + \pi)p^T(A + w\ell^T) \) (in coherence with old classical economists, wages are supposed to be paid in advance). The ensuing rate of profit is the maximum rate of profit, \( \Pi \).
the system, and in a partially different way: other disciplines, like political or social sciences, economic history, etc., can here usefully support the economic investigation. For example it is obvious that relative prices affect the composition of final output by affecting the demand of the various commodities. But these effects may change according to the historical or social circumstances; moreover, they are not univocal, and can partially compensate each other. For all these reasons it is preferred to consider the intermediate data \((i), (ii)\) and \((iii)\) of old classical economy or \((I), (II)\) and \((III)\) of modern classical political economy as determined separately from the endogenous variables of the core; formally this is realized by consider the intermediate data as given when one is studying the forces that determines the endogenous variables of the ‘core’: \((a), (b)\) and \((c)\) or \((A), (B)\) and \((C)\).

There are cases where some specific institutional problems could be analysed in a formal way: for example the study of gravitation of market prices around production prices, or the study of the accumulation process in Ricardian frameworks. But, again, the formalization concerns the specific problem at hand, not the working of the entire economic system. Hence, in these stages of analysis other variables are kept frozen at

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8 As we will see later, Luigi Pasinetti has proposed an analogous, although not coincident, ‘separation’ between a theoretical stage of analysis (to be faced by the deductive methods of pure economic theory) and an institutional stage, (to be faced with the support of other disciplines).

9 Interesting enough is a letter on this issue sent by Sraffa to Arun Bose (SP, C32/3). I reproduce it here integrally.

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Cambridge, 9th December, 1964

Dear Arun,

I am sorry to have kept your MS so long – and with so little result. The fact is that your opening sentence is for me an obstacle which I am unable to get over. You write: “It is a basic proposition of the Sraffa theory that prices are determined exclusively by the physical requirements of production and the social wage-profit division, with consumers demand playing a purely passive role.”

Never have I said this: certainly not in the two places to which you refer in your note 2. Nothing, in my view, could be more suicidal than to make such a statement. You are asking me to put my head on the block so that the first fool who comes along can cut it off neatly.

Whatever you do, please do not represent me as saying such a thing. This initial and to me quite maddening obstacle has prevented me, in spite of many attempts, from reading understandingly your article. You must find a more detached reader to advise you about it. I am very sorry to seem so unhelpful, but I have spent quite a lot of time upon your work, to no purpose. I do not think that it would be any good keeping it longer, so I now return it to you.

Yours sincerely,
some given levels: for example, the wage rate is taken as given when one studies how market prices gravitate around natural prices in consequence of capital mobility (see, for example, Boggio, 1990, p. 48, or Duménil and Lévy, 1993, chapter 5, or Garegnani, 1990, p. 333); again, the level of the wage rate is taken as given when one studies how profits are accumulated and new plots of land are cultivated (see Bellino, 2014, in particular § 3, and Kurz and Salvadori, 2006, pp. 110-1).

This attitude to analyse the working of an economic system in different stages, each for a specific problem or situation, sometimes in formal terms, in other cases by using the instruments of social, historical and institutional analysis, is in sharp contrast with the attitude followed by neoclassical economists, which normally refer to a general model.¹⁰

3. Structural Economic Dynamics

Luigi Pasinetti is probably the scholar that more than other succeeded in making a sort of bridge between the Keynesian major instances and the modern classical reappraisal led by Sraffa. Pasinetti’s model of structural change grounds in fact on Keynes principle of effective demand and on classical value theory of distribution and value. He presented this model in his PhD dissertation in 1963; then it was published in various versions in (1965, 1981, and 1993).

This analysis has a twofold objective:

i) to study the consequences of structural change of technology and of final demand on output, value and employment;

ii) to study the conditions that have to be satisfied in order to accomplish the potential of the system concerning growth, employment and the satisfaction of final wants.

In brief, we could say that objective i) is descriptive, while objective ii) is normative. To handle objective i) Pasinetti starts from Sraffa’s price equations and extends them to the

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¹⁰This is, probably, one of the reasons that explains the difficulties in communicating between the two approaches. In principle, however, also Neoclassical theory adopts the same methodological choice to consider some (other) magnitudes as given when it studies how its endogenous variables are determined: in general equilibrium analysis, for example, preferences, endowments, technology and property rights are taken as given when prices and allocations are determined. The obvious links between these groups of variables and data are intentionally not analysed.
case of an economic system undergoing a process of structural change, i.e. a change in the proportions of the various industries. Hence, some of the magnitudes that have been kept as given in the price equations, here must be left free to change. But, as we will see immediately, this extension is done in line with the methodological requirements imposed by the logical structure of surplus theories.

The price equations considered by Pasinetti are

\[
\begin{bmatrix}
0 & \cdots & 0 & \pi + \delta_1 & \cdots & 0 & l_i \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \pi + \delta_c & l_c \\
0 & \cdots & 0 & 0 & \lambda_i & v_i \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \lambda_c & v_c \\
c_1 & \cdots & c_c & j_1 & \cdots & j_c \\
\end{bmatrix}
\begin{bmatrix}
p_1 \\
\vdots \\
p_c \\
v_1 \\
\vdots \\
v_c \\
w \\
\end{bmatrix}
= 
\begin{bmatrix}
p_i \\
\vdots \\
p_c \\
q_i \\
\vdots \\
q_c \\
w + \pi \sum_{c=1}^{C} v_c c_c \\
\end{bmatrix}.
\]

We are considering an economic system where \( C \) final commodities are produced. 1 unit of commodity \( c \) requires \( \ell_c \) units of labour and 1 unit of a capital good, specific to the commodity, \( c = 1, \ldots, C \) (for this reason we will call briefly ‘capital good \( c \)’ the capital good employed in the production of commodity \( c \));\(^{11}\) in each production period a constant proportion, \( \delta_c \), of capital good \( c \) wears out. 1 unit of capital good \( c \) is produced by \( \lambda_c \) units of labour. Let \( c_c \) be the units of final good \( c \) required by each individual as final consumption; let \( j_c \) be the individual demand of capital good \( c \) by the final sector (net investment). Let \( p_c \) and \( q_c \) be the prices of commodity \( c \) and of its specific capital good. System (6) contains \( 2C + 1 \) equations in \( 2C + 2 \) unknowns: \( p_1, \ldots, p_C, v_1, \ldots, v_C, w \) and \( \pi \). The first \( 2C \) equations can be written as:

\[
p_c = (\delta_c + \pi)v_c + w\ell_c, \quad c = 1, \ldots, C, \quad (6p)
\]

\[v_c = w\lambda_c, \quad c = 1, \ldots, C. \quad (6v)\]

\(^{11}\) Following Pasinetti, the capital good used to produce 1 unit of commodity \( c \) can be considered as a composite commodity, that he calls ‘productive capacity of final good \( c \)’. In this way we can denote by the same single magnitude a set of heterogeneous means of production. The advantage of this procedure is that a change of the physical form of productive capacity of a final commodity, induced for example by technical change, can be ultimately reduced to a decrease in quantity of vertically integrated labour necessary to produce the commodity (for further details, see Pasinetti, 1973, § 15).

Moreover, here we are considering the case where capital goods are produced only by labour. The general case, where capital goods are produced by labour and other capital goods, is presented in (Pasinetti, 1981, chp. II, sect. 7).
After having chosen the *numéraire*, it remains one degree of freedom which express, as usual, the fact that income distribution is determined outside the price equations. For example, if we chose commodity ‘1’ as *numéraire*, i.e. if we set \( p_1 = 1 \), we obtain after substitution \( 1 = p_1 = (\delta_1 + \pi)w\lambda_1 + w\ell_c \), which originates the following inverse relation between the rate of profit the wage rate expressed in terms of commodity ‘1’:

\[
\frac{1}{w} = \frac{1}{(\delta_1 + \pi)\lambda_1 + \ell_1}.
\]

There remain, however, a further equation in system (6), the last one,

\[
p^Tc + v^Tj = w + \pi v^Tc,
\]

which expresses the condition that all incomes (wages + profits) must be entirely spent.

In addition to the price system we have a quantity system. Let \( x_c \) and \( k_c \) be the quantities produced of final good \( c \), and of its productive capacity; let \( x_N \) be the quantity of labour employed in all production activities. The quantity equations are

\[
\begin{bmatrix}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0 \\
\delta_1 & \cdots & 0 & \cdots & 0 \\
I_1 & \cdots & I_c & \cdots & \lambda_c
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_c \\
\vdots \\
x_N
\end{bmatrix}
= \begin{bmatrix}
x_1 \\
\vdots \\
x_c \\
\vdots \\
x_N
\end{bmatrix}.
\]

System (7) contains \( 2C + 1 \) equations in \( 2C + 1 \) unknowns: \( x_1, ..., x_c, k_1, ..., k_C, x_N \). It is a homogeneous system; once the condition to ensure not-trivial solution is satisfied\(^{12}\) we have one degree of freedom; the quantity which is more adapt to be fixed from outside is the employment level, \( x_N \); if we want to guarantee full employment we fix

\[
x_N = N,
\]

where \( N \) is the amount of the labour force.

The last equation of system (7) is

\[
\ell^Tc + \lambda^Tj = N.
\]

\(^{12}\) For details see Pasinetti (1981, chp. II, § 3).
Equations (6N) and (7N) together express the Keynesian principle of effective demand: wages and profits must be entirely spent in order to ensure that labour requirements employ the available amount of labour force. As we will see in brief this condition is never automatically satisfied.

Pasinetti assumes that all magnitudes entering as ‘parameters’ in the description of the industrial system of the previous section are now allowed to change. In particular, Pasinetti supposes that technical coefficients, final demand coefficients and population vary according to the following exponential functions

\[ \ell_c(t) = \ell(0)e^{\rho \ell}, \quad \lambda_c(t) = \lambda(0)e^{\rho \lambda}, \quad c_c(t) = c(0)e^{\rho c}, \quad N(t) = N(0)e^{nt}. \]  (9)

There remains a set of coefficients in the quantity equations (7) whose dynamics has not yet been specified: they are the coefficients of net investment in capital good \( c, k_{cN} \). In coherence with his normative attitude, Pasinetti fix them in such a way that the productive capacity of each commodity increases in line with the evolution of final demand for that commodity. This amount to fix\(^{13}\)

\[ k_{cN}(t) = (n + r_c)c_c(t). \]  (10)

The dynamics of parameters envisaged by equations (9) and (10) entails a structural change for the endogenous variables of the system considered: for prices, for sectoral output, for sectoral employment, as well as a macro-dynamic for the aggregate level of employment. This is a relevant result for a growth model. We do not enter here into details (see Pasinetti, 1981, chp. V).

The second objective pursued by Pasinetti in this investigation derives from interpreting the equilibrium conditions just as relations describing an ideal (efficient) situation, where the ‘potential’ of the economic system concerning growth, employment and satisfaction of final wants is realized at best, independently of the study of the forces that will take care to realize these conditions in an actual system. This allows him to ‘separate’ a level of analysis where the various conditions that must be satisfied in the ideal or ‘natural’ configuration of the economic system are described and a level of analysis where the institutional mechanisms to achieve these conditions are described.

\(^{13}\) For details see Pasinetti (1981, chp. V, sect. 4).
and compared.\textsuperscript{14} We could summarize the relations that must be satisfied in the ‘natural’ system in five points as follows:

(N1) a price system which guarantees the reproducibility of the various commodities, identified by the first 2\(C\) equations of system (6);

(N2) a set of output levels which satisfy the final demand of each commodity, identified by the first 2\(C\) equations of system (7);

(N3) the ‘macro–economic condition’, i.e., equations (6N), (7N) and (8), which together guarantees full employment of labour force;

(N4) an income distribution configuration which guarantees the growth of the productive capacity in each sector in line with the growth of the final demand of the respective commodity. Pasinetti envisages the fulfillment of this goal by a set of differentiated rates of profit, called ‘natural rates of profit’, determined by the total rate of growth of the final demand of the correspondent commodity \((\pi_c = n + r_c, c = 1, \ldots, C)\); it is possible to prove that in this case the price of each commodity becomes proportional to the quantity of labour which is necessary i) to reproduce the commodity, ii) to reproduce its means of production and iii) to expand these means according the growth rate of final demand of that commodity. A renewed form for the theory of labour value takes thus shape in this case. Consequently, each individual receives a fraction of the net product equal to the proportion of the quantity of labour he contributes with respect to the total labour of the system (‘labour principle’ of income distribution);

(N5) a natural rate of interest, which guarantees that the debt and credit relationships among individuals do not distort the income distribution process from its ‘labour principle’.

\textsuperscript{14} It is interesting to observe that a similar perspective has been suggested by Sraffa himself in a note written in 1942:

This paper deals with an extremely elementary problem; so elementary indeed that its solution is generally taken for granted. The problem is that of ascertaining the conditions of equilibrium of a system of prices & the rate of profits, independently of the study of the forces which may bring about such a state of equilibrium. Since a solution of the second problem carries with it a solution of the first, that is the course usually adopted in modern theory. The first problem however is susceptible of a more general treatment, independent of the particular forces assumed for the second; & in view of the unsatisfactory character of the latter, there is advantage in maintaining its independence (D3/12/15: 2).
The connection of the ‘natural’ relations with actual systems is seen by Pasinetti as an ‘institutional’ problem. In particular, as regards the achievement of goals listed above as (N1)-(N5) we can observe\textsuperscript{15} that goals (N1) and (N2) are normally fulfilled in capitalist systems: free competition ensures that prices tend towards their normal levels while the Keynesian principle of effective demand aligns the output of each commodity to its demand.\textsuperscript{16} On the contrary, goal (N3) is not automatic in modern capitalist economies; moreover, the phenomenon of structural change is for Pasinetti a force that contrasts with the fulfillment of this objective; it can be pursued by means of several policies, according to the specific situation of the system (introduction of new goods, search of new markets, public investments, monetary policies, etc). Goals (N4) and (N5) are probably never feasible, or only partially feasible, in capitalist systems; nevertheless, they may represent a sort of benchmark levels, to evaluate how far an actual system is far from its ‘natural’ configuration.\textsuperscript{17}

4. Two directions, one framework

To carry out our comparison between the surplus approach and the structural dynamics research programmes it is convenient to start from two (quite long) quotations from our two authors. Garegnani writes:

[W]hy take as given some magnitudes that the theory has also to determine, and are therefore ultimately in the nature of unknowns? We shall see ... below how this method of ‘intermediate data’ has its basis in the distinction, implicit in the application of the notion of surplus to a market economy, between two fields of inquiry and the corresponding different methods of analysis. On the one hand, we have the necessary quantitative relations, which competition entails between commodity prices and distributive variables and, which, in their comparative simplicity, are of a nature allowing for a mainly deductive treatment. On the other, we have the circumstances determining what we have described as the

\textsuperscript{15} For further details see Pasinetti (1981) and Bellino (2011).
\textsuperscript{16} It could also be argued that goals (N1) and (N2) are both fulfilled by competition only (without the need to invoke the Keynesian principle of effective demand): the gravitation process envisaged by classical economists presupposes a joint movement of prices and quantities towards their normal position.
\textsuperscript{17} An interesting perspective through which to penetrate the meaning of the natural configuration comes from the fact that the same commodity receives normally a different price according to the vertically hyper-integrated subsystem which uses it as a means of production. This result, which regards the general case—not considered here—where capital goods are produced by capital goods and labour, emerges clearly in Pasinetti (1988, §3). The apparent oddness of this result disappears if one recognizes that while the uniformity of the rate of profit as well as the law of one price are typical of competitive systems, in the natural configuration prices measure the difficulty of reproduction of the various commodities. Within the vertically hyper-integrated subsystem this difficulty will be suitably modified according to the diversified structure of the growth rates of the final demand of each specific commodity. This diversification is at the basis of the different pricing of the same commodity.
‘intermediate data’: the subsistence or, more generally, the wage, the outputs, the technical conditions of production. These circumstances were seen to be closely related to institutional and historical factors, which, because of their complexity and variability according to circumstances, prevented deducing the corresponding variables from a few basic principles as was possible for prices and profits in the ‘core’. Those intermediate data rather required, for their study, methods of a more inductive kind. This distinction, concerning both contents and methods, which underlies the notion of surplus, appears to be what has entailed the separation between the two fields of analysis and the corresponding logical construct of the ‘intermediate data’ (Garegnani, 2007, p. 186).

Pasinetti writes:

In the previous chapters, the analysis has been concentrated on the theoretical scheme of what has been called the ‘natural’ economic system of a simple [...] production economy. The natural economic system represents so to speak the framework skeleton of the present theoretical construction. It is a set of relations that possess characteristics of analytical relevance and logical consistency, with strong normative properties.

But the natural economic system does not come into existence automatically. For any actual economic system, the problem arises of inventing and setting up those organizational devices – in other words those ‘institutions’ – which put into motion processes actually able to bring the natural economic system into existence. [...] The institutional problem does not need to have a unique solution, nor does it emerge once for all. By being a problem of construction of organizational devices (the institutions) in order to achieve certain results (the natural economic system), it is obviously susceptible of being faced in different ways, from place to place, from time to time, and at the variation of many external circumstances, without mentioning that the organizational field is itself subject to continuous evolution and innovation.

Moreover, an economic system does not come about in a vacuum. It presupposes a complex network of political, juridical, and legal institutions. These institutions may have been shaped through different historical processes or according to different traditions in different countries, sometimes with even stronger requirements than those behind economic institutions. With this wider institutional framework, the economic institutions must merge and intermingle, while carrying out the task entrusted to them (Pasinetti, 1993, pp. 117-8).

In both authors we observe a ‘separation’ of the analysis into a more restricted theoretical level of relations (the ‘core’ and the ‘natural system’) and a second level which includes in both cases the institutional level. This ‘separation’ does not overlap for the two authors. Pasinetti’s natural system is a theoretical construction that includes the essential relations among sectoral outputs, relative prices, employment and wages, profits and interest that must be satisfied in an economic system in order to exploit at best its potential, independently of the institutional set-up to enforce these relations. Garegnani’s core is a subset of relations among the same set of magnitudes in the case of a specific institutional set-up: capitalism.
The purposes of these logical constructions are different; they are not alternative but complementary one another: the core is a descriptive device, the natural system has a normative purpose. Yet, there are two fundamental characteristics that are common to both the approaches:

1) the study of the relations among the magnitudes belonging to the ‘core’ as well as those belonging to the natural system are carried out, in both cases, by ‘necessary quantitative relations’ (i.e. equations), while the relations outside of the core as well as those concerning the institutional system are better studied in a separated way, in connection with social, historical, political disciplines;

2) all parameters considered as given by Sraffa and Garegnani (see the previous quotation from Sraffa, 1960, p. 6) are supposed to change in Pasinetti’s model. But, there is a peculiarity, that avoids that these changes let enter by the back door what was left out of the front door: the co-determination of prices and quantities and, consequently, the return to a ‘mechanical’ determination of income distribution based on supply and demand forces. The return is prevented by the choice to assume that the rates of change of labour inputs, of final demand and of population ($n$, $\rho_c$, $\rho_j$, and $r_c$) are independent of any other endogenous variable of the model. This formal independence is just a way to preserve the theoretical choice to provide a separate determination of some variables, which appear more subject to the influence of institutional factors. The methodology of ‘given quantities’ and ‘given technology’ is thus now extended to the case of an evolving economy: final consumption and technical coefficients are assumed to evolve according to a given pattern of evolution. This is a methodological
aspect induced by a specific theoretical need. On this point Pasinetti highlights that

The [...] innovative methodological line of research consists in separating sharply the distinction between variables and constants from the distinction between unknowns and data. In traditional economic analysis these two distinctions tend to coincide because of the essentially static approach which is adopted: those magnitudes which are considered as unknowns are also considered as variables, and those magnitudes that are considered as data are also considered as constants. But in a dynamic context, to insist upon this coincidence makes no sense. Or rather, to insist on this coincidence is equivalent to frustrating the purpose of any investigation into dynamics (Pasinetti, 1993, p. 11).

In solving a particular problem there is thus no contradiction in considering as given some magnitudes, even though they may change as time goes by—as in Pasinetti’s structural change model—or they may be affected by changes of the endogenous variables they concur to determine—as in the core of the surplus approach: this kind of feedbacks can be disregarded when there are reasons to think that they are non-univocal, non-systematic or non-persistent.

A common methodology is thus adopted to analyse different aspects: the description of the workings of capitalist economies and the design an ‘ideal’ configuration to be considered as a norm to tend to.

5. Nine main characteristics of classical-Keynesian analysis

In a recent work of his Pasinetti has listed what he considers the main features of the entire ‘Cambridge Keynesian School’ (see Pasinetti, 2007, 217-37). I report here the list for convenience:

1. Reality (and not simply abstract rationality) as the starting point of economic theory.
2. Economic logic with internal consistency (and not only formal rigour).
3. Malthus and the Classics (not Walras and the Marginalists) as the major inspiring source in the history of economic thought.
4. Non-ergodic (in place of stationary, timeless) economic systems.
5. Causality vs. interdependence.
7. Disequilibrium and instability (not equilibrium) as the normal state of the industrial economies.

8. Necessity of finding an appropriate analytical framework for dealing with technical change and economic growth.

9. A strong, deeply felt social concern.

It is evident how the overwhelming majority of these characteristics is shared in deep by the group of scholars leaded by Pierangelo Garegnani that developed and systematized the modern version of surplus approach.

1. Reality: the pure logic which supports the Sraffa framework is never conceived as an exercise for its own sake; Sraffa’s idiosyncrasy against mathematical formalism are known to any of his scholars.\(^{18}\) Additionally, the interpretation of Sraffa prices as the ‘normal positions’ of the economy is a clear attempt to establish a correspondence between theoretical and observable variables (on this, see Garegnani, 2007).

2. Internal consistency. Self-evident!

3. Classics, not the Marginalists. Self-evident!

4. Non-ergodicity. The interpretation of Sraffa’s framework as a ‘steady state’ has been rejected in several occasions: for example, the assumption of uniformity of the rate of profit describes the normal position around which actual (marker) prices ‘gravitate’. This does not exclude the possibility to conceive a dynamics of normal positions (on this see, for example, Cesariatto, 1995).

5. Causality. In several occasions in these pages we have seen how specific features of the surplus approach have been obtained by a suitable choice of asymmetrical links among the relevant magnitudes. For further details on this point see Bellino-Nerozzi (2014).

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\(^{18}\) A simple but meaningful example is the way Sraffa handled the case of self-reproducing non-basics having a physical rate of surplus lower than the average rate of the other (basic) commodities. Newman (1962) was inclined to consider this case as symmetrical to the case of a self-reproducing non-basic commodity with a physical rate of surplus higher than the average. Sraffa, in his reply, argued how the former situation can be regarded as exceptional, on the basis of ‘reality’ arguments (for the entire exchange between Sraffa and Newman see Sraffa, 1970).
6. Macroeconomics before microeconomics. The distinction between macro- and micro-economics is extraneous to the classical tradition. Pasinetti’s intention, however, is that of recalling that

[...] the Cambridge economists caught very clearly the principle that the behaviour of the economic system as a whole is not reducible to, in the sense that it does not emerge as the exclusive result of, the sum of its single individual parts [...] there are many examples of fallacy of composition that the Cambridge School have highlighted, as against the attempts to extend what is true for the single individual to the behaviour of the economic system as a whole (Pasinetti, 2007, pp. 227-8).

The works of Garegnani’s and of other surplus theorists evidently fulfil this characteristic.

7-8. Disequilibrium and instability + technical change and economic growth. These are, probably, two points where the methodology of modern surplus theorists diverges but – I argue – only partially from that of the other members of the ‘Cambridge Keynesian School’. It is undeniable the preference of surplus theorists for the Marshallian ‘short chains of reasoning’, to investigate “economic change step by step” (Cesaratto, 1995, p. 274). On the contrary Pasinetti, on the basis of Frisch’s (1935-36) notion of ‘moving equilibrium’19, has studied how an equilibrium configuration changes as a consequence of a change in one or more parameters. As we saw (see Section 3 above) the relevant parameters (population, technology and final demand) are supposed to change as time goes. The possibility to slip into a model of full interdependence (like a Walrasian model) where all variables are mechanically determined is avoided by a specification of the dynamics of these parameters on the basis of a given pattern, which replicates in a reasonable way long-term historical tendencies—like the supposition of reducing labour input coefficients due to technical change, or an evolution of final demand coefficients as that described by Engel—rather than by a ‘microfoundation’ of the variations of the parameters.

9. Social concern. Self evident!

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19 I owe this reference to Ariel Wirkierman: Pasinetti quotes Frisch’s paper only in his mathematical formulation of Ricardo’s system (Pasinetti, 1960, p. 84, fn. 3).
The fundamental characteristics of the ‘Cambridge Keynesian School’ seem thus to be substantially satisfied by the surplus approach theory.

6. Some concluding remarks

Modern classical analysis has often been perceived as a school which has been compact in its critical attitude towards neoclassical economics. Yet, on the positive side, the positions appear quite un-homogeneous and discordant. The effort undertaken in these page has been to constructively collect and combine those elements, which are at the basis of the research pursued by Pierangelo Garegnani and Luigi Pasinetti, who undoubtedly are the most authoritative exponents of this school. The analysis shows that the elements at the basis of the surplus approach as described by Garegnani are not contradictory with the analysis of structural change of industrial systems developed by Pasinetti. Clearly, Pasinetti adopts a perspective which in some way is pre-institutional. He is interested in describing the conditions that an industrial system, undergoing a process of structural change, must satisfy in order to accomplish its potential concerning growth, employment and the satisfaction of final wants. Furthermore, the logical separation, emphasized by Garegnani, of the analysis of prices and distribution from that the quantities produced is preserved in Pasinetti’s analysis in the choice to formulate the evolution of final demand (and of technology) independently of the price system.

We have thus not two (or more) ways to develop Sraffa’s framework, but a unified ‘core’ to describe the economic workings of capitalist societies and to orientate the system towards its ideal ‘natural’ configuration.

Recently Pasinetti expressed his concern with a negative attitude developed by the members of Cambridge school—which he called “Cambridge *prima donna* syndrome”—to disregard, or to ignore the works of the others members, too worried to emphasize the peculiarities of their own contributions (see Pasinetti, 2007, p. 46, fn. 18). Without disregarding the differences and the specificities of the two approaches here presented, I hope that my present work may help to converge the research undertaken towards a common *modern classical-Keynesian school*. 
Appendix – The relations with Keynes’ analysis

It seems here the right place where to hint at a further issue, which has often seemed to be an obstacle to the integration of the approaches here discussed: the rôle given to Keynes’s analysis. On the one hand the Keynesian principle of effective demand is recognized by all members of the modern classical approach: Pasinetti devoted an entire essay to explain it in its ‘pure’ version, i.e. not contaminated by the Walrasian interpretation based contained in the Neoclassical synthesis (see Pasinetti, 1974, Essay II), while Garegnani based upon it a theoretical and empirical research concerning the Italian post-war system (see Garegnani, 1962\(^20\)). There are however two aspects that see the surplus approach scholars on one side of the debate and the post-Keynesian growth theorists on the other side.

The first aspect concerns the theory which is behind Keynes’ marginal efficiency of capital curve. Garegnani underlines how this curve is deeply rooted in the theory of marginal productivity:

> However, the price which Keynes has to pay for the traditional strand in his thought becomes clear with respect to the schedule of the marginal efficiency of capital (Garegnani, 1964-65, part II, p. 78),

and this link has significantly reduced the importance of Keynes’ critique to the traditional theory of employment:

> The critique of the traditional theory of interest becomes then the key to an acceptance of Keynes’ arguments—and the concept of the marginal efficiency of capital proves to be the Achille’s heel of that very critique (Garegnani, 1964-65, part II, p. 78)

Most likely, the critique would have been more effective if it had been paired with one of the fundamental results of the capital debates of the Sixties, i.e. the non-existence, in general, of a monotonic and inverse relation between the rate of profit and the capital-labour ratio. This result would have contributed to disprove the misleading idea that Keynes results were essentially due to rigidities and that a suitable flexibility of price factors (wages and profits) is sufficient to restore full-employment.

On the other hand Pasinetti is more possibilist and maintains that:

> the marginal-efficiency-of-capital schedule, which might, at a first superficial look, appear as belonging to the marginal economic analysis, when examined more

\(^{20}\)The theoretical part of this work has been published in Italian in Garegnani (1964-65).
deeply turns out to have a rather different origin. Keynes’ ranking of all investment projects in a decreasing order of profitability is more akin to Ricardo’s ranking of all lands in a decreasing order of fertility than to any marginal economic elaboration. And in any case, there is absolutely no need to consider Keynes’s marginal-efficiency-of-capital schedule as an expression of the marginal productivity theory of capital (Pasinetti, 1974, p. 43, emphasis added).

More recently Pasinetti returned on the issue specifying his position:

Keynes was not able, or was not in time, to take advantage of Sraffa’s ongoing critical elaborations. But we are in a position now to state the results of the critique of the neoclassical production function, which would have been needed to debunk the demand-for-investment side of the orthodox theory. [...] The conclusions [of the reswitching of techniques controversy] are strictly logical and devastating. The downward-sloping investment-demand function, to the extent that it relies on a continuous process of substitution of capital for labour, as the rate of interest falls, is theoretically unsound; it has no logical foundations. [...] The ‘reswitching’ result only means that, if such a downward-sloping relation exists, it cannot be explained by a process of substitution of capital for labour (i.e. by a neoclassical production function); it cannot be explained by more and more capital-intensive techniques as the rate of interest falls. Such a relation, if it exists, must be explained by something else – by some other theory or circumstance.

It is to this effect that we must logically search for a meaning (non-orthodox meaning) of Keynes’s notion of the ‘marginal efficiency of capital’ (Pasinetti, 1997, pp. 203-4, emphasis in the original).

As we can see the interpretative disagreement on this point does not undermine the acceptance of the principle of effective demand.

The second disagreement concerns the result entailed by the Cambridge equation; the critical position was expressed in particular by Pierangelo Garegnani and Ferdinando Vianello, which argued against the compulsory negative relationship that the Cambridge equation establishes between the growth rate and the real wage rate. They aim to break the idea that a higher growth rate entails a lower real wage. An explicit statement of this idea can be found, for example, in Kaldor:

[t]he theory thus serves to explain the long-observed fact [...] that distributive shares are constant over long periods whilst they fluctuate over shorter periods [...] as well as the fact that in fast-growing economies the share of profits is generally appreciably greater than in economies which grow at a relatively slow rate (Kaldor, 1966, p. 561, emphasis added).

Marglin is still more explicit:

[i]n the short run, fluctuations in investment demand are reflected in fluctuations in output; the rate of capacity utilization changes in accordance with aggregate demand. The distributional conflict between capitalists and workers is, as it were, a non-zero-sum game. [...] But in the long run, the period with which neo-Keynesian analysis concerns itself, there is no excess capacity to accommodate investment
demand. Distribution must bear the brunt of adjusting aggregate demand to supply. In contrast with the short period, the long-run conflict is a zero-sum game—at least in the absence of technological substitution or technological change (Marglin, 1984, pp. 474-5).

The surplus approach school envisages the additional resources for accumulation in the variations of the rate of capacity utilization: this is possible because the productive capacity of firms is never fully utilized, even in the long run (entrepreneurs prefer to leave a margin of available productive capacity, to face unexpected peaks in demand). Hence, a higher rate of accumulation does not require higher profits (and lower wages) as long as the degree of utilization of productive capacity can be increased: the emphasis is thus placed on the forces of demand for their activating power in creating new and permanent increments of income along purely Keynesian lines (see Vianello, 1985, 1996 and Garegnani, 1992).

Undoubtedly this is a point of view in opposition to that expressed by Kaldor and, mainly, by Pasinetti. Nevertheless, there are no apparent reasons which prevent the integration of the possibility to vary the degree of utilization of productive capacity into the post-Keynesian theories of income distribution. Moreover, the normative meaning given by Pasinetti to the Cambridge equation allows one to read it along two perspectives: i) it sets a minimal level under which the profit rate cannot fall, i.e. a maximum level that the wage rate cannot exceed if the economic system has to grow at a given rate; but, at the same time, ii) it identifies a reference level for the rate of profit: it identifies a threshold level to evaluate when a rate of profit is no more justifiable on the basis of the accumulation needs of the system.

References


Duménil G. and Lévy D (1993), The Economics of the Profit Rate: Competition, Crises and Historical Tendencies in Capitalism, Edward Elgar, Aldershot.


