Munich Personal RePEc Archive

On the number of licenses under symmetric versus asymmetric information with signaling

Antelo, Manel and Sampayo, Antonio

University of Santiago de Compostela

2014

Online at https://mpra.ub.uni-muenchen.de/60759/
MPRA Paper No. 60759, posted 19 Dec 2014 18:44 UTC

On the number of licenses under symmetric versus asymmetric information with signaling*

Manel Antelo† and Antonio Sampayo‡

December, 2014

Abstract

We analyze a two-period licensing game in which a non-producer upstream patent holder licenses an innovation to either one or two downstream licensees for a payment based on the licensee's expected per-period profit. Licensees have private information about the innovation's value, and their period-1 output may signal that value. We find that two licensees are more likely to be preferred under asymmetric information with signaling than under symmetric information.

Keywords: Licensing, symmetric and asymmetric information, profit-based payments, monopoly, duopoly

JEL Classification: D45

---

* The authors gratefully acknowledge financial support from Xunta de Galicia research program GPC2013-045 partly funded the European Regional Development Fund (ERDF). The second author also acknowledges the aid received from the Spanish Ministry of Economy and Competitiveness research program ECO2013-48884-C3-1-P.

† Departamento de Fundamentos da Analise Economica, Universidade de Santiago de Compostela, Spain. E-mail: manel.antelo@usc.es

‡ Departamento de Fundamentos da Analise Economica, Universidade de Santiago de Compostela, Spain. Corresponding author. Address: Departamento de Fundamentos da Analise Economica. Facultade de CC. Economicas e Empresariais. Avda. Burgo das Nacións, s/n, 15782, Santiago de Compostela (Spain). E-mail: ar.sampayo@usc.es

1. Introduction

Consider a non-producer upstream patent holder that is obliged to sell licenses of an innovation (to one or several downstream firms). The literature has shown that, if the downstream firms have private information about the market value of the innovation, then more licenses may be sold by the patent holder than when the upstream patent holder knows the downstream firms' types; see, for instance, Schmitz (2002, 2007). We argue in this paper that a related result can be obtained within a signaling framework.

We build a two-period signaling game of licensing in which each potential user of an innovation owned by the patent holder has private information about the effect of the innovation on production costs (which can take one of two possible values, high or low), and where licensees can use their period-1 production as a signal. If two licenses are granted then the downstream firms engage in Cournot competition. We also assume that payments for each license are formed by the per-period expected profit of each firm.

Under incomplete but symmetric information about how the cost of production of each licensee is affected by the innovation, the existence of a monopoly (or a duopoly) in the downstream market depends on the trade-off between two well-known sampling and rent dissipation effects. On one hand, issuing more licenses makes it more likely that there will be a low-cost licensee; on the other hand, more licenses reduce the licensor's expected profits by increasing market competition. Which effect dominates depends on (i) the difference between high- and low cost of production and (ii) the probability of licensees being low-cost type.

Here we show that, under asymmetric information, there is also a signaling effect giving informational rents to the downstream firms. In order to minimize the fee paid to the patent holder in period 2, a single downstream user would signal itself to the (upstream) patent holder as a high-cost producer by reducing its first-period output. When there are two users, however, Cournot competition also induces a "horizontal" incentive in each licensee to signal a low production cost to the rival. We find that, in the Bayesian separating equilibrium of least cost, competition (i.e., the issuing of two licenses) in the downstream market occurs within a range of parameters—the difference between high and low realization of production costs and the probability of being a low-cost user—for which only one license would be issued under symmetric information. Competition in the downstream market allows the upstream patent holder to reduce in period 1 the informational rents paid to the licensees.

Licensing under asymmetric information has been addressed extensively in the literature. However, the focus is usually on the relative merits of various contract arrangements (Beggs, 1992; Kamien, 1992; Antelo, 2013); the effects of adverse selection and signaling on the number of licenses granted are generally overlooked. Schmitz (2002) builds an adverse selection model,[1] where the licensee is better informed (than is the licensor) about the innovation's value, and examines the consequences of this asymmetric information for the number of licenses granted. He shows that, in some states of the world, an efficient mechanism involves granting two licenses instead of only one. Our results complement this finding in that we consider a signaling framework where the licensee(s) can use period-1 production level to signal innovation value as a means to minimizing the period-2 fee. We find that, compared with the case of incomplete but symmetric information, the patent holder under asymmetric information is more likely to prefer the competition of two licensees in order to countervail such opportunistic behavior on their part.

We remark that, in Schmitz's framework, more licenses need not imply more competition in the downstream market because there is always the possibility that a licensee will fail to develop the innovation into a new product. Absent that possibility, the patent holder has no incentive to grant more than one license. Our approach differs from that of Schmitz's because we incorporate signaling and assume that each licensee is a successful producer.

The rest of the paper is structured as follows. Section 1 describes the model while Section 2 presents the main results under symmetric and asymmetric information. Section 4 concludes and proofs are relegated to the Appendix.

2. The model

We take the case of an upstream patent holder licensing an invention that will become worthless after two production periods. Having no production capacity, the patent holder is obliged to license the invention to users capable of using it to manufacture a product. The market demand for such a product in each period $t\,(t = 1, 2)$ is public information and is given by

$$p_t = \begin{cases} 1 - q_t & \text{if } q_t < 1, \\ 0 & \text{if } q_t \geq 1; \end{cases} \tag{1}$$

[1] Schmitz (2007) undertakes a similar analysis in a moral hazard setting.

where q_t denotes total output produced by one or two firms $(q_t = q_t^A + q_t^B)$ depending on the number of licenses granted by the patent holder. In the case of two licenses, the licensees compete à la Cournot.

The marginal cost of production of a licensee, or the innovation's value, is known only to the licensee; other players (i.e., the patent holder and perhaps a second licensee) have only a prior assessment of such cost or value. In particular, the marginal cost of production is represented by an independent random variable $\tilde{c}$ such that

$$\tilde{c} = \begin{cases} 0 & \text{with probability } \mu, \\ c & \text{with probability } 1-\mu, \end{cases} \tag{2}$$

where $0 < \mu < 1$.[2] Throughout the paper we assume that $0 < c < 0.5$, so that the existence of an equilibrium with positive production by both users is always possible irrespective of their cost type and any party's beliefs about those costs. Finally, we assume that all players are risk neutral and there is no factor discount on profits achieved by each user.

3. Results

3.1. Symmetric information

If the value of the innovation is uncertain but information about that value is symmetric at the licensing game's beginning, then the optimal allocation of the innovation can be formalized as follows.

Proposition 1. *Under symmetric information, it is optimal for the patent holder to grant one license if parameters* μ *and* c *satisfy* $1-(1-\mu)c[2-(1-32\mu)c]>0$. *Otherwise, the patent holder should grant two licenses (i.e., enable creation of a duopoly).*

Proof. See Appendix.

Proposition 1 is based on the trade-off between two effects that arise in the licensing process: the rent dissipation effect and the sampling effect. Under the rent dissipation effect, more licenses reduce each licensee's benefits and hence the patent holder's income. Moreover, this effect increases as the production costs of individual licensees converge because market

[2] The same results would follow if we assumed the uncertainty to lie rather in the size of market demand.

competition is then even fiercer. The sampling effect refers to the increased likelihood of there being low-cost users—and thus higher profits for licensee and patent holder both, under our assumed payment scheme—when more licenses are granted.

3.2. Asymmetric information and signaling

We now assume that each licensee has private information about its own production cost, which is known with certainty.[3] Prior to period-1 production, both the patent holder and the competitor (if any) know only that this production cost is a random variable given by equation (2). These parties can also observe, at the end of period 1, the licensee's production in period 1 and then use that information (and Bayes' rule) to update their beliefs about the production cost. As in Antelo (2013), the patent holder offers one-period contracts to either one or two licensees at the beginning of period 1 in exchange for a payment based on expected profits. After observing the period-1 production of each licensee, the patent holder then offers another contract (at the start of period 2) in exchange for a different payment. Note that it is suboptimal for the patent holder to offer a two-period contract at the start of period 1, since that would preclude using first-period production as a signal to identify licensee type.

Because the patent holder cannot distinguish among licensee types before observing the period-1 production, payment under the first licensing agreement is not a function of type. After observing the first-period production, however, the patent holder can craft a second-period licensing agreement whose payment stipulations reflect the production data. This setup leads to a signaling game in which users (the licensees) have incentives to behave opportunistically.

We show in Lemma 1 that a single user has a vertical incentive to be perceived as inefficient by the patent holder in order to pay less for the renewal of the license in period 2; hence also a truly inefficient producer has an incentive to signal its high production cost.

Lemma 1. *If the patent holder grants a single license, then in the separating equilibrium of least cost the inefficient user's first-period production is lower than it would be if information were symmetric in order to signal that the innovation is of low value.*

[3] This assumption is intended to capture a common situation: the patent holder remains largely unaware of the innovation's downstream development; the licensee, with more experience in the field, is better positioned to recognize the innovation's true value.

Proof. See Appendix.

In order to be perceived as inefficient by the patent holder, an inefficient producer needs to produce less than it would under symmetric information in order to guarantee that an efficient producer does not find it profitable to behave likewise. Of course, the first-period informational rents of an inefficient producer are zero. Yet those of an efficient producer are positive, and they are higher the lower is the inefficient firm's production.

Our next lemma proves that the patent holder can counteract licensees' opportunistic behavior (i.e., informational rents of efficient users) by granting two licenses, although doing so does not eliminate the inefficient producer's incentive to signal its (high) costs.

Lemma 2. *If the patent holder grants two licenses, then the following statements hold.*

(i) The incentive of each user to be perceived (by the patent holder) as an inefficient user remains but is less than under the single-license scenario.
(ii) There is a separating equilibrium of least cost where period-1 production for each inefficient user is lower than under symmetric information.

Proof. See Appendix.

Lemmas 1 and 2 are used to prove Proposition 2, which—in combination with Proposition 1—allows us to claim that the patent holder prefers granting two licenses in some regions of the (μ, c)-space where a single license would be sold under symmetric information.

Proposition 2. *In the separating equilibrium of least cost, it is optimal for the patent holder to grant two licenses if* $E\pi_{DD}^{\text{PH}} \geq E\pi_{MM}^{\text{PH}}$, *where* $E\pi_{DD}^{\text{PH}}$ *and* $E\pi_{MM}^{\text{PH}}$ *are as given in Appendix by equations* (A.27) *and* (A.29), *respectively. Otherwise, a single license is granted.*

Proof. See Appendix.

Proposition 2 shows that competition between license users leads to an increase in period-1 production of each licensee in order to increase its period-2 market share; as a result competition becomes more attractive to the patent holder. On one hand, when the probability

μ of having efficient users is high enough, vertical incentives for efficient users to be seen as inefficient are low because the license fee tends to be high in any case. Furthermore, vertical incentives for inefficient users to signal their inefficiency are increasing in μ. Therefore, if the probability of efficient users is high then the patent holder has less need to discourage efficiency misrepresentation and so is less likely to prefer a duopoly. On the other hand, the lower is the production cost c, the greater is the difference between monopoly and duopoly profits; this effect reduces the advantages of a duopoly from the patent holder's perspective.

A comparison of Propositions 1 and 2 leads to the following result.

Proposition 3. *Granting two licenses is more likely to occur under asymmetric information than under symmetric information.*

Figure 1 illustrates this finding. The plots reveal that, overall, asymmetric information widens the range of parameters for which two licenses result in equilibrium.

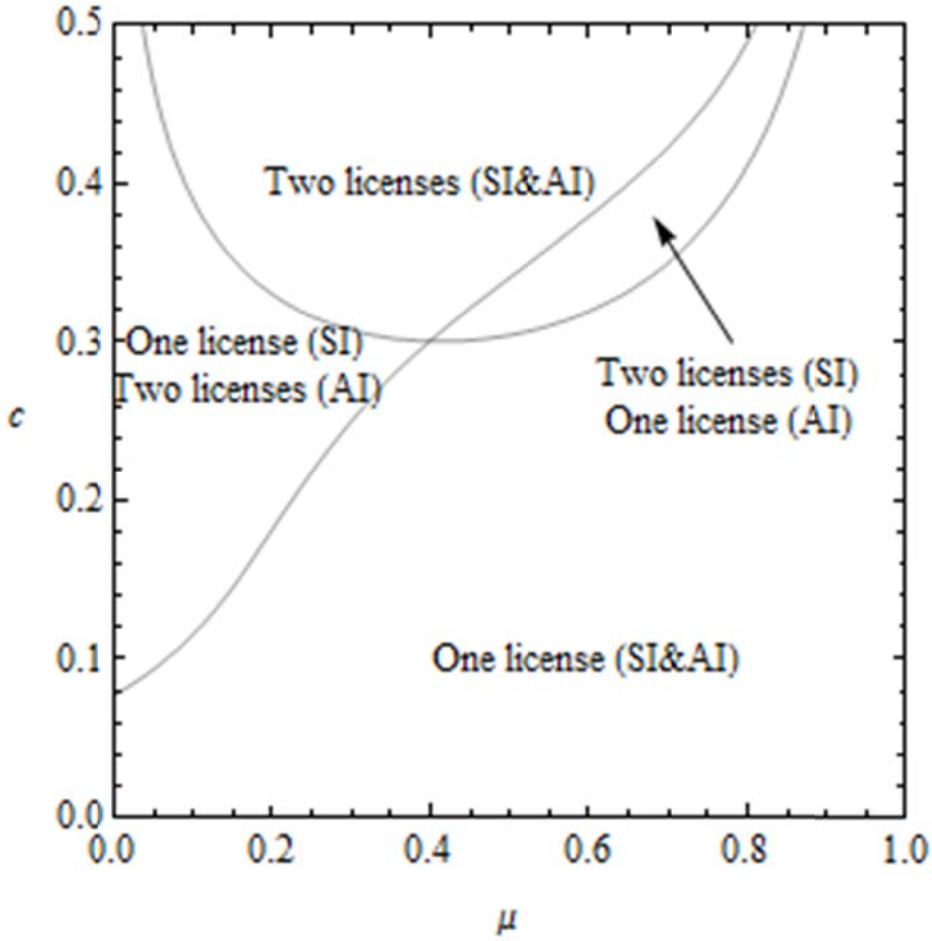

Figure 1. Number of licenses when information is symmetric (SI) and asymmetric (AI).

Conclusions

For a licensing framework in which each licensee has private information about the value of an innovation to be used for production, we have shown that competition between licensees may serve to alleviate the effects of their opportunistic behavior. We compute a separating equilibrium with signaling where, for a certain range of parameters, the upstream patent holder sells two licenses under asymmetric information but—with the same parameters—only a single license under symmetric information. In this case, competition serves to reduce the informational rents that the patent holder is obliged to grant to efficient users to induce downstream producers to reveal their private information. And this reduction on informational rents, added to the sampling effect, outweighs the dissipation effect of competition.

Appendix: Proofs

Proof of Proposition 1. The monopolist's per-period level of profit-maximizing production in the downstream market is

$$q_t = \begin{cases} \frac{1}{2} & \text{if } \tilde{c} = 0, \\ \frac{1-c}{2} & \text{if } \tilde{c} = c. \end{cases} \quad \text{(A.1)}$$

Therefore, the patent holder's expected profits (i.e., the license fee) from a single two-period license are equal to the licensee's expected profits:

$$E\pi_{MM}^{\mathrm{PH}} = \frac{1}{2}\big(1 - (1-\mu)c(2-c)\big). \quad \text{(A.2)}$$

If the patent holder instead grants two licenses lasting two periods each, then the result is a duopoly in the downstream market and the two licensees engage in Cournot competition. The profit-maximizing production level for each firm $i, j = A, B$ in period $t = 1, 2$ is

$$q_{it} = \begin{cases} \frac{1}{3} & \text{if } (\tilde{c}_i, \tilde{c}_j) = (0,0), \\ \frac{1+c}{3} & \text{if } (\tilde{c}_i, \tilde{c}_j) = (0,c), \\ \frac{1-2c}{3} & \text{if } (\tilde{c}_i, \tilde{c}_j) = (c,0), \\ \frac{1-c}{3} & \text{if } (\tilde{c}_i, \tilde{c}_j) = (c,c), \end{cases} \quad \text{(A.3)}$$

and the patent holder's two-period expected profits are

$$E\pi_{DD}^{PH} = \frac{4}{9}(1 - (1-\mu)c[2 - (1+4\mu)c]). \quad \text{(A.4)}$$

The statement of the proposition follows after comparing equations (A.2) and (A.4).

Proof of Lemma 1. Let $v_2^L(\tilde{c})$ denote the licensee's second-period monopoly profits in the downstream market, where $\tilde{c} \in \{0, c\}$. Given the per-period profit-maximizing quantities stipulated in (A.1), we have

$$v_2^L(\tilde{c}) = \begin{cases} \frac{1}{4} & \text{if } \tilde{c} = 0, \\ \frac{(1-c)^2}{4} & \text{if } \tilde{c} = c. \end{cases} \quad \text{(A.5)}$$

Let $\pi_2^L(\tilde{c}/\tilde{c}^b) = v_2^L(\tilde{c}) - f_2(\tilde{c}^b)$ denote second-period monopoly profits net of license fee, where $v_2^L(\tilde{c})$ is given by (A.5), $f_2(\tilde{c}^b) = v_2^L(\tilde{c}^b)$ is the license fee, and $\tilde{c}, \tilde{c}^b \in \{0, c\}$ are, respectively, licensee's production costs and patent holder beliefs (b) about them. These profits are given by

$$\pi_2^L(\tilde{c}/\tilde{c}^b) = \begin{cases} \frac{c(2-c)}{4} & \text{if } \tilde{c} = 0 \text{ and } \tilde{c}^b = c, \\ 0 & \text{if } \tilde{c} = 0 \text{ and } \tilde{c}^b = 0, \\ 0 & \text{if } \tilde{c} = c \text{ and } \tilde{c}^b = c, \\ -\frac{c(2-c)}{4} & \text{if } \tilde{c} = c \text{ and } \tilde{c}^b = 0. \end{cases} \tag{A.6}$$

From (A.6) it is clear that $\pi_2^L(\tilde{c}/c) \geq \pi_2^L(\tilde{c}/0)$ for all $\tilde{c} \in \{0, c\}$, which means that licensees would prefer being perceived as inefficient. In a separating equilibrium, the efficient licensee is charged the highest possible fee in the second period. A costly signal of her low cost is therefore of no use to her and so she produces the profit-maximizing quantity in every period, $q_1(0) = q_2(0) = \frac{1}{2}$; thus $v_2^L(0) = \frac{1}{4}$ and $\pi_2^L(0/0) = 0$ (for ease of exposition, we refer to the efficient and inefficient licensees as "she" and "he", respectively). The inefficient licensee can pay a lower fee if he uses his first-period production, $q_1(c)$, to signal low efficiency. This licensee would then be charged a fee that allows him to earn zero net profits also in the second period, $\pi_2^L(c/c) = 0$. For there to be an equilibrium, each licensee's incentive compatibility constraint must be verified. The inefficient producer's profits computed over two periods under signaling must be no less than his profits from paying the fee that an efficient licensee is charged and sending no signal—that is, from producing profit-maximizing quantities in both periods:

$$\left(1 - c - q_1(c)\right)q_1(c) + \pi_2^L(c/c) \geq \frac{(1-c)^2}{4} - \frac{c(2-c)}{4}. \tag{A.7}$$

For the efficient licensee, profits from sending no signal must be no less than paying the fee that an inefficient licensee is charged and sending a signal:

$$\frac{1}{4} + \pi_2^L(0/0) \geq \left(1 - q_1(c)\right)q_1(c) + \frac{c(2-c)}{4}. \tag{A.8}$$

In a separating equilibrium with signaling, the patent holder can charge different fees to each licensee that match the corresponding benefits, from which it follows that $\pi_2^L(0/0) = \pi_2^L(c/c) = 0$. The value of $q_1(c)$ that maximizes the inefficient licensee's profits and verifies constraints (A.7) and (A.8)—the former with strict inequality—is $q_1^s(c) = \frac{1}{2} - \frac{\sqrt{c(2-c)}}{2}$. Hence the efficient licensee produces $q_1(0) = \frac{1}{2}$, which coincides with her production under complete information; the inefficient licensee produces a lower quantity than under complete information, $q_1^s(c) = \frac{1}{2} - \frac{\sqrt{c(2-c)}}{2} < \frac{1-c}{2}$, to signal his high production cost.

Proof of Lemma 2. The proof proceeds in two steps. We first show that licensees' incentives to be perceived as inefficient by the patent holder dominate their incentives to be seen as efficient by a rival. Then we prove the existence of a separating equilibrium with signaling.

Step 1. Each firm has an incentive to be perceived as efficient by its rival. We can use $v_2^i(\tilde{c}_i, \tilde{c}_j)$ to denote firm i's profit in period 2; here $i, j = A, B$ when the two firms compete à la Cournot in the downstream market and $\tilde{c}_i, \tilde{c}_j \in \{0, c\}$ denote the production costs for firm i and firm j (respectively), which are common knowledge. Then

$$v_2^i(\tilde{c}_i, \tilde{c}_j) = \begin{cases} \frac{1}{9} & \text{if } \tilde{c}_i = 0 \text{ and } \tilde{c}_j = 0, \\ \frac{(1+c)^2}{9} & \text{if } \tilde{c}_i = 0 \text{ and } \tilde{c}_j = c, \\ \frac{(1-2c)^2}{9} & \text{if } \tilde{c}_i = c \text{ and } \tilde{c}_j = 0, \\ -\frac{(1-c)^2}{9} & \text{if } \tilde{c}_i = c \text{ and } \tilde{c}_j = c. \end{cases} \tag{A.9}$$

We shall now denote firm i's profit in period 2 as $v_2^i(\tilde{c}_i/\tilde{c}_j^b, \tilde{c}_j)$, where $\tilde{c}_i$ and $\tilde{c}_j$ are (as before) the production costs for i and j and where $\tilde{c}_j^b \in \{0, c\}$ denotes firm j's beliefs about firm i's production costs. When the rival's beliefs are not correct, it is easy to check that $v_2^i(\tilde{c}_i/\tilde{c}_j^b, \tilde{c}_j)$ is given by

$$v_2^i(\tilde{c}_i/\tilde{c}_j^b, \tilde{c}_j) = \begin{cases} \frac{(2-c)^2}{36} & \text{if } \tilde{c}_i = 0,\ \tilde{c}_j = 0 \text{ and } \tilde{c}_j^b = c, \\ \frac{(2+c)^2}{36} & \text{if } \tilde{c}_i = 0,\ \tilde{c}_j = c \text{ and } \tilde{c}_j^b = c, \\ \frac{(2-3c)^2}{36} & \text{if } \tilde{c}_i = c,\ \tilde{c}_j = 0 \text{ and } \tilde{c}_j^b = 0, \\ \frac{(2-c)^2}{36} & \text{if } \tilde{c}_i = c,\ \tilde{c}_j = c \text{ and } \tilde{c}_j^b = 0. \end{cases} \tag{A.10}$$

If j's beliefs about i's costs are correct then firm i's profits coincide with common knowledge, $v_2^i(\tilde{c}_i/\tilde{c}_j^b = \tilde{c}_i, \tilde{c}_j) = v_2^i(\tilde{c}_i, \tilde{c}_j)$, and are given by (A.9). It can then be easily checked that, for $\tilde{c}_j \in \{0, c\}$, we have $v_2^i(0/c, \tilde{c}_j) < v_2^i(0, \tilde{c}_j)$ and $v_2^i(c/0, \tilde{c}_j) > v_2^i(c, \tilde{c}_j)$; this shows that each firm has horizontal incentives to be perceived as an efficient producer.

Next we show that each firm has an incentive to be perceived as inefficient by the patent holder. Licensees' second-period net profits are given by $\pi_2^i = v_2^i - f_2^i$, where f_2^i denotes the fixed fee that the patent holder charges to firm i in period 2. If there is common knowledge then $f_2^i(\tilde{c}_i, \tilde{c}_j) = v_2^i(\tilde{c}_i, \tilde{c}_j)$, so $\pi_2^i(\tilde{c}_i, \tilde{c}_j) = 0$ for all $\tilde{c}_i, \tilde{c}_j \in \{0, c\}$. In the absence of common knowledge, however, license fees depend on the patent holder's beliefs about licensee production costs. We denote the patent holder's beliefs about firm i's production cost as $\tilde{c}_{\mathrm{PH}}^b \in \{0, c\}$. Then the license fee is given by

$$f_2^i(\tilde{c}_{\mathrm{PH}}^b, \tilde{c}_j) = \begin{cases} \frac{(1-2c)^2}{9} & \text{if } \tilde{c}_{\mathrm{PH}}^b = c \text{ and } \tilde{c}_j = 0, \\ \frac{(1-c)^2}{9} & \text{if } \tilde{c}_{\mathrm{PH}}^b = c \text{ and } \tilde{c}_j = c, \\ \frac{1}{9} & \text{if } \tilde{c}_{\mathrm{PH}}^b = 0 \text{ and } \tilde{c}_j = 0, \\ \frac{(1+c)^2}{9} & \text{if } \tilde{c}_{\mathrm{PH}}^b = 0 \text{ and } \tilde{c}_j = c; \end{cases} \tag{A.11}$$

here we can see that $f_2^i(c, \tilde{c}_j) < f_2^i(0, \tilde{c}_j)$, which means that the licensee prefers to be perceived as inefficient by the patent holder.

Finally, we prove that vertical incentives to be seen as inefficient dominate horizontal incentives to be seen as efficient. Since both the patent holder and the rival have the same information and since both parties update their beliefs according to Bayes' rule, it follows that their beliefs must match: $\tilde{c}_{\mathrm{PH}}^b = \tilde{c}_j^b = \tilde{c}_{-i}^b \in \{0, c\}$. Thus

$$\pi_2^i(\tilde{c}_i / c_{-i}^b, \tilde{c}_j) = v_2^i(\tilde{c}_i / \tilde{c}_{-i}^b, \tilde{c}_j) - f_2^i(\tilde{c}_{-i}^b, \tilde{c}_j).$$

Given this equality, we can combine (A.10) and (A.11) to obtain

$$\pi_2^i(\tilde{c}_i / \tilde{c}_{-i}^b, \tilde{c}_j) = \begin{cases} \frac{c(4-5c)}{12} & \text{if } \tilde{c}_i = 0,\ \tilde{c}_j = 0 \text{ and } \tilde{c}_{-i}^b = c, \\ \frac{c(4-c)}{12} & \text{if } \tilde{c}_i = 0,\ \tilde{c}_j = c \text{ and } \tilde{c}_{-i}^b = c, \\ -\frac{c(4-3c)}{12} & \text{if } \tilde{c}_i = c,\ \tilde{c}_j = 0 \text{ and } \tilde{c}_{-i}^b = 0, \\ -\frac{c(4+c)}{12} & \text{if } \tilde{c}_i = c,\ \tilde{c}_j = c \text{ and } \tilde{c}_{-i}^b = 0. \end{cases} \tag{A.12}$$

In (A.12) it can be checked that $\pi_2^i(0/c, \tilde{c}_j) > \pi_2^i(0, \tilde{c}_j) = 0$ and $\pi_2^i(c/0, \tilde{c}_j) < \pi_2^i(c, \tilde{c}_j) = 0$ for $\tilde{c}_j \in \{0, c\}$. This proves that every licensee prefers being perceived as inefficient because vertical incentives are the dominant ones.

Step 2. Let the index $i_{\tilde{c}} \in \{A_{\tilde{c}}, B_{\tilde{c}}\}$ denote firm i with production cost $\tilde{c} \in \{0, c\}$. Let $q_{1_s}^{i_{\tilde{c}}}$ and $q_1^{i_{\tilde{c}}}$ denote, respectively, the signaling and nonsignaling first-period production of firm $i_{\tilde{c}}$. We shall prove that, in a separating equilibrium, it is optimal for (a) an efficient licensee to pay the high fee and do not send a costly signal and (b) an inefficient licensee to signal his high cost (and thus be charged a lower fee in period 2).

(a) An efficient firm i_0 sending no signal in the first period seeks to solve the following problem:

$$\max_{q_1^{i_0}} Ev_1^{i_0}(0,\tilde{c}_j) = \max_{q_1^{i_0}}\left[1 - q_1^{i_0} - \left(\mu q_1^{j_0} + (1-\mu)q_{1_s}^{j_c}\right)\right]q_1^{i_0}, \qquad \text{(A.13)}$$

where $j_0 \in \{A_0, B_0\}$, $j_c \in \{A_c, B_c\}$. The solution is

$$q_1^{i_0} = \frac{1-(1-\mu)q_{1_s}^{j_c}}{2+\mu}. \qquad \text{(A.14)}$$

(b) An inefficient firm i_c that does not signal high-cost production (assuming that other players do not change their equilibrium behavior) seeks to solve

$$\max_{q_1^{i_c}} Ev_1^{i_c}(c,\tilde{c}_j) = \max_{q_1^{i_c}}\left[1 - q_1^{i_c} - \left(\mu q_1^{j_0} + (1-\mu)q_{1_s}^{j_c}\right) - c\right]q_1^{i_c}; \qquad \text{(A.15)}$$

the solution to this problem is

$$q_1^{i_c} = \frac{2(1-c)-\mu c-2(1-\mu)q_{1_s}^{j_c}}{2(2+\mu)}. \qquad \text{(A.16)}$$

Finally, a separating equilibrium requires that the following two incentive compatibility constraints be verified:

- If firm's cost is low, then

$$\mu\left(1 - q_1^{i_0} - q_1^{j_0}\right)q_1^{i_0} + (1-\mu)\left(1 - q_1^{i_0} - q_{1_s}^{j_c}\right)q_1^{i_0} \geq \mu\left[\left(1 - q_{1_s}^{i_c} - q_1^{j_0}\right)q_{1_s}^{i_c} + \frac{c(4-5c)}{12}\right] + (1-\mu)\left[\left(1 - q_{1_s}^{i_c} - q_{1_s}^{j_c}\right)q_{1_s}^{i_c} + \frac{c(4-c)}{12}\right], \qquad \text{(A.17)}$$

  that is, the efficient licensee prefers to produce her nonsignaling expected profit-maximizing output, in which case her second-period profits will be zero after deducting the license fee charged to an efficient firm, rather than to produce the signaling output and hence pay the license fee charged to an inefficient firm—namely, the fee given by the first two rows of (A.11)—and thus to earn the corresponding second-period net profits, which are given by the first two rows of (A.12).

- If firm's cost is high, then

$$\mu\left(1 - c - q_{1_s}^{i_c} - q_1^{j_0}\right)q_{1_s}^{i_c} + (1-\mu)\left(1 - c - q_{1_s}^{i_c} - q_{1_s}^{j_c}\right)q_{1_s}^{i_c}$$

$$\geq \mu\left[\left(1-c-q_1^{i_c}-q_1^{j_0}\right)q_1^{i_c}-\frac{c(4-3c)}{12}\right]+(1-\mu)\left[\left(1-c-q_1^{i_c}-q_{1_s}^{j_c}\right)q_1^{i_c}-\frac{c(4+c)}{12}\right]; \quad \text{(A.18)}$$

in this case, the inefficient licensee prefers to produce his signaling expected profit-maximizing output, in which case his second-period profits will be zero after deducting the license fee charged to an inefficient firm, rather than to produce the non-signaling output and hence pay the license fee charged to an efficient firm—namely, the fee given by the last two rows of (A.11)—and thus to earn the corresponding second-period net profits, which are given by the last two rows of (A.12).

Solving (A.17) and (A.18) as equalities and then using (A.14) and (A.16), we obtain

$$q_{1_s}^{i_c}=\frac{1}{3}\pm\frac{(2+\mu)\sqrt{3[4c-(1+4\mu)c^2]}}{18}, \quad \text{(A.19)}$$

$$q_{1_s}^{i_c}=\frac{1-c}{3}-\frac{\mu c}{6}\pm\frac{(2+\mu)\sqrt{3[4c+(1-4\mu)c^2]}}{18}. \quad \text{(A.20)}$$

The negative root in (A.20) is the lowest value (corresponding to the least production), followed by the negative root in (A.19), the positive root in (A.20), and finally the positive root in (A.19).

The unconstrained profit-maximizing production for the inefficient producer solves the following problem:

$$\max_{q_{1_{un}}^{i_c}} Ev_1^{i_c}(c,\tilde{c}_j)=\max_{q_{1_{un}}^{i_c}}\left[1-q_{1_{un}}^{i_c}-\left(\mu q_{1_{un}}^{j_0}+(1-\mu)q_{1_{un}}^{j_c}\right)-c\right]q_{1_{un}}^{i_c}. \quad \text{(A.21)}$$

The solution for a Cournot equilibrium is

$$q_{1_{un}}^{i_c}=\frac{1}{3}-\frac{(2+\mu)c}{6}. \quad \text{(A.22)}$$

A separating equilibrium requires the inefficient firm to choose—in a way that maximizes profits—between production as given by (A.19) and as given by (A.20). It can easily be verified that

$$\frac{1}{3}-\frac{(2+\mu)\sqrt{3[4c-(1+4\mu)c^2]}}{18}<\frac{1}{3}-\frac{(2+\mu)c}{6}<\frac{1-c}{3}-\frac{\mu c}{6}+\frac{(2+\mu)\sqrt{3[4c+(1-4\mu)c^2]}}{18}. \quad \text{(A.23)}$$

The positive root in (A.20) does not verify inequality (A.17), yet the negative root in (A.19) does verify (A.17) with equality and (A.18) with strict inequality. So in a separating equilibrium, the inefficient firm chooses a level of production given by the negative root in (A.19) and, by (A.14), the efficient firm chooses

$$q_1^{i_0} = \frac{1}{3} + \frac{(1-\mu)\sqrt{3[4-(1+4\mu)c]c}}{18}. \tag{A.24}$$

It follows from (A.23) that the inefficient firm's equilibrium production under signaling is lower than the unconstrained equilibrium production characterized by (A.22).

Proof of Proposition 2.

(i) *One license.* We start by computing the first-period patent fee, which we denote by $f_1(\tilde{c})$ in the monopoly case. Because the patent holder has no information about licensee costs, there can be only one license fee; that fee is set equal to the firm's profits according as whether it is efficient or inefficient. Yet in a signaling equilibrium the fee charged must equal the inefficient firm's profits, for otherwise that firm would have negative profits. Therefore, $f_1\,(\tilde{c}) = f_1(c) = v_1^L(c)$. Taking $q_1^s(c)$ from Lemma 1 now yields

$$f_1(c) = v_1^L(c) = (1 - c - q_1^s(c)\,)q_1^s(c) \;= \frac{1}{4}\left(1 - 4c + c^2 + 2c\sqrt{c(2-c)}\right), \tag{A.25}$$

and first-period licensee profits are $\pi_1^L(c) = v_1^L(c) - f_1(c) = 0$; for the efficient firm, $\pi_1^L(0) = v_1^L(0) - f_1(c)$. Using the quantity produced, $q_1(0)$, from Lemma 1 yields $v_1^L(0) = (1 - q_1(0)\,)q_1(0) \;= \frac{1}{4}$. After accounting for the license fee given by (A.25), we obtain

$$\pi_1^L(0) = \frac{c}{4}\left(4 - c - 2\sqrt{c(2-c)}\right). \tag{A.26}$$

From (A.26) it follows that $\pi_1^L(0) > 0$, which shows that the efficient firm earns informational rents in the first period.

When it grants only one license, the patent holder's expected profits are

$$E\pi_{MM}^{\mathrm{PH}} = v_1^L(c) + \mu\, v_2^L(0) + (1-\mu)v_2^L(c), \tag{A.27}$$

where $v_1^L(c)$, $v_2^L(0)$ and $v_2^L(c)$ are as given by (A.25) and (A.5).

(ii) *Two licenses.* Again we start by computing the first-period license fee. As in the monopoly case, the patent holder has no information about the licensees type. So in order to verify the participation constraint for both firms, the patent holder must set a fee equal to an inefficient firm's expected profits; doing so makes the first-period expected profits (net of the license fee) equal to zero for that firm type and, at the same time, gives positive informational rents to a firm of the efficient type. Note that a higher fee would violate the inefficient firm's participation constraint. For $f_1^i(\tilde{c}_i, \tilde{c}_j)$, the first-period license fee, our previous reasoning implies that $f_1^i(\tilde{c}_i, \tilde{c}_j) = f_1^j(\tilde{c}_i, \tilde{c}_j) = f_1^i(\tilde{c}_j) = f_1^j(\tilde{c}_i)$ and hence that this fee is given by

$$f_1^i(\tilde{c}_j) = \mu\big(1 - c - q_{1_s}^{i_c} - q_1^{j_0}\big)q_{1_s}^{i_c} + (1-\mu)\big(1 - c - q_{1_s}^{i_c} - q_{1_s}^{j_c}\big)q_{1_s}^{i_c}; \qquad \text{(A.28)}$$

here $q_{1_s}^{i_c} = q_{1_s}^{j_c}$ and $q_1^{j_0} = q_1^{i_0}$, equalities that are given by the negative root in (A.19) and (A.24), respectively.

In the duopoly case, the patent holder's expected profits are

$$E\pi_{DD}^{\mathrm{PH}} = 2\left(\mu\, v_1^i(c,0) + (1-\mu)\, v_1^i(c,c)\right) + 2\left(\mu^2 v_2^i(0{,}0) + \mu(1-\mu)\left(v_2^i(0,c) + v_2^i(c,0)\right) + (1-\mu)^2 v_2^i(c,c)\right). \qquad \text{(A.29)}$$

In (A.29), the first- and second-period profits are given by (A.28) and (A.9), respectively. In a separating equilibrium with two licenses, second-period profits coincide with those of a Cournot equilibrium with symmetric information, which are given by (A.9).

References

Antelo, M. (2013), "Duration and payment of licensing contracts for users to reveal what they know", *Economics of Innovation and New Technology* 22, 127-151.

Beggs, A. W., (1992), "The licensing of patents under asymmetric information", *International Journal of industrial Organization*, 10, 171-191.

Kamien, M., (1992), "Patent Licensing", in Aumann, R. J., and Hart, S. (Eds.), *Handbook of Game Theory*, Chapter 11.

Schmitz, P.W., (2002), "On Monopolistic Licensing Strategies under Asymmetric Information," *Journal of Economic Theory*, 106 (1), 177—189.

Schmitz, P. W., (2007), "Exclusive versus Non-exclusive Licensing Strategies and Moral Hazard", *Economic Letters,* 97 (3)*,* 208-214.