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Abstract
Savin *et al*. (2007) and Lo *et al*. (2000) analyse the predictive power of head-and-shoulders (HS) patterns in the U.S. stock market. The algorithms in both studies ignore the relative position of the HS pattern in a price trend. In this paper, a filter that removes invalid HS patterns is proposed. It is found that the risk-adjusted excess returns for the HST pattern generally improve through the use of our filter.

Keywords: Technical analysis; Head-and-shoulders pattern; Kernel regression.

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1. Introduction

Previous studies on technical analysis have concentrated on indicator-based and model-based trading rules. For example, Brock et al. (1992) find significant excess returns for moving average trading rules in the U.S. stock market. Gencay (1998) shows that non-parametric model-based trading rules outperform the buy-and-hold strategy. Compared with the work on these two trading rules, studies on the profitability of pattern-based trading rules are relatively rare. Among the limited scholarship that exists, Bulkowski (1997) provides definitions for some prevailing patterns. Lo, Mamaysky and Wang (2000) (hereafter referred to as LMW) apply the non-parametric kernel regression to recognize technical patterns. In a more recent work, Savin, Weller and Zvingelis (2007) (hereafter referred to as SWZ) apply the kernel-smoothing algorithm of Lo, Mamaysky and Wang (2000) to analyse the predictive power of head-and-shoulders top (HST) patterns in the U.S. stock market. Their results show that the pattern-based trading rules generate significant risk-adjusted excess returns. Both studies use the non-parametric kernel smoothing procedure and apply different filtering criteria to detect the HST pattern. However, the relative position of the HST pattern is ignored in their analysis. As a result, their algorithms might wrongly identify such patterns at the bottom of the market. Moreover, they do not report the results for the head-and-shoulders bottom pattern.

This paper complements the previous studies by proposing a filter to remove the invalid patterns. In addition, we will also analyze the head-and-shoulders bottom (HSB) patterns not covered by SWZ. The rest of this paper is organized as follows. Section 2 discusses the methodology used in this paper. The work of Savin, Weller and Zvingelis (2007) is revisited, and an improved pattern recognition procedure is proposed. Section 3 discusses the data and defines the returns used in this paper. Section 4 presents our results and Section 5 concludes the paper.

2. Methodology and Procedures

The pattern recognition algorithm consists of two steps: (1) to remove the noise of the data using a smoothing function and (2) to detect the HS patterns from the smoothed data.

2.1 Data Generation Process, Rolling Windows and Kernel Regression

To begin with, a nonparametric regression is estimated to smooth the price data. We assume that the price data are generated by

$$P_i=m(X_i)+e_i \qquad 1<i<T \tag{1}$$

where $m(X_i)$ is a smooth function of time and e_i's are zero i.i.d. random errors with zero mean and constant variance. In our case, X_i is the time index.

The algorithm for pattern identification is applied to a rolling window of span n.[2] Following Savin, Weller and Zvingelis (2007), a rolling window of n=63 days is used. The prices series within each window of span n is smoothed using the Nadaraya–Watson kernel estimator, defined as

[2] The window sizes of Lo, Mamaysky and Wang (2000) and Savin, Weller and Zvingelis (2007) are 38 and 63 days, respectively.

$$m_{i,n}(x) = \frac{\sum_{j=i}^{i+n-1} P_j K(\frac{x - X_j}{h_{i,n}})}{\sum_{j=i}^{i+n-1} K(\frac{x - X_j}{h_{i,n}})} \tag{2}$$

where *m(x)* is the smoothed price function, X_j is the x-axis index near the data point *x*, within *i*-th windows with window size *n*, *P* is the original price and *K(·)* is the kernel function. The bandwidth *h* controls the magnitude of the smoothing function. Increasing *h* makes the price curve smoother.[3] In this paper, we use the multiples (1.5, 2 and 2.5) of the optimal bandwidth chosen by the leave-one-out cross-validation (LOOCV). Figure 1 shows a snapshot of the kernel regression.

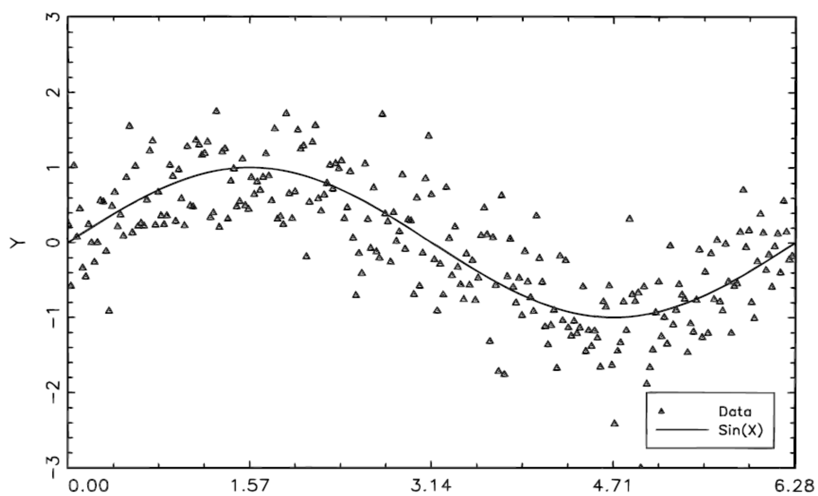

Figure 1. Kernel regression snapshot from Lo et al. (2000)

[3] Härdle (1990) argues that it is the choice of bandwidth rather than the kernel function that determines the performance of the non-parametric regression. Savin, Weller and Zvingelis (2007) adopt the leave-one-out cross-validation (LOOCV) of Stone (1977a and 1977b) to estimate the optimal bandwidth.

2.2 Extrema and Algorithms

Bulkowski (1997, 2000) provide definitions for both the head-and-shoulders top (HST) and the head-and-shoulders bottom (HSB) pattern. The HST pattern is a bearish pattern that signals the reversal of an uptrend and the beginning of a downtrend. The HSB pattern is a mirror image of the HST pattern. After a non-parametric regression has been estimated, a computational algorithm is used to detect the extrema, which are local maxima or local minima of the price graph. We will revisit the LMW and SWZ algorithms in this paper.

The filtering algorithm of Lo, Mamaysky and Wang (2000) is specified in Figure 2 and Table 1, where E_i (i=1,2,…) represents the extrema found.

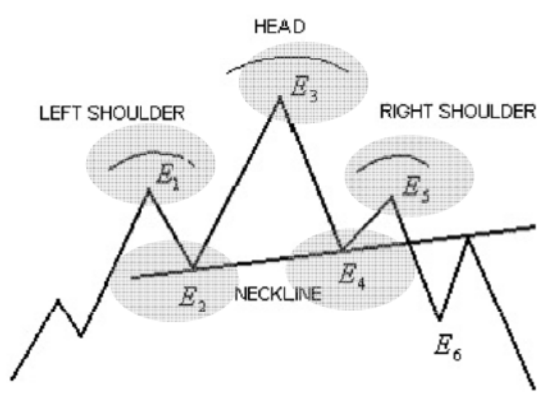

Figure 2. HST pattern under the LMW algorithm

Table 1. LMW algorithm (Lo et al., 2000)

Restrictions	Implications	
E_1 is a maximum	Start with a left shoulder	(R1)
$E_3 > E_1$	The head should be higher than the left shoulder	(R2)
$E_3 > E_5$	The right shoulder should be lower than the head	(R3)
$\max_i \mid (E_i - \overline{E}) \mid \leq 0.015 \times \overline{E}$, i =1, 5 where $\overline{E} = (E_1 + E_5)/2$	Restrict the magnitude of the shoulders	(R4)
$\max_i \mid (E_i - \overline{E}) \mid \leq 0.015 \times \overline{E}$, $i = 2,4$ where $\overline{E} = (E_2 + E_4)/2$	Restrict the magnitude of the troughs	(R5)

A trading signal will be generated when E_5 is observed and if all of the above criteria are satisfied. Savin, Weller and Zvingelis (2007) extend the work of Lo, Mamaysky and Wang (2000) by modifying the criteria for recognizing the HST pattern. Table 2 provides a description of each extension. Conditions (R4a), (R5a), (R6), (R7), (R8) and (R9) are referred to as the Bulkowski restrictions.

Table 2. SWZ algorithm (Savin et al., 2007)

Restrictions	**Implications**	
$\max_i \lvert (E_i - \overline{E}) \rvert \leq 0.04 \times \overline{E} \quad i = 1, 5$	Allow greater magnitude of the shoulders and troughs	(R4a)
$\max_i \lvert (E_i - \overline{E}) \rvert \leq 0.04 \times \overline{E} \quad i = 2, 4$		(R5a)
$\frac{[(E_1 - E_2) + (E_5 - E_4)]}{E_3 - (E_2 + E_4)/2} \leq 0.7$	Restrict the range of the proportion between the average magnitude of the shoulders and the magnitude of the head	(R6)
$\frac{[(E_1 - E_2) + (E_5 - E_4)]}{E_3 - (E_2 + E_4)/2} \geq 0.25$		(R7)
$\frac{[E_3 - (E_2 + E_4)/2]}{E_3} \geq 0.03$		(R8)
$\max_i \lvert (X_{i+1} - X_i) - \overline{X} \rvert \leq 1.2 \times \overline{X}$ where i = 1,..,4, $\overline{X}$ is the average deviation between consecutive points	Restrict the horizontal asymmetry	(R9)
neckline crossing restriction	A minimum is discovered below the neckline after E_5	(R10)

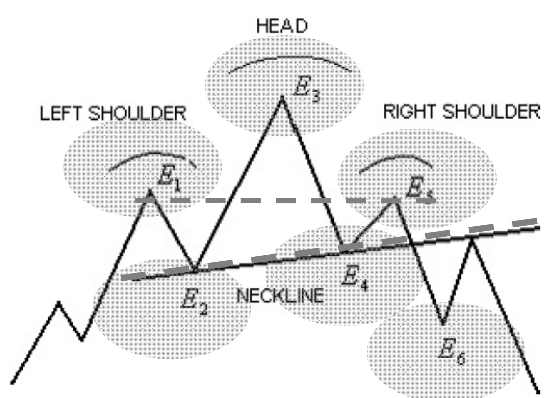

Figure 3. HST pattern under the SWZ algorithm

Figure 3 indicates the major features of HS patterns captured by the SWZ filtering rule. After the neckline crossing condition (R10) and all the other criteria mentioned have been satisfied, a short position is opened three days after the first minimum (E_6) is observed.

2.3 Head-and-Shoulders Bottom

Savin, Weller and Zvingelis (2007) only cover the HST pattern. In this paper, an analysis of the HSB pattern is also conducted to complement their work. Our filtering rules for the HSB pattern are as follows:

$\boldsymbol{E_1}$ is the minimum. **(R1a)**

$\boldsymbol{E_3 < E_1}$. **(R2a)**

$\boldsymbol{E_3 < E_5}$. **(R3a)**

Most of the conditions for the detection of the HSB pattern are the same as those for the HST pattern, except for (R1) to (R3). The same modifications are applied to both the LMW and the SWZ pattern recognition algorithm.[4]

[4] During the implementation of the computational algorithm, integrated solutions were not available in either Matlab or Stata. Such statistical software allows the kernel regression and cross-validation to be conducted separately. For Stata, a module for the bandwidth selection in the kernel density estimation (KDE) was available (Salgado-Ugarte and Pérez-Hernández, 2003), but heavy customization of the Stata codes is needed to transform them into a kernel regression with LOOCV. Alternatively, an approximation of the kernel regression might be obtained by applying the WARP approach (Härdle, 1991; Scott, 1992). Users of the programming language "R" might employ the "np" package (Hayfield and Racine, 2008).

2.4 Removal of Wrong Patterns

This paper improves the algorithm of SWZ by employing simple moving averages (SMA) to filter out the invalid patterns. The N-day simple moving average at time *t* is defined as

$$SMA_N(t) = \frac{\sum_{i=1}^{N} P(t-i+1)}{N} \quad (3)$$

The *SMA(·)* is used to filter out the invalid pattern located in a wrong position in the price trend; the 250-day and 150-day long-term moving averages will be employed for the analysis. The former is commonly used to determine whether the market is in a bull or a bear state. For the HST pattern to be valid, we require that for i=1,…, 6,

$$event(E_i \geq SMA_{150/250}) \geq 3 \quad \textbf{(R10a)}$$

The *event(·)* function indicates the number of times that the event occurs, as stated in brackets. The above filter rule requires at least three of the extrema (E_1 to E_6) to be above the moving average line. The corresponding rule for the HSB pattern is:

$$event(E_i \leq SMA_{150/250}) \geq 3 \quad \textbf{(R10b)}$$

In addition, instead of investigating the HST and HSB patterns separately, we also report the risk-adjusted excess return by combining (R10a) and (R10b). In this case, we can evaluate the trading performance considering head-and-shoulders patterns as a whole. However, simply combining (R10a) and (R10b) might produce misleading results. The combined rules could capture two opposite patterns that occur consecutively within a very short time period. Since HST is a bearish pattern while HSB is a bullish pattern, we should eliminate one of the patterns in the aforementioned situation. With (R10c), we apply a more restrictive filter rule that requires the first five extrema to be located on one side of the SMA. The chances of mistakenly capturing a wrong pattern can be significantly reduced.

$$\begin{cases} E_i \geq SMA & \text{for } i=1,..,5 \Rightarrow \text{detect HST pattern} \\ E_i \leq SMA & \text{for } i=1,..,5 \Rightarrow \text{detect HSB pattern} \end{cases} \qquad \textbf{(R10c)}$$

(R10c) requires the first five extrema found to be above (below) the SMA for the HST (HSB) pattern.

3. Data

3.1 Data

For ease of comparison with Savin, Weller and Zvingelis (2007), this paper uses daily stock price data of the S&P 500 and the Russell 2000 for analysis, covering the period from January 1990 to December 1999. The data are drawn from the database of the Center for Research in Security Prices (CRSP), accessed through the Wharton Research Data Services (WRDS). Using the constituent list from Savin, Weller and Zvingelis (2007), 484 stocks are used for the S&P 500, while 2,000 stocks are used for the Russell 2000. The two sets of stocks are chosen as a means of testing the robustness of the strategies' performance in different classes of stocks and the stock prices are adjusted for stock dividends. The daily three-month Treasury bill rates are taken from the CEIC database.

3.2 Procedures for Calculating Excess Returns

Conditional on the detection of HS patterns as trading signals, we measure the return of the trading strategy as shown below:

$$r_{i,c} = \ln(\frac{P_{i+n+c}}{P_{i+n}}), \tag{4}$$

where c = 20, 60 are the days after a trading signal is identified.

The c-day exit condition represents the duration of the holding period before a position is closed. In this paper, we adopt the 20-day and 60-day exit conditions (20-day-exit, 60-day-exit). After the holding period, the position is closed. We assume that the transaction cost is negligible. The excess return is then calculated by subtracting the daily compounded three-month Treasury bill rate.

Note that a profitable trade is associated with a negative excess return for HST, while it is associated with a positive excess return for HSB.

3.2 Risk-Adjustment of the Excess Returns

The monthly returns of the different strategies are measured by compounding the captured corresponding daily returns. Following Savin, Weller and Zvingelis (2007), the Carhart (1997) four-factor model is used to analyse the risk-adjusted monthly return. We estimate the following model:

$$EXR_t = \alpha + \beta_{mkt} EXMKT_t + \beta_{HML} HML_t + \beta_{SMB} SMB_t + \beta_{MOM} MOM_t + \varepsilon_t \tag{5}$$

where

EXR_t is the excess return conditional on detecting an HS pattern when the span of rolling windows is n=63 and then subtracted by the three-month Treasury bills' daily interest rate.[5]

$EXMKT_t$ is the excess market return factor,

HML_t is the book-to-market factor at time t.

SMB_t is the size factor,

MOM_t is the momentum factor at time t.

The intercept α provides the risk-adjusted excess return.

4. Results

4.1. Head-and-Shoulders Bottom as a Reversal Pattern

Tables 3a and 3b show the empirical results for the HSB trading strategy without the moving average filter.

(INSERT TABLE 3a)

(INSERT TABLE 3b)

For the S&P 500, negative risk-adjusted excess returns are found in all cases in Table 3a, which indicate that the strategy is not profitable. The results are similar for the Russell 2000. Tables 4a to 4h present the results when the moving average filter is imposed. The results with and without the use of the SMA restriction are compared. Tables 4a to 4d are for the S&P 500, while Tables 4e to 4h are for the Russell 2000.

[5] Details can be found at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

(INSERT TABLE 4a)

(INSERT TABLE 4b)

(INSERT TABLE 4c)

(INSERT TABLE 4d)

For the HST pattern detection in the S&P 500 data – with the original set of pattern detection criteria suggested by Lo, Mamaysky and Wang (2000) – the 60-day-exit risk-adjusted excess return in Table 4a drops to -0.25% per month from -0.12% for the unit bandwidth multiple, after adding the 150-day MA as a further restriction. Similar results are found when the 250-day MA filter is used. Since profitable trade is associated with a negative excess return for HST, the use of a moving average enhances the trading performance. For the HSB pattern, all the risk-adjusted excess returns are negative. Although profitable trade is associated with a positive excess return for HSB, the use of a moving average still improves the performance of the trading rule by making the excess returns less negative in most cases in Tables 4c and 4d. The use of the 150-day MA and the 20-day-exit strategy significantly improves the monthly return by 0.11% from -0.18% to -0.07% for the unit bandwidth. Most of the risk-adjusted excess returns in the other cases are also improved.

(INSERT TABLE 4e)

(INSERT TABLE 4f)

(INSERT TABLE 4g)

(INSERT TABLE 4h)

For the Russell 2000, a slight improvement of the results is found after applying the

150-day MA restriction to the HST patterns. Most of the results improve, with the greatest improvement of 0.05% (from -0.46% to -0.51%) in the monthly return in Table 4e. The performance for the 250-day MA restriction is less impressive.

Tables 5a and 5b report the empirical results of the combined rule. A more positive return implies a higher level of profitability. Monthly returns ranging from -0.07% to 0.35% are reported in Table 5a. Similarly, for the Russell 2000, the combined rule does not perform well either. A negative risk-adjusted excess return of -0.56% is found in Table 5b.

(INSERT TABLE 5a)

(INSERT TABLE 5b)

4.2. Head-and-Shoulders Bottom as a Continuation Pattern

The aforementioned combined rule method is based on the general perception that the head-and-shoulders pattern is a reversal pattern. In this subsection, we also provide the results obtained when assuming the HSB to be a continuation pattern; i.e., a short position instead of a long position is taken when an HSB pattern is observed.

(INSERT TABLE 6a)

(INSERT TABLE 6b)

For the S&P 500, for both the LMW and the SWZ algorithm, most combinations of the bandwidth multiples and exit-day conditions are improved and a monthly risk-adjusted excess return as high as 0.38% (or 4.56% per year) is captured. For the Russell 2000, a higher risk-adjusted excess return is found in all cases. In Table 6b, a significant monthly risk-adjusted excess return of 1.6% (or 19.2% per year) is found for the SWZ algorithm, unit bandwidth multiple and 60-day-exit condition. Surprisingly, the trading performance improves when we treat the HSB pattern as a continuation pattern.

5. Conclusion

While pattern recognition is a major component of technical analysis, it is an understudied topic compared with the extensive literature on indicator-based trading rules. This paper revisits the head-and-shoulders (HS) pattern studied by Lo, Mamaysky and Wang (2000) and Savin, Weller and Zvingelis (2007). We complement the previous studies with several sets of empirical results. First, a modified pattern recognition algorithm is developed to filter out invalid HST patterns. Second, the case for the HSB pattern is examined. The Carhart four-factor model is employed to assess the profitability of the HS trading rules under risk adjustment. Most of the risk-adjusted excess returns for the HST pattern are improved through the use of our filters. Our study raises several issues for future research along this line. For example, one might examine other smoothing methods (e.g., local polynomial regression) to address the boundary problem present in the kernel regression (Hastie and Loader, 1993). To test the robustness of the performance of our trading strategy, our algorithm might also be applied to exchange rates and other markets. Finally, our results, as well as those of Savin, Weller and Zvingelis (2007), are based on the fixed-window exit strategy. It will be of interest to examine the results of a more practical exit strategy used by market practitioners.

Table 3a Regression coefficients for HSB in the four-factor regression: S&P 500, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
20 days							
LMW	1	-0.0018**	0.4414**	0.0665**	0.1595**	-0.0579**	13983
LMW	2.5	-0.0009	0.4382**	0.0040	0.1093	-0.0433**	3423
SWZ	1	-0.0020**	0.4404**	0.0785**	0.1556**	-0.0622**	8666
SWZ	2.5	-0.0003	0.4278**	0.0021	0.1198	-0.0514**	4628
60 days							
LMW	1	-0.0019**	0.6648**	0.1078**	0.2313**	-0.0774**	13983
LMW	2.5	-0.0012**	0.7236**	0.0510**	0.2331**	-0.0862**	3423
SWZ	1	-0.0015	0.6581**	0.1186**	0.2137**	-0.0857**	8666
SWZ	2.5	-0.0009	0.7404**	0.0749**	0.265**	-.1381**	4628

The table reports the regression results in the four-factor linear model, where the dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window. Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 3b Regression coefficients for HSB in the four-factor regression: Russell 2000, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
20 days							
LMW	1	-0.0027**	0.4049**	0.2896**	0.2038**	-0.0381**	20482
LMW	2.5	-0.0044**	0.3716**	0.3239**	0.2282**	-0.0161	4127
SWZ	1	-0.0043**	0.3821**	0.4403**	0.1519**	-0.0361**	18575
SWZ	2.5	-0.0032**	0.3544**	0.4075**	0.1842**	-0.0605**	3459
60 days							
LMW	1	-0.0047**	0.5883**	0.5405**	0.2656**	-0.045**	20482
LMW	2.5	-0.0057**	0.5791**	0.5470**	0.2842**	-0.0112**	4127
SWZ	1	-0.0062**	0.6103**	0.6682**	0.1715*	-0.0809**	18575
SWZ	2.5	-0.0062**	0.5943**	0.7046**	0.2537**	-0.025**	3459

The table reports the regression results in the four-factor linear model, where the dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window. Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4a Regression coefficients for HST detected by the LMW algorithm in the four-factor regression: S&P 500, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Observation Number
20 days								
No	LMW	1	0.0009**	0.4494**	0.0856**	0.1030**	-0.0849**	14318
250MA	LMW	1	-0.0002	0.4235**	0.0636**	0.0967**	-0.0054*	11181
150MA	LMW	1	-0.0004	0.4256**	0.0735**	0.1024**	-0.0087**	11341
No	LMW	2.5	0.0012**	0.4568**	0.0162**	0.0998**	-0.0554**	3564
250MA	LMW	2.5	-0.0002	0.4498**	0.0178	0.1284	0.0758**	2699
150MA	LMW	2.5	0.0000	0.4253**	-0.0035	0.0816	0.0458**	2669
60 days								
No	LMW	1	-0.0012**	0.706**	0.0743**	0.2622**	-0.082**	14318
250MA	LMW	1	-0.0019	0.6829**	0.0924**	0.2525**	0.0098	11181
150MA	LMW	1	-0.0025**	0.6934**	0.0811**	0.2743**	0.0053	11341
No	LMW	2.5	-0.0018	0.7233**	0.1161**	0.3247**	-0.0593**	3564
250MA	LMW	2.5	-0.0022	0.7126**	0.0972**	0.289	0.0511	2699
150MA	LMW	2.5	-0.0029	0.7085**	0.086**	0.2723	0.0375	2669

The table reports the regression results in the four-factor linear model with the imposition of the 150-day and 250-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HST pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4b Regression coefficients for HST detected by the SWZ algorithm in the four-factor regression: S&P 500, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	SWZ	1	0.0013*	0.439**	0.0826**	0.0931**	-0.0785**	8712
250MA	SWZ	1	0.0005	0.4096**	0.034**	0.0656*	-0.0002	6701
150MA	SWZ	1	0.0001	0.4112**	0.0433**	0.0672*	-0.0019	6832
No	SWZ	2.5	0.0003	0.4771**	-0.004	0.1092**	-0.0512**	2474
250MA	SWZ	2.5	-0.0007	0.4784**	0.004	0.1431	0.0624**	1852
150MA	SWZ	2.5	-0.0005	0.4582**	-0.0219	0.1073	0.035*	1843
				60 days				
No	SWZ	1	-0.0008	0.7019**	0.109**	0.2502**	-0.1069**	8712
250MA	SWZ	1	-0.0020	0.6842**	0.0902**	0.2549*	0.0151	6701
150MA	SWZ	1	-0.0023	0.6953**	0.0896**	0.2511*	-0.0028	6832
No	SWZ	2.5	-0.0024	0.7411**	0.0943**	0.3413**	-0.0736**	2474
250MA	SWZ	2.5	-0.0027	0.7304**	0.0875**	0.3009	0.0445	1852
150MA	SWZ	2.5	-0.0036	0.726**	0.073*	0.2846	0.0374	1843

The table reports the regression results in the four-factor linear model with the imposition of the 150-day and 250-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HST pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window. Results for the SWZ algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4c Regression coefficients for HSB detected by the LMW algorithm in the four-factor regression: S&P 500, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	LMW	1	-0.0018**	0.4414**	0.0665**	0.1595**	-0.0579**	13983
150MA	LMW	1	-0.0007**	0.4419**	0.0713**	0.1845**	-0.1268**	9665
No	LMW	2.5	-0.0009	0.4382**	0.004	0.1093	-0.0433**	3423
150MA	LMW	2.5	-0.0020	0.4403**	0.0334**	0.1191	-0.133**	1585
				60 days				
No	LMW	1	-0.0019**	0.6648**	0.1078**	0.2313**	-0.0774**	13983
150MA	LMW	1	-0.0012	0.6686**	0.1258**	0.2576**	-0.1666**	9665
No	LMW	2.5	-0.0012**	0.7236**	0.051**	0.2331**	-0.0862**	3423
150MA	LMW	2.5	-0.0002	0.7135**	0.0635**	0.28**	-0.1739**	1585

The table reports the regression results in the four-factor linear model with the imposition of the 150-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4d Regression coefficients for HSB detected by the SWZ algorithm in the four-factor regression: S&P 500, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Observation Number
				20 days				
No	SWZ	1	-0.0020**	0.4404**	0.0785**	0.1556**	-0.0622**	8666
150MA	SWZ	1	-0.0009**	0.4369**	0.1156**	0.1906**	-0.1427**	4628
No	SWZ	2.5	-0.0003	0.4278**	0.0021	0.1198	-0.0514**	2308
150MA	SWZ	2.5	-0.0015	0.3815**	0.0108	0.08	-0.1621**	1090
				60 days				
No	SWZ	1	-0.0015	0.6581**	0.1186**	0.2137**	-0.0857**	8666
150MA	SWZ	1	-0.0012	0.6651**	0.1553**	0.2452**	-0.1748**	4628
No	SWZ	2.5	-0.0009	0.7404**	0.0749**	0.265**	-0.1381**	2308
150MA	SWZ	2.5	-0.0007	0.741**	0.0447**	0.2746**	-0.2175**	1090

The table reports the regression results in the four-factor linear model with the imposition of the 150-day and 250-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the SWZ algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4e Regression coefficients for HST detected by the LMW algorithm in the four-factor regression: Russell 2000, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	LMW	1	-0.0026**	0.3596**	0.353**	0.1083**	-0.065**	22196
250MA	LMW	1	-0.0016	0.3536**	0.3012**	0.0806	0.0055	13863
150MA	LMW	1	-0.0025*	0.3402**	0.3176**	0.0952	0.0154*	13844
No	LMW	2.5	-0.0031**	0.3246**	0.4321**	0.0828	0.0025	3698
250MA	LMW	2.5	-0.0030**	0.3179**	0.3801**	0.035	0.0686**	2473
150MA	LMW	2.5	-0.0032**	0.3646**	0.3815**	0.0757	-0.0086	2475
				60 days				
No	LMW	1	-0.0043	0.5676**	0.5613**	0.2489*	-0.0978**	22196
250MA	LMW	1	-0.0037	0.5507**	0.5359**	0.2287*	0.0365*	13863
150MA	LMW	1	-0.0041	0.5544**	0.5304**	0.2349	0.0117	13844
No	LMW	2.5	-0.0046	0.5179**	0.6404**	0.1549	-0.1447**	3698
250MA	LMW	2.5	-0.0034	0.496**	0.5795**	0.1063	-0.0433**	2473
150MA	LMW	2.5	-0.0051*	0.4984**	0.5774**	0.1349	-0.0632**	2475

The table reports the regression results in the four-factor linear model with the imposition of the 150-day and 250-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HST pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4f Regression coefficients for HST detected by the SWZ algorithm in the four-factor regression: Rusell 2000, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	SWZ	1	-0.0036**	0.3827**	0.4142**	0.0943**	-0.0781**	20953
250MA	SWZ	1	-0.0018**	0.3818**	0.37**	0.0845**	-0.0027**	12745
150MA	SWZ	1	-0.0037**	0.3644**	0.4056**	0.1237**	0.0131**	12887
No	SWZ	2.5	-0.0028	0.3347**	0.5151**	0.051	-0.0463**	4478
250MA	SWZ	2.5	-0.0026**	0.3174**	0.448**	0.0058	0.0286**	2976
150MA	SWZ	2.5	-0.0028	0.3388**	0.4255**	0.0129	-0.0031	2962
				60 days				
No	SWZ	1	-0.0061**	0.6127**	0.6467**	0.2009*	-0.1214**	20953
250MA	SWZ	1	-0.0055**	0.608**	0.6095**	0.2008**	0.0327**	12745
150MA	SWZ	1	-0.0062**	0.5958**	0.6029**	0.1932**	0.0097	12887
No	SWZ	2.5	-0.0039	0.5879**	0.7234**	0.2195	-0.1672**	4478
250MA	SWZ	2.5	-0.0035	0.5728**	0.6237**	0.2339	-0.0453**	2976
150MA	SWZ	2.5	-0.0040	0.5776**	0.6183**	0.2263	-0.0645*	2962

The table reports the regression results in the four-factor linear model with the imposition of the 150-day and 250-day moving average restrictions. The dependent variables consist of monthly excess return conditional on detecting an HST pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the SWZ algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4g Regression coefficients for HSB detected by the LMW algorithm in the four-factor regression: Russell 2000, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	LMW	1	-0.0027**	0.4049**	0.2896**	0.2038**	-0.0381**	20482
150MA	LMW	1	-0.0024**	0.3948**	0.2733**	0.2216**	-0.0758**	9664
No	LMW	2.5	-0.0044**	0.3716**	0.3239**	0.2282**	-0.0161	3459
150MA	LMW	2.5	-0.0044**	0.3675**	0.3743**	0.3173**	-0.0087	1726
				60 days				
No	LMW	1	-0.0047**	0.5883**	0.5405**	0.2656**	-0.045**	20482
150MA	LMW	1	-0.0051**	0.5732**	0.5442**	0.2757**	-0.1088**	9664
No	LMW	2.5	-0.0057**	0.5791**	0.547**	0.2842**	-0.0112**	3459
150MA	LMW	2.5	-0.0054**	0.5665**	0.5833**	0.3072**	-0.0887**	1726

The table reports the regression results in the four-factor linear model with and without the 150-day moving average restriction. The dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 4h Regression coefficients for HSB detected by the SWZ algorithm in the four-factor regression: S&P 500, 1990-1999.

Moving Average	Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
				20 days				
No	SWZ	1	-0.0043**	0.3821**	0.4403**	0.1519**	-0.0361**	9307
150MA	SWZ	1	-0.0044**	0.364**	0.4394**	0.1521**	-0.1059**	8737
No	SWZ	2.5	-0.0032**	0.3544**	0.4075**	0.1842**	-0.0605**	4127
150MA	SWZ	2.5	-0.0040**	0.4097**	0.4198**	0.2399**	-0.1626**	2107
				60 days				
No	SWZ	1	-0.0062**	0.6103**	0.6682**	0.1715*	-0.0809**	9307
150MA	SWZ	1	-0.0065**	0.5557**	0.7084**	0.1581	-0.1846**	8737
No	SWZ	2.5	-0.0062**	0.5943**	0.7046**	0.2537**	-0.025**	4127
150MA	SWZ	2.5	-0.0070**	0.6065**	0.7048**	0.2674**	-0.1159**	2107

The table reports the regression results in the four-factor linear model with and without the 150-day moving average restriction. The dependent variables consist of monthly excess return conditional on detecting an HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the SWZ algorithm, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 5a Regression coefficients for combined rules in the four-factor regression: S&P 500, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
			20 days				
LMW	1	0.0008	0.0077	0.0235**	0.0924	-0.1766**	15632
LMW	2.5	0.0004	0.0829**	0.112**	0.1476**	-0.2452**	985
			60 days				
LMW	1	0.0018	-0.0317	0.0432**	-0.018	-0.2323**	15632
LMW	2.5	0.0024	-0.0394	0.1629**	0.0627	-0.245**	985
			20 days				
SWZ	1	-0.0001	0.0204	0.0962**	0.1392	-0.2163**	9480
SWZ	2.5	-0.0007	-0.0007	0.2244**	0.182**	-0.1878**	1260
			60 days				
SWZ	1	0.0021	-0.0284	0.0851**	-0.0279	-0.2652**	9480
SWZ	2.5	0.0035	-0.0645	0.2118**	0.0342	-0.2939**	1260

The table reports the regression results in the four-factor linear model. The dependent variables consist of monthly excess return conditional on detecting a HST or HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 5b Regression coefficients for combined rules in the four-factor regression: Russell 2000, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
			20 days				
LMW	1	-0.0004	0.0729	-0.0490**	0.1214	-0.1084**	20866
LMW	2.5	-0.0056**	0.1117**	0.0937**	0.2406**	-0.0345**	2670
			60 days				
LMW	1	-0.0007	0.0040	0.0190	0.0372	-0.1543**	20866
LMW	2.5	-0.0006	0.0968**	-0.0096	0.1804**	-0.2094**	2670
			20 days				
SWZ	1	-0.0029**	-0.0365	0.0985**	0.0381	-0.1494**	17516
SWZ	2.5	-0.0006	0.1670**	-.1053**	0.1965**	-0.3195**	2370
			60 days				
SWZ	1	-0.0031**	-0.0627**	0.1159**	-0.0637**	-0.2618**	17516
SWZ	2.5	-0.0029	0.1048	-0.097**	0.1291	-0.1221**	2370

The table reports the regression results in the four-factor linear model. The dependent variables consist of monthly excess return conditional on detecting a HST or HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 6a Regression coefficients for continuation combined rule in the four-factor regression: S&P 500, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
			20 days				
LMW	1	0.0005	-0.8581***	-0.1624***	-0.2992***	0.1413***	15632
LMW	2.5	0.0005	-0.7881***	-0.0716***	-0.2433**	0.0586***	985
			60 days				
LMW	1	0.0036	-1.3525***	-0.2246***	-0.5179***	0.1872***	15632
LMW	2.5	0.0038*	-1.3702***	-0.1187***	-0.4424***	0.1748***	985
			20 days				
SWZ	1	0.0009***	-0.8415***	-0.1861***	-0.2817***	0.1468***	9480
SWZ	2.5	0.0008	-0.8579***	-0.0554***	-0.2332***	0.1564***	1260
			60 days				
SWZ	1	0.0040	-1.3532***	-0.257***	-0.5036***	0.1882***	9480
SWZ	2.5	0.0050	-1.3984***	-0.1473***	-0.4564***	0.1641***	1260

The table reports the regression results in the four-factor linear model. The dependent variables consist of monthly excess return conditional on detecting a HST or HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

Table 6b Regression coefficients for continuation combined rule in the four-factor regression: Russell 2000, 1990-1999.

Simtype	Bandwidth Multiple	Risk-adjusted Excess Return	Excess market return factor	Size factor	Book-to-market factor	Momentum factor	Number of patterns
			20 days				
LMW	1	0.0063***	-0.7589***	-0.6156***	-0.3861***	0.0185***	20866
LMW	2.5	0.0069***	-0.7261***	-0.7139***	-0.32*S**	0.0465***	2670
			60 days				
LMW	1	0.0100**	-1.1601***	-1.0963***	0.0372	-0.1543**	20866
LMW	2.5	0.0062*	-0.8364***	-0.9248***	-0.2986*	0.4122***	2670
			20 days				
SWZ	1	0.0097***	-0.6937***	-0.9077***	-0.2712***	0.0652***	17516
SWZ	2.5	0.0062*	-0.8364***	-0.9248***	-0.2986*	0.4122***	2370
			60 days				
SWZ	1	0.0160***	-1.1723***	-1.3145***	-0.3079	0.2126***	17516
SWZ	2.5	0.0107	-1.2806***	-1.3498***	-0.5682*	0.4565***	2370

The table reports the regression results in the four-factor linear model. The dependent variables consist of monthly excess return conditional on detecting a HST or HSB pattern when the span of the rolling windows is n=63. The returns are reported for 20- and 60-day window, Results for the LMW and SWZ algorithms, and different bandwidth multiples (1 and 2.5) are shown. An autocorrelation and heteroskedasticity-consistent covariance matrix estimator is used for estimation. "*" denotes that the coefficient is significant at the 10% level, "**" denotes that the coefficient is significant at the 5% level, "***" denotes that the coefficient is significant at the 1% level.

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