Search Deterrence

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Abstract

This paper studies sales techniques which discourage consumer search by making it harder or more expensive to return to buy after a search for alternatives. It is unilaterally profitable for a seller to deter search under mild conditions, but sellers can suffer when all do so. When a seller cannot commit to its policy, it exploits the inference that those consumers who try to buy later have no good alternative, and in many cases the outcome is as if the seller could only make an exploding offer. Search deterrence results in sub-optimal matching of products to consumers and often raises the price consumers pay.

Keywords: Consumer search, price discrimination, sequential screening, exploding offers, sales techniques.

1 Introduction

When feasible, a seller often makes it cheaper to purchase its product at the first opportunity, and if a potential customer wishes to return to buy later the price is higher or purchase might even be impossible. The motive for this form of price discrimination might be *strategic*, and as a kind of high-pressure sales tactic the seller offers a "buy-now" discount or makes an exploding offer when it first meets the buyer to increase its chance of making the sale. Alternatively, the seller might *opportunistically* raise its price when a buyer tries to purchase later, if it infers that the buyer has discovered in the meantime

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that her other options are poor. Regardless of the seller's motive, a buyer who anticipates the price will rise if she investigates other options is more inclined to buy immediately, and search is deterred.

Because inducements for quick decisions are usually offered casually during the course of a one-to-one sales encounter, and because opportunist price hikes are not publicly announced, it is hard to obtain empirical evidence about this form of price discrimination. In his account of sales practices, Cialdini (2001, page 208) provides examples of exploding offers: "A door-to-door magazine solicitor might say that salespeople are in the customer's area for just a day; after that, they, and the customer's chance to buy their magazine package, will be long gone. A home vacuum cleaner operation I infiltrated instructed its sales trainees to claim that, 'I have so many other people to see that I have the time to visit a family only once. It's company policy that even if you decide later that you want this machine, I can't come back and sell it to you'." In a labor market context, Roth and Xing (1994, page 1001) discuss exploding job offers. For instance, judges make offers for clerkships which are withdrawn if not accepted quickly, perhaps during the telephone call itself. Law journals operate a system whereby an author can submit an article simultaneously to several journals, and a journal offers to publish provided the author agrees quickly—before she has heard back from other, perhaps more prestigious, journals.[1]

A less extreme tactic is to allow a buyer to purchase later but to make it more expensive to do so, so that a "buy-now" discount is offered. Bone (2006, pages 71–73) describes how a home improvement company he infiltrated offers its potential customers a buy-now discount if the customer signs the contract immediately. Robinson (1995) discusses other examples of these discounts, such as a prospective tenant who is offered an apartment for $900 per month but to whom the landlord offers $850 if she agrees immediately, or a car dealer trying to close a deal who offers a further $500 off the price if the buyer accepts now, so (as he claims) he can make his sales quota for that month. A recent UK review

[1] The submission page of the *Harvard Law Review* currently has a link to a letter from a number of prominent law journals committing them to give an author seven days to decide whether to accept an offer to publish. The letter states "In recent years, many law journals have adopted the practice of issuing "exploding offers"—giving scholars only a couple of days, hours or even minutes to accept an offer of publication. The reasoning behind these offers was simple: we each hoped to secure the best articles for our own journal before others could identify them and make competing offers." See *harvardlawreview.org/submissions* [visited 26 September 2014].

of cosmetic surgery "was concerned about reports of patients being offered discounts for surgery if they sign a binding contract at the end of the first consultation".[2]

Airlines provide a rich source for examples of dynamic pricing. Most relevant to this paper are allegations that airfares can rise if a traveller leaves a travel website and then returns, while the price returns to its original level if the computer's "cookies" are deleted or a different computer is used.[3]

To implement these strategies a seller needs to be able to recognize customers, in the sense that it can distinguish potential customers it meets for the first time from those who have returned. Usually this is simply not possible. (A supermarket, for instance, keeps no track of a consumer's exit from the store.) Nevertheless, in many markets—notably those which depend on personal interaction between buyers and sellers—customer recognition is feasible. A sales assistant might discern from a potential customer's questions or demeanor whether this is her first visit to the store for the relevant product. A telephone or doorstep seller can be confident when the first encounter with a prospective customer occurs. Sometimes—as with job offers, automobile sales, housing rentals, tailored consumer financial products, medical or life insurance, cosmetic surgery, academic journals, or home improvements—a consumer needs to interact with a seller to discuss specific requirements, and this process reveals the consumer's identity. In online markets, a retailer may be able to tell if a visitor using the same computer has visited its website before.

After discussing related literature, in section 3 we present a simple example which illustrates a seller's incentive to deter search. Here, a seller attempts to sell its product to a buyer whose utility from the product is common knowledge. The buyer will subsequently be able to find an alternative for the product, although how good that alternative is is not known when the buyer first meets the seller. If the seller allows free recall—so that its price is unchanged after the buyer searches for the alternative—the buyer will search and return to buy whenever the payoff from that alternative is disappointing. The seller can

[2]See paragraph 5.8 of *Review of the Regulation of Cosmetic Interventions (Final Report)*, Department of Health (UK), April 2013. A similar observation was made in a recent investigation by the UK consumer right body, *Which?*, into the selling of laser eye surgery. See www.which.co.uk/news/2014/08/undercover-we-expose-laser-eye-surgery-clinics-376378 [visited 2 October 2014].

[3]For instance, type "deleting cookies flights" into a search engine to see various allegations (and denials) in the media. Of course, even if airlines never actual engage in this practice, the fact that many travellers believe they do will nevertheless act to deter search.

increase its profits, however, if it commits to an exploding offer, while the buyer is harmed by this tactic. If the seller cannot commit to its buy-later policy, it nevertheless has an incentive to induce the buyer to purchase immediately with a low "buy-now" price.

In section 4, we study a richer model in which search is costly and the buyer has private information about her utility from the seller's product. In section 4.1 we show it is more profitable to offer a buy-now discount than allow free recall when the relevant demand curve is log-concave. This form of price discrimination has novel features: *both* buy-now and buy-later prices can rise relative to the uniform price with free recall, and the seller charges a lower price to buyers who value its product more. If the seller is constrained to make an exploding offer or to allow free recall, an exploding offer is more profitable in the more restrictive case where demand is concave.

Buy-now discounts and exploding offers are simple tactics, which are easy to communicate and rationalize to buyers.[4] Moreover, these tactics are informal, in the sense they do not require a potential buyer to sign a contract for the right to buy later. (For instance, while one can envisage a sales assistant offering a discount on a camera "before I end my shift", it is harder to imagine the assistant offering a non-refundable deposit contract for the right to buy later.) Nevertheless, the seller may do better still by using more ornate selling schemes. In section 4.2 we derive the *optimal* way to sell in this environment, which allows the buyer to buy immediately at a low price, to return later to buy at a high price, and it also offers a menu of option contracts (where by paying a non-refundable deposit the buyer obtains the right to buy later at a specified price). The optimal mechanism involves search deterrence, in the sense that more buyers purchase without discovering their outside option than would be the case with free recall. In a numerical example, this scheme does only modestly better than the buy-now discount scheme.

In section 4.3 we discuss the seller's policy when it cannot commit to its buy-later price. A customer who returns to buy later reveals she has found no attractive alternative, and the seller often has an incentive to raise its price to this customer. When there is

[4] It may be important for the seller to be able to rationalize its sales policy, to give the policy more credibility. For example Bone (page 71) writes that to justify the buy-now discount customers were told "the company had so many appointments that it was difficult for our salespeople to cover them all ... if we went back to everyone twice we wouldn't see nearly as many people and would generate a lot less business."

an intrinsic cost of returning to the seller after search, the equilibrium involves no buyers returning to the seller after search, and the outcome is as if the seller could only make an exploding offer.

We extend our analysis to duopoly in section 5, where one seller's offer constitutes the other seller's outside option. In section 5.1, the sellers supply differentiated products, and a buyer searches both for a low price and a good product. Here, each seller has an incentive to deter search, but when both do this industry profit can fall relative to the free-recall regime. Search deterrence not only leads to a less efficient match between buyers and products, but can also induce sellers to set even their discounted buy-now price above the free-recall benchmark. Market performance with these sales techniques is poor: sellers are usually worse off relative to free recall, and buyers obtain a less suitable product in return for a higher price. In section 5.2, sellers compete to supply a homogeneous product and buyers search only for a low price. When sellers allow free recall the equilibrium involves price dispersion. However, free recall is not an equilibrium when sellers can recognize buyers who return after the first encounter, while an equilibrium exists in which sellers make exploding offers at the monopoly price. Here, the ability of sellers to discriminate against buyers who do not purchase at the first opportunity boosts industry profit—to the collusive level—and denies consumers the benefits of competition.

The paper concludes by discussing fruitful directions for further research as well as some thoughts about consumer policy to combat search deterrence.

2 Related Literature

This paper studies a new form of price discrimination, where price is conditioned on whether the buyer has searched or not, and we find that the ability to discriminate often leads *both* prices to rise relative to the uniform-price benchmark. In the context respectively of monopoly and duopoly, Nahata, Ostaszewski, and Sahoo (1990) and Corts (1998) study when this phenomenon can happen. Both papers study third-degree discrimination, where there are no cross-price effects in demand for the two products. The former paper requires profit functions not to be single-peaked, while the latter needs sellers to have different views about which is the "strong" market (i.e., the market with the higher price when price discrimination is used), to obtain this outcome. In our framework, cross-price effects are present, and both buy-now and buy-later prices rise due to the artificial search frictions introduced with price discrimination.

The basic mechanism in our model, that a higher cost of returning to buy later makes agents less inclined to search, has been studied before. For instance, Karni and Schwartz (1977) and Janssen and Parakhonyak (2014) show that an agent will cease her search sooner than she would with free recall if the ability to return to a previous option is uncertain or costly. The uncertainty or cost of return is exogenous in these papers, rather than determined endogenously by a seller's sales tactics as in our paper.

Few previous papers relate directly to strategic search deterrence. One that does is Ellison and Wolitzky (2012), who offer a model with homogeneous products and price dispersion in which a seller deters search by making onward search more costly, rather than by making it more expensive to buy later. They suppose that a buyer's incremental search cost increases with her cumulative search effort, and if a seller increases its in-store search cost—for instance, by instructing its sales staff to engage buyers in lengthy discussions or complicating its price scheme—this will make further search less attractive. As with our models in section 5, even if intrinsic frictions are small a market can still suffer from substantial endogenous search frictions and high prices in equilibrium.

Our analysis of search deterrence without commitment relates to two classic papers. Diamond (1971) shows how even small search frictions can lead to monopoly pricing in a market where sellers supply a homogenous product. In its starkest form, hold-up can cause the market to shut down altogether, and a closely related argument drives one main result in section 4.3. In the literature on durable good pricing initiated by Coase (1972), when a buyer does not buy quickly, she reveals she has a low valuation for the product. Because of this adverse selection, the seller has an opportunistic incentive to reduce its price to buyers who buy later. In our model, by contrast, if a buyer tries to buy later, she reveals she has searched but found her outside option was disappointing. Due to this *advantageous* selection, our seller often sets a higher price to buyers who buy later.[5]

[5] Zhu (2012) examines equilibrium pricing in a market for over-the-counter financial securities. His model has a single seller who searches sequentially for a high price offer among a number of potential buyers. A buyer's price is valid only for the initial contact, and if the seller rejects the initial offer and contacts that buyer a second time, the buyer suggests a new price. Zhu shows that a buyer lowers the

This paper is part of a wider literature which studies how sellers determine the information their potential buyers possess at the time they decide whether to purchase. Most of this literature studies a situation in which buyers learn, or do not learn, their idiosyncratic value for the product, while our model concerns the extent to which buyers are informed about the various offers available to them in the market. Lewis and Sappington (1994) discuss a situation where the seller can control how much the buyer learns about her valuation before purchase. One strategy allows the buyer to discover her valuation before agreeing to buy, in which case she purchases when the valuation is above the price. A second strategy, akin to an exploding offer in our framework, forces the buyer to buy without knowing her valuation. Lewis and Sappington show that the seller either reveals all or none of the information, and the second strategy is more profitable than the first when the production cost is small.[6] Essentially the same insight underlies the observation that an exploding offer boosts profit in our simple model in section 3.

Sellers can affect how informed their buyers are by influencing how early they buy, since a buyer plausibly has less precise information about her eventual value when she buys in advance of consumption. There is now a large literature on "advance selling", mostly within marketing, much of which concerns yield management. In a paper more focussed on price discrimination motives for advance selling, Nocke, Peitz, and Rosar (2011) analyze a seller's incentive to offer an "advance-purchase discount". Nocke *et al.* assume the seller can commit to its price path, and find conditions under which an advance-purchase discount is more profitable than either making all buyers buy in advance (akin to an exploding offer) or allowing all buyers to buy at the time of consumption (akin to free recall). In their model, which involves a binary valuation structure, the seller can do no better with more ornate schemes than to offer an advance-purchase discount.

With a richer information structure, though, the seller will be able to squeeze more profit out of buyers by using more complicated selling mechanisms. Such mechanisms offer the buyer a menu of option contracts, which involve paying a deposit for the right to buy

offered price if the seller makes a second approach, since the buyer infers the seller did not find an attractive price quote from other buyers.

[6] Wang (2014) extends this model so that buyers can discover their true valuation by incurring a cost. He finds that the seller might wish *partially* to reveal product information, so as to deter buyers from discovering their exact valuation.

later at a specified price (or equivalently, specifying a partial refund if the buyer eventually does not want the product). An early contribution to this literature on optimal sequential screening is Courty and Li (2000), who characterize the most profitable menu of refund contracts. Our study of the optimal way to sell in section 4.2 with a single seller uses the techniques developed in this sequential screening literature.

In formal terms, an information structure in which a buyer initially has an imprecise estimate of her eventual value for the product, which is made precise at the time of consumption, is similar to a situation in which a buyer initially sees a product of known value from the seller and can then discover an uncertain alternative elsewhere. However, in our model the buyer only discovers her net valuation if she decides to search, while in the sequential screening literature the buyer's information usually evolves exogenously over time.[7] Relative to the sequential screening literature, our focus is more on casual sales encounters, and for this reason we study *ad hoc* but simple sales techniques as well as the optimal way to sell. Our analysis reveals the importance of the shape of the relevant demand curve and the possibility that all prices rise relative to the uniform pricing case, neither of which were apparent in earlier contributions. We also analyze additional settings, such as when the seller does not have commitment power or when the outside option is provided by a second strategic seller.

Indeed, in an oligopoly context it makes a major difference whether uncertainty concerns a buyer's valuation for the product, common to all sellers, or the offer made by a rival. An extension from monopoly to oligopoly can be straightforward in the former case, and indeed competition might then eliminate a seller's incentive to make early offers.[8] When

[7]In this respect our analysis is closer to Krahmer and Strausz (2011). They consider a procurement setting where the agent can choose to invest costly effort to discover the actual cost of implementing a project after signing the contract. As in our paper, they find that the optimal mechanism features a fixed price and a menu of option contracts, and it not only aims to screen agents with different *ex ante* private information but also takes into account the incentive for agents to acquire information.

[8]For instance, consider the model in Armstrong and Vickers (2001, section 4). Two symmetric sellers supply a horizontally differentiated product, and buyers are heterogenous both in terms of their brand preference, x, and their taste for the product, θ. Armstrong and Vickers assume buyers know θ when they choose their seller, and show that an equilibrium exists in which each seller offers a cost-based two-part tariff, provided that the market is covered and x is uncorrelated with θ. However, the same argument applies even if sellers have the opportunity to make early offers, before buyers know the realization of their θ. So long as a buyer's prior distribution for θ is uncorrelated with x, when its rival offers the cost-based two-part tariff (available early or late), a seller's best response is to do the same. Thus, in this framework sellers have no incentive to make early offers.

the buyer's uncertainty concerns the offer she might receive from a rival, though, a seller has an incentive to induce early purchase, and this introduces endogenous search frictions into the market. These frictions enable sellers to raise their prices and, where relevant, cause sub-optimal matching of product to consumer.[9]

3 An Illustrative Example

A seller costlessly supplies a product to a risk-neutral buyer. The buyer is known to value a unit of this product at $u > 0$, and if she pays price p for the product her net surplus is $u-p$. The buyer has an alternative source for the product which gives her exogenous net surplus v. This net surplus is a continuous random variable with cumulative distribution function (CDF) $G(v)$ and support $[0, v_{\max}]$, and this distribution is commonly known by both seller and buyer. (Uncertainty in v might stem from uncertainty about the match utility of the alternative product or its price.) In this example we assume that $0 < v_{\max} \leq u$.[10] The buyer does not know the realization of v when she first encounters the seller, and needs to leave the seller to discover v. However, the buyer incurs no intrinsic search cost to find out v, nor to return to the seller after search. The seller never observes the realization of v, and so cannot make its offer to the buyer contingent on her outside option. The key assumption we make throughout this paper is that the seller can distinguish whether it is meeting the buyer for the first time or after a previous encounter.

In most markets, a seller's price does not depend (at least in the short term) on when the buyer decides to purchase, and the buyer has "free recall" of the seller's offer. To analyze this uniform price benchmark, suppose the seller offers price p to the buyer, regardless

[9] Möller and Watanabe (2014) develop a duopoly model of advance-purchase discounts which is related to our discussion of competitive buy-now discounts in section 5.1. In their model, a buyer knows by how much she prefers one product to the other but initially she is uncertain about which product will be the better one. An advance-purchase discount is attractive to buyers who do not care much about getting the better product, while "choosier" buyers prefer to wait and pay more for the product they prefer. Relative to a monopoly scenario where one firm sells both products, they show that competition leads to a deeper advance-purchase discount and a worse match between product and consumer.

[10] If the buyer consumes the seller's product *and* the outside option, suppose her gross utility is $\max\{u, v\}$, in which case our assumption $v_{\max} \leq u$ implies that the buyer never gains additional utility from consuming the outside option alongside the seller's product. One situation where $v_{\max} \leq u$ is if the buyer's two options involve the same homogeneous product (as we study in section 5.2), so that $v = u - P$ where P is the alternative price.

of when the buyer decides to purchase. With free recall, the buyer will investigate the outside option, and return to buy from the seller when $u - p \geq v$. Therefore, the seller's expected profit with free-recall price p is $pG(u-p)$, and let p_F denote the most profitable free-recall price. The expected net surplus of the buyer when free recall is allowed is $\mathbb{E}_v[\max\{u - p_F, v\}]$, where $\mathbb{E}_v[\cdot]$ denotes taking expectations with respect to v.

However, the seller can do better than this by committing to an exploding offer, where the buyer must decide whether to purchase before she can discover the outside option. (If she chooses to search, the seller will refuse to sell the product if she returns later.) The buyer accepts the exploding offer with price p if $u - p \geq \bar{v} = \mathbb{E}_v[v]$, and by choosing the price which just makes this inequality bind the seller can obtain profit $u - \bar{v}$. To see that the seller's profit is greater with an exploding offer, observe that $u - \bar{v}$ satisfies

$$u - \bar{v} > u - \mathbb{E}_v[\max\{u - p_F, v\}] = \mathbb{E}_v[\min\{p_F, u - v\}] \geq p_F G(u - p_F) \ . \tag{1}$$

Here, the first inequality holds because $\max\{u - p_F, v\} \geq v$, with strict inequality when v is small, while the second follows from the assumption that $v \leq u$ for sure.[11] When $u = 1$ and v is uniformly distributed on the interval $[0, 1]$, for instance, the seller's profit doubles when it makes an exploding offer. (In this case, $p_F = u - \bar{v} = \frac{1}{2}$, i.e., the seller's price is the same in the two regimes, but the buyer always purchases the product with an exploding offer while she only buys half the time with free recall.)

The profit $u - \bar{v}$ from the exploding offer is the profit the seller could obtain if it could observe v before setting its own price (in which case it would set price $p = u - v$), and this represents the maximum profit the seller can extract using *any* mechanism. The reason why an exploding offer boosts profit and harms the buyer is the same as the more familiar insight that a seller would like to sell to a buyer before the latter learns her uncertain valuation for the item (provided the valuation is sure to be above the seller's cost), and this is because the seller then captures the entire consumer surplus rather than merely a "rectangle" under her demand curve. With an exploding offer, the buyer obtains expected net surplus $\bar{v}$, which is strictly below her surplus in the free-recall regime, $\mathbb{E}_v[\max\{u - p_F, v\}]$.

Finally, suppose the seller cannot commit to its buy-later price when it first meets the buyer. That is, the seller makes an initial offer which is good for that time only, and it

[11] Since $u \geq v_{\max}$, it follows that $\mathbb{E}_v[\min\{p_F, u - v\}] \geq p_F \times \Pr\{p_F \leq u - v\} = p_F G(u - p_F)$.

cannot make a credible commitment to its price should the buyer return later. Consider the subgame in which the buyer rejects the initial offer made by the seller (whatever that may be) and chooses to discover v. Since it is costless for the buyer to return to the seller, we suppose she does so regardless of v to see what buy-later price the seller offers, and will accept the seller's buy-later price p_2 if $u - p_2 \geq v$. Therefore, the subgame-perfect buy-later price is just the free-recall price, p_F, which maximizes $p_2 G(u - p_2)$. If she searches, then, the rational buyer anticipates she will obtain expected surplus $\mathbb{E}_v[\max\{u - p_F, v\}]$, and so is just willing to pay a "buy-now" price p_1 which satisfies $u - p_1 = \mathbb{E}_v[\max\{u - p_F, v\}]$ and to purchase without search. Naturally, this buy-now price is below p_F, since the buyer needs to be rewarded if she is to forego the option value of the alternative. This strategy yields the seller the profit $u - \mathbb{E}_v[\max\{u - p_F, v\}]$, which from (1) is weakly above the profit generated when it induces the buyer to search, $p_F G(u - p_F)$. (The profit is *strictly* higher whenever the buyer sometimes consumes the outside option with free recall, i.e., if $u - p_F < v_{\max}$. This is the case if $v_{\max}$ is close enough to u or if the density for v at $v_{\max}$ is small.) Hence, even if the seller cannot commit to its sales policy, it usually has an incentive to deter search.[12] In the example where $u = 1$ and v is uniformly distributed on $[0, 1]$, the buy-now price which just deters search is $p_1 = \frac{3}{8}$, and the seller's price rises to $p_F = \frac{1}{2}$ if the buyer rejects the initial offer. Here, the seller's profit rises by 50% if it deters search, instead of doubling when it could commit to an exploding offer.

The intuition for why search deterrence is profitable even without commitment is as follows. The joint surplus available to the seller and buyer with a given selling mechanism is a weighted sum of u (when the buyer purchases from the seller) and the average value of v conditional on the buyer taking the outside option. Since $v \leq u$, this joint surplus is maximized when the buyer always purchases from the seller. When the seller cannot commit to its buy-later policy, it is constrained to deliver the same surplus to the buyer as it does with free recall, but can keep the additional joint surplus resulting from search deterrence. When the seller can commit, though, it can enjoy this extra joint surplus but

[12] There is a stark contrast between this model and the similar model in section I of Aghion and Bolton (1986) where the seller has no incentive to induce the buyer to purchase early. This difference stems from our assumption that the seller does not observe v if the buyer searches. (If the seller can observe v when the buyer searches, as discussed previously it would set its buy-later price equal to $u - v$ and make expected profit $u - \bar{v}$, and it can do no better than this with another selling mechanism.)

also reduce the buyer's surplus by making search less attractive.

In the next section we extend this framework so that the buyer has private information about her value for the seller's product, u, which is sometimes below the value of the outside option, v, and it is costly for the buyer to discover the outside option.

4 A Monopoly Model

A seller offers a product with constant marginal cost which is normalized to zero. The product yields random utility $u \geq 0$ to a risk-neutral buyer. This utility u is observed by the buyer when she first meets the seller and is unchanged over time, but in contrast to the previous section u is not observed by the seller. The distribution of u has support $[0, u_{\max}]$, and has CDF $F(\cdot)$ and density function $f(\cdot)$ which is continuous on its support. There is an outside option which yields uncertain net surplus $v \geq 0$ to the buyer. For simplicity, v is assumed to be distributed independently of u.[13] The distribution of v has support $[0, v_{\max}]$, and has CDF $G(\cdot)$ and a continuous density function $g(v)$ on $(0, v_{\max}]$. (We allow for an "atom" at $v = 0$, reflecting the possibility that the buyer does not find an outside option.) The buyer does not know the realization of v when she first encounters the seller, and must incur a search cost $s \geq 0$—and leave the seller—to reach the outside option and discover its value. The seller never observes the realization of v. For simplicity, we assume for now that the buyer incurs no further search cost if she comes back and buys from the seller after investigating the outside option.[14]

If the buyer consumes neither the seller's product nor the outside option, her payoff is zero. However, this is irrelevant if the search cost satisfies

$$s < \bar{v} \ , \tag{2}$$

where $\bar{v}$ is the expected value of the outside option v. Condition (2), which we assume henceforth, ensures that the buyer prefers investigating the outside option to buying noth-

[13] This implies that buyer and seller have the same information about v when they first meet. Correlation between u and v might induce a non-monotonic stopping rule for the buyer which complicates the analysis.

[14] In most cases, buyers do face an intrinsic cost of returning to a previously visited seller. In most of our analysis, introducing a small intrinsic returning cost does not affect results qualitatively, but complicates the analysis, and we assume it away. However, when we discuss the situation without commitment in section 4.3, whether or not there is an intrinsic return cost will make an important difference.

ing. The buyer gains no extra utility if she consumes *both* the product and the outside option, and her gross utility with both items is equal to $\max\{u, v\}$.

Let $Q(p) \equiv 1 - F(p)$ denote the demand curve faced by the seller in the hypothetical case when the outside option v is zero. We assume

$$\log Q(p) \text{ is strictly concave when } Q(p) > 0 \; . \tag{3}$$

When the buyer has a deterministic outside option v, this condition ensures that the seller's demand at price p, $Q(p+v)$, is more elastic when v is larger.[15]

4.1 Simple ways to sell

In this section we analyze three simple ways the seller in this environment can supply its product.

Free recall: We first analyze the uniform price benchmark where a seller's price does not depend on when the buyer decides to purchase.

Suppose the seller offers price p to the buyer. If the buyer obtains utility u from the seller's product, her net surplus if she buys immediately is $u - p$, while her expected net surplus if she investigates the outside option is $\mathbb{E}_v[\max\{v, u - p\}] - s$. If she investigates the outside option she incurs the search cost but can then consume the superior option. Let

$$S(x) \equiv \mathbb{E}_v[\max\{x, v\}] - x = \int_x^{v_{\max}} (1 - G(v))dv \tag{4}$$

denote the expected benefit of search when the buyer has free recall of payoff x at the seller. Here, $S(\cdot)$ is decreasing and convex. With free recall the buyer will purchase the seller's product without search whenever $u - p \geq \mathbb{E}_v[\max\{u-p, v\}] - s$, i.e., if $S(u-p) \leq s$ or $u \geq p + a$, where a solves

$$S(a) = s \; .$$

(From (2), such an a exists, and it is unique, strictly positive and decreases with s.) The parameter a, which depends only on the distribution of the outside option and the cost of

[15] Condition (3) rules out the boundary case with a loglinear demand curve (i.e., the case where u is exponentially distributed). In that case, none of the search deterring sales tactics discussed in this paper can strictly improve the seller's profit relative to the free-recall benchmark.

its discovery, represents the net surplus the buyer needs to obtain from the seller in order to forgo search when there is free recall. For the remainder of the paper we impose the following mild condition:

$$a \leq u_{\max} \ . \tag{5}$$

Assumption (5) rules out the uninteresting case in which the buyer searches even if she has the highest possible valuation $u_{\max}$ and the seller charges the lowest possible price of zero. (Since $a \leq v_{\max}$, a sufficient condition for this to hold is that $v_{\max} \leq u_{\max}$.)

If the buyer decides to investigate the outside option, i.e., if $u \geq p + a$, she will return to buy from the seller when the outside option turns out to be worse than the seller's offer, i.e., when $v < u - p$. The buyer's purchase decision for the various realizations of (u, v) is summarized in Figure 1. Given this pattern of consumption, the firm's total demand with free recall and price p, denoted $q(p)$, is

$$q(p) = \mathbb{E}_v[Q(p + \min\{v, a\})] \ . \tag{6}$$

The most profitable free-recall price, denoted p_F, therefore maximizes $pq(p)$.

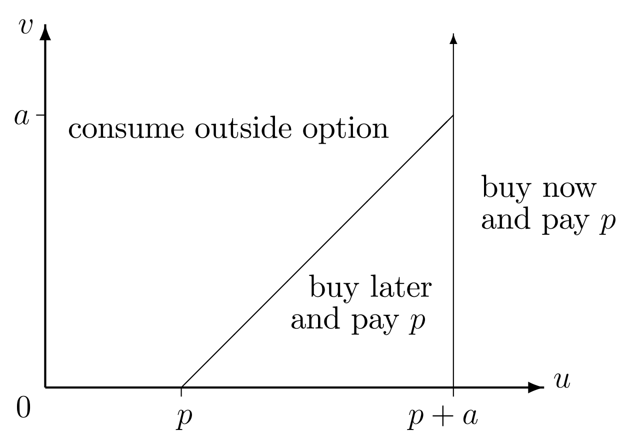


Figure 1: Demand with free recall

As shown on the figure, demand can be divided into "buy-now" demand, denoted $q_1(p)$, and "buy-later" demand $q_2(p)$. Buy-now demand is the fraction of buyers who purchase immediately without search, i.e., $q_1(p) = Q(p + a)$. Buy-later demand consists of those buyers who purchase after investigating the outside outside option, i.e., $q_2(p) = q(p) - q_1(p)$. Buy-later demand can *increase* with price: an increase in p shifts the region of buy-later demand uniformly to the right on Figure 1, and so q_2 increases with p if the density of u increases with u, i.e., if $Q(\cdot)$ is concave.

More generally, buy-now demand is more elastic than total demand (and buy-later demand), which is the fundamental reason why the seller with commitment power wishes to discriminate against buyers who wish to purchase after investigating their outside option.[16]

Lemma 1 *With free recall, when there is some buy-now demand it is more elastic than total demand.*

Proof. From (6), we have

$$q'(p) = \mathbb{E}_v[Q'(p + \min\{v, a\})] > \mathbb{E}_v\left[\frac{Q'(p+a)}{Q(p+a)}Q(p + \min\{v, a\})\right] = \frac{q_1'(p)}{q_1(p)}q(p) \ ,$$

where the inequality follows from the strict log-concavity of Q in (3) and the fact that $\min\{v, a\} \leq a$ with strict inequality when $v < a$. This proves the claim. ■

Intuitively, buy-now demand $Q(p + a)$ can be regarded as the demand from a buyer with known outside option a. From (6), total demand is an average of demands, each of which has a weakly lower outside option $\min\{v, a\}$. Condition (3) therefore implies that buy-now demand is more elastic than total demand. A similar argument to that used in Lemma 1 shows that total demand $q(p)$ is more elastic than monopoly demand $Q(p)$, which implies that p_F is lower than the monopoly price p^* which maximizes $pQ(p)$.

Buy-now discounts: Next suppose that the seller engages in price discrimination, and offers distinct prices p_1 and $p_2 \geq p_1$ if a buyer respectively purchases immediately or after search.[17] For now, we assume the seller announces and commits to its buy-later price p_2 when it first encounters the buyer. Figure 2 depicts the pattern of demand with buy-now price p_1 and buy-later price p_2. If the buyer's utility is u, she prefers to buy without search

[16] The fact that demand from buyers who buy at the first opportunity is more elastic than buy-later demand is also important in Armstrong, Vickers, and Zhou (2009). That paper only studied free recall, and considered the impact of one seller being encountered first by all buyers. This seller sets a lower price than its less prominent rivals, since more of its demand consists of (elastic) buy-now demand.

[17] In many cases it makes sense to rule out situations where $p_2 < p_1$. For instance, it seems plausible that the seller cannot verify if the buyer has actually engaged in search rather than merely left the seller and returned. In this case, if $p_2 < p_1$, the buyer would never buy at the higher price p_1, and the outcome is as if the seller allowed free recall at price p_2. However, even if it was possible to charge a lower buy-later price, the following analysis shows that the seller does not wish to do so.

if $u - p_1 \geq \mathbb{E}_v[\max\{u - p_2, v\}] - s$, i.e., if $S(u - p_2) \leq s + p_2 - p_1$ or $u \geq p_2 + S^{-1}(s + p_2 - p_1)$. If she does search, she will return to buy later if $u - p_2 \geq v$.[18]

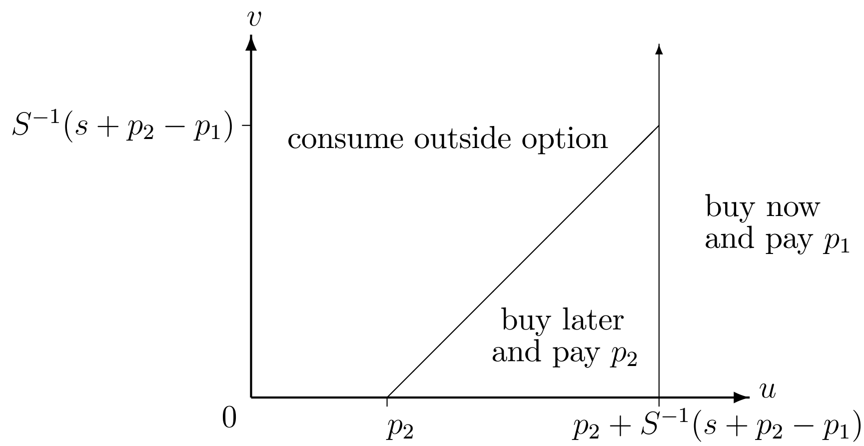


Figure 2: Demand with buy-now price p_1 and buy-later price p_2

Given Lemma 1 it is intuitive that the seller wishes to set a lower price to buyers who purchase at the first opportunity. The next result verifies this intuition.

Proposition 1 *Offering a buy-now discount is more profitable than allowing free recall.*

For a given buy-later price $p_2 = p$, comparing Figures 1 and 2, and recalling that $S^{-1}(s) = a$, shows that the impact on total demand of introducing a buy-now discount $\tau \geq 0$ (so that $p_1 = p - \tau$) is precisely as if the exogenous search cost increased from s to $s + \tau$ in the free-recall regime. Thus, the firm is able to boost its demand by increasing endogenous search frictions, but at the revenue cost of offering a buy-now discount to its buyers. Proposition 1 demonstrates that the former positive effect outweighs the latter negative effect, at least for a small buy-now discount. Since buyers who purchase immediately value

[18] In fact, the buyer has a third option, which is to buy the product immediately *and* search for the outside option. Following such a strategy yields the type-u buyer expected surplus $\mathbb{E}_v[\max\{u, v\}] - p_1 - s$, since she pays the buy-now price p_1 and the search cost, and then can consume the better of u and v. When $p_1 \approx p_2$, so that the buy-now discount is small, this strategy is dominated by the "buy-later" strategy which yields surplus $\mathbb{E}_v[\max\{u - p_2, v\}] - s$, and so Proposition 1 is valid. In addition, this "buy and search" strategy is never followed in optimal mechanism discussed in the next section. More generally, if prices for the product are high (because of production costs, for instance, which are normalized to zero in our analysis), or if the nature of the product is such that consuming the product and the outside option together is unrealistic (e.g., it is rarely possible for someone to accept two distinct jobs simultaneously), the buyer will never follow this buy-and-search strategy.

the seller's product more highly than those who buy later, Proposition 1 implies that the seller charges a *lower* price to those buyers with higher valuations, which is not the typical situation with price discrimination.

Exploding offers: In some situations, it may not be practical for the seller to offer a buy-now discount, and its only alternative to allowing free recall is to make an exploding offer. For instance, the only credible story a salesman can tell might be that "he is in the area for today only", or similar. Suppose, then, that the seller makes an exploding offer, i.e., it forces the buyer to decide whether to buy its product before she is able to discover v.[19] If the seller's price is p and the buyer values the seller's product at u, then her net surplus is $u - p$ if she buys immediately without search, while her expected surplus if she rejects the offer is $\bar{v} - s$. Therefore, the buyer will buy the product if and only if $u \geq p + \bar{v} - s$, and the seller's demand with an exploding offer at price p is $Q(p + \bar{v} - s)$. The next result describes when an exploding offer is more, or less, profitable than allowing free recall.

Proposition 2
(i) If $Q(\cdot)$ is convex, free recall is (weakly) more profitable than making an exploding offer;
(ii) If $Q(\cdot)$ is concave whenever $Q > 0$ and there is some buy-now demand with free recall, making an exploding offer is (weakly) more profitable than free recall.

Proof. If $Q(\cdot)$ is convex, Jensen's Inequality implies that $q(p)$, the seller's demand with free recall defined in (6), satisfies

$$q(p) = \mathbb{E}_v[Q(p + \min\{v, a\})] \geq Q(p + \mathbb{E}_v[\min\{v, a\}]) = Q(P + \bar{v} - s) \ ,$$

where the second equality above follows from the identity

$$\mathbb{E}_v[\min\{v, a\}] = \mathbb{E}_v[v + a - \max\{a, v\}] = \bar{v} - S(a) = \bar{v} - s \ .$$

Therefore, for any given price making an exploding offer reduces the seller's demand relative to free recall, which proves part (i).

[19] Of course, an exploding offer is a special case of a buy-now discount tariff, where the buy-later price is high. Buy-later demand vanishes when the discount $p_2 - p_1$ is large enough, and from Figure 2 this is the case when $s + p_2 - p_1 \geq S(0) = \bar{v}$. The outcome then is as if the seller makes an exploding offer at price p_1.

For part (ii) the reverse argument can be made, provided we take care that demand is always positive. (The function $Q(\cdot)$ cannot be concave for all $p \geq 0$ since it is either zero for large p if $u_{\max}$ is finite, or it approaches zero asymptotically.) Specifically, suppose that at the free-recall price p_F there is positive buy-now demand, i.e., $Q(p_F + a) > 0$. Then the demand $Q(p_F + \min\{v, a\})$ is positive for all v. Jensen's Inequality then implies that making an exploding offer at the free-recall price p_F will boost the seller's demand and profits provided $Q(\cdot)$ is concave whenever it is positive. ■

Intuitively, making an exploding offer induces more buyers with relatively high u to purchase immediately, but prevents those buyers with moderate u from returning to purchase after search. Which effect is stronger depends on the relative proportions of high- and low-valuation buyers. When the density of u is increasing (i.e., when Q is concave), there are proportionally more high-valuation buyers in the population, and so an exploding offer is more profitable than free recall.

An example: To illustrate this analysis, consider an example where both u and v are uniformly distributed on $[0, 1]$ and the search cost is $s = \frac{1}{18}$. It follows that $Q(p) = 1 - p$, $S^{-1}(x) = 1 - \sqrt{2x}$ (for $0 \leq x \leq \frac{1}{2}$) and $a = S^{-1}(s) = \frac{2}{3}$. Figure 1 implies that free-recall total demand at price p is $q(p) = \frac{5}{9} - p$. The optimal free-recall price is therefore $\frac{5}{18}$, which also equals total demand. Of this demand, a fraction 20% comes from buyers who buy without search (so there is some buy-now demand) and the remaining 80% consists of those who search and then return to buy. The seller's profit with free recall is $(\frac{5}{18})^2 \approx 0.077$ while aggregate consumer surplus can be calculated to be about 0.511.

Suppose next that the seller offers a buy-now discount, where p is its buy-now price and $p + \tau$ its buy-later price. With linear demand Q, one can check that τ does not affect the seller's total demand, which is again equal to $q = \frac{5}{9} - p$.[20] The seller's profit is therefore

$$p\left(\tfrac{5}{9} - p\right) + (\tau \times \text{buy-later demand}) .$$

Buy-later demand is the area of the triangle in Figure 2, which is $\frac{1}{2}(S^{-1}(s+\tau))^2$. Thus, the seller's profit is additively separable in the buy-now price p and the buy-later premium τ. As such, the optimal buy-now price is again $p = \frac{5}{18}$, while the optimal buy-later premium

[20] In general, one can show that introducing a buy-later premium $\tau \geq 0$ weakly reduces total demand if Q is convex and weakly increases total demand if Q is concave (and there is some buy-now demand).

can be calculated to be $\tau \approx 0.121$, which is about 43% of the buy-now price. Note that when the seller engages in this form of price discrimination, *both* its prices weakly increase, which contrasts with the usual case in monopoly price discrimination where the optimal uniform price typically lies between the discriminatory prices. Here, about 70% of the seller's customers buy without search, and so this sales tactic significantly deters search. The seller's profit with a buy-now discount is about 13% higher than it would be with free recall, while consumer surplus is 2% lower.

Finally, suppose the seller makes an exploding offer at price p. A type-u buyer will buy if and only if $u - p \geq \bar{v} - s = \frac{4}{9}$. Therefore, the seller's demand with an exploding offer at price p is the same as with free recall (as implied by Proposition 2), and so the optimal price with an exploding offer is also $p = \frac{5}{18}$ and the seller makes the same profit using the two sales techniques. By definition, all of the seller's customers buy without search. Buyers are now about 6% worse off relative to free recall.[21]

In this section we discussed two simple ways to deter search—buy-now discounts and exploding offers—and found conditions which ensure they generate higher profits than free recall. These procedures are easy to communicate and rationalize to buyers, and so merit analysis in their own right. However, while it has been demonstrated that the seller has an incentive to depart from free recall, the *ad hoc* nature of these sales tactics makes it hard to obtain attractive results about the most profitable version of either tactic. For instance, it is unclear beyond the uniform example whether the optimal buy-now discount tariff induces more buyers to forego search than would be the case with free recall. In the next section we derive the optimal selling mechanism. As well as being a useful benchmark with which to compare profits in the *ad hoc* schemes, the description of the optimal way to sell is more transparent in terms of the primitives of the model.

4.2 The optimal way to sell

This section studies the seller's optimal selling procedure. As will be seen, the most profitable mechanism incorporates a buy-now discount, in that a buyer has the option

[21] In this example with buy-now discounts or exploding offers, one can check that the "buy and search" option mentioned in footnote 18 is dominated either by buying without search or by consuming the outside option for sure.

either to buy the product immediately at a relatively low price or return to buy later at a relatively high price (in fact, at the monopoly price p^* which maximizes $pQ(p)$). However, the mechanism also uses an additional instrument, which is that the buyer can choose an option contract, where by paying a non-refundable deposit on her first visit she obtains the right to return to buy later at a specified exercise price. In this mechanism, a buyer with high u will choose to buy without search, a buyer with low u will never purchase the item, while a buyer with intermediate u will search but pay for an option to return later if the outside option is unsatisfactory.

The form of the optimal selling scheme is presented in the following result, the proof of which is in the appendix.

Proposition 3 *Let*

$$\lambda(u) \equiv u - \frac{1 - F(u)}{f(u)} \tag{7}$$

be the agent's "virtual utility" (which is increasing under condition (3)), and let p^* *satisfy* $\lambda(p^*) = 0$ *and* $\hat{u}$ *satisfy* $\lambda(\hat{u}) = a$. *Then the optimal selling procedure satisfies:*
(i) if $u < p^*$, *the type-u buyer consumes the outside option for sure;*
(ii) if $p^* \leq u \leq \hat{u}$, *the type-u buyer pays the seller a non-refundable deposit*

$$D(u) = \int_{p^*}^{u} G(\lambda(\tilde{u}))(\lambda'(\tilde{u}) - 1)d\tilde{u} \ , \tag{8}$$

investigates the outside option, and returns to buy the seller's product later if $v < \lambda(u)$ *at the incremental price*

$$p(u) = u - \lambda(u) = \frac{1 - F(u)}{f(u)} \ ; \tag{9}$$

(iii) if $u > \hat{u}$, *the type-u buyer purchases the seller's product without search at price*

$$P = D(\hat{u}) + p(\hat{u}) \ . \tag{10}$$

Note that when $u = p^*$, the associated contract has $D(u) = 0$ and $p(u) = p^*$, so a buyer always has the option to return to buy at the monopoly price without paying a deposit. More generally, in the range $[p^*, \hat{u}]$ the buy-later price $p(u)$ decreases with u, while the corresponding deposit $D(u)$ increases with u. (This is because a buyer with a higher u anticipates she is more likely to buy later, and hence she has more at stake in securing a low buy-later price.) The pattern of demand in this optimal selling mechanism is illustrated in Figure 3.

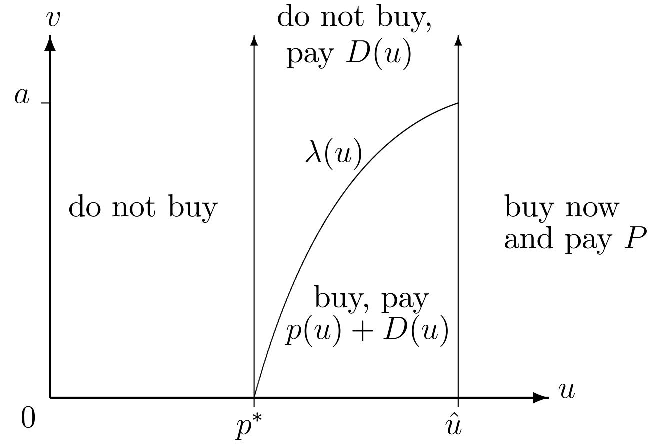


Figure 3: Demand in the optimal mechanism

Intuition for the form of the optimal mechanism, and for the direction in which consumption is distorted relative to the first-best benchmark, goes as follows. Suppose, as in section 3, the seller *can* observe a buyer's valuation, u. It is intuitive (and can be shown formally) that the most profitable way to sell to the type-u buyer is either: (i) to prevent the buyer from searching, or (ii) to allow the buyer to search but charge her for the option to buy later at the efficient price of zero. The maximum profit the seller can obtain with (i) involves making an exploding offer such that the buyer is just indifferent between accepting and going on to search (which yields the buyer surplus $\bar{v} - s$), and this generates profit

$$s + u - \bar{v} \ .$$

With strategy (ii), the largest deposit D the seller can charge the type-u buyer for the right to return to buy later for free satisfies $\mathbb{E}_v[\max\{u, v\}] - D = \bar{v}$, and the resulting profit is

$$\mathbb{E}_v[\max\{u, v\}] - \bar{v} = S(u) + u - \bar{v} \ .$$

We deduce that the no-search strategy (i) is more profitable if and only if $s \geq S(u)$, i.e., if $u \geq a$. Otherwise, the buyer is instructed to search, she returns to buy when $v < u$, and hence purchases with probability $G(u)$.

When u is the buyer's private information, however, the seller wishes to reduce the buyer's information rent. It does so by choosing a probability of purchase which is weakly *below* this first-best level, except for the highest valuation buyer. Given assumption (5), therefore, the type-$u_{\max}$ buyer purchases without search in the optimal second-best mechanism, as do all buyers with $u \geq \hat{u}$, although the downward distortion implies that fewer buyers purchase without search than in the first-best procedure (that is, $\hat{u} > a$). Those who search either buy with probability zero (if $u < p^*$) or with probability $G(\lambda(u))$ (if they retain the right to buy later), and in either case this is below the corresponding first-best probability.

Proposition 3 also implies the following results:

Corollary 1 *In the optimal selling mechanism: (i) it is cheaper for the buyer to buy immediately than to buy after search (i.e., $P \leq D(u) + p(u)$), and (ii) more buyers buy without search than would be the case with free recall.*

Proof. Part (i) follows from the facts that $P = D(\hat{u}) + p(\hat{u})$ and $D(u) + p(u)$ decreases with u in the range $p^* \leq u \leq \hat{u}$. (To see the latter, note that the derivative of $D(u) + p(u)$ equals $(\lambda'(u) - 1)(G(\lambda(u)) - 1)$, which is negative since (3) implies $\lambda' > 1$.) Now we prove part (ii). If there is no buy-now demand with free recall, it is trivially the case that there must be more buy-now demand in the optimal mechanism. Therefore, suppose there is positive buy-now demand with free recall. We know that in the free-recall regime the buyer buys immediately when $u \geq a + p_F$, where p_F is the optimal free-recall price. In the optimal selling scheme, the buyer buys immediately when $u \geq \hat{u} = a + \hat{p}$, where $\hat{p}$ maximizes $pQ(p+a)$. (The price $\hat{p}$ that maximizes $pQ(p+a)$ satisfies $\lambda(a+\hat{p}) = a$, and so $\hat{u} = a+\hat{p}$.) However, since $Q(p+a)$ is buy-now demand in the free-recall regime with price p, and since Lemma 1 shows that buy-now demand is more elastic than total demand in the free-recall regime, it follows that $p_F > \hat{p}$, and so $a + p_F > a + \hat{p}$, i.e., the seller deters search in the optimal mechanism. ■

Returning to the linear demand example where u and v are uniformly distributed on $[0, 1]$ and $s = \frac{1}{18}$, we have $\lambda(u) = 2u - 1$ and so the buy-now threshold in the optimal mechanism is $\hat{u} = \frac{5}{6}$, while a buyer will never purchase the seller's product if $u < p^* = \frac{1}{2}$. As with all the earlier *ad hoc* schemes, total demand is $\frac{5}{18}$. Of those who eventually buy the product, exactly 60% do so without search. Expression (10) implies that the buy-now price is again $P = \frac{5}{18}$, while the seller's profit from the optimal mechanism is about 0.09. This is 16% greater than the profit with free recall, but only 3% more than the profit generated with the simple buy-now discount scheme. Aggregate consumer surplus here is about 4% below that obtained with free recall. Although total output is identical in all four regimes—that is, with free recall, buy-now discounts, exploding offers, and the optimal mechanism—the pattern of consumption in (u, v)-space is different in each case. Since the buy-now price is the same in all regimes, in this example buyers are best off with free recall (as that regime has an unrestricted buy-later policy) and worst off with an exploding offer (which entirely removes their ability to buy later).

4.3 Search deterrence without commitment

So far in this section we have assumed that the seller can commit to its buy-later policy when it first meets the buyer, and we now discuss the outcome when this assumption is relaxed. Suppose that the seller sets one price at the initial meeting with the buyer and then a second price if the buyer returns to buy after discovering the outside option. We assume the seller can make no commitment about the buy-later price at the first meeting, and the actual buy-later price can be discovered only after she returns to the seller. We assume that buyers are rational, and foresee the seller's equilibrium price if they return later. (At the end of this section we also discuss how lack of commitment affects the seller's ability to use the optimal mechanism.)

Here, unlike the rest of section 4, it makes an important difference whether or not the buyer faces an intrinsic cost of returning to the seller after search. In the simple case in section 3, where there were no search costs of any form, we showed how the seller had an incentive to offer a low buy-now price to prevent the buyer searching, even when it could not commit to its buy-later price. A similar analysis carries over to the situation where u is the buyer's private information and search costs are present.[22] In this case, however, high-u buyers will purchase immediately, as in Coase (1972), leaving a pool of relatively

[22] For instance, in the running example where u and v are uniformly distributed on $[0, 1]$ and the search cost is $s = \frac{1}{18}$, the seller's most profitable time-consistent strategy is to offer the buy-now price $p_1 \approx 0.251$, which induces the subgame-perfect buy-later price $p_2 \approx 0.286$.

low-u buyers who might purchase later. This adverse selection gives rise to a countervailing incentive to set a low price for buy-later demand, and only when the search cost is not too large will buy-now discounts be offered in equilibrium.[23]

The outcome is more clear-cut in the situation in which a buyer incurs an exogenous cost $r > 0$ to return to the seller after search. (Indeed, in the following we need not assume that Q is logconcave, nor that u and v are independently distributed.) With an intrinsic returning cost $r > 0$, no matter how small, there is no equilibrium with positive buy-later demand. The fact that the buyer has chosen to incur the return cost to buy reveals that her outside option was poor, and gives the seller an incentive to set a high buy-later price. In essence, the seller's buy-later demand is perfectly inelastic around the buyer's anticipated buy-later price. As a result, any equilibrium involves a buyer either buying immediately at the initial price, or searching but never returning. The seller will then choose its initial price to maximize its profit, anticipating that no buyer will return, and the outcome is as if it must make an exploding offer.[24]

Proposition 4 *Suppose a buyer incurs a positive cost of returning to the seller after search. Suppose the seller sets an initial price on a buyer's first visit, but cannot commit to the price it will offer if the buyer decides to search and return later. Then the unique equilibrium outcome is as if the seller makes an exploding offer.*

Proof. Suppose by contrast that an equilibrium where some buyers return to purchase after search exists. Consider a buyer who does not accept the initial price but returns to try to buy from the seller after investigating the outside option. By revealed preference, such a buyer must have taste parameters (u, v) and have anticipated a buy-later price p_2^e such

[23] If a buyer can pretend to search (without incurring the search cost), then in a situation where the buy-later price is lower than the buy-now price, all buyers can purchase at the low price and the outcome is as if there is free recall.

[24] One can also derive the seller's policy under the alternative assumption that buyers are *naive*, in the sense that they expect to be offered the same price when they return after search as they were initially offered. When the seller sets initial price p_1, therefore, a naive buyer who returns to try to buy later has parameters (u, v) such that $u - p_1 - r \geq v$. Thus, the seller has an incentive to raise its price by at least r. However, the incentive to inflict a surprise price rise on naive buyers does not depend on there being an intrinsic returning cost. Without a return cost, Figure 1 implies that buy-now demand from naive buyers with initial price p_1 is $Q(p_1 + a)$, and given p_1 the firm chooses its subsequent price p_2 to maximize profit given that remaining buyers have type $u \leq p_1 + a$. In our running example with uniform u and v and $s = \frac{1}{18}$, one can check that the firm's optimal policy is to set $p_1 \approx 0.262$ followed by $p_2 \approx 0.310$.

that $u - p_2^e - r > v$. Suppose the seller actually sets a slightly higher buy-later price, say $p_2^e + \varepsilon$. Though surprised, the buyer is willing to accept this offer if $\varepsilon < r$. Therefore, the seller has an incentive to raise its buy-later price above p_2^e, which contradicts the buyer's belief, and there can be no equilibrium in which the buyer returns to buy later if she does not accept the initial price. Hence, the only possible equilibrium outcome in this setting is as if the seller makes an exploding offer, and such an equilibrium exists under suitable assumptions about the seller's beliefs about the buyer's preferences if the latter does return to buy. (For example, suppose the seller believes that a buyer who returns to purchase later has taste parameters $u = u_{\max}$ and $v = 0$, in which case it would charge $p_2 = u_{\max}$. Anticipating this, no buyer would incur the cost r to return to the seller.) ■

The seller makes greater profit with an exploding offer than with free recall only under restrictive conditions. (Proposition 2 shows that when the demand curve is convex, free recall is the more profitable strategy.) As a commitment device, therefore, the seller might welcome consumer protection policy which requires it to allow free recall.

In practice sellers themselves can usually find ways to commit, at least partially, to future prices if the buyer returns later. For example, the seller may be able to commit to an *upper bound* on its buy-later price when a buyer first visits. (In a store, this upper bound might be the price label on the product, and a sales assistant has no authority to raise the price above the displayed price.) Then, using the same argument as for Proposition 4, the only equilibrium outcome is that the seller charges a buy-later price equal to this upper bound if the buyer does not accept its initial offer. In this case, the buy-now discount regime analyzed in section 4.1 can be implemented.

Likewise, if the buyer pays a deposit in the optimal mechanism, the seller may be legally obliged to sell to the buyer later at the specified price. Suppose, though, that a buyer did not pay the required deposit on her first visit, so that the seller has made no commitment to the price she will be offered if she attempts to buy later. By the same argument as in Proposition 4, the only equilibrium is that the seller's price for this returning buyer is so disadvantageous that it is not worthwhile for the buyer to return. Thus, the optimal mechanism studied in section 4.2 remains credible even if the seller cannot commit to refuse to serve a buyer who did not contract for the right to buy later.

5 Search Deterrence in Oligopoly

Up to this point the buyer's outside option has been exogenous. However, the most natural interpretation of the outside option is that it is an offer made by a rival seller. In this final section we extend the analysis to a duopoly market with strategic competitors, each of which considers deterring search. Compared to our monopoly model, an important difference here is that a seller also meets buyers who have already encountered the rival and know their outside option. We assume that a seller cannot tell whether or not a buyer it meets for the first time has already visited its rival, although we maintain the assumption that a seller can distinguish whether a buyer is visiting it for the first or for the second time. The extension to duopoly allows us to consider the impact on equilibrium profit and welfare when this form of price discrimination is used. (At least with commitment, a monopoly seller can only benefit from an ability to deter search, and in the monopoly model we could not calculate welfare when the buyer consumes the outside option, as the profit of the supplier of the outside option was not specified.)

As mentioned in section 3, uncertainty about the outside option v might concern the match utility of the alternative product or its price. In the following, we present two models: one with horizontally differentiated products with uncertain match quality and deterministic prices, and another with perfect substitutes and uncertain prices. In each case, we show that search deterrence arises in equilibrium.

5.1 Differentiated products

There are two symmetric sellers, 1 and 2, and a buyer's utility from a seller's product is an independent random draw from a common distribution with CDF $F(u)$ and support $[0, u_{\max}]$. As before, assume that monopoly demand $Q(p) \equiv 1 - F(p)$ satisfies (3). We consider a game where sellers first choose their selling procedures and prices, and then buyers search sequentially. A buyer discovers one seller's match utility and selling policy for free, but needs to incur the search cost s to discover the second seller's utility and selling policy. Unless stated otherwise, we assume there is no intrinsic cost to return to a previously visited seller and that a seller can commit to its selling policy when it meets a buyer for the first time.

We analyze the Perfect Bayesian Equilibrium in this market. Buyers do not observe a seller's actual choice of price and sales policy before they start searching, but hold rational expectations of firms' strategies. Information unfolds as the search process goes on, but buyers' beliefs about the offer made by the second seller are unchanged even if they see an off-equilibrium offer from the first seller. We focus on symmetric pure-strategy equilibria in which sellers choose the same selling strategy and buyers visit sellers in a random order (with half the buyers meeting seller 1 first, and the remainder first meeting seller 2).

Free recall: If both sellers allow free recall, the situation is a duopoly version of Wolinsky (1986)'s model of search. Define

$$V(p) \equiv \mathbb{E}_u[\max\{0, u-p\}] = \int_p^{u_{\max}} Q(u)du \ , \tag{11}$$

so that $V(p) - s$ is the expected net benefit of incurring search cost s to visit a monopolist who charges p for its product. We assume that

$$s < V(p^*) \ , \tag{12}$$

where, as before, p^* is the price which maximizes $pQ(p)$. Condition (12) implies a buyer is willing to incur the cost s to visit a monopolist which charges the monopoly price.

Let p_F be the symmetric equilibrium price in the free-recall regime, when a symmetric equilibrium exists. Suppose for now that this equilibrium price is below the monopoly price p^*, and so a buyer is willing to investigate the rival seller if the first offer was disappointing. To derive the equilibrium price we need to calculate a seller's demand if it sets a different price, and suppose seller i chooses price p while seller j sets the equilibrium price p_F.

Consider a buyer who visits seller i first and finds match utility u_i there. If she buys immediately, her net surplus is $u_i - p$, while if she chooses to search and visit seller j, her expected net surplus is

$$\mathbb{E}_{u_j}[\max\{u_i - p, u_j - p_F\}] - s = u_i - p + V(u_i + p_F - p) - s \ .$$

If we define A by

$$V(A) \equiv s \ , \tag{13}$$

then the buyer will buy immediately from seller i if and only if $u_i \geq A + p - p_F$. Here, A is the threshold match utility which induces immediate purchase when the two sellers offer the same price. (Since $V(\cdot)$ is a decreasing function, (12) and (13) imply that $A > p^*$.) If the buyer chooses to investigate the second seller, she will then buy from the seller with the greater net surplus, provided that surplus is non-negative. This pattern of demand is depicted on Figure 4a, where the shaded area represents those buyers who purchase neither product. On the other hand, if a buyer first visits the rival seller j, this buyer anticipates that seller i will offer the equilibrium price p_F and so she buys immediately from j if $u_j \geq A$, and otherwise she investigates i and then chooses the superior option (if that net surplus is positive). This case is depicted on Figure 4b.

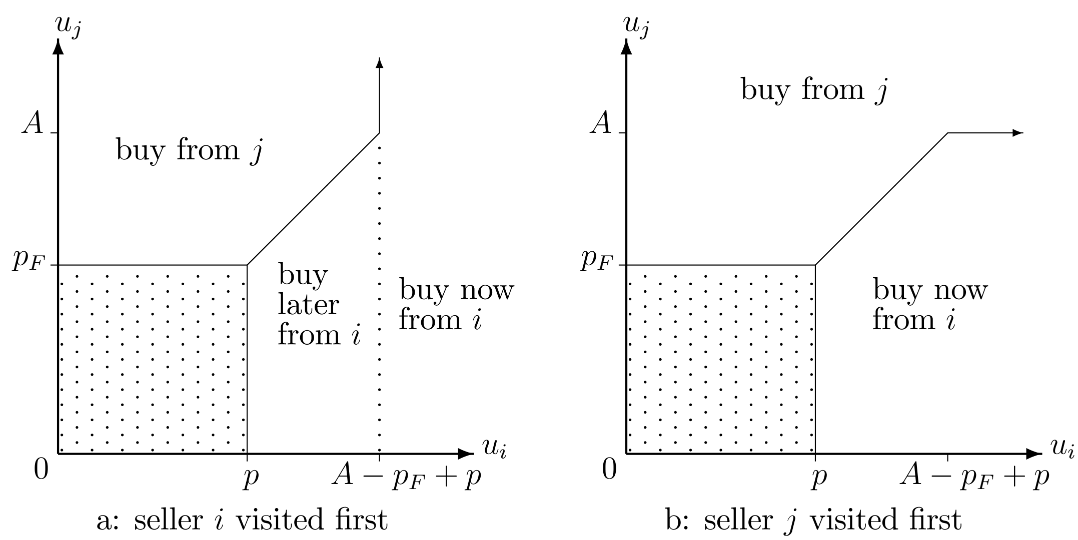


Figure 4: Demand in duopoly with free recall

In the appendix we derive the first-order condition for the symmetric equilibrium price p_F. One can show that a symmetric equilibrium exists if the monopoly profit function, $pQ(p)$, is concave, and that the equilibrium price is indeed below the monopoly price p^*. Unlike the monopoly setting, here there is always some buy-now demand in symmetric equilibrium (provided $s > 0$), since if a buyer finds the highest possible match utility $u_{\max}$ at the first seller, she does not expect to do better at the rival and will not search.

Search deterrence: Starting from the free-recall equilibrium, each seller's incentive to introduce a buy-now discount or an exploding offer is as in the monopoly setting in section 4.1. An argument similar to that in Lemma 1 shows that a seller's buy-now demand in Figure 4 is more elastic than its total demand given condition (3). Arguments similar to Propositions 1 and 2 then demonstrate when a seller has an incentive to introduce a buy-now discount or make an exploding offer, as stated in the following result.[25,26]

Proposition 5 *Starting from a free-recall equilibrium:*
(i) a seller has a unilateral incentive to offer a buy-now discount;
(ii) if $Q(\cdot)$ is concave whenever $Q > 0$ a seller has a unilateral incentive to make an exploding offer.

In the appendix we discuss how to derive the first-order conditions for tariffs when both sellers use buy-now discounts or exploding offers. In the regime with exploding offers, again a symmetric equilibrium exists if the monopoly profit function, $pQ(p)$, is concave, and the equilibrium price is then below the monopoly price p^*. In the regime of buy-now discounts, though, it seems hard to provide a general condition which ensures the existence of symmetric equilibrium. In either case, search deterrence arises whenever a symmetric equilibrium exists, i.e., more buyers purchase immediately at the first seller they find than would be the case with free recall.[27]

Similarly to our earlier Proposition 4, if sellers cannot commit to the buy-later price but buyers incur an intrinsic cost of returning to a previously visited seller, a seller has an incentive to hold-up a returning customer with a high buy-later price, with the result that the only equilibrium is as if the sellers could only make exploding offers.

One can also investigate the outcome when both sellers can use general (deterministic) selling mechanisms, parallel to the monopoly analysis in section 4.2. However, the required

[25] A buy-later premium or an exploding offer made by seller i only affects those buyers who visit it first. For them, their outside option when they first meet seller i is $\max\{0, u_j - p_F\}$ where $j \neq i$. Since the arguments in the monopoly model did not rely on the distribution of the outside option, the same results hold with duopoly.

[26] As shown in Armstrong and Zhou (2011), this result holds with any finite number of sellers. In the limiting case with an infinite number of sellers, though, with free-recall buyers never return once they leave a seller. Therefore, there is no buy-later demand and no way for sellers to deter search by making it harder to buy later.

[27] In the free-recall regime, a buyer will purchase immediately at the first seller if $u \geq A$. In the regime with exploding offers and a symmetric price $p < p^* (< A)$, a buyer with match utility u at her first seller will accept the exploding offer if $u - p \geq V(p) - s$. Hence the threshold utility for immediate purchase is $p + V(p) - s = p + V(p) - V(A) < A$, where the inequality arises since $V(\cdot)$ is a decreasing function with slope greater than -1. In a symmetric equilibrium with buy-now discount τ, Figure 7a in the appendix shows that the threshold utility for immediate purchase is $V^{-1}(s + \tau) + \tau$, which is also smaller than $A = V^{-1}(s)$ since $V^{-1}(\cdot)$ is a decreasing function with slope less than -1.

analysis for duopoly is substantially more complex than that for monopoly. The main reason is that when a buyer comes to the seller for the first time, she has another dimension of private information which is whether or not she has already visited the rival and obtained her outside option (and if so, what that outside option is), which entails solving a multidimensional screening problem.[28] Similarly to Proposition 3 in the monopoly model, it turns out that a seller can restrict attention to a simple family of mechanisms:

Proposition 6 *Suppose sellers use general (deterministic) selling mechanisms. Then it is without loss of generality to look for a symmetric equilibrium where each seller offers a buy-now price and a menu of deposit contracts.*

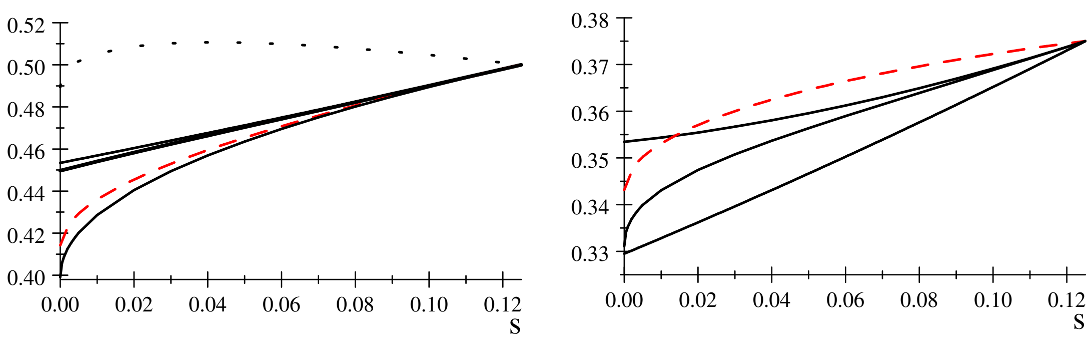
(The proof of this proposition is contained in a separate online appendix. There we also provide the procedure to derive the symmetric equilibrium with general selling mechanisms, if it exists, and the details of the equilibrium mechanism in the uniform example presented below.[29] As in Corollary 1 it is also shown that search deterrence occurs whenever a symmetric equilibrium exists.)

As with the regime of buy-now discounts it is hard to derive a general condition which ensures the existence of a symmetric equilibrium in which sellers use general selling mechanisms. For this reason, in the remainder of this section we focus on the example of a uniform distribution, so that $Q(p) = 1 - p$, in which case symmetric equilibrium exists in all four regimes. Condition (12) requires $s < \frac{1}{8}$ in this example, and so in the following figures outcomes are plotted as a function of the search cost over the range $0 \leq s \leq \frac{1}{8}$.

[28] Deb and Said (2014) study a related model in the context of monopoly. They extend Courty and Li (2000) by introducing a group of buyers who arrive at the second period, alongside first-period buyers who may have chosen to delay their purchase. The seller therefore faces a multidimensional screening problem in the second period. Like us, for reasons of tractability they focus on deterministic mechanisms.

[29] In more detail, the mechanism each seller uses is as follows. A seller asks a buyer it meets whether or not she already knows her outside option. If she reports she does not, she participates in a mechanism of the form used in the monopoly analysis (restricted to be deterministic). If she reports she does know her outside option already, she is asked to report both the value of the outside option and her utility at the seller, and contingent on that report she either obtains the seller's product at a specified price, or she obtains nothing from the seller. Of course, the buyer must be induced to report truthfully whether she already knows her outside option. In equilibrium, a seller cannot perfectly infer whether a first-time visitor has already visited the rival or not. Both the high-valuation buyers who visit the seller first and the buyers who come after visiting the rival (if they eventually buy from the seller) choose the buy-now price.

Figure 5a depicts the equilibrium prices in the four regimes. The free-recall price is depicted as the dashed curve, and the three solid curves respectively as we move upwards represent the buy-now price in the equilibrium general mechanism, the buy-now price in the buy-now discount scheme, and the exploding-offer price. When buy-now discounts or exploding offers are used, even the buy-now price is higher than the free-recall price, so buyers end up paying more. The intuition is that search deterrence adds to frictions in the market, and this allows sellers to charge higher prices. When sellers compete with general mechanisms, the buy-now price is slightly lower than the free-recall price, but the buy-later price (at which a buyer can return to buy a seller's product without paying a deposit) is significantly higher. (The latter is depicted as the dotted line on the figure.) The effect of search deterrence on market prices is the most significant when the search cost is small. When the search cost approaches its maximum level which allows search to occur, buy-later demand becomes negligible even with free recall, and all prices converge to the monopoly price $p^* = \frac{1}{2}$.



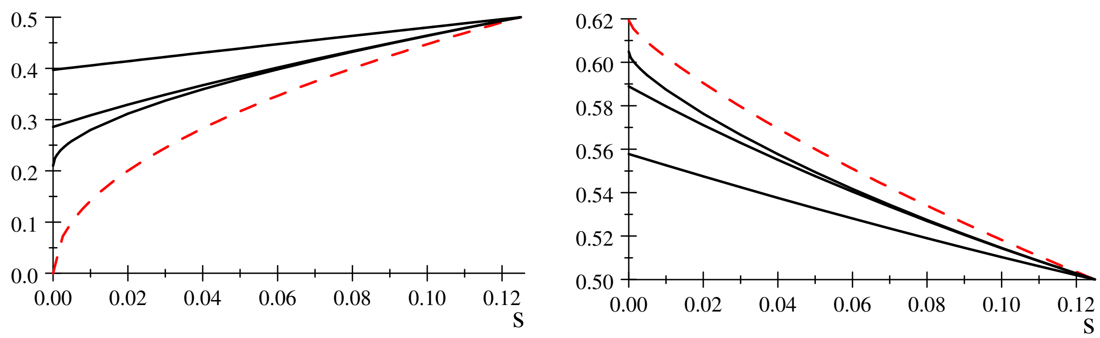
a: Prices

b: Industry profit

Figure 5: Buy-now prices and profits in four regimes

Figure 5b compares industry profit in the four regimes. Profit with free recall is represented by the dashed curve, while the three solid curves depict respectively as we move downwards the regimes of buy-now discounts, general mechanisms and exploding offers. The use of exploding offers and general mechanisms always reduces industry profit, while buy-now discounts reduce profits unless the search cost is small. Search deterrence reduces profit because it leads to high prices which exclude too many buyers.[30] Because profit usually falls when sellers induce buyer to purchase immediately, sellers might welcome a consumer protection policy which prevents their use of these tactics, if such regulation was feasible and applied to all sellers in the market.[31]

Figure 6a shows the proportion of all buyers who purchase immediately at the first seller—i.e., the fraction who do not search—in this example. (From top to bottom, the curves correspond to the regimes with exploding offers, buy-now discounts, general mechanisms, and free recall.) As one would expect, this fraction increases with the intrinsic search cost s in each case. Again, the impact of search deterrence is most marked when s is small. Here, few buyers buy immediately with free recall, since there is usually some chance they will find a better offer from the second seller, but with an exploding offer 40% of buyers purchase without search.



a: Fraction who buy immediately

b: Welfare

Figure 6: Search deterrence and welfare in four regimes

Thus, at least in this example, when sellers can recognize customers and pursue the various sales tactics, prices rise *and* search is deterred. As a result buyers are harmed twice: the chosen product is on average a less good match with their tastes, and they

[30] In the regime of general mechanisms, the fraction of buyers who leave the market without purchasing anything is the product of the buy-now price and the buy-later price. As we have seen, the former is slightly lower but the latter is much higher than the free-recall price, and net effect is that more buyers are excluded from the market.

[31] Recall from the introduction how a number of law journals collectively agreed to stop making exploding offers to authors.

pay more for this product. (With general mechanisms, although those buyers who buy immediately without search pay a slightly lower price, consumer surplus is still lower than in the free-recall regime.) Figure 6b shows total welfare—consumer surplus plus industry profit—in the four regimes. (From bottom to top, the curves correspond to exploding offers, buy-now discounts, general mechanisms, and free recall.) As is intuitive, these sales tactics cause limited harm in a market with significant intrinsic search frictions, since there is then little buy-later demand even with free recall. When intrinsic frictions are small, though, the extra friction induced by these tactics lead to significant welfare losses.

5.2 A homogeneous product

Suppose there are two sellers with costless production which compete to supply a homogeneous product. All buyers have the same value u for a unit of this product. One way to generate price uncertainty in an equilibrium model is to suppose that a fraction of buyers see only one seller's price, while others see more prices, in which case sellers choose their prices according to a mixed strategy. (For instance, see Varian (1980) for a model of this form.) In this spirit, suppose each buyer is sure to find one random seller (with each seller equally likely to be found first), and with exogenous probability $0 < \sigma < 1$ she can discover the second seller as well. A buyer searches sequentially through her available options, but without incurring search costs. We assume she does not know if she will be able to find the second seller at the time she meets the first seller.

Free recall: Consider first the outcome when sellers permit a potential buyer to purchase later at the same price originally offered. Since there are no search costs, when sellers allow free recall we assume that each buyer will search through all available options and buy from the seller with the lower price. A seller will be visited first by half of the buyers, and a fraction σ of them will find the second seller. It follows that each seller has $\frac{1-\sigma}{2}$ captive buyers, while σ buyers visit both sellers. Then, as is familiar from the previous literature, the assumption $0 < \sigma < 1$ implies that in equilibrium sellers choose prices according to a continuous mixed strategy with support $[p_{\min}, u]$, where $p_{\min}$ is the smallest price chosen in equilibrium by a seller.

In more detail, suppose the equilibrium mixed strategy is for a seller to choose its price to be above p with probability $X(p)$. Then the condition for a seller to be indifferent between all prices in the range $[p_{\min}, u]$ is that

$$\left[\frac{1-\sigma}{2} + \sigma X(p)\right] p = \frac{1-\sigma}{2} u \ . \tag{14}$$

The left-hand side is a seller's profit when it chooses price p, where the term $[\cdot]$ is the number of buyers who buy from this seller given that its rival chooses a higher price with probability $X(p)$. The right-hand side is the profit when the seller charges the monopoly price u (in which case only its captive buyers buy). The lower bound of the price distribution is

$$p_{\min} = \frac{1-\sigma}{1+\sigma} u \ .$$

If there are limited search frictions, in the sense that $\sigma \approx 1$, then equilibrium profit is small and prices are usually close to zero.

Note that if a seller allows free recall and chooses the monopoly price u, it is sure to achieve at least the free-recall level of profit $\frac{1-\sigma}{2} u$, regardless of the selling policies followed by its rival. (When it does this, it sells at least to those buyers it meets who are captive.) Thus, in contrast to the previous model with product differentiation, here there can be no equilibrium with search deterrence in which sellers make lower profit than with free recall.

Search deterrence: Given that its rival allows free recall and follows the mixed pricing strategy defined in (14), a seller with commitment power has a strict incentive to make an exploding offer. Free recall, therefore, is not an equilibrium strategy in this market when sellers are able to discriminate against buyers who wish to purchase after search.

Proposition 7 *Suppose sellers can commit to a buy-later policy when they meet a buyer for the first time. Then both sellers allowing free recall is not an equilibrium.*

Proof. In the free-recall regime with mixed pricing strategy in (14) with support $[p_{\min}, u]$, a seller's potential buyers can be divided into two groups: those who meet it first (and who may or may not be able to find the rival subsequently) and those who already have an offer from the rival. Suppose a seller deviates and makes an exploding offer with price $p > p_{\min}$. The latter group of buyers are not affected, and will buy with probability $X(p)$, just as with free recall at price p. So the effect of making an exploding offer stems from the first group alone. Given the exploding offer, a buyer who visits the seller first will accept it if her surplus $u - p$ is greater than her expected surplus from searching for the (possible) second option, $\sigma \mathbb{E}_{\tilde{p}}[u - \tilde{p}]$, which must be strictly less than $u - p_{\min}$. Therefore, when p is close to $p_{\min}$ this buyer will accept the exploding offer for sure, while with free recall at the same price the buyer accepts only with probability $1 - \sigma + \sigma X(p) < 1$. If a seller makes an exploding offer with price sufficiently close to $p_{\min}$, we deduce that a seller's profit strictly increases relative to when it allows free recall at any price in the range $[p_{\min}, u]$. ■

When sellers can use any selling mechanism, it is complicated in this model to fully investigate all possible equilibria. However, one simple equilibrium is that each seller makes an exploding offer at the monopoly price u. Given a buyer holds this expectation, she will then buy from the first seller she meets since she does not expect to find a strictly better offer and there is the chance $1 - \sigma$ she will end up with nothing if she rejects.[32] Given this search behaviour, the only buyers a seller meets are those who have it first in their search order. If this seller makes an exploding offer at price u, it makes monopoly profit u from each buyer it meets. This is the maximum profit it could extract if these buyers had no outside option at all, and so this seller cannot do better with any other selling scheme. In essence, the artificial search frictions introduced by exploding offers causes the Diamond Paradox to re-emerge, and prices to rise to the monopoly level.[33]

In this equilibrium with exploding offers, industry profit rises and consumer surplus falls, relative to the situation with free recall. This equilibrium corresponds to the outcome where sellers collude to maximize industry profits, and so search deterrence provides a decentralized means to achieve collusion in this market.

[32] Here, as usual with unit demand models, we are assuming that when a buyer is indifferent between accepting a seller's offer and buying nothing, she accepts the offer. The buyer would *strictly* prefer to purchase at the monopoly price for sure, rather than have a chance of being left with nothing, in a richer model where buyers had positive surplus at the monopoly price (e.g., if buyers had downward-sloping demand or heterogeneous values u for the homogeneous product).

[33] There is no price below the monopoly price which is an equilibrium with exploding offers. To see this, consider a hypothetical equilibrium where sellers makes exploding offers at price $p < u$. Suppose one seller now unilaterally raises its price to $\tilde{p} > p$. This seller only encounters buyers for whom it is first in their search order, and such a buyer is willing to accept the higher price if $u - \tilde{p} > \sigma(u - p)$, which is satisfied for $\tilde{p}$ close to p given $\sigma < 1$. Therefore, $p = u$ is the unique equilibrium price with exploding offers.

6 Conclusions

The analysis in this paper suggests that search deterrence is privately profitable, yet plausibly leads to welfare losses. Artificial search frictions drive up prices and (where relevant) reduce the quality of the match between product and consumer. These two kinds of harm can induce fewer buyers to buy, with the result that sellers in equilibrium can also be made worse off when these sales tactics are used.

Public policy might attempt to limit the use of such tactics. For instance, the *Unfair Commercial Practices Directive,* adopted in 2005 across the European Union, prohibits sellers in all circumstances "falsely stating that a product will only be available for a very limited time, or that it will only be available on particular terms for a very limited time, in order to elicit an immediate decision and deprive consumers of sufficient opportunity or time to make an informed choice". Nevertheless, the enforcement of such policies will inevitably be difficult given the casual nature of much sales interaction and the frequency with which discounts from regular prices are offered. A less direct—but possibly more effective—method to control aggressive sales techniques is to require a "cooling off" period for specified products, as is currently done in many jurisdictions. If a salesman manages to convince a consumer to buy immediately, through whatever means, the consumer then has the ability to cancel the deal within a specified period if she discovers a better deal elsewhere.

Our analysis could usefully be extended in a number of directions. In this paper, search deterrence required that a seller be able to recognize its customers. However, one could investigate if other selling techniques can profitably deter search when consumers are anonymous. "Flash sales", or short-run discounts, are a common marketing tactic, and a number of "daily deal" websites operate on the internet. A discount which is known to be short-lived can deter search, since consumers may be unable to take advantage of it if they take the time needed to search. In a similar vein, a seller which suggests that its price is likely to rise soon may deter search. For example, an airline's website might report there is "only one seat left at this price" when a traveller checks for current fares.

A way to deter search, which involves a more standard kind of customer recognition than studied in this paper, is to offer dynamic quantity discounts. Suppose a buyer potentially purchases the seller's product repeatedly, and can find alternatives to the product over time as well. If the buyer agrees to a long-term contract such that subsequent units are relatively cheap, she becomes less inclined to search for an alternative. For instance, if the buyer agrees to buy from the seller for two periods, she has less incentive to search for an alternative between periods, relative to a situation where the seller offers a sequence of one-period contracts. The analysis in this paper suggests that it will often be profitable, though not necessarily welfare-enhancing, for a seller to induce a buyer to accept a long-term contract in ignorance of her subsequent offers.[34]

One could also consider situations where a seller has some knowledge of the buyer's outside option. If the seller has information that the outside option is bad for the consumer, it may disclose this information to deter search. For example, a gas station might display a sign stating "last fuel for 20 miles", or "cheapest fuel in town". (The credibility of such statements will determine how consumers react to them.) On the other hand, a seller who knows the outside option is likely to be attractive might make an exploding offer to prevent its discovery by the buyer. When a seller's choice of buy-later policy can be made contingent on the outside option, a savvy buyer might then use the seller's policy as a signal of her outside option.

Although we analyzed a model with rational buyers, it is natural to consider the psychological impact of these sales tactics. Behavioural factors might make these tactics more or less effective than predicted in our model. If some buyers are susceptible in general terms to "sales pressure", it may be that these tactics are even more effective than our model with rational buyers suggests. (For instance, a seller might attempt to make a potential customer "feel bad" if the latter suggests she wants to get a second quote before deciding to buy.) On the other hand, some buyers may be antagonized by aggressive selling. Brown, Viriyavipart, and Wang (2014) describe an experiment in which human sellers face either human or robot buyers, and choose their price and whether their offer involves free recall or an exploding offer. Buyer valuations were randomly chosen so that exploding offers were optimal if buyers were rational (as the robots were programmed to be). However, human

[34] A similar issue has been explored in the labour market context, where a dynamic wage contract in which the wage increases with employment tenure can act to deter on-the-job search efforts by employees. For instance, see Stevens (2004).

buyers rejected exploding offers more often than rational buyers would, and sellers learned that exploding offers were less profitable than when playing against robot buyers. Just as buyers who shop around induce sellers to set low prices in conventional models of search, buyers who refuse to engage with high-pressure sales tactics may induce sellers to adopt a less aggressive approach.

APPENDIX: Proofs and Omitted Analysis

Proof of Proposition 1: Let p_F be the optimal free-recall price. As we can see from Figure 1, buy-now demand vanishes if and only if $p_F + a \geq u_{\max}$. Consider first the situation with positive buy-now demand in the free-recall regime, so that $p_F + a < u_{\max}$. (Clearly, this requires condition (5) to hold.)

Since p_F maximizes $p[q_1(p) + q_2(p)]$, it follows that

$$\frac{-p_F q_1'(p_F)}{q_1(p_F)} \frac{q_1(p_F)}{q_1(p_F) + q_2(p_F)} + \frac{-p_F q_2'(p_F)}{q_2(p_F)} \frac{q_2(p_F)}{q_1(p_F) + q_2(p_F)} = 1 \ ,$$

so that an average of the two demand elasticities equals 1. (Since $q_i(p_F) > 0$, the weight on each elasticity is positive.) Lemma 1 implies that q_1 is strictly more elastic than q_2, and so it follows that q_2 has elasticity less than 1 at p_F, i.e.,

$$q_2(p_F) + p_F q_2'(p_F) > 0 \ . \tag{15}$$

For the purposes of this proof, let $x_1(p_1, p_2)$ and $x_2(p_1, p_2)$ denote respectively the buy-now and buy-later demands with (non-uniform) prices p_1 and p_2, so that $q_i(p) \equiv x_i(p, p)$. Then

$$\begin{aligned} \frac{\partial}{\partial p_2} [p_1 x_1(p_1, p_2) + p_2 x_2(p_1, p_2)] &= p_1 \frac{\partial x_1(p_1, p_2)}{\partial p_2} + p_2 \frac{\partial x_2(p_1, p_2)}{\partial p_2} + x_2(p_1, p_2) \\ &= p_1 \frac{\partial x_2(p_1, p_2)}{\partial p_1} + p_2 \frac{\partial x_2(p_1, p_2)}{\partial p_2} + x_2(p_1, p_2) \ . \end{aligned}$$

Here, the second equality follows from Slutsky symmetry $\partial x_1/\partial p_2 \equiv \partial x_2/\partial p_1$, which is an implication of rational consumer choice. (This can be verified directly using Figure 2.) Setting $p_1 = p_2 = p_F$ in the above yields

$$\left. \frac{\partial}{\partial p_2} [p_1 x_1(p_1, p_2) + p_2 x_2(p_1, p_2)] \right|_{p_1 = p_2 = p_F} = p_F q_2'(p_F) + q_2(p_F) > 0 \ ,$$

where the inequality follows from (15). We deduce that, starting from the most profitable free-recall price p_F, increasing the buy-later price will boost profit.

Next, we show the result also holds if buy-now demand is zero, so that $p_F + a \geq u_{\max}$, provided condition (5) holds. Suppose the seller offers buy-later price $p_2 = p_F$ and buy-now price p_1 which satisfies $u_{\max} = p_F + S^{-1}(s + p_F - p_1)$. From Figure 2 we see that the type-$u_{\max}$ buyer is just indifferent between buying immediately and keeping searching. In other words, this buy-now-discount scheme generates the same demand pattern and profit as the free-recall price p_F. Since $S(a) = s$, we can write p_1 explicitly as

$$p_F - p_1 = \int_{u_{\max}-p_F}^{a} (1 - G(v))dv \ . \tag{16}$$

Note that $0 < p_1 \leq p_F$ given that $p_F + a \geq u_{\max}$ and that $a \leq u_{\max}$ in (5).[35]

Suppose the seller further decreases its buy-now price by a small ε (such that the buy-now price is still positive). Then those buyers with u close to $u_{\max}$ now buy immediately. Figure 2 implies that the buyer type who is indifferent between buying now and buying later is $u(\varepsilon) = p_F + S^{-1}(s + p_F - p_1 + \varepsilon)$. The seller's profit is then

$$\pi(\varepsilon) \equiv (p_1 - \varepsilon)Q(u(\varepsilon)) + p_F \int_{p_F}^{u(\varepsilon)} G(u - p_F)dF(u) \ ,$$

where $Q(u(\varepsilon))$ is the buy-now demand and the integral term is the buy-later demand. Using the facts $u(0) = u_{\max}$ and $u'(\varepsilon) < 0$, one can check that $\pi'(0)$ has the same sign as

$$p_1 - p_F G(u_{\max} - p_F) \ .$$

(Intuitively, this is the benefit from inducing a buyer with $u \approx u_{\max}$ to buy immediately: the seller earns p_1 if the buyer buys immediately, and if she searches on she will return with probability $G(u_{\max} - p_F)$ in which case the seller earns p_F.) Using (16) and the condition $a \leq u_{\max}$, one can check that this expression is positive. Therefore, even if there is no buy-now demand in the free-recall regime, offering a buy-now discount is more profitable than allowing free recall.

Proof of Proposition 3: Consider a two-stage direct mechanism consisting of the functions $\{x(u'); (q_0(u'), t_0(u')); (q(u', v'), t(u', v'))\}$. In the first stage, the buyer is required to

[35] Given $a \leq u_{\max}$, we have $p_F - p_1 = S(u_{\max} - p_F) - s \leq S(a - p_F) - s$. One can then check that the "buy and search" option discussed in footnote 18 is irrelevant.

report her valuation for the seller's product, u'. She is then instructed either to cease her search or to investigate the outside option.[36] With probability $x(u')$, she is instructed to stop searching, in which case the buyer pays $t_0(u')$ and obtains the product with probability $q_0(u')$. With probability $1 - x(u')$, she is instructed to investigate the outside option. In that case she is required to report the realization of the outside option, v'. Then contingent on the two reports, the buyer pays $t(u', v')$ and obtains the product with probability $q(u', v')$.

Given this mechanism, define

$$U(u, u'; v, v') \equiv \max\{u, v\}q(u', v') + v(1 - q(u', v')) - t(u', v') - s \; . \tag{17}$$

This is the type-(u, v) buyer's expected surplus if she is instructed to search for the outside option after reporting u' in the first stage and then reports v' in the second stage. (If the buyer obtains both the product and the outside option, she consumes the better option.) According to the revelation principle in a dynamic setting (see Myerson, 1986), without loss of generality we can focus on direct mechanisms such that (i) the buyer reports truthfully in the second stage *if* she has been truthful in the first stage and has been instructed to search, so that

$$U(u, u; v, v) = \max_{v'} : U(u, u; v, v') \; ; \tag{18}$$

and (ii) the buyer reports truthfully in the first stage, so that

$$\begin{aligned} \Phi(u) &\equiv x(u)[uq_0(u) - t_0(u)] + (1 - x(u))\mathbb{E}_v[U(u, u; v, v)] \\ &= \max_{u'} : x(u')[uq_0(u') - t_0(u')] + (1 - x(u'))\mathbb{E}_v[\max_{v'} U(u, u'; v, v')] \; . \end{aligned} \tag{19}$$

Note that if the buyer has lied in the first stage ($u' \neq u$), she is able—and in general has an incentive—to lie again in the second stage (so $v' \neq v$).[37] Here, $\Phi(u)$ defined in (19) is

[36] We assume that the seller is able to monitor whether the buyer does as instructed, and the buyer is not able to "step out the door and back in again" without incurring the search cost. However, the solution to the seller's problem assuming this monitoring ability—which is as described in Proposition 3—does not in fact require that the seller monitors search activity, and so this assumption plays no role.

[37] In the literature on sequential screening, a "strong truthtelling" condition is often imposed, which requires that the agent reports truthfully even if she previously lied. For example, see Courty and Li (2000), Krahmer and Strausz (2011), and section 5 of Pavan, Segal, and Toikka (2014). This is because in their settings, an agent's reporting incentives depend only on her previous reports, but not on whether those reports were truthful. Using our notation, this would be the case if U did not depend on the true u.

the buyer's surplus from participating in the mechanism. It is clear that Φ is an increasing function, and one can also show it is convex. From (19), the envelope theorem implies that

$$\Phi'(u) = x(u)q_0(u) + (1 - x(u))\, \frac{\partial}{\partial u}\mathbb{E}_v[U(u,u';v,v)]\bigg|_{u'=u}$$

$$= x(u)q_0(u) + (1 - x(u)) \int_0^u q(u,v)dG(v) \ , \tag{20}$$

where the second equality follows the definition of U in (17) which implies $\frac{\partial}{\partial u}U(u,u';v,v) = q(u',v)$ if $u > v$, and $\frac{\partial}{\partial u}U(u,u';v,v) = 0$ otherwise. Here, (20) is the probability that the type-u buyer consumes the seller's product, i.e., that the product is given to the buyer and that she prefers the product to the outside option if the latter is discovered.

The seller's problem is to choose $\{x, q_0, t_0, q, t\}$ in order to maximize its profit

$$\int_0^{u_{\max}} \{x(u)t_0(u) + (1 - x(u))\mathbb{E}_v[t(u,v)]\}dF(u)$$

subject to the pair of incentive constraints (18)–(19) and the participation constraint $\Phi(u) \geq \bar{v} - s$. (It is without loss of generality to assume that the seller offers a mechanism which is accepted by all buyers, as the non-participation option can be made available within the mechanism.)

We first solve a "relaxed" problem by imposing the participation constraint and only the local incentive compatibility constraint (20). From the definition of U in (17) and $\Phi(u)$ in (19), the profit from the type-u buyer is

$$x(u)t_0(u) + (1 - x(u))\mathbb{E}_v[t(u,v)] =$$

$$x(u)uq_0(u) + (1 - x(u))\left\{\int_0^u (u - v)q(u,v)dG(v) + \bar{v} - s\right\} - \Phi(u) \ . \tag{21}$$

Therefore, the seller's profit can be written as

$$\pi = \int_0^{u_{\max}} \left[x(u)uq_0(u) + (1 - x(u))\left\{\int_0^u (u - v)q(u,v)dG(v) + \bar{v} - s\right\} - \Phi(u)\right] dF(u) \ .$$

In such cases, the incentive for a type-u' buyer to report truthfully in the second stage on the equilibrium path implies that a type-u buyer will report truthfully in the second stage even after reporting u' (rather than u) in the first stage. Because of this difference we cannot impose the strong truthtelling constraint in our model. Without a strong truthtelling constraint, the incentive compatibility condition in the first stage is more complicated than in the usual case since we need to consider the optimal lying strategy in the second stage.

Using

$$\int_0^{u_{\max}} \Phi(u)dF(u) = \Phi(0) + \int_0^{u_{\max}} \Phi'(u)[1-F(u)]du$$

and (20), we can rewrite this profit as

$$\int_0^{u_{\max}} \left[x(u)\lambda(u)q_0(u) + (1-x(u))\left\{ \int_0^u (\lambda(u)-v)q(u,v)dG(v) + \bar{v} - s \right\} \right] dF(u) - \Phi(0) \ , \tag{22}$$

where $\lambda(\cdot)$ is given in (7).

This expression can be maximized point-wise with respect to x, q_0 and q, where each of these probabilities is constrained to lie between 0 and 1. This implies that $q_0(u) = 1$ if and only if $\lambda(u) \geq 0$, i.e., if $u \geq p^*$, where p^* is the monopoly price. This also implies

$$q(u,v) = \begin{cases} 1 & \text{if } v \leq \lambda(u) \\ 0 & \text{otherwise} \end{cases} \ . \tag{23}$$

In particular, both q_0 and q are zero when $u < p^*$, and in this range the buyer never obtains the product. From (22), in this range it is therefore optimal to set $x(u) = 0$. Substituting $q(u,v)$ in (23) we obtain

$$\int_0^u [(\lambda(u)-v)q(u,v)]dG(v) = \int_0^{\lambda(u)} [\lambda(u)-v]dG(v) = \int_0^{\lambda(u)} G(v)dv \ .$$

For convenience, we introduce the following notation:

$$R(x) \equiv \mathbb{E}_v[\max\{v,x\} - v] = S(x) + x - \bar{v} = \int_0^x G(v)dv \ . \tag{24}$$

Here, $R(x)$ measures the *ex ante* benefit to the buyer of being able to return to obtain surplus x from the seller once she has discovered her outside option v.

Then the seller's profit (22) simplifies to

$$\pi = \int_{p^*}^{u_{\max}} \left[x(u)\lambda(u) + (1-x(u))\left\{ R(\lambda(u)) + \bar{v} - s \right\} \right] dF(u) + F(p^*)(\bar{v}-s) - \Phi(0) \ . \tag{25}$$

From (24) we know that $R(\lambda(u)) = S(\lambda(u)) + \lambda(u) - \bar{v}$, and so the $\{\cdot\}$ term in the integrand is greater than $\lambda(u)$ if and only if $\lambda(u) \leq a$, i.e., if $u \leq \hat{u}$ where $\lambda(\hat{u}) = a$. Therefore, it is optimal to set $x(u) = 0$ if $u \leq \hat{u}$ and otherwise to set $x(u) = 1$. Together with the participation constraint $\Phi(0) = \bar{v} - s$, this implies that profit in (25) is

$$\pi = \int_{p^*}^{\hat{u}} \left[R(\lambda(u)) + \bar{v} - s \right] dF(u) + \int_{\hat{u}}^{u_{\max}} \lambda(u)dF(u) - (1-F(p^*))(\bar{v}-s) \ . \tag{26}$$

Since

$$\int_{\hat{u}}^{u_{\max}} \lambda(u)dF(u) = \hat{u}(1 - F(\hat{u})) \ ,$$

(26) simplifies further to

$$\pi = \int_{p^*}^{\hat{u}} R(\lambda(u))dF(u) + (1 - F(\hat{u}))(\hat{u} - (\bar{v} - s)) \ . \tag{27}$$

The profit (27) achieved in the relaxed problem is an upper bound on the profit achievable when we impose all incentive constraints. We show next that the selling mechanism described in Proposition 3 is implementable and generates profit equal to (27). We can then conclude that the selling mechanism in Proposition 3 is indeed the optimal selling mechanism.

If buyers behave as described in the statement of Proposition 3, the seller's profit is

$$\hat{\pi} \equiv (1 - F(\hat{u}))P + \int_{p^*}^{\hat{u}} \left[D(u) + p(u)G(\lambda(u))\right] dF(u) \ . \tag{28}$$

Using $R(\cdot)$ defined in (24), we can rewrite $D(u)$ in (8) as

$$D(u) = R(\lambda(u)) - \int_{p^*}^{u} G(\lambda(t))dt \ . \tag{29}$$

Then

$$\begin{aligned} \int_{p^*}^{\hat{u}} D(u)dF(u) &= \int_{p^*}^{\hat{u}} R(\lambda(u))dF(u) - \int_{p^*}^{\hat{u}} \left(\int_{p^*}^{u} G(\lambda(t))dt\right) dF(u) \\ &= \int_{p^*}^{\hat{u}} R(\lambda(u))dF(u) - \int_{p^*}^{\hat{u}} p(u)G(\lambda(u))dF(u) + (1 - F(\hat{u})) \int_{p^*}^{\hat{u}} G(\lambda(u))du \ . \end{aligned}$$

(The second equality is from integration by parts and the definition of $p(u)$ in (9).) Substituting this into (28) and using the expression for $P = D(\hat{u}) + p(\hat{u}) = D(\hat{u}) + \hat{u} - a$ yields

$$\begin{aligned} \hat{\pi} &= \int_{p^*}^{\hat{u}} R(\lambda(u))dF(u) + (1 - F(\hat{u})) \left(\int_{p^*}^{\hat{u}} G(\lambda(u))du + D(\hat{u}) + \hat{u} - a\right) \\ &= \int_{p^*}^{\hat{u}} R(\lambda(u))dF(u) + (1 - F(\hat{u}))(\hat{u} - (\bar{v} - s)) \ . \end{aligned}$$

(The second equality used the expression for $D(\hat{u})$ in (29) and $\bar{v} - s = a - R(a)$.) This is exactly the profit in (27).

The final step in our argument is to show that the type-u buyer chooses the appropriate contract in the statement of Proposition 3. Denote by $\phi(u', u)$ a type-u buyer's expected surplus if she chooses a deposit contract $(D(u'), p(u'))$ with $u' \in [p^*, \hat{u}]$. Then

$$\begin{aligned}\phi(u', u) &= \mathbb{E}_v[\max\{v, u - p(u')\}] - D(u') - s \\ &= R(u - p(u')) - D(u') + \bar{v} - s\ .\end{aligned}$$

It follows that

$$\frac{\partial \phi(u', u)}{\partial u'} = p'(u')\left[G(u' - p(u')) - G(u - p(u'))\right]\ . \tag{30}$$

First, consider a buyer with $u > \hat{u}$. Her surplus from buying the seller's product immediately without search is $u - P$. If she chooses one of the deposit contracts, her maximum surplus will be $\phi(\hat{u}, u)$ since (30) is positive for $u' \in [p^*, \hat{u}]$ and $u > \hat{u}$. But

$$\begin{aligned}\phi(\hat{u}, u) &= R(u - p(\hat{u})) - D(\hat{u}) + \bar{v} - s \\ &= a - D(\hat{u}) + \int_a^{u - p(\hat{u})} G(v)dv \\ &< u - p(\hat{u}) - D(\hat{u}) = u - P\ .\end{aligned} \tag{31}$$

(The second equality used $\bar{v} - s = a - R(a)$, the inequality follows because $u - p(\hat{u}) > \lambda(\hat{u}) = a$, and the last equality used the fact that $P = D(\hat{u}) + p(\hat{u})$.) Therefore, this buyer has no incentive to choose a deposit contract. On the other hand,

$$u - P > \hat{u} - P = \hat{u} - p(\hat{u}) - D(\hat{u}) = \lambda(\hat{u}) - D(\hat{u}) > a - R(a) = \bar{v} - s\ .$$

(The second inequality follows from $D(\hat{u}) < R(\lambda(\hat{u})) = R(a)$.) So this buyer has no incentive to consume the outside option for sure either.[38]

Second, consider a buyer with type $u \in [p^*, \hat{u}]$. Then (30) implies that $\phi(u', u)$ is single-peaked in u' and maximized by setting $u' = u$. One can also check that $\phi(u, u) \geq \bar{v} - s$, so this buyer does not wish to consume the outside option for sure. In addition, we have $\phi(u, u) \geq \phi(\hat{u}, u) \geq u - P$. (The last step here follows the same logic as in (31), but now

[38]Finally, one can check that these buyers do not wish to pursue the "buy and search" strategy. If such a buyer purchases the item at price P and searches, her expected surplus is $\mathbb{E}_v[\max\{u, v\}] - P - s$, and this is below her surplus if she buys without further search, $u - P$, when $s \geq \mathbb{E}_v[\max\{u, v\}] - u = S(u)$, i.e., if $u \geq a$. However, this is the case since $\hat{u} \geq a$.

$u - p(\hat{u}) \leq a$ and so the inequality is reversed.) Hence, this buyer does not wish to buy immediately either.

Finally, consider a buyer with type $u < p^*$. If she chooses a deposit contract, the maximum surplus is $\phi(p^*, u) = \bar{v} - s$ since (30) is negative for $u' \in [p^*, \hat{u}]$ and $u < p^*$. Likewise,

$$u - P < p^* - P = p^* - p(\hat{u}) - R(a) + \int_{p^*}^{\hat{u}} G(\lambda(t))dt < \hat{u} - p(\hat{u}) - R(a) = \bar{v} - s \ .$$

(The first equality used $P = D(\hat{u}) + p(\hat{u})$ and (29), and the second equality used $\hat{u} - p(\hat{u}) = \lambda(\hat{u}) = a$.) Therefore, the optimal choice for a buyer with $u < p^*$ is to consume the outside option for sure.

Analysis for free recall, buy-now discounts and exploding offers in section 5.1:

Free recall: By examining Figure 4, one sees that seller i's demand, when it deviates to p and seller j charges the equilibrium price p_F, is

$$\tfrac{1}{2}\left(1 + F(A)\right)\left(1 - F(A - p_F + p)\right) + \int_{p_F}^{A} F(u) f(u + p - p_F) du \ .$$

In equilibrium it equals $\frac{1}{2}(1 - F(p_F)^2)$ since a fraction $F(p_F)^2$ of buyers buy from neither seller. The first-order condition for p_F to be the equilibrium price is

$$\frac{1 - F\left(p_F\right)^2}{p_F} = f(A)(1 + F(A)) - 2\int_{p_F}^{A} F(u) f'(u) du \ . \tag{32}$$

As we showed in our working paper, Armstrong and Zhou (2011, Lemma 1), conditions (3) and (12) imply that (32) has a solution which satisfies $p_F < p^*$. Moreover, if $pQ(p)$ is concave, the first-order condition is also sufficient for p_F to be the equilibrium price.

In the uniform distribution example with $F(p) = p$, condition (32) becomes $1 - p_F^2 = (1 + A)p_F$, where $A = 1 - \sqrt{2s}$, and solving this equation yields the price and profit shown above in Figure 5.

Buy-now discounts: Here we derive the symmetric equilibrium tariff (P, τ), where P is the buy-now price and τ is the buy-later premium (so the buy-later price is $P + \tau$). Suppose seller i deviates to (P_i, τ_i). It is without loss of generality that we consider deviations restricted to $\tau_i \leq V(P) - s$.[39] For a buyer who visits seller i first and values its product at u_i, her surplus is $u_i - P_i$ if she buys immediately. If she chooses to search and visit seller j, her expected surplus is

$$\mathbb{E}_{u_j}[\max\{u_i - (P_i + \tau_i), u_j - P\}] - s = u_i - (P_i + \tau_i) + V(u_i - (P_i + \tau_i) + P) - s \ .$$

Therefore, this buyer will buy immediately if and only if $\tau_i > V(u_i - (P_i + \tau_i) + P) - s$, i.e., if $u_i > V^{-1}(s + \tau_i) + \tau_i + P_i - P$. If she visits both sellers, she will return to buy from i if $u_i - (P_i + \tau_i) > u_j - P$. The pattern of demand for these buyers who first visit seller i is depicted on Figure 7a. Buyers who first visit seller j hold equilibrium beliefs about seller i's pricing strategy, and so their demand is as shown on Figure 7b.

With the help of these figures, one can obtain expressions for a seller's total demand and buy-later demand, and use these to obtain the first-order conditions for (P, τ) to be an equilibrium within the set of buy-now discount pricing schemes. (See Armstrong and Zhou, 2011, section 3.2 for more detail.) Consider the case of a uniform distribution example, where $F(p) = p$ and $V^{-1}(x) = 1 - \sqrt{2x}$. As in the monopoly case, in this example seller i's buy-later demand does not depend on its buy-now price P_i and its total demand does not depend on its buy-later premium τ_i. As a result, seller i's profit is additively separable in τ_i and P_i. Here, the absence of cross-price effects greatly simplifies the profit function and enables us to show that equilibrium exists.

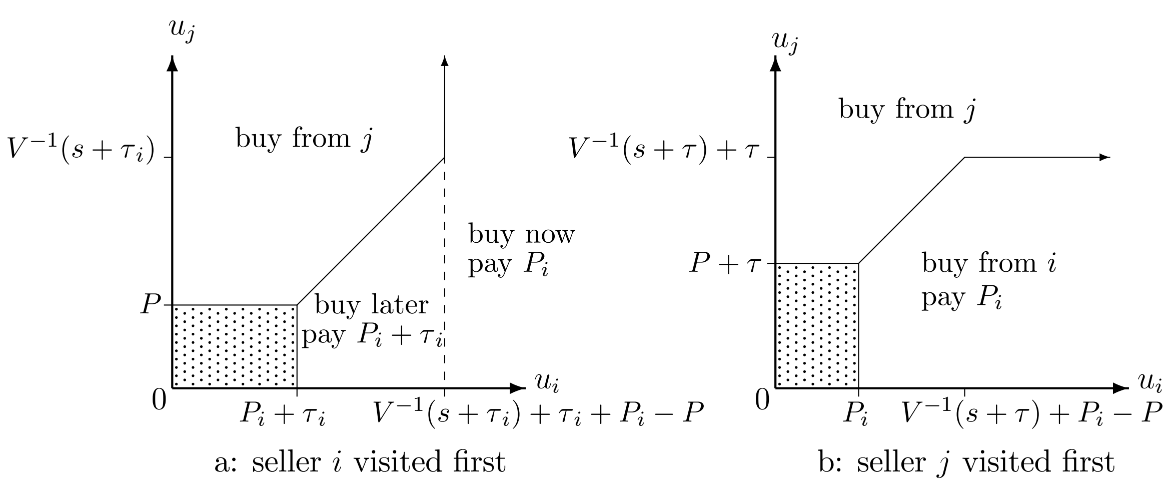


Figure 7: Demand in duopoly with a buy-now discount

[39] As can be seen from Figure 7a, when $\tau_i > V(P) - s$, buy-later demand disappears and i's profit is independent of τ_i. Hence, our restriction to $\tau_i \leq V(P) - s$ is without loss of generality.

Figure 7 shows that in symmetric equilibrium a fraction $P(P+\tau)$ of buyers buy nothing, and each seller's total demand is $\frac{1}{2}(1-P(P+\tau))$. Similarly to the free-recall case, the first-order condition for the buy-now price P is

$$1 - P(P+\tau) = (1 + V^{-1}(s+\tau) + \tau)P \ .$$

Since a seller's choice of τ_i does not affect its total demand, but only the proportion of buy-later demand, a seller chooses τ_i to maximize its revenue from those who buy later. Figure 7a implies that the volume of seller i's buy-later demand is $\frac{1}{2}((V^{-1}(s+\tau_i))^2 - P^2)$, and so the first-order condition for the buy-later premium τ is

$$(V^{-1}(s+\tau))^2 - P^2 = -2\tau V^{-1}(s+\tau)(V^{-1})'(s+\tau) \ .$$

Numerically solving this pair of first-order conditions yields the prices and profits shown above on Figure 5.

Exploding offers: Finally, we examine the equilibrium prices when exploding offers are made. Suppose the equilibrium price is P. If seller i deviates and sets price P_i, its total demand is

$$\tfrac{1}{2}Q(P_i + V(P) - s) + \tfrac{1}{2}[1 - Q(P + V(P) - s)]Q(P_i) \ . \tag{33}$$

Here, the first term represents demand from those buyers who first visit seller i: if they have match utility u_i, they will accept i's exploding offer if $u_i - P_i \geq V(P) - s$. The second term is the demand from those buyers who first visit the rival: a buyer will reject the rival's exploding offer if $u_j - P < V(P) - s$, since they anticipate that seller i offers the equilibrium price P, and then they buy from i if $u_i \geq P_i$. The first-order condition for P to be a symmetric equilibrium price is therefore

$$Q(P + V(P) - s) + (1 - Q(P + V(P) - s))Q(P) +$$

$$P\left[Q'(P + V(P) - s) + (1 - Q(P + V(P) - s))Q'(P)\right] = 0$$

As shown in Armstrong and Zhou (2011, Lemma 3), conditions (3) and (12) imply that this first-order condition has a solution which satisfies $P < p^*$. If $pQ(p)$ is concave, the first-order condition is sufficient for P to be the equilibrium price.

In the uniform example, where $V(P) = \frac{1}{2}(1-P)^2$, the first-order condition for equilibrium price P simplifies to

$$P(2 - 2s + P^2) = 1 \ .$$

Solving this first-order condition yields the price and profit shown above on Figure 5.

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Online Appendix (not for publication)

This online appendix reports the details for the general selling mechanism in the duopoly model in section 5.1. The key step in solving for a symmetric equilibrium mechanism is to study a seller's best response when the rival offers the equilibrium mechanism and buyers hold equilibrium beliefs.[40] This is the same as studying the optimal selling mechanism in an *extended* monopoly model where the seller faces two groups of buyer: one group visits the seller first and do not know their outside option yet, while the other group knows their outside option when they first visit the seller. We will show that in this extended monopoly model it is without loss of generality to look for the optimal mechanism among the class of mechanism which consists of a buy-now price and a menu of deposit contracts.

Studying the optimal mechanism in the extended monopoly model is substantially more difficult than that in the baseline monopoly model. This is because when a buyer comes to the seller for the first time, she has another dimension of private information which is whether or not she already has her outside option in hand (and if so, what that outside option is). This implies that we must solve a multidimensional screening problem. To mitigate this difficulty we consider only deterministic mechanisms.

Before we proceed to the details, consider a hypothetical situation where the seller is able to tell the buyers who visit it first from those who already know the outside option. Then the seller can treat the two groups of buyer separately. The optimal mechanism for the first group is the same as in the baseline monopoly model, and it consists of a buy-now price P and a menu of deposit contracts. With the second group the seller can do not better than to offer a fixed price $\hat{P}$. In general, of course, $P \neq \hat{P}$ and so a simple combination of these two mechanisms is not implementable when the seller cannot distinguish between

[40] In general, the outside option can even depend on what the seller itself is offering if the rival is using a contingent mechanism (i.e., if what the rival is offering depends on the seller's offer, e.g., through a price matching scheme). This possibility is ruled out, as the existing literature on competing mechanism often did, because studying competing mechanism design which allows for contingent mechanisms is a hard problem in general. However, the symmetric equilibrium we are considering continues to be an equilibrium even if we allow for contingent mechanisms. This is because in a (pure strategy) Nash equilibrium, each seller correctly anticipates the mechanism used by the rival, and so they do not have a strict incentive to ask buyers to report the mechanism adopted by the rival. Of course, allowing for contingent mechanisms may generate other equilibria.

these groups. But this suggests that we should look for the optimal mechanism among those which consist of a buy-now price and a menu of deposit contracts.

Proof of Proposition 6: Consider first an extended monopoly model as discussed above. Suppose α buyers visit the seller first, as in section 4.2. When such a buyer meets the seller, she learns her valuation $u \in [0, u_{\max}]$ for the seller's product. But she does not know the realization of her outside option which is denoted by $v \in [0, v_{\max}]$. To reach and learn the outside option, she has to search and incur a cost $s \geq 0$. There is a second group of buyers, with measure β, who already know their outside option when they first visit the seller. Denote their outside option by $\hat{v} \in [0, \hat{v}_{\max}]$. Let G be the CDF of v as before, and let $\hat{G}$ be the CDF of $\hat{v}$. (We allow the outside option to differ across groups, since the second group of buyers will only come to the seller if $\hat{v}$ is not too large.) The seller is unable to distinguish these two groups of buyer. The basic monopoly model in section 4.2 corresponds to the case $\beta = 0$.

A seller's best response, given its rival is using the equilibrium mechanism and buyers hold the equilibrium beliefs, is the optimal mechanism in this extended monopoly model with $\alpha = \frac{1}{2}$ and β, G, $\hat{G}$ determined by the equilibrium mechanism. Thus, it suffices to show that in this extended monopoly model it is without loss of generality to look for the optimal mechanism among the class of mechanism which consists of a buy-now price and a menu of deposit contracts.

By the dynamic revelation principle, we can focus on direct mechanisms which induce truthful report on equilibrium path. A direct mechanism in the extended monopoly model has two parts:

$$\mathcal{M} = \{x(u'); (q_0(u'), t_0(u')); (q(u', v'), t(u', v'))\}$$

is designed for the α buyers, and

$$\hat{\mathcal{M}} = \{\hat{q}(u', \hat{v}'), \hat{t}(u', \hat{v}')\}$$

is designed for the β buyers. When a buyer comes to the seller, the seller first asks whether she already has her outside option. If she reports yes, sub-mechanism $\hat{\mathcal{M}}$ applies. Based on her report of her valuation u' for the seller's product and her valuation $\hat{v}'$ for the outside option, she pays $\hat{t}(u', \hat{v}')$ and gets the seller's product with probability $\hat{q}(u', \hat{v}')$. If she reports no, then the sub-mechanism $\mathcal{M}$ applies, which is as in the basic monopoly case. Based on her report of u', she will be instructed whether to stop searching or not. With probability $x(u')$, she will be instructed to stop searching, in which case she pays $t_0(u')$ and gets the product with probability $q_0(u')$. With probability $1 - x(u')$, she will be instructed to look for the outside option, in which case she will be further required to report her valuation v' for the outside option. The allocation and the payment scheme will then depend on both u' and v'.[41]

The seller's profit, given that buyers report truthfully, is

$$\alpha \mathbb{E}_u \{x(u)t_0(u) + (1 - x(u))\mathbb{E}_v[t(u, v)]\} + \beta \mathbb{E}_{u,\hat{v}}[\hat{t}(u, \hat{v})] \ .$$

To ensure truthful reporting, the seller faces the following constraints from each group of buyers.

The α buyers. Consider a buyer who encounters the seller first. Let

$$U(u, u'; v, v') \equiv \max\{u, v\}q(u', v') + v(1 - q(u', v')) - t(u', v') - s$$

be a type-(u, v) buyer's surplus if she chooses $\mathcal{M}$ and reports u' in the first period and v' in the second period when she searches for the outside option. Let $U(u; v) \equiv U(u, u; v, v)$ be the buyer's truthful reporting surplus. Let

$$\Phi(u, u') \equiv x(u')[uq_0(u') - t_0(u')] + (1 - x(u'))\mathbb{E}_v[\max_{v'} U(u, u'; v, v')]$$

be a type-u buyer's expected surplus if she chooses $\mathcal{M}$ and reports u' in the first period. The first part here is from when the buyer stops searching, and the second part is from when she continues to search. (Notice that if the buyer lies in the first period, she usually has an incentive to lie again in the second period.) Let $\Phi(u) \equiv \Phi(u, u)$ be this buyer's truthful reporting surplus. Truthful reporting on equilibrium path requires:

(a) A buyer reports truthfully in the second stage if she has reported truthfully in the first stage:

$$U(u; v) = \max_{v'} : U(u, u; v, v') \tag{IC-1}$$

[41] The natural timing in our model is that once a buyer encounters the seller, she immediately learns her valuation for the seller's product. So she can mis-report both u and whether she already knows the outside option simultaneously.

for all u and v.

(b) She reports truthfully in the first stage:

$$\Phi(u) = \max_{u'} : \Phi(u, u') \tag{IC-2}$$

and

$$\Phi(u) \geq \max_{u', \hat{v}'} : u\hat{q}(u', \hat{v}') + (\bar{v} - s)(1 - \hat{q}(u', \hat{v}')) - \hat{t}(u', \hat{v}') \tag{34}$$

for all u. Condition (IC-2) ensures that the buyer reports her valuation for the seller's product truthfully if she has chosen $\mathcal{M}$. The right-hand side of condition (34) is the surplus if this buyer instead chooses $\hat{\mathcal{M}}$ and reports u' and $\hat{v}'$. (Here, $\bar{v}$ is the expected value of v, taken with respect to the CDF G.) So condition (34) ensures the buyer truthfully reports that she does not have an outside option yet. As we will see, (34) takes a simpler form once we have studied the β buyers' incentive compatibility conditions. Notice that (IC-1) and (IC-2) imply that

$$\Phi(u) = x(u)[uq_0(u) - t_0(u)] + (1 - x(u))\mathbb{E}_v[U(u; v)] \ .$$

Since an α buyer can always continue to search for the outside option after rejecting the seller's mechanism, the participation constraint is

$$\Phi(u) \geq \bar{v} - s \tag{IR}$$

for all u.

The β buyers. Now consider a buyer who already knows her outside option $\hat{v}$ when she encounters the seller. If $u \leq \hat{v}$, the buyer has no incentive to participate in the seller's mechanism. So without loss of generality, we can specify $\hat{q}(u, \hat{v}) = 0$ and $\hat{t}(u, \hat{v}) = 0$ for $u \leq \hat{v}$, and focus on the buyers with $u > \hat{v}$. Let

$$\hat{\Phi}(u, \hat{v}) \equiv u\hat{q}(u, \hat{v}) + \hat{v}(1 - \hat{q}(u, \hat{v})) - \hat{t}(u, \hat{v})$$

be a type-$(u, \hat{v})$ buyer's surplus if she reports truthfully. (If she gets the seller's product, she consumes it since $u > \hat{v}$. Otherwise, she consumes the outside option.) Truthful reporting requires

$$\hat{\Phi}(u, \hat{v}) = \max_{u', \hat{v}'} : u\hat{q}(u', \hat{v}') + \hat{v}(1 - \hat{q}(u', \hat{v}')) - \hat{t}(u', \hat{v}') \tag{35}$$

and

$$\hat{\Phi}(u,\hat{v}) \geq \max_{u',v'} : x(u')[uq_0(u') + \hat{v}(1-q_0(u')) - t_0(u')] \tag{36}$$
$$+(1-x(u'))[uq(u',v') + \hat{v}(1-q(u',v')) - t(u',v')]$$

for all u and $\hat{v}$. Condition (35) ensures truthful report of her valuations when the buyer chooses the correct sub-mechanism $\hat{\mathcal{M}}$, while condition (36) ensures that she does not lie about whether she already has her outside option. (The right-hand side of (36) is the surplus if a type-$(u,\hat{v})$ β buyer chooses mechanism $\mathcal{M}$ and reports u' in the first stage and v' in the second stage.)

Since a β buyer can always consume the outside option at hand by rejecting the seller's mechanism, the participation condition is

$$\hat{\Phi}(u,\hat{v}) \geq \hat{v} \tag{37}$$

for all u and $\hat{v}$.

The problem for the β buyers is two dimensional. However, the following observation helps to simplify (35)-(37).

Claim 1 *Condition (35) implies that if $u_1 - \hat{v}_1 = u_2 - \hat{v}_2$, then $\hat{q}(u_1,\hat{v}_1) = \hat{q}(u_2,\hat{v}_2)$ and $\hat{t}(u_1,\hat{v}_1) = \hat{t}(u_2,\hat{v}_2)$. Hence, we can focus on $\hat{\mathcal{M}} = \{\hat{q}(z),\hat{t}(z)\}$, where $z \equiv u - \hat{v}$ is a β buyer's net valuation for the seller's product.*

Proof. There is a one-to-one correspondence between $(u,\hat{v})$ and $(z,\hat{v})$. So we can suppose the sub-mechanism takes the form $\hat{\mathcal{M}} = \{\hat{q}(z,\hat{v}),\hat{t}(z,\hat{v})\}$. In this new system, (35) becomes

$$\hat{\Phi}(z) \equiv \hat{\Phi}(u,\hat{v}) - \hat{v} = \max_{z',\hat{v}'} : z\hat{q}(z',\hat{v}') - \hat{t}(z',\hat{v}') \ . \tag{38}$$

The outcome of the maximization problem in the right-hand side depends on z only, which is why we can define the left-hand side as a function only of z. Moreover, since the objective function in (38) is linear in z, $\hat{\Phi}(z)$ must be convex in z and so is differentiable almost everywhere. Consider $(z_1,\hat{v}_1)$ and $(z_2,\hat{v}_2)$ with $z_1 > z_2$. Then (38) requires that

$$\hat{\Phi}(z_1) - \hat{\Phi}(z_2) \geq (z_1 - z_2)\hat{q}(z_2,\hat{v}_2)$$

and

$$\hat{\Phi}(z_2) - \hat{\Phi}(z_1) \geq (z_2 - z_1)\hat{q}(z_1,\hat{v}_1) \ .$$

Dividing each side by $z_1 - z_2$ and letting $z_2 \to z_1$ yields

$$\hat{q}(z_1, \hat{v}_2) \leq \hat{\Phi}'(z_1) \leq \hat{q}(z_1, \hat{v}_1) \ .$$

A similar argument for $(z_1, \hat{v}_2)$ and $(z_2, \hat{v}_1)$ generates the opposite inequality. Therefore, we must have $\hat{q}(z_1, \hat{v}_2) = \hat{q}(z_1, \hat{v}_1)$, i.e., $\hat{q}(z, \hat{v})$ is independent of $\hat{v}$. Since $\hat{t}(z, \hat{v}) = z\hat{q}(z, \hat{v}) - \hat{\Phi}(z)$, $\hat{t}(z, \hat{v})$ is independent of $\hat{v}$ too. ■

Given this result, we can rewrite the conditions (35)-(37) as:

$$\hat{\Phi}(z) = \max_{z'} : z\hat{q}(z') - \hat{t}(z') \ , \tag{IC\^{}-1}$$

$$\hat{\Phi}(z) \geq \max_{u', v'} : x(u')[zq_0(u') - t_0(u')] + (1 - x(u'))[zq(u', v') - t(u', v')] \ , \tag{IC\^{}-2}$$

and

$$\hat{\Phi}(z) \geq 0 \ . \tag{IR\^{}}$$

Meanwhile, the incentive compatibility condition (34) for the α buyers becomes

$$\Phi(u) \geq \max_{z'} : u\hat{q}(z') + (\bar{v} - s)(1 - \hat{q}(z')) - \hat{t}(z') \ . \tag{IC-3}$$

In sum, the seller's problem is to maximize

$$\alpha \mathbb{E}_u\{x(u)t_0(u) + (1 - x(u))\mathbb{E}_v[t(u, v)]\} + \beta \mathbb{E}_z[\hat{t}(z)]$$

subject to (IC-1), (IC-2), (IC-3), (IR), (IC^-1), (IC^-2) and (IR^). The principal challenge in this problem stems from the two new incentive compatible conditions (IC^-2) and (IC-3) which ensure that a buyer truthfully reports whether or not she already knows her outside option. For these constraints the usual first-order approach does not work, and as in most multidimensional screening problems it is *ex ante* difficult to identify for which u and z they will bind in the solution.

From now on, we restrict attention to *deterministic* mechanisms, i.e., mechanisms with $\hat{q}, x, q_0, q \in \{0, 1\}$. We first analyze the mechanism $\hat{\mathcal{M}}$ for the β buyers. The next result says that $\hat{\mathcal{M}}$ is incentive compatible and individually rational only if it takes the form of a fixed price.

Claim 2 *Constraints (IC^-1) and (IR^) imply that there exists a threshold $k \geq 0$ such that*

$$\hat{q}(z) = \begin{cases} 1 & \textit{if } z > k \\ 0 & \textit{otherwise} \end{cases} \tag{39}$$

and

$$\hat{t}(z) = \begin{cases} k & \textit{if } z > k \\ 0 & \textit{otherwise.} \end{cases} \tag{40}$$

Proof. By a standard argument, (IC^-1) implies $\hat{q}(z)$ weakly increases with z. Since $\hat{q}(z) \in \{0, 1\}$, (39) follows immediately. If the seller does not allocate its product, it cannot ask the buyer to make a positive payment because of the participation constraint (IR^). If the seller allocates its product to two buyers z_1 and z_2, it cannot ask for different payments (otherwise, one buyer would mis-report). So $\hat{t}(z)$ must be a constant for $z > k$. It remains to show $\hat{t}(z) = k$ for $z > k$. If $\hat{t}(z) > k$, then buyers with z slightly above k would rather give up the product. If $\hat{t}(z) < k$, then buyers with z slightly below k would mis-report $z' > k$ in order to get the product and obtain a positive surplus. Therefore, a deterministic mechanism $\hat{\mathcal{M}}$ which satisfies (IC^-1) and (IR^) must satisfy (39) and (40). ■

Turn next to the mechanism $\mathcal{M}$ aimed at the α buyers. We first explore the implications of the constraints for $x(u)$, $q_0(u)$ and $t_0(u)$. The following result says that in an incentive compatible and individually rational mechanism $\mathcal{M}$, any α buyer with u greater than some threshold will be instructed to stop searching and get the product immediately at a fixed price, and those with u below the threshold will be instructed to search for the outside option.

Claim 3 *The constraints (IC-1), (IC-2) and (IR) imply there exists a threshold $\hat{u}$ and constant $t_0 \leq \hat{u} - (\bar{v} - s)$ such that*

$$x(u) = \begin{cases} 1 & \textit{if } u > \hat{u} \\ 0 & \textit{otherwise} \end{cases} \tag{41}$$

$$q_0(u) = \begin{cases} 1 & \textit{if } u > \hat{u} \\ 0 & \textit{otherwise} \end{cases} \tag{42}$$

and

$$t_0(u) = \begin{cases} t_0 & \textit{if } u > \hat{u} \\ 0 & \textit{otherwise.} \end{cases} \tag{43}$$

Proof. The key step is to show (41). Since we are considering deterministic mechanisms with $x(u) \in \{0,1\}$, it suffices to show that $x(u)$ is weakly increasing. We initially make two observations. First, notice that both $U(u,u';v,v')$ and $\Phi(u,u')$ are convex in u. So (IC-1) and (IC-2) together imply that $\Phi(u)$ is convex, and by an envelope argument we have

$$\Phi'(u) = x(u)q_0(u) + (1-x(u))\int_0^u q(u,v)dG(v) \ . \tag{44}$$

The convexity of $\Phi(u)$ requires that $\Phi'(u)$ weakly increases in u. Second, notice that $q_0(u) = 1$ if $x(u) = 1$. (If an α buyer anticipates that she will be asked to stop searching but will not get the product when she reports truthfully, she will either not participate into the mechanism or mis-report her type.)

Now consider $u_2 < u_1$, and suppose $x(u_2) = 1$ and $x(u_1) = 0$ in contrast to the claim. Then we have $q_0(u_2) = 1$ from the second observation above, and so (44) implies $\Phi'(u_2) = 1$. Since $\Phi'(u)$ must be weakly increasing, we have

$$\Phi'(u_1) = \int_0^{u_1} q(u_1,v)dG(v) = 1 \ . \tag{45}$$

But this is impossible as we show below.

If $u_1 < v_{\max}$, it is clear that (45) cannot hold. Now consider $u_1 \geq v_{\max}$, in which case (45) requires $q(u_1,v) = 1$ for all possible v. Given the requirement of truthful reporting in the second stage, this implies that $t(u_1,v)$ should be independent of v. Let $t_1 = t(u_1,v)$. Then $\Phi(u_1) = u_1 - t_1 - s$. If this buyer reports u_2, her surplus will be $u_1 - t_0(u_2)$. So truthful reporting requires

$$t_1 + s \leq t_0(u_2) \ .$$

Meanwhile, the equilibrium surplus of the buyer with u_2 is $\Phi(u_2) = u_2 - t_0(u_2)$. If she deviates and reports u_1, her expected surplus will be $\mathbb{E}_v[\max\{u_2,v\}] - t_1 - s$. So truthful reporting requires

$$t_1 + s \geq t_0(u_2) + \mathbb{E}_v[\max\{u_2,v\}] - u_2 > t_0(u_2) \ .$$

This leads to a contradiction, and so completes the proof for (41).

The other two results (42) and (43) just follow from (41) and the second observation above. (Notice that for $u \leq \hat{u}$, it does not matter how to specify $q_0(u)$ and $t_0(u)$ since the buyer will be asked to search anyway.) Finally, the restriction $t_0 \leq \hat{u} - (\bar{v} - s)$ follows because of condition (IR) for $u > \hat{u}$. ■

We further explore the implications of the constraints for the second-stage mechanism $\{q(u,v), t(u,v)\}$.

Claim 4 *Constraint (IC-1) implies there exist functions $\rho(u) \leq u$ and $t(u)$ such that*

$$q(u,v) = \begin{cases} 1 & \text{if } v \leq \rho(u) \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

and

$$t(u,v) = \begin{cases} t(u) + u - \rho(u) & \text{if } v \leq \rho(u) \\ t(u) & \text{otherwise.} \end{cases} \tag{47}$$

This claim says that in an incentive compatible mechanism $\mathcal{M}$, if a type-u buyer searches for the outside option, she will be given the product if and only if her outside option is below a threshold $\rho(u)$. She pays at least $t(u)$ in either case, but when she gets the product she pays an extra amount $u - \rho(u)$. This corresponds to a situation where the buyer pays a non-refundable deposit $t(u)$ in order to search, and an extra payment $u - \rho(u)$ if she returns to buy later.

Proof. Condition (IC-1) implies that $U(u;v)$ is convex in v since the objective function $U(u,u;v,v')$ itself is convex in v. By an envelope argument we have

$$\frac{\partial}{\partial v} U(u;v) = \begin{cases} 1 - q(u,v) & \text{if } v \leq u \\ 1 & \text{otherwise.} \end{cases}$$

The convexity of $U(u,v)$ in v requires that $q(u,v)$ be weakly decreasing in v when $v \leq u$. Since $q(u,v) \in \{0,1\}$ in a deterministic mechanism, it follows that there exists $\rho(u) \leq u$ such that

$$q(u,v) = \begin{cases} 1 & \text{if } v \leq \rho(u) \\ 0 & \text{if } \rho(u) < v \leq u \end{cases}.$$

To ensure truthful reporting, the payment scheme must take the form

$$t(u,v) = \begin{cases} t(u) + u - \rho(u) & \text{if } v \leq \rho(u) \\ t(u) & \text{if } \rho(u) < v \leq u \end{cases}.$$

It is easy to understand that in each interval the payment must be independent of v (otherwise some buyers would mis-report v given the same allocation outcome). The difference $u - \rho(u)$ between the two payments is because the buyer with $v = \rho(u)$ should be indifferent about whether she gets the seller's product or not.

For a buyer with $v > u$, she will consume v regardless of whether she gets the seller's product or not. Hence, for this buyer to report her v truthfully, $t(u, v)$ must be independent of v for $v > u$, and denote it by $\tilde{t}(u)$. If this buyer mis-reports $v' \in (\rho(u), u]$, her surplus will be $v - t(u)$. So truthful reporting requires $\tilde{t}(u) \le t(u)$. Similarly, for a buyer with $v \in (\rho(u), u]$ not to mis-report $v' > u$, we need $t(u) \le \tilde{t}(u)$ (independent of the value of $q(u, v')$). Therefore, $\tilde{t}(u) = t(u)$. In the same time, if $q(u, v) = 1$ for $v > u$, then those buyers with $v \le \rho(u)$ will have an incentive to mis-report $v' > u$ in order to get the product at a lower price. Thus, we need $q(u, v) = 0$ for $v > u$. ■

So far we have not used the constraints (IC-3) and (IC^-2) which ensure that the buyers will truthfully report about whether they already have their outside option.

Claim 5 *When $\hat{u} < u_{\max}$ (where $\hat{u}$ is introduced in Claim 3), constraints (IC-3) and (IC^-2) imply that*

$$k = t_0 \ , \tag{48}$$

where k and t_0 are defined in (39) and (43) respectively.

Proof. When $\hat{u} < u_{\max}$, (41), (42) and (43) imply that $\Phi(u) = u - t_0$ for $u > \hat{u}$. Then (IC-3) requires

$$u - t_0 \ge u - k \ .$$

In the same time, for $z > k$, (39) and (40) imply $\hat{\Phi}(z) = z - k$. Then (IC^-2) requires

$$z - k \ge z - t_0 \ .$$

These two inequalities imply $k = t_0$. ■

In sum, we have shown that any incentive compatible and individually rational (deterministic) mechanism $\{\mathcal{M}, \hat{\mathcal{M}}\}$ must satisfy all the conditions stated in Claims 2–5.[42] However, any mechanism which satisfies these conditions can be replicated by a mechanism which consists of a buy-now price P and a menu of deposit contracts $\{D(u), p(u)\}$. For a

[42] These conditions are not sufficient for incentive compatibility and individual rationality. For sufficiency, further conditions need to be imposed on k, $\hat{u}$, t_0, $\rho(u)$ and $t(u)$.

mechanism with $\hat{u} < u_{\max}$, we can use the following indirect mechanism with a buy-now price

$$P = k = t_0$$

and a menu of deposit contracts

$$\{D(u) = t(u), p(u) = u - \rho(u)\} \text{ for } u \leq \hat{u} \ .$$

It is not difficult to see that this mechanism leads to the same choice outcome as the original direct mechanism. (The only non-trivial part is when a type-u buyer picks a deposit contract $(D(u'), p(u'))$ and chooses to search. Suppose she finds out that her valuation for the outside option is v. Then if she returns to buy the seller's product, her surplus will be $\max\{u, v\} - (t(u') + u' - \rho(u')) - s$. If she chooses not to buy the seller's product, her surplus will be $v - t(u') - s$. In the direct mechanism if the buyer reports u' in the first stage, she will face exactly the same two options, depending on whether she reports $v' \leq \rho(u')$ or not in the second stage.) For a mechanism with $\hat{u} = u_{\max}$, then q_0 and t_0 do not matter, and we can use the same indirect mechanism with $P = k$.

We conclude that in the extended monopoly model, it is without loss of generality to look for the optimal (deterministic) mechanism among the class of mechanism which consists of a buy-now price and a menu of deposit contracts. This completes the proof.

Equilibrium analysis for general mechanisms.

Extended monopoly. To study the equilibrium mechanism in the duopoly model, we first study the optimal mechanism in the extended monopoly model as discussed in the proof of Proposition 6. We know that if we focus on deterministic mechanisms, it is without loss of generality to consider mechanisms which consist of a buy-now price P and a menu of deposit contracts $\{D(p), p\}$, where if a buyer pays a deposit $D(p)$, she has the right to buy the product later at an incremental price p.

Suppose that for all buy-later prices p we have $P \leq D(p) + p$ (which will be verified later). Then the β buyers who already know their outside option will buy at the buy-now price P if and only if $u - \hat{v} > P$. So the profit from each β buyer is $\Pi(P) \equiv P(1 - H(P))$, where H is the CDF of net surplus $u - \hat{v}$.

Now consider the α buyers who visit the seller first. If a type-u buyer chooses a deposit contract $(D(p), p)$ and continues to search, her expected surplus is

$$\mathbb{E}_v[\max\{v, u-p\}] - D(p) - s = R(u-p) - D(p) + \bar{v} - s \ ,$$

where

$$R(x) \equiv \mathbb{E}_v[\max\{v, x\} - v] = S(x) + x - \bar{v} = \int_0^x G(v)dv$$

is the *ex ante* benefit to the buyer of being able to return to obtain surplus x from the seller once she has discovered her outside option v. Clearly, $R(x) = 0$ if $x \leq 0$, and R is convex and increasing.

If the buyer decides to choose one of the deposit contracts, she will choose the best contract from the menu, with resulting surplus

$$\phi(u) \equiv \max_p : \{R(u-p) - D(p) + \bar{v} - s\} \ .$$

Since the function $R(\cdot)$ is increasing and convex, $\phi(\cdot)$ is also increasing and convex and hence differentiable almost everywhere. Let $p(u)$ be the optimal choice of buy-later price for the type-u buyer (which is uniquely determined almost everywhere). Since $R(\cdot)$ is convex, a revealed preference argument shows $p(u)$ must weakly decrease with u. The envelope theorem implies that

$$\phi'(u) = R'(u - p(u)) = G(u - p(u)) \ , \tag{49}$$

and the deposit payment associated with the buy-later price $p(u)$ is

$$D(u) = R(u - p(u)) + \bar{v} - s - \phi(u) \ . \tag{50}$$

Expression (49) implies that the surplus from choosing a deposit contract increases with u but with slope less than 1. By contrast, the surplus from leaving the firm altogether, $\bar{v} - s$, does not depend on u, and the surplus from immediate purchase, $u - P$, increases with slope 1. Thus, for small u, say for $u < \check{u}$, the buyer leaves the seller immediately, for intermediate u, say $\check{u} \leq u \leq \hat{u}$, the buyer chooses one of the deposit contracts, and for $u > \hat{u}$ the buyer buys immediately without search.[43] For the buyer to be indifferent

[43] By choosing $\check{u}$ and $\hat{u}$ appropriately one can allow only a subset of the three strategies to be made available. For instance, setting $\check{u} = \hat{u}$ means that no option to buy later is made available and the seller

between the relevant options at the points $\check{u}$ and $\hat{u}$, we require

$$\phi(\check{u}) = \bar{v} - s \ ; \ \phi(\hat{u}) = \hat{u} - P \ . \tag{51}$$

Using this and (49), we have

$$\phi(u) = \bar{v} - s + \int_{\check{u}}^{u} G(\tilde{u} - p(\tilde{u}))d\tilde{u} \ . \tag{52}$$

If the type-u buyer chooses a deposit contract with buy-later price p, she will return to buy later when $v \leq u - p$, i.e., with probability $G(u-p)$.

The seller's profit is then

$$\alpha \left[P(1-F(\hat{u})) + \int_{\check{u}}^{\hat{u}} \{D(u) + p(u)G(u-p(u))\} \, dF(u) \right] + \beta\Pi(P) \ , \tag{53}$$

where $D(u)$ is given in (50). The first portion is the profit from the α buyers, and the second portion is the profit from the β buyers. Integrating by parts and using (49) yields

$$\int_{\check{u}}^{\hat{u}} \phi(u)dF(u) = \phi(\check{u})(1-F(\check{u})) - \phi(\hat{u})(1-F(\hat{u})) + \int_{\check{u}}^{\hat{u}} G(u-p(u))(1-F(u))du \ .$$

Using this and (51), we can rewrite the term $[\cdot]$ in (53) as

$$(\hat{u}-(\bar{v}-s))(1-F(\hat{u})) + \int_{\check{u}}^{\hat{u}} \left\{ R(u-p(u)) + G(u-p(u)) \left(p(u) - \frac{1-F(u)}{f(u)} \right) \right\} dF(u) \tag{54}$$

which is a function of $(\check{u}, \hat{u}, p(u))$. Using (52), we have

$$P = \hat{u} - \phi(\hat{u}) = \hat{u} - (\bar{v} - s) - \int_{\check{u}}^{\hat{u}} G(u-p(u))du \ . \tag{55}$$

So $\Pi(P)$ in (53) is also a function of $(\check{u}, \hat{u}, p(u))$.

We first ignore the constraint that $p(u)$ should be weakly decreasing and derive the optimal $p(u)$ given $\check{u}$ and $\hat{u}$. Substituting (54) and (55) into (53) and maximizing the profit with respect to $p(u)$, we can check that the candidate $p(u)$ satisfies[44]

$$p(u) = \frac{1-F(u)}{f(u)} + \frac{1}{f(u)}\frac{\beta}{\alpha}\Pi'(\hat{u} - (\bar{v}-s) - \int_{\check{u}}^{\hat{u}} G(\tilde{u} - p(\tilde{u}))d\tilde{u}) \ . \tag{56}$$

makes an exploding offer, while if $\hat{u} = u_{\max}$ no consumer buys without search. In the baseline monopoly model with $\beta = 0$, given the assumption $a < u_{\max}$, we must have $\hat{u} < u_{\max}$ and $\check{u} > 0$. While in the extended monopoly model we may have a corner solution with $\hat{u} = u_{\max}$. This case can be treated by a modified analysis. However, as we will see later, this does occur in the duopoly case with two symmetric sellers.

[44] This can be done by the standard technique of calculus of variations. It can also be checked that given $\check{u} < \hat{u}$, (56) is also sufficient for $p(u)$ to be the solution if $\Pi(\cdot)$ is concave.

(When $\beta = 0$, the solution is consistent with the buy-later price in Proposition 3.) To deal with this functional equation, let $K = \frac{\beta}{\alpha}\Pi'(\cdot)$ in (56). Then

$$p(u) = \frac{1 - F(u) + K}{f(u)} \ , \tag{57}$$

where K solves the equation

$$K = \frac{\beta}{\alpha}\Pi'(P) \tag{58}$$

with

$$P = \hat{u} - (\bar{v} - s) - \int_{\check{u}}^{\hat{u}} G(u - \frac{1 - F(u) + K}{f(u)})du \ . \tag{59}$$

Substituting (57) into the profit function and maximizing it with respect to $\check{u}$ and $\hat{u}$ yields

$$\check{u} - p(\check{u}) = 0 \tag{60}$$

and

$$\hat{u} - p(\hat{u}) = a \ , \tag{61}$$

where a solves $S(a) = s$ as in the baseline monopoly model. (When $u - p(u)$ is increasing, the profit function is single-peaked in $\check{u}$ and $\hat{u}$.)

If $p(u)$ in (57) is decreasing, the optimal mechanism is characterized by (57)-(61). It can also be shown that this system has a unique solution if in addition $\Pi(\cdot)$ is concave.[45] Unlike in the baseline monopoly model, here the logconcavity condition (3) is not enough for $p(u)$ in (57) to be decreasing. Due to the endogeneity of the constant K (which might be positive or negative), it is hard to derive a primitive condition for this to hold. But $p(u)$ is indeed decreasing at least in the uniform distribution example where $p(u) = 1 - u + K$.

When $p(u)$ is decreasing, one can check that the α buyers will indeed choose the correct deposit contract. Our analysis is also predicated on $P \leq D(u) + p(u)$ for $u \in [\check{u}, \hat{u}]$ such that β buyers have no incentive to choose a deposit contract. Using (50), we have

$$D(u) + p(u) = p(u) + R(u - p(u)) - \phi(u) + \bar{v} - s \ .$$

[45] If $p(u)$ is decreasing, then $u - p(u)$ is increasing, and (60) and (61) uniquely determine $\check{u}$ and $\hat{u}$ as functions of K. We also have $\check{u} < \hat{u}$ and both of them increase in K. Using the second observation, one can verify that P in (59) increases in K. Then the right-hand side of (58) decreases in K if Π is concave. This ensures that (58) has a unique solution.

Using (51) and (61), one can verify that $D(\hat{u})+p(\hat{u}) = P$. On the other hand, the derivative of $D(u) + p(u)$ with respect to u is

$$p'(u)(1 - G(u - p(u))) \ .$$

Hence, if $p(u)$ is decreasing, $D(u) + p(u)$ decreases in u and so $P \leq D(u) + p(u)$ for $u \in [\check{u}, \hat{u}]$. In the same time, as in the baseline monopoly model we also have $D(\check{u}) = 0$ and $p(\check{u}) = \check{u}$ (which can be verified by using (50), (51) and (60)), and so a buyer always has the option to return to buy at price $\check{u}$ without paying a deposit.

Duopoly. We now solve the equilibrium mechanism in the duopoly model. The extra complication in the duopoly case is that the outside options v and $\hat{v}$ are endogenous and depend on the equilibrium selling mechanism. Let

$$(P, \{D(u), p(u)\}_{u \in [\check{u}, \hat{u}]})$$

be the symmetric equilibrium mechanism. Given seller j is using the equilibrium mechanism and buyers hold equilibrium beliefs, seller i's problem is the same as the extended monopoly model with the following specifications:

(i) Half the buyers visit seller i first (so $\alpha = \frac{1}{2}$). If a buyer decides to visit seller j without paying a deposit, she will either buy seller j's product at price P (given $P \leq D(u) + p(u)$) or leave the market with nothing. So this buyer's outside option is $v = \max\{0, u_j - P\}$, and its CDF is

$$G(v) = F(v + P) \tag{62}$$

for $v \in [0, u_{\max} - P]$. (Notice that v has a mass point at zero, but this does not affect our analysis in the extended monopoly model.) The mean of v is then

$$\bar{v} = \int_P^{u_{\max}} [1 - F(v)] dv \ . \tag{63}$$

The reservation surplus a solves

$$s = \int_{a+P}^{u_{\max}} [1 - F(v)] dv \ ,$$

so

$$a = A - P \ , \tag{64}$$

where A is defined in (13) and is the threshold of match utility for immediate purchase in the free-recall benchmark. (Both $\bar{v}$ and a decrease in P because a higher P makes the outside option v less attractive.)

(ii) The other half of the buyers visit seller j first, and $F(\hat{u})$ fraction of them will continue on to visit seller i (so $\beta = \frac{1}{2}F(\hat{u})$). For those who value seller j's product at $u_j \in [\check{u}, \hat{u}]$, they pay a deposit $D(u_j)$ to seller j and then visit seller i, and so act as if they have an outside option $\hat{v} = u_j - p(u_j)$. Those who value seller j's product below $\check{u}$ also visit seller i but without paying a deposit to seller j, so they have outside option $\hat{v} = 0$. Then the CDF of the net valuation $z = u_i - \hat{v}$, conditional on a buyer coming to seller i after visiting seller j, is

$$\begin{aligned} H(z) &= \Pr(u_i - \hat{v} \le z | u_j \le \hat{u}) \\ &= \frac{1}{F(\hat{u})}\left(F(z)F(\check{u}) + \int_{\check{u}}^{\hat{u}} F(z + u_j - p(u_j))dF(u_j)\right) . \end{aligned} \tag{65}$$

(In equilibrium, we have $\hat{u}-p(\hat{u}) = a$ and $\check{u}-p(\check{u}) = 0$, and so the support of z is $[-a, u_{\max}]$.) If seller i sets buy-now price $\tilde{P}$, its profit from each β buyer is $\Pi(\tilde{P}) \equiv \tilde{P}(1 - H(\tilde{P}))$. The derivative of this function Π, evaluated at $\tilde{P} = P$ for convenience later, can be shown to be

$$\Pi'(P) = \frac{1}{F(\hat{u})}\left(\gamma(P)F(\check{u}) + \int_{\check{u}}^{\hat{u}} \gamma(P + u - p(u))dF(u)\right) , \tag{66}$$

where

$$\gamma(x) \equiv 1 - F(x) - xf(x) .$$

Notice that $\Pi(\tilde{P})$ is concave if $\gamma(x)$ is decreasing, i.e., if $x(1 - F(x))$ is concave, as is now assumed in the following discussion.

Seller i's best response to j's equilibrium selling mechanism can then be calculated as in the extended monopoly model with $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}F(\hat{u})$, G as defined in (62), and H as defined in (65). Therefore, the buy-later price is

$$p(u) = \frac{1 - F(u) + K}{f(u)} \tag{67}$$

for $u \in [\check{u}, \hat{u}]$, where $\check{u}$ and $\hat{u}$ are determined by

$$\check{u} - p(\check{u}) = 0 , \tag{68}$$

and

$$\hat{u} - p(\hat{u}) = A - P \ , \tag{69}$$

respectively. The remaining two parameters K and P solve

$$K = \gamma(P)F(\check{u}) + \int_{\check{u}}^{\hat{u}} \gamma(P + u - p(u))dF(u) \ , \tag{70}$$

and

$$P = \hat{u} - (\bar{v} - s) - \int_{\check{u}}^{\hat{u}} F(P + u - p(u))du \ . \tag{71}$$

(Equation (70) is from (58), $\frac{\beta}{\alpha} = F(\hat{u})$, and (66). Equation (71) is from (59) and $G(\cdot)$ defined in (62).) In addition, notice that $\bar{v}$ depends on P as indicated in (63).

The symmetric equilibrium mechanism is characterized by the system of (67)–(71) if it has a solution such that $p(u)$ is decreasing. Suppose this is true. Then following a similar argument as in the monopoly case, we have $P \leq D(u) + p(u)$ for $u \in [\check{u}, \hat{u}]$. In particular, the equality holds at $u = \hat{u}$. Then from (69) we have $\hat{u} = A - D(\hat{u}) < A$. That is, more buyers buy immediately at the first visited seller than in the free-recall case. Meanwhile, as in the monopoly model we have $D(\check{u}) = 0$ and $p(\check{u}) = \check{u}$, so $\check{u}$ is also the buy-later price when a buyer does not pay a deposit.

However, as in the extended monopoly model, it is hard to find a general condition under which the system of (67)–(71) has a solution with $p(u)$ decreasing. However, this is the case with the uniform example with $F(u) = u$. In that case, we have $p(u) = 1 - u + K$ (which is decreasing), $A = 1 - \sqrt{2s}$, and $\bar{v} = \frac{1}{2}(1 - P)^2$. To have active search, the search cost s should not exceed $\frac{1}{8}$. The four parameters $(\check{u}, \hat{u}, P, K)$ then solve the following system of simultaneous equations:

$$\begin{aligned}
\check{u} &= \tfrac{1}{2}(1 + K) \\
\hat{u} &= \tfrac{1}{2}(2 - \sqrt{2s} - P + K) \\
K &= \hat{u}(1 - 2P) - 2\int_{\check{u}}^{\hat{u}} (2u - 1 - K)du \\
P &= \hat{u} - \tfrac{1}{2}(1 - P)^2 + s - \int_{\check{u}}^{\hat{u}} (P + 2u - 1 - K)du \ .
\end{aligned}$$

Substituting the first two equations into the latter two, one can derive that

$$K = \frac{1}{1 + 2P}(P^2 - 3P + 1 + \sqrt{2s} - 2s)$$

and

$$2P^3 + 3P^2 + (6 - 4s)P + 2s - 2\sqrt{2s} - 3 = 0 \ .$$

The second equation has a unique real solution P for $s \in [0, \frac{1}{8}]$, which is depicted on Figure 5a, and so the system of four equations has a unique solution. Numerical simulations indicate that $0 < \check{u} < \hat{u} < 1$ for $s < \frac{1}{8}$, and the following table reports some parameters of interest for a range of search costs s:

$s =$	0	0.05	0.1	0.125
$\check{u}$	0.489	0.511	0.505	0.5
$\hat{u}$	0.790	0.621	0.536	0.5
P	0.399	0.464	0.489	0.5
K	-0.021	0.021	0.009	0