Nominal Term Structure and Term Premia. Evidence from Chile

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Abstract

The downward trend exhibited in Chile’s nominal term structure since 2003 has been a common pattern shared by other developed and developing economies. To understand the behavior of the nominal yield curve in Chile, we rely on an affine dynamic term structure model (DTMS) which allows to decompose the term structure into the expected short-term premium (related to the monetary policy expectation) and a term premia. We show that most of the fall of long-term interest rates as well as its dynamics are related to the term premia rather than the expected short-term interest rate. With this, we report that the term premia is driven primarily by nominal uncertainty, i.e. the uncertainty for expected inflation and the US term premia.

1 Introduction

The conventional monetary policy (changes in the short-term interest rate) seeks to influence the expectations of market participants about the future path of the short-term interest rate, affecting the relevant interest rates for consumers and firms and therefore the level of output in the economy. This mechanism assumes that changes in the monetary policy rate affect the long-term interest rate through the expected path of short interest rates, which relies on the expectation hypothesis (EH) of the term structure of interest rates. The EH proposes that the long-term rate is determined only by the current and expected path of short interest rates. Thus, central banks can affect long rates through changes in market participants expectations of future monetary policy.

However, while central banks certainly play a key role in determining the behavior of short-term interest rates, the impact on long-term interest rates is not clear. Bernanke (2013) stated that the behavior of long-term interest rates might by driven by others factors (term premia) than the expected short-term. Thus, a better understanding of the drivers that move the long interest rates are a relevant issue to evaluate the current stance of policy and also for thinking about how rates may evolve. For instance, if long interest rates are driven mainly by premia, the standard monetary policy tool might be ineffective as long as the premiums determinants are unchanged.

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In recent years, policy makers have been concerned with the determinants behind the downward trend exhibited in long interest rates in developed and developing countries (see Appendix A). Moreover, an effort to make a decomposition of long interest rates into the expected short term rate and the term premia components has been carried out. For instance, Bernanke (2013) and Adrian et al. (2013) indicate that the behavior of long interest rates in the US have been mostly driven by the term premia rather than by the expected short-term interest rate. The opposite result has been found in the UK (Guimares (2012)) where the falling of the ten-year nominal bond (since the onset of the financial crisis) have reflected a lower expectation of short-term interest rates. In the case of Chile, since 2003 we have seen that the long-term interest rates have exhibited a downward trend since 2003. Figure 1 shows the behavior of long-term interest rates in Chile: (a) the 10 year nominal interest rate and (b) the five years forward interest rate in five-year (5y5).

**Figure 1: Long term interest rates in Chile**

![Graph showing long-term interest rates in Chile](image)

The 10y (blue solid line) is the ten-year yield of nominal bonds in Chile. The 5y5 (red dashed line) is the five-year forward rate in five years. Monthly data taken from Central Bank of Chile database.

The aim of this work is to analyze the behavior of expected long-term interest rates in Chile, focusing on their decomposition into the expected short-term rate (related to the monetary policy) and the term premia component. A correct identification of the expected short-term interest rate and the term premia components allows a better comprehension of the real stance of monetary policy and the adequate tools that the monetary authority may use in order to affect the long interest rate. To this, we apply the standard affine dynamic term structure models (DTSM) standard in the macro finance literature since Ang & Piazzesi (2003). Based on a DTSM with unspanned macroeconomic variables, we evaluate different model specifications in order to decompose long interest rates in Chile. Once the term premia are identified, we proceed to evaluate the main drivers that move them using alternative measures of nominal and real uncertainty.

Our results suggest that the dynamics of term premia depend on samples considered being more parsimonious in 2005-2007 and more volatile during the financial crisis of 2008-2010 and episodes of large changes in long interest rates are caused by changes in term premia rather than the
expected short term interest rate. Therefore, the behavior of term premia is a relevant element to be considered. In this regard, we analyze the response of term premia to nominal and real uncertainty shocks as well the US term premia. Our findings suggest that in 10-year term premia, the nominal uncertainty has a significant and positive impact, especially the inflation expected two years ahead (the relevant horizon for monetary policy). Another interesting result is the positive and significant effect of the US premia on the Chilean premia.

The paper is organized as follows: section 2 presents the literature review of the dynamic term structure models. Section 3 describes the data used in this paper. In section 4 the DTMS methodology is explained as well as the use of macroeconomic variables as unspanned factors. Section 5 presents the findings related to the decomposition of long-term interest rates in Chile, and the analysis of the key drivers that move term premia. Finally, some conclusions are presented in section 6.

2 Literature review

To figure out what is the term premium, the literature has focused in understanding and interpreting the yield curve (the relation between the maturities of zero-coupon bonds and their yields in a given moment)\textsuperscript{1}. The first model that tries to characterize the yield curve is the Expectation Hypothesis (henceforth EH). The basic idea is that, under a framework of risk-neutral investors, any $n$-period bond interest rate is the average of expected future short-term interest rates until period $n$, plus a constant term that just depends on maturity but not on timing. With this definition and considering that real-world investors are not risk neutral, there exists a difference between actual long-term yields and the average expected future short-term interest rates (what is predicted by the EH). This difference is known as the risk premium or term premium, and it is not time varying in the case of the EH. Generally speaking, regardless of the model or assumptions considered, any difference between actual yields and actual expectations is defined as a risk premium.

The EH has been tested by many authors following different methodologies. For example, Campbell & Shiller (1991) proposed two tests to evaluate the validity of the EH. The idea is when the yield curve is steeper than usual both short and long-term rates must be expected to rise (analogous when it is flatter). Their first test compares the return on lending an $n$-period bond and the expected average return on lending on sub-periods of length $m < n$. If the EH is valid, the return of these two strategies should be the same. Running this test is equal to running a regression on the difference of rolling yields constructed under these two strategies and testing if the slope is equal to one. The second test proposed by these authors is based on the implications of the EH that the expectation of future interest rates between periods $m$ and $n$ should be the forward rate over that period. Again, the test is based on the slope of the regression between those rates. These tests have found that the slope of these regressions are, in general, negative and become more negative on $n$. Other authors, such as Fama & Bliss (1987), Backus et al. (2001), Duffee (2002) and Cochrane & Piazzesi (2005, 2008), regress excess returns on holding $n$-period bonds for $m < n$ periods over the return on holding an $m$-period bond over the whole period. If EH is true, then term premia should be time-invariant and the expected excess return should be constant, implying that right-hand variables should be jointly equal to zero. Once again, these tests indicate that EH

\textsuperscript{1}This section is based on Gürkaynak & Wright (2012).
is rejected. As we can see, in general, the hypothesis has been consistently rejected. Some authors have tested the hypothesis in different segments of the yield curve, finding mixed evidence about its validity (see Rudebusch (1995) and Longstaff (2000)). Finally, most of the EH using postwar data reject it, while those who have used earlier sample periods have obtained mixed results (see Hardouvelis (1994) and Mankiw & Miron (1986)). The weak evidence in favor of the EH as an explanation of the term structure of interest rates and its features have been related to one of its most controversial assumptions: the constancy of risk premia in time. It is for this reason that a new wave of literature related to the term structure has taken into account this element, proposing models with time-varying risk premia.

Given the importance of characterizing a time-varying risk premia, affine term structure models have become a very popular framework to achieve this. The key idea behind this type of models is the relevance of the predictability of interest rates in a framework that rules out the possibilities of arbitrage. The most basic elements of affine term structure models will be explained later, but here we will mention their components. First of all, we assume a law of motion of factors, that could be observed or latent. Given these factors, the short-term interest rate is assumed affine (linear plus a constant) on those factors (Duffie & Kan (1996)). We also assume that the pricing kernel (which is a strictly positive random variable that price any asset at any point in time) is log-normal, conditional on the short-term interest rate and the factors. These assumptions make that yields at maturities are all affine on the factors. Three branches of literature have worked with the basic model and modified its assumptions.

The first family of models that have worked with the affine structure assumes that factors are a (non)linear combination of the interest rates, that could be observed or latent, which characterize the cross-section of the yield curve. Most authors have worked with three factors and have estimated the model via both Maximum Likelihood and the Kalman filter. The literature that has used these model is extensive. Several models that use unobserved latent variables as factors are Duffie & Kan (1996), Dai & Singleton (2000, 2002), Duffee (2002), Kim & Wright (2005) and Kim & Orphanides (2011), among others. In general, three principal components of yields are used to explain bond yields. One particular interesting example in this regard is Christensen et al. (2010), who consider a three-factor model, imposing restrictions in the risk-neutral version of the model, so the short-term rate has the same functional form of Nelson & Siegel (1987). These models have shown a good fit of the observed interest rates using a small number of variables (see Backus & Wright (2007), Rudebusch et al. (2007) and Christensen et al. (2010)). The results of these kinds of models are similar among them. For example, Kim & Wright (2005) find a stable term premia in long rates that fluctuates between zero and 2%. The drop of these long-rates occurs because of a decrease in the term premium. Similar conclusions have been found by Kim & Lin (2012). On the other hand, Backus & Wright (2007) find that the term premia has shown high volatility and a decreasing trend since 1986. Until the mid-1990s term premia fluctuate between 2 and 4% but in 2007 they reached a value of 1%. Again, the fall in long-term rates is due to the fall in premia. Using data from 1970 to 2010, Christensen et al. (2010) show a high volatility in the term premia until 1985, reaching a value or premia around 5%. Then, a drop in premia is observed, to values of 0 to 1% in the 2005-2010 period. These models are not very tractable given the number of parameters that have to be estimated and have no economic intuition behind the movements of the yield curve.

\footnote{The methodologies employed to this end have faced several drawbacks. In particular, there exist econometric issues related, for example, to the short period of time for samples (Bekaert & Hodrick (2001), Bekaert et al. (2001)).}
A second family of affine structure models incorporates macroeconomic variables to explain yields. Models like the one proposed by Ang & Piazzesi (2003), Bernanke et al. (2004), Diebold et al. (2006), Hördael et al. (2006) and Smith & Taylor (2009) allow the yield curve to respond to macroeconomic shocks, giving them more intuition that makes the model more useful for policy recommendations. In particular, they have considered measures of inflation, activity (GDP growth and output growth), Fed-funds rates and expectations, among other variables. For example, Ang & Piazzesi (2003) use inflation and economic growth factors jointly with unobserved latent factors to explain the dynamics of the yield curve. In their analysis, most of the variation in bond yields is due to macroeconomic factors, affecting mostly the short and medium term of the curve, while latent factors explain the long section of the curve. Meanwhile Hördael et al. (2006) use a three-equation model to characterize the New Keynesian Phillips curve, the aggregate demand and the Taylor rule. On the other hand, Diebold et al. (2006) include capacity utilization, the Fed funds rate and annual price inflation, combined with three latent factors (the same that were proposed by Nelson & Siegel (1987)). Some authors have restricted their models in order for the response of macroeconomic variables to yield shocks to be zero (unidirectional propagation of shocks), which is not necessarily true in real life (see Rudebusch & Wu (2008), Ludvigson & Ng (2009), Orphanides & Wei (2012) and Joslin et al. (2014)). However, their main drawback is related to the following idea. If macroeconomic variables are true factors of the yield curve, as is proposed by the affine structure of these papers, then a regression of yields on macroeconomic variables should report a very good adjustment of macroeconomic variables used in the literature, which has been consistently rejected.3 This failure of the affine model motivates the imposition of restriction on the relation between yields and macroeconomic factors. In particular, it is assumed that yields of all maturities have loadings equal to zero, so it is not possible to recover macroeconomic factors from yield data. Even in those cases, changes in macroeconomic factors can affect future yield curves and expectations of future short-term interest rates, but they have an offsetting impact on premia. In that way, the two effects cancel out and the contemporaneous term structure is unchanged. In these situations we say that macroeconomic variables are unspanned factors.

In the latter, which corresponds to the third family of models, affine structure models have worked with the idea of unspanned factors, in which macroeconomic factors are used to adjust the short-term interest rate but not the observed yields at longer maturities. This allows that macroeconomic variables affect expected future short-term interest rates but not term premia. This feature permits that the effect of macroeconomic variables observed on both expected future rates and term premia–previously used in the literature–do not cancel out. This last family of models–on which our work is based–has been proposed by Joslin et al. (2014). Other papers related to this framework are Wright (2011) and Bauer et al. (2014).

For Chile, there is no evidence of the application of this type of models. However, some authors have attempted to evaluate the premium implicit in the break-even inflation (see Jervis (2007), Chumacero & Opazo (2008)) but no economic interpretation nor term premia structure was analyzed. Ochoa (2006) analyzes the interest rate’s factor of real bonds based on a no-arbitrage term structure model. The author concludes that the dynamics of yields are explained by two latent factors, but no identification of term premia was examined. Finally, the economic iteration between the term structure and macroeconomic variables has been reported extensively in Chile (see Ochoa

3In general, the $R^2$ is quite moderated.
(2006), Morales (2010), Alfaro et al. (2011) and (Ceballos 2014)) although there is no term premia derivation. Thus, to the best of our knowledge, this is the first paper attempting to decompose and analyze the term premia in the nominal interest rates in Chile.

3 Data

The interest rates data used in this paper correspond to interest rate swaps for maturities shorter than one year, and nominal government bonds denoted as “Banco Central de Chile in pesos” (BCP). The maturities considered are 3, 6, 12, 24, 60 and 120 months. The nominal yield curve in Chile slopes up on average implying that an investor who holds long-term bonds must be compensated for taking such position.

The macroeconomic factors used in this paper are the standard in the macro-finance DTSM, which are variables related to economic growth and inflation. In particular, we consider the effective annual growth of core inflation and GDP growth measured by the IMACEC indicator (denoted as \( \pi_{\text{core}} \) and \( Y \) respectively). We also consider other potential candidates, such as the information reported in the Economic Expectation Survey (EES) reported in monthly frequency by the Central Bank of Chile, which asks for the expectation of private agents and academics about economic growth and inflation. Also, we consider the output gap and the standard deviation of inflation from its target of 3%.

Finally, we consider a set of indicators that may be related to nominal and real uncertainty that could affect the dynamics of the term premia. In particular, we consider the dispersion informed by the Economic Expectation Survey (EES) related to the questions of expected inflation one and two years ahead (nominal uncertainty), as well as expected economic growth (growth of GDP) for current and next year (real uncertainty). Appendix B explains in detail how the different variables were constructed as well as their sources.

4 Dynamic Term Structure Model

4.1 Model Specification

The standard affine Gaussian Dynamic Term Structure Model (henceforth DTSM) in the literature since Ang & Piazzesi (2003) is determined by the existence of \( N \) risk factors, \( X_t \), which follow a first-order VAR under the probability measure \( \mathbb{P} \):

\[
X_{t+1} = \mu + \phi X_t + \Sigma \varepsilon_{t+1}
\]

The short-term interest rate \( r_t \) (which in the rest of the work corresponds to the 3-month rate) is an affine function of the risk factors:

\[4\text{Ceballos (2014) presents the main empirical facts for the Chilean nominal term structure.}
\[5\text{Our measure of core inflation is the consumer price index (CPI) excluding energy and food. The 236 products included in this index total a combined 72.29}\%\text{ of the total weight of the CPI.}
\[6\text{Computed as the cyclical component of the IMACEC derived from a Hodrick-Prescott filter with smoothing parameter of 14400.}
\[7\text{Ceballos & Romero (2014) report that these components are relevant in the determination of output growth and inflation.}
\[8\text{This section uses standard notation in the literature, specifically based on Bauer et al. (2012).} \]
Finally, there exists a stochastic discount factor (SDF) that prices all assets under no arbitrage, which is affine as in Duffee (2002):

\[ r_t = \delta_0 + \delta_1' X_t \]  

(2)

where the vectors of risk prices (\( \lambda \)) are affine to the same risk factors as follows \( \lambda_t = \lambda_0 + \lambda_1 X_t \). Under the risk-neutral probability measure \( Q \), the price of an \( n \)-period zero coupon bond is determined by

\[ P^n_t = E^Q_t (\exp \left( -\sum_{h=1}^{n-1} r_{t+h} \right)) \]

(3)

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\[ \lambda_t = \lambda_0 + \lambda_1' X_t \]

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\[ P^n_t = E^Q_t (\exp \left( -\sum_{h=1}^{n-1} r_{t+h} \right)) \]

(4)

where \( \mu^Q = \mu - \Sigma \lambda_0 \) and \( \phi^Q = \phi - \Sigma \lambda_1 \). With this, the price of bonds at different maturities can be summarized into \( P^n_t = \exp \left( A_n + B_n' X_t \right) \), where \( A_n \) and \( B_n \) follow the recursions:9

\[ A_{n+1} = A_n + \left( \mu^Q \right)' B_n + \frac{1}{2} B_n' \Sigma \Sigma' B_n - \delta_0 \]

(5)

\[ B_{n+1} = \left( \phi^Q \right)' B_n - \delta_1 \]

(6)

with initial values \( A_0 = B_0 = 0 \). Thus, the model-implied yields are \( y^n_t = \frac{-\log(P^n_t)}{n} = A_n + B_n' X_t \), with \( A_n = \frac{A_n}{n} \) and \( B_n = \frac{B_n}{n} \). On the other hand, the risk-neutral yield (the observed yield if investors would price bonds under risk neutrality) corresponds to

\[ \tilde{y}^n_t = \tilde{A}_n + \tilde{B}_n' X_t \]

(7)

\[ \tilde{A}_n = \frac{A_n}{n} (\mu, \phi, \delta_0, \delta_1, \Sigma) \]

(8)

\[ \tilde{B}_n = \frac{B_n (\phi, \delta_1)}{n} \]

(9)

The risk-neutral yield denoted in (7) are related mainly to the expected path of the future monetary policy rate, and therefore, reflects the part of the interest rates that are driven by expectations. Furthermore, the derivation of the expected short rate allows us to identify the term premia (\( tp \)) component, which corresponds to the difference between the implied yield and the risk-neutral yield, as follows:

\[ tp^n_t = y^n_t - \tilde{y}^n_t \]

(10)

Calibration of this model requires the computation of parameters \( \theta = (\mu, \phi, \mu^Q, \phi^Q, \delta_0, \delta_1, \Sigma) \) which can be obtained using maximum likelihood (ML). However, we follow the normalization approach reported by Joslin et al. (2014). This normalization allows to calibrate the DTSM model under

\[ \frac{1}{n} \sum_{t=1}^{n} (y_t - \tilde{y}_t) = \frac{1}{n} \sum_{t=1}^{n} (\tilde{A}_t + \tilde{B}_t' X_t - \tilde{A}_t - \tilde{B}_t' X_t) = 0 \]

9This result is standard in the literature. See the appendix in Ang & Piazzesi (2003) for a proof.
a two-step estimation procedure: (1) the conditional likelihood of \( X \) depends on \((\mu, \phi, \Sigma)\), which can be estimated easily via OLS and (2) the conditional likelihood of yield under \( Q \) depends on \((r_Q, \lambda_Q, \Sigma)\), where \( r_Q \) is the risk-neutral unconditional mean of the short-term interest rate, and \( \lambda_Q \) are the eigenvalues of \( \Phi_Q \). This approach allows an efficient estimation of the model, in particular, the convergence to global optimum is faster than ML regardless of the number of risk factors considered.

An important issue with this class of models is the high persistence of yield bonds when the VAR estimation in (1) is done. Thus, conventional estimates of DTSM parameters are biased, and therefore a misleading interpretation of expected short-term rate and term premia arises. Following Bauer et al. (2012), a bias-correction is made in the parameter estimation.\(^{10, 11}\)

### 4.2 Unspanned macroeconomic factors

The typical modeling of DTSM models (Ang & Piazzesi (2003), Rudebusch & Wu (2008), among others) assumes that the relation between bond yield and macroeconomic factors can be inverted, so the risk factors may be represented as linear combinations of yields. This is known as macroeconomic spanning variables, which according to Joslin et al. (2014) implies that macroeconomic variables are not informative about risk premiums. Empirically, the opposite results have been reported.

Thus, the risk factors \( X \) may be separated, for instance as \( X_t = [X^1_t \; X^2_t] \), where factors \( X^1_t \) are relevant for explaining the historical interest yield and has a good fit in observed interest rates, and \( X^2_t \) affects both the future expected path of short-term interest rates and the term premia. In other words, \( X^2_t \) factors are relevant for forecasting future interest rates, but changes in those factors have no effects on yield bonds, which are denoted as unspanned factors.

In particular, in this work we consider as risk factors the following variables in vector \( X \): \( PC1, \; PC2, \; PC3, \; \pi, \; Y, \; \rho \). The \( PC \) terms correspond to the first, second and third principal components of interest rates, commonly denoted as the level, slope and curvature of the yield curve (Litterman & Scheinkman (1991)). The variables \( \pi \) and \( Y \) are the annual growth of core inflation and economic growth, respectively, and \( \rho \) is related to international factors relevant for the local yield curve like the \( VIX \) risk index. With this, we consider as unspanned macroeconomic variables \([\pi, Y, \rho]\).

### 4.3 Robustness

The DTSM model estimation and the information related to the expected short-term rate as well as the term premia, may be potentially affected by (1) the short sample estimation and (2) the risk factors considered. In the first point, the sample considered covers the period from 2003 to 2014 in monthly frequency (the data section provides further analysis of data considered). The second issue is related of how the term premia and expected short-term rate change when different risk factors are taken into account. Rudebusch et al. (2007) compare five-term premia estimates from different DTSMs, concluding that they may differ in the level of term premia, but exhibit a similar

\(^{10}\)A further analysis and explanation of potential misleading of standard estimates as well as the bias correction used in this paper is extensively explained in that paper.

\(^{11}\)The estimation process was done using the Matlab code of Bauer et al. (2014) publicly available at https://www.aeaweb.org/articles.php?doi=10.1257/aer.104.1.323
downward trend since 1990.

For robustness, we proceed to compute the term premia for long interest rate (10 years nominal bond) under different combination of the unspanned risk factors ($X_t^2$). The risk factors $X_t^1$ are the same because those factors have a good fit in capturing the empirical observed yield. For this, we consider some alternative specifications, described in Table 1. In general terms, we attempt to control for different measures of local inflation and economic growth. In particular, we evaluate historical, expected and gap measures of these variables. Also, we control for international factors that may affect local interest rates ($VIX$).

### Table 1: Alternative DTSMs risk factors

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
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<tr>
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<td>✓</td>
<td>✓</td>
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<td>✓</td>
</tr>
<tr>
<td>Factors used to forecast the short-term interest rate ($X_2$)</td>
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<tr>
<td>$\pi_{Core}$</td>
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<tr>
<td>$Y$</td>
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<td></td>
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<tr>
<td>$EES(\pi_{1y})$</td>
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<td>✓</td>
<td>✓</td>
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<tr>
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<tr>
<td>$\pi_{Core} - \bar{\pi}$</td>
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<tr>
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<tr>
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</tbody>
</table>

Each column represents a different specification to compute term premia. See Table 3 for the definition of each variable.

In our second robustness check, our baseline DTSMs model is computed using a recursive estimation procedure starting from January 2007 to the last observation in order to quantify the sample effect of the DTSM estimation. This is done to evaluate the sensitivity of the term premia computation. For all the estimations, we report the 10, 50 and 90 percentile for the term premia in 10 years interest rates.

### 5 Results

#### 5.1 Decomposing long interest rates: term premia

In Figure 2 we present our estimated term premia.\textsuperscript{12} \textsuperscript{13} The first panel of the figure reports the term premia estimated for all maturities in time series, while the second panel shows the average term premia structure for different samples. Several elements can be noted from these

\textsuperscript{12}In Appendix C we report the estimated parameters.

\textsuperscript{13}Similar results are obtained using the methodology of Adrian et al. (2013) for Chile. We are working on an extended version considering this methodology and including other developing countries in the sample.
figures. First of all, in the first panel we observe three different regimes of the premiums. In the period 2005-2007, we observe a stable pattern for premia, which fluctuates between 0 and 2%, depending on the maturity that we analyze. This period of stability is followed by a turbulent phase by the middle of 2008 to the middle of 2010, when the global financial crisis started. In this period, while short-term interest rates (three to six months) show a stable process around 0% of premia, interest rates between one and ten years increase significantly. In particular, the one-year interest rate rose from zero premia to 2% of premium, while longer interest rates showed an increase of almost 100%, passing from 2% to 4% of premia. As we can see, this phenomenon was observed just in the middle of the crisis period and for medium to long-term interest rates. This is reasonable because the monetary policy rate (which should rule the short-term rates) was at minimum values after the crisis (2009-2010), not affecting short-term rates but longer rates because of the financial situation. After 2010, we observe a clear return to the pre-crisis values in the case of one-and two-year premia, and smaller values for premia between five and ten years. It is also interesting to note that at the end of the sample we observe a decrease in premiums between one and two years, while we also observe an increase in premiums between five and ten years.

As is clear from the first panel of the figure, we observe three different regimes for term premia. In order to see where are the biggest changes, we show in the second panel of Figure 2 the mean of the cross section values of the term premium in each in different sub-samples, compared with the full-sample estimates. As we mentioned before, the first sub-sample (2005-2007) shows a more parsimonious behavior that is reflected by the less volatile behavior of the term premium. The crisis period (2008-2009) showed a steeper slope from the one-year rate onwards. Finally, as is logical, the mean value across the full sample is an average between the two curves. From this figure, several conclusions could be obtained. First of all, there is no big difference in the short section of the yield curve premia, i.e. between the three-month yield and one-year yield. Second, the section of the yield curve where the slope is bigger is between two and five years. In those years the changes in term premia are maximum. Third, while the changes in term premia between two and five years are quite similar between the first sub-sample and the whole sample curve, this is not the case for the 2008-2009 curve for premia. In the first case we observe a change of around 50 basis points while in the crisis period this change amounts to 250 basis points. Finally, in the last section of the curve we observe an increasing pattern in all the samples but the crisis, where almost no change is observed between five-and ten-year premia.

**Figure 2: Term premium estimation in Chile**
To understand the historical behavior of the ten-year nominal bond we proceed to compare the interest yield observed in some relevant episodes with the pre-crisis period detailed in Table 2. The first sample spans from January 2005 to December 2007 which we refer to as the pre-crisis period. In this period, the model decomposition for the ten-year bond suggests that almost 71% of the interest rate average corresponds to the expected short-term interest rate and 60% to the term premia.
The second period is related to the sample from Jan-08 to Jun-09 which according to the National Bureau of Economic Research was when the US recession began. This period was characterized by the global recession started in 2007 with the emergence of mortgage-loan losses and reached its financial turmoil peak with the bankruptcy event of Lehman Brothers in Sep-08. Locally, shortly after the collapse of Lehman Brothers, the Central Bank of Chile began reducing the policy rate from an initial level of 7.25 percent annually until it reached the effective lower bound of 50 basis points in July 2009. The rate was kept there for one year until June 2010. During this period, the long-term interest rate went up to 6.41% (34 bps higher than the pre-crisis level) explained by an increase in the term premia of about 113 bps, whereas the expected short-term interest rate decreased almost 80 bps, in line with the recent monetary policy response to the global financial crisis.

Also, we consider a normalization period covering the period from July 2009 to December 2011, which was characterized by a fall in risk indicators after their peak reached during the financial crisis and a recovery in GDP growth. During that time, the Central Bank also took unconventional measures aimed at providing liquidity to the financial markets. For instance, the Bank offered financial institutions access to a term liquidity program (FLAP) at the monetary policy rate of 0.5 percent. Again, the long-term interest rate decomposition suggests that the rise in long-term interest rates by 16 bps was explained by higher term premia, which compensated the lower expected short-term interest rate.

Finally, the ten-year nominal bond dropped to its all-time low, beginning with a sharp decline in May 2013 when officials of the Federal Reserve System first began to talk of the possibility of the U.S. central bank tapering its securities purchases (tapering talk). Since May 2013, the ten-year bond has fallen 60 bps, where the expected short-term interest rate accounted for more than 90% of the movement. The latter has been in line with recent monetary policy actions taken by the Central Bank of Chile, reducing the monetary policy rate by 150 bps to 3.5%. Also, comparing the current ten-year interest rate with the values observed during the pre-crisis period, the model decomposition suggests that 40% of the fall in long-term interest rates is related to lower short-term interest rates, and 60% to a fall in the term premia.

5.2 What drives term premia?

In this section we test some potential factors explaining term premia and their evolution. Specifically, we proceed to evaluate the relevance of uncertainty as a possible driver of the term premium as some authors have argued. Wright (2011) and Bauer et al. (2014) find that the uncertainty relative to inflation and GDP are important drivers of term premium behavior.

Why does uncertainty matter? Because nominal bonds are riskier when the expected inflation is more volatile, investors will demand a positive compensation for holding such bonds to hedge against inflation risk. Empirically, Rudebusch et al. (2006) founding that many affine term structure models suggest that the lower trend exhibited in term premia is because inflation uncertainty also shows a downward trend. Similar results are reported by Wright (2011) for ten developed economies.

Another reason supporting the idea of considering the uncertainty as a term premia driver, is that it has been extensively reported that the impact of nominal and real uncertainty in the real economy is significant (Fountas & Karanasos (2007), Kim & Lin (2012), among others). In the
case of Chile, Ceballos & Romero (2014) reported that the impact of inflation uncertainty was especially relevant in the period where the inflation-targeting (IT) framework became explicit. Thus, we evaluate empirical benchmarks presented in Ceballos & Romero (2014), which are related to nominal and real uncertainty as the dispersion of economic survey of expectations related to inflation and economic growth (GDP), respectively, informed monthly by the Central bank of Chile through the Economic Expectation Survey (EES).

With this, we estimate the joint dynamics between uncertainty and term premia using a VAR framework considering two lags according to most lag criteria selection (Akaike, Schwarz and Hannan-Quinn). We proceed to evaluate the dynamic between the term premia in Chile and nominal and real uncertainty considering two alternatives: (1) VAR between term premia and nominal and real uncertainty related to the one-year expected inflation and expected economic growth to actual year, and (2) VAR between term premia and nominal and real uncertainty related to the two-year expected inflation and expected economic growth for next year. In both cases, we consider the term premia estimate for the US according to Adrian et al. (2013) as a control for international behavior of premia that might affect local term premia. Figure 4 shows the impulse-response from shocks to nominal and real uncertainty to the estimated term premia as well as the US term premia, considering the generalized impulse-response function. In the 10-year term premia, the nominal uncertainty has a significant and positive impact, especially the inflation expected two years ahead (the relevant horizon for monetary policy) within a one-year horizon. Another interesting result is the positive and significative effect of the US premia on the Chilean premia.
Figure 4: Impulse-response function: responses of Chilean term premia to different shocks

Impulse responses from inflation and economic growth uncertainty, and US term premia to Chilean term premia. All responses are expressed in percentage terms. Red solid lines correspond to shocks in the expected inflation one year ahead and the economic growth (GDP) expected the current year, while dashed red lines show 90% confidence interval. The black dotted line corresponds to shocks in the expected inflation two year ahead and economic growth (GDP) expected next year. Grey area shows the 90% confidence interval.

6 Conclusions

The objective of this paper is to decompose the long-term interest rates in Chile (the 10-year interest rates in Chile) into the expected short-term interest rate and the term premia component to analyze their behavior and main drivers. Under a dynamic term structure model we estimate the premia for Chile in the period 2003-2014 and proceed to compute the term premia using alternatives of risk factors that may affect the expected short-term interest rate (and therefore the term premia) including different measures of local inflation and economic growth (historical, expected and gaps) and controlling for international factors such as the VIX.

The decomposition of long interest rates shows three interesting facts: (1) the term premia structure is sloped up, (2) the dynamics of term premia depend on the sample considered being more parsimonious in 2005-2007 and more volatile during the financial crisis in 2008-2010, and (3) in episodes of financial turmoil the term premia is a key driver of long-term interest rate dynamics. Therefore, the behavior of term premia is a relevant element to consider. In this regard, we analyze
the response of term premia to nominal and real uncertainty shocks (related to the dispersion in the Expectation Economic Survey in inflation and GDP) as well as the US term premia. Our findings suggest that in the 10-year term premia, the nominal uncertainty has a significant and positive impact, especially the inflation expected two years ahead (the relevant horizon for monetary policy). Another interesting result is the positive and significant effect of the US premia on the Chilean premia.

Our results are relevant to consider in practice for effectiveness of monetary policy. The policy implication of decomposing long-term interest rates might be summarized as follows. First, it provides feedback for central bank policy decisions about the market’s expectations regarding future changes in the short interest rate and term premia. Thus, the monetary authority may monitor whether movements in long interest rates are in line with monetary policy or not. Second, it can serve as an indicator of relevant and real monetary stance in the economy. For instance, a non-expected loose monetary policy does not necessarily lead lower long interest rates if term premium drivers are unchanged or going opposites direction. Finally, in some circumstances monetary policy should control long interest rates via public statements or communications regarding the control of future inflation, and therefore reducing uncertainty of market participants, leading a reduction in the drivers that might affect term premia at long interest rates.
References


Kim, D. & Wright, J. (2005), ‘An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates’. 


Appendix A  Long-term interest rates

The figure shows the level factor under the Nelson-Siegel model. This factor is related to the level of the whole term structure because changes in these factors implied a parallel movement in all interest rates at different maturities.

**Figure 5:** Long-term interest rates for selected economies
Appendix B  Data description

The table shows all the variables considered in this work. Particularly, the table describes the macroeconomic variables used to compute the DTSM using both local and international economic factors, a brief description and the source where the data is available.

Table 3: Data description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{Core}$</td>
<td>Annual inflation CPI core</td>
<td>CBCh</td>
</tr>
<tr>
<td>$Y$</td>
<td>Economic growth (IMACEC)</td>
<td>CBCh</td>
</tr>
<tr>
<td>$EES(\pi_{1Y})$</td>
<td>Inflation expectation 1 year ahead reported by EES</td>
<td>CBCh</td>
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<tr>
<td>$EES(Y)$</td>
<td>GDP expectation 1 year ahead reported by EES</td>
<td>CBCh</td>
</tr>
<tr>
<td>$\pi - \pi$</td>
<td>Inflation gap: CPI minus inflation target (3%)</td>
<td>CBCh</td>
</tr>
<tr>
<td>$Y - Y$</td>
<td>Output gap: IMACEC minus HP trend</td>
<td>CBCh</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>Inflation uncertainty measured by the dispersion reported by EES</td>
<td>CBCh</td>
</tr>
<tr>
<td>$\sigma_{Y}$</td>
<td>Output uncertainty measured by the dispersion reported by EES</td>
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</tr>
<tr>
<td>VIX</td>
<td>VIX</td>
<td>Bloomberg</td>
</tr>
</tbody>
</table>

Core CPI is the consumer price index (CPI) excluding energy and food. The 236 products included in this index total a 72.29% of the total weight of the CPI. CBCh: Central Bank of Chile. All data in monthly frequency.
The table reports the coefficient for the DTSM model explained in section 4.1.

**Table 4: Baseline model parameter estimates**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
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<td>$\delta$</td>
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<td>$\mu$</td>
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</table>
Appendix D  Robustness estimations

In Figure 6 the term premia for the 10-year bonds are shown. The range corresponds to the maximum and minimum values for all different DTSM model specifications. The solid line is the baseline estimate for term premia. The alternative models are explained in section 4.3, where we estimate DTSM using different risk factors.

Figure 7 shows the recursive estimation starting in January 2009. In this figure, the 10, 50 and 90 percentiles are reported.

**Figure 6:** Computing term premia for long-term interest rates in Chile: robustness exercises

**Figure 7:** Computing term premia for long-term interest rates in Chile: rolling estimates