An unobvious dynamics of rolled over time banking deposits under a shift in depositors' preferences: whether a decrease of weighted average maturity of deposits is indeed an early warning liquidity indicator?

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25 December 2014

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December 2014

Version 1

Working paper

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Abstract

A continuous-time deterministic model for analytical simulation of an impact of changes in credit turnover, term to maturity structure and rollover rate on balances of time banking deposits, i.e., preferences of depositors, is developed. The model allows taking into account an attraction of new deposits and rolling over the maturing deposits. It is shown some deceptive and unobvious regimes of depositing when the deposit balances increase in the beginning and then fall down and vice versa. It is presented an equilibrium money conservation law for banks. Besides, the examples of calculations of continuous-time deposit dynamics are given. It is shown that such a Basel early warning liquidity indicator as a decrease of weighted average maturity of liabilities is necessary but not sufficient. It is proposed that to make more accurate ALM decisions and avoid serious managerial errors a bank should rely not only on a change in deposit balances but on changes in turnovers, term structure of deposits and rollover rate. At long-term lending, a bank should orient on minimal deposit balances in a short-term period and long-term, steady state deposit balances, employing for this an equilibrium money conservation law.

Key words: bank, time deposit, retail, balance, credit turnover, debit turnover, term to maturity, rollover rate, dynamics, money conservation law, liquidity, early warning indicator, Basel iii, asset liability management (ALM), Volterra integral equation, Laplace transform

JEL Classifications: G21

1. INTRODUCTION

Retail deposits are an important source of banks' funds to finance loan expansion. Banks often make decisions about funding its operations and allocating resources basing on whether its deposit portfolio increases or decreases. But orienting only on a change in deposit balances at ALM decisions may lead to serious managerial errors. It should be noted that the change in the deposit balances may be deceptive and unobvious. So, in some cases, changes in a turnover and a term structure of deposits affect a deterioration of liquidity in a short-term period while the bank's liquidity will really improve in a relatively long-term period and vice versa. Wherein, a bank may handle a short-term decline of its deposit balances as a prevailing long-term trend and respectively as a signal to stop lending or to increase an interest rate on deposits protect from the further fall of its deposits.

The above-mentioned cases deal with transient liquidity regimes that are very distinctive for banks. So, Voloshyn (2005) showed that time of transition from one liquidity regime to another may reach from 5 to 7 years. So, taking into account namely transient liquidity regime is very important to effectively manage assets and liabilities.

Note that a change in deposit balances depends on both attraction of new deposits and rolling over the maturing deposits. Each of these processes is characterized by a turnover, a term to maturity structure of deposits and a given rollover rate. Namely these parameters describe a transient liquidity regime. Therefore, sound ALM decisions would be based on changes in turnover, term structure of deposits, and rollover rate but not only on changes in deposit balances.

The aim of this paper is to study dynamics of retail time banking deposits under sudden stepwise changes in depositors' preferences in term and amount of deposits.

A continuous-time deterministic model is chosen as a tool of investigation in deposit dynamics (Voloshyn, 2004, 2014; Freedman, 2004; Selyutin and Rudenko, 2013). Because, "transition from a discrete-time model to a continuous-time one allows employing a rich store of methods of functional analysis, giving a qualitative picture of a bank's business activity, discovering general regularities of bank's dynamics" (Linder, 1998).

Such a study is central to understand a situation with liquidity when the deposit portfolio gets recovered after a bank run. Then the next urgent question appears: "What volume of attracted deposits could bank direct to long-term lending".

Another vital question for banks consists in selecting appropriate early warning liquidity indicators. So, BIS (2008) recommends such early warning indicators as:

- a decrease of weighted average maturity of liabilities;
- increasing retail deposit outflows.

In this paper, such early warning indicators are shown to be insufficient.

2. A CONTINUOUS-TIME MODEL

We will consider a continuous deterministic dynamics of retail time banking deposits that are rolled over by depositors. A deposit principal is matured at the end of each deposit agreement. The depositors are assumed to roll over only their deposit principals but not interests. Thus, they withdraw all interests on their deposits. Besides, we will ignore early withdrawal effect.

The first derivative of the time deposit balances with respect to time is equal to the difference of the instantaneous credit and debit turnovers in the deposit accounts:

$$\frac{dB(t)}{dt} = Ct(t) - Dt(t), \tag{1}$$

where *Ct(t)* and *Dt(t)* denote the instantaneous credit and debit turnovers in the deposit accounts, respectively. Wherein,

$$Ct(t) = \int_0^{\infty} mCt(t,w) \cdot dw, \tag{2}$$

where *mCt(t, w)* is the instantaneous partial (relating to the term to maturity w) credit turnover, i.e., the intensity of the attraction of the deposits with the term to maturity *w*. The partial credit turnover *mCt(t, w)* is assumed to be the sum:

$$mCt(t, w) = mCt_{new}(t, w) + mCt_{ro}(t, w), \tag{3}$$

where $mCt_{new}(t, w)$ is the partial credit turnover of the new attracted deposits, $mCt_{ro}(t, w)$ is the partial credit turnover of rolled over deposits.

Further, let the partial credit turnovers $mCt_{new}(t, w)$ and $mCt_{ro}(t, w)$ be represented through the next functions with separated variables, respectively:

$$mCt_{new}(t, w) = Ct_{new}(t) \cdot f(w), \tag{4}$$

$$mCt_{ro}(t, w) = ROR_1 \cdot Dt(t) \cdot f(w), \tag{5}$$

where $Ct_{new}(t)$ is the total (across all terms to maturity) credit turnover in the new deposit accounts, i.e., the intensity of the attraction of the new deposits (attracted after $t > 0$), f(w) is the term to maturity structure of deposits, ROR_1 is the rollover rate of new and rolled over deposits. Wherein,

$$\int_0^{\infty} f(w) \cdot dw = 1, \tag{6}$$

The dynamics of the debit turnover in the deposit accounts is described by the next equation (Voloshyn, 2004, 2014; Freedman, 2004; Selyutin and Rudenko, 2013):

$$Dt(t) = \varphi(t) + \int_0^t mCt(s, t-s) \cdot ds, \tag{7}$$

where $\varphi(t)$ is the repayment schedule of deposits, existing at time $t = 0$. The second term in the right part of (7) is the repayment schedule of deposits attracting or rolling over after $t > 0$.

Substituting (4) and (5) in (3) and the obtained express in (7) we will get:

$$Dt(t) = \varphi(t) + \int_0^t Ct_{new}(s) \cdot f(t-s) \cdot ds + \int_0^t ROR_1 \cdot Dt(s) \cdot f(t-s) \cdot ds, \tag{8}$$

Note that to find the debit turnover $Dt(t)$ it needs to solve the equation (8) that is the Volterra equation of the second kind.

For simplicity but without loss of generality, the term to maturity structure of the new and rolled over deposits is assumed to be same and to obey the exponential distribution law respect to term to maturity w:

$$f(w) = \frac{1}{W_1} \cdot \exp\left(\frac{-w}{W_1}\right), \tag{9}$$

where W_1 is the weighted average maturity of new and rolled over deposits.

Let the repayment schedule of old deposits, existing at time $t = 0$, be:

$$\varphi(t) = \frac{Ct_0}{1 - ROR_0} \cdot \exp\left(-\frac{t}{W_0}\right), \tag{10}$$

where Ct_0 is the constant credit turnover of deposits attracted at time $t \leq 0$, W_0 is the weighted average maturity of these deposits, ROR_0 is the rollover rate of old deposits.

Let

$$Ct_{new}(t) = Ct_1, \tag{11}$$

be a constant, i.e., not dependent on time.

The non-stationary solution

Thus, at time t = 0, we have some repayment profile (10). At time t > 0, the term structure of attracted deposits and their total turnover are suddenly stepwise changed and then remain constant. These changes are caused by a shift of depositors preferences.

To solve the integral equation (8) in the unknown debit turnover Dt(t) we will apply the Laplace transform method (D'Azzo and Houpis, 1988).

Taking into consideration (9-11), the Laplace transform images of terms of the integral equation (8) are given in Table 1.

Replacing the original terms in equation (8) by the Laplace transform images from Table 1, we will obtain an algebraic equation for the Laplace transform images:

$$F(s) = \frac{Ct_0 \cdot W_0}{(1 - ROR_0) \cdot (1 + W_0 \cdot s)} + \frac{Ct_1}{s + W_1 \cdot s^2} + \frac{ROR_1 \cdot F(s)}{1 + W_1 \cdot s}, \tag{12}$$

or after simple algebraic transformation:

$$F(s) = \frac{Ct_0 \cdot W_0 \cdot (1 + W_1 \cdot s)}{(1 - ROR_0) \cdot (1 + W_0 \cdot s) \cdot (1 - ROR_1 + W_1 \cdot s)} + \frac{Ct_1}{s \cdot (1 - ROR_1 + W_1 \cdot s)}, \tag{13}$$

Table 1. The original functions and their Laplace transform images

#	The original functions	The Laplace transform images
1	$Dt(t)$	$F(s)$
2	$\varphi(t) = \frac{Ct_0}{1-ROR_0}\cdot\exp\left(-\frac{t}{W_0}\right)$	$\frac{Ct_0\cdot W_0}{(1-ROR_0)\cdot(1+W_0\cdot s)}$
3	$Ct_1\cdot\int_0^t\frac{1}{W_1}\cdot\exp\left(-\frac{t-\tau}{W_1}\right)d\tau$	$\frac{Ct_1}{s+W_1\cdot s^2}$
4	$ROR_1\cdot\int_0^t\frac{Dt(\tau)}{W_1}\cdot\exp\left(-\frac{t-\tau}{W_1}\right)d\tau$	$\frac{ROR_1\cdot F(s)}{1+W_1\cdot s}$

The inverse Laplace transforms of the images (13) in Table 2 give the desired debit turnover $Dt(t)$:

$$Dt(t)=\frac{\frac{Ct_0}{1-ROR_0}\cdot\left\{(W_1-W_0)\cdot\exp\left(-\frac{t}{W_0}\right)+ROR_1\cdot W_0\cdot\exp\left(-\frac{(1-ROR_1)\cdot t}{W_1}\right)\right\}}{W_1-(1-ROR_1)\cdot W_0}+$$

$$+\frac{Ct_1}{1-ROR_1}\cdot\left(1-\exp\left(-\frac{1-ROR_1}{W_1}\cdot t\right)\right)$$

. (14)

Knowing the debit (13) and credit (11) turnovers and taking into account (1-6) we will get the deposit balance from the next differential equation:

$$\frac{dB(t)}{dt}=Ct_1-(1-ROR_1)\cdot Dt(t), \tag{15}$$

Note that $\lim_{t\to\infty} Dt(t)=\frac{Ct_1}{1-ROR_1}$ and $\lim_{t\to\infty}\frac{dB(t)}{dt}=0$.

In other words, at constant credit turnover and after a long time we will have zero-increment of deposit balances. Therefore, in order to ensure a continuous growth of deposit portfolio, an uninterrupted growth of credit turnover is needed. Wherein, there is a dangerous pitfall consisting in the fact that under a strong competition for deposits a bank tries to support a deposit growth by increasing the interest rate on deposits. A reluctance of the bank to lower net interest margin may lead to denial from deposit growth strategy.

Table 2. The Laplace transform images and their original functions

#	The Laplace transform images	The original functions
1	$\dfrac{\dfrac{Ct_0}{1-ROR_0}\cdot W_0\cdot(1+W_1\cdot s)}{(1+W_0\cdot s)\cdot(1-ROR_1+W_1\cdot s)}$	$\dfrac{\dfrac{Ct_0}{1-ROR_0}\cdot(W_1-W_0)\cdot\exp\left(-\dfrac{t}{W_0}\right)}{W_1-(1-ROR_1)\cdot W_0}+$ $+\dfrac{\dfrac{Ct_0}{1-ROR_0}\cdot ROR\cdot W_0\cdot\exp\left(-\dfrac{(1-ROR_1)\cdot t}{W_1}\right)}{W_1-(1-ROR_1)\cdot W_0}$
2	$\dfrac{Ct_1}{s\cdot(1-ROR_1+W_1\cdot s)}$	$\dfrac{Ct_1}{1-ROR_1}\cdot\left(1-\exp\left(-\dfrac{1-ROR_1}{W_1}\cdot t\right)\right)$

The stationary solution

In order to get the stationary solution for the debit turnover *Dt(w)* it is conveniently to employ equation (8) that has the next equivalent differential form, setting $\dfrac{dDt(t,w)}{dt}=0$:

$$-\frac{dDt(w)}{dw}=\left(Ct_1-\left(1-ROR_1\right)\cdot Dt(0)\right)\cdot f(w), \tag{16}$$

at the next initial condition:

$$Dt(0)=\frac{Ct_1}{1-ROR_1}, \tag{17}$$

which was obtained from (15), setting:

$$\frac{dB(t)}{dt}=0, \tag{18}$$

Integrating equation (16) in view of (8) at the initial condition (17) leads to the following stationary distribution of the debit turnover through w:

$$Dt(w) = \frac{Ct_1}{1 - ROR_1} \cdot \exp\left(-\frac{t}{W_1} \right), \tag{19}$$

Then, the steady state deposit balances will be equal to:

$$\lim_{t \to \infty} B(t) = B(0) + \int_0^\infty Dt(w) \cdot dw = \frac{Ct_1 \cdot W_1}{1 - ROR_1}. \tag{20}$$

This is a marginal volume of deposit portfolio. Then, growth of deposits is ceased.

It can be shown the validity of the next equilibrium money conservation law for deposit transition from one steady state to another: *the equilibrium change in deposit balances is equal to*:

$$\Delta B = \frac{Ct_1 \cdot W_1}{1 - ROR_1} - \frac{Ct_0 \cdot W_0}{1 - ROR_0}. \tag{21}$$

Thus, the change in the deposit balances depends on the changes in the credit turnovers, the weighted average terms to maturity and the rollover rates. It should be paid attention that the law (20) is valid only under steady state condition.

3. EXAMPLES OF CALCULATIONS

We will consider the next set of the case studies that are given in Table 3. Note that in Table 3 the credit turnovers Ct_0 and Ct_1 have dimension 'bln USD per year', and the weighted average maturities W_0 and W_1 have dimension 'year'.

The results of calculations are given on Fig. 1-3. Under case study #1 (Table 3), an decrease of credit turnover with a simultaneous increase of the weighted average maturity leads to a drop of deposit balances in a short-term period but to an unobvious growth of them up to equilibrium value 2.75 in a long-term period (Fig. 1). Wherein, a bank may handle a short-term decline of its deposit balances as a prevailing long-term trend and respectively as a signal to stop lending. If the bank uses Basel increasing retail deposit outflows as an early warning indicator (BIS, 2008), perhaps, it could increase interest rate on deposits to protect from the further falling down its deposits. Although in fact the deposit balances will further rise greater than the initial level in 2.5 bln USD.

Table 3. Parameters of the old (attracted before t = 0), new attracted and rolled over (after t > 0) deposits

Case studies	Parameters of old deposits	Parameters of new and rolled over deposits	Rollover rates	Changes in deposit balances, ΔB, at t →∞ (see formula (21))
# 1	$Ct_0 = 1$ $W_0 = 0.25$	$Ct_1 = 0.55$ $W_1 = 0.5$	$ROR_0 = 0.9$ $ROR_1 = 0.9$	$\frac{0.55 \cdot 0.5}{1-0.9} - \frac{1 \cdot 0.25}{1-0.9} =$ $= 2.75 - 2.5 = +0.25$
# 2	$Ct_0 = 1$ $W_0 = 0.5$	$Ct_1 = 1.8$ $W_1 = 0.25$	$ROR_0 = 0.9$ $ROR_1 = 0.9$	$\frac{1.8 \cdot 0.25}{1-0.9} - \frac{1 \cdot 0.5}{1-0.9} =$ $= 4.5 - 5.0 = -0.5$
# 2	$Ct_0 = 1$ $W_0 = 0.5$	$Ct_1 = 3.0$ $W_1 = 0.25$	$ROR_0 = 0.9$ $ROR_1 = 0.9$	$\frac{3 \cdot 0.25}{1-0.9} - \frac{1 \cdot 0.5}{1-0.9} =$ $= 7.5 - 5.0 = +2.5$

On the contrary, under case study #2 (Table 3), an increase of credit turnover with a simultaneous decrease of the weighted average deposit maturity leads to a growth of deposit balances in a short-term period but to an unobvious fall of them up to equilibrium value 4.5 in a long-term period (Fig. 2). Herewith, a bank may handle a short-term increase of its deposit balances as a prevailing long-term trend and respectively as a signal to extend its lending programs. Although in fact the deposit balances will further fall down lower than the initial level in 5.0 bln USD. Perhaps, the bank could decrease interest rate on deposits to protect from the further growing its deposits.

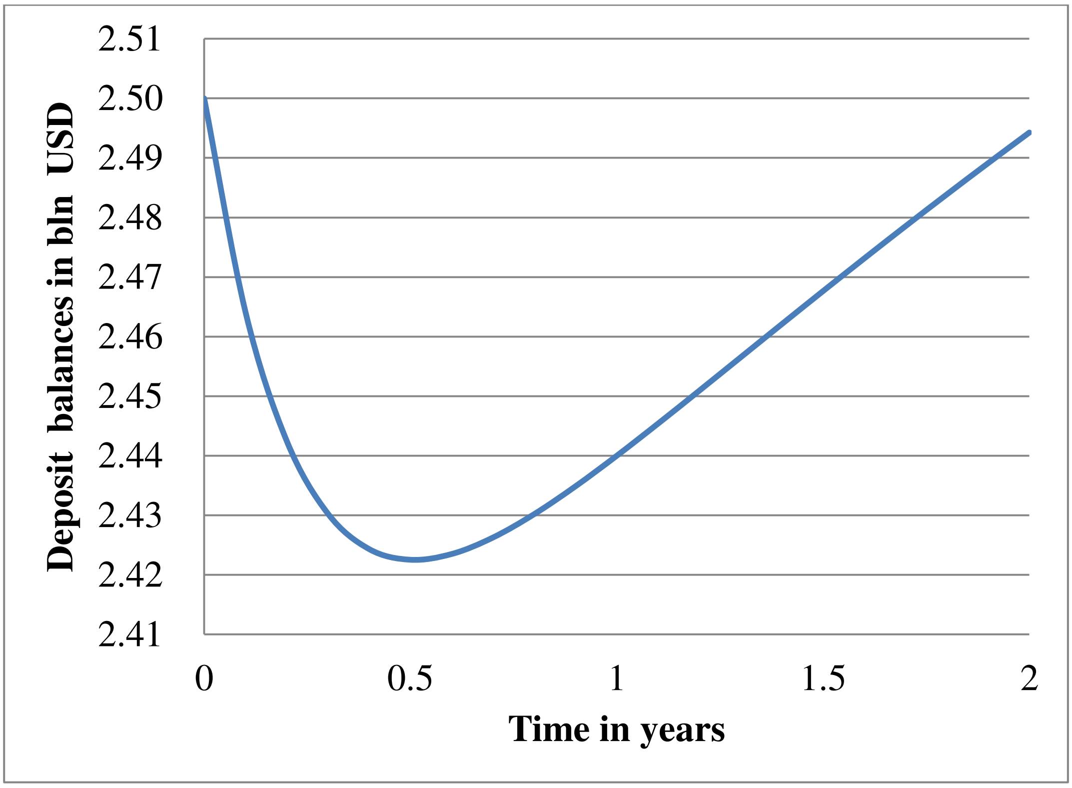

Fig. 1. Dynamics of deposit balances under case study #1:

$B(0) = 2.5,\ \lim_{t\to\infty} B(t) = 2.75,\ \Delta B = +0.25$ in bln USD

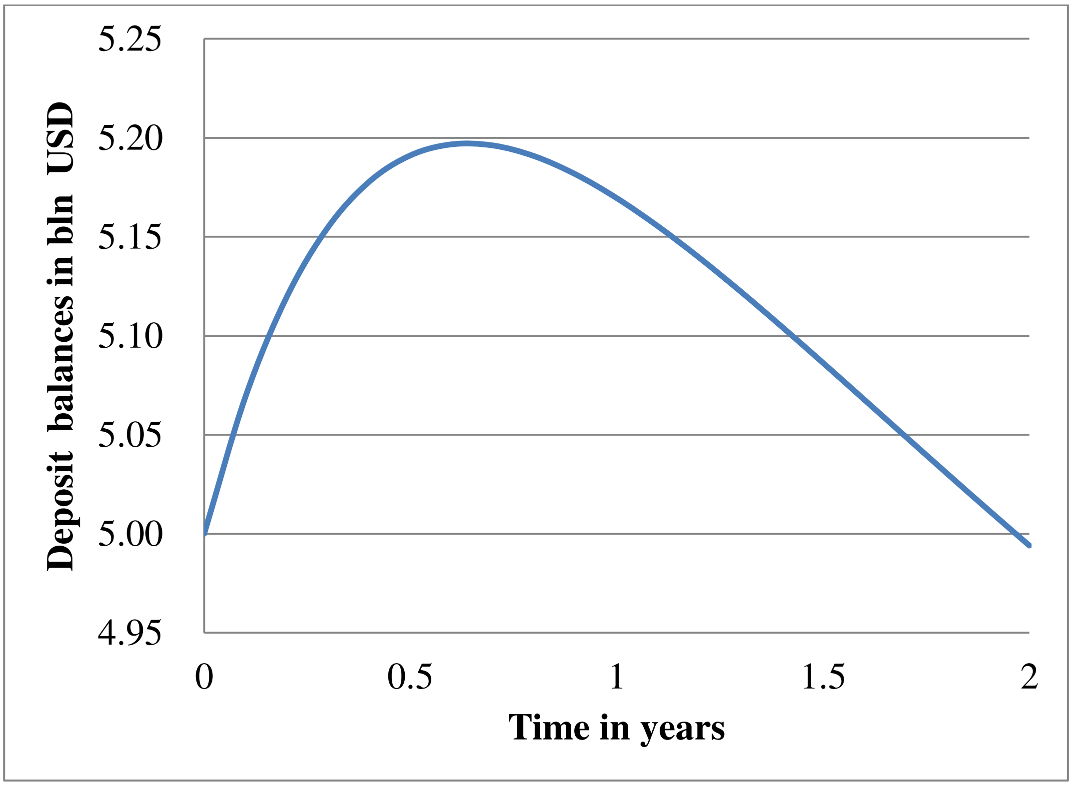

Fig. 2. Dynamics of deposit balances under case study #2:

$B(0) = 5.0,\ \lim_{t\to\infty} B(t) = 4.5,\ \Delta B = -0.5$ in bln USD

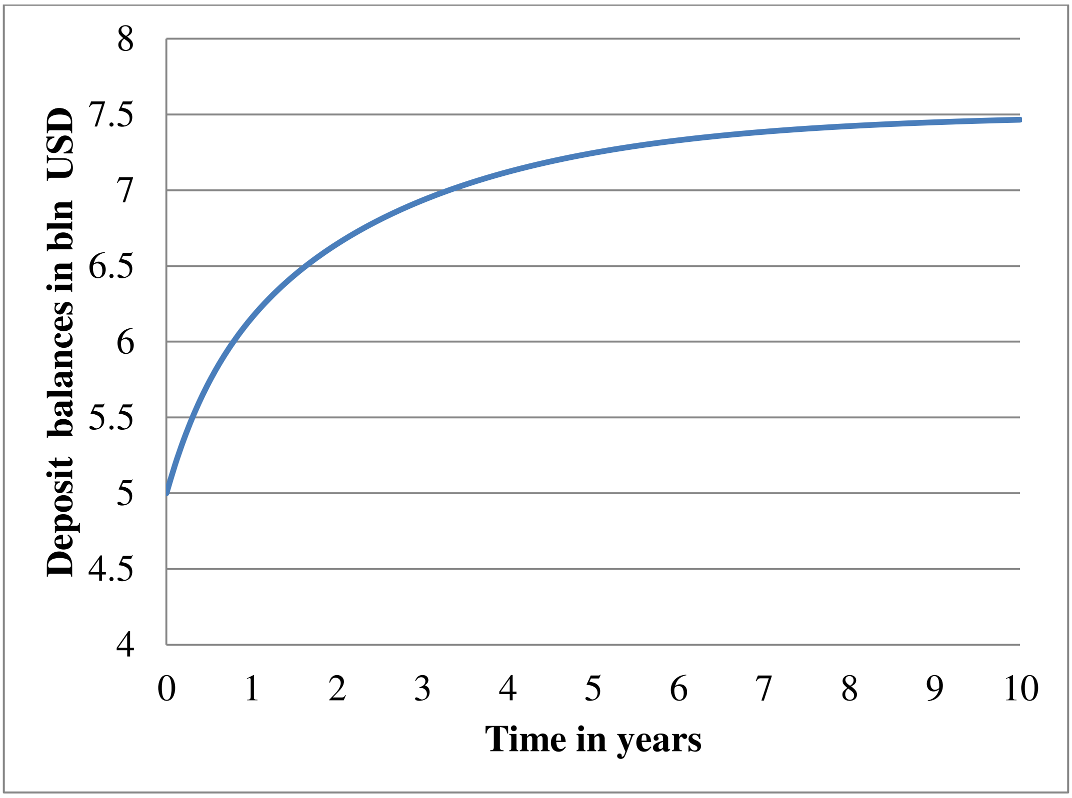

Fig. 3. Dynamics of deposit balances under case study #3:

$B(0)$ = 5.0, $\lim_{t\to\infty} B(t)$ =7.5, ΔB = +2.5 in bln USD

The case study #3 (Fig. 3) demonstrates that such a Basel early warning indicators as a decrease of weighted average maturity of liabilities (BIS, 2008) is insufficient. In fact, a weighted average maturity of deposits was decreased from 0.5 to 0.25 year (Table 3) nevertheless deposit balances continuously grew. Thus, decreasing a weighted average maturity of deposits does not always result to deterioration in liquidity.

In order to make correct ALM decisions and avoid serious managerial errors a bank should rely not only on change in deposit balances but on changes in turnovers, term structure of deposits and rollover rate. At long-term lending, a bank should orient on minimal deposit balances in a short-term period and on long-term or steady state deposit balances, employing for this an equilibrium money conservation law.

4. SUMMARY

After a long time and at constant credit turnover, a bank will have zero-increment of its deposit balances. Therefore, in order to a banking deposit portfolio will continuously rise, an uninterrupted growth of credit turnover is needed. Wherein, there is a dangerous pitfall consisting in the fact that under strong competition for deposits a bank tries to support a deposit growth by gradual increasing the interest rate on deposits. Reluctance of bank to lower its net interest margin can lead to denial from deposit growth strategy.

It is shown that a change in deposit balances may be deceptive and unobvious. In some cases, changes in a turnover and a term structure of deposits deteriorate bank's liquidity in a short-time period but may really improve in a relatively long-term period and vice versa.

To make more accurate ALM decisions and avoid serious managerial errors a bank should base not only on change in deposit balances but on changes in turnovers, term structure of deposits and rollover rate. Therefore, such a Basel early warning liquidity indicator as a decrease of weighted average maturity of liabilities is necessary but not sufficient.

At long-term lending, a bank should orient on minimal deposit balances in a short-term period and on long-term, steady state deposit balances, employing for this an equilibrium money conservation law.

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