The Verdoorn Law: Some Evidence from Non-parametric Frontier Analysis

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8 The Verdoorn Law: Some Evidence from Non-parametric Frontier Analysis

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INTRODUCTION

In his inaugural lecture, Kaldor (1966) refers to what he terms Verdoorn's Law, the statistical relationship between the rate of growth of labour productivity and the rate of growth of output, as evidence of the pervasive existence in industrial economies of static and dynamic economies of scale. Since this contribution, it has often been suggested that attempts at estimating the law (including, of course, Kaldor's own one) suffer from serious specification problems. As is well put by McCombie and Thirlwall (1994, p. 167), "... the debate over the Verdoorn Law would make a good textbook example of the problems that can beset statistical inference!". As can be seen from the surveys in Bairam (1987a) and in McCombie and Thirlwall (1994, Ch. 2), problems with estimating the law are related to three major issues.

First, it has been pointed out that if no variable measuring the stock of capital (or its growth) is included in the estimated specification, then no definite conclusion can be made about the nature of returns to scale, unless strong assumptions are made about the evolution of the capital-output ratio. Second, when estimating the Verdoorn Law using OLS, there are problems in positing that either inputs or output are exogenous variables, since most models of economic development imply that neither of them can fulfil this requirement. Third, the statement has often been made that the Verdoorn Law might spuriously arise from some other relationship present in the data. Thus, if different observations experience different rates of growth for exogenous technological progress, and the estimation procedure does not allow for these differences, the Verdoorn Law might just be the spurious result of posing output growth rates *inclusive of exogenous technical progress* against input growth rates. Another problem arises because of the presence of an accounting identity in the data relevant for estimation of the Verdoorn relationship. Since total value added must by definition be equal to the sum of labour and non-labour incomes, the question arises as to whether an estimated relationship between output and (labour and capital) inputs merely reflects this identity or has some behavioural content. Finally, if the law is estimated using time-series data, it is maintained that the positive relationship between the rate of growth of labour productivity and the rate of growth of output might just reflect the existence of labour hoarding in the short run (the so-called Okun Law).

A further issue is that typically different values for the returns to scale are found when estimating the law in level (static) or rate-of-growth (dynamic) terms. Various explanations have been put forward for this static-dynamic paradox, ranging from the impact of measurement errors to the fact that the correct static model could be a non-linear technical progress function rather than the usually adopted Cobb-Douglas production function (McCombie, 1982a).

In the present work, we show how the nature of economies of scale can be assessed using a set of procedures based on non-parametric frontier analysis[2]. We suggest that this exercise is a useful addition to the existing literature on the law, as these techniques allow a novel approach to the issues of simultaneity and spuriousness. It should also be pointed out immediately that through non-parametric frontier analysis it is possible to characterise qualitatively the nature of returns to scale for each observation, yielding important information on the heterogeneity of observations across both time and space. Indeed, even an approximate quantitative measure of returns to scale can be produced for each observation. By way of application, we assess economies of scale across a sample of 52 countries, taken from the Penn World Table, mark 5.6.

Note that, although in principle non-parametric frontier analysis can deal with variables both in levels and rates of growth, in our opinion the treatment of the latter requires a more complex analysis and shall be taken up in future work. Also, while the empirical procedures here proposed have in our view some strong advantages, it is fair to say at the outset that they also entail drawbacks, which shall be duly laid forth. Yet, these drawbacks are on their way of being solved in the literature on non-parametric frontier analysis, and it is hoped that the present work might be considered as a useful first attempt, showing the relevance of this literature for the problem at hand, and fostering further analytical developments.

The rest of the work proceeds as follows. In the next section, we expound more fully the specification issues briefly recalled above. Next, we briefly describe the set of procedures taken from non-parametric frontier analysis which are to be used in the present work, while in the following section we argue in favour of their expediency in the present context. Then, we describe the data utilised in the empirical example and present the empirical results, while the last section offers some concluding remarks.

INTERPRETING AND ESTIMATING THE VERDOORN LAW

It is convenient to couch our discussion of the following issues in terms of a simple algebraic representation of the law, which mirrors the formulation given in Dixon and Thirlwall (1975). Let us consider first a linear representation of Kaldor's technical progress function (Kaldor, 1957):

$$r = d + \alpha_0 k \qquad (1)$$

where r is the rate of growth of output per worker, d is the rate of disembodied technical progress and k is the rate of growth of capital per worker. Now, positing:

$$d = \alpha_1 + \beta_1 q \qquad (2)$$

$$k = \alpha_2 + \beta_2 q \qquad (3)$$

where β_1 is a measure of static and dynamic economies of scale[3], β_2 is an "accelerator" parameter and q is the rate of growth of output, one can obtain Kaldor's formulation of the Verdoorn Law, by substituting equations (2) and (3) into equation (1):

$$r = \lambda_0 + \lambda_1 q \qquad (4)$$

where $\lambda_0 = (\alpha_1 + \alpha_0 \alpha_2)$, and $\lambda_1 = (\beta_1 + \alpha_0 \beta_2)$.

This formulation makes it clear that the Verdoorn relationship exists even in the absence of economies of scale ($\beta_1 = 0$), provided that $\alpha_0 \beta_2$ and k differ from zero (the latter condition holds out of steady-state growth). Accordingly, equation (4) may be an interesting expression for the modelling of growth and development, but does not allow us to appraise unconditionally the degree of economies of scale. The latter might be assessed in the following expression where only equation (2) is substituted in equation (1):

$$r = \alpha_1 + \beta_1 q + \alpha_0 k \qquad (5)$$

The above formulation complements the more usual arguments[4] as to why no definite conclusion can be drawn about the nature of returns to scale, unless explicit allowance is made for the capital stock. This result also suggests some reasons for the lack of parameter stability of equation (4). Indeed, it is unlikely that equation (3) may satisfactorily represent the determination of k, and hence even if the other behavioural relationships are stable, equation (4) - but not equation (5) - must show lack of parameter stability.

In the present work, we take the (not very controversial) view that if Kaldor brought to the fore in his inaugural lecture what he termed Verdoorn's Law, it was mainly because he was interested in favourable evidence for the existence of static and dynamic economies of scale. Hence, it will be equation (5), and not equation (4), that constitutes the focus of the empirical analysis, and it will be this relation which we will refer to as the Verdoorn Law. This is a fundamental point in the research strategy adopted here, because, as will become clearer in the next section, while non-parametric frontier analysis is likely to yield interesting insights about the degree of economies of scale in equation (5), it is rather ill suited to the estimation of equation (4).

A point which has often been made in Verdoorn’s Law literature[5] is that in estimating the Verdoorn Law by OLS, there are problems in deciding which are the correct regressors, since most models of economic development imply that neither of r, q or k can be exogenous variables. Hence neither equation (5), nor its reparameterisation as:

$$n = \alpha'_1 + \beta'_1 q + \alpha'_0 k \qquad (5a)$$

where n is the rate of growth of the labour input (and there is no risk of spurious correlation between the left and right hand side of the equation) are likely to yield unbiased and consistent estimates for their parameters when estimated by OLS.

This is also true of:

$$q = \alpha''_1 + \beta''_1 n + \alpha''_0 k \qquad (5b)$$

(which is often referred to as the Rowthorn specification). Related to this is the finding that while specifications like equation (5a) provide evidence in favour of the existence of economies of scale, equation (5b) most often does not[6]. A natural econometric solution to this conundrum is the adoption of instrumental variable techniques, but, by and large, this does not provide any conclusive evidence[7]. Another solution could be the adoption of cointegration analysis, which seems to yield evidence in favour of increasing returns to scale[8]. As the latter analysis is carried out using time-series data, its discussion can be better addressed below.

It has been recalled above that the Verdoorn Law has often been alleged to arise spuriously from some other relationship present in the data. First of all, if different observations experience different rates of growth for exogenous technological progress, the α_1 of equation (2), and the estimation procedure does not allow for these differences, the Verdoorn Law might just be the spurious result of regressing productivity growth against output growth rates *inclusive of differing rates of exogenous technical progress*. Consider the case depicted in Figure 1

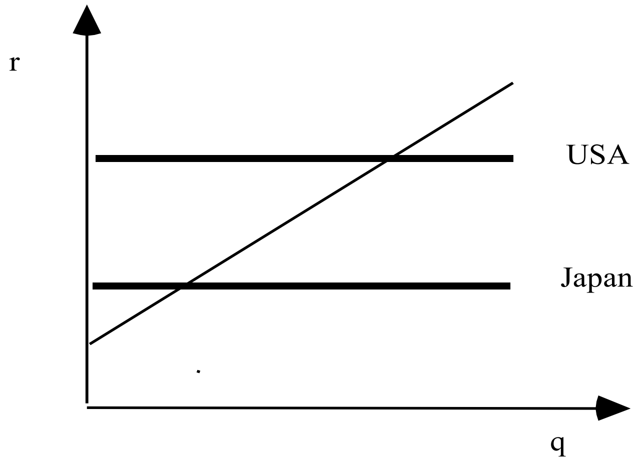

FIGURE 1

By hypothesis, in Figure 1 the rate of growth of productivity does not depend on the rate of growth of output in each given country, and the corresponding Verdoorn relationship is given by an horizontal line. Yet, exogenous technical progress may differ across countries, yielding different values for r. A spurious Verdoorn relationship is obtained when r is regressed on q using cross-country data, and the countries with the higher exogenous technical progress also have the higher q, either coincidentally or because α_1 is a source of output growth. Thus, unless the estimation procedure controls for the possibility that α_1 might differ across observations, the Verdoorn relationship does not mean much. A similar argument applies to specifications in levels, where the role of different rates of exogenous technical progress must however be taken by different levels of technology.

Attempts to deal explicitly with this problem include Gomulka (1983) and Bairam (1986), who use dummy variables to model differences in exogenous technical progress. Yet, as acknowledged in Bairam (1987a, p. 26), this procedure is costly in terms of degrees of freedom, which might result in inefficient estimates and the impossibility to choose the appropriate specification. Another possible solution is the use of appropriate data-sets. Thus, McCombie and de Ridder (1983, 1984) and Bairam (1987b, 1988c) estimated the law on cross-regional data, under the hypothesis that differences in technology across regions in a given country must be small. This, however, might not always be true (consider for instance, all the references in the Mezzogiorno literature to the technological differences - at least in a broad sense - between Northern and Southern Italy).

It has also been suggested that estimating the law on time-series data for a given country might also circumvent this difficulty. This is only true if exogenous technical progress does not occur through time, which seems a rather tall assumption, or, again, if one controls for exogenous technical progress. It is not clear, for instance, whether the time-series works quoted above (Harris and Lau, 1998, Harris and Liu, 1999) are impervious to this critique, since their specifications do not include variables representing exogenous technical progress. Indeed, estimating the law using time-series data might also present other problems. It has often been pointed out that such an exercise incurs the risk of mixing the (long-run) Verdoorn relationship with the so-called Okun Law, reflecting the existence of labour hoarding in the short run (see on this McCombie and Thirlwall, 1994, pp. 197-200). This would allegedly provide spurious evidence in favour of the law, but, as pointed out in Jefferson (1988), the presence of adjustment costs for other inputs (in particular, for the capital stock) is likely to produce biases with the opposite sign. In any case, all these arguments signal a broad consensus about the need to assess the law on the long-run (low-frequency) component of the data, filtering out other components through appropriate time-series techniques. Again, this argument is just as relevant to specifications in levels as it is to rate-of-growth ones.

Finally, a problem of identification arises because an accounting identity underlies the data relevant for estimation of the Verdoorn relationship (see McCombie and Thirlwall; 1994, pp. 212-216). Since total value added must by definition be equal to the sum of labour and non-labour incomes, the question arises as to whether an estimated relationship between output and (labour and capital) inputs merely reflects this identity or has some behavioural content. Indeed, for any given country (or region), value added is defined by:

$$Q_t = w_t N_t + r_t K_t \qquad (6)$$

In terms of rates of growth (or of natural logs), the above formula becomes:

$$q_t = a\,\varphi_t + (1 - a)\,\phi_t + a\,n_t + (1 - a)\,k_t \qquad (7)$$

where φ and ϕ are the rates of growth of real wages and of the real rental price of capital, and it is assumed for simplicity (this is not strictly necessary for the argument, but it simplifies the functional forms considered; see McCombie and Dixon, 1991) that factor shares are constant. Now, consider the following relation (which could be a Cobb-Douglas production function in dynamic terms, or a Rowthorn-like reparameterisation of the Verdoorn Law; for expositional purposes we here assume that there is no loss of generality in doing so):

$$q_t = \lambda + \alpha\, n_t + \beta\, k_t \tag{8}$$

If the sum $[\, a\, \varphi_t \;+\; (1 - a)\, \phi_t \,]$ can be expressed by a constant, ω (as is often the case, see McCombie and Dixon, 1991), then the estimation of equation (8) will just reflect the underlying identity, equation (7).

Matters differ to some extent for the usual cross-country (or cross-region) set-up adopted for estimating the law. In this case (always assuming constant factor shares), the underlying identity can be represented by:

$$q_i = a\, \varphi_i + (1 - a)\, \phi_i + a\, n_i + (1 - a)\, k_i \tag{9}$$

or, if $\omega_i \;=\; [\, a\, \varphi_i \;+\; (1 - a)\, \phi_i \,]$, by:

$$q_i = \omega_i + a\, n_i + (1 - a)\, k_i \tag{9a}$$

On the other hand, the law can be represented as:

$$q_i = \lambda + \alpha\, n_i + \beta\, k_i \tag{10}$$

or, if the degree of returns to scale is represented by ν, as:

$$q_i = \lambda + \nu \, [\, a\, n_i \;+\; (1 - a)\, k_i \,] \qquad (11)$$

Now, across countries it is no longer appropriate to suppose that ω_i is a constant term identical for each country. On the other hand, consider the growth in total factor productivity, $\theta_i = q_i - [\, a\, n_i + (1 - a)\, k_i \,]$. We can write:

$$\theta_i = \omega_i = \lambda + (\nu - 1)\, [\, a\, n_i + (1 - a)\, k_i \,] \qquad (11a)$$

$$\theta_i = \omega_i = \lambda/\nu + [\, (\nu - 1)/\nu \,]\, q_i \qquad (11b)$$

Hence, the accounting identity equation (9) underlies the assessment of returns to scale that can be carried out through estimation of equation (10) or equation (11). Consequently, what can be ascertained through estimation of the latter is whether a faster weighted growth of real input prices turns out to be associated with a faster growth of output (or with a faster weighted growth of inputs). This is of interest, but does not allow us to assess the role that increasing returns to scale, exogenous technical progress, or even capital accumulation might have in this correlation. To do so, and in particular to disentangle the role of returns to scale, we require *a priori* knowledge on the magnitude and bias of technical change, as well as on the determination of income distribution in the economies under examination.

NON-PARAMETRIC FRONTIER ANALYSIS. A BRIEF SURVEY

An Overview

A unifying feature of virtually all the empirical studies carried out so far on the Verdoorn Law is that they have been couched in terms of the econometric estimation of a constant-parameter function (be it considered as a technical progress function or a more traditional production function) fitted to the whole sample. Naturally, this presumes that a constant-parameter function can adequately represent the technology of all productive units being examined, an assumption which has rarely left been unchallenged when put to an empirical test[9]. Utilising a constant-parameter function is not however the only way in which a productive technology can be modelled. A varying-parameter function could be estimated, or, even more fundamentally, some important characteristics of the technology under examination (including the degree of returns to scale) could be assessed without representing this technology through a given functional form. In such a case, mathematical programming techniques are used in order to build a production set which must satisfy some properties (usually strong disposability and convexity). We choose to follow here the second alternative, which, as will be argued below, has some important advantages in the present ambit.

To start with, since these so-called non-parametric methods provide estimates of the upper boundary of a production set (the so-called production frontier) without supposing the existence of a functional relationship between inputs and outputs, they need only a limited number of restrictive assumptions about to the production process. Beginning with the seminal contribution of Farrell (1957), these techniques are used to build the frontier of a production set (satisfying some properties which are specified a priori). The frontier is supported by some of the observed producers, which are defined as efficient. It is of paramount importance to stress that non-parametric techniques share the hypothesis that the distance of non-efficient producers from the frontier must be entirely explained by a factor (or a set of factors), traditionally termed inefficiency, which obeys a one-sided statistical distribution.

Non-parametric methods are usually divided between those directly related to Farrell's contribution (usually gathered under the label of Data Envelopment Analysis, or DEA) and those based on the Free Disposal Hull (FDH) approach first proposed in Deprins *et al*. (1984). In the latter case, the only property imposed on the production set is strong input and output disposability, while in DEA the additional hypothesis of convexity is made.

More formally, in FDH, for a given set of producers Y_0 , the reference set[10] $Y(Y_0)$ is characterised, in terms of an observation i, by the following postulate:

$$(\mathbf{X}^i, \mathbf{Y}^i) \text{ observed, } (\mathbf{X}^i + \mathbf{a}, \mathbf{Y}^i - \mathbf{b}) \in Y(Y_0), \qquad \mathbf{a}, \mathbf{b} \geq 0$$

where **a** and **b** are vectors of free disposal of input and output, respectively. In other words, due to the possibility of free input and output disposability, the reference set includes all the producers which are using the same or more inputs and which are producing the same or less output in relation to observation i.

Let us take as an example Figure 2, where we are considering a technology with one input (X) and one output (Y). The input-output pairs correspond to a cross-section of producers examined at a given point in time. Beginning with observation B, we define every observation located at its right and/or below it (i.e. with more input and same output, or with less output and same input; or else with more input and less output, as F) as dominated by B. As for E, it is not dominated by B, because it uses more input but it is also producing more output, but it is dominated by A. On the other hand, C and D are not dominated by either A or B, because they produce less output but are also using less input. Similarly, A is not dominated by any observation, because it uses more input but it is also producing more output. As a matter of fact, A, B, C and D are not dominated by any producer belonging to the reference set.

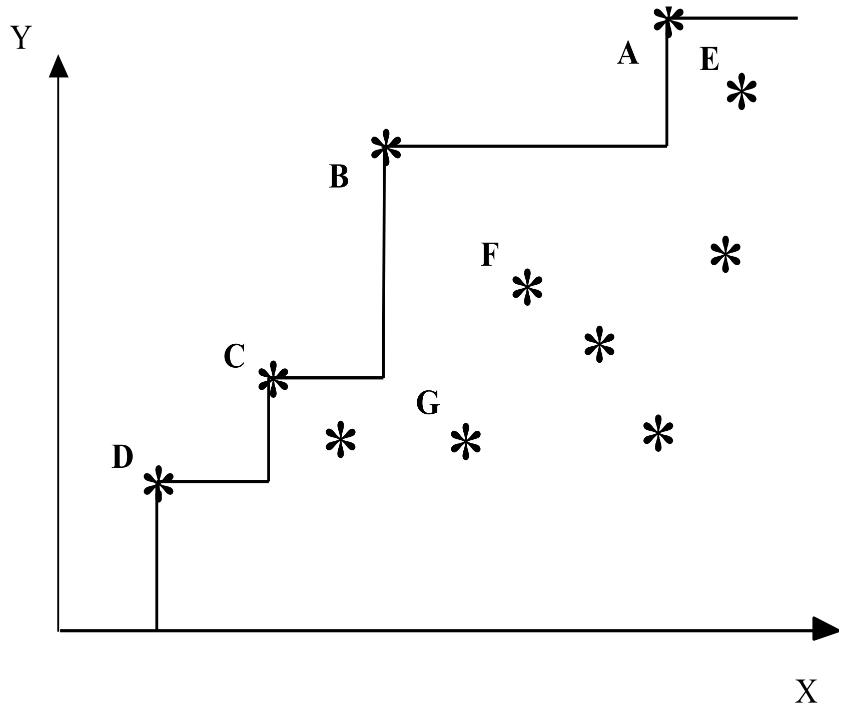

FIGURE 2

In the FDH approach, this procedure of comparison is carried out for every observation, and the observations dominated by other producers are considered as inefficient. Those units which are not dominated by any other observation are considered instead as efficient producers, belonging to the frontier of the reference set.

In DEA, identification of the production frontier is carried out through the construction of a convex hull around the production set, based upon the a priori specification of strong input and output disposability *and convexity* for the production technology. In order to build this convex hull, appropriate linear programming procedures are used. In Figure 3, examples of three kinds of convex hulls typical of the application of DEA are shown for a one-input one-output technology. Again, the points on the Y-X planes are input-output pairs corresponding to a cross-section of producers taken at a given point in time. In DEA-CRS, the identification of the convex hull is based on the hypothesis of constant returns to scale, while in DEA-NIRS the (less restrictive) hypothesis adopted is that of non-increasing returns to scale, and, lastly, in DEA-VRS the convex hull is allowed to show a particular type of variable returns to scale (increasing first, and decreasing afterwards)[11].

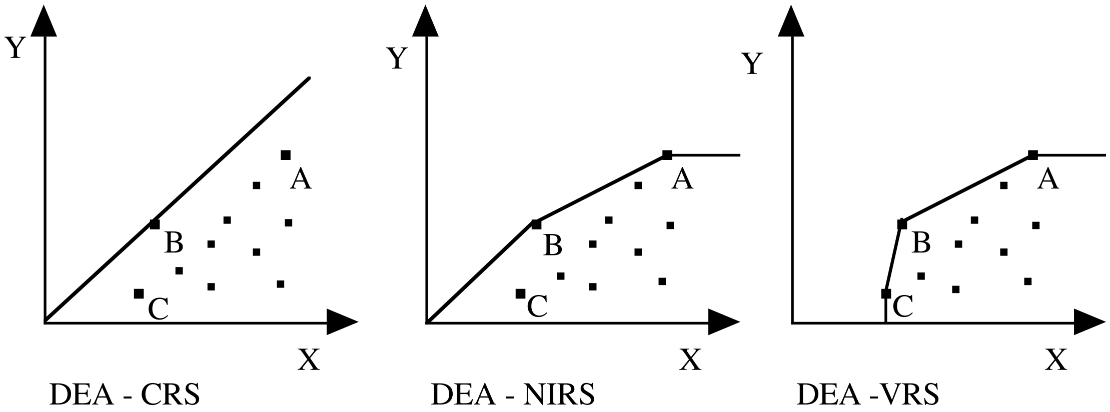

FIGURE 3

In both FDH and DEA the distance of producers from the frontier is deemed to give their measures of technical efficiency, or, for short, their efficiency scores. Typically, the (output-oriented or input-oriented) measure of Debreu-Farrell is used. If the measure is output-oriented, technical efficiency is given by the relative output expansion needed to bring a producer on the frontier, for given inputs. A producer which is technically efficient (and which is therefore on the frontier of reference) will not be able to attain such an expansion, achieving an efficiency score equal to one. If the measure of Debreu-Farrell is input-oriented, it is given by the relative input contraction needed to bring a producer on the frontier, for given outputs[12].

Clearly, the adoption of FDH allows us to leave behind the hypothesis of convexity of the production set typical of DEA. This means that the frontier obtained through FDH is likely to fit more closely the data than the one obtained through DEA, if the reference set is characterised (at least locally) by the existence of non-convexities. Moreover, it is important to emphasise that, unlike DEA, (where the inefficient producers are dominated by virtual observations located on the convex hull of the production set), in FDH an inefficient producer is necessarily dominated by at least another well identified, actually existing producer.

The possibility of building a frontier of the reference set based on actually existing producers, and to make direct comparisons between these and the units dominated by them, can be regarded as one of the major advantages of FDH. For instance, in the context of cross-country comparisons, this allows to identify some "clubs" of countries constituted by a dominant unit and the relative dominated units, which must share relatively similar technologies. Also, as the frontier of the reference set is made up of actually existing units (rather than by a convex hull), FDH will be less sensitive to the presence in the reference set of outliers (or of erroneously measured values) than DEA. More precisely, the section of the frontier influenced by the presence of the outlier will be smaller with FDH than with DEA.

Yet, a drawback of the definition of reference frontier in the FDH approach is that a producer can belong to it without dominating any other observation. Such a producer (like D in Figure 2) could be defined as efficient only because is located in an area of the production set where there are no other observations with which it could be compared.

Non-parametric Frontier Analysis and Returns to Scale

The treatment of returns to scale in non-parametric frontier analysis has been first treated for convex technologies like the one depicted in Figure 4

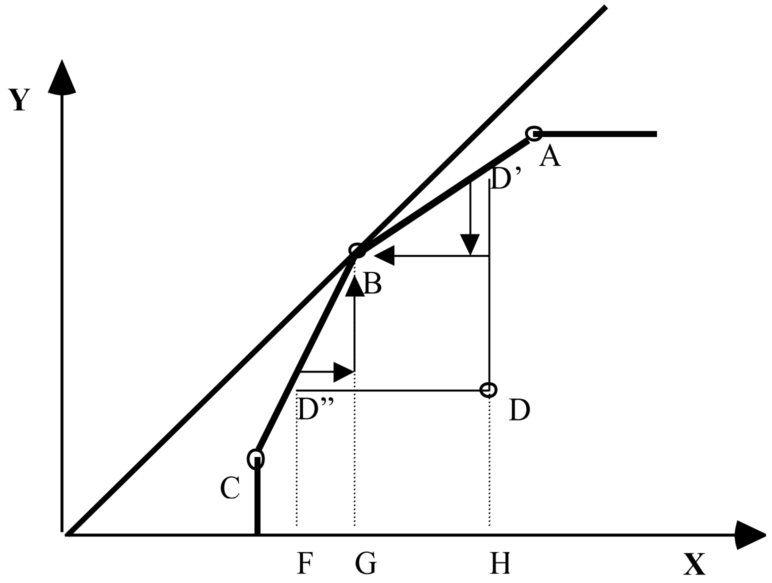

FIGURE 4

We can see here that according to the DEA-VRS technology, producers A, B and C are all efficient. However, B only takes advantage of the scale of production consistent with the maximum productivity (or, correspondingly, the minimum average cost). Since the scale of production of B is the most productive scale size, this producer can be defined as being scale-efficient. On the other hand, A is scale-inefficient because it is too large, and C is scale-inefficient because it is too small. It can be easily noticed that B is the only producer which is efficient according to both DEA-CRS and DEA-VRS. Scale efficiency (the distance of DEA-VRS from DEA-CRS) is obtained for each producer as the ratio between the distance from the DEA-CRS frontier to the distance from the DEA-VRS frontier (in common parlance, the ratio between the CRS and VRS efficiency scores).

Note that scale inefficiency *per se* does not define the nature of the returns to scale characterising the frontier at given points. Still, there are various methods in non-parametric frontier analysis to assess the nature of returns to scale on the frontier point relevant for any given producer (see the discussions in Førsund, 1996a, or in Kerstens and Vanden Eeckaut, 1999). Basically, one must ascertain whether the frontier point relevant for an inefficient producer according to the variable-returns-to-scale technology must be scaled up or down to obtain the frontier point relevant for an inefficient producer according to the constant-returns-to-scale technology. In the first case, the frontier exhibits increasing returns to scale, while the contrary holds true in the opposite case. If the two frontier points coincide, the frontier exhibits constant returns to scale.

Note that, as a consequence of the hypothesis of variable returns to scale, the frontier point relevant for an inefficient producer might exhibit increasing returns to scale in the sense of input reduction, and decreasing returns to scale in the sense of output expansion. Also, this way to characterise returns to scale implies that at least one producer must exhibit constant returns to scale. But this may not make much sense if all the scale-inefficient units to which this producer can be compared are either larger or smaller than it (that is in the case in which it might be impossible to define a technically optimal scale of production). These possibilities, as well as their empirical relevance, will be taken up again in what follows.

Naturally, one could also ask whether this qualitative assessment of the nature of returns to scale might depend on statistically insignificant discrepancies between the constant-returns-to-scale and the variable-returns-to-scale technology. Strictly speaking, the answer to this depends on the possibility of constructing, for each observation, confidence intervals for the efficiency scores obtained under the alternative returns-to-scale assumptions. Now, the construction of such intervals is not straightforward, although appropriate bootstrap procedures are in the process of being developed (Simar and Wilson, 2000). However, Banker (1996) suggests some simple procedures, appearing to have reasonable small-sample properties, through which it is possible to test whether the production set is best characterised by DEA-CRS or by DEA-VRS (indeed, DEA-VRS, which is the less constrained model, can also provide the null hypothesis for DEA-NIRS or for DEA-NDRS).

There exist also some procedures that allow the derivation of quantitative measure of returns to scale from non-parametric frontier analysis. Some of these procedures (Banker and Thrall, 1992) assume the convexity of the production set. The method suggested in Førsund and Hjalmarsson (1979) and Førsund (1996a) does not share this stricture, but can only be applied to inefficient producers. Cooper *et al.* (2000, Ch. 5) suggest a simpler and more general approach, which will be adopted in the empirical analysis presented below. This method consists in evaluating the percentage input and output variations between the variable-returns-to-scale frontier point relevant to any given observation and the corresponding most productive scale size. For example, taking observation D in Figure 4, one first singles out the variable-returns-to-scale frontier points (D' in the output-increasing sense, and D'' in the input-reducing sense) and the most productive scale size (B, in both senses). Then, the output-orientated measure of scale elasticity is found dividing the percentage output variation between D'' and B by the corresponding percentage input variation. Analogously, the input-orientated measure of scale elasticity is found dividing the percentage output variation between D' and B by the corresponding percentage input variation. Finally, an average measure of scale elasticity is obtained as the weighted average of the input- and output-orientated measures (weights being given by the relative amplitude of the relevant input variations – FG and GH in this case). When dealing with multi-input multi-output technologies, all the above applies to radial (equiproportional) input and output variations. Naturally, this procedure yields indeterminate results for observations (like B) already located at their most productive scale size. This entails no loss of information, as the elasticity of scale of such producers is, by definition, equal to one.

Recently, a class of models has been proposed (Bogetoft, 1996) that preserves most of the flexibility of FDH, while increasing the scope for comparability among producers. These models are of particular interest in the present analysis, because they allow an explicit treatment of returns to scale within non-convex production sets. In line with Kerstens and Vanden Eeckaut (1999), we see these models as refinements of FDH. In FDH-CRS (Figure 5) any producer is compared with proportional rescalings of all other producers (still allowing for non-convexities that can arise from the advantages of specialising in given output- or input-mixes[13]).

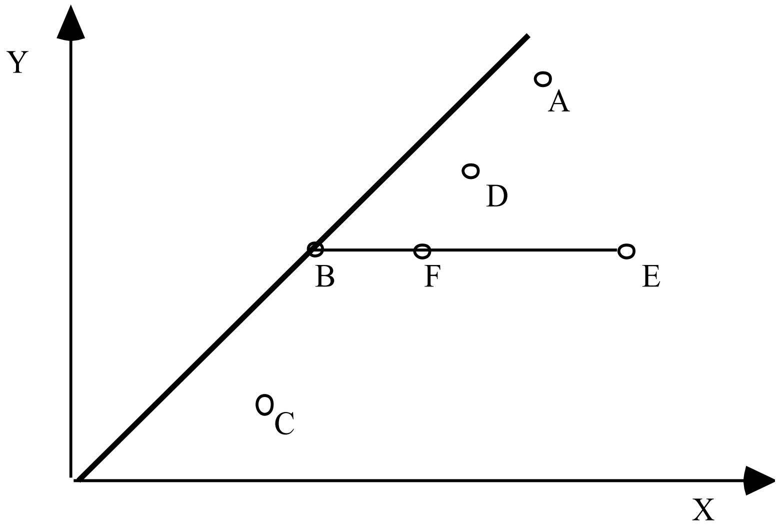

FIGURE 5

In FDH-NIRS (Figure 6), any producer is compared with smaller proportional rescalings of all other producers; in FDH-NDRS (Figure 7) the opposite is true and producers can only be compared to larger proportional rescalings of other producers.

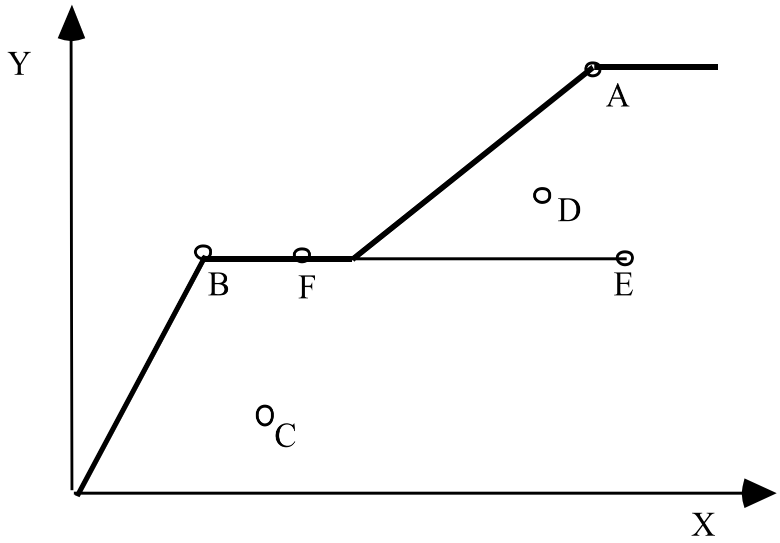

FIGURE 6

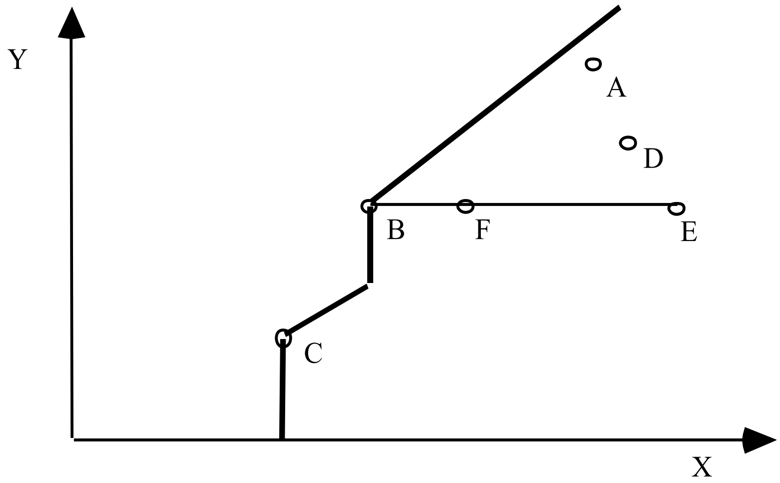

FIGURE 7

In this manner, while comparison is no longer restricted to actual producers, any observation is still dominated by a clearly identifiable rescaling of another producer. What is more for present purposes, the assessment of returns to scale can be carried out (along the above suggested lines) in a framework no longer restricted by the assumption of convexity. These considerations suggest the adoption of these models for the empirical application to be presented below, also because, as explained in Kerstens and Vanden Eeckaut (1999), one can define and construct a FDH-VRS model which is constituted by the intersection of the FDH-NIRS and FDH-NDRS models.

NON-PARAMETRIC FRONTIER ANALYSIS AND THE VERDOORN LAW

In our opinion, non-parametric frontier analysis has some distinctive strong points for the assessment of returns to scale. To start with, it seems important to be able to carry out such an assessment without making any assumption about the functional form. Also, the possibility to appraise the nature of returns to scale for each observation yields potentially important information on the heterogeneity of observations across both time and space. However, there are other arguments in favour of non-parametric frontier analysis, even more closely linked to the empirical debate over the law.

Consider first the (closely related) problems of measurement errors and simultaneity bias. They are very serious issues for econometric analysis, especially in the cross-section set-up so typical for the estimation of the law. Most probably, they also explain why different values are obtained for returns to scale when either inputs or output are taken as dependent variables. In non-parametric analysis these problems are, in some sense, assumed away without great loss for the scope of the analysis. To start with, non-parametric output- or input-orientated estimates must single out the same production frontier[14]. But much more importantly, it has been recently shown[15] that DEA and FDH provide consistent estimators for the production frontier under fairly general conditions. The latter include some typical properties of the production set (strong disposability and, for DEA, convexity) and the assumption that all observed producers reflect feasible choices. In other words, there are no measurement errors, noise, etc. and the distance of non-efficient producers from the frontier must be entirely explained by a factor (or a set of factors), traditionally termed inefficiency, following a one-sided statistical distribution.

Indeed, this would seem a very stringent condition, and, besides, consistency is an asymptotic property. But recall that we have already argued that in any case the law must be assessed over the long-run component of the data, filtering out other components through appropriate time-series techniques. Now, virtually any filter (ranging from simple moving averages to kernel smoothers) that can be devised in order to take out the cyclical component from the data is also suitable to the extraction of noise, and can do so with usually very little computational cost. As far as small-sample bias is concerned, available evidence suggests that it affects the position (rather than the shape) of the frontier, and that it is not likely to be substantial for a one-output two-input production set like the one traditional for empirical analysis of the law (Park et al., 1997; Kneip et al., 1998; Gijbels et al., 1999). In any case, were this problem thought to be serious, one could devise consistent bootstrap procedures to deal with it (Simar and Wilson, 2000). Finally, as far as the existence of outliers is concerned, it is probably best dealt in any case within non-parametric techniques, due to their flexibility (this is true for FDH in particular).

Consider now the claims that the Verdoorn Law might spuriously arise from some other relationship present in the data. In the previous section, it has been shown that one of the reasons behind these claims is that most available estimation procedures do not allow to separate in an efficient, data-based, manner producers endowed with different states of technology or with different rates of exogenous technical progress. But in non-parametric analysis the frontier is built by comparing any given producer with other (relatively similar) producers. This is all the more true for FDH, where this comparison is carried out only between any producer and its dominated observations.

To be sure, similarity is only defined in terms of observable inputs and outputs, but, as will be seen in the empirical exercise presented below, *when the production set is specified in terms of input and output levels*, this seems enough to prevent comparisons between producers widely believed to be characterised by different states of technology. Matters are considerably less straightforward when production relationships are specified in terms of rates of growth. Some preliminary estimates did not show in this case any shaping up of comparison groups (constituted by a dominant unit and the relative dominated units) sharing relatively similar technologies. In our opinion, the treatment of rate-of-growth models within non-parametric frontier analysis requires a more complex framework[16] and shall be taken up in future work.

It was also shown above that, since total value added must by definition be equal to the sum of labour and non-labour incomes, the question arises as to whether an estimated relationship between output and (labour and capital) inputs merely reflects this identity or has some behavioural content. But this is only true if one posits the existence of a functional relationship between outputs and inputs. Clearly, this is not the case for non-parametric frontier analysis, which consequently provides means to deal with also this source of spurious relationships.

To sum up, in our view, non-parametric frontier analysis allows us to assess the nature of economies of scale approaching in a novel and promising manner the issues of simultaneity and spuriousness affecting parametric estimates of the law. Also, through this analysis it is possible to characterise qualitatively the nature of returns to scale, and even to produce an approximate quantitative measure of returns to scale, for all producers. Relevant drawbacks of this approach are that fully fledged statistical inference is not readily obtainable, and that a more complex framework seems to be needed to take care of rate-of-growth specifications of the law. However, taking care of these two issues is not out of the reach of the non-parametric approach (although this requires a considerable complication of the analysis) and suggests obvious developments of the present paper.

It was already noted that ambiguities might arise in the present framework because the frontier point relevant for an inefficient producer might exhibit increasing returns to scale in the sense of input reduction, and decreasing returns to scale in the sense of output expansion. In principle, these ambiguities can be dealt with through another analytical extension, the adoption of graph efficiency measures, simultaneously allowing for input reduction and output expansion[17]. However, it is also interesting to assess the scope for these ambiguities to arise, and in the present paper we will be content with such an exercise. Finally, recall that the characterisation of returns to scale adopted here implies that at least one producer must exhibit constant returns to scale. This may not make much sense if all the scale-inefficient units to which this producer can be compared are either larger or smaller than it. Indeed, in this case the very definition of a technically optimal scale of production might not make sense. Again, the practical relevance of this event will be considered in the empirical exercise that follows.

THE DATA

The procedures presented above have been adopted to appraise the nature of returns to scale in 52 countries throughout the 1965-92 period. The data used in this application have been taken from the Penn World Table (mark 5.6). A drawback of the Penn World Table is that it does not contain series for manufacturing or industry, but only data for the whole economy. Yet, we have chosen these data, because:

a) They are widely available, and they have been used (much as other economy-level data) in empirical research related to the Verdoorn Law. Thus our relatively new analysis can be applied to a rather well known data-set.

b) These data provide estimates for the stock of capital. As pointed out in the previous sections, we may be confident about the appropriateness of frontier analysis only if it is applied to production relationships, and reduced forms excluding the stock of capital might also depend on other behavioural relationships (e.g. an investment function).

c) Estimates for the stock of capital and GDP are given in PPP terms. The utilisation of a common unit of measurement is indeed crucial to the comparison of different producers needed to build a production frontier.

The Penn World Table data have been elaborated by a group of researchers co-ordinated by Robert Summers and Alan Heston, building on the benchmark studies of the United Nations' International Comparison Program and the national economic accounts of the countries examined. The procedures adopted in constructing the data bank are described in detail in Summers and Heston (1991) and in a file annexed to the Table, mark 5.6. The main point however is that the series of the expenditure variables are estimated at international prices, expressed in one currency, common to all countries. In this way, it is possible to use these data to carry out quantitative comparisons not only through time, but also across countries.

The data bank contains series on 147 countries throughout the 1950-92 period. However, not all variables are available for all the countries and for the whole period. As a matter of fact, data on the stock of capital are not available before 1965, and they do not exist for all countries. Among those countries for which data on the stock of capital starting from 1965 exist, a further selection has been made according to the following criteria. First of all, we have excluded the countries for which the quality of the data has obtained the minimum score according to the scale of judgement presented in Summers and Heston (1991)[18], because the data of these countries are likely to be biased by important errors of measurement. The only exception to this criterion has been made for Taiwan, owing to the particular analytical importance of this country and because it has been possible to use for it in the Penn World Table 5.6 the (purchasing power parity) national accounting estimates elaborated by Yotopoulos and Lin (1993). Secondly, we have left out Nepal and Yugoslavia, because for all variables these countries have a high number of missing values at the end of the sample period. Finally, all countries with a population of less than one million inhabitants, as well as Iran, Nigeria and Venezuela (all of them OPEC member countries), have been excluded from the analysis in order to reduce the influence of non-measurable idiosyncratic factors on the estimates for the returns to scale[19]. In the end, the sample used in the empirical analysis includes the 52 countries listed in the first column of Table 1.

The output variable that has been used is Q, a measure of GDP at 1985 international prices, whereas the input variables are K, the capital stock (net of residential construction) at 1985 international prices and N, the number of workers in employment. The employment series mostly come from census data (gathered by the ILO) on active population: the data in between the census years have been mainly obtained through interpolation. Indeed, for many countries, the 1991 and 1992 values for this variable were not available from the Penn World Table; however, we utilised simple time-series techniques to extrapolate these values from the existing ones. Finally, in order to estimate the stock of capital, the perpetual inventory method has been applied to the series of gross fixed investments in equipment, machinery and non-residential construction, obtained by multiplying some benchmark estimates for the various countries for the respective growth rates.

In order to consider only the long-run component in these variables, that is to take out the cyclical component and the noise from the data (the latter being a requirement of non-parametric frontier analysis), we take average values over periods not shorter than five years. To maintain some comparability with past empirical works on the law, the periods chosen are 1965-73, 1974-79, 1980-86, 1987-92.

THE EMPIRICAL RESULTS

In this section, we provide an application of non-parametric frontier analysis to the data from the Penn World Table. As already stated above, we choose to base our empirical exercise on the non-convex models examined in Bogetoft (1996) and in Kerstens and Vanden Eeckaut (1999). More precisely, we define and construct a FDH-VRS model, which, as explained in Kerstens and Vanden Eeckaut (1999), is constituted by the intersection of the FDH-NIRS and FDH-NDRS models[20]. To repeat, in FDH-VRS actual producers are not compared just to actual producers, but also to their rescalings. Hence, the assessment of returns to scale can be carried in a framework not restricted by the assumption of convexity of the production set.

First of all, we will provide evidence about the relations of dominance in FDH-CRS. The nature of these relations is crucial in order to understand whether the assessment of returns to scale is based on comparisons between relatively similar producers. This means that the relations of dominance should not associate producers widely believed to be characterised by different states of technology. Then we present the results relevant for the qualitative assessment of returns to scale, as well as the quantitative approximate measures of returns to scale. Attention will be paid to the possibility that input- and output-orientated estimates might give rise to ambiguous results. Also, the results will be appraised from the inferential point of view. We close the section comparing the results from non-parametric frontier analysis with some OLS estimates.

In Table 1 we provide evidence about the relations of dominance in FDH-CRS. As can be seen, for all the four periods under consideration these relations do not associate producers too different from each other. Hence, in virtually no case can the assessment of returns to scale be said to rely on the comparison between producers widely believed to be characterised by different states of technology. Table 2 spells out the evidence about the qualitative assessment of returns to scale. It can be easily seen that increasing returns to scale are very pervasive, not only among developed countries, for all the four periods. Input- and output-orientated estimates give rise to ambiguous results, but not in very many cases. More precisely, this happens in 1 case for 1965-73, 2 cases for 1974-79, 6 cases for 1980-86 and 7 cases for 1987-92. While this warrants future research on graph efficiency measures, it does not appear to be enough to weaken the above reached conclusions on the pervasiveness of increasing returns. On the other hand, there are large scale-efficient economies which always dominate smaller economies (the U.S.A. and the U.K. are instances of this). In such a case, perhaps, the very definition of a technically optimal scale of production might be called into question (and these economies characterised by the same kind of returns to scale as the economies they FDH-CRS dominate). These considerations must be however developed in future work, and they shed no doubt on the conclusions reached about the other economies.

In Table 3 we give some appraisal of the qualitative measures of returns to scale from the inferential point of view. Note that strictly speaking the tests of returns to scale proposed in Banker (1996) rely on the assumption of convexity of the production set. We feel however that their application in the present context is not arbitrary, as the consistency of non-convex estimators has also been proved in the literature (Korostelev et al., 1995a; b), and as Banker himself (1996, p. 151) vouchsafes for the good small-sample performance of his test when applied to non-convex production sets. Table 3 supports the conclusions suggested by Table 2. The data favour a FDH-VRS characterisation of the production set (we recall that FDH-VRS is obtained as the intersection of FDH-NDRS and FDH-NIRS), and increasing returns to scale rule the roost in dictating the departure of the production set from a situation of constant returns to scale: if FDH-NIRS is tested against FDH-CRS, more often than not the efficiency scores of the two models do not differ significantly.

Both Tables 2 and 3 suggest that the evidence in favour of increasing returns to scale, although rather strong, slightly weakens over time. The number of countries characterised by non-increasing returns to scale is higher in the third and fourth period. Much the same message is conveyed by the quantitative measures of returns to scale given in Table 4. Here, in order to ease presentation, countries are divided in four groups, Africa (countries from Ivory Coast to Zimbabwe in Table 1), Non-OECD America (from Dominican Rep. to Peru), Non-OECD Asia (from Hong Kong to Thailand), OECD (from Australia to USA), and mean values of elasticity of scale per period and group are presented. Also, purely as a descriptive device, a one-sided t-test (with unit elasticity of scale as the null hypothesis) is applied to the period-, group- and overall mean values of the elasticity of scale. While the mean values of the returns to scale are not too high in terms of the Verdoorn Law literature (but for Non-OECD America), nonetheless they always significantly differ from unity at the 5% significance level when all countries are considered. In fact, the evidence against increasing returns is clearly circumscribed to Africa and Non-OECD Asia. On the other hand, the evidence in favour of increasing returns is less decisive in the third and fourth period, consistently with the slight rise in the number of countries characterised by non-increasing returns to scale. While we have no explanation for these phenomena, we believe they highlight the capability of non-parametric analysis of yielding important information on the heterogeneity of observations across both time and space.

All the above evidence, pointing to the pervasive existence of increasing returns to scale, contrasts with the results obtained from the OLS estimates of a Cobb-Douglas production function. Here, as is customary, the hypothesis of constant returns to scale can never be rejected (see Table 5), when the dependent variable is (the natural log of) output. Relying on the arguments expounded above, we ascribe these differences to problems of simultaneity and spuriousness which are best dealt within non-parametric frontier analysis.

CONCLUDING REMARKS

In this paper we have provided estimates of returns to scale for a sample of 52 countries, using a non-parametric frontier approach. In our view, this approach allows a novel and promising treatment of the issues of simultaneity and spuriousness affecting parametric estimates of the Verdoorn Law. Also, through this analysis it is possible to characterise qualitatively the nature of returns to scale, and to produce an approximate measure of elasticity of scale, for each observation. Drawbacks of the present approach are that fully fledged statistical inference is not readily available, and that a more complex framework seems to be needed to take care of rate-of-growth specifications of the law. However, taking care of these two issues is not out of the reach of the non-parametric approach (although it requires a considerable complication of the analysis) and suggests obvious developments of the present paper.

The results obtained in the present application point to the pervasive existence of increasing returns to scale across developed and developing countries, in sharp variance with traditional parametric evidence obtained on the same data set. It is hoped that the present work might be considered as a useful first attempt, showing the relevance of the non-parametric frontier literature for the problem at hand, and fostering further analytical developments.

TABLE 1 The Countries in the Sample and the Relations of Dominance in FDH-CRS

COUNTRIES	**Dominant observation FDH-CRS 1965-73**	**Dominant observation FDH-CRS 1974-79**	**Dominant observation FDH-CRS 1980-86**	**Dominant observation FDH-CRS 1987-92**
IVORY COAST	PARAGUAY	PARAGUAY	PARAGUAY	PARAGUAY
KENYA	PARAGUAY	PARAGUAY	PARAGUAY	PARAGUAY
MADAGASCAR	MOROCCO	PARAGUAY	PARAGUAY	PARAGUAY
MALAWI	PARAGUAY	PARAGUAY	PARAGUAY	PARAGUAY
MOROCCO	MOROCCO	MOROCCO	PARAGUAY	MOROCCO
SIERRA LEONE	SIERRA LEONE	SIERRA LEONE	SIERRA LEONE	SIERRA LEONE
ZAMBIA	GUATEMALA	GUATEMALA	PARAGUAY	PARAGUAY
ZIMBABWE	ARGENTINA	ARGENTINA	CHILE	GUATEMALA
DOMINICAN REP.	DOMINICAN REP.	GUATEMALA	GUATEMALA	HONG KONG
GUATEMALA	GUATEMALA	GUATEMALA	GUATEMALA	GUATEMALA
HONDURAS	ARGENTINA	GUATEMALA	GUATEMALA	GUATEMALA
JAMAICA	ARGENTINA	ARGENTINA	GUATEMALA	GUATEMALA
MEXICO	SPAIN	U.K.	U.K.	HONG KONG
PANAMA	SPAIN	U.K.	U.K.	HONG KONG
ARGENTINA	ARGENTINA	ARGENTINA	ARGENTINA	HONG KONG
BOLIVIA	ARGENTINA	ARGENTINA	CHILE	HONG KONG
CHILE	ARGENTINA	ARGENTINA	CHILE	HONG KONG
COLOMBIA	SPAIN	ARGENTINA	ARGENTINA	HONG KONG
ECUADOR	ARGENTINA	ARGENTINA	U.K.	HONG KONG
PARAGUAY	PARAGUAY	PARAGUAY	PARAGUAY	PARAGUAY
PERU	ARGENTINA	ARGENTINA	HONG KONG	HONG KONG

HONG KONG	SPAIN	ARGENTINA	HONG KONG	HONG KONG
INDIA	PARAGUAY	PARAGUAY	PARAGUAY	MOROCCO
ISRAEL	U.S.A.	U.S.A.	U.S.A.	U.K.
KOREA, REP.	ARGENTINA	ARGENTINA	HONG KONG	HONG KONG
PHILIPPINES	GUATEMALA	GUATEMALA	GUATEMALA	GUATEMALA
SRI LANKA	ARGENTINA	ARGENTINA	HONG KONG	HONG KONG
SYRIA	SPAIN	ARGENTINA	MEXICO	HONG KONG
TAIWAN	ARGENTINA	ARGENTINA	U.S.A.	U.S.A.
THAILAND	MOROCCO	GUATEMALA	GUATEMALA	HONG KONG
AUSTRALIA	U.S.A.	U.S.A.	U.S.A.	U.S.A.
AUSTRIA	U.K.	U.S.A.	U.S.A.	U.S.A.
BELGIUM	U.S.A.	U.S.A.	U.S.A.	U.S.A.
CANADA	U.S.A.	U.S.A.	U.S.A.	U.S.A.
DENMARK	U.S.A.	U.S.A.	U.S.A.	U.S.A.
FINLAND	U.S.A.	U.S.A.	U.S.A.	U.S.A.
FRANCE	U.S.A.	U.S.A.	U.S.A.	U.S.A.
GERMANY, WEST	U.S.A.	U.S.A.	U.S.A.	U.S.A.
GREECE	U.K.	SPAIN	U.S.A.	U.K.
IRELAND	SPAIN	U.K.	U.K.	U.K.
ITALY	U.S.A.	U.S.A.	U.S.A.	U.S.A.
JAPAN	SPAIN	U.S.A.	U.S.A.	U.S.A.
NETHERLANDS	U.S.A.	U.S.A.	U.S.A.	U.S.A.
NEW ZEALAND	U.S.A.	U.S.A.	U.S.A.	U.S.A.
NORWAY	U.S.A.	U.S.A.	U.S.A.	U.S.A.
PORTUGAL	ARGENTINA	ARGENTINA	HONG KONG	HONG KONG
SPAIN	SPAIN	SPAIN	U.S.A.	U.S.A.
SWEDEN	U.S.A.	U.S.A.	U.S.A.	U.S.A.
SWITZERLAND	U.S.A.	U.S.A.	U.S.A.	U.S.A.
TURKEY	GUATEMALA	ARGENTINA	CHILE	HONG KONG
U.K.	U.K.	U.K.	U.K.	U.K.
U.S.A.	U.S.A.	U.S.A.	U.S.A.	U.S.A.

TABLE 2 Qualitative Assessment of Returns to Scale

1965-73	**FDH-VRS Output-orientated**		
FDH-VRS Input-orientated	*Decreasing Returns to Scale*	*Constant Returns to Scale*	*Increasing Returns to Scale*
Decreasing Returns to Scale	Ivory Coast, Kenya, Mexico, India, Philippines, Thailand, Japan, Turkey		
Constant Returns to Scale		Morocco, Sierra Leone, Dominican Rep., Guatemala, Argentina, Paraguay, Spain, UK, USA	
Increasing Returns to Scale	Malawi		The other 34 countries

1974-79	**FDH-VRS Output-orientated**		
FDH-VRS Input-orientated	*Decreasing Returns to Scale*	*Constant Returns to Scale*	*Increasing Returns to Scale*
Decreasing Returns to Scale	Ivory Coast, Kenya, Madagascar, India, Philippines, Thailand		
Constant Returns to Scale		Morocco, Sierra Leone, Guatemala, Argentina, Paraguay, Spain, UK, USA	
Increasing Returns to Scale	Malawi, Korea Rep.		The other 36 countries

1980-86	**FDH-VRS Output-orientated**		
FDH-VRS Input-orientated	*Decreasing Returns to Scale*	*Constant Returns to Scale*	*Increasing Returns to Scale*
Decreasing Returns to Scale	Ivory Coast, Kenya, Morocco, India, Korea Rep., Philippines, Thailand, Turkey		
Constant Returns to Scale		Sierra Leone, Guatemala, Argentina, Chile, Paraguay, Hong-Kong, UK, USA	
Increasing Returns to Scale	Malawi, Madagascar, Zambia, Peru, Sri Lanka, Portugal		The other 30 countries

1987-92	**FDH-VRS Output-orientated**		
FDH-VRS Input-orientated	*Decreasing Returns to Scale*	*Constant Returns to Scale*	*Increasing Returns to Scale*
Decreasing Returns to Scale	Ivory Coast, Kenya, Mexico, Argentina, Colombia, India, Korea Rep., Philippines, Thailand, Turkey		
Constant Returns to Scale		Morocco, Sierra Leone, Guatemala, Paraguay, Hong-Kong, UK, USA	
Increasing Returns to Scale	Malawi, Madagascar, Zambia, Zimbabwe, Peru, Sri Lanka, Portugal		The other 28 countries

TABLE 3 Some Inference on the Qualitative Measures of Returns to Scale.

	H1:FDH-VRS		**H1:FDH-NDRS**		**H1:FDH-NIRS**	
	Input - orientated model	Output - orientated model	Input - orientated model	Output - orientated model	Input - orientated model	Output - orientated model
1965-73	2.55	2.42	1.95	1.61	1.14	1.20
1974-79	2.30	2.21	1.69	1.43	1.18	1.29
1980-86	2.61	2.30	1.51	1.26	1.29	1.50
1987-92	3.24	2.70	1.76	1.36	1.28	1.42

The efficiency scores are assumed to follow a half-normal distribution.
The critical values are F (52, 52): *1.43* (10%) *1.59* (5%) *1.92* (1%)
H0 is rejected if the test statistics are larger than the critical values

	H1:FDH-VRS		**H1:FDH-NDRS**		**H1:FDH-NIRS**	
	Input - orientated model	Output - orientated model	Input - orientated model	Output - orientated model	Input - orientated model	Output - orientated model
1965-73	2.09	1.95	1.73	1.58	1.11	1.14
1974-79	2.01	1.91	1.64	1.51	1.12	1.16
1980-86	1.95	1.83	1.48	1.32	1.20	1.26
1987-92	2.51	2.20	1.72	1.48	1.23	1.28

The efficiency scores are assumed to follow an exponential distribution.
The critical values are F (104, 104): *1.29* (10%) *1.38* (5%) *1.58* (1%)
H0 is rejected if the test statistics are larger than the critical values

NB: The Banker (1996) Test of Returns to Scale compares (under various distributional assumptions) the efficiency scores from the restricted model (under H0) with the efficiency scores from the unrestricted model (under H1). If the efficiency scores under H0 are significantly lower than the efficiency scores under H1, H0 is rejected. In the above table, the restricted model (H0) is always given by FDH-CRS.

TABLE 4 Quantitative Measures of Returns to Scale. A Summing-up

	1965-73	*1974-79*	*1980-86*	*1987-92*	***All Periods***	p-value
Africa	1.13	1.05	0.88	0.95	**1.00**	0.4974
Non-OECD America	1.16	1.24	1.23	1.70	**1.33**	0.0012
Non-OECD Asia	1.07	1.03	1.03	1.18	**1.08**	0.1002
OECD	1.09	1.10	1.06	1.09	**1.08**	0.0000
All Groups	**1.11**	**1.11**	**1.07**	**1.23**	***1.13***	*0.0000*
p-value	0.0016	0.0001	0.0214	0.0174		

NB: the quantitative measures of returns to scale given in the table above are cell means of the country values calculated as explained in the text.
The p-values relate to one-sided t-tests
(H0: elasticity of scale = 1, H1: elasticity of scale > 1)
carried out on the period-, group- and overall mean values of these measures.

TABLE 5 OLS Estimates

Dependent variable : ln Q

N=52		coeff.	t-ratio
1965-73	CONSTANT	4.07	7.55
	ln N	0.44	6.02
	ln K	0.57	11.82
1974-79	CONSTANT	3.93	7.39
	ln N	0.40	5.74
	ln K	0.59	12.79
1980-86	CONSTANT	3.54	7.26
	ln N	0.36	5.69
	ln K	0.63	15.21
1987-92	CONSTANT	3.34	7.01
	ln N	0.36	5.93
	ln K	0.64	16.73

Dependent variable: ln N

N=52		coeff.	t-ratio
1965-73	CONSTANT	-4.00	-3.88
	ln K	-0.26	-1.96
	ln Q	0.97	6.02
1974-79	CONSTANT	-4.12	-3.85
	ln K	-0.29	-2.00
	ln Q	1.00	5.74
1980-86	CONSTANT	-4.06	-3.73
	ln K	-0.40	-2.43
	ln Q	1.11	5.69
1987-92	CONSTANT	-3.61	-3.30
	ln K	-0.48	-2.88
	ln Q	1.16	5.93

LEGEND:

ln Q = natural log of GDP (1985 prices)
ln K = natural log of capital stock (1985 prices)
ln N = natural log of employment.

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[2] Useful introductions to this body of techniques can be found in Lovell (1993), in the other essays contained in the same book (Fried *et al.*, 1993), and in Cooper *et al.* (2000).

[3] Dixon and Thirlwall refer to this term as a "learning by doing" coefficient, but there is no reason why it should not also reflect *static* economies of scale.

[4] See for instance Bairam (1987a, pp. 30-32) or McCombie and Thirlwall (1994, pp. 175-180), who both rely on a dynamic Cobb-Douglas function. The present argument has the advantage of yielding qualitatively unchanged conclusion even if the technical progress function becomes non-linear.

[5] This issue was perhaps first brought out by Rowthorn (1975a).

[6] A partial exception to this is found in Leon-Ledesma (1999).

[7] See for instance McCombie (1981, 1985a), Bairam (1986, 1987b, 1988c).

[8] Harris and Lau (1998), Harris and Liu (1999).

[9] See McCombie and Thirlwall (1994, pp. 171-173). Vaciago (1975) is an early example of non-linear specification of the law, while Pieper (2000) provides estimates based on local regression analysis. Pugno (1996a), although not directly concerned with the law, also provides relevant evidence in this respect.

[10] This reference set can be indifferently a production set, an input requirement set (for given outputs) or a production possibilities set (for given inputs).

[11] Note that a DEA-NDRS model also exists in the literature, but has been very seldom applied. See on this Seiford and Thrall (1990).

[12] If the technologies considered have more than one output or one input, then the two measures of Debreu-Farrell are equal to, respectively, the radial (equiproportional) expansion of all outputs needed to bring a producer on the frontier, for given inputs, and to the radial contraction of all inputs needed to bring a producer on the frontier, for given outputs.

[13] Clearly, different output-mixes can exist only in the presence of multi-output production sets.

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[14] To see this, consider that, if constant return to scale are posited, one obtains numerically identical values for output- and input-orientated efficiency scores for any given observation.

[15] Banker (1996); Korostelev *et al*. (1995a; b).

[16] Different rates of total factor productivity growth could be measured through a Malmquist index. However, if the production technology does not display constant returns to scale, there is no consensus in the literature on how to account for scale effects (see on this Färe *et al*.,1994; Färe *et al*., 1997; Ray and Desli, 1997). In the present context, this means that static and economies of scale may be confused with exogenous technical progress. Førsund (1996b) provides evidence for Norwegian manufacturing establishments suggesting a positive correlation between some Malmquist indices of productivity and output growth. However, no attempt is made in that study to distinguish between exogenous technical progress and scale effects.

[17] See on this Färe *et al*. (1985).

[18] This scale of judgement, relative to PWT mark 5, ranges from an A as the highest score to a D as the lowest one.

[19] This choice replicates a similar one made in Mankiw *et al*.(1992).

[20] The software for estimating these models was developed and generously made available to me by Antonio Pavone, from Istat, Rome.

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