A simple new test for slope homogeneity in panel data models with interactive effects

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Abstract

We consider the problem of testing for slope homogeneity in high-dimensional panel data models with cross-sectionally correlated errors. We consider a Swamy-type test for slope homogeneity by incorporating interactive fixed effects. We show that the proposed test statistic is asymptotically normal. Our test allows the explanatory variables to be correlated with the unobserved factors, factor loadings, or with both. Monte Carlo simulations demonstrate that the proposed test has good size control and good power.

Key words: Cross-sectional dependence, Endogenous predictors, Slope homogeneity

JEL Classification Codes: C12, C23.

1 Introduction

This paper considers the testing of slope homogeneity for high-dimensional panel data models. Testing slope homogeneity is useful for empirical studies. In finance, for example, testing asset pricing models, including the capital asset pricing model[1] and the multiple factor pricing models,[2] is related to testing homogeneity.[3] There are a number of studies on testing for slope homogeneity in panel data models, including Pesaran et al. (1996), Phillips and Sul (2003), Pesaran and Yamagata (2008), Blomquist and Westerlund (2013), and Su and Chen (2013).

Pesaran and Yamagata's (2008) testing procedure is useful in the sense that it treats a high-dimensional panel data model where the number of cross-sectional units N and the time series dimension T are large. However, the test does not allow cross-sectionally, serially correlated errors. To deal with the serially correlated errors, Blomquist and Westerlund (2013) extended their test to the case when the errors are heteroskedastic and/or serially correlated in an unknown fashion. However, the test still does not deal with the practically relevant case of cross-sectional dependence. Moreover, these tests do not allow dependence between the set of predictors and unobservable errors.

To deal with these problems, we propose a new simple test that accommodates cross-sectional dependence by using the results of Bai (2009), Song (2013), and Ando and Bai (2014). These studies considered panel data models with interactive fixed effects. Our proposed test statistic, denoted by $\hat{\Gamma}$, is a modified version of Swamy's (1970) test statistic, similar to that of Pesaran and Yamagata (2008). An advantage of our testing procedure is that it provides a robust test under cross-sectionally correlated errors with heteroskedasticity. Furthermore, the proposed test works even when the set of predictors and the unobservable errors that contain the factor structure are correlated. We investigate the asymptotic distribution of our test statistic, and show that the test has a standard normal distribution as $N, T \to \infty$ such that $\sqrt{T}/N \to 0$. Monte Carlo experiments show that the proposed test tends to have the correct size and satisfactory power as $N, T \to \infty$.

Recently, Su and Chen (2013) proposed a residual-based LM test for slope homogeneity in high-dimensional panel data models with interactive fixed effects. In general, Su and Chen's test works well. But there is a tendency of size distortion when N is much larger than T. One possible explanation is that their assumptions $N^{3/4}/T \to 0$ and $T^{2/3}/N \to 0$ imply a relatively narrow band between N and T. Usually, these assumptions are sufficient conditions, and they are not necessarily required in practice. But Monte Carlo experiments do reveal that these assumptions between N and T ap-

[1]See, e.g., Sharpe (1965) and Lintner (1965).

[2]See, e.g., Fama and French (1992) and Carhart (1997).

[3]In this case, it involves testing intercept homogeneity, which is a special case of the test considered in this paper.

pear to be crucial. In contrast, the proposed test in this paper provides the correct size control even when N is much larger than T. Our test also exhibits very good powers.

Notation. Let $\|A\| = [tr(A'A)]^{1/2}$ be the usual norm of the matrix A, where "tr" denotes the trace of a square matrix. The equation $a_n = O(b_n)$ states that the deterministic sequence a_n is at most of order b_n, $c_n = O_p(d_n)$ states that the random variable c_n is at most of order d_n in terms of probability, and $c_n = o_p(d_n)$ is of a smaller order in terms of probability. All asymptotic results are obtained under a large number of units N and a large number of time periods T.

The outline of the rest of the paper is as follows. Section 2 provides a literature review of the slope homogeneity test for high-dimensional panel data models. Section 3 proposes the $\hat{\Gamma}$ test statistic, a modified version of Swamy's (1970) test statistic, and derives its asymptotic distribution. In Section 4, we conduct Monte Carlo experiments to evaluate the finite sample performance of the proposed test. Some concluding remarks are provided in Section 5.

2 Literature review

Consider the following high-dimensional panel data model, with a large number of cross-sectional units N and a large number of time periods T

$$\boldsymbol{y}_i = X_i \boldsymbol{\beta}_i + \boldsymbol{u}_i, \quad i = 1, \ldots, N, \tag{1}$$

where $\boldsymbol{y}_i = (y_{i1}, ..., y_{iT})'$ $X_i = (\boldsymbol{x}_{i1}, ..., \boldsymbol{x}_{iT})'$, $\boldsymbol{u}_i = (u_{i1}, ..., u_{iT})'$. Here, each $\boldsymbol{x}_{it}$ is a $p \times 1$ vector of observable predictors, $\boldsymbol{\beta}_i$ is a $p \times 1$ vector of unknown slope coefficients, and u_{it} is an idiosyncratic error. The null hypothesis of interest in this paper is

$$H_0 \ : \ \boldsymbol{\beta}_1^0 = \boldsymbol{\beta}_2^0 = \cdots = \boldsymbol{\beta}_N^0 = \boldsymbol{\beta}^0 \ \text{ for some } \boldsymbol{\beta}^0.$$

The alternative hypothesis is

$$H_1 \ : \ \boldsymbol{\beta}_i^0 \neq \boldsymbol{\beta}_j^0 \ \text{ for a nonzero fraction of pairwise slopes for } i \neq j.$$

There are several procedures that can be used to test the null hypothesis. Although one may consider the standard F statistic, this test is valid for a fixed N, while this paper focuses on high-dimensional panel data models with large N and T. In this section, we provide a literature review of the test of slope homogeneity for high-dimensional panel data models.

2.1 Δ test

To check the slope homogeneity assumption, Pesaran and Yamagata (2008) considered a panel data model with fixed effects and heterogeneous slopes $y_{it} = \alpha_i + \boldsymbol{\beta}_i' \boldsymbol{x}_{it} + u_{it}$, where each $\boldsymbol{x}_{it}$ is a $p \times 1$ vector of observable predictors, $\boldsymbol{\beta}_i$ is a $p \times 1$ vector of unknown slope coefficients, and u_{it} is an error term. Under the assumption that ε_{it} are mutually uncorrelated over i and t, they proposed a standardized version of Swamy's test of slope homogeneity. Using the individual slope estimator $\hat{\boldsymbol{\beta}}_{i,FE} = (X_i' M_1 X_i)^{-1} X_i' M_1 \boldsymbol{y}_i$, with $M_1 = \boldsymbol{I} - \boldsymbol{1}\boldsymbol{1}'/T$ and the weighted fixed effects pooled estimator $\hat{\boldsymbol{\beta}}_{WFE} = (\sum_{i=1}^N X_i' M_1 X_i / \hat{\sigma}_i^2)^{-1} \sum_{i=1}^N X_i' M_1 \boldsymbol{y}_i / \hat{\sigma}_i^2$ with $\hat{\sigma}_i^2 = (\boldsymbol{y}_i - X_i \hat{\boldsymbol{\beta}}_{i,FE})' M_1 (\boldsymbol{y}_i - X_i \hat{\boldsymbol{\beta}}_{i,FE})/(T-p-1)$, Pesaran and Yamagata (2008) proposed the Δ tests

$$\hat{\Delta} = \sqrt{N} \left(\frac{N^{-1}\hat{S} - p}{\sqrt{2p}} \right), \tag{2}$$

where $\hat{S}$ is given as

$$\hat{S} = \sum_{i=1}^N (\hat{\boldsymbol{\beta}}_{i,FE} - \hat{\boldsymbol{\beta}}_{WFE})' \left(\frac{X_i' M_1 X_i}{\hat{\sigma}_i^2} \right) (\hat{\boldsymbol{\beta}}_{i,FE} - \hat{\boldsymbol{\beta}}_{WFE}).$$

Under large N and T, and $\sqrt{N}/T \to 0$, the test statistic asymptotically follows the standard normal distribution under the null hypothesis $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}_i$ for all i.

In addition to the $\hat{\Delta}$ test statistic, Pesaran and Yamagata (2008) also considered the following modified version

$$\tilde{\Delta} = \sqrt{N} \left(\frac{N^{-1}\tilde{S} - p}{\sqrt{2p}} \right), \tag{3}$$

where $\tilde{S}$ is given as

$$\tilde{S} = \sum_{i=1}^N (\hat{\boldsymbol{\beta}}_{i,FE} - \bar{\boldsymbol{\beta}}_{WFE})' \left(\frac{X_i' M_1 X_i}{\tilde{\sigma}_i^2} \right) (\hat{\boldsymbol{\beta}}_{i,FE} - \bar{\boldsymbol{\beta}}_{WFE}),$$

where instead of $\hat{\sigma}_i^2$, $\tilde{\sigma}_i^2 = (\boldsymbol{y}_i - X_i \hat{\boldsymbol{\beta}}_{FE})' M_1 (\boldsymbol{y}_i - X_i \hat{\boldsymbol{\beta}}_{FE})/(T-1)$ is used. Here, $\hat{\boldsymbol{\beta}}_{FE} = (\sum_{i=1}^N X_i' M_1 X_i)^{-1} \sum_{i=1}^N X_i' M_1 \boldsymbol{y}_i$, and the weighted FE estimator is computed using $\tilde{\sigma}_i^2$, $\bar{\boldsymbol{\beta}}_{WFE} = (\sum_{i=1}^N X_i' M_1 X_i / \tilde{\sigma}_i^2)^{-1} \sum_{i=1}^N X_i' M_1 \boldsymbol{y}_i / \tilde{\sigma}_i^2$. Similar to $\hat{\Delta}$, the test statistic $\tilde{\Delta}$ asymptotically follows the standard normal under the null. However, it is shown that this claim holds under $\sqrt{N}/T^2 \to 0$, which is weaker than the $\hat{\Delta}$ test statistic.

Although Pesaran and Yamagata (2008) considered modified versions of the $\hat{\Delta}$ test statistic, one of their crucial assumptions is that the error terms are cross-sectionally and serially independent. Moreover, they also assume that the p-dimensional predictors are strictly exogenous. In the presence of interactive effects (factor errors), the Δ test works when the regressors are uncorrelated with the factor errors, but will not work when correlations are allowed. As will be shown in our Monte Carlo results, their Δ test suffers from the size distortion problem in such situations. This motivates us to develop a new slope homogeneity test. It can be shown that our proposed test is able to overcome these issues.

2.2 HAC version of Δ test

The Δ test by Pesaran and Yamagata (2008) for slope homogeneity in large panels has become very popular in the literature. However, Blomquist and Westerlund (2013) pointed out that the test cannot deal with the practically relevant case of heteroskedastic and serially correlated errors. To overcome this difficulty, Blomquist and Westerlund (2013) proposed a generalized test that accommodates both features.

The HAC version of $\hat{\Delta}$ in (2) is given by

$$\hat{\Delta}_{\text{HAC}} = \sqrt{N}\left(\frac{N^{-1}\hat{S}_{\text{HAC}} - p}{\sqrt{2p}}\right),$$

where $\hat{S}_{\text{HAC}}$ is given as

$$\hat{S}_{\text{HAC}} = \sum_{i=1}^{N}(\hat{\boldsymbol{\beta}}_{i,OLS} - \hat{\boldsymbol{\beta}}_{\text{HAC}})'\left(X_i'M_1X_i\hat{V}_i^{-1}X_i'M_1X_i/T^2\right)(\hat{\boldsymbol{\beta}}_{i,OLS} - \hat{\boldsymbol{\beta}}_{\text{HAC}})$$

with $\hat{\boldsymbol{\beta}}_{\text{HAC}} = (\sum_{i=1}^{N} X_i'M_1X_i\hat{V}_i^{-1}X_i'M_1X_i/T^2)^{-1}\sum_{i=1}^{N} X_i'M_1X_i\hat{V}_i^{-1}X_i'M_1\boldsymbol{y}_i/T$ and $\hat{\boldsymbol{\beta}}_{i,OLS}$ is the OLS estimator for cross-sectional unit i. The heteroskedasticity and serial correlation are treated by the HAC estimator of $\hat{V}_i$

$$\hat{V}_i = \hat{\Gamma}_i(0) + \sum_{j=1}^{T-1} K\left[1/M_{i,T}\right]\left[\hat{\Gamma}_i(j) + \hat{\Gamma}_i(j)'\right],$$

where $\hat{\Gamma}_i(j) = T^{-1}\sum_{t=j+1}^{T}\hat{\boldsymbol{u}}_{it}\hat{\boldsymbol{u}}_{it-j}'$, $\hat{\boldsymbol{u}}_{it} = (\boldsymbol{x}_{it} - \bar{\boldsymbol{x}}_i)\hat{\varepsilon}_{it}$, $\bar{\boldsymbol{x}}_i = \sum_{t=1}^{T}\boldsymbol{x}_{it}/T$, and $\hat{\varepsilon}_{it} = y_{it} - \bar{y}_i - \hat{\boldsymbol{\beta}}_{\text{HAC}}'(\boldsymbol{x}_{it} - \bar{\boldsymbol{x}}_i)$. The kernel function $K(\cdot)$ and the bandwidth parameter $M_{i,T}$ are assumed to satisfy some regularity conditions.

However, similar to Pesaran and Yamagata's (2008) Δ test, the HAC version of the Δ test does not work when the regressors are correlated with the unobservable factor errors.

2.3 A residual-based Lagrangian multiplier test

Recently, Su and Chen (2013) considered a residual-based Lagrangian multiplier (LM) test for slope homogeneity in high-dimensional panel data models with the interactive fixed effects of Bai (2009), $\boldsymbol{y}_i = X_i\boldsymbol{\beta}_i + F\boldsymbol{\lambda}_i + \boldsymbol{\varepsilon}_i$, $i = 1, ..., N$, where F is a $T \times r$ matrix of unobservable common factors, $\boldsymbol{\lambda}_i$ is the factor loading, and $\boldsymbol{\varepsilon}_i$ are idiosyncratic errors. A key idea is that, under the null hypothesis of homogenous slopes, the p-dimensional predictors do not contain any useful information about the residuals.

In their testing procedure, a restricted model is first estimated by imposing slope homogeneity. Under the null, the model becomes $\boldsymbol{y}_i = X_i\boldsymbol{\beta} + F\boldsymbol{\lambda}_i + \boldsymbol{\varepsilon}_i$, which can be estimated by using Bai's (2009) procedure. Given the number of common factors, the parameters $\{\boldsymbol{\beta}, F, \Lambda\}$ are estimated by minimizing the least-squares objective function $\{\hat{\boldsymbol{\beta}}, \hat{F}, \hat{\Lambda}\} = \text{argmin}_{\{\beta,F,\Lambda\}} \sum_{i=1}^{N} \|\boldsymbol{y}_i - X_i\boldsymbol{\beta} - F\boldsymbol{\lambda}_i\|^2$.

Then, the heterogeneous panel regression of the restricted residuals is $\hat{\boldsymbol{\varepsilon}}_i = X_i\boldsymbol{\phi}_i + \boldsymbol{\eta}_i$, where for each unit, the coefficient of predictors $\boldsymbol{\phi}_i$ can be regarded as the slope parameter. Under the null, it is expected that $\boldsymbol{\phi}_i = \mathbf{0}$. Assuming that $\boldsymbol{\eta}_i$ are independent and identically distributed with $N(0, \sigma^2)$, across i, they maximize the Gaussian quasi log-likelihood of the restricted residuals. This is equivalent to finding $\{\hat{\boldsymbol{\phi}}_1, ..., , \hat{\boldsymbol{\phi}}_N\} = \text{argmin}_{\{\phi_1,...,,\phi_N\}} \sum_{i=1}^{N} \|\hat{\boldsymbol{\varepsilon}}_i - X_i\boldsymbol{\phi}_i\|^2$. The test of slope homogeneity can be based on the LM statistic

$$LM_{NT} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\boldsymbol{\varepsilon}}_i' X_i (X_i' X_i)^{-1} X_i' \hat{\boldsymbol{\varepsilon}}_i.$$

Su and Chen (2013) showed that, under the null hypothesis H_0,

$$J_{NT} = (LM_{NT} - B_{NT})/V_{NT}^{1/2} \tag{4}$$

asymptotically follows the standard normal distribution. Here B_{NT} and V_{NT} are estimated by

$$\hat{B}_{NT} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_{it}^2 \hat{h}_{i,tt} \quad \text{and} \quad \hat{V}_{NT} = \frac{4}{T^2 N} \sum_{i=1}^{N} \sum_{t=2}^{T} \left(\hat{\boldsymbol{\varepsilon}}_{it} \hat{b}_{it} \sum_{s=1}^{t-1} \hat{b}_{is} \hat{\boldsymbol{\varepsilon}}_{is} \right),$$

where $\hat{h}_{i,tt}$ denotes the t-th diagonal element of $\hat{H}_i = M_{\hat{F}} X_i (X_i' X_i)^{-1} X_i' M_{\hat{F}}$, and $\hat{b}_{it} = (X_i' X_i / T)^{-1/2} (\boldsymbol{x}_{it} - \sum_{s=1}^{T} \hat{\boldsymbol{f}}'_t \hat{\boldsymbol{f}}_s \boldsymbol{x}_{it})$.

One of the differences between Su and Chen's (2013) test and our proposed test is that their test has a narrower tolerance to the relationship between N and T. Their conditions $N^{3/4}/T \to 0$ and $T^{2/3}/N \to 0$ as $N, T \to \infty$ are stronger than ours, $\sqrt{T}/N \to 0$. Using Monte Carlo experiments, we demonstrate that our proposed test has the correct size and satisfactory power, while that of Su and Chen (2013) suffers size distortion. The LM type of test in the presence of interactive effects appears to be sensitive to the configurations between N and T.

3 A new procedure for testing slope homogeneity

3.1 Model

Consider a high-dimensional panel data model, with a large number of cross-sectional units N, and a large number of time periods T, $\boldsymbol{y}_i = X_i \boldsymbol{\beta}_i + \boldsymbol{u}_i$ in (1). In this paper, we assume that the error term contains multifactor structures:

$$\boldsymbol{u}_i = F\boldsymbol{\lambda}_i + \boldsymbol{\varepsilon}_i, \quad , i = 1, ..., N, \tag{5}$$

where

$$F = \begin{pmatrix} \boldsymbol{f}_1' \\ \boldsymbol{f}_2' \\ \vdots \\ \boldsymbol{f}_T' \end{pmatrix}, \ \boldsymbol{\lambda}_i = \begin{pmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{ir} \end{pmatrix}, \ \boldsymbol{\varepsilon}_i = \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{pmatrix},$$

where $\boldsymbol{f}_t$ is an $r \times 1$ vector of unobservable common factors, $\boldsymbol{\lambda}_i$ is the factor loading, and ε_{it} are the idiosyncratic errors. In the next section, we describe the assumptions under the null and alternative hypotheses.

3.2 Assumptions

We state the assumptions needed for the asymptotic analysis.

Assumption A: Common factors

The common factors satisfy $E\|\boldsymbol{f}_t\|^4 < \infty$. Furthermore, $T^{-1}\sum_{t=1}^{T} \boldsymbol{f}_t \boldsymbol{f}_t{}' \to \Sigma_F$ as $T \to \infty$, where Σ_F is an $r \times r$ positive definite matrix.

Assumption B: Factor loadings

The factor-loading matrix $\Lambda = [\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_N]'$ satisfies $E\|\boldsymbol{\lambda}_i^4\| < \infty$ and $\|N^{-1}\Lambda'\Lambda - \Sigma_\Lambda\| \to \mathbf{0}$ as $N \to \infty$, where Σ_Λ is an $r \times r$ positive definite matrix.

Assumption C: Error terms

There exists a positive constant $C < \infty$ such that for all N and T,
(1): $E[\varepsilon_{it}] = 0$, $E[|\varepsilon_{it}|^8] < C$ for all i and t;
(2): ε_{it} and ε_{js} are independent, for $i \neq j$ and $t \neq s$.
(3): For every (s,t), $E[|N^{-1/2}\sum_{i=1}^{N}(\varepsilon_{is}\varepsilon_{it} - E[\varepsilon_{is}\varepsilon_{it}])|^4] < C$.
(4): ε_{it} is independent of $\boldsymbol{x}_{js}$, $\boldsymbol{\lambda}_i$, and $\boldsymbol{f}_s$ for all i, j, t, s.

Assumption D: Predictors

We assume $E\|\boldsymbol{x}_{it}\|^4 < C$. The $p \times p$ matrix $\frac{1}{T}[X_i' M_{F^0} X_i]$ is positive definite, where $M_F = I - F(F'F)^{-1}F'$, and M_{F_0} is equal to M_F evaluated at the true common factors F^0. Furthermore, we define $A_i = \frac{1}{T}X_i' M_F X_i$, $B_i = (\boldsymbol{\lambda}_i \boldsymbol{\lambda}_i') \otimes I_T$, $C_i = \frac{1}{\sqrt{T}}\boldsymbol{\lambda}_i' \otimes (X_i' M_F)$. Let $\mathcal{A}$ be the collection of F such that $\mathcal{A} = \{F : F'F/T = I\}$. We assume

$$\inf_{F \in \mathcal{A}} \left[\frac{1}{N}\sum_{i=1}^{N} E_i(F)\right] \quad \text{is positive definite,} \tag{6}$$

where $E_i(F) = B_i - C_i' A_i^- C_i$ and A_i^- is the generalized inverse of A_i.

Assumption E: Central limit theory

We assume

$$\frac{1}{\sqrt{T}} X_i' M_{F^0} \boldsymbol{\varepsilon}_i \to_d N(\mathbf{0}, \Omega_i),$$

where Ω_i is the probability limit of (as T goes to infinity)

$$\frac{1}{T} E[X_i' M_{F^0} \boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i' M_{F^0} X_i].$$

Remark 1 Assumptions A and B above are commonly imposed on the panel data model (1). The full rank assumptions of Σ_F and Σ_Λ imply the number of common factors is r. Assumption C allows heteroskedasticity in the idiosyncratic errors $\boldsymbol{\varepsilon}_i$. Assumptions D and E above are imposed for deriving the asymptotic distributions of the slope coefficients (see Bai (2009), Song (2013), Ando and Bai (2014)).

We consider estimating the model (1) with factor structure (5) under the null and alternative hypotheses, respectively. Under the null hypothesis, we can estimate the common slope coefficient $\boldsymbol{\beta}$ by using the procedure in Bai (2009). Under the alternative, we employ the estimation procedure in Song (2013) and Ando and Bai (2014). Given the number of common factors r, we minimize the least-squares objective function

$$\ell(\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_N, F, \Lambda) = \sum_{i=1}^{N} \|\boldsymbol{y}_i - X_i \boldsymbol{\beta}_i - F \boldsymbol{\lambda}_i\|^2 \tag{7}$$

subject to the constraints on the factors and its loadings (see Connor and Korajczyk (1986), Bai and Ng (2002), Bai (2009)). The number of common factors can be selected by the C_p criterion, proposed in Ando and Bai (2014). Thus, we can compare the restricted and unrestricted estimators of the slope coefficients.

3.3 A new slope homogeneity test

To test slope homogeneity, we consider Swamy's test statistic. Swamy's (1970) test of slope homogeneity calculates the dispersion of individual slope estimates from a suitable pooled estimator (also see Pesaran and Yamagata (2008)). In our setting, Swamy's test statistic applied to the slope coefficients can be written as

$$\hat{\Gamma} = \frac{T(\hat{\boldsymbol{\beta}} - \bar{\boldsymbol{\beta}}_{1_N})' \left(\hat{S} - \frac{1}{N}\hat{L}'\right) \hat{\Omega}^{-1} \left(\hat{S} - \frac{1}{N}\hat{L}\right) (\hat{\boldsymbol{\beta}} - \bar{\boldsymbol{\beta}}_{1_N}) - Np}{\sqrt{2Np}}, \tag{8}$$

where $\hat{\boldsymbol{\beta}}' = (\hat{\boldsymbol{\beta}}_1', ..., \hat{\boldsymbol{\beta}}_N')$, $\bar{\boldsymbol{\beta}}_{1_N}' = (\bar{\boldsymbol{\beta}}', ..., \bar{\boldsymbol{\beta}}')$, $\bar{\boldsymbol{\beta}} = \sum_{i=1}^{N} \hat{\boldsymbol{\beta}}_i / N$, and $\hat{\boldsymbol{\beta}}_i$ are obtained by minimizing (7), $\hat{S}$ is an $Np \times Np$ block diagonal matrix with ith block $(X_i' M_{\hat{F}} X_i)/T$, and

$\hat{L}$ is an $Np \times Np$ matrix with ij-th block $\hat{a}_{ij}(X_i' M_{\hat{F}} X_j)/T$ with $\hat{a}_{ij} = \hat{\boldsymbol{\lambda}}_j'(\hat{\Lambda}'\hat{\Lambda}/N)^{-1}\hat{\boldsymbol{\lambda}}_j$ and $\hat{\Omega}$ is the variance–covariance estimator of Ω given as

$$\Omega = \begin{pmatrix} \Omega_1 & & & \\ & \Omega_2 & & \\ & & \ddots & \\ & & & \Omega_N \end{pmatrix}, \tag{9}$$

and Ω_i is the variance–covariance matrix of $X_i' M_{F_0} \boldsymbol{\varepsilon}_i/\sqrt{T}$, $i = 1, ..., N$. The following theorem provides the asymptotic distribution of $\hat{\Gamma}$ under the null hypothesis.

Theorem 1 *Suppose that Assumptions A–E and $\sqrt{T}/N \to 0$ hold. Then, under H_0,*

$$\hat{\Gamma} \to N(0,1) \quad \text{in distribution,}$$

as $T, N \to \infty$.

The proof of Theorem 1 is provided in the Appendix. The proposed test is simple to implement as it has a limiting $N(0,1)$ distribution. This result holds even under the cross-sectional correlations and heteroskedasticity in $\boldsymbol{u}_i$.

Remark 2 Under the null H_0, we need the value of the true common slope coefficients $\boldsymbol{\beta}^0$, for which we use $\bar{\boldsymbol{\beta}} = \sum_{i=1}^N \hat{\boldsymbol{\beta}}_i/N$ where $\hat{\boldsymbol{\beta}}_i$ are obtained by minimizing (7). Note that we can also employ Bai's (2009) estimator $\hat{\boldsymbol{\beta}}$. Because these two provide similar results, we thus report only the use of $\bar{\boldsymbol{\beta}}$.

Remark 3 To calculate $\hat{\Gamma}$, we need to estimate the variance–covariance matrix of $X_i' M_{F_0} \boldsymbol{\varepsilon}_i/\sqrt{T}$, Ω_i in (9). The following provides a practical calculation method for Ω_i.
Case 1: Homoskedastic errors over i and t
In this case, Ω_i is given as $\Omega_i = \sigma^2 S_{ii}$ with $\sigma^2 = \mathrm{Var}(\varepsilon_{it})$. In the absence of serial correlation and heteroskedasticity, the common variance can be estimated by

$$\hat{\sigma}^2 = \frac{1}{NT - Np - (N+T)r} \sum_{i=1}^N \sum_{t=1}^T (y_{it} - \boldsymbol{x}_{it}'\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{f}}_t'\hat{\boldsymbol{\lambda}}_i)^2.$$

Case 2: Heteroskedastic errors over i
The i-th block diagonal element of Ω, Ω_i, is given as $\Omega_i = \sigma_i^2 S_{ii}$ with $\sigma_i^2 = \mathrm{Var}(\varepsilon_{it})$. The variance can be estimated as

$$\hat{\sigma}_i^2 = \frac{1}{T-p} \sum_{t=1}^T (y_{it} - \boldsymbol{x}_{it}'\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{f}}_t'\hat{\boldsymbol{\lambda}}_i)^2.$$

Case 3: Heteroskedastic errors over i and t
If the idiosyncratic errors ε_{it} are heteroskedastic over i and t (i.e., $E[\varepsilon_{it}] = \sigma_{it}^2$), then $\hat{\Omega}_i = T^{-1}\sum_{t=1}^T \hat{\boldsymbol{x}}_{it}\hat{\boldsymbol{x}}_{it}'\hat{\varepsilon}_{it}^2$, where $\hat{\boldsymbol{x}}_{it}$ is the t-th row of $M_{\hat{F}} X_i$. Furthermore, with serial correlation, we can also use the method of Blomquist and Westerlund (2013) to compute Ω_i.

Remark 4 A simpler version of the test statistic is

$$\tilde{\Gamma} = \frac{\sum_{i=1}^{N} T(\hat{\boldsymbol{\beta}}_i - \bar{\boldsymbol{\beta}})' S_{ii} \hat{\Omega}_i^{-1} S_{ii} (\hat{\boldsymbol{\beta}}_i - \bar{\boldsymbol{\beta}}) \left[1 - \hat{\boldsymbol{\lambda}}_i' (\hat{\Lambda}'\hat{\Lambda})^{-1} \hat{\boldsymbol{\lambda}}_i\right]^2 - Np}{\sqrt{2Np}}. \tag{10}$$

Under Assumptions A–E and $\sqrt{T}/N \to 0$, the $\tilde{\Gamma}$ asymptotically follows the standard normal distribution under the null H_0. The proof of this claim is provided in the Appendix.

4 Simulation

In this section, we conduct a Monte Carlo simulation to evaluate the finite sample performance of our testing procedure. As a performance comparison, we considered Pesaran and Yamagata's (2008) Δ test statistics $\hat{\Delta}$ in (2) and $\tilde{\Delta}$ in (3), and Su and Chen's (2013) residual-based LM test in (4). Pesaran and Yamagata (2008) assume that u_{it} are mutually uncorrelated over i and t. Although Su and Chen (2013) allow cross-sectional dependence through the factor structure among u_{it}, their conditions on the relationship between N and T are stronger than ours.

4.1 Data generating processes

GDP1: The first data generating process considered is $y_{it} = \boldsymbol{x}_{it}'\boldsymbol{\beta}_i + u_{it}$ and $u_{it} = \boldsymbol{f}_t'\boldsymbol{\lambda}_i + \varepsilon_{it}$, where the $r(=2)$-dimensional factor $\boldsymbol{f}_t$ is a vector of $N(0,1)$ variables, each element of the factor-loading vector $\boldsymbol{\lambda}_i$ follows $N(0, I)$, and the noise term ε_{it} is also generated from $N(0,1)$. Setting $p = 2$, each of the elements of X_i is generated from the uniform distribution over $[-2, 2]$. Under the null H_0, the true parameter vectors $\boldsymbol{\beta}_i$ were set to $\boldsymbol{\beta}_i = (-1/2, 1/2)'$, $i = 1, ..., N$. Under the alternative H_1, the true parameter vectors $\boldsymbol{\beta}_i$ were set to $\boldsymbol{\beta}_i = (\beta_{i1}, 1/2)'$ with β_{i1} being generated from the uniform distribution over $[1, 1.5]$.

GDP2: As the second example, we investigated the performance of the proposed testing procedure when the predictors and the unobservable factor structures have dependency. We generated the predictors as follows:

$$x_{it,1} = 0.2 + 0.3\boldsymbol{f}_t'\boldsymbol{\lambda}_i + \varepsilon_{it,1}^x, \quad \text{and} \quad x_{it,2} = 0.5 + 0.5\boldsymbol{f}_t'\boldsymbol{\lambda}_i + \varepsilon_{it,2}^x,$$

where $\varepsilon_{it,1}^x$ and $\varepsilon_{it,2}^x$ are independently generated from the standard normal distribution. The other variables are defined as before. The key feature of this model is that the noise $\boldsymbol{u}_i$ and predictors X_i are correlated.

4.2 Results

We consider various configurations of (N, T). For a given configuration of (N, T), we generate 2,000 replications from each of the two data generating models. Our test statistics $\hat{\Gamma}$ in (8) and $\tilde{\Gamma}$ in (10) are calculated under the true number of common factors $r = 2$ in our testing procedure, as it can be identified by the C_p criterion of Ando and Bai (2014).

Theorem 1 suggests that our test statistic $\hat{\Gamma}$ in (8) is asymptotically normal with mean 0 and standard deviation 1, when the null hypothesis of slope homogeneity is satisfied. Therefore, we reject the null hypothesis if the absolute value of our test statistic exceeds the critical value at α based on the normal distribution. We focus on the rejection frequency at an $\alpha = 5\%$ nominal level for our test across 2,000 simulations. Furthermore, we check the finite sample power rejection frequency of our testing procedure under the alternative.

The finite sample properties of the proposed test under each of the data generating processes are summarized in Tables 1–3. Each column reports the rejection frequency (= the number of rejections/2,000) under the null H_0 and alternative H_1. Table 1 provides the results for the first data generating process, and gives the size and power for a wide range of N and T. The results for $\hat{\Delta}$ and $\tilde{\Delta}$ are in line with those of Pesaran and Yamagata (2008). Furthermore, the LM results are in line with those of Su and Chen (2013).

Table 1 suggests that the level of our test behaves reasonably well as the size of the panel increases $N, T \to \infty$. When the null hypothesis does not hold, Table 1 suggests our test statistics $\hat{\Gamma}$ and $\tilde{\Gamma}$ have higher power than those of $\hat{\Delta}$ and $\tilde{\Delta}$ under small T and N.

However, we can see the clear advantages of our testing procedure in Table 2. Under the null, Pesaran and Yamagata's (2008) testing procedure rejects the null in almost all cases. In contrast, our testing procedure has nice size control and power for all combinations of N and T. This difference arises because $\boldsymbol{x}_{it}$ and the factor structure are correlated in the second data generating process. Our test permits this correlation.

Su and Chen's (2013) procedure works well in general. There is, however, a tendency of size distortion when N is relatively large with respect to T, for example, $T = 50$ and $N = 200$. A possible reason is that their test imposes the following conditions: $N^{3/4}/T \to 0$ and $T^{2/3}/N \to 0$ as $N, T \to \infty$. In general, these conditions are sufficient, and they are not necessary. Nevertheless, they imply a relatively narrow band between N and T. To verify this, we further compared the size and power of all tests under large N. Under much larger N relative to T, the finite sample properties of the proposed test are summarized in Table 3. Under the second data generating process with $(N, T) = (200, 20)$, the finite sample rejection frequencies at an $\alpha = 5\%$ nominal level for our test were 0.06 under the null H_0 and 0.72 under the alternative H_1. However, those of Su and Chen's (2013) test are 0.35 under the null H_0 and 0.96 under the alternative H_1. It can be seen that our proposed test still provides correct size, while a size distortion is observed with Su and Chen's (2013) test. This difference arises because of the relationship between N and T. Similar observations can be made for Pesaran and Yamagata's (2008) $\hat{\Delta}$ test statistic. As $\tilde{\Delta}$ needs the weaker condition $\sqrt{N}/T^2 \to 0$, the $\tilde{\Delta}$ procedure has nice size control under the first DGP, but not under the second DGP, as before. Our test continues to work well under much larger N than T.

5 Conclusion

In this paper we examined the problem of testing slope homogeneity in a high-dimensional panel data model. We developed testing procedures based on the Swamy's (1970) test principle. Our testing procedure allows cross-sectional dependence as well as the dependence between the predictors and unobservable factor structures. Monte Carlo experiments suggest that the proposed method is a useful testing procedure for detecting slope homogeneity in high-dimensional panel data in which both the time dimension and the cross-sectional dimension are large.

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Appendix 1: Proof of Theorem 1. Let $\hat{\boldsymbol{\beta}}_i$ be obtained by minimizing the least-squares objective function in (7). Let $S_{ii} = (X_i' M_{F^0} X_i)/T$, $L_{ij} = a_{ij}(X_i' M_{F^0} X_j)/T$ with $a_{ij} = \boldsymbol{\lambda}_j^{0\prime}(\Lambda^{0\prime}\Lambda^0/N)^{-1}\boldsymbol{\lambda}_j^0$ and $\boldsymbol{\zeta}_i = X_i' M_{F^0}\boldsymbol{\varepsilon}_i/\sqrt{T}$. Song (2013) rigorously showed that under $\sqrt{T}/N \to 0$,

$$\sqrt{T}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0) = S_{ii}^{-1}\boldsymbol{\zeta}_i + S_{ii}^{-1}\frac{1}{N}\sum_{j=1}^{N} L_{ij}\sqrt{T}(\hat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_j^0) + o_p(1), \quad i = 1, ..., N,$$

which implies

$$\sqrt{T}\begin{pmatrix} \hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1^0 \\ \hat{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2^0 \\ \vdots \\ \hat{\boldsymbol{\beta}}_N - \boldsymbol{\beta}_N^0 \end{pmatrix} = \begin{pmatrix} S_{11}^{-1} & & & \\ & S_{22}^{-1} & & \\ & & \ddots & \\ & & & S_{NN}^{-1} \end{pmatrix}\begin{pmatrix} \boldsymbol{\zeta}_1 \\ \boldsymbol{\zeta}_2 \\ \vdots \\ \boldsymbol{\zeta}_N \end{pmatrix}$$
$$+\frac{1}{N}\begin{pmatrix} S_{11}^{-1} & & & \\ & S_{22}^{-1} & & \\ & & \ddots & \\ & & & S_{NN}^{-1} \end{pmatrix}\begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \ddots & L_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{pmatrix}\begin{pmatrix} \sqrt{T}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1^0) \\ \sqrt{T}(\hat{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2^0) \\ \vdots \\ \sqrt{T}(\hat{\boldsymbol{\beta}}_N - \boldsymbol{\beta}_N^0) \end{pmatrix} + o_p(1).$$

We can express the above formula as

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}^0) = S^{-1}\boldsymbol{\zeta} + \frac{1}{N}S^{-1}L\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}^0) + o_p(1), \tag{11}$$

where $\hat{\boldsymbol{\beta}}' = (\hat{\boldsymbol{\beta}}_1', ..., \hat{\boldsymbol{\beta}}_N')$, $\boldsymbol{\beta}^{0\prime} = (\boldsymbol{\beta}_1^{0\prime}, ..., \boldsymbol{\beta}_N^{0\prime})$, $\boldsymbol{\zeta}' = (\boldsymbol{\zeta}_1', ..., \boldsymbol{\zeta}_N')$, S is an $Np \times Np$ block diagonal matrix with ith block S_{ii}, and L is an $Np \times Np$ matrix with ij-th block L_{ij}.

Let Ω be the variance–covariance matrix of $\boldsymbol{\zeta}$, that is block diagonal matrix,

$$\Omega = \begin{pmatrix} \Omega_1 & & & \\ & \Omega_2 & & \\ & & \ddots & \\ & & & \Omega_N \end{pmatrix}, \tag{12}$$

where Ω_i is the variance–covariance matrix of $\boldsymbol{\zeta}_i$.

From (11), we then have

$$\Omega^{-1/2}\left(S - \frac{1}{N}L\right)\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}^0) = \Omega^{-1/2}\boldsymbol{\zeta} + o_p(1),$$

which implies

$$T(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}^0)'\left(S - \frac{1}{N}L'\right)\Omega^{-1}\left(S - \frac{1}{N}L\right)(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}^0) = \boldsymbol{\zeta}'\Omega^{-1}\boldsymbol{\zeta} + o_p(1).$$

From Assumption (E), $\boldsymbol{\zeta}'\Omega^{-1}\boldsymbol{\zeta}$ in the last line asymptotically follows a chi-squared distribution with Np degrees of freedom. Because the noise terms are cross-sectionally independent as in Assumption (C), by the central limit theorem, we have

$$\frac{\boldsymbol{\zeta}'\Omega^{-1}\boldsymbol{\zeta} - Np}{\sqrt{2Np}} \to N(0,1).$$

It can be shown that replacing $\boldsymbol{\beta}^0$ by $\bar{\boldsymbol{\beta}}_{1_N}$, and the unknown elements in S, L, and Ω by their estimators, the same asymptotic representation holds. This completes the proof of Theorem 1.

Appendix 2: Proof of (10). The test statistic $\hat{\Gamma}$ is derived by correcting bias for the whole system (joint bias). If we correct the bias for each individual parameter $\hat{\boldsymbol{\beta}}_i$, then using

$$\frac{1}{N}S_{ii}^{-1}L_{ii} = \frac{1}{N}a_{ii}I_p = \boldsymbol{\lambda}_i^{0\prime}(\Lambda^{0\prime}\Lambda^0)^{-1}\boldsymbol{\lambda}_i^0 I_p,$$

we have

$$\sqrt{T}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0) = S_{ii}^{-1}\boldsymbol{\zeta}_i + \frac{1}{N}a_{ii}\sqrt{T}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0) + o_p(1).$$

The reason that this approximation works is that the weighted average of $(\hat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_j^0)$ for $j \neq i$,

$$S_{ii}^{-1}\frac{1}{N}\sum_{j \neq i} L_{ij}\sqrt{T}(\hat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_j^0),$$

is of small magnitude, and is also uncorrelated with the leading term ζ_i. This leads to

$$S_{ii}\sqrt{T}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0)(1 - a_{ii}/N) = \boldsymbol{\zeta}_i + o_p(1),$$

or

$$\Omega_i^{-1/2} S_{ii}\sqrt{T}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0)(1 - a_{ii}/N) = \Omega_i^{-1/2}\boldsymbol{\zeta}_i + o_p(1).$$

We then have

$$T(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0)' S_{ii}\Omega^{-1} S_{ii}(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}_i^0)(1 - a_{ii}/N)^2 = \boldsymbol{\zeta}_i'\Omega_i^{-1}\boldsymbol{\zeta}_i + o_p(1),$$

for $i = 1, ..., N$. Again, $\boldsymbol{\zeta}_i'\Omega_i^{-1}\boldsymbol{\zeta}_i$ follows a chi-squared distribution with p degrees of freedom from Assumption (E). Using Assumption (C) and the central limit theorem, we have $\tilde{\Gamma} \to N(0, 1)$. This completes the proof.

Table 1: Finite sample properties of the proposed test under the first data generating process. Each column reports the rejection frequency under the null H_0 and alternative H_1. Critical level is set as $\alpha = 5\%$. $\hat{\Gamma}$: the proposed test statistic in (8), $\tilde{\Gamma}$: the proposed test statistic in (10), $\hat{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (2), $\tilde{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (3) and LM: the residual-based Lagrangian Multiplier test of Su and Chen (2013) in (4).

Rejection frequency under H_0

T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
50	50	0.02	0.02	0.02	0.04	0.06
50	100	0.02	0.02	0.02	0.07	0.07
50	200	0.03	0.03	0.03	0.09	0.09
100	50	0.02	0.02	0.03	0.05	0.04
100	100	0.03	0.02	0.02	0.04	0.05
100	200	0.03	0.02	0.03	0.07	0.06
200	50	0.01	0.01	0.02	0.02	0.03
200	100	0.02	0.02	0.02	0.03	0.05
200	200	0.03	0.03	0.03	0.04	0.05

Rejection frequency under H_1

T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
50	50	0.91	0.91	0.42	0.57	0.92
50	100	1.00	1.00	0.71	0.88	1.00
50	200	1.00	1.00	0.96	1.00	1.00
100	50	1.00	1.00	0.92	0.95	1.00
100	100	1.00	1.00	1.00	1.00	1.00
100	200	1.00	1.00	1.00	1.00	1.00
200	50	1.00	1.00	1.00	1.00	1.00
200	100	1.00	1.00	1.00	1.00	1.00
200	200	1.00	1.00	1.00	1.00	1.00

Table 2: Finite sample properties of the proposed test under the second data generating process. Each column reports the rejection frequency under the null H_0 and alternative H_1. Critical level is set as $\alpha = 5\%$. $\hat{\Gamma}$: the proposed test statistic in (8), $\tilde{\Gamma}$: the proposed test statistic in (10), $\hat{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (2), $\tilde{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (3) and LM: the residual-based Lagrangian Multiplier test of Su and Chen (2013) in (4).

Rejection frequency under H_0

T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
50	50	0.02	0.02	1.00	1.00	0.02
50	100	0.03	0.03	1.00	1.00	0.07
50	200	0.03	0.03	1.00	1.00	0.10
100	50	0.01	0.01	1.00	1.00	0.02
100	100	0.03	0.03	1.00	1.00	0.04
100	200	0.03	0.04	1.00	1.00	0.07
200	50	0.01	0.01	1.00	1.00	0.02
200	100	0.02	0.02	1.00	1.00	0.03
200	200	0.03	0.03	1.00	1.00	0.04

Rejection frequency under H_1

T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
50	50	0.73	0.74	1.00	1.00	0.86
50	100	0.96	0.96	1.00	1.00	1.00
50	200	1.00	1.00	1.00	1.00	1.00
100	50	1.00	1.00	1.00	1.00	1.00
100	100	1.00	1.00	1.00	1.00	1.00
100	200	1.00	1.00	1.00	1.00	1.00
200	50	1.00	1.00	1.00	1.00	1.00
200	100	1.00	1.00	1.00	1.00	1.00
200	200	1.00	1.00	1.00	1.00	1.00

Table 3: Finite sample properties of the proposed test under large number of units N compared with the length of time series T. Each column reports the rejection frequency under the null H_0 and alternative H_1. Critical level is set as $\alpha = 5\%$. $\hat{\Gamma}$: the proposed test statistic in (8), $\tilde{\Gamma}$: the proposed test statistic in (10), $\hat{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (2), $\tilde{\Delta}$: the test procedure of Pesaran and Yamagata (2008) in (3) and LM: the residual-based Lagrangian Multiplier test of Su and Chen (2013) in (4).

Rejection frequency under H_0

	T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
DGP 1	20	200	0.02	0.02	0.03	0.43	0.24
	50	500	0.04	0.04	0.01	0.15	0.10
	50	800	0.04	0.04	0.02	0.31	0.29
DGP 2	20	200	0.06	0.06	1.00	1.00	0.35
	50	500	0.04	0.04	1.00	1.00	0.14
	50	800	0.04	0.04	1.00	1.00	0.25

Rejection frequency under H_1

	T	N	$\hat{\Gamma}$	$\tilde{\Gamma}$	$\tilde{\Delta}$	$\hat{\Delta}$	LM
DGP 1	20	200	0.87	0.87	0.29	0.95	0.97
	50	500	1.00	1.00	1.00	1.00	1.00
	50	800	1.00	1.00	1.00	1.00	1.00
DGP 2	20	200	0.72	0.72	1.00	1.00	0.96
	50	500	1.00	1.00	1.00	1.00	1.00
	50	800	1.00	1.00	1.00	1.00	1.00