MPRA
Munich Personal RePEc Archive

Sensitivity analysis of scenario models for operational risk Advanced Measurement Approach

Chaudhary, Dinesh

29 December 2014

Online at https://mpra.ub.uni-muenchen.de/60996/
MPRA Paper No. 60996, posted 29 Dec 2014 16:02 UTC

Sensitivity analysis of scenario models for operational risk Advanced Measurement Approach

Dinesh Chaudhary[1]

Dec, 2014

Abstract

Scenario Analysis (SA) plays a key role in determination of operational risk capital under Basel II Advanced Measurement Approach. However, operational risk capital based on scenario data may exhibit high sensitivity or wrong-way sensitivity to scenario inputs. In this paper, we first discuss scenario generation using quantile approach and parameter estimation using quantile matching. Then we use single-loss approximation (SLA) to examine sensitivity of scenario based capital to scenario inputs.

1. Introduction

As per the Basel II capital guidelines, banks may compute capital requirement for operational risk using one of the three approaches viz. Basic Indicator Approach (BIA), The Standardized Approach (TSA) and Advanced Measurement Approach (AMA). AMA is a risk-sensitive approach that requires regulatory capital estimation at 99.9^{th} quantile of the annual loss distribution using internal models. Due to paucity of empirical annual loss data, annual loss distribution is generated through convolution of annual frequency and individual loss severity distribution using parametric models.

Regulatory AMA guidelines state that internal capital model must incorporate four data elements viz. internal loss data (ILD), relevant external loss data (ELD), scenario analysis (SA) and business environment and internal control factors (BEICF). As there is no standard modelling approach acceptable across the industry, different modelling approaches use these four data elements in a variety of ways (BCBS, 2009a, Table 18A, 18B, 18C). There are also multiple ways of conducting scenario analysis, which can be broadly classified into three categories viz. Individual scenario approach, percentile/quantile approach, bucket/interval approach (BCBS, 2009b, Table S1).

Guidelines also require banks to identify non-overlapping units viz. Operational Risk Categories (ORC) that share similar risk profile. Typically, ORCs are identified as combination of banking business lines such as retail banking, commercial banking, trading and sales, payment and settlement etc. and operational risk event types such as internal fraud, external fraud, damage to physical assets, business disruption and system failure etc.

In this paper we examine Value at Risk (VaR) sensitivity of AMA models that use scenario analysis as a direct model input. We demonstrate that scenario output may exhibit significant volatility due to small changes in scenario input data and this can have practical implications for banks migrating to AMA.

2. Literature Review

Rosengren (2006) summarises implications of scenario structuring choices and challenges in validation of scenario based models. Chaudhury (2010) reviews practical issues with operational risk capital modelling including scenario analysis. RMA (2012) industry position paper summarises industry practices in use of SA in AMA implementation in United States. Shevchenko and Wüthrich (2006) demonstrate the use of Bayesian techniques for combining scenario data with loss data, alongwith multiple ad-hoc procedures.

[1] Dinesh Chaudhary works at Asymmetrix Solutions in India. Email: Dinesh.chaudhary@asymmetrix.co.in

To the best of author's knowledge, there is a lack of work in public domain discussing AMA model sensitivity for scenario based models. Colombo and Desando (2008) discuss development and implementation of scenario models alongwith model sensitivity. For LDA models, Opdyke and Cavallo (2012) demonstrate wrong-way VaR sensitivity to small losses where parameters are estimated based on loss data using MLE.

3. Scenario Generation

Scenario analysis involves elicitation of expert opinions for forward-looking estimates of likelihood and impact of plausible operational risk events. Quantile approach is a common approach for scenario analysis, resulting in expert estimates for specific percentiles of the frequency and severity distribution. The approach typically involves elicitation of mean frequency (MF), most likely severity (MS) and worst-case severity (WS) estimates from subject matter experts in following manner:

Frequency

No.	Target Statistics/Quantile	Question for the experts	Illustrative answer
1	MF	What is the expected average number of loss events in a year?	10

Individual loss severity

No.	Target Statistics/Quantile	Question for the experts	Illustrative answer (Million)
2	MS	What is the most likely impact of this scenario?	5
3	WS	One of following approaches: • Time-based elicitation: What would you judge to be the impact of the single largest event over the next 't' years? • Count-based elicitation: What would you judge to be the impact of the single largest loss out of 'x' such loss events?	 50 (1 in 10 years scenario) 50 (1 worst out of 100 losses)

There are multiple ways of eliciting worst-case severity. Count-based method allows elicitation of a pre-determined quantile of the individual loss severity distribution. For instance, single largest loss out of 100 losses relates to 99^{th} quantile of individual loss severity distribution. Time-based elicitation of tail quantile interlinks severity and frequency distribution. Probability associated with worst-case severity (referred as 'WS probability' in the paper) in such cases is dependent on the mean frequency as well as frequency associated with worst-case loss. With 'Fx' as the distribution function for severity distribution and Poisson(λ) as frequency of a loss greater than zero, frequency of losses above a threshold 'u' can be calculated as:

$$\lambda_u = \lambda * \left(1 - F_x(u)\right)$$

Therefore, probability that a loss less than or equal to worst-case loss would occur is:

$$F_x(u) = \left(1 - \frac{\lambda_u}{\lambda}\right)$$

$$\text{ie, } WS\ Probability = \left(1 - \frac{WS\ Frequency}{\lambda}\right)$$

with λ_u=1/t where 't' is the horizon for worst-case scenario.

For instance, for 1 in 10 years scenario, annual frequency of worst-case loss would be 0.10. This is referred to as the 'WS frequency' in rest of the paper and it implies the frequency with which a loss greater than or equal to worst-case loss would occur during the horizon. With average frequency of 10 events per annum, 1 in 10 years worst-case event represents 99^{th} quantile of the severity distribution. As can be observed, an increase in average frequency would increase the WS probability and vice-versa.

3.1. Distribution fitting to scenario inputs

Quantile matching is a logical method for fitting continuous distributions to severity data collected using quantile approach. The method involves parameter estimation by minimizing the squared difference between empirical quantiles (as elicited from experts) and theoretical quantiles (defined by the inverse cumulative distribution function of the selected distribution). The objective is to ensure that the fitted distribution has the same quantiles as the expert opinion, by minimizing the following objective function for two or more quantiles:

$$(Param\_1 \ldots Param\_N) = arg\ min \sum_{i=1}^{N} \left(q_i - F^{-1}(P_i)\right)^2$$

For purpose of our analysis, we assume that most likely severity (MS) is interpreted as median of the individual loss severity distribution. We consider only sub-exponential distributions from shape-scale family for sensitivity analysis, as these are commonly used in operational risk modelling and are suggested in AMA guidelines. Sub-exponential distributions are those distributions with slower tail decay than exponential distribution. This class includes weibull distribution (shape<1), lognormal distribution, pareto, burr distribution etc. We have not considered gamma distribution and weibull distribution (shape>1) as these are thin-tailed distributions. We have also not considered cases where pareto shape parameter declines below 1, as infinite-mean distribution would result in unrealistic capital figures.

Parameter estimates are arrived at as follows:

Distribution	CDF	Scale Parameter	Shape Parameter
Lognormal	$N\left(\frac{\ln(x)-\mu}{\sigma}\right)$ σ>0	$\mu = \ln(MS)$	$\sigma = \frac{\ln(WS) - \ln(\text{MS})}{Z1}$ with Z1 = $N^{-1}(WS\ probability)$
Weibull	$1 - exp^{-(x/\theta)^k}$ $\theta > 0, k > 0$	$\theta = \frac{MS}{\ln(2)^{1/k}}$	$k = \frac{ln(-ln(1 - WS\ Prob)/ln(2))}{\ln(\text{WS/MS})}$
Pareto	$1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha}$ $\theta > 0, \alpha > 0$	Numerical methods	

MS is the Median Severity and WS is the worst-case severity elicited from the experts. (1-WS Probability) is the probability that a loss higher than worst-case loss would occur. N(x) is the standard normal distribution function and $N^{-1}(p)$ is the quantile function for standard normal distribution.

Closed-form solution for lognormal and weibull are arrived on basis of quantile function evaluated at cumulative probability equal to 50% and WS probability. Parameters for pareto distribution are estimated using numerical methods.

4. OpVaR computation

For sub-exponential distributions, VaR maybe approximated using following closed-form solution, known as single-loss approximation (Böcker & Klüppelberg, 2005). The expression shows that VaR based on convolution of frequency and severity distribution can be approximated by computing a higher quantile of the individual loss severity distribution.

$$OpVaR_{ci} = F^{-1}\left(1 - \frac{1-ci}{\lambda}\right)$$

where F^{-1} is the quantile function of individual loss severity distribution, ci is the confidence level, λ is the mean frequency ie, average number of events in a time period. With mean correction (Böcker & Sprittulla, 2008), the refined approximation is:

$$OpVaR_{ci} = F^{-1}\left(1 - \frac{1-ci}{\lambda}\right) + (\lambda - 1) * E(X)$$

Degen (2010) shows that for heavy-tailed, finite mean severity distributions, the approximation can be further improved to:

$$OpVaR_{ci} = F^{-1}\left(1 - \frac{1-ci}{\lambda}\right) + \lambda * E(X)$$

where E(X) is the mean of the individual loss severity distribution. The first term represents the Unexpected Loss (UL) and the second term represents the Expected Loss (EL) of aggregate loss distribution. In the rest of the paper, we refer to VaR as the Unexpected Loss component only ie, we examine sensitivity of UL component only.

VaR approximation would be as follows at 99.9% confidence level for our candidate distributions:

Distribution	Quantile Function	99.9% VaR
Lognormal	$x = exp^{\mu+\sigma*N^{-1}(p)}$	$VaR = exp^{\mu+\sigma*N^{-1}(1-1/(1000*\lambda))}$
Weibull	$x = -\theta * \ln{(1-p)}^{\frac{1}{k}}$	$VaR = \theta * \ln{(1000*\lambda)}^{\frac{1}{k}}$
Pareto	$x = \theta * \left[\left(\frac{1}{1-p}\right)^{\left(\frac{1}{\alpha}\right)} - 1\right]$	$x = \theta * \left[(1000*\lambda)^{\left(\frac{1}{\alpha}\right)} - 1\right]$

For lognormal distribution, substituting estimates of shape and scale on basis of scenario output, we get:

$$99.9\%\ OpVaR = MS * exp^{\frac{\ln(WS)-\ln(\mathrm{MS})}{Z1} * Z2}$$

with

$$Z2 = N^{-1}\left(1 - \frac{1}{1000 * \lambda}\right)$$

5. Wrong-way Sensitivity of OpVaR to Median Severity

Parameter estimates for lognormal distribution show that with a decrease in MS, scale parameter would decline (reducing VaR) and the shape parameter would increase (increasing VaR). With opposing impact on scale and shape parameter, it is difficult to predict if OpVaR would increase or decrease with change in median severity. In the following section, we show that for probable scenarios, OpVaR would always increase due to decline in median severity and vice-versa.

Differentiating lognormal VaR with respect to MS, we get:

$$\frac{\partial OpVaR}{\partial MS} = (1 - c) * exp^{\sigma * Z2}$$

and

$$\frac{\partial^2 OpVaR}{\partial MS^2} = \frac{-c}{MS} * (1 - c) * exp^{\sigma * Z2}$$

with

$c = \frac{Z2}{Z1}$ or in expanded form

$$c = \frac{N^{-1}\left(1 - \frac{1}{1000 * \lambda}\right)}{N^{-1}\left(1 - \frac{WS\ Frequency}{\lambda}\right)}$$

It is unlikely that banks would focus on worst-case severities with associated frequency of less than 0.001 ie, if worst-case loss occurs less frequently than 1 in 1000 years. It maybe observed that c would be greater than 1 if WS frequency is greater than 0.001. First derivative of OpVaR with respect to MS is negative for cases where c>1, indicating that OpVaR would increase with decrease in MS. Second derivative of VaR with respect to MS is positive with c>1. With worst-case annual frequency exactly equal to 0.001, VaR would be equal to WS and shows no sensitivity to change in MS.

It is concluded that VaR would increase with decline in MS as increase in lognormal shape parameter would more than offset the benefit due to reduction in scale parameter.

For ‘a%’ change in MS, percentage change in VaR for lognormal distribution would be:

$$\frac{VaR_1}{VaR_0} - 1 = \frac{MS(1 + a\%) * exp^{\sigma_1 Z2}}{MS * exp^{\sigma_0 Z2}} - 1$$

which results in:

$$\frac{VaR_1}{VaR_0} - 1 = (1 + a\%)^{1-c} - 1$$

For decrease in MS, VaR change would be positive and vice-versa.

For weibull distribution, percentage change in VaR would be:

$$\frac{VaR_1}{VaR_0} - 1 = \frac{(1 + a\%) * \ln(1000 * MF)^{-c1*\ln(1+a\%)}}{\ln(2)^{-c1*\ln(1+a\%)}} - 1$$

with

$$c1 = \frac{1}{ln(-ln(1 - WS\ Prob)/ln(2))}$$

This shows that percentage change in VaR depends on percentage change in MS, value of MF and WS probability, rather than on absolute level of MS, WS or WS/MS ratio. VaR sensitivity to MS reduces with increase in WS probability ie, sensitivity is lower for scenarios with low frequency associated with worst-case loss.

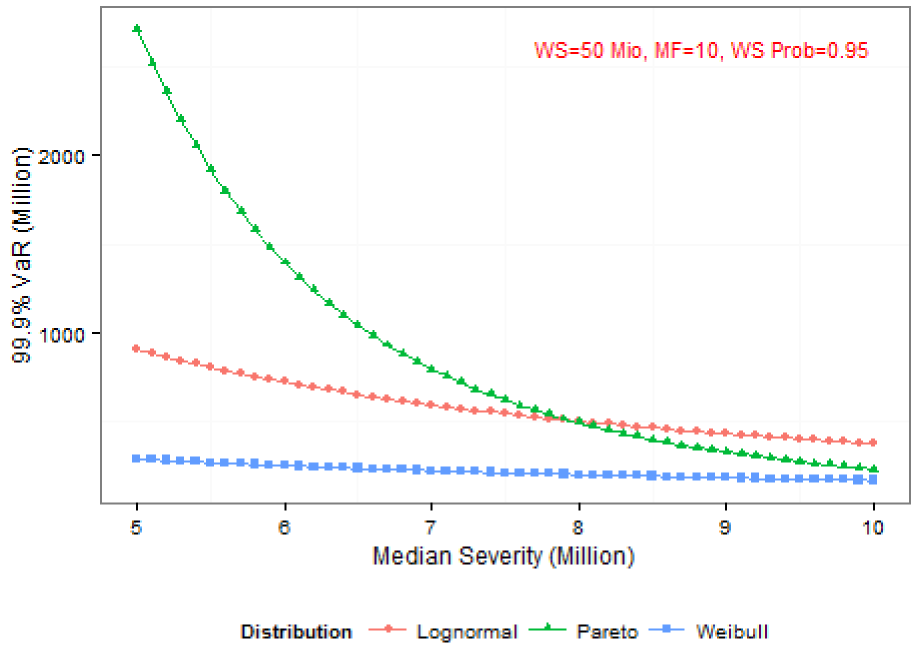
5.1. Numerical results

We use a test scenario, with Median Severity of 5 million and WS of 50 Million as basecase in Year-1. Due to an improvement in risk-profile of the Bank and improvement in control environment, the Bank revises Median severity estimate from 5 million in Year-1 to 4 million in Year-2. However, this leads to an increase in AMA capital. Conversely, an increase in Median severity due to deterioration in risk profile would lead to a capital saving. The illustration assumes that other scenario inputs are not changed.

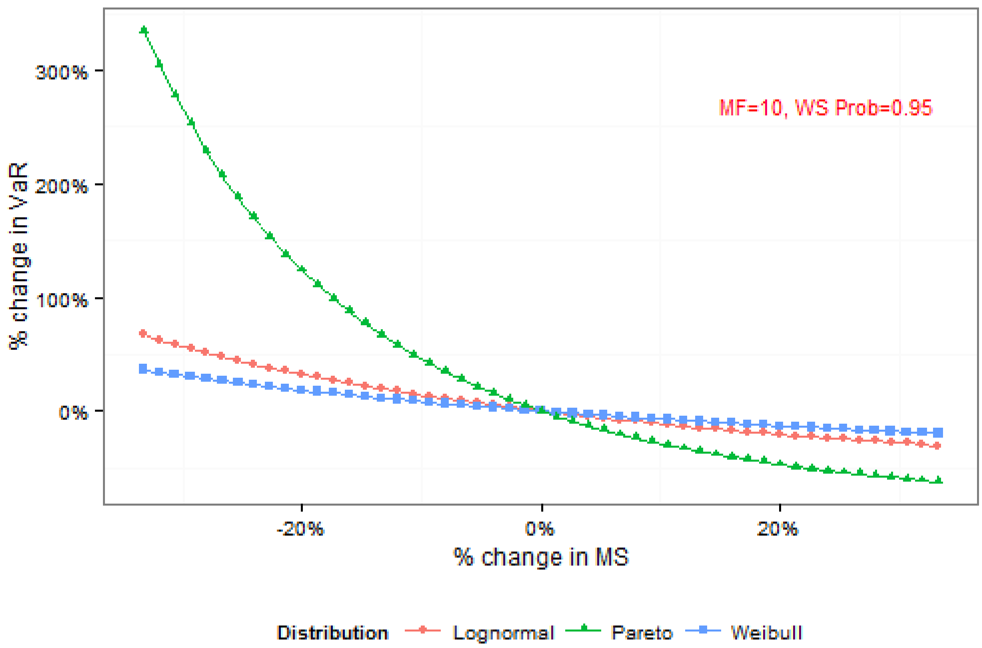
Mean Frequency	Median Severity (Million)	Worst-case Severity (Million)	WC-probability	OpVaR (Million)	OpVaR: % change from basecase
10	1	50	0.95	6940	661%
10	2	50	0.95	2896	218%
10	3	50	0.95	1737	90%
10	4	50	0.95	1208	32%
10	5	50	0.95	912	-
10	6	50	0.95	725	-21%
10	7	50	0.95	597	-35%
10	8	50	0.95	504	-45%
10	9	50	0.95	435	-52%
10	10	50	0.95	381	-58%

5.2. VaR sensitivity to MS for different distributions

The following chart shows VaR sensitivity to MS for three sub-exponential distributions for the above illustration:



The chart below shows percentage change in VaR due to change in MS.



5.3. VaR sensitivity to lower quantile

This is a generalisation of the previous results where the lower quantile is taken as the median. Scenario maybe designed in a manner such that both the quantiles are in the tail region of severity distribution or the first quantile is in the mid region and second quantile is in the tail region. VaR sensitivity to the lower quantile is still wrong-way as shown below:

$$\frac{VaR_1}{VaR_0} - 1 = (1 + a\%)^{1-c'} - 1$$

with

$$c' = \frac{Z2 - Z0}{Z1 - Z0}$$

Where Z0 is the standard normal quantile at probability associated with lower quantile. With c'>1, VaR would increase due to decline in MS and vice-versa, which would be the case if frequency associated with worst-case scenario is greater than 0.001.

Following are the practical implications of the above results:

- While use of SA in AMA capital models might be preferred for reasons of conservatism and forward-looking estimates, high importance to SA as direct input in AMA model may lead to undesired consequence in form of volatile capital estimate. For instance, a change in median severity by 1 million (from 2 million to 1 million) pushes up capital requirement by 4.04 billion.
- Due attention should also be given to median severity, rather than focussing solely on the worst-case severity estimates. High median severity estimates than justifiable on basis of empirical data (internal and/or external) should be properly supported. This should be done to prevent 'gaming' of the scenario output.
- It may not be obvious to experts that for a given worst-case severity, lower median severity would translate into higher capital and vice-versa.
- Banks should keep in mind that improvement/deterioration in risk profile may not have a logically consistent impact on OpVaR. This is also true for models where expert opinion is elicited for only for worst-case severity and median severity is derived from ILD. A decline is ILD median would increase OpVaR and vice-versa.
- Choice of distribution for SA has non-trivial implications on VaR and VaR sensitivity. VaR is lowest for weibull, followed by lognormal and then pareto distribution. The only exception in the above illustration is when MS exceeds 8 million where pareto VaR declines below lognormal VaR. It is observed that VaR sensitivity is lowest for weibull (thinnest tail), followed by lognormal and pareto distribution. For each distribution, sensitivity increases as distribution tail becomes thicker ie, with increase in shape parameter for lognormal and decline in shape parameter for weibull and pareto.

6. Right-way Sensitivity of OpVaR to Worst-case Severity

Differentiating lognormal VaR with respect to WS, we get:

$$\frac{\partial OpVaR}{\partial WS} = \frac{c * MS}{WS} * exp^{\sigma * Z2}$$

$$\frac{\partial^2 OpVaR}{\partial WS^2} = \frac{c * MS}{WS^2} * (c - 1) * exp^{\sigma * Z2}$$

Both first and second derivatives are positive for scenarios with worst-case frequency greater than 0.001. This shows that OpVaR changes in same direction as worst-case severity.

For 'b%' change in WS, percentage change in VaR for lognormal distribution would be:

$$\frac{VaR_1}{VaR_0} - 1 = \frac{exp^{\sigma_1 Z2}}{exp^{\sigma_0 Z2}} - 1$$

which results in:

$$\frac{VaR_1}{VaR_0} - 1 = (1 + b\%)^c - 1$$

For weibull distribution, percentage change in VaR would be:

$$\frac{VaR_1}{VaR_0} - 1 = \frac{\ln{(1000 * MF)}^{c1*\ln{(1+b\%)}}}{\ln{(2)}^{c1*\ln{(1+b\%)}}} - 1$$

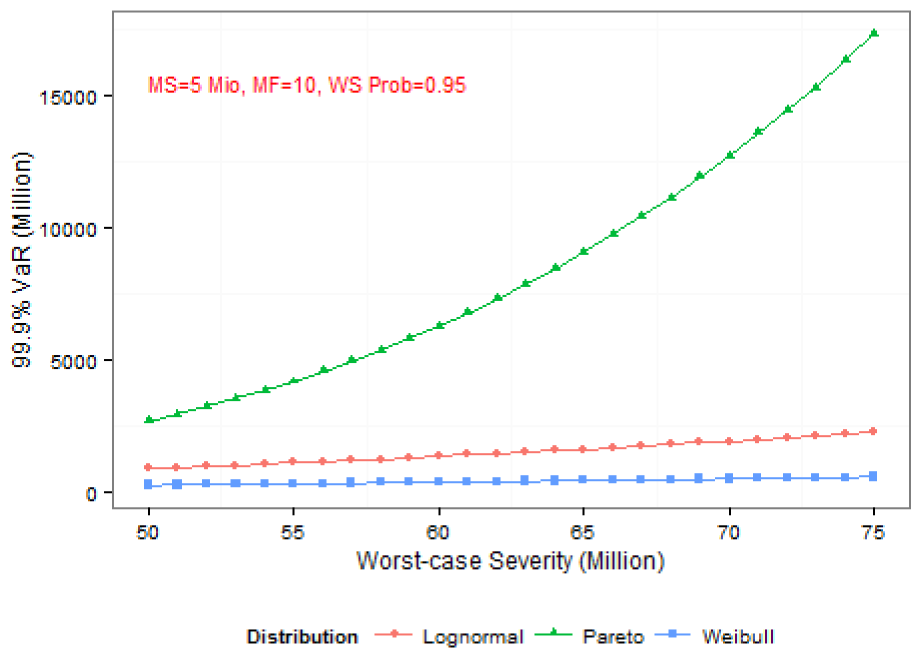
with

$$c1 = \frac{1}{ln(-ln(1 - WS\ Prob)/ln(2))}$$

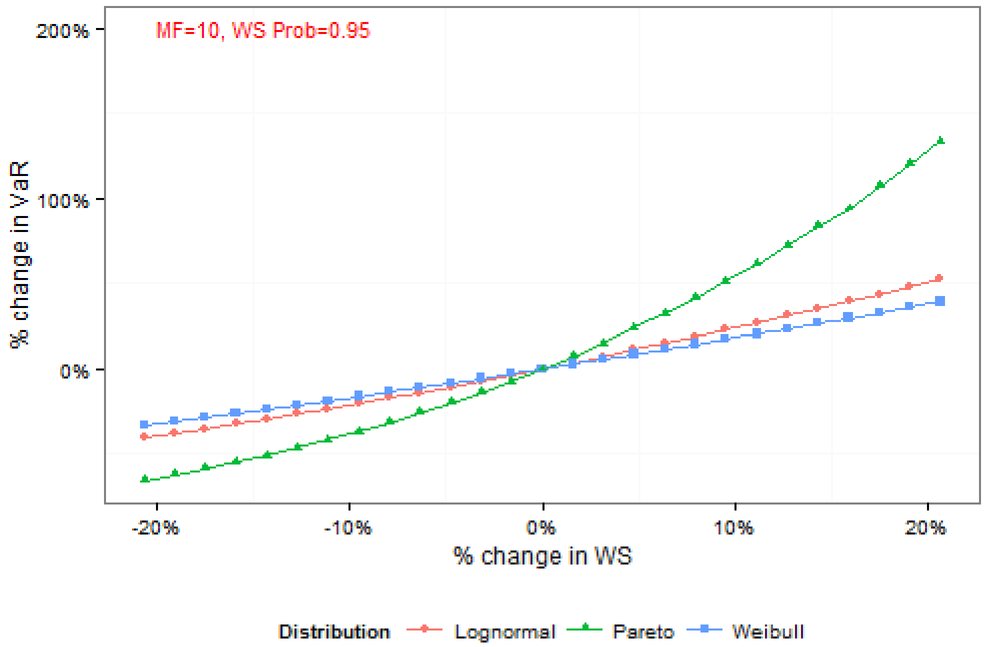
This shows that percentage change in VaR depends on percentage change in WS, value of MF and WS probability, rather than on absolute level of MS, WS or WS/MS ratio. Further, absolute percentage change in VaR would be greater than absolute percentage change in WS, if c>1.

6.1. VaR sensitivity to WS for different distributions

VaR changes in same direction as WS for lognormal, weibull and pareto distribution. VaR and VaR sensitivity is lowest for weibull, followed by lognormal and then pareto distribution. For each distribution, VaR sensitivity increases as distribution tail becomes thicker.



The chart below shows percentage change in VaR at different percentage change in WS.



6.2. VaR sensitivity to upper quantile

Following would be the VaR sensitivity to 'b%' change in upper quantile, where lower quantile may not be the median:

$$\frac{VaR_1}{VaR_0} - 1 = (1 + b\%)^{c'} - 1$$

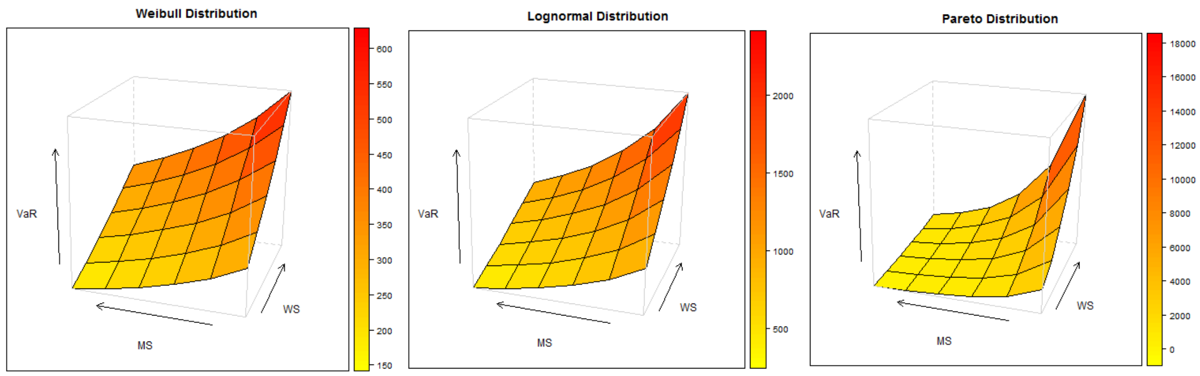
with

$$c' = \frac{Z2 - Z0}{Z1 - Z0}$$

Where Z0 is the standard normal quantile at probability associated with lower quantile. With c'>1, VaR would increase due to increase in WS and vice-versa, which would be the case if frequency associated with worst-case scenario is greater than 0.001.

7. VaR sensitivity to MS and WS

The following chart show impact of simultaneous change in WS and MS on VaR for three sub-exponential distributions.



For lognormal distribution, it maybe shown that for 'a%' change in MS and 'b'% change in WS, percentage change in VaR would be:

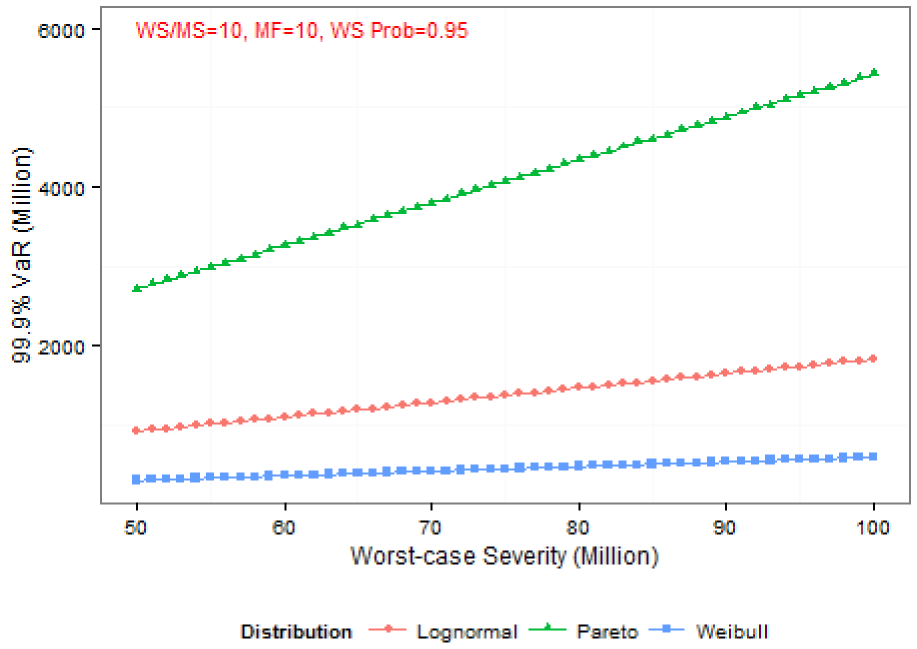
$$\frac{VaR_1}{VaR_0} - 1 = (1 + a\%)^{1-c} * (1 + b\%)^c - 1$$

This shows that for same percentage change in MS and WS (ie, a%=b%), change in VaR would also be a%. ie, if WS and MS are changed by a scalar such that WS/MS ratio is held constant, then VaR changes by the same scalar. At constant WS/MS ratio, shape parameter remains constant. Therefore, VaR changes linearly with change in scale parameter. This can be shown for lognormal distribution where MS and WS are scaled by a scalar 's', resulting in new VaR which is 's' times that of existing VaR:

$$\frac{VaR_{new}}{VaR_{old}} = \frac{s * MS * exp^{\frac{\ln(s*WS) - \ln(s*\mathrm{MS})}{Z0} * Z2}}{MS * exp^{\frac{\ln(WS) - \ln(\mathrm{MS})}{Z0} * Z2}}$$

$$\frac{VaR_{new}}{VaR_{old}} = s$$

Following chart shows VaR at various WS and MS levels such that WS/MS ratio is 10.



8. Wrong-way sensitivity of OpVaR to Mean Frequency for certain scenarios

For scenarios with time-based elicitation of worst-case severity, VaR is dependent on MF in two opposing ways:

- MF has a direct relationship with OpVaR as VaR is the 'MF'-fold convolution of the individual loss severity distribution.
- For time-based elicitation, probability associated with worst-case severity (WC probability) has direct relationship with mean frequency. Therefore, a decline (increase) in mean frequency would result in a decline in WC probability and an increase (decline) in OpVaR. This sensitivity is not applicable for count-based scenario elicitation, as the worst-case severity is a pre-determined percentile of the individual loss severity distribution that does not change due to change in MF.

Indirect MF impact on VaR through 'WC probability' would offset the direct impact of MF on VaR, leading to wrong-way VaR sensitivity to MF. For lognormal distribution, percentage change in VaR due to change in MF would be:

$$\frac{VaR_1}{VaR_0} - 1 = \left(\frac{WS}{MS}\right)^{c_{new} - c} - 1$$

where

$$c\_new = \frac{N^{-1}\left(1 - \frac{1}{1000 * \lambda\_new}\right)}{N^{-1}\left(1 - \frac{WS\ Frequency}{\lambda\_new}\right)}$$

VaR sensitivity to MF would remain same for a constant (WS/MS) ratio. Sensitivity to MF increases with an increase in WS/MS ratio.

Illustrative results are shown below, with MF=10 considered as basecase:

Mean Frequency	Median Severity (Million)	Worst-case Severity with WCF =0.5 p.a. (Million)	WC probability (1-WCF/MF)	OpVaR (Million)	OpvaR: % change from basecase
5	5	50	0.9	2893	217%
10	5	50	0.95	912	0%
15	5	50	0.967	605	-34%
20	5	50	0.975	483	-47%
25	5	50	0.98	416	-54%

Following are the practical implications of the above results:

- Probability associated with a worst-case loss may change due to changes in MF, even when the horizon associated with worst-case loss does not change. For instance, a 1 in 25 years event would be 99^{th} quantile of severity distribution if MF is 4 and would be 96^{th} quantile of severity distribution if MF is 1. Models that link scenario MF with ILD/ELD mean frequency may exhibit logically inconsistent results over time ie, VaR may increase with decline in empirical MF and vice-versa.
- Linkage between MF and WS probability may not be clear to the experts during scenario exercise. For rare events with low average arrival rate, experts' opinion about WS may not relate to a high quantile of severity distribution even though experts might perceive otherwise. For instance, a '1 in 25 years' worst-case loss of 100 million would be just the 80^{th} quantile of severity distribution if MF is '1 in 5 years'. This may lead to unrealistic capital figures for certain rare event scenarios. Conversely, for certain ORCs a '1 in 2 years' worst-case loss maybe sufficiently in the tail if MF is very high. Therefore, for time-based elicitation it is critical that time horizon is carefully selected to ensure that worst-case loss is sufficiently in the tail region.

9. Conclusion

The objective of the paper was to highlight scenario model sensitivities for the benefit of practitioners. We have shown that AMA models using scenario analysis as a direct input may exhibit significant volatility due to changes in scenario inputs. Many of the model sensitivities may not be obvious to the experts during scenario elicitation exercise. For instance, experts might believe that lower median severity would result in lower capital, for same worst-case severity. Similarly, impact of a decline in mean frequency on capital due to decline in probability associated with worst-case loss may not be obvious in time-based elicitation. Another important result is that capital sensitivity increases with choice of a conservative risk-curve for fitting to scenario data.

Further work needs to be done to examine VaR sensitivity where EL term is included in the VaR approximation and where more than two severity quantiles are elicited. Future studies are also needed to examine overall capital sensitivity to scenario inputs, for models that combines scenario and loss data using various approaches such as body-tail splice or Bayesian methods.

References

Basel Committee on Banking Supervision. (2009a). Observed range of practice in key elements of advanced measurement approaches (AMA), 61-64

Basel Committee on Banking Supervision. (2009b). Results from the 2008 loss data collection exercise for operational risk, Annex E, 17

Böcker, K. & Klüppelberg, C. (2005). Operational VAR: a closed-form approximation, *Risk Magazine*, 90-93

Böcker, K. & Sprittulla, J. (2008). Operational VAR: meaningful means, *Risk Magazine*, 96-98

Chaudhury, M. (2010), A review of the key Issues in operational risk capital modelling, 14-19

Colombo, A. & Desando, S. (2008). Developing and implementing scenario analysis models to measure operational risk at Intesa Sanpaolo, Retrieved from http://in.mathworks.com/company/newsletters/articles/developing-and-implementing-scenario-analysis-models-to-measure-operational-risk-at-intesa-sanpaolo.html

Opdyke, J. & Cavallo, A. (2012), Estimating operational risk capital: The challenges of truncation, the hazards of MLE, and the promise of robust statistics, 30

Risk Management Association (2012), Scenario analysis practices

Rosengren, E. (2006). Operational risk scenario analysis workshop: Scenario analysis and the AMA [Powerpoint slides]. Retrieved from https://www.boj.or.jp/en/announcements/release_2006/data/fsc0608be9.pdf

Shevchenko, P.V. & Wüthrich, M.V. (2006). The structural modelling of operational risk via Bayesian inference: combining loss data with expert opinions, *The Journal of Operational Risk 1*(3), 3-26