Problems of utility and prospect theories. Certainty effect near certainty

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Certainty Effect near Certainty

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A need for experiments on the certainty effect near the certainty (near the probability $p = 1$) is stated in this paper. The need supported by the Aczél–Luce question whether Prelec's weighting function $W(p)$ is equal to 1 at $p = 1$, by the purely mathematical restrictions and the "certain–uncertain" inconsistency of the random–lottery incentive experiments. The results of the experiments of the certainty effect near the certainty show that Prelec's (probability) weighting function can be discontinuous at the probability $p = 1$. There is a need for new experiments at probabilities which are closer to $p=1$, e.g., at probabilities $p=.99$ and $p=.999$.

Introduction

The purpose of the present paper is to investigate a so-called Certainty Effect. The paper sketches very roughly a future article. It will be included into a group of three articles. Supposed titles of the articles are: "A possible discontinuity of Prelec's function," "Certainty Effect near Certainty" and "A "certain-uncertain" inconsistency of the random-lottery incentive system."

There are a number of theories concerned with one or another concept of utility. They include, e.g., Bernoullian expected utility, von Neumann–Morgenstern expected utility, subjective expected utility, subjectively weighted utility theories (see, e.g., a review by Schoemaker, 1982); prospect theory (see Kahneman and Tversky, 1979) and cumulative prospect theory (see Tversky and Kahneman, 1992) or, in other terminology, original prospect theory and prospect theory; the salience theory of choice under risk (see Bordalo, Gennaioli, Shleifer, 2012); expected uncertain utility theory (Gul and Pesendorfer, 2014); etc.

In the present paper these theories are referred to as utility and prospect theories.

Ever since Bernoulli (1738), the problems with the theory of utility exposed by the Saint Petersburg paradox have been investigated. Von Neumann and Morgenstern (1947) promised the feasibility of a correct and, naturally, rational foundation of economic theory with their book, *Theory of Games and Economic Behavior*. But these promises were dashed by the Allais paradox (Allais, 1953).

Kahneman and Thaler (2006) pointed out that the basic problems of utility and prospect theories, including the paradoxes of Allais (Allais, 1953) and Ellsberg (Ellsberg, 1961), have not yet been adequately solved.

1. Clarification

Kahneman and Tversky (1979) stated at page 263: "people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses."

A second type of the certainty effect should be mentioned (see, e.g., Halevy, 2008): people prefer the certainty of the present to the uncertainty of the future.

In this paper, we shall consider only the first type of the certainty effect: people prefer the certainty (the probability which is equal to 1 or guaranteed outcomes) to the uncertainty (the probability which is less than 1) at the same averages of distribution.

2. Reasons

2.1. The Aczél–Luce question

An essential part of problems of utility and prospect theories consists in the problems that are connected with a probability weighting (see, e.g., Tversky and Wakker, 1995). A probability weighting means that subjects treat the probability p by a probability weighting function $W(p)$ which is not equal to p (see also uncertainty perception in (Gul and Pesendorfer, 2014) as an example of perception variety). I define the function $W(p)$ both for uncertain (probable) and certain outcomes. Prelec's weighting function (Prelec, 1998) is one of the most popular probability weighting functions.

One possible way to solve the above problems is to consider the vicinities of the borders of the probability scale, e.g. at $p \sim 1$ (see, e.g., Aczél and Luce, 2007).

Aczél and Luce (2007) emphasized a fundamental question: whether $W(1) = 1$ (whether Prelec's weighting function $W(p)$ (see Prelec, 1998) is equal to 1 at $p = 1$). In this article, I refer to this question as the Aczél–Luce question (or Luce question).

There is a deal of evidence for the existence of a qualitative difference between subjects' treatment of the probabilities of uncertain (probable) and certain outcomes (see, e.g., Kahneman and Tversky, 1979; McCord and de Neufville, 1986; Gneezy, List and Wu, 2006; Halevy, 2008). Therefore, in the general case, one should distinguish between the values of the probability weighting function $W(p)$ of a certain outcome and the limit of the probability weighting function $W(p)$ of uncertain outcomes as the probability of uncertain outcomes tends to 1.

Let us additionally define or specify a value $W_{Certain}$ of the probability weighting function $W(p)$ for a certain outcome. At that, $W_{Certain}$ may be assumed to be equal to 1. Otherwise, other values of $W(p)$ may be normalized by $W_{Certain}$.

Let us here additionally specify a value $W(1)$ as the limit of the probability weighting function $W(p)$ for a probable (uncertain) outcome as p tends to 1

$$W(1) \equiv \lim_{p \to 1} W(p) \ .$$

If $W(1) = W_{Certain}$, then $W(p)$ is continuous (at $p = 1$). This is usually assumed by default. But this has not been proven for the general case. So, if $W_{Impossible}$ is defined for the impossible case, then, similar to Aczél and Luce (2007),

$$W(p) = \begin{cases} W_{\mathrm{Im}\,possible} & p = 0 \\ W(p) & p \in]0,1[\\ W_{Certain} & p = 1 \end{cases}$$

and $W(p)$ can be continuous or discontinuous.

One may modify the Aczél–Luce question whether $W(1) = 1$ into the question whether $W(1) = W_{Certain}$ or whether $W(p)$ is continuous at $p = 1$. Note, various aspects of continuity have been discussed in the literature. See, e.g., Aczél and Luce (2007), Kothiyal, Spinu and Wakker (2011), Delbaen, Drapeau and Kupper (2011), Spinu and Wakker (2013).

To answer the modified question and to prove or disprove the continuity of $W(p)$ at $p=1$ one should determine and measure the difference

$$W_{Certain} - W(1) = ?$$

The answer $W(1) \neq W_{Certain}$ to the modified Aczél–Luce question means that the function $W(p)$ has a discontinuity at $p=1$. This is not a quantitative but a qualitative, moreover, a topological feature. So, the answer to the question can qualitatively change the situation in utility and prospect theories, at least in their mathematical aspects.

2.2. Purely mathematical restrictions

Another possible way to solve the problems of utility and prospect theories has been widely discussed, e.g., in Schoemaker and Hershey (1992); Hey and Orme (1994); Chay, McEwan and Urquiola (2005); Butler and Loomes (2007); Galaabaatar and Karni (2013); Chambers and Hayashi (2014). Its essence consists in a proper attention to noise, uncertainty, imprecision, incompleteness, unforeseen contingencies and other reasons that might cause dispersion, scattering, a spread of the data.

A purely mathematical investigation (see, e.g., Harin, 2010, 2012) has synthesized these two different ways. That is, it considers the dispersion of the data (or the influence of the dispersion of the data) near the borders of the probability scale.

Purely mathematical theorems (see, e.g., Harin, 2010, 2012) prove that the probability p cannot attain 1 under the condition of a non-zero dispersion of the data.

This signifies that under the condition of a non-zero dispersion of the data, the probability p cannot take on the value 1, i.e., $p < 1$. It cannot even be arbitrarily close.

As a matter of fact, a non-zero dispersion of data can be caused, e.g., by non-zero noise, which is practically unavoidable in economics.

One may additionally suppose here, that if the probability weighting function $W(p) \leq p$ at $p > 3/4$ and $W_{Certain} = 1$, then this opens up a possibility of the existence of a discontinuity of $W(p)$ at $p = 1$. So, one may say the theorems predict the possibility of the existence of a discontinuity of the probability weighting function $W(p)$ at the probability $p = 1$, under the condition of a non-zero dispersion of the data.

2.3. The "certain–uncertain" inconsistency of the random–lottery incentive experiments

The discontinuity of the probability weighting function $W(p)$ at the probability $p = 1$ is not evidently supported by experiments. But the prevalent experimental procedure in utility and prospect theories is the random-lottery incentive system (see, e.g., Starmer, 2000 and Baltussen et al., 2012; etc.).

Moreover, in the random-lottery incentive system, the choices of certain (sure) outcomes are stimulated by uncertain lotteries (see Harin 2014). Because of this "certain–uncertain" inconsistency, the deductions from a random-lottery incentive experiment that includes a certain outcome cannot be unquestionably correct, especially at $p \sim 1$.

The well-known experiment of Starmer and Sugden (1991) evidently supports this inconsistency.

3. Results

So, investigations of the certainty effect near the certainty (near the probability $p = 1$) may be useful and crucial for utility and prospect theories.

One can see below, that experiments at probabilities $p \sim 1$ support the possibility of the existence of a discontinuity of the probability weighting function $W(p)$ at the probability $p = 1$.

3.1. The thought experiment of Halevy

Halevy (2008) considered the thought experiment:

“The two temporal choice problems translate into a choice between prospects:

Problem 19. ($100, 1) or ($110, 0.96);

Problem 29. ($100, 0.77) or ($110, 0.74),

where prospect (*$x, p*) represents a lottery that pays *$x* with probability *p* and *$0* with probability *1-p*. It has been well documented in experiments (Kahneman and Tversky 1979) that although expected utility theory predicts making the same choice ($100 or $110) in both, many subjects exhibit the certainty effect: they overweigh certain outcomes relative to very likely but not completely certain outcomes. As a result, they prefer $100 in Problem 19 and $110 in Problem 29.”

3.2. The experiment of Starmer and Sugden

The well-known experiment of Starmer and Sugden (1991):

“Page 974: “For groups A and D, this page began with an underlined text stating that question 22 would be played for real. For groups B and C, the corresponding text stated that one of the two questions would be played for real and that which question was to played out would be decided at the end of the experiment in the following way. The subject would roll a six-sided die. If the number on the die was 1, 2, or 3, then question 21 would be played; if the number was 4, 5, or 6, question 22 would be played.”

“One problem, which we shall call P', required a choice between two lotteries R' (for "riskier") and S' (for "safer"). R' gave a 0.2 chance of winning £10.00 and a 0.75 chance of winning £7.00 (with the residual 0.05 chance of winning nothing); S' gave £7.00 for sure.”

So, in the R'-S' problem, R' gives *£10.00*0.2+£7.00*0.75 = £7.25*. S' gives *£7.00*1 = £7.00*. Here *R' = £7.25>S' = £7.00*.

Let us consider the results from table 2 on Page 976, those are of interest here (the **boldface** is my own):

- Group = B, Incentive = Random lottery, R':S' = 19:21
- Group = C, Incentive = Random lottery, R':S' = 22:18
- Group = D, Incentive = **P' real**, R':S' = **13:27**”

One can see that the experiment supports the possibility of the existence of a discontinuity of $W(p)$ at the probability $p = 1$.

4. Need for new experiments

Unfortunately, the vast majority of experiments are performed at probabilities which are below $p=.9$.

There is a need for new experiments at probabilities $p\sim 1$. The experiments may be, e.g., analogous of that of Starmer and Sugden (1991). The probabilities under investigation in such experiments should be moved forward to $p=1$, namely to, e.g., $p=.99$ or $p=.999$.

Conclusions

Experiments on the certainty effect play an essential part in utility and prospect theories. A need for experiments on the certainty effect near the certainty (near the probability $p = 1$) is supported by the Aczél–Luce question whether Prelec's weighting function $W(p)$ is equal to 1 at $p = 1$, by the purely mathematical restrictions and the "certain–uncertain" inconsistency of the random–lottery incentive experiments.

The results of the experiments of the certainty effect near the certainty show that Prelec's (probability) weighting function can be discontinuous at the probability $p = 1$.

Due to these reasons, there is a need for new experiments at probabilities $p\sim 1$. The experiments may be, e.g., analogous of that of Starmer and Sugden (1991). The probabilities under investigation in such experiments should be moved forward to $p=1$, namely to, e.g., $p=.99$ or $p=.999$.

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