Peculiar Results and Theoretical Inconsistency of New Keynesian Models

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Abstract. In this paper, several flaws of the basic no-capital/labor-only New Keynesian model, as in Galí (2008), are discussed. Some flaws were left undiscovered because mass of varieties n in Dixit-Stiglitz aggregator is often considered as not affecting overall outcomes. Only when $n = 1$ would ordinary results of the basic New Keynesian model hold. To save the theory, we consider the case where production function exhibits constant return to scale for its input labor, then concludes that linear production function itself leads to other sets of problems. The aforementioned results are proven by checking several limit cases of the basic New Keynesian model, which itself is the limit case model of several New Keynesian models. Then we show some problems with applying transversality condition to consumption Euler equation of the model.

1. Introduction

Since Real Business Cycle and Dynamic Stochastic General Equilibrium revolution, concerns of many economists have focused on amending Real Business Cycle models to exhibit short-run non-neutrality of money in data, as can be seen in Romer and Romer (1989). Out of these efforts came New Keynesian models, which incorporated both imperfect competition and sticky price, sometimes also with sticky wage, that demonstrate the power of monetary policy (Galí (2008)). Today, central banks around the world use New Keynesian models to aid monetary policy.

New Keynesian models themselves brought up theoretical concerns, such as identification problems and possible conflicts with Lucas Critique, as in Cochrane (2011) and Chari, Kehoe and McGrattan (2009). With recent financial crisis, moves towards better microfoundations, incorporating recent developments in microeconomics, that match with reality, such as financial frictions, as in Christiano, Motto, Rostagno (2014), have become important objective for New Keynesian economists.

We will demonstrate that there is a deeper theoretical issue that is not much mentioned in economics circles.

When a New Keynesian model uses Dixit-Stiglitz CES aggregator, as first expounded in Dixit and Stiglitz (1977), and reduces to the basic New

Keynesian model, as in Galí (2008), by taking limit of some parameters to constants, inconsistency in the basic New Keynesian model leads to inconsistency in the model. This is done by first taking limit of the basic New Keynesian model to classical monetary model, which does not end up being inconsistent only when mass of varieties $0 < n \in \mathbb{R}$ in Dixit-Stiglitz aggregator is 1. As n should not affect consistency when $0 < n$, we instead choose to sacrifice output elasticity in labor/technology-only Cobb-Douglas production function and make labor's output elasticity as 1, leading to constant return-to-scale linear production function. It is then shown that with the limiting case of zero natural rate of output and technology growth, inconsistency arises again.
It is also shown that if we impose zero-profit-for-firms condition (for every period) to classical monetary model, then output elasticity of Cobb-Douglas production function has to come out as 1, and if output elasticity is 1 then there is no profit for imperfectly-competitive firms every period in classical monetary model. As classical monetary model is the limiting case of the basic New Keynesian model, if zero-profit condition is imposed to the limiting case, then the conclusion that output elasticity of Cobb-Douglas production function is 1 must follow even for the basic New Keynesian model.
That capital is missing from the basic New Keynesian model cannot be said to be the sole reason for inconsistency - if limit of amount of capital K at single period t is taken to zero, for the model we consider below, K remains zero for every period along with investment I remaining zero also, reducing the model to the basic New Keynesian model. Introduction of firm-specific capital, as in Woodford (2003), does not help also, because then limit of every firm's firm-specific capital to zero can be taken, reducing the model again to the basic labor-only New Keynesian model.

2. Inconsistency of the Basic Labor-only New Keynesian Model: Dixit-Stiglitz Aggregation

We will follow Galí (2008)'s basic labor-only New Keynesian model.
The representative household solves the following optimization problem:

$$\max_{C_t, N_t, B_t} E_0 \sum_{k=0}^{\infty} \beta^t U(C_t, N_t)$$

$$U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

with budget constraint $\int_0^n P_t(i)C_t(i)\, di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$ and $\lim_{T\to\infty} E_t\{B_T\} \geq 0$, where $C_t(i)$ represents consumption of firm i's goods, N_t represents hours worked and B_t represents the quantity of one-period, nominal riskless discount bonds purchased, which pays one unit of money at maturity, Q_t is the price of bonds, P_t is price level, W_t is nominal wage, D_t is dividends, which equal firms' profits. (We assume that all profits of firms are not kept and given away as dividends) n represents mass of varieties factor in Dixit-Stiglitz CES aggregation. Usually, $n = 1$ is chosen, but this is not a fundamental condition required. We will see how $n = 1$ hides inherent problems in the model.

Firms in this model are monopolistically competitive and are modelled by Dixit-Stiglitz. Aggregate consumption level C_t is given by

$$C_t \equiv \left(\int_0^n C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The household also maximizes C_t given fixed expenditure level Z_t, which is given by the following Lagrangian:

$$\mathcal{L}_1 = \left[\int_0^n C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda_t\left(\int_0^n P_t(i)C_t(i)\, di - Z_t\right)$$

Regardless of n, the first-order condition is given by

$$C_t(i)^{\frac{-1}{\varepsilon}} C_t^{\frac{1}{\varepsilon}} = \lambda_t P_t(i)$$

Therefore, $C_t(i) = (\lambda_t P_t(i))^{-\varepsilon} C_t$. Recalling the expression of C_t above,

$$C_t = \left[\int_0^n \left((\lambda_t P_t(i))^{-\varepsilon} C_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_t = \lambda_t^{-\varepsilon} C_t \left(\int_0^n P_t(i)^{1-\varepsilon}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\lambda_t = \left(\int_0^n P_t(i)^{1-\varepsilon}\, di\right)^{\frac{1}{\varepsilon-1}} = \frac{1}{P_t}$$

Thus, regardless of what n is, the following holds:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

Now let us take limit $\epsilon \to \infty$ and price stickiness factor $\theta \to 0$. For now, the full specification of price stickiness is not needed as long as we recognize that we are taking the limit to frictionless perfect competition model. Therefore,

in this limit $P_t(i)$ and $C_t(i)$ will be the same for $\forall i$.
Some calculations bring us the result

$$\lim_{\varepsilon\to\infty,\theta\to 0} C_t = nC_t(i)$$

$$\lim_{\varepsilon\to\infty,\theta\to 0} \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} = \frac{1}{n}$$

taking $C_t(i)$ as constant for $\forall i$.
Production function for each firm i is:

$$Y_t(i) = C_t(i) = A_t N_t(i)^{1-\alpha}$$

Labor market clearing requires $N_t = \int_0^n N_t(i)\ di$ and

$$N_t = \int_0^n \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di$$

$$N_t = \int_0^n \left[\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t}{A_t}\right]^{\frac{1}{1-\alpha}} di$$

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^n \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} di$$

In $\theta \to 0$ limit, $P_t(i)$ can be thought as being constant for all firms, and therefore,

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} n \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}}$$

As long as $1-\alpha > 0$, the limit $\lim_{\varepsilon\to\infty,\theta\to 0}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}$ can be used, and therefore,

$$\lim_{\varepsilon\to\infty,\theta\to 0} N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} n\frac{1}{n} = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$$

This confirms our intuition that regardless of n, the basic New Keynesian model's production function collapses to classical monetary monetary model's production function when $\varepsilon \to \infty$ and $\theta \to 0$ which is given by $Y_t = A_t N_t^{1-\alpha}$.
Note that we used good markets clearing equation $Y_t(i) = C_t(i)\ \forall i$.
This suggests that we have two equal representations of Y_t for limit

$\lim_{\varepsilon\to\infty,\theta\to 0}$:

$$Y_t = A_t N_t^{1-\alpha} = A_t \left[\int_0^n N_t(i)\ di\right]^{1-\alpha}$$

$$Y_t = \left[\int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\ di\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\int_0^n \left(A_t N_t(i)^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}}\ di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

We did not yet take the limit for the second form of Y_t - we will do so. As $N_t(i)$ is constant in this limit, for the first form of Y_t,

$$Y_t = A_t n^{1-\alpha} N_t(i)^{1-\alpha}$$

For the second form of Y_t,

$$Y_t = \lim_{\varepsilon\to\infty,\theta\to 0} A_t \left[\int_0^n N_t(i)^{\frac{(1-\alpha)(\varepsilon-1)}{\varepsilon}}\ di\right]^{\frac{\varepsilon}{\varepsilon-1}} = \lim_{\varepsilon\to\infty,\theta\to 0} A_t n^{\frac{\varepsilon}{\varepsilon-1}} N_t(i)^{1-\alpha}$$

$$Y_t = A_t n N_t(i)^{1-\alpha}$$

Therefore, in this limit,

$$Y_t = A_t n^{1-\alpha} N_t(i)^{1-\alpha} = A_t n N_t(i)^{1-\alpha}$$

which suggests $n^{1-\alpha} = n$, unless $A_t = 0$ or $N_t(i) = 0$ - which cannot happen as this will make output zero all the time.

If α is considered as any arbitrary constant $0 \le \alpha \le 1$, as often done, then the only way of not violating $n^{1-\alpha} = n$ is when $n = 1$, because $n > 0$.

We now see the first trouble with the New Keynesian model. When $n = 1$, as often set by many authors, the problem never reveals itself. However, when $n \neq 1$, the model will have to be inconsistent, unless α is set to 0, as that is the only value where $n^{1-\alpha}$ collapses to n. If $\alpha = 0$, then any $n > 0$ can be used, and the basic New Keynesian model for now is saved.

One may say the result only implies that the basic New Keynesian model does not become classical monetary model in the limit, but our result has nothing to do with convergence to classical monetary model. In fact, for our proofs, there was no reference to classical monetary model.

Also, $n = 1$ choice has no deep theoretical basis - it is chosen out of convenience. Our problematic results are thus relevant.

Our result does raise a suspicion that there is a problem with labor clearing equation $\int_0^n N_t(i)\ di = N_t$. However, it is worth mentioning that the equation is not just derived from the intuition that N_t is the sum of all inputs to firms. In this basic New Keynesian model, labor market is perfectly competitive with perfect labor substitutability. And assuming an aggregator similar to Dixit-Stiglitz, and taking analogy of Y_t to N_t, the result does collapse to $N_t = \int_0^n N_t(i)\, di$. What can be done to remedy the problem will not be discussed in this section.

3. A Peculiar Result of the Basic New Keynesian Model with Linear Production Function

In the previous section, we showed that $\alpha = 0$ is needed to save the basic New Keynesian model. Let us impose this condition, which makes the production function of each firm in the model $Y_t(i) = A_t N_t(i)$. This can also be viewed as a consistency check by limiting case of $\alpha = 0$.

As $Y_t = \left[\int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, when $\theta \to 0$, $Y_t = n^{\frac{\varepsilon}{\varepsilon-1}} Y_t(i) = n^{\frac{\varepsilon}{\varepsilon-1}} A_t \frac{N_t}{n} = n^{\frac{1}{\varepsilon-1}} A_t N_t$.

Let us assume the limit $\theta \to 0$ for now. We first consider the case where by log-linear approximation, $Y_t = A_t N_t$, as done in Galí (2008). This is true without using approximation when $n = 1$. We will abuse using $n = 1$ for now, as showing inconsistency only requires proving for one particular n.

When $n = 1, \theta \to 0$, marginal product of labor, or MPN_t is $MPN_t = A_t$, as can be seen by taking partial derivative. Let us defined real marginal cost as RMC_t. Then,

$$RMC_t = \frac{W_t/P_t}{MPN_t} = \frac{W_t/P_t}{A_t} = \frac{W_t N_t}{P_t Y_t}$$

As $RMC_t = (\varepsilon - 1)/\varepsilon$ at $\theta = 0$,

$$\frac{\varepsilon - 1}{\varepsilon} Y_t = \frac{W_t N_t}{P_t}$$

By the accounting logic of the model, or by the budget constraint equality for maximization of utility,

$$P_t Y_t = W_t N_t + Profit_t = W_t N_t + D_t$$

$$Y_t = \frac{W_t N_t}{P_t} + \frac{D_t}{P_t}$$

$$\frac{D_t}{P_t} = \frac{1}{\varepsilon} Y_t$$

Maximizing $\max_{C_t, N_t, B_t} E_0 \sum_{k=0}^{\infty} \beta^t U(C_t, N_t)$ under constraints requires first-order condition of

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

Thus,

$$\frac{\varepsilon-1}{\varepsilon}Y_t = Y_t^{\sigma}N_t^{\varphi+1}$$

$$Y_t^{1-\sigma} = \frac{\varepsilon}{\varepsilon-1}N_t^{\varphi+1}$$

$$Y_t = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\sigma}} N_t^{\frac{\varphi+1}{1-\sigma}} = A_t N_t$$

$$N_t^{\frac{\varphi+\sigma}{1-\sigma}} = A_t\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{-1}{1-\sigma}}$$

Let us now take the limit $\varepsilon \to \infty$ also. Then,

$$\lim_{\varepsilon\to\infty} Y_t = N_t^{\frac{\varphi+1}{1-\sigma}} = A_t N_t$$

We are at the classical limit again. Now we take advantage of measurement invariance that this classical limit must satisfy. That is, even if it is possible to scale output Y_t for $\forall t$ by constant factor to $Y_t^1 = Y_t/k\ \forall t$ where $k > 0$ and $k \in \mathbb{R}$, important conclusions of an economic model should remain invariant. This is because for $n = 1$, $Y_t = A_t N_t$ and $Y_t(i) = A_t N_t(i)$. And as $N_t(i)$ is equal for every firm and $\int_0^1 N_t(i)\ di = N_t$, scaling A_t by constant factor will not disrupt consistency. After all, if Y_t is indeed affected by scale factor, then we have to be very careful every time we measure real output, despite the fact that goods in this model are infinitely divisible. We now show that $A_j = A_0 - \nu_0$ must be 1, if natural technology factor growth rate exists.

$$A_t = A_j g^t \nu_t$$

$$Y_t = A_j g^t \nu_t N_t$$

$$Y_t^1 \equiv Y_t/A_j = g^t \nu_t N_t$$

where A_j is constant factor that represents initial $t = 0$ technology factor A_t without ν_t technology shock, g represents natural growth factor of A_t and ν_t is random technology shock such that $\ln \nu_t$ is assumed to follow $\mathcal{N}(0, \sigma_a^2)$. We can modify the specification of $\ln \nu_t$ to allow technology shock inertia, but the specification will not matter much for our purpose, as long as $\lim_{T\to\infty} E_t[\nu_T] = 1$ for $\forall t$.
Now check marginal product of labor for both Y_t and Y_t^1, and we get:

$$MPN_t = A_t = A_j g^t \nu_t = \frac{W_t}{P_t} = Y_t^{\sigma} N_t^{\varphi}$$

$$MPN_t^1 = g^t \nu_t = \frac{W_t^1}{P_t^1} = \left(Y_t^1\right)^{\sigma} N_t^{\varphi}$$

This leads to:

$$\nu_t = \frac{\left(Y_t^1\right)^{\sigma} N_t^{\varphi}}{g^t} = \frac{Y_t^{\sigma} N_t^{\varphi}}{A_j g^t}$$

$$\left(Y_t^1\right)^{\sigma} \equiv \frac{Y_t^{\sigma}}{A_j^{\sigma}} = \frac{Y_t^{\sigma}}{A_j}$$

As A_j cannot be zero, $A_j = 1$ in order for the equation to be consistent. Now let us take the limiting case $g \to 1$.

$$\lim_{T\to\infty, g\to 1} E_t\left[Y_T\right] = \lim_{T\to\infty} E_t\left[\nu_T N_T\right] = \lim_{T\to\infty} E_t\left[N_T\right] = \lim_{T\to\infty} E_t\left[N_T^{\frac{\varphi+1}{1-\sigma}}\right]$$

The only case that does not restrict on the value of φ and σ is when $N_t = 1$. This suggests that in zero growth rate, $\lim_{T\to\infty} E_t\left[Y_T\right] = 1, \forall t$. While this is not entirely unjustifiable answer, the intuition that we can start from some steady output level that is not 1 has to be given up. This destruction of intuition seems very unjustifiable.

While there may be different options on not restricting N_t by restricting the possible values of σ and φ, the only option that does not require any other restrictions is when $(\varphi+1)/(1-\sigma) = 1$. Then, $-\sigma = \varphi$. This restriction is unjustifiable.

We often impose the following conditions to $U(C_t, N_t)$: $U_{C,t} > 0, U_{CC,t} \leq 0, U_{N,t} \leq 0, U_{NN,t} \leq 0$. These conditions will not be satisfied by $-\sigma = \varphi$ when the utility form is $U = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$.

As a side note, for the result that $\lim_{T\to\infty} E_t\left[\nu_T N_T\right] = \lim_{T\to\infty} E_t\left[N_T\right]$, the long-run understanding of natural rate is invoked.

Because this is the limiting case, the results here affect the entire basic New Keynesian model.

4. Deriving $\alpha = 0$ with no profit assumption in classical monetary model

The limit $\theta \to 0, \varepsilon \to \infty$ model of the basic New Keynesian model, classical monetary model, does not impose long-run zero profit condition. Here, we go to the extreme and prove that if we impose zero profit condition at $\forall t$, then $\alpha = 0$. If firms earn zero profit, then by the accounting or budget constraint logic and specification of production function,

$$Y_t = \frac{W_t N_t}{P_t} = A_t N_t^{1-\alpha}$$

$$MPN_t = \frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

Thus,

$$A_t N_t^{1-\alpha} = (1-\alpha) A_t N_t^{1-\alpha}$$

Therefore, $\alpha = 0$.
The assumption of short-run zero profit is not entirely unjustifiable - if economic agents realize that just by forming a firm would give them free lunch, economic agents will enter the business quickly when without entry costs. In such case, agents will work as workers but also as firm manager, because this limiting model does not have capital nor requirement of a firm manager to "manage" workers. Therefore,the only case where this is prevented is when firms earn zero profit.

5. A New Keynesian model with capital and the limiting case of the basic New Keynesian model

We now show that existence of capital does not destroy our results by presenting a New Keynesian model with capital that in $\theta \to 0, K \to 0$ reduces to the basic labor-only New Keynesian model. The model presented below is slightly modified from Duffy and Xiao (2011).
Utility maximization remains the same as the basic New Keynesian model, except that it is now under the following budget constraint:

$$C_t + Q_t \frac{B_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \frac{R_t}{P_t} K_t + \frac{B_{t-1}}{P_t} + \frac{D_t}{P_t}$$

where $R_t K_t / P_t$ represents real capital rental income and I_t represents new investment. The following equation is imposed on new investment:

$$I_t = I\left(\frac{K_{t+1}}{K_t}\right) K_t$$

where $I(\cdot)$ represents capital adjustment cost function of K_{t+1}/K_t, as in Woodford (2003).
Production function form each firm i, in Duffy and Xiao is:

$$Y_t(i) = K_t(i)^\alpha N_t(i)^{1-\alpha}$$

But we modify it to:

$$Y_t(i) = A_t K_t(i)^\kappa N_t(i)^{1-\alpha}$$

where $1 + \kappa - \alpha \leq 1$ (constant or decreasing returns to scale) and the same Dixit-Stiglitz aggregator for Y_t out of $Y_t(i)$ is in place. This makes the original production function model the limiting case. While the solutions will have to be different if $\alpha \neq \kappa$, the structure of the model is not changed, and therefore there is no problem with this choice.

Note that in this model, $Y_t = C_t + I_t$. Therefore, for the result $Y_t(i) = ()^{-\varepsilon} Y_t$, C_t cannot be simply substituted in place of Y_t. The result is justified, if we consider that agents maximize $Y_t - I_t$ given expenditure level Z_t. As the first order condition will remain the same except that now C_t is Y_t and $C_t(i)$ is $Y_t(i)$, the result follows.

Because this model exhibits perfectly competitive capital market with perfect substitutability, $K_t = \int_0^n K_t(i)\ di$.

In this model, we cannot simply take the limit $\kappa \to 0^+, K \to 0^+$; if we set $K = 0$, then perform limit $\kappa \to 0^+$, then the limit will simply be zero, resulting in zero output. What should be done instead is getting a special case model of $\kappa = 0$. This results in a model where capital does not really take part of production of goods, but produced goods are used as investment to replenish depreciating capital. But utility maximization implies that in equilibrium, $K = 0$. Thus, without a need to enforce directly $K \to 0$, $\kappa = 0$ model is equivalent to the model with limit $K \to 0$ and $\kappa = 0$. The I_t equation suggests that $I_t \to 0$, as $K \to 0$, and $\lim_{K_t \to 0} K_t(i)^0 = 1$. Thus $Y_t = C_t$, $Y_t = A_t N_t(i)^{1-\alpha}$, and the budget constraint reduces to the labor-only New Keynesian model's budget constraint.

By the argument that if an allowed special case exhibits inconsistency then a model in general is inconsistent, our results in the previous sections are not reversed by the introduction of capital.

6. Troubles with Consumption Euler Equation

Consumption Euler equation can be derived from the basic New Keynesian model. We will skip the derivation for spacing reasons. The basic consumption Euler equation and its derivation are well-known and can be found easily. For this section, we will ignore the problems derived in the previous sections.

The equation in log-linear consumption gap, or output gap in the basic New Keynesian model is given by:

$$\tilde{c_t} = E_t\left[\tilde{c_{t+1}}\right] - \sigma^{-1}\tilde{r_t}$$

where $\tilde{r_t} = i_t - E_t[\pi_{t+1}] - r_t^n$, r_t^n is natural real interest rate, i_t is nominal short-run interest rate set by central bank, π_{t+1} represents inflation rate at $t+1$, and $\tilde{c_t}$ represents consumption gap from its natural consumption level. In the basic model, $\tilde{c_t} = \tilde{y_t}$. Every lowercase variable in the consumpion Euler equation is in log-linear form and every lowercase variable with tilde represents gap from its natural rate.

Often transversality condition $\lim_{t\to\infty} E_t[\tilde{y_t}] = 0$ and the nominal interest rate rule given by Taylor rule, $i_t = r + \phi_\pi \pi_t + \phi_y \tilde{y_t} + \eta_t$ where r is constant factor and $\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_t^\eta$ with ε_t^η following $\mathcal{N}(0, \sigma_\eta^2)$ is AR(1) monetary policy shock, are imposed to solve the solution for $c_t = y_t$. But it is clear that if central bank targets real interest rate $r_t \leq r_t^n$ for $\forall t$ instead of Taylor rule, then transversality condition cannot be satisfied (even zero output will not save the transversality condition). As other parts of the model are microfounded, at least these parts themselves, even with non-Taylor monetary rule, should be valid. This leaves only one big option to save transversality condition: central bank cannot follow constant real interest rate. But how? As we will see, this is not possible at least inside the New Keynesian model without Taylor rule.

For simplification, let us assume that $\phi_y = 0$. Central bank only responds to change in inflation, and by Blanchard-Khan condition, $\phi_\pi > 1$ is assumed. While Woodford (2003) shows more of global multiplicity of equilibria problem, we will use Cochrane (2011)'s flexible example as suitable. After all, the flexible limit of the New Keynesian model exists, and unless one believes that stickiness allows for determination of inflation rate and flexible price allows for infinitely many equilibria, our limiting case is a fine use.

Flexible and sticky case only differ by the following: Fisher equation is $i_t = r_t + E_t[\pi_{t+1}]$, and in flexible case, as in Cochrane (2011), $r_t = r$, but this equality is not satisfied when stickiness is introduced, even though $\lim_{T\to\infty} E_t[\tilde{r_T}] = 0$. When flexible price is assumed, the resulting equation is:

$$\pi_{t+1} = \phi_\pi \pi_t + \eta_t + \delta_{t+1}$$

where δ_{t+1} is sunspot random variable with $E_t[\delta_{t+1}] = 0$. In this equation, every equilibrium solution except one solution exhibits inflation explosion. Cochrane (2011) argues that there is no reason to eliminate explosive solutions, as transversality condition is used only for eliminating real explosions. One representative New Keynesian response to Cochrane (2011) can be seen in Wren-Lewis (2013), referencing Woodford (2003). Simon Wren-Lewis argues that central bank's intended inflation target known to agents is good enough to make agents return to central bank's intended inflation target, which is given by the non-explosive inflation equilibrium. In New Keynesian model, agents first form expectation of future inflation rates from $t+1$ to ∞. Then central bank sets nominal interest rate which equilibrates $\tilde{y}_t$ and π_t and possibly adjust future inflation expectations that are consistent with Taylor rule, since $\pi_t = \beta E_t[\pi_{t+1}] + \psi\tilde{y}_t$ (New Keynesian Phillips Curve), where ψ is constant value, and Taylor rule has factor $\phi_\pi\pi_t$. For π_t equation, what is not determined is $\tilde{y}_t$ which is dependent on nominal interest rate i_t, and nominal interest rate rule itself is dependent on π_t, by Taylor rule. The current values of $\tilde{y}_t$ and π_t all depend on future and current real interest rates.

Now for our frictionless case, $\tilde{y}_t = 0$, and therefore, future inflation expectations determines π_t completely. Central bank may influence this future inflation expectation by changing ϕ_π, but unless central bank does change the reaction coefficient, only ε_t^η may change future inflation expectation. This shows an important difference between frictionless and sticky case when dealing with nominal interest rate rule. In sticky case, following Taylor rule requires that nominal interest rate equilibrates output gap and inflation rate, and therefore output effect of monetary policy that feeds back to current inflation rate becomes very important. But as we are in frictionless case we do not have to consider this possibility.

This suggests that nominal interest rate rule given by Taylor rule is passive, given "rational" expectations or beliefs of future inflation rates, expected nominal interest rate can be determined. The fact that central bank follows Taylor rule, combined with inflation target announcement and continuously increasing inflation rate, is enough to show, according to Simon Wren-Lewis, that agents have wrong beliefs of an economy.

But this response requires that intended inflation target be achievable, and the only mean for central bank to enforce the target in the basic New Keynesian model is by the control on short-run nominal interest rate. Unless central bank has control on nominal interest rate unconditionally, there is no rational reason to believe in central bank's inflation target.

With this observation, consider the alternate interest rule of constant real interest rate for $\forall t$. Assume that both central bank and agents know Taylor rule and the constant rate rule, but agents initially expect Taylor rule-styled policy even when central bank announces its constant real interest rate rule. Central bank has full knowledge of inflation expectations. Agents first set expectation for future inflation rates, which form forward-looking and the only inflation part of the right-hand side of New Keynesian Phillips curve, based on Taylor rule. With these expectations set, central bank sets nominal interest rate without much concern for equilibrium. It does not care whether disequilibrium results in due to its policy. It just sets nominal interest rate i_t such that $i_t = r_1 + E_t[\pi_{t+1}]$, with the knowledge that agents will change their future expectations by the belief that central bank has to follow Taylor rules - suggesting that agents treat any deviation from non-stochastic Taylor rule as being induced by ε_t^{η}. Therefore, central bank considers not initial future inflation expectation, but rather final future inflation expectation from initial future inflation expectation, as $E_t[\pi_{t+1}]$. Notice that there is no π_t in Fisher equation. This process will continuously upset agents, until agents realize that central bank does indeed not follow Taylor rule and does have power to commit to constant real interest rate rule.

Agents may rely on market to form their expectation, but as long as central bank controls currency, i_t will still be set by central bank. Therefore, agents either have to abandon currency or have to assume constant real interest with stochastic variations announced by central bank. The former requires a model to back up and is definitely outside the realm of New Keynesian models. When constant real interest rate is assumed by every agent, transversality condition $\lim_{T\to\infty} E_t[\tilde{y_T}] = 0$ is no longer satisfied, if $r_1 \neq r_t^n$ ($r_1 = r_t$ in this rule). (Furthermore, note that while one may decide to eliminate transversality condition, the basic consumption Euler equation will go in conflict with some variants of the basic New Keynesian Phillips curve, such as Sticky-Information Phillips curve, as in Mankiw and Reis (2002).) In such a case, agents form rational expectation for future inflation rates and central bank set just nominal interest rate based on these expectations. Of course under New Keynesian solution-picking assumptions, the model is unsolvable. After all, natural rate hypothesis will need to be abandoned in any equilibrium solution, not just a standard New Keynesian equilibrium.

The violation of natural rate suggests that it may necessarily need to add either long-run adjustment or other anchoring mechanisms directly. For example, instead of other interest rates anchored to the interest rate related to central interest rate, which are assumptions of the New Keynesian model, expected long-run interest rates set by market may be more of reference even for short-run interest rates. The other possibility goes with replacing natural rate hypothesis with some expectation anchoring assumption, such as Farmer (2013)'s nominal GDP anchoring, expressed by $E_t[\Delta x_{t+1}] = \Delta x_t$. This expectation will not be rational expectation. In a similar vein, Sumner (2014) argues for nominal GDP target, instead of interest rate/inflation targeting.

7. Conclusion

We have demonstrated that log-linearized equations of New Keynesian models hide the fact that the model may have actually failed to establish equilibrium and is under disequilibrium. To make the models establish equilibrium, two choices can be considered. One may modify labor clearing equation $\int_0^n N_t(i)\ di = N_t$ to the form of equations that involve a different labor aggregator which does not reduce to the original labor clearing equation in any limiting case. This implies that the definition of N_t has to change - it is no longer the sum of all labor inputs applied. Instead of modifying labor clearing equation, Dixit-Stiglitz aggregator may be replaced by a different aggregator. Again, though, there is no purpose of modifying an aggregator of a model, if the aggregator reduces to Dixit-Stiglitz aggregator.

We also have shown that transversality condition $\tilde{y}_t = 0$ in the basic New Keynesian model cannot be enforced when central bank actively engages to set constant real interest rate that deviates from natural output level y_n, and central bank can do this by not following Taylor rule and actively engaging. While constant non-natural interest rate may make market form a new currency, it must first be suspected that more equations, particularly related to expectation, are needed to enforce long-run natural rate of output independent of central bank policy - that is, the model is incomplete abstraction of reality. Anchoring may be done by anchoring expected future nominal output to current nominal output, which may or may not replace output gap transversality condition.

If we limit the purpose of New Keynesian models as explaining only very short-run phenomena, then the issues related to consumption Euler equation may not be actual problems, but this of course requires explicating how long long-run adjustments would not be dominant.

Looking from historical context, it may also be necessary to introduce money directly into the basic New Keynesian model. In AD-AS-IS-LM model, monetary expansion leads to LM curve shifting right, with temporarily lower real/nominal interest rate and higher real output. But this causes shift in AD curve to right also, causing higher price level. This higher price level must be reflected back into LM curve, along with AS-side adjustments, which means AS curve shift left and LM curves shifts left to equilibrate at natural output rate. The key point of this model is that adjustment takes time, and this allows short-run non-neutrality. But in long-run, equilibration at natural output and interest rate must occur. Or, from a different angle, one may imagine disequilibrium process that eventually establishes natural rate equilibrium. It is true that AD-AS-IS-LM is outdated, but it may be possible to take cues from price level adjustment that allows adjustment back to natural rate. We wish that future progress show up to clarify these matters.

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