A SumDiscount method for appraising firms:An illustrative example

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Abstract. This paper presents a new way of valuing firms and measuring residual income. The method, originally introduced in Magni (2000a, 2000b, 2000c, 2001), is here renamed lost-capital paradigm. In order to enhance comprehension the presentation relies on a very simple numerical example which shows that the new paradigm of residual income enjoys a property of abnormal earnings aggregation, according to which the NPV (and therefore the market value) of the firm does not change if each residual income changes, as long as the (uncapitalized) sum of all residual incomes do not change. While radically different from the standard residual income, the difference between the two notions is equal to the interest accrued on the past cumulated standard residual incomes, which has interesting implications for incentive compensation.

Keywords. Firm valuation, residual income, lost capital, Discount&Sum, Sum&Discount, incentive compensation

JEL codes. G11, G12, G30, G31, M2, M41

Introduction

As widely known, residual income (RI) (also known as abnormal earnings) is defined as the difference between actual income and the "would-be" income corresponding to the best alternative opportunity open to the investor. Such a would-be income is often called 'capital charge':

$$\text{RI} = \text{Income} - \text{capital charge}. \tag{1}$$

Traditionally, the standard way of measuring capital charge is to multiply the capital employed by the opportunity cost of capital:

$$\text{Capital charge} = \text{Income} - \text{cost of capital*invested capital} \tag{2}$$

(see Preinreich 1936, 1938; Edwards and Bell, 1961; Solomons, 1965; Peasnell, 1981, 1982). According to the perspective taken (entity or proprietary) and the capital selected (book value or market value) eq. (2) gives rise to different metrics: *Economic Value Added* (Stewart, 1991),

Economic Profit,[1] also known as *Edwards-Bell-Ohlson* model (see Ohlson, 1989, 1995; Fernández, 2002; Arnold, 2005), *Created Shareholder Value* (Fernández, 2002), *CFROI* (Madden, 1999) etc.[2]

This paper proposes a different way to compute the capital charge and therefore a different way of measuring residual income. The method is here named "lost capital paradigm" and has been introduced and thoroughly studied from several points of view in a vast array of contributions (Magni, 2000a, 2000b, 2000c, 2001, 2003, 2004, 2005, 2006, 2007a; Ghiselli Ricci and Magni, 2006): This paper takes a corporate finance perspective and represents the new notion by focusing on two interesting aspects:

(1) the lost-capital paradigm is consistent with the NPV (and the market value of the firm) but the procedure to obtain the latter is opposite to the standard one: Residual incomes are first summed, and then discounted back to present. This means that the lost-capital paradigm enjoys a significant property of abnormal earnings aggregation
(2) the lost-capital paradigm, though different from the standard paradigm, may be viewed as stemming from a cumulation of past standard residual incomes.

Although we rely on numerical examples for illustrative purposes, all results do not depend on the examples selected and hold for any possible situation. The reader interested in rigorous formal proofs of the properties here illustrated may find it useful to browse through the following contributions: Magni (2000a, 2004, 2005, 2007b).

1. Consistency with the NPV and abnormal earnings aggregation

Firm X is incorporated at time 0 to undertake a project which generates sales equal to 500, 190, 600 at time 1, 2, 3 respectively. These are assumed to be the dividends paid to the shareholders. The capital infused by shareholders is 1000 (Net Assets). The firm is unlevered. At time 3, after payment to shareholders of the 600 dollars, the firm shuts down (it is assumed zero scrap value). Let us assume that the depreciation charges for the Net Assets are 400, 100, 500, respectively. For simplicity, let us assume: No working capital, no production costs, no general expenses, no tax. The net profit, the Return On Equity (ROE), the Equity Cash Flow (ECF), the Internal Rate of Return of the project (IRR) are easily obtained in the following Table:

Time	0	1	2	3
Net Assets	1000	600	500	0
Equity	1000	600	500	0
Sales		500	190	600
Depreciation		400	100	500
Net Profit		100	90	100
ROE=RONA		10%	15%	20%
ECF	−1000	500	190	600
IRR	13.41%			

[1] Economic profit here refers to a proprietary perspective, but it is sometimes used as a generic synonym of residual income with no particular reference to the perspective adopted.
[2]Strictly speaking, CFROI is a rate of return, not a residual income measure. However, a residual income measure may be naturally obtained by subtracting the WACC and multiplying by the capital invested (the latter is obtained with the recurrence equation End-of-period capital=Beginning-of-period capital*(1+CFROI)−Free Cash Flow to Firm).

The equity cash flows are obviously equal to sales, given the simplified assumptions.

Suppose that the shareholders may alternatively choose to invest in a comparable financial asset whose rate of return is 9%. The latter is obviously the equity cost of capital.

The Net Present Value of the project is

$$\text{NPV} = -1000 + \frac{500}{1.09} + \frac{190}{1.09^2} + \frac{600}{1.09^3} = 81.94$$

Denoting with Δ a variation, the book value of the capital invested may be found by making use of the following relation:

$$\Delta \text{ capital} = \text{net profit} - \text{ECF} \tag{3}$$

which is known, in management accounting, cleans surplus relation (see Brief and Peasnell, 1996). Equation (3) may be rewritten

$$\text{end-of-period capital} = \text{beginning-of-period capital}*(1+\text{ROE}) - \text{ECF}$$

(see Table below).

Time	Capital invested	
0	1000	
1	600	= 1000*(1+0.1) −500
2	500	= 600*(1+0.15) −190
3	0	= 500*(1+0.2) −600

The standard residual incomes are

$RI_1 = 1000*(0.1-0.09) = 10$
$RI_2 = 600*(0.15-0.09) = 36$
$RI_3 = 500*(0.2-0.09) = 55.$

Given that the accounting rate of return is ROE=ROA, the residual income computed is, at the same time, *Economic Value Added* (entity perspective) and Economic Profit (proprietary perspective).

Let us first discount back the RIs:

$RI_1/(1.09) = 10/(1.09) = 9.17$
$RI_2/(1.09)^2 = 36/(1.09)^2 = 30.3$
$RI_3/(1.09)^3 = 55/(1.09)^3 = 42.47$

and then let us sum:

$$9.17+30.3 + 42.47 = 81.94=\text{NPV}.$$

This procedure may be called a *Discount&Sum* procedure.

A different notion of residual income may be found if one computes the net profits and the capital invested assuming that shareholders invest the 1000 euros in the financial asset and withdraw from that asset the cash flows 500, 190, 600 at time 1, 2, 3 respectively. Using the clean surplus relation one finds

Time	Capital invested
0	1000
1	590 = 1000*(1+0.09) −500
2	453.1 = 590*(1+0.09) −190
3	−106.12 = 453.1*(1+0.09) −600

Such a capital is the capital that shareholders could have owned at the various dates (and invested in the following period) if they had initially invested 1000 dollars in the financial asset. In other terms, they could have realized the same pattern of cash flows as the project's (−1000, 500, 190, 600) by investing in the financial asset (the negative terminal capital indicates that the alternative opportunity is not worth undertaking because it brings about an arbitrage loss of −106.12). Given that shareholders invest in the project rather than in the financial asset, they *lose* the opportunity of owning that pattern of capitals: It is therefore the *lost capital*. O'Hanlon and Peasnell (2002) use the expression *unrecovered capital* to mean the same concept. The lost capital gives rise to a lost income, because if shareholders had invested in the financial asset, their income would have been

Time	Lost Income
1	0.09*1000 = 90
2	0.09*590 = 53.1
3	0.09*453.1 = 40.78

This income is *lost*, because shareholder cannot recover it.
Let us define *lost-capital residual income* (LCRI) the difference between the actual income and the lost income:

$LCRI_1 = 100-90 = 10$
$LCRI_2 = 90-53.1 = 36.9$
$LCRI_3 = 100-40.78 = 59.22$

Let us first sum the LCRIs:

$$10 + 36.9 + 59.22 = 106.12$$

and then discount back to present

$$106.12/(1.09)^3 = 81.94 = NPV$$

This procedure may be called a *Sum&Discount* procedure. It is the reverse of the standard procedure.

Note that the difference between the terminal capital of the firm (0) and the terminal lost capital (−106.12) coincides, in absolute value, with the sum of the LCRIs:

$$0-(-106.12) = 106.12$$

This amount represents the so-called Net Final Value (NFV), or excess return (see Young and O'Byrne, 2001, p. 31):

$$NFV=NPV(1.09)^3$$
$$106.12=81.94(1.09)^3$$

As a result:

(i) the LCRI is consistent with the NPV, which means that it is a legitimate method for valuing firms and projects
(ii) the LCRI enjoys a property of abnormal earnings aggregation. That is, it does not matter what a residual income amounts to and it does not matter when it is generated: For determining the market value, only the total dollar abnormal earnings matter. This means that not only different permutations of the residual incomes does not affect the firm value, but the firm value is not even affected by different values of each residual income, as long as the sum does not change.

To illustrate the latter point, we now consider all the possible permutations of the standard residual incomes. Let us assume that there exists firms A, B, C, D, and E that produce the same residual incomes as firm A, but they are generated in different periods. In the following Table the NPV is computed for each firm by making use of the standard paradigm.

	Residual income				
Firm	Time 1	Time2	Time 3	NPV via standard RI (Discount&Sum)	Equity market value (NPV+equity book value)
X	10	36	55	81.94	1081.94
A	10	55	36	83.26	1083.26
B	55	36	10	88.48	1088.48
C	55	10	36	86.67	1086.67
D	36	10	55	83.91	1083.91
E	36	55	10	87.04	1087.04

The NPV and the equity market value changes as the distribution of residual incomes changes.

Let us re-calculate NPV and equity market value for each firms by making use of the lost-capital paradigm:

	Residual income				
Firm	Time 1	Time2	Time 3	NPV via LCRI (Sum&Discount)	Equity market value (NPV+equity book value)
X	10	36.9	59.22	81.94	1081.94
A	10	59.22	36.9	81.94	1081.94
B	59.22	36.9	10	81.94	1081.94
C	59.22	10	36.9	81.94	1081.94
D	36.9	10	59.22	81.94	1081.94
E	36.9	59.22	10	81.94	1081.94

This elegant mathematical property has practical usefulness as well as theoretical significance. If the notion of residual income employed is the lost-capital one, one does not have to forecast each and every RI and associate it to each and every date. Because only the total dollar abnormal earnings matter, one can rely on the notion of average (abnormal) earning power (see Penman, 1992). Such an abnormal earning power (AEP) may be referred to a span of t periods and is found by computing the arithmetic average of the past t abnormal earnings. For example, letting LCRI_k be the lost-capital abnormal earnings at time $k = 1, 2, \ldots, t-1$, the AEP is given by

$$\text{AEP} = \frac{\sum_{k=1}^{t-1} \text{LCRI}_k}{t}.$$

This AEP is an average (lost-capital) residual income. If the AEP is regarded as a reliable indicator of the prospective abnormal earnings power, it may be used as a proxy for future residual incomes and therefore for computing the NPV of the firm and the equity market value.

The property of aggregation may also be used for comparative valuations among firms or business units: For example, if two or more business units' forecasted LCRIs are equal (similar), their value must be equal (similar), even if the residual incomes of same amount are generated in different periods.

The LCRI may be used as a control method as well: After valuations of firms or business units have been made via DCF methods or standard RI method, the LCRIs of each firm (business unit) could be computed: If the total amount of LCRIs are similar (or equal) for the firm under examination, then the values of the firms (business units) should not be too different. If, instead, values have been found which considerably differ, then there is a lack of consistency in valuation.

2. Property of cumulation of residual incomes

While radically different from the standard RI, the LCRI is not disconnected from the former. Barring for the first period, where the two RIs coincide, the difference between the two RIs in any period is equal to the periodic interest on the sum of past compounded standard RIs:

$$\text{LCRI}_t - \text{standard RI}_t = i\sum_{k=1}^{t-1} (\text{standard RI}_k)(1+i)^{t-1-k} \tag{4a}$$

or, equivalently,

$$\text{LCRI}_t = \text{standard RI}_t + i\sum_{k=1}^{t-1} (\text{standard RI}_k)(1+i)^{t-1-k} \tag{4b}$$

where i is the cost of capital.

The following Table shows this result for the simplified example illustrated above:

	$\text{LCRI}_t - \text{standard RI}_t$	$0.09\sum_{k=1}^{t-1}(\text{standard RI}_k)(1+0.09)^{t-1-k}$
$t = 1$	0	-
$t = 2$	0.9	0.9
$t = 3$	4.221	4.221

This property shows some features of the lost-capital RI when used for incentive compensation. Thanks to eq. (3), to use the lost-capital RI for incentive compensation is equivalent to using a compensation plan based on the past standard residual incomes: In particular, and index which is equal to the current standard residual income and the interest accrued on past cumulated standard residual incomes. In such a way, current compensation depends on the past performance as well as the current one. In other terms, measuring performance with the current lost-capital RI is equivalent to measuring performance with an index dependent on all past standard RIs as well as the current one. Given the compounding process in eq. (4), this index is found in a way which is interestingly similar to a bonus bank system (see Young and O'Byrne, 2001).

The dependence of compensation on the past performances implies that past positive performances reverberate positively on current performance, while negative performances will reverberate negatively on current performance. In other terms, the philosophy of reward is as follows: If a manager's past performances are positive, then his current standard RI is increased by the interest earned on the past cumulated standard RIs, so that the latter act as an insurance bonus for those periods where current performance is negative. If a manager's past performances are negative, then the interest on past cumulated standard RIs are negative and the latter act as a penalty (see some examples and discussion in Magni, 2007b).

Concluding Remarks

This note has presented some numerical examples aimed at showing some features of the lost-capital paradigm, a new method of measuring residual income and valuing firms. This method has been first proposed in Magni (2000a, 2000b, 2001) with the name of *Systemic Value Added*, and has been investigated from several different perspectives: Theoretical, formal, economic and cognitive (Magni, 2003, 2004, 2005, 2006, 2007a, 2007b; Ghiselli Ricci and Magni, 2006). The method, here renamed *lost-capital paradigm*, is consistent with the NPV and the market value of the firm, but the NPV is reached with a procedure which is opposite to the standard one: If one employs the standard paradigm, the NPV is obtained by first discounting RIs and then summing (Discount&Sum procedure); if one employs the lost-capital method, the NPV is reached by first summing the RIs and then discounting back to present (Sum&Discount). This is the property of (abnormal) earnings aggregation. Another interesting property has also been illustrated: The lost-capital residual income is equal to the current standard RI plus the interest accrued on past standard residual incomes cumulated. This implies that if compensation is based on the LCRI, the reward for the manager is the same as if it were based on a function of all standard past residual incomes cumulated. The use of this system amplifies performance with respect to the standard one: If past standard performances are positive, then performance in the LC paradigm is higher with respect to the standard RI, if past standard performances are negative, then performance in the LC paradigm is smaller.

The lost-capital paradigm may be considered not necessarily an alternative to the standard residual income, but an enrichment of the toolbox of the corporate finance theorist and practitioner, which is enlarged by a new method of (i) measuring residual income, (ii) valuing firms, (iii) compensating managers.

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