Propensity to search: common, leisure, and labor models of consumer behavior

Malakhov, Sergey

Pierre-Mendès France University

January 2015

Sergey MALAKHOV

Ph.D., Applied Economics,

Pierre-Mendès-France University,

Grenoble, France

Propensity to search: common, leisure, and labor models of consumer behavior.

Abstract

The analysis of the propensity to search specifies the "common" or the ordinary model of consumer behavior based on the synthesis of the neoclassical approach with satisficing concept and "leisure" and "labor" models of behavior that represent different combinations of conspicuous consumption, leisure, and labor. Some of these combinations result in phenomena of Veblen effect and money illusion. While "the common model" of behavior demonstrates a moderate propensity to search, "leisure" and "labor" models of consumer behavior manifest vigorous propensities to search. The analysis of trends in allocation of time in the USA during last decades assumes that women have followed "common model" of behavior while men have exhibited both "leisure" and "labor" models of behavior.

Introduction

The previous papers on the optimal consumption-leisure choice under price dispersion have demonstrated the importance of the concept of propensity to search, i.e., to substitute labor for search for beneficial price (Malakhov 2012, 2013, 2014a, 2014b, 2014c). The basic assumption on the concept of the propensity to search is that labor and search always "move" in opposite direction, or the value $\partial L/\partial S$ is always negative because labor and search represent different sources of income. However, the analytical significance of this concept needs a particular effort that could summarize results of comparative analysis of different models of consumer behavior produced by different propensities to search. And that effort is realized in the paper presented here. It is organized as follows. Part I describes the moderate propensity to search in the "common model" of behavior, i.e., in the model that explains the everyday economic behavior on the basis of the synthesis of the methodology of optimization with the satisficing approach. Part II illustrates the vigorous propensity to search in the "leisure model" of behavior that could produce such anomaly as Veblen effect. Part III describes the reverse manifestation of the

vigorous propensity to search when labor itself becomes conspicuous. And this "labor model" of consumer behavior could result in such anomaly as money illusion that is analyzed in Part IV.

Part I. Propensity to search in "common model" of behavior

The static optimal consumption-leisure choice can be described by the Cobb-Douglas utility function $U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S}$ subject to the equality of marginal savings on search to its marginal costs:

$$max\ U(Q,H)\ subject\ to\ w\frac{\partial L}{\partial S}=Q\frac{\partial P}{\partial S} \quad (1.1)$$

$$\Lambda = U(Q,H) - \lambda\left(w - \partial P/\partial S\frac{Q}{\partial L/\partial S}\right) \quad (1.2)$$

$$\frac{\partial U}{\partial Q}=\lambda\frac{\partial P/\partial S}{\partial L/\partial S} \quad (1.3)$$

$$\frac{\partial U}{\partial H}=-\lambda Q\frac{\partial P/\partial S}{(\partial L/\partial S)^2}\partial^2 L/\partial S\partial H=-\lambda\frac{w}{\partial L/\partial S}\partial^2 L/\partial S\partial H \quad (1.4)$$

where the value of price reduction or marginal savings on purchase $\partial P/\partial S$ is given by a location and price settings of a store, the value ($-\partial L/\partial S$) of propensity to search, i.e., of the propensity to substitute labor time L for search S is equal to the share of non-leisure time in the time horizon of the consumption leisure choice ($-\partial L/\partial S=(L+S)/T$), the value ($-\partial H/\partial S$) is equal to the share of leisure time ($-\partial H/\partial S=H/T$) and the value of the time horizon T is equal to the time until the next purchase or to the commodity lifecycle (Fig.1):
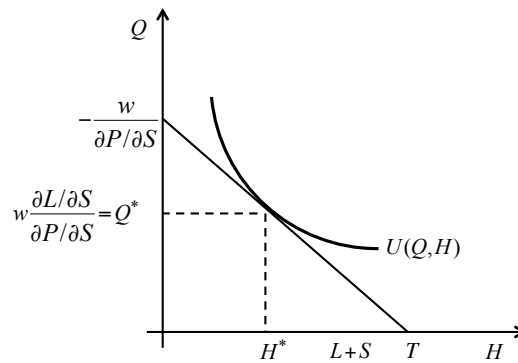

Fig.1. "Common model" of behavior

The optimization problem results in the "common model" of behavior. The key attribute of the "common model" of behavior is the moderate propensity to search $\partial L/\partial S$. Here it is limited, as we can see at Fig.1, by the [-1;0] interval. While the value of the propensity to search can by literally described by the "whiskey-soda-ice" metaphor, when ice (search) displaces both

whiskey (labor) and leisure (soda) in the glass, it can be directly derived from the optimization problem:

$$\frac{Q^* = w\,\partial L/\partial S \Big/ \partial P/\partial S}{w(\partial L/\partial S = -1)\Big/\partial P/\partial S} = \frac{L+S}{T} \Rightarrow \frac{\partial L}{\partial S} = -\frac{L+S}{T} \quad (2)$$

The static resolution of the utility maximization problem gives the way to the comparative static analysis of the satisficing decision where one part of the constraint, the value $\partial P/\partial S$, is softened and the consumer reserves the labor income and takes a chance to search the pre-determined quantity in different places of purchases where he finally finds the satisficing price P_P that equalizes marginal costs of search with its marginal benefit and therefore maximizes the utility of the consumption leisure choice with respect to the given wage rate w and to the chosen place of purchase $\partial P/\partial S$ (Malakhov 2014a). If we re-arrange the presentation of the propensity to search we can easily show that its derivative with respect to leisure time is equal to the inversed value of time horizon:

$$\frac{\partial L}{\partial S} = -\frac{L+S}{T} = \frac{H-T}{T} \quad (3.1)$$

$$\partial^2 L/\partial S\partial H = 1/T \quad (3.2)$$

The utility maximization problem and the satisficing decision procedure becomes interconnected by the equilibrium price P_e where $P_e = w(L+S) > P_P = wL$, that enters into the marginal rate of substitution, of leisure for consumption in the following form:

$$-\frac{dQ}{dH} = \frac{\partial U/\partial H}{\partial U/\partial Q} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{T\partial P/\partial S} = \frac{w}{w(L+S)} = \frac{w}{P_e} \quad (4)$$

Part II. Propensity to search in "leisure model" of behavior

As we can see, the "common model" of behavior takes place when search plays a supporting role with regard to labor. Here the search only adjusts labor costs to the satisficing level. It happens because when $\partial L/\partial S > -1$, the constraint in Equation (1.1) produces the "common" relationship between the wage rate and marginal savings on purchase $w > Q|\partial P/\partial S|$. But if the consumer can get from the search marginal savings greater than the wage rate, he could change his model of behavior. The relationship $w < Q|\partial P/\partial S|$ results in vigorous propensity to search $\partial L/\partial S < -1$. However, the vigorous propensity to search changes the relationship between search and leisure. This relationship becomes positive $\partial H/\partial S > 0$ due to very simple reasoning:

$$L+S+H = T \Rightarrow \partial L/\partial S + 1 + \partial H/\partial S = 0 \quad (5)$$

However, the positive $\partial H/\partial S$ relationship changes the sign of the second derivative $\partial^2 L/\partial S\partial H$. It becomes negative – the increase in leisure time decreases the value of propensity to search $\partial L/\partial S$

and increases its absolute value $|\partial L/\partial S|$. It happens because here either the fall in labor supply contributes to both search and leisure or the increase in labor supply reduces both search and leisure.

Unfortunately, it is very difficult to find the natural algorithm for this kind of the redistribution of time like the "whiskey-soda-ice" metaphor makes it for the "common model" of behavior. However, we could try to derive the geometrical algorithm for the $\partial L/\partial S<-1;\ \partial H/\partial S>0$ relationships.

First, we should agree in accordance with the Equation (4) that the negative $\partial^2 L/\partial S\partial H$ value changes the relationship between consumption and leisure. It becomes positive, or $dQ/dH>0$. Second, we accept the fact that a consumer cannot buy the target consumption level Q^* within the "common model" even if he spends all time on working because the initial $\partial P_0/\partial S_0$ value is very high and it corresponds to extremely high price with respect to the consumer's wage rate. When consumer starts the search he discovers that achievable marginal savings will be less than at the start of the search but nevertheless could be still greater than his wage rate ($|\partial P_0/\partial S_0|>|\partial P/\partial S|>w$). However, at the beginning of the search the consumer realizes that he needs more time to search and evidently he needs some leisure to consume the target consumption level because at the limit of the "common model" ($H=0; \partial L/\partial S=-1$) he has neither enough search time nor leisure time to get the target level of consumption. (We take here for illustrative purposes the zero level of leisure time but the same logic can be used if we presuppose some fixed physical minimum of leisure.) (Fig.2):
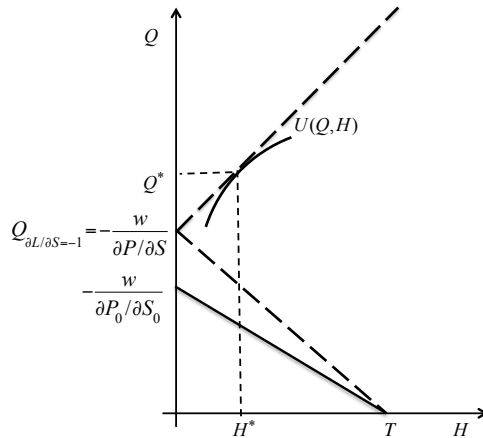
Fig.2. Search for great marginal savings on purchase of big-ticket item

The consumer vigorously exploits the positive $\partial H/\partial S$ relationship and finally he buys the desired item. Hence, the conversion of the dQ/dH ratio changes only the direction of constraint line but it

does not change its final slope. And the result of search can be graphically presented in the following form (Fig.3):
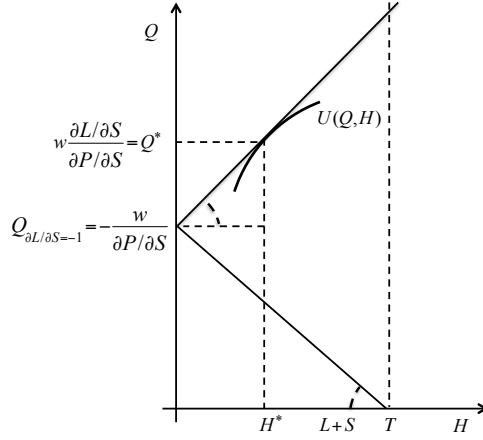
Fig.3. "Leisure model" of behavior

The utility is determined here by the same equation $U(Q,H)=Q^{-\partial L/\partial S}H^{\partial H/\partial S}$. The change in the shape of the utility curve happens because the value $(-\partial H/\partial S)$ really becomes negative.

If we re-arrange the constraint of the model with respect the Fig.3, keeping in mind that the Equation (5) always tells us that $\partial L/\partial S=-(1+\partial H/\partial S)$, we get the following result:

$$Q^* = \frac{w}{\partial P/\partial S}\partial L/\partial S = -\frac{w}{\partial P/\partial S}(1+\partial H/\partial S) = -\frac{w}{\partial P/\partial S}(1+\frac{H}{T}) \quad (6)$$

It looks like the consumer cannot get the target level of consumption if he spends all time only for labor and search ($T=L+S; H=0; \partial L/\partial S=-1; Q_{\partial L/\partial S=-1}<Q^*$). However, when search is more efficient than labor and marginal savings on purchase are greater than the wage rate, the consumer can cut labor time in favor of both search and leisure. And the increase in leisure time provides him with a missing quantity of consumption $dQ(H)$:

$$Q^* = Q_{\partial L/\partial S=-1} + dQ(H) = -\frac{w}{\partial P/\partial S}(1+\frac{H}{T}) = -\frac{w}{\partial P/\partial S}-\frac{w}{\partial P/\partial S}\frac{H}{T} = Q_{\partial L/\partial S=-1} + dH\frac{\partial Q}{\partial H} \quad (7)$$

Unfortunately, Fig.2 and Fig.3 presume that consumption is "bad". The increase in consumption decreases the utility level. The utility can be raised only if the increase in consumption is supported by the more noticeable increase in leisure time. However, this fall in the Q/H ratio can be also reproduced by the respective change in $\partial P/\partial S$ value, more precisely, by the increase in the absolute $|\partial P/\partial S|$ value. The increase in the absolute $|\partial P/\partial S|$ value changes the slope of the constraint line and makes it flatter. However, this way opens the door to the increase in utility (Fig.4):
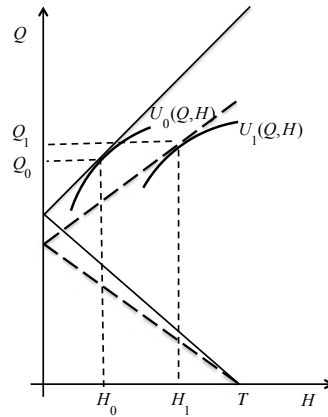
Fig.4. Veblen effect

But the increase in the absolute $|\partial P/\partial S|$ value means the increase in the price of purchase. And the Veblen effect takes place.

However, we should not forget that Fig.2, Fig.3, and Fig.4 describe very particular case of "bad" consumption. If we come back to the values of marginal utilities of consumption and leisure, we can see that with the negative $\partial^2 L/\partial S\partial H$ value consumption becomes "bad" and leisure stays normal only when the value of the Lagrangian multiplier λ, i.e., the value of the marginal utility of money income $MU_w = \lambda$ becomes negative (Malakhov 2013). There is very simple reasoning for this. In the "leisure model" of behavior search and labor change their roles. Now search becomes a leading actor and it leaves for labor only a supporting role.

The understanding of the negative marginal utility of income comes in the situation when an increase in wage rate is followed by a greater increase in the absolute value of marginal savings. If the increase in wage rate moves consumers from low-price stores to high-price stores, the indirect utility function $V=V(w,\partial P/\partial S(w))$ takes place. [1] Indeed, consumers really can get greater marginal savings on purchase in high-price stores. Thus, the increase in wage rate produces ambitious wishes to buy an important item and to keep the reputation of a smart-shopper. i.e., to buy that item with great discount. In this sense the increase in wage rate is not good and the marginal utility of labor income is really negative.

And when the increase in marginal savings is greater than the increase in the wage rate the constraint line becomes flatter. The flat constraint line results in the decrease in utility in the "common model" of behavior. The utility can be raised only in the "leisure model".

---

[1] The comparative static analysis of the utility function $V=V(w,\partial P/\partial S(w))$ is presented in (Malakhov 2014c).

Part III. Propensity to search in "labor model" of behavior

If the value f the Lagrangian multiplier λ, i.e., the value of the marginal utility of money income $MU_w = \lambda$ as well as consumption stays positive, the negative $\partial^2 L/\partial S\partial H$ value produces "bad" leisure. Here we come to another model of behavior. This new model is opposite to the "leisure model" in the sense that when the propensity to search is important ($\partial L/\partial S<-1; \partial H/\partial S>0$) the consumer decreases both search and leisure in favor of labor. This model was previously analyzed in (Malakhov 2012) but that analysis missed graphical presentations and resulted in the confusion with the "leisure model" of behavior.

Now in order to avoid that confusion and to present "bad" leisure we should change axis for it. In addition, the change of axis modifies the shape of the utility function (Fig.5):
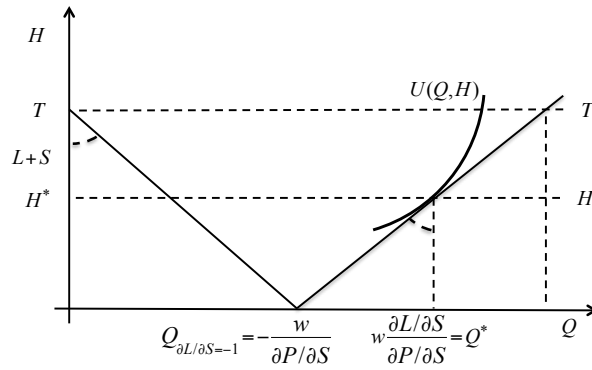

Fig.5. Normal consumption and "bad" leisure

Here we meet again the positive dQ/dH relationship but now it tells us that consumption is normal good and leisure is "bad". We see that Equations (6) and (7) also work for this model of behavior. However, if the increase in the wage rate results in moderate increase in marginal savings we get another picture (Fig.6):
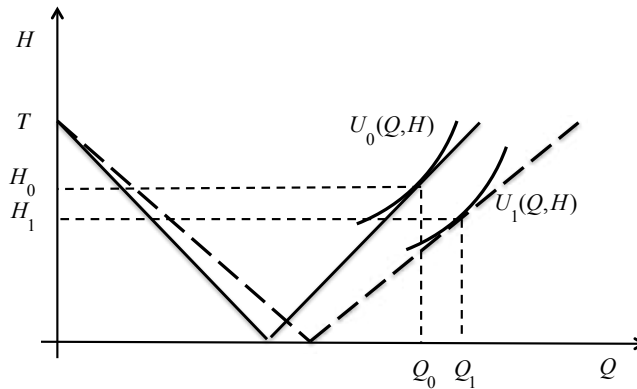

Fig.6. "Labor model" of behavior

We see that the moderate increase in marginal savings after the increase in wage rate results in the decrease in "bad" leisure and the increase in consumption. In addition, this increase in utility is followed by the decrease in the time of search due to positive ∂H/∂S relationship.

The last consideration requires the specification of this model of consumer behavior. While the redistribution of time discovers here an important propensity to search (the value ∂L/∂S is still ∂L/∂S<-1), it works in the opposite direction and therefore could hardly be esteemed as the "leisure model". The specification of the "labor model" seems to be more compatible with this type of behavior because here the consumer cuts both leisure and search in favor of labor.

This *laborholic* type of behavior is well known. Moreover, sometimes it looks like the manifestation of conspicuous labor (Bellezza et al.). Sales are organized for that kind of people because well-advertised sales save time of *laborholics* and keep their reputation of smart-shoppers. (Today this tradition is well developed by online shopping.) Unfortunately, and our favorite example of the table tennis bought on sales and got in a season its proper place in the garage confirms it, this kind of behavior leaves no time for consumption. And this is the fact of purchase and the following possession of a status item, a boat, may be, that becomes symbolic and in that sense conspicuous. This is the reason why sometimes the idea of the restriction on working hours seems to be an appropriate tool for the reduction of welfare losses of conspicuous consumption. However, the restriction on working hours raises the level of "bad" leisure and again it can be depreciated only by the respective increase in consumption (Fig.7):
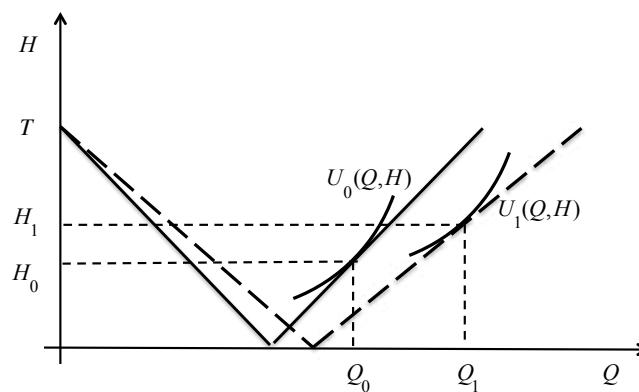

Fig.7. "Good" suits and "bad" parties

Fig.7 illustrates the other favorite example when a consumer likes suits but he dislikes parties. Here the "labor model" becomes almost undistinguishable from the "leisure model" of behavior because here consumers again increase both search and leisure time.

The dichotomy "normal consumption – "bad" leisure" has another important application. This model of behavior can illustrate the phenomenon of money illusion.

Part IV. Money illusion

The phenomenon of money illusion from the point of view of the "common model" means the change in consumer's preferences at the moment of purchase. If the consumer decides to buy more potatoes in the local store for the given time horizon he intensifies the consumption of potatoes.[2]

While the $w/\partial P/\partial S$ ratio is given by the choice of the local store, the change in preferences means the transformation and the following shift of the utility curve $U(Q,H)=Q^{-\partial L/\partial S}H^{\partial H/\partial S}$ in the north-west direction along the constraint line because the decision to buy more potatoes changes the respective $\partial L/\partial S$ and $\partial H/\partial S$ values when T and $w/\partial P/\partial S$ values stay constant.

The analysis of the transformation of utility under changes in the allocation of time discovers the reference point U_0 where the utility gets its minimum level for given T and $w/\partial P/\partial S$ values. It is easy to demonstrate that the north-west shift of the utility function from U_C to U_0 decreases the utility level. However, the shift from U_0 to U_S increases the utility level (Fig.8):
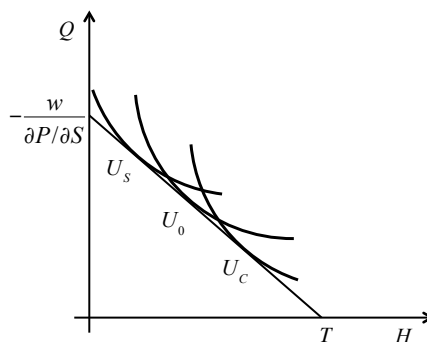

Fig.8. Net leisure substitutes and net leisure complements

It can happen because in the U_C area consumption complements leisure. And the decrease in leisure time is not compensated by the increase in consumption that results in the decline in utility level. Otherwise, if consumption substitutes leisure in the U_S area the increase in consumption is reasonable and it raises the utility level. Here the trade-off between net complements and net substitutes looks like the corresponding trade-off between net borrowing and net saving around the endowment point. We can also find an indifference curve for the $U_S;U_C$ values that discovers the relative nature of utility with regard to leisure complementarity/substitutability. However, from the point of view of money illusion things look

---

[2] Here we pay attention to the fact that the time horizon is given or $T{\neq}T(Q)$. The analysis of the choice under $T=T(Q)$ assumption stays beyond the scope of this paper. Partially that analysis was presented in the examination of the phenomenon of sunk costs sensitivity (Malakhov 2014 b)

rather different. When all prices and wage rates rise in the same proportion we should meet the unit elasticity of absolute marginal savings with respect to the wage rate, or $e_{|\partial P/\partial S|,w} = 1$. The change in all prices and wage rates does not change the constraint of the consumer's choice. And, if money illusion occurs it takes places only for leisure substitutes but not for leisure complements. The purchase of more potatoes after the proportional increase in prices and wage rates decreases the utility level while the purchase of more hours in *aqualand* is reasonable.

There is another group of consumers who can really get more from the increase in prices and wages. There are persons who spend on market and non-market work significantly more than a half of time. And while home production can be presented as the form of search, and Aguiar and Hurst (2007b) came to that assumption, when they compared home production with shopping with the filter of price reduction, we can expect that *home laborholics* can really get advantage from the simultaneous increase in prices and wages. Unfortunately, this group is not representative. According to Aguiar and Hurst (2007a) the greatest total non-leisure time (market work + non-market work+ child care) was obtained during last five decades in the USA in 1965, when men and women spent on non-leisure time 61.74 and 62.69 hours per week respectively.

Hence, the "common model" leaves too little space for reasonable money illusion. However, this is not true for the "labor model" of behavior. We can see that the shift of the utility curve along the constraint line, here in the north-east direction, can really increase the utility level from U_0 to U_1 (Fig.9):
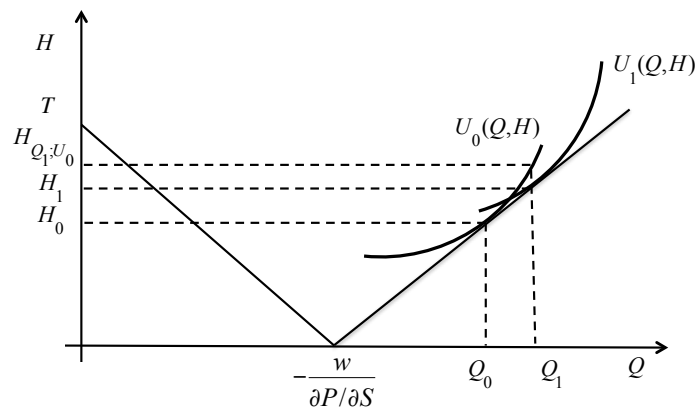
Fig.9. Money illusion

Indeed, this is the shape of the utility function that produces the increase of utility with the shift along the constraint line. This shift results in the increase in normal consumption form Q_0 to Q_1 and the increase in "bad" leisure from H_0 to H_1. However, if a consumer moves along the original indifference curve, he gets the same increase in consumption with greater amount $H_{Q1,U0}$ of "bad" leisure.

This consideration results in the reverse question – if money illusion is really an evident phenomenon, may be, the "labor model" of behavior is a dominant? Unfortunately, it was mentioned above that when the "labor model" demonstrates the decrease in labor in favor of both search and leisure, it looks like the "leisure model". The difference between two models in this case can be found only if we agree with relativity of the concept of negative marginal utility of money. Intuitively, the evident waste of money manifests their negative marginal utility and, therefore, the "leisure model" of behavior but the question of waste of money is very subjective. When we make the comparison of the allocation of time in 1965 and in 2003 based on the data from Aguiar and Hurst (2007a), we see that during this period women increased the time for total market work and the time for leisure, while they decreased the time for non-market work. Hence, we could suppose that during that period women generally followed the "common model" of behavior. But when we take the data for men, we see the decrease in the market work and the increase in both leisure and non-market work. And we cannot definitely answer whether it was the manifestation of the *home laborholic* trend, specified in Malakhov (2012) as "gardening aboard the boat effect", or it covered the waste of money under the "leisure model" of behavior.

Conclusion

The very profound analysis of welfare effects of conspicuous consumption and conspicuous/inconspicuous leisure, presented in Arrow and Dasgupta (2009), discovered different relationships of both consumption and labor supply with a social optimum. In particular, the combination of conspicuous consumption with inconspicuous leisure results in consumption and labor supply over the social optimum. That conclusion corresponds to properties of the "labor model" of behavior. Arrow and Dasgupta also paid attention to the ambiguity of a welfare effect when both consumption and leisure were conspicuous. The paper presented here explains that ambiguity when it recognizes the possibility of visual resemblance of "labor" and "leisure" models of behavior. To makes things really divisible one needs to accept the relativity of the concept of the optimum quantity of money with respect to different consumption patters and different living standards in order to explain the waste of money and therefore their negative marginal utility even on low social levels.

References

1. Aguiar, M., and Hurst, E. (2007a) 'Measuring Trends in Leisure: The Allocation of Time Over Five Decades' *Quarterly Journal of Economics*, 122 (3), 969-1006

2. Aguiar, M., and Hurst, E. (2007b) 'Life-Cycle Prices and Production', *American Economic Review*, 97(3), 1533-1559

3. Arrow, R.J.,Dasgupta P.S. (2009) 'Conspicious Consumption, Inconspicious Leisure', *Economic Journal*, 119 (541), 497-516

4. Bellezza, S., Keinan, A., Paharia, N. (2014) 'Conspicuous Consumption of Time: When Busyness at Work and Lack of Leisure Time Become a Status Symbol' Manuscript (http://www.rotman.utoronto.ca/-/media/Files/Programs-and-Areas/Marketing/papers/BELLEZZA-TheRedSneakersEffect2.pdf)

5. Malakhov, S. (2012) 'Veblen effect, Search for Status Goods, and Negative Utility of Conspicuous Leisure', *Journal of Institutional Studies*, 4(3), 6-21 (http://works.bepress.com/sergey_malakhov/5/)

6. Malakhov, S. (2013) 'Money Flexibility and Optimal Consumption-Leisure Choice' *Theoretical and Practical Research in Economic Fields*, IV (1), 77-88. (http://www.asers.eu/asers_files/tpref/TPREF%20Volume%20IV%20Issue%201_7_%20Summer%202013_last.pdf)

7. Malakhov, S. (2014a) 'Satisficing Decision Procedure and Optimal Consumption-Leisure Choice.' *International Journal of Social Science Research,* 2 (2), 138-151 (http://dx.doi.org/10.5296/ijssr.v2i2.6158)

8. Malakhov, S. (2014b). 'Sunk Costs of Consumer Search: Economic Rationality of Satisficing Decision', *Theoretical and Practical Research in Economic Fields*. V: 1(9), 53-62 (http://www.asers.eu/asers_files/tpref/TPREF%20Volume%20V%20Issue%201(9)%20Summer%202014.pdf)

9. Malakhov, S. (2014c) 'Willingness to Overpay for Insurance and for Consumer Credit: Search and Risk Behavior Under Price Dispersion', *Expert Journal of Economics*, 2(3), 109-119(http://economics.expertjournals.com/wp-content/uploads/EJE_213malakhov2014pp109-119.pdf)