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Abstract. The Walrasian model has played a central role in all aspects of economics. The purpose of this paper is to propose a general modeling of differential information in the spirit of the Arrow-Debreu model and following the Radner tradition but away from the usual measurability conception. We provide a characterization of better informed agents in order to firstly formalize an essential feature known as part of some oral tradition: a decisionmaker prefers more information to less.

Key words: Uncertainty, differential information, information sets, informational feasibility and better informed agents

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1 Introduction.

Uncertainty and information are central issues in economic analysis: economic decisions, contracts among agents and, in fact, all economic activities take place under conditions of general uncertainty and involve incomplete information. Research in economic theory has long recognized their importance and has shown that, regarding the introduction of uncertainty and information into general equilibrium analysis, the Arrow-Debreu model has turned to be a solid framework.

The incorporation of exogenous uncertainty into the classical Walrasian equilibrium model was made by means of a set of possible states of nature that is built into the definition of a commodity. Modeling uncertainty in this way and the resulting "state-contingent" approach to uncertainty was first introduced by Arrow (1953) and further detailed by Debreu (1959) in Chapter 7 of his "Theory of Value" and, precisely, was made famous in the late 1960s with the seminal work of Radner (1968) in which differential information was considered in a model of general equilibrium for the first time.

"Asymmetric in information" emerges when information is incomplete and lack of information exhibits different degrees from one agent to the other. Every agent is assumed to have her own private information structure concerning exogenous uncertainty, that is, a mechanism which she uses to perceive the world and only these observation results allow the agent to draw conclusions about the states of nature. So agents do not necessarily know which state of nature has actually occur and what each agent observes is element of her information set, so-called an event, containing the realized state of nature.

Radner's work (1968) is particularly relevant in this context and gives rise to a growing literature in equilibrium theory. These works, usually referred as differential information economies, consider that an agent's information is described either by a $\sigma-$ algebra or, for the case of finitely many states of nature, by a partition of the set of states. All these models of economies with asymmetric information share the same assumption regarding the impact of information on an agent's decisions: incomplete information restricts an agent's choice of action plans in such a way that her admissible plans have to be measurable with respect to her own private information. Up until now, all studies has focused on this Radner-type approach (e.g. Yannelis

(1991), Koutsougeras and Yannelis (2003) or Glycopantis, Muir and Yannelis (2003, 2004)). As Glycopantis and Yannelis (2004) recently note "*we believe that the natural and intuitive way to proceed is to analyze concepts in terms of measurability of allocations*".

Our analysis will develop in this context of general equilibrium in the spirit of Radner but differs from the standard measurability outline. This paper deals with different approach to differential information where information is not about measurability. How private information describes the realizations an agent can discern? This question leads us to concentrate on highlight that an agent's private information on the environment does not permit discrimination among the states belonging to any element of her information structure. We consider two alternative scenarios-the set of states of the world can either be a set of points or a measurable space. In either case, our modeling of information goes to a step further: private information is considered as an application mapping states of nature to future realizations or results, usually referred in the literature as events. One of the contributions of this paper is to provide a unified framework in which both outlines-partitions and σ-algebras- can be accommodated.

Our aim is to analyze how imperfect information places a restriction on the feasible action plans. We present a new specification of this constraint by introducing the notion of the set of admissible action plans determined by an agent's information. In this fashion, we set up a partial order on the information of the agents that empowers us to establish not only when an agent is at least or even better informed than the others but also that better informed agents, due to asymmetric information constraints, choose their plans in a bigger subspace.

All along this paper we consider a simple example which enables us not only to illustrate to bear witness to a very recent work by Dubra and Echenique (2004). In that paper, they propose a modeling of information based on the use of σ-algebras and provide an example from which they conclude that in a differential information model "a decision maker prefers less information to more". Being cognizant of their wrong argument and their completely erroneous conclusion, we finally formalize an essential principle in the differential information analysis: an agent is going to be better off endowed with more information.

The rest of the paper is organized as follows: in Section 2 our modeling of information is developed. The notion of uncertainty realizations is defined in such a way that all the states the agent is not able to distinguish conform a realization. Thus, an agent's information describes which realizations this agent can discern. In Section 3 it is discussed that impact of information on an agent's decisions. We present a new specification of this constraint by introducing the concepts of informational feasibility and the set of admissible action plans determined by an agent's information. In Section 4 we state a characterization of economic agents by their information structures that permit us to decide who the better informed agents are and to firstly formalize the basic proposition that more information is preferred to less. Finally, in Section 5 we present different examples that permit us to illustrate our approach.

2 The information of the agents.

As mentioned in the introduction, the analysis of economies with asymmetric information tackles two essential elements. Agents are assumed to be incomplete informed and also differences in information among economic agents are allowed. This concept of information is often interpreted in the literature in the following way: every agent is assigned an information structure on the set of states of the world and if an element of her information set comprises several states of nature, the agent will not be able to discriminate among those states. Being mindful of this formulation, let us now introduce our differential information framework.

Let T be the set of agents and let P_t denote the information set of each agent $t \in T$. Each agent $t \in T$ has access only to a mechanism which she uses to perceive the world. Let (Ω, P_t) denote the realizations of the states that agent t observes in accordance with her own information P_t. The idea is that the states contained in an element of (Ω, P_t) cannot be made out under P_t. As different agents are assumed to have different information, any other agent t' with private information $P_{t'}$ beholds $(\Omega, P_{t'})$ that can be distinct from (Ω, P_t) "recognized" by agent t.

Firstly, we consider that exogenous uncertainty is represented by a finite set of states of nature and let Ω be a set of points. In this context, the private information of an agent t is usually represented in the literature by an application $P_t : \Omega \to 2^{\Omega}$. For each $\omega \in \Omega$, denote by $P_t(\omega)$ the element of her information structure that contains ω. Then, $P_t(\omega) = P_t(\omega')$ whenever the states ω and ω' are indistinguishable by agent t under information P_t. Thus, private information P_t

arranges the states of the world by means of an equivalence relation $\sim_t$ in such a way that two states of nature ω and ω' are perceived by agent t as the same uncertainty realization whenever $P_t(\omega) = P_t(\omega')$. Therefore, the equivalence relation $\sim_t$ is defined by P_t as follows:

$$\omega \sim_t \omega' \text{ if and only if } P_t(\omega) = P_t(\omega').$$

Note that this equivalence relation classifies the set Ω as equivalence classes: if ω is the true state of nature, agent t observes the event $[\omega]$ which represents the equivalence class of ω. Hence, the agent's information is defined by an application $P_t : \Omega \to (\Omega, P_t)$ that describes the set of realizations this agent can discern in Ω as the set of equivalence classes, i.e., the quotient set $\Omega/\sim_t = \{[\omega]_t = \{\omega' \; ; \; \omega' \sim_t \omega\} \; ; \; \omega \in \Omega\}$. So $(\Omega, P_t) = \Omega/\sim_t$.

Now suppose that uncertainty is modelled by a measurable space $(\Omega, \mathcal{B}, \mu)$ where $\mathcal{B}$ is a σ-algebra denoting the set of all possible results and μ is probability measure on $\mathcal{B}$. In this context, agent t recognizes the possible future outcomes of uncertainty by using a σ-algebra $\mathcal{B}_t$ which is coarser than $\mathcal{B}$. The fact that $\mathcal{B}_t$ is coarser than $\mathcal{B}$ implies that the identity map $P_t : (\Omega, \mathcal{B}, \mu) \to (\Omega, \mathcal{B}_t, \mu)$ is measurable [1]. Therefore, the environment realizations observed by agent t when her information is P_t are represented by a measurable space $(\Omega, \mathcal{B}_t, \mu)$ so the set of events she realizes is $(\Omega, P_t) = (\Omega, \mathcal{B}_t, \mu)$.

Note that, in both differential information contexts, each agent $t \in T$ with private information P_t perceives the environment as realizations (Ω, P_t). So a *private information structure maps states of the world to uncertainty realizations, so-called events.* We remark precisely that our definition of information provides a single approach and allows us for taking into account both aspects of uncertainty and information.

At this point, we drew our attention to the example from Dubra and Echenique (2004). There, *the state of the world can be a real number between* 0 *and* 1*... a decisionmaker can choose either be perfectly informed, so that she gets to know the exact value of* ω *or only be told if the true* ω *is smaller or larger that* $1/2$. From now on, this situation is referred in our work as Example 1 and, as we will show in Section 5, more information is going to be preferred to less which contradicts the result of Dubra and Echenique.

[1] Mathematically, a function $f : (\Omega, \mathcal{B}, \mu) \to (\Omega, \mathcal{B}_t, \mu)$ is measurable if and only if $f^{-1}(A) \in \mathcal{B}$ for any open set A in $\mathcal{B}_t$.

Example 1. Let $\Omega = [0,1]$ be the set of possible states of nature. We consider two agents, 1 y 2, whose information is P_1 and P_2. Agent 1 is assumed to be incompletely informed about Ω and only knows if the realized state of nature is smaller or larger than 1/2. Agent 2 is supposed to be perfectly informed and knows the state ω that has been exactly realized.

Agent 1 under $P_1 : [0,1] \to (\Omega, P_1)$, understands uncertainty as two equivalence classes $[\omega_1]_1 = [\omega < 1/2]_1 = [0, 1/2)$ and $[\omega_2]_1 = [\omega \geq 1/2]_1 = [1/2, 1]$. Then, her information P_1 leads to know two events and $(\Omega, P_1) = \Omega/\sim_1 = \{[\omega_1]_1, [\omega_2]_1\}$.

Agent 2, with her information $P_2 : [0,1] \to (\Omega, P_2)$, is able to detect all states of the world, i.e., $\omega \sim_2 \omega'$ if and only if $\omega = \omega'$. In this case, the set of events (Ω, P_2) coincides with the set of states of nature, i.e., $(\Omega, P_2) = \Omega/\sim_2 = \{\{\omega\}\,;\, \omega \in [0,1]\} = [0,1]$.

3 The impact of information on action plans. The informational feasibility constraint.

In this section we analyze the basic assumption regarding the impact of information on agents decisions.

The essential feature in Debreu's approach to uncertainty is that each state of the world determines the entire history of all aspects of the economy that are beyond the control of any of the agents. Then, action plans are indexed by the state of nature and all agents are supposed to choose their plans in the same state-contingent commodity space. Let B denote the set of commodities in each state. An action plan f is defined by a function that maps states of nature (Ω or $(\Omega, \mathcal{B}, \mu)$ depending on the uncertainty representation) into B.

Once the asymmetries in information are incorporated, every agent that is incompletely informed must make a decision without being able to discern all the states: she cannot choose different plans on those states she is not able to distinguish. The agents are then supposed to choose her action plans in a subset of the contingent commodity space defined as the set of all state-dependent commodities that are compatible with her own information. Therefore, in contrast with a large part of literature on general equilibrium, due to the information constraints, their choice is limited to smaller subsets that indeed differ from agent to agent. Keeping in mind this basic feature we firstly introduce a new notion of information feasibility for admissible action plans:

Definition 3.1 *An action plan f is* ***informationally feasible*** *for an agent t with information P_t if and only if f is factorized by P_t, i.e , if*

$$\begin{array}{ccccc} & & f & & \\ \{\textit{states of nature}\} & & \longrightarrow & & B \\ & & & & \\ P_t & \searrow & & \nearrow \tilde{f} & \qquad f = \tilde{f} \circ P_t \\ & & (\Omega, P_t) & & \end{array}$$

is a commutative diagram.

The idea of our characterization is that each agent's private information factorizes state-dependent action into those that are informationally compatible so the admissible action plans for the generic agent t are interpreted as a correspondence from the set of realizations (Ω, P_t) she realizes under P_t into B. Note that our definition differs from that formulation in terms of measurability introduced by Radner (1968, p.37) but provides a common framework that comprises both settings regarding the exogenous uncertainty representation.

We now turn our attention to the factorization mechanism running. Let us first move to our initial outline and suppose that Ω is the set of states of the world. In this context, private information of agent t is represented by $P_t : \Omega \to \Omega/\sim_t$ and an action plan defined by $f : \Omega \to B$ is said to be informationally feasible for agent t if and only if there exists $\tilde{f} : \Omega/\sim_t \to B$ such that $f = \tilde{f} \circ P_t$, where $P_t(\omega) = [\omega]_t$

$$\begin{array}{ccccc} & & f & & \\ \Omega & & \longrightarrow & & B \\ & & & & \\ P_t & \searrow & & \nearrow \tilde{f} & \\ & & (\Omega, P_t) = \Omega/\sim_t & & \end{array}$$

In the measurable space background, $(\Omega, \mathcal{B}, \mu)$ represents the possible states of nature and $(\Omega, \mathcal{B}_t, \mu)$ shows the set of events that agent t can discern under P_t. An action plan $f : (\Omega, \mathcal{B}, \mu) \rightarrow B$ is said to be compatible with her own private information if and only if there exists $\tilde{f} : (\Omega, \mathcal{B}_t, \mu) \rightarrow B$ such that $f = \tilde{f} \circ P_t$

$$
\begin{array}{ccc}
(\Omega, \mathcal{B}, \mu) & \stackrel{f}{\longrightarrow} & B \\
P_t \quad \searrow & & \nearrow \tilde{f} \\
 & (\Omega, P_t) = (\Omega, \mathcal{B}_t, \mu) &
\end{array}
$$

In this case, her information represented by $P_t : (\Omega, \mathcal{B}, \mu) \rightarrow (\Omega, \mathcal{B}_t, \mu)$ is the identity map so $P_t(\omega) = \omega$. Note that for P_t to be well defined, i.e, for $f = \tilde{f} \circ P_t$ to be well defined, it is necessary that if any two functions f y g are $\mathcal{B}$-measurable and such that $[f] = [g]$ with respect to the σ-algebra $\mathcal{B}$, it should be verified that $[\tilde{f}] = [\tilde{g}]$ and, precisely, it is enough that $\tilde{f}$ and $\tilde{g}$ are $\mathcal{B}_t$-measurable. Therefore, it is required that the identity $P_t : (\Omega, \mathcal{B}, \mu) \rightarrow (\Omega, \mathcal{B}_t, \mu)$ is measurable.

This position leads us to incorporate the information constraint in each agent's action plans set so at this point we introduce an essential concept in our work: the *subspace of action plans that are informationally feasible.*

Let $\mathbb{P}_t = \{f : \Omega \rightarrow B \ ; \ \exists \ \tilde{f} : (\Omega, P_t) \rightarrow B \text{ such that } f = \tilde{f} \circ P_t\}$ denote the set of action plans that are compatible with the information P_t of agent t. Note that in $\mathbb{P}_t$ the following two operations are hold:

(i) (*Addition*) For all $f, g \in \mathbb{P}_t$ the sum $f + g \in \mathbb{P}_t$.

(ii) (*Scalar multiplication*) For all $f \in \mathbb{P}_t$ and any scalar λ, $\lambda \cdot f \in \mathbb{P}_t$.

so the set $\mathbb{P}_t$ is a vector space[2]. Observe that not any vector space is betoken for defining the

[2]It is easy to prove that if $f, g \in \mathbb{P}_t$ there exit $\tilde{f} : (\Omega, P_t) \rightarrow B$ and $\tilde{g} : (\Omega, P_t) \rightarrow B$ such that $f = \tilde{f} \circ P_t$ and

subspace of action plans satisfying the information feasibility assumption. A basic proposition regarding the subspace $\mathbb{P}_t \subset \{f : \Omega \to B\}$ is that, for representing the set of action plans that are informationally attanaible for a generic agent t, it must contain constant functions [3] This assumption is justified by the fact that in this context an agent can choose identical actions contingent on all the states of the world. We stress that constant functions are always informationally feasible and, in fact, are the only one elements in the set $\mathbb{P}_t$ if agent t cannot discriminate any state in the environment set. In this case, if $\mathbb{P}_t = \{f : \Omega \to B\,;\ f \text{ constant}\}$ then set of events (Ω, P_t) comes down to a point, i.e., either $\Omega/\sim_t = \{[\omega]_t\}$ or the σ-algebra $\mathcal{B}_t$ is the trivial σ-álgebra $\mathcal{B}_t = \{\Omega, \emptyset\}$ depending uncertainty model.

A case in point is that where $\Omega = \{\omega_1, \ldots, \omega_k\}$ is a finite set. In this position the achievable action plans for an agent t are given by

$$\mathbb{P}_t = \{(x(\omega_1), \ldots, x(\omega_k)) \in B^k\ ;\ x(\omega_j) = x(\omega_{j'}) \text{ si } P_t(\omega_j) = P_t(\omega_{j'})\}.$$

Note that if $x = (x(\omega_1), \ldots, x(\omega_k)) \in \mathbb{P}_t$ then it follows that $x + \lambda \cdot \bar{1} = (x(\omega_1) + \lambda, \ldots, x(\omega_k) + \lambda) \in \mathbb{P}_t$, for any λ.

There again, suppose that agent t makes out the state ω_j, i.e., $P_t(\omega_j) = [\omega_j]_t = \{\omega_j\}$. This implies that this agent's subspace of admissible action plans in the informational sense has none restriction on this state ω_j. Formally, if $(x(\omega_1), \ldots, x(\omega_j), \ldots, x(\omega_k)) \in \mathbb{P}_t$ then it follows that $(x(\omega_1), \ldots, x, \ldots, x(\omega_k)) \in \mathbb{P}_t$ for any $x \in B$.

Hence, the requirement of informational consistency obviously implies that the subspace of action plans achievable for an agent t who is incompletely informed about the states of nature is restricted to the set $\mathbb{P}_t$ that we characterize as the subspace of action plans that are informationally feasible and, therefore, compatible with the private information P_t.

$g = \tilde{g} \circ P_t$ respectively. Then, $f + g = (\tilde{f} + \tilde{g}) \circ P_t \in \mathbf{P}_t$ and $\lambda \cdot f = \lambda \cdot (\tilde{f} \circ P_t) = (\lambda \cdot \tilde{f}) \circ P_t \in \mathbf{P}_t$.

[3] If f is a constant function then $f(\omega) = k$ for all ω so $\tilde{f} : (\Omega, P_t) \to B$ where $\tilde{f}(\cdot) = k$ is well defined and verifies that $f = \tilde{f} \circ P_t$ and, therefore, $f \in \mathbf{P}_t$.

4 Better informed agents.

Our main concern in this section is to balance private information of different economic agents by providing an order in their information sets. However, it is an essential feature that this order must be necessarily partial on account off that information structures may not be comparable. Note that the informationally feasible action plans common for different economic agents can be restricted to constant functions that, as we have already point out, are always attainable in the informational sense (see Example B).

Precisely, this setting in which different economic agents' information sets are comparable is our starting point. Our claim is that in the position of more information, an agent recognizes more realizations into the set of possible states of the world and this, in point of fact, allows for more action plans contingent on discernable events. On a first stage our aim is to define an application relating the set of uncertainty realizations that information structures describe for each agent and that enable us to check whether an agent is better informed than any other one. Secondly, we show that better informed agents make their choices in a broader set of admissible action plans. This permit us to demonstrate that in this position an agent is better off so we conclude that an economic agent always prefers more information to less in contradiction with the work of Dubra and Echenique (2004).

We consider two agents t and t' with private information P_t and $P_{t'}$ respectively.

Definition 4.1 *Agent t is at least as informed as t' if and only if environment realizations that t is not able to discern cannot be either distinguished by agent t'.*

This definition expresses the idea that what a better informed agent cannot recognized is not either discriminated by a worse informed agent.

In the measurable space uncertainty setting, Definition 4.1 implies that agent t is going to discriminate more events than t', so her σ-algebra is finer than that belonging to agent t'. Therefore, in this case agent t is said to be at least as informed as t' only if the identity map given by $i : (\Omega, \mathcal{B}_t, \mu) \rightarrow (\Omega, \mathcal{B}_{t'}, \mu)$ is measurable.

If Ω is a set of points, the idea is that two any states of nature ω and ω' that are not recognized by agent t cannot be either discerned by individual t'. Note that if t cannot discriminate between ω and ω' then $P_t(\omega) = P_t(\omega')$ so agent t is said to be at least as informed as any other t' whenever $P_t(\omega) = P_t(\omega')$ and $P_{t'}(\omega) = P_{t'}(\omega')$. This implies that the application

$$\begin{array}{lcccl} i: & (\Omega, P_t) & \rightarrow & (\Omega, P_{t'}) & \\ & [\omega]_t & \rightarrow & i([\omega]_t) & = [\omega]_{t'} \end{array}$$

is well defined. Observe that the application i is well defined if for any two states of nature ω y ω' such that $[\omega]_t = [\omega']_t$, it is satisfied that $[\omega]_{t'} = [\omega']_{t'}$. Therefore, it is the existence of $i : (\Omega, P_t) \rightarrow (\Omega, P_{t'})$ what determines that t is at least as informed as t'. The fact is that if two any states ω and ω' are understood by t as the same realization, i.e., $P_t(\omega) = P_t(\omega')$, then it must occur that $i([\omega]_t) = i([\omega']_t)$ which only helds if $[\omega]_{t'} = [\omega']_{t'}$, that is, only if $P_{t'}(\omega) = P_{t'}(\omega')$.

Next we state a basic characterization:

Proposition 4.1 *Agent t is at least as informed as agent t' if and only if $P_{t'}$ is factorized by P_t, that is, if the application given by $i : (\Omega, P_t) \rightarrow (\Omega, P_{t'})$ such that $P_{t'} = i \circ P_t$ is well defined.*

$$\begin{array}{ccccc} & & P_{t'} & & \\ & \Omega & \longrightarrow & (\Omega, P_{t'}) & \\ P_t & \downarrow & \nearrow & & i \\ & (\Omega, P_t) & & & \end{array}$$

Really, as we have already mentioned, it is the existence of the application $i : (\Omega, P_t) \rightarrow (\Omega, P_{t'})$ what enables us to say that agent t is at least as informed as t'. Let us specify the intuition underlying our factorization of $P_{t'}$ by P_t or, equivalently, that i is well defined: if (Ω, P_t) is a measurable space, this factorization means the measurability of application i; if (Ω, P_t) is the set of equivalence classes, the factorization implies that the identity map $i : [\omega]_t \in (\Omega, P_t) \rightarrow i([\omega]_t) = [\omega]_{t'} \in (\Omega, P_{t'})$ is well defined, i.e., the image of the equivalence class does not depend on the representant considered.

Draw to a close, we now turn over in our minds the examples stated in Section 2. There, the private information of each agent describes the events $(\Omega, P_1) = \Omega/\sim_1 = \{[\omega_1]_1, [\omega_2]_1\}$ and

$(\Omega, P_2) = \Omega/\sim_2 = \{\{\omega\}\ ;\ \omega \in \Omega\}$. In this case,

$$\begin{array}{rccc} i: & (\Omega, P_2) & \to & (\Omega, P_1) \\ & [\omega]_2 & \to & i([\omega]_2) = [\omega]_1 \end{array}$$

is a well defined application and, therefore, agent 2 is better informed than 1.

Here and now, agent t is supposed to be at least as informed as t'. Our next goal is to show that agent t selects actions in the subspace $\mathbb{P}_t$ of plans satisfying the informational feasibility assumption. Obviously, if $\mathbb{P}_{t'} \subset \mathbb{P}_t$ agent t can choose all those action plans that are available for agent t' and, in the case of strict inclusion, is able to opt even for another plans that are not achievable for agent t'. Hence, our main concern is to show that better informed agents have more chances to choose her action plans than those agents who are worse informed.

Theorem 4.1 *Agent t is at least as informed as any other individual t' if and only if $\mathbb{P}_{t'} \subset \mathbb{P}_t$.*

Demonstration.

Let t and t' be two agents whose private information is P_t and $P_{t'}$ respectively. Now assume that agent t is at least as informed as agent t'. By Proposition 4.1, this is equivalent to the existence of $i : (\Omega, P_t) \to (\Omega, P_{t'})$ such that $P_{t'} = i \circ P_t$

$$\begin{array}{ccccccc} & & & & f & & \\ & \Omega & & & \longrightarrow & B & \\ & & \searrow & & & & \\ P_t & \downarrow & & P_{t'} & & \uparrow & \tilde{f} \\ & & & & \searrow & & \\ & (\Omega, P_t) & & & \longrightarrow & (\Omega, P_{t'}) & \\ & & & & i & & \end{array}$$

The fact $f \in \mathbb{P}_{t'}$ implies that $f = \tilde{f} \circ P_{t'} = \tilde{f} \circ i \circ P_t$. Then, f is factorized by P_t via $\tilde{f} \circ i$, and this leads to $f \in \mathbb{P}_t$.

Q.E.D.

This Theorem highlights that better informed agents are less restricted, due to the information restrictions, when choosing their actions than those other worse informed agents. Then, chances of agent t' are all included in $\mathbb{P}_t$ and indeed are all possible and attainable for agent t. Extreme cases are those situations in which an agent can either be completely informed or not informed at all: in the case of perfect information, the agent has no constraint, beside the classical ones, when choosing her plans; on the contrary, if the agent is not able to discern any state, she is going to perceive the set of possible states of the world as a point and, in this case, her achievable plans are in the set of constant functions.

In this final section, we want to complete our view over our characterization of better informed agents by illustrating Theorem 4.1. Let us turn once again to the the setting provided in Example 1. There, agent 1 is completely informed and realizes Ω as the two events defined by $[\omega_1]_1$ and $[\omega_2]_1$; her set of admissible action plans is $\mathbb{P}_1 = \{f : [0,1] \to B \ ; \ \exists \ \tilde{f} : (\Omega, P_1) \to B$ such that $f = \tilde{f} \circ P_1\}$ so $f = \tilde{f} \circ P_1$ if and only if $f(z) = f(z')$ for all $z, z' \in [\omega_i]_1$ with $i = 1, 2$. Then, $f \in \mathbb{P}_1$ only if f is constant on the two events she perceives, i.e., $\mathbb{P}_1 = \{f : [0,1] \to B \ ; \ f(z) = a$ for all $z \in [0, 1/2)$ y $f(\hat{z}) = b$ for all $\hat{z} \in [1/2, 1]\}$. Nevertheless, agent 2 is completely informed and exactly knows the realized state of nature so she will choose her action plans in the set $\mathbb{P}_2 = \{f : [0,1] \to B \ ; \ \exists \ \tilde{f} : (\Omega, P_2) \to B$ such that $f = \tilde{f} \circ P_2\} = \{f : [0,1] \to B\}$. Consequently, any function $f : [0,1] \to B$ is an informationally admissible action plan for agent 2. Obviously, $\mathbb{P}_1 \subset \mathbb{P}_2$.

Therefore, by considering the same setting as that provided by Dubra and Echenique (2004) we have come to the opposite conclusion. Their point is the use of σ-algebras as a model of information based on the fact that finer partitions need not generate finer σ-algebras. This is in direct conflict to a well know part of some oral tradition in the literature on "asymmetric in information". The results stated and proved in this paper allow us to firstly formalize the following basic proposition that, although it is well known, has not appeared in print yet: finer information allows, in accordance with the informational constraint, for more action plans contingent on events each agent can discern and then, the more information an agent has, the larger is the subspace of informationally action plans attainable. We formally draw the following conclusion: an agent always prefers more information to less.

5 Some examples

To complete our view over original modeling of information in the spirit of the Radner approach, we provide some examples. We eye up all possible situations about the representation of information depending on whether the uncertainty is represented by a measurable space, a finite set of the states of the world and even the unit real interval $[0,1]$.

Example A. Consider that the possible states of nature are represented by a measurable space $([0,1], \mathcal{B}, \mu)$ where $\mathcal{B}$ is the Borel σ-algebra and μ is the Lebesgue measure.

Let $\mathcal{B}_t$ be a sub-σ-algebra of $\mathcal{B}$. An agent t who is incompletely informed perceives the realized state of nature by $\mathcal{B}_t$ so $(\Omega, P_t) = ([0,1], \mathcal{B}_t, \mu)$.

In this context, admissible action plans in the informational sense for an agent who is completely informed on the possible states of the world are given by any measurable function such that $f : ([0,1], \mathcal{B}, \mu) \to B$. Now, suppose that there exists an agent t who, in accordance to her private information P_t, recognizes the realizations $([0,1], \mathcal{B}_t, \mu)$, so the set of action plans that are informationally admissible is

$$\mathbb{P}_t = \{f : ([0,1], \mathcal{B}, \mu) \to B \ ; \ \exists\, \tilde{f} : ([0,1], P_t) \to B \text{ such that } f = \tilde{f} \circ P_t\}.$$

Here, factorization means that $P_t : ([0,1], \mathcal{B}, \mu) \to ([0,1], \mathcal{B}_t, \mu)$ is measurable and, therefore, $\mathbb{P}_t = \{f : ([0,1], \mathcal{B}, \mu) \to B \ ; \ f \text{ is } \mathcal{B}_t - \text{measurable}\}$. If f and g are measurable functions then $\{\omega \in [0,1] \mid f(\omega) \neq g(\omega)\}$ is also measurable. Note that two action plans f y g are identical for agent t whenever f and g are $\mathcal{B}_t$-measurable functions that differ in a set of measure zero. That is,

$$f \sim_t g \iff g \in [f] = \begin{array}{l} \{h : ([0,1], \mathcal{B}_t, \mu) \to B \ \mathcal{B}_t\text{-measurable} \\ \quad \text{such that } \mu\{\omega \in [0,1], h(\omega) \neq f(\omega)\} = 0\}. \end{array}$$

Note that in this setting, it might occur that two $\mathcal{B}_t$ -measurable functions f and g can be indiscernible for agent t as $\mu\{\omega \in [0,1] \mid f(\omega) \neq g(\omega)\} = 0$. At the same time, for any other agent $t' \neq t$ either function f or g may not be admissible action plans in the way that they are not $\mathcal{B}_{t'}$ -measurable.

The subspace $\mathbb{P}_t$ of attainable action plans for agent t is the set of (classes of) measurable (or integrable, etc.) functions $f : (\Omega, \mathcal{B}, \mu) \to B$. If agent t is better informed than t', the identity map $i : (\Omega, \mathcal{B}_t, \mu) \to (\Omega, \mathcal{B}_{t'}, \mu)$ is measurable and, consequently, $\mathbb{P}_{t'} = \{f : (\Omega, \mathcal{B}, \mu) \to B \; ; \; f \text{ is } \mathcal{B}_{t'} - \text{measurable}\} \subseteq \mathbb{P}_t = \{f : (\Omega, \mathcal{B}, \mu) \to B \; ; \; f \text{ is } \mathcal{B}_t - \text{measurable}\}$.

Example B. Consider a three agents economy, $T = \{1, 2, 3\}$, with four states of nature $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. Let P_1, P_2 and P_3 denote the private information of each agent.

Consider the following information structure: P_1 does not allow agent 1 to make a distinction either between ω_1 and ω_2 or ω_3 and ω_4; agent 2 cannot distinguish any state under P_2 and agent 3 with P_3 identifies ω_1 with ω_3 and ω_2 with ω_4.

Let us see how private information establishes for each agent an equivalence relation among all those states she is not able to discern. In the case of agent 1, as $P_1(\omega_1) = P_1(\omega_2)$ and $P_1(\omega_3) = P_1(\omega_4)$, her information $P_1 : \Omega \to (\Omega, P_1)$ defines the following equivalence relations $\omega_1 \sim_1 \omega_2$ and $\omega_3 \sim_1 \omega_4$. Therefore, agent 1 knows two events in Ω given by two equivalence classes

$$[\omega_1]_1 = [\omega_2]_1 = \{\omega_1, \omega_2\} \text{ and } [\omega_3]_1 = [\omega_4]_1 = \{\omega_3, \omega_4\}$$

Agent 1 then observes $(\Omega, P_1) = \Omega/\sim_1 = \{[\omega_1]_1, [\omega_3]_1\}$ so $\Omega/\sim_1$ represents the two uncertainty realizations she is able to discern in Ω.

Agent 2 under the information $P_2 : \Omega \to (\Omega, P_2)$ cannot discriminate any state of the world so $P_2(\omega_1) = P_2(\omega_2) = P_2(\omega_3) = P_2(\omega_4)$, which implies that $\omega_1 \sim_2 \omega_2 \sim_2 \omega_3 \sim_2 \omega_4$. Thus, her information determines a set of events $\Omega/\sim_2$ with a unique element, that is, $(\Omega, P_2) = \Omega/\sim_2 = \{[\omega_1]_2\}$ where $[\omega_1]_2 = [\omega_2]_2 = [\omega_3]_2 = [\omega_4]_2$.

The information available for agent 3 is given by $P_3 : \Omega \to (\Omega, P_3)$ and implies that $P_3(\omega_1) = P_3(\omega_3)$ and $P_3(\omega_2) = P_3(\omega_4)$. So $\omega_1 \sim_3 \omega_3$ y $\omega_2 \sim_3 \omega_4$. In this case $(\Omega, P_3) = \Omega/\sim_3 = \{[\omega_1]_3, [\omega_2]_3\}$, where $[\omega_1]_3 = [\omega_3]_3 = \{\omega_1, \omega_3\}$ and $[\omega_2]_3 = [\omega_4]_3 = \{\omega_2, \omega_4\}$ are the two environment results that agent 3 makes out.

An action plan $f : \Omega \to B$ is now given by $f = (f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4))$. For being attainable for agent 1 with private information P_1, it must be an element of $\mathbb{P}_1 = \{f : \Omega \to B \ ; \ \exists \ \tilde{f} : (\Omega, P_1) \to B \text{ such that } f = \tilde{f} \circ P_1\}$ which implies that

$$f(\omega_1) = \tilde{f} \circ P_1(\omega_1) = \tilde{f}([\omega_1]_1)$$

$$f(\omega_2) = \tilde{f} \circ P_1(\omega_2) = \tilde{f}([\omega_2]_1) = \tilde{f}([\omega_1]_1)$$

$$f(\omega_3) = \tilde{f} \circ P_1(\omega_3) = \tilde{f}([\omega_3]_1)$$

$$f(\omega_4) = \tilde{f} \circ P_1(\omega_4) = \tilde{f}([\omega_4]_1) = \tilde{f}([\omega_3]_1).$$

Note that $f(\omega_1) = f(\omega_2)$ y $f(\omega_3) = f(\omega_4)$ so an action f is informationally admissible for agent 1 if and only if it takes at most two different values. Therefore, $\mathbb{P}_1 = \{(f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4)) \in B^4 \ ; \ f(\omega_1) = f(\omega_2) \text{ and } f(\omega_3) = f(\omega_4)\}$.

In the case of agent 2, an action $f = (f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4))$ is compatible with her information P_2 only if there exists $\tilde{f} : (\Omega, P_2) \to B$ such that $f = \tilde{f} \circ P_2$, so

$$f(\omega_1) = \tilde{f} \circ P_2(\omega_1) = \tilde{f}([\omega_1]_2)$$

$$f(\omega_2) = \tilde{f} \circ P_2(\omega_2) = \tilde{f}([\omega_2]_2) = \tilde{f}([\omega_1]_2)$$

$$f(\omega_3) = \tilde{f} \circ P_2(\omega_3) = \tilde{f}([\omega_3]_2) = \tilde{f}([\omega_1]_2)$$

$$f(\omega_4) = \tilde{f} \circ P_2(\omega_4) = \tilde{f}([\omega_4]_2) = \tilde{f}([\omega_1]_2)$$

Therefore, the set of informationally feasible action plans for agent 2 is given by $\mathbb{P}_2 = \{f : \Omega \to B \ ; \ \exists \ \tilde{f} : (\Omega, P_2) \to B \text{ such that } f = \tilde{f} \circ P_2\} = \{(f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4)) \in B^4 \ ; \ f(\omega_1) = f(\omega_2) = f(\omega_3) = f(\omega_4)\}$ which is a line in B^4 (i.e., the set of multipliers of any vector $x \in B^4$ with $x \neq 0$).

An action plan $f = (f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4))$ is allowable for agent 3 if and only if there exists $\tilde{f} : (\Omega, P_3) \to B$ such that $f = \tilde{f} \circ P_3$, so

$$f(\omega_1) = \tilde{f} \circ P_3(\omega_1) = \tilde{f}([\omega_1]_3)$$

$$f(\omega_2) = \tilde{f} \circ P_3(\omega_2) = \tilde{f}([\omega_2]_3)$$

$$f(\omega_3) = \tilde{f} \circ P_3(\omega_3) = \tilde{f}([\omega_3]_3) = \tilde{f}([\omega_1]_3)$$

$$f(\omega_4) = \tilde{f} \circ P_3(\omega_4) = \tilde{f}([\omega_4]_3) = \tilde{f}([\omega_2]_3)$$

As $f(\omega_1) = f(\omega_3)$ and $f(\omega_2) = f(\omega_4)$, it follows that an action plan f for agent 3 is in accordance with the two observations she realizes through her private information P_3. Then her set of action plans that are informationally feasible is $\mathbb{P}_3 = \{(f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4)) \in B^4 \ ; \ f(\omega_1) = f(\omega_3)$ and $f(\omega_2) = f(\omega_4)\}$.

We stress that the private information endowments of agent 1 and 3 are not comparable: each of these agents according to her own information observes Ω as a set of two events and the set $\mathbb{P} = \mathbb{P}_1 \bigcap \mathbb{P}_3$ of action plans held in common is given by

$$\mathbb{P} = \{(f(\omega_1), f(\omega_2), f(\omega_3), f(\omega_4)) \in B^4 \ ; \ f(\omega_1) = f(\omega_2) = f(\omega_3) = f(\omega_4)\},$$

and, therefore, the only admissible action plans are constant vectors on B^4. However, information structures of agents 1 and 3 are comparable with the information of agent 2. Note that agents 1 and 3 observes two possible realizations while agent 2 is not able to discern any state and realizes Ω as a unique point; therefore, the classification of agent 2 is coarser than that that of the other agents. Indeed, it is easy to see that those states that cannot be distinguished by 1 or 3 are not either observed by agent 2.

Let us first consider agents 1 and 3. On one hand, the application

$$\begin{array}{rlcll} i: & (\Omega, P_1) & \rightarrow & (\Omega, P_3) & \\ & [\omega]_1 & \rightarrow & i([\omega]_1) & = [\omega]_3 \end{array}$$

is not well defined as $[\omega_1]_1 = [\omega_2]_1$ but $i([\omega_1]_1) = [\omega_1]_3$ and $i([\omega_2]_1) = [\omega_2]_3$ where $[\omega_1]_3 \neq [\omega_2]_3$. On the other hand, the application

$$\begin{array}{rlcll} i: & (\Omega, P_3) & \rightarrow & (\Omega, P_1) & \\ & [\omega]_3 & \rightarrow & i([\omega]_3) & = [\omega]_1 \end{array}$$

is not well defined either. Then, neither agent 1 is better informed than agent 3 nor agent 3 is better informed than individual 1.

However, each of the following applications

$$\begin{array}{rlcll} i: & (\Omega, P_1) & \rightarrow & (\Omega, P_2) & \\ & [\omega]_1 & \rightarrow & i([\omega]_1) & = [\omega]_2 \end{array}$$

and

$$\begin{array}{rccl} i: & (\Omega, P_3) & \rightarrow & (\Omega, P_2) \\ & [\omega]_3 & \rightarrow & i([\omega]_3) \quad = [\omega]_2 \end{array}$$

is well defined. Consequently, we conclude that agents 1 and 3 are better informed than 2. The choice of admissible action plans for agents 1 and 3 is made in broader sets that the choice of agent 2 who has none information on Ω and, therefore, her set $\mathbb{P}_2$ is limited to those action plans constant on B^4. So $\mathbb{P}_2 \subset \mathbb{P}_1$ and $\mathbb{P}_2 \subset \mathbb{P}_3$.

Example C. Suppose that there is a finite set of agents $T = \{1, 2, \ldots, N\}$ and the set of possible states of the world is the interval $[0, 1]$. The information of each agent $t \in \{1, 2, \ldots, N\}$ is assumed to be a partition of $[0, 1]$ given by $P_t = \{0 < x_1^t < \ldots < x_{n(t)}^t = 1\}$.

Each agent t "classifies" the uncertainty set Ω as $n(t)$ events so $(\Omega, P_t) = \{[\omega_1]_t, \ldots, [\omega_{n(t)}]_t\}$, where $[\omega_1]_t = [0, x_1^t), \ldots, [\omega_{n(t)}]_t = [x_{n(t)-1}^t, x_{n(t)}^t]$. In this case, the set of events (Ω, P_t) is formed by $n(t)$ elements.

The set of action plans satisfying the informational feasibility constraint for an agent t is $\mathbb{P}_t = \{f : [0, 1] \rightarrow B \ ; \ \exists \ \tilde{f} : (\Omega, P_t) \rightarrow B \text{ tal } \ f = \tilde{f} \circ P_t\}$. The set of results she realizes is $(\Omega, P_t) = \{[\omega_1]_t, \ldots, [\omega_{n(t)}]_t\}$, where for each $i \in \{1, \ldots, n(t)\}$, $[\omega_i]_t = \{z \ ; \ z \in [x_{i-1}^t, x_i^t)\}$. Then $f = \tilde{f} \circ P_t$ if and only if $f(z) = f(z')$ for all $z, z' \in [\omega_i]_t$ and for any $i \in \{1, \ldots, n(t)\}$. That is, $f \in \mathbb{P}_t$ whenever f is a step function which takes at most $n(t)$ values and whose steps are intervals $[x_{i-1}^t, x_i^t)$. That is, $\mathbb{P}_t$ is the space of functions $f : [0, 1] \rightarrow B$ that are constant on each interval $[x_{i-1}^t, x_i^t)$.

Let us consider two agents t and t'. Agent t through her information $P_t : [0, 1] \rightarrow (\Omega, P_t)$ distinguishes $n(t)$ events given by the set $(\Omega, P_t) = \{[\omega]_t \ ; \ \omega \in [0, 1]\} = \{[\omega_1]_t, \ldots, [\omega_{n(t)}]_t\}$, where $[\omega_1]_t = [0, x_1^t), \ldots, [\omega_{n(t)}]_t = [x_{n(t)-1}^t, x_{n(t)}^t]$. Agent t', in accordance with her information $P_{t'} : [0, 1] \rightarrow (\Omega, P_{t'})$, realizes $n(t')$ events defined by the set $(\Omega, P_{t'}) = \{[\omega]_{t'} \ ; \ \omega \in [0, 1]\} = \{[\omega_1]_{t'}, \ldots, [\omega_{n(t')}]_{t'}\}$ where $[\omega_1]_{t'} = [0, x_1^{t'}), \ldots, [\omega_{n(t')}]_{t'} = [x_{n(t')-1}^{t'}, x_{n(t')}^{t'}]$.

In this position, the application

$$\begin{array}{rccl} i: & (\Omega, P_t) & \rightarrow & (\Omega, P_{t'}) \\ & [\omega]_t & \rightarrow & i([\omega]_t) \quad = [\omega]_{t'} \end{array}$$

is well defined if and only if for any $x_j^{t'}$ where $j \in \{1, \ldots, n(t')\}$, there exists $k \in \{1, \ldots, n(t)\}$ such that $x_j^{t'} = x_k^t$, i.e,

$$\{x_1^{t'}, \ldots, x_{n(t')}^{t'}\} \subset \{x_1^t, \ldots, x_{n(t)}^t\}$$

To make clear this point we consider that private information of agent t is the finite partition $P_t = \{0 < 1/3 < 1/2 < 3/4 < 9/10 < 1\}$ and for agent t' is $P_{t'} = \{0 < 1/3 < 1/2 < 1\}$. Information P_t determines for agent t the set $(\Omega, P_t) = \{[\omega]_t \ ; \ \omega \in [0,1]\} = \{[\omega_1]_t, [\omega_2]_t, [\omega_3]_t, [\omega_4]_t, [\omega_5]_t\}$, so she can discern five events given by $[\omega_1]_t = [0, 1/3)$, $[\omega_2]_t = [1/3, 1/2)$, $[\omega_3]_t = [1/2, 3/4)$, $[\omega_4]_t = [3/4, 9/10)$ y $[\omega_5]_t = [9/10, 1]$. Agent t', under the information $P_{t'}$, observes $(\Omega, P_{t'}) = \{[\omega]_{t'} \ ; \omega \in [0,1]\} = \{[\omega_1]_{t'}, [\omega_2]_{t'}, [\omega_3]_{t'}\}$ where $[\omega_1]_{t'} = [0, 1/3)$, $[\omega_2]_{t'} = [1/3, 1/2)$ y $[\omega_3]_{t'} = [1/2, 1]$. If this is the case, the application $i : (\Omega, P_t) \rightarrow (\Omega, P_{t'})$ is well defined and, therefore, agent t is said to be better informed than t'. However, the application $i : (\Omega, P_{t'}) \rightarrow (\Omega, P_t)$ is not well defined since for all $\omega \in [\omega_3]_{t'} = [1/2, 1]$ while $i([\omega]_{t'}) = [\omega]_t$ depends on representant of the class $[\omega]_t$ that is considered.

In this setting, agent t is said to be better informed than t' if and only if $\{x_1^{t'}, \ldots, x_{n(t')}^{t'}\} \subset \{x_1^t, \ldots, x_{n(t)}^t\}$. Then, $\mathbb{P}_{t'} \subset \mathbb{P}_t$ which implies that the set of attainable action plans is wider for agent t and in fact includes all those action plans informationally feasible for agent t'.

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