Handbook on DSGE models: some useful tips in modeling a DSGE models

Valdivia, Daney

10 January 2015

Handbook on DSGE models: some useful tips in modeling a DSGE models

Daney David Valdivia Coria[ϕ]

V3.2015

Abstract

Despite there are useful books and text books from recognized authors about modeling macroeconomics through various types of methods and methodologies, "Some Useful tips in Modeling a DSGE models" try to add special features through an economist can use to model macro and micro relations to explain different scenarios in an specific economy.

In this sense, this work begin since basic conceptions of difference equations to build a Dynamic Stochastic General Equilibrium model covering special topics like rule – of – thumb consumers, monetary and fiscal policies, sticky prices, investment and problem of the firms, topics in Dynare and others.

JEL Classification: A33, C00, E1

Keywords: Differential equation, dynamic stochastic general equilibrium refinements, policy instruments.

[ϕ] Daney Valdivia is professor of macroeconomics and DGSE models at Military School of Engineering and Andean University Simon Bolivar, senior researcher consultant and currently works at the Tax Authority Challenge.

This product arises from the knowledge transmitted by my professor Carlos Garcia in my in graduate studies and research done in later years developed during my professional life.

The author is solely responsible for any error or omission in the present notes, **but not for transcription.**

Contact Info: dvaldivia@ait.gob.bo, daneyvaldivia@yahoo.com, ddvecon@gmail.com

Blog: http://ddvcecon.blogspot.com/

Web: https://sites.google.com/site/ddvcecon

1. INTRODUCTION

Most of the recent literature in macroeconomics is referred to develop the new vintage of macroeconomic models, incorporating the principal advantage: all variables are around a steady state in the Dynamic Stochastic General Equilibrium Models (DSGE) – natural levels. In this sense, I will show some tips that sophisticate these kinds of models in order to bring them to reality and evaluate an economy against different shocks.

Despite there are useful books from recognized authors about models in Macroeconometrics and the way how can be implemented, “Some Useful tips in Modelling a DSGE models” add special features through an economist can use to model macro and micro relations to explain the response from the economy to different kind of shocks.

Therefore, the following structure is follows:

2. What is a DSGE model?
3. Linear difference equations and high – order linear models, where I introduce basic concepts about how to overcome it and expand single models to multiple equations;
4. Log – linearizationz, RBC and RBC in practice, in part I is introduced log – linearizationz in order to get a variable around a steady state and introduces a simple general equilibrium model to a DSGE Model and how to resolved it;
5. DYNARE, in this part I introduce to the lecturer to program in this software created by Michell Juliard to compute DSGE models;
6. Rule of thumb consumers, here we bring up to the reality in describe two type of households;
7. Long run labor supply and the elasticity of intertemporal substitution for consumption, in this part I pointed out the effect of agents to choose labor supply and the influence on the Euler equation;
8. Labor supply and indivisible labor, permits us to bring the results comparable with micro data;
9. The problem of the firm, introduces how to maximize the benefits of shareholders and introduce the Tobin’s Q;
10. Investment, describe in a deep manner the relation of Tobin’s Q and the structure of a DSGE model;
11. Advanced Picks in DYNARE, we refine a DGSE model;
12. Sticky price model, introduces the model of the New Keynesian Phillips Curve and how it performs the comprehension of inflation dynamics;

13. Flexible Vs Sticky Prices introduce a comparison about these two types of model and the effects over the economy;
14. Individual maximization in a monetary model, it's introduced two things: i) money demand and ii) basic type of Taylor Rule;
15. Fiscal Policy, it discusses the effects of government purchases in the economy and how we model it, fiscal stance and debt policies;
16. Optimal Monetary Policy, this chapter discusses the effects of monetary policy on controlling the inflation and the tradeoff between output and inflation;
17. Is monetary policy a science?, introduces some tips about how to conduct monetary policy and depicts some troubles on implementing it;

2. WHAT IS A DSGE MODEL?

The history of this type of models is largely and complex. DSGE models are in the vein of the called “new macroeconomic vintage” (around 2005 – 2007) new Keynesian models. The introduction of this models were hard, since we have some advances from 1995 and the popular “first” formal DSGE model done by Smets and Wouters (2002), “ An estimated stochastic dynamic general equilibrium model for the Euro Area”. Representatives of this type of work are: Marco Del Negro, Lawrence Christiano, Martin Eichenbaum, Jordi Galí, Tommaso Monacelli, Frankn Shorfeide among others.

But what are the main characteristics of these models?

- A DSGE model can help us to find a “unique” and complete equilibrium for a particular economy, support by its structure and parameter foundation.
- It can help us to distinguish intratemporal and intertemporal effect, e.g. decision between work hours and consumption, the path of consumption.
- Mostly and ideally they should be microfounded.

Among macroeconometric models, they have some differences:

- The equilibrium in called a “natural level” despite of potential, full employment equilibrium, tendency level, etc.
- The equilibrium is solved around “certainly” levels.
- Depending on deeply parameters (well calibrated or estimated separately or structurally before use Bayesian econometric techniques) this type of models can simulate the principal moments of main and fundamentals variables despite the model doesn’t know historical data.

Additionally we can:

- Make structural forecasting.
- Assessing ex-ante and ex-post policy and compare with empirical data.
- Understand economic process and causality between fundamentals.

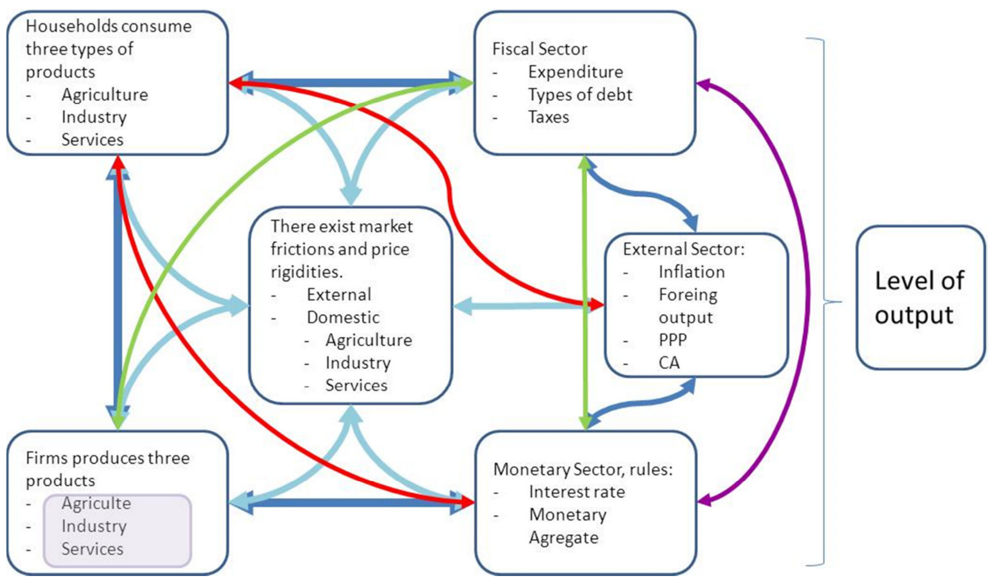
On the other hand, these types of models have some weaknesses:

- It needs much information, microeconomic and macroeconomic data.
- Well knowledge and managing of continuous and discrete differential equations.
- Knowledge about how to manage microeconometric and macroeconometric techniques.

Despite we perceive similar structure to CGE models, a DGSE model have the ability of use current data, replicate business cycles and recently some authors are working to introduce environment, natural resources, etc., e.g. Pieschacon. Modeling is also hard and it's still developing, e.g. Canova and Sala (2009); Komunjer and Ng (2011); Iskrev (2010).

Here is an example of building blocks of DGSE model that I developed in my paper called "Sectorial Fluctuations and economic growth impact".

Example of DSGE structure



Source: Valdivia (2012)

In the example above we can see three sources of shocks: agriculture, industry and services; any of this can move the equilibrium and the final result we look for the response of output. Since we have monetary and policy sectors, one of them should react to fight to, e.g. inflation pressures, or work jointly (policy coordination).

Besides and complementary to DGSE models it's useful to use comovements in order to understand in how many periods answer variables to movements of others.

3. LINEAR DIFFERENCE EQUATIONS AND HIGH – ORDER LINEAR MODELS

Linear difference equations

Linear difference equations are useful to compute DGSE models. Since most of the relationships are representations of rational expectation equations, this technique help us to compute them. The block construction takes a multi equation structure that helps us to determine relations and correlations (contagion) between variables. The compute solutions also allow observing the transmission mechanism of different shocks in the economy. More important, its impact (in terms of deviation of some level called potential, natural, steady state or some like these).

As R. Farmer(1999) describe, let us suppose a model structure generated by the following equations:

$$Y_t = E_t f(Y_{t+1}, X_t, \mu_t) \qquad 1$$

$$X_t = g(X_{t-1}, v_t) \qquad 2$$

WhereE_t represent the so-called belief of agents on Y_{t+1} and X_t is an autoregressive process with μ_t and $v_t$called random shocks following an i.i.d. process with$(0, \sigma^2)$. One important assumption in order to avoid biased is that they are hypothetically uncorrelated, otherwise conclusions and interpretations are not valid.

If we assume that E_t represents rational expectations, so it shows the true probability distribution of $Y_{t+1}|t$, that is the forward values of Y_{t+i} are conditional of information available on time t.

Example 1

Given $Y_t = E_t f(Y_{t+1})$and its steady state is $\bar{Y} = f(\bar{y})$ obtain the dynamics of variable Y_t (Should be an Euler equation).

One solution is taking a Taylor expansion from the difference equation (1), so we can represent it around a steady state:

Let be,

$$dY_t = E_t b\, dY_{t+1}$$

Where $$b = \left.\frac{df}{dY}\right|_{Y=\bar{Y}}$$

$$dY_t = Y_t - \bar{Y}$$

Remember that a Taylor Approximation (T.A.) is: $\boldsymbol{f(x) = f(\bar{x}) + f'(\bar{x})(x - \bar{x})}$

T.A. implies work with cycle component of the variables

Computing the equation 1 around a T.A. we have:

$Y_t = E_t f(Y_{t+1})$

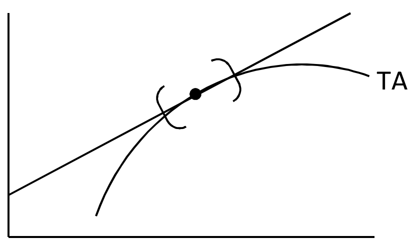
$Y_t = \bar{Y} + \dfrac{df(\ )}{d\bar{Y}}(Y_{t+1} - \bar{Y})$

$Y_t - \bar{Y} = \dfrac{df(\ )}{d\bar{Y}}(Y_{t+1} - \bar{Y})$

$\boldsymbol{dY_t = b\, E_t dY_{t+1}}$

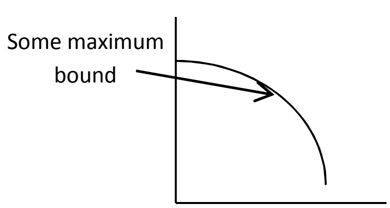
The last result show that the path of Y_t is explained by a "rate" b and future values of Y_t. This simple difference equation

One important thing about T.A. is that it's useful only when the variable Y_t is closer and the neighborhood of the steady state.



Definition *"An economic equilibrium is a sequence of probability distributions for the endogenous variable Y_t which satisfies $dY_t = b\, E_t dY_{t+1}$ plus some bounding or transversality condition*".

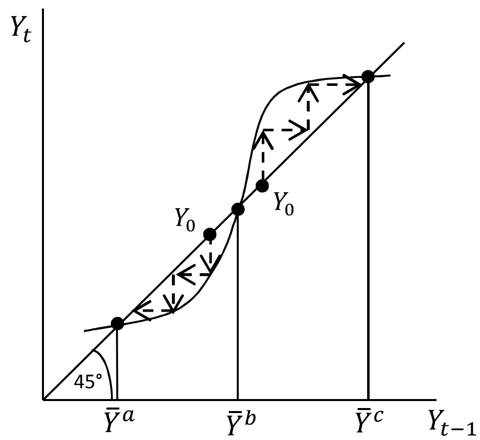
The bounding condition ensure that values of parameters do not take values out of range of economic theory.



$$\left.\begin{array}{c} dY_t = b\, E_t dY_{t+1} \\ Y_0 = \bar{Y}_0 \end{array}\right\} \text{E.g. defining stocks}$$

The transversality condition implies that in the $\lim_{t\to\infty}(Y_t) < \infty$. The limit Y_t must converges to some value, let us see, e.g., the future evolution of prices.

Let suppose that in equilibrium$\bar{Y} = Y_t = Y_{t-1}$ (on the 45° line) and the evolution of Y_t is given by $Y_t = f(Y_{t+1}, X_t, t)$. Assume that X_t is constant, X .



From the graph, we can see:

1st $f(\circ)$ Doesn't depend on t explicitly, it

implies autonomy

2nd $f(\circ)$ Has three fixed points that represent solutions to the steady state.

Then, given the initial value, Y_0, we can have two stable points and one instable. If we linearize our function the two stable points are $\bar{Y}^a \wedge \bar{Y}^c$.

For our equation it can be represented by Phillips curve, $dY_t = bdY_{t+1}$, important for conduct Monetary Policy.

Solutions to the difference equation

$b < 1$ Regular case

We need to pick an arbitrary initial value of Y_1 close to $\bar{Y}$ to generate Y_t through Markov process[1].

The regular case violated the convergence condition because it explode

$$dY_t = \frac{1}{b} dY_{t-1} + \eta_t^*$$

η_t^* is random variable. Only exists one condition where we are in equilibrium, $Y_1 = \bar{Y}$, in this case we don't violate the transversality condition and η_t is removed.

Then, beliefs must themselves be functions of fundamental economic parameters (deep parameters) of the model.

[1] A random process is when future probabilities are determined by its most recent values.

b > 1 ***Irregular case***

In this case the model converges

$$dY_t = \frac{1}{b} dY_{t-1} + \eta_t^*$$

Let assume η_t^* is small and we have beliefs that the model will converge to $\bar{Y}$, Why? Since η_t^* is small, Y_t will be associated with sequences of probability distributions that converge to a stationary distribution that contains the fixed point $\bar{Y}$, the so called self – fulfilling prophecies.

E.g.

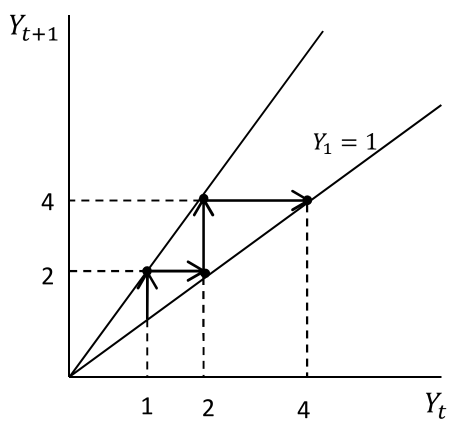
$$dY_t = b\, E_t dY_{t+1} + \eta \rightarrow \text{Shock}$$

$$dY_{t-1} = bdY_t + \eta$$

$$dY_t = \frac{1}{b} dY_{t-1} + \frac{1}{b}\eta$$

$dY_t = \frac{1}{b} dY_{t-1} + \eta^*$, when we aggregate shocks, expectations disappear.

Let be

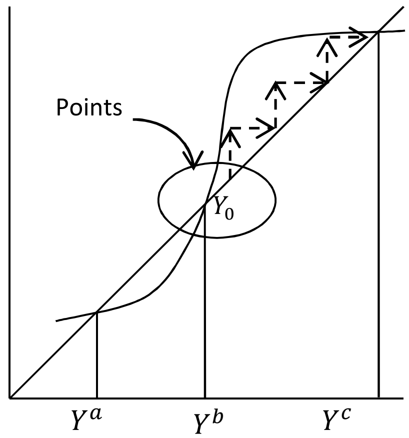


Then, there is no sequence $\{Y_t\}_{t=1}^{\infty}$ that will be consistent with the equilibrium.

E.g. $Y_t = 2Y_{t-1}$; $Y = 0$

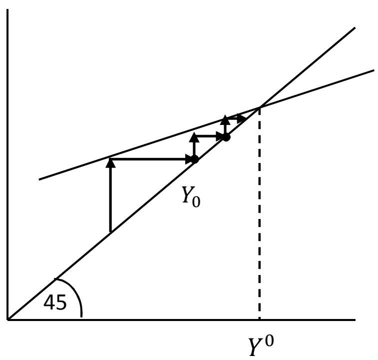
Remember that from equation $Y_t = f(Y_{t+1}, X_t, t)$ we know that

1. $f(\circ)$ Doesn't depend explicitly on t →is autonomous
2. $f(\circ)$ Has three fixed points
3. In steady state (SS) Y^a, $Y^c$are stable and Y^bnot



Points around Y^a, Y^c are good candidates to linealize around SS.

E.g.



Around the fixed points the linear approximation is a good approximation similar to Taylor Expansion.

E.g. $Y_t = b + aY_{t-1}$ $\leftarrow$ this is around SS

Where $a = f_y(\bar{x}, \bar{y})$ $b = \bar{Y} - a\bar{Y}$

The Taylor Expansion will be: (be in mind X_t =constant $\rightarrow fx = 0$)

$$Y_t = \bar{Y} + fy(\bar{X}, \bar{Y})(Y_{t-1} - \bar{Y})$$

$$Y_t = \underbrace{Y - a\bar{Y}}_{b} + aY_{t-1}$$

From de last we have two cases:

$\{X_t\}$ Is a non trivial function of time $\rightarrow$ no autonomous $\rightarrow$ $\{X_t\}$ isn't a constant

$\{X_t\}$ Is a random variable (r.v.)

In any of the two cases:

$$Y_t = \bar{Y} + f_y(\bar{X}, \bar{Y})(Y_{t-1}, \bar{Y}) + f_x(\bar{X}, \bar{Y})(X_t - \bar{X})$$

$$Y_t = \bar{Y} - a\bar{Y} - c\bar{X} + aY_{t-1} + cX_t \qquad (3)$$

Then, the equation (3) isn't autonomous.

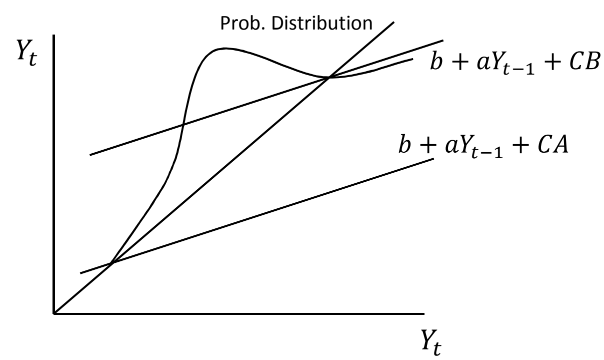
Two options for solving first – order linear models

We are interested in a model where steady state value is continuously buffeted by a random disturbance, and then we have two options:

1. First – Order deterministic equation
   Given (3) if $c = 0$ we have
   $|a| < 1$ ← System is stable as $t \to \infty$
   $|a| > 1$ ← Unstable and divergent $Y_t \to \infty$
   $a = 1$ ← We always is SS
2. First order stochastic equation and $c \neq 0$
   Given(3):
   i. $|a| < 1$ → Y_t might be stable.
   ii. (3) is in function of the distribution of X_t, we need that its probability distribution be invariant through time.

Therefore Y_t Converges to an invariant probability distribution.

Y_t moves through stables ranges



If we have more than one equation then we have to apply simultaneous equations, but if one is unstable, so the processY_t will be unstable.

Higher – order linear models

High order linear equations are represented by matrix and vectors because, in this case, we are interested in finite number of variables in order to describe the dynamics of an economy.

Let be:

$$\underset{n\times 1}{Y_t} = \underset{n\times 1}{b} + \underset{n\times 1}{A}\underset{n\times 1}{Y_{t-1}} + \underset{n\times 1}{c}\underset{n\times 1}{X_t} \qquad 4$$

In order to compute the equation 4, we must be familiarized with the terms "**eigenvalues"** and **"eigenvectors".** Hoffman and Kunze (1971) called them characteristic roots or characteristic values. Marcus and Minc (1988) used the term proper values or latent roots. These eigenvalues represent solution of the model.

The behavior of first – order vector is formed by decomposing the matrix system into a set of first order equations which are uncoupled in the sense that equation describes evolution of a single variable that does not depend on the other variable in the set.

$|\lambda^a|, |\lambda^b| < 1 \rightarrow$ Stable

Any $|\lambda > 1|$unstable: $|\lambda^a| < 1 \wedge |\lambda^b| > 1 \rightarrow$ find a saddle point

Stochastic vector difference equations

$$Y_t = b + AY_{t-1} + cX_t$$

X_t has to be draws from a invariant probability distribution through time:

Let $\quad AY_t = \lambda Y_t$

$$(A - \lambda)Y_t = 0 \;\; \Rightarrow \;\; (A - \lambda I)Y = 0$$

We need that Y_t must not be zero

$ax = 0$ Solution

$a \neq 0 \Rightarrow$ Must exist $x = \frac{0}{a} = \frac{1}{a}$

For having a solution $\Rightarrow \;(A - \lambda I) = 0$

e.g.

$$\left|\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right| = 0$$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0$$

$$a_{11}a_{22} - a_{11}\lambda - a_{22}\lambda + \lambda^2 - a_{12}a_{21} = 0$$

$$\lambda^2 = \lambda(a_{11} + a_{22}) + a_{12}a_{21} - a_{11}a_{22} = 0$$

$$\lambda^2 - \lambda A + C = 0$$

Then, λ^a , λ^b eigenvalues

e.g.

$$A = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix}$$

$$\lambda^2 - \lambda(2+1) + 2 - 2 = 0$$

$$\lambda^2 - \lambda(3) = 0 \Rightarrow \lambda(\lambda - 3) = 0$$

$$\lambda^a = 3$$
$$\lambda^b = 0$$

$$\lambda^a = 3$$

We know $(A - \lambda I)Y = 0$

$$\left|\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\right| = \vec{0} \Rightarrow \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} y_1^a \\ y_2^a \end{pmatrix} = \vec{0}$$

$$\left.\begin{matrix} -Y_1^a + 2Y_2^a = 0 \\ Y_1^a \ - \ 2Y_2^a = 0 \end{matrix}\right\} Y_1^a = 2Y_2^a$$

Since one Y_i is arbitrary, there will be an infinite number of eigenvectors that satisfy the equation.

$$Y_1 = 2Y_2 \Rightarrow \ Y_2 = 1 \ \wedge \ \ Y_1 = 2$$

The roots $A_{2\times 2}$ are two solutions for $\lambda^a \ \wedge \ \lambda^b$ to $AY^i = \lambda^i Y^i Y^a$, Y^b be eigenvectors

How to get the roots?

From $\lambda^2 = \lambda(a_{11} + a_{22}) + a_{12}a_{21} - a_{11}a_{22} = 0$ we can get eigenvalues.

Let $$A\underbrace{[Y^aY^b]}_{Q} = \underbrace{[Y^aY^b]}_{Q}\begin{bmatrix} \lambda^a & 0 \\ 0 & \lambda^b \end{bmatrix}$$

$$A = Q\lambda Q^{-1}$$

Suppose that $\underset{n\times 1}{Y_t} = \underset{n\times n}{A}\ \underset{n\times 1}{Y_{t-1}}$

$$\Rightarrow\ Y_t = Q\lambda Q^{-1}Y_{t-1} \qquad ||Q^{-1}$$

$$Q^{-1}Y_t = \underbrace{Q^{-1}Q}_{1}\lambda Q^{-1}Y_{t-1}$$

And transform $Z_t = \lambda Z_{t-1}$ is an independent model

$$\Rightarrow\ \lambda = \begin{bmatrix} \lambda_a & 0 \\ 0 & \lambda_b \end{bmatrix}$$

$$\Rightarrow\ {Z^1}_t = \lambda^a Z_{t-1}^1$$

$${Z^2}_t = \lambda^b Z_{t-2}^2$$

This model is stable because of roots are < 1

Unstable when any is > 1

Stochastic vector difference equation

Let $Y_t = b + AY_{t-1} + cX_t$

Period 1 $\quad Y_1 = b + AY_0 + cX_1$

Period 2 $\quad Y_2 = b + cX_2 + A(b + AY_0 + cX_1)$

Period t

$$Y_t = \sum_{s=0}^{t} A^s(b + CX_{t-s}) + A^tY_0$$

$$A^s = Q\Lambda Q^{-1} = Q\Lambda^s Q^{-1}$$

Λ The stability condition

If all of eigenvalues are around the unit circle, so we can write

$$Y_t = (I - A)^{-1}b + C\sum_{s=0}^{\infty} A^s X_{t-s}$$

Linear rational expectation models

Include belief about the future induced dynamics through beliefs by the effects of accumulating stocks from past.

Assume:

$$\begin{matrix} Y_t^1 \\ Y_t^2 \end{matrix} = A\begin{pmatrix} Y_{t+1}^1 \\ Y_{t+1}^2 \end{pmatrix} + B\begin{pmatrix} v_{t+1} \\ w_{t+1} \end{pmatrix}$$

$$w_{t+1} = E(Y_{t+1}) - Y_{t+1} \ \leftarrow \text{Error expectation}$$

So we have:

$$Y_t = \begin{pmatrix} Y_t^1 \\ Y_t^2 \end{pmatrix}$$

$$Y_t^1 \in R^{n_1} \ \wedge \ Y_t^2 \in R^{n_2}$$

$$\Rightarrow \ n_1 + n_2 = n$$

Y_t are state variables.

Factors that influence economic behavior at date t

Y_t can be partitioned into those variables:

Capital stock	predetermined (with initial condition)
Real value of money supply	free

Shocks or disturbances

Fundamental disturbances are i.i.d. through time

$$v_t \in R^m$$

Example: preference shocks, endowments and technology.

4. LOG LINEARIZATION, RBC AND RBC IN PRACTICE

Log linearization, is a first Taylor approximation around steady state

$$\hat{x}_t = \log x_t - \log \bar{x} = SS$$

To $\log x_t$ apply a TA

Remember: $x_t = \bar{x} + x'(x_t - \bar{x})$

$$\log x_t = \log(\bar{x}) + \frac{1}{x}(x_t - \bar{x})$$

$$\log x_t - \log \bar{x} = \frac{x_t - \bar{x}}{\bar{x}}$$

We can express like:

$$\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}}$$

$$\Rightarrow \quad \log x_t - \log \bar{x} = \hat{x}_t$$

Then, log linearizationz:

$$x_t = \bar{x}(1 + \hat{x}_t)$$

$$x_t = \bar{x}e^{\hat{x}_t}$$

Let be

$$x_t^a = (x_t)^a \;\; \Rightarrow \; \left(\bar{x}e^{\hat{x}_t}\right)^a = \bar{x}^a e^{a\hat{x}_t}$$

$$x_t^a = \bar{x}^a(1 + a\hat{x}_t) \tag{1}$$

Proof of Taylor expansion

$$\bar{x}^a e^{a\hat{x}_t} = \bar{x}^a e^{a0} + \bar{x}^a e^{a0}(a\hat{x}_t - 0)$$

$$\bar{x}^a e^{a\hat{x}_t} = \bar{x}^a(1 + a\hat{x}_t)$$

If:

$a x_t = a\left(\bar{x}e^{\hat{x}_t}\right) = a\bar{x}(1 + \hat{x}_t)$

$$\therefore \qquad a x_t = a\bar{x}(1 + \hat{x}_t)$$

Product of two variables

$$x_t y_t = \bar{x}e^{\hat{x}_t}\bar{y}e^{\hat{y}_t} = \bar{x}\bar{y}e^{\hat{x}_t+\hat{y}_t} = \bar{x}\bar{y}(1+\hat{x}_t+\hat{y}_t)$$

$$x_t y_t = \bar{x}\bar{y}(1+\hat{x}_t+\hat{y}_t) \qquad (3)$$

Adding two variables

$$x_t + y_t = \bar{x}e^{\hat{x}_t} + \bar{y}e^{\hat{y}_t} = \bar{x}(1+\hat{x}_t) + \bar{y}(1+\hat{y}_t) \qquad (4)$$

Example : Given a single utility consumption function:

$$\underset{\{c_t,k_{t+1}\}}{\text{Max}} E_t \sum_{1}^{\infty} \beta^{t-1} U(C_t)$$

$$U(c_t) = \log c_t$$

St. $\quad C + I = Y$ and $K_{t+1} = (1-\delta)K_t + I_t$

We have to replace the capital law in the consumption constraint.

$$C_t + K_{t+1} = (1-\delta)K_t + s_t K_t^{\alpha}$$

Let suppose a Cobb Douglas production function

$$Y_t = s_t K_t^{\alpha} \Rightarrow$$

$\alpha < 1 \rightarrow Y$ Concave

And the technology following a AR(1) process

$$s_t = s_{t-1}^{\rho}\gamma_t \qquad 0 \leq \overbrace{\rho}^{\text{persistence}} \leq 1$$

$\beta_t(0,1) \rightarrow$ Discount rate

δ Depreciation rate

γ_t~iid

FOC

$$d/dC_t \quad : \qquad \beta^{t-1}\frac{1}{C_t} = \beta^{t-1}\lambda_t \qquad (1)$$

$$d/dK_{t+1}: \qquad -\beta^{t-1}\lambda_t + \lambda\beta^t\lambda_{t+1}(1-\delta) + \alpha\beta^t\lambda_{t+1}K_{t+1}^{\alpha-1}s_{t+1} = 0$$

$$F^K_{t+1} = PMgKtal$$

$$\beta^{t-1}\lambda_t = \beta^t\lambda_{t+1}[\alpha\, s_{t+1}K^{\alpha-1}_{t+1} + 1 - \delta]$$

$$\lambda_t = \beta\lambda_{t+1}[1 - \delta + \alpha\, s_{t+1}K^{\alpha-1}_{t+1}] \qquad (2)$$

Building the model

Euler equation

$1 \rightarrow 2$

$$\frac{1}{C_t} = E_t\frac{\beta}{C_{t+1}}\left(1 - \delta + \underbrace{\alpha\, s_{t+1}K^{\alpha-1}_{t+1}}_{F^K_{t+1}}\right) \qquad (1)$$

Restriction

$$C_t + I_t = Y_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$Y_t = s_tK^\alpha_t \qquad \wedge \qquad s_t = s^\rho_{t-1}N_t$$

$$\Rightarrow\ C_t + K_{t+1} = s_tK^\alpha_t + (1-\delta)K_t$$

1^{st} Log – lin

$$-\frac{1}{C}\hat{C}_t = \frac{\beta}{C}F^K\left(\hat{F}^K_{t+1} - \hat{c}_{t+1}\right) + \frac{\beta}{C}(1-\delta)(-\hat{c}_{t+1})$$

We know that

$$R_{t+1} = (1-\delta) + F^k_{t+\alpha} \rightarrow \hat{r}_{t+1} = \hat{s}_{t+1} + (1-\alpha)\widehat{K}_{t+1}$$

1^{st} log –lin (1)

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}}\underbrace{\left(1 - \delta + \underbrace{\alpha\, s_{t+1}K^{\alpha-1}_{t+1}}_{F^K_{t+1}}\right)}_{R_{t+1}}$$

$$C_{t+1} = C_t\beta(R_{t+1})$$

$$c\left(1 + \hat{C}_{t+1}\right) = \beta cR\left(1 + \hat{C}_t + \hat{R}_{t+1}\right)$$

$$\hat{C}_t = E\left(\hat{C}_{t+1}\right) - \hat{r}_{t+1} +\ w \qquad (a)$$

$$F^K\left(1+\hat{f}^K_{t+1}\right)=\alpha s K^{\alpha-1}\left(1+\hat{s}_{t+1}+(\alpha-1)\widehat{K}_{t+1}\right)$$

$$\hat{f}^K_{t+1}=\hat{s}_{t+1}+(1-\alpha)\widehat{K}_{t+1} \qquad (b)$$

Log – lin capital law

$$K_{t+1}=(1-\delta)K_t+I_t \qquad \leftarrow \qquad \begin{aligned} C+I&=Y\\ I&=Y-C\end{aligned}$$

$$K_{t+1}=(1-\delta)K_t+s_tK_t^{\alpha}-C_t$$

$$K\left(1+\widehat{K}_{t+1}\right)=(1-\delta)K\left(1+\widehat{K}_t\right)+sK^{\alpha}\left(1+\hat{s}_t+\alpha\widehat{K}_t\right)-C\left(1+\hat{C}_t\right)$$

$$K_{t+1}=\underbrace{\frac{(1-\delta)K+sK^{\alpha}}{K}}_{a_1}\widehat{K}_t+\underbrace{\frac{sK^{\alpha}}{K}}_{a_2}\hat{s}_t-\underbrace{\frac{C}{K}}_{a_3}\hat{C}_t$$

$$\widehat{K}_{t+1}=a_1\widehat{K}_t+a_2\hat{s}_t-a_3\hat{C}_t$$

Shock

$s_t=s_{t-1}^{\rho}\cdot\gamma_t \equiv s_{t+1}=s_t^{\rho}\cdot\gamma_t$ Butγ_{t+1},γ_t are similar

$$s(1+\hat{s}_{t+1})=s^{\rho}\gamma(1+\rho\hat{s}_t+\hat{\gamma}_t)$$

$$\hat{s}_{t+1}=\rho\hat{s}_t+\hat{\gamma}_t$$

Built our system

$$(1)\hat{C}_t=E_t\left(\hat{C}_{t+1}-\hat{r}_{t+1}\right)$$

$$(1.1)\hat{C}_t=E_t\hat{C}_{t+1}-\hat{s}_{t+1}-(1-\alpha)\widehat{K}_{t+1}$$

$$\hat{C}_t=a_1\hat{C}_{t+1}+a_2\hat{s}_{t+1}-a_3\widehat{K}_{t+1}$$

$$(2)\widehat{K}_{t+1}=b_1\widehat{K}_t+b_2\hat{s}_t-b_3\hat{C}_t \qquad ||b$$

$$(3)\hat{s}_{t+1}=\rho\hat{s}_t+\hat{\gamma}_t$$

$$C_t=\overbrace{a_1}^{1}\hat{C}_{t+1}+a_2\hat{s}_{t+1}-a_3\widehat{K}_{t+1}$$

$$K_t=\frac{1}{b_1}\widehat{K}_{t+1}-\frac{b_2}{b_1}\hat{s}_t-\frac{b_3}{b_1}\hat{C}_t$$

$$s_t=\frac{1}{\rho}s_{t+1}-\hat{\gamma}_t$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \left(\frac{b_3}{b_1}\right) & \left(\frac{1}{b_1}\right) & \left(\frac{b_2}{b_1}\right) \\ 0 & 0 & \frac{1}{\rho} \end{bmatrix} \begin{bmatrix} C_t \\ K_t \\ s_t \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{t+1} \\ K_{t+1} \\ s_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & -1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{Y}_{t+1} \\ w_{t+1}^{C} \\ w_{t+1}^{K} \\ w_{t+1}^{s} \end{bmatrix}$$

$$\begin{bmatrix} \hat{C}_t \\ \hat{K}_t \\ s_t \end{bmatrix} = A \begin{pmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \\ s_{t+1} \end{pmatrix} + \beta \begin{pmatrix} \hat{V}_{t+1} \\ w_{t+1}^{C} \\ w_{t+1}^{K} \\ w_{t+1}^{s} \end{pmatrix}$$

Steady State

$$\frac{1}{C} = \frac{1}{C}\beta(R) \Rightarrow \beta = \frac{1}{R}$$

$$Y_t = s_t K_t^{\alpha} \quad || \frac{Y}{K} = \underbrace{sK^{\alpha-1}}_{F^K}$$

$$F^K = \alpha s K^{\alpha-1} \Rightarrow \frac{F^K}{\alpha} sK^{\alpha-1}$$

$$\Rightarrow \ \alpha\frac{Y}{K} = F^K$$

$$\frac{1}{C} = \frac{\beta}{C}\left((1-\delta) + F^K\right)$$

$$1 = \beta(1 - \delta + F^K)$$

$$F^K = \frac{1}{\beta} + \delta - 1 \qquad\qquad \beta = \frac{1}{1+r}$$

$$F^K = 1 + r + \delta - 1$$

$$F^K = r + \delta$$

$$\Rightarrow \ \alpha\frac{Y}{K} = r + \delta$$

$$Y = \left(\frac{r+\delta}{\alpha}\right)K$$

Capital law of notion

$$K = (1-\delta)K + sK^{\alpha} - C$$

$$K = K - \delta K + sK^{\alpha} - C$$

$$\delta K = Y - C$$

$$C = \frac{Y}{K} - \delta$$

$$C + I = Y$$
$$\frac{Y}{C} = 1 + I$$

$$\frac{K}{Y} \cdot C = 1 - \delta \frac{K}{Y}$$

$$\frac{K}{Y}(C + \delta) = 1$$

$$\frac{K}{Y} = \frac{1}{C + \delta}$$

$$C = \left(\frac{r + \delta}{\alpha}\right) K$$

Different types of rational expectations

- If the number of $\lambda's$ =number of variables free initial condition$\Rightarrow \exists!$ eqq
- $\lambda's >$number of free variables conditions $\Rightarrow \nexists$eqq
- $\lambda's <$number of free variables conditions $\Rightarrow$ many eqq

Let see our model

$$\hat{C}_t = \hat{C}_{t+1} - a_2 \hat{s}_{t+1} - a_3 \hat{K}_{t+1}$$

$$K_t = b_1 \hat{K}_t + b_2 \hat{s}_t + b_3 \hat{C}_t$$

$$\hat{s}_{t+1} = \rho\, \hat{s}_t + \hat{v}_t$$

We have predetermined variables

K, s

Free variable γ_t

$\uparrow \gamma_t \to \uparrow \underbrace{s_{t+1}} \to \uparrow \underbrace{K_{t+1}}$

Impulse response and paths generated

Then we can reach a unique path for:

$\hat{C}_t \leftarrow$ That will represent a unique rational model

Let consider the system:

$Y_t = Y_{t+1} - R_{t+1}$ (IS)

$\pi_t = \beta\pi_{t+1} + \lambda Y_t + w_{t+1}$ (Phillips curve)

$R_{t+1} = \phi\pi_t + v_{t+1}$ (Taylor rule)

$v_{t+1} = \rho v_t + \mu_{t+1}$

w_{t+1}: Error expectations

$w_{t+1} = E_t\pi_{t+1} - \pi_{t+1}$

Building the model:

$Y_t = Y_{t+1} - \phi\pi_t - \gamma_{t+1}$

$\pi_t = \beta\pi_{t+1} + \lambda Y_t + w_{t+1}$

$\gamma_t = \rho\gamma_{t-1} + \mu_t$

$Y_t + \phi\pi_t = Y_{t+1} - \gamma_{t+1}$

$-\lambda Y_t + \pi_t = \beta\pi_{t+1} + w_{t+1}$

$$\gamma_t = \rho v_t + \mu_{t+1}$$

Y_t, π_t Stock we have initial conditions

γ_t Free

w_{t+1} free

This equilibrium is unique?

Depends on $\phi \begin{matrix} < \\ > \end{matrix}$

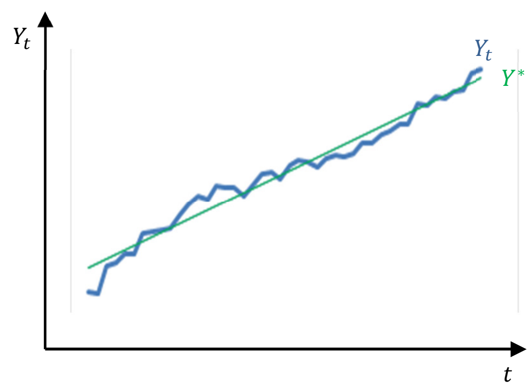
RBC and RBC in practice

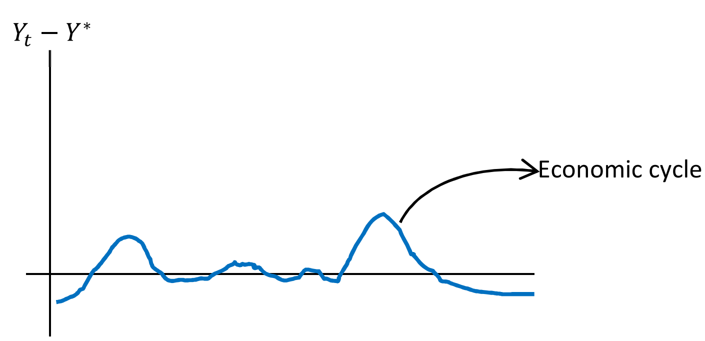
- ✓ Most of researchers have skepticism that technology shock is the source of business fluctuations.
- ✓ But, there is evidence that larger technology shocks to produce RBC.
- ✓ O the other hand, great variation of productivity amplify effect of technology shocks, so produce real business cycles

But, how the cycle is measure?

We have many options:
HP Hodrick and Prescott filter
CF Christiano Fitsgerald filter
BP Band Pass Filter
VAR model
Kalman filter
Nadaraya Watson Filter

The typical used by most of researchers is the HP filter, based in its tractability and common use.





Example from US

$$\sigma_{C\text{ non durable}} < \sigma_y$$

$$\sigma_{C\text{ durable}} > \sigma_y$$

$$3\sigma_I \geq \sigma_y$$

$$\sigma_G = \sigma_y$$

$$\sigma_{\text{hour worked}} = \sigma_y$$

$\sigma_K < \sigma_y$, $\sigma_K {}^{w}/_{utilization} > \sigma_y$

$$\sigma_{labor} = \sigma_y$$

$\sigma_{hrs\times worker} < \sigma_y$ $\Rightarrow$ The most cyclical variation in total hours worked is from changes in employment

$$\sigma_{\left({}^{L}/_{Y}\right)} < \sigma_y$$

$$\sigma_{\left({}^{w}/_{P}\right)} < \sigma_y$$

At this point we have to introduce the comovement term. It is a correlation between the actual and future period. Then, we have that the Most series are pro cyclical

Example ${}^{w}/_{P}$, G, K cyclical

R_t a cyclical/ counter cyclical

Persistence → most display $(\rho \cong 0.9)$

Implications:

$3\sigma_I \geq \sigma_y$ Animal spirits

$\sigma_K < \sigma_y$ Used to abstract change in capital

$\sigma_{hours} > \sigma_y$ Labor market is the key to understand business fluctuations

$\sigma_{\left({}^{w}/_{P}\right)} < \sigma_y$ Wage isn't important to allocate labor in BC

Let consider:

$$Max\ E_0 \sum_{0}^{\infty} \beta^t U(C_t, L_t)$$

$U(\ )$ is a concave function ⇒ refers to permanent income hypothesis

$$N_t + L_t = 1$$

If profit $\pi_t = 0 \Leftarrow Y_t = A_t F(K_t, N_t) = W_t N_t + Z_t K_t$

$$P_t C_t + P_t I_t = Y_t + \pi_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$K_0 > 0$$

$$Max\ \pi_t = Y_t - W_t N_t + Z_t K_t \ \Rightarrow\ Y_t = W_t N_t + Z_t K_t$$

Built blocks

$$Max\ E_0 \sum_{0}^{\infty} \beta^t U(C_t, 1 - N_t)$$

$$C_t + I_t = \frac{1}{P_t} Y_t + \cancelto{0}{\frac{\pi_t}{P_t}}$$

FOC

$$d/_{dC_t}: \beta^t D_1 U(C_t, 1 - N_t) = \lambda_t \beta^t$$

$$d/_{dN_t}: -\beta^t D_2 U(C_t, 1 - N_t) = \beta^t \lambda_t A_t D_2 F(K_t, N_t)$$

$$d/_{dK_{t+1}}: \beta^t \lambda_t = \beta^{t+1} E_t[\lambda_{t+1} A_{t+1} D_1 F(K_{t+1}, N_{t+1})] + \beta^{t+1} E_t\big(\lambda_{t+1}(1-\delta)\big)$$

Labor supply (labor decision)

$$\frac{U_N}{U_C} = \frac{D_2 U(C_t, 1 - N_t)}{D_1 U(C_t, 1 - N_t)} = A_t D_2 F(K_t, N_t)$$

Intertemporal effect, the last effect can help us to assess the intertemporal effects of consumption

$$\uparrow A_t \rightarrow \uparrow [A_t D_2 F(\ )] \Rightarrow \downarrow L_t \wedge \downarrow C_t \Rightarrow \left(\frac{D_2 U(\ )}{D_1 U(\ )}\right) \uparrow$$

Euler consumption decision

$$D_1U(C_t, 1-N_t) = \beta E_t[D_1U(C_{t+1}, 1-N_{t+1})\{A_{t+1}D_1F(K_{t+1}, N_{t+1}) + 1 - \delta\}]$$

Intertemporal effect

$\uparrow A_t \rightarrow \uparrow D_1U_t(\quad) \rightarrow \uparrow D_1U_{t+1}(\quad)by\uparrow [A_{t+1}D_1F_{t+1}(\ )]$

We know:

$$A_tD_2F(K_t, N_t) = \left(\frac{W}{P}\right)_t$$

HOUSEHOLD

Labor supply

$$\frac{D_2U\ (\ )}{D_1U\ (\ )} = \left(\frac{W}{P}\right)_t$$

$$A_tD_1F(\quad) = \left(\frac{Z}{P}\right)_t$$

Euler

$$D_1U_t(\quad) = \beta E_t\left(D_1U_{t+1}\left\{\left(\frac{Z}{P}\right)_{t+1} + 1 - \delta\right\}\right)$$

FIRMS dealing with competitive market with $\pi = 0$

$$Max\ \ P_tY_t - W_tN_t - Z_tK_t$$

St. $Y_t = A_tF(K_t, N_t)$

FOC

$$\left.\begin{array}{l} K_t:\ A_tD_1F(\quad) = \left(\frac{Z}{P}\right)_t \\ N_t:\ A_tD_2F(\quad) = \left(\frac{W}{P}\right)_t \end{array}\right\}\text{MRS (marginal rate of substitution)}$$

Give functional form to find a solution

Let $\quad U(C_t, L_t) = \log C_t + \theta \log(1 - N_t)$

St.

$$P_tC_t + P_tI_t = \left(\frac{W}{P}\right)_t N_t + \left(\frac{Z}{P}\right)_t K_t + \frac{\pi_t}{P_t}$$

FOC

$$C_t: \beta^t \frac{1}{C_t} = \beta^t \lambda_t \qquad (1)$$

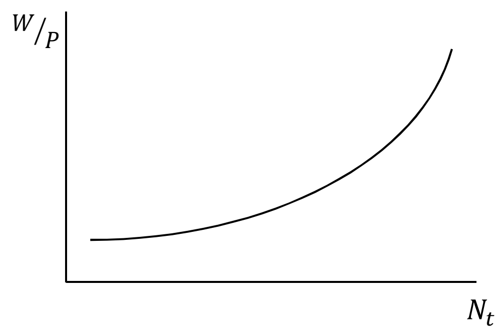
$$L_t: \beta^t \frac{\theta}{L_t} = -\lambda_t \left(\frac{W}{P}\right)_t$$

First

$$\frac{d \log L_t}{dL_t} \frac{dL_t}{dN_t} = \frac{\theta}{L_t}(-1) = -\frac{\theta}{1 - N_t}$$

$$N_t: -\beta^t \frac{\theta}{1 - N_t} = \lambda_t \left(\frac{W}{P}\right)_t \qquad (2)$$

$${}^{(2)}\!\big/_{(1)}: \ \frac{\theta C_t}{1 - N_t} = \left(\frac{W}{P}\right)_t \ \Delta: \uparrow \frac{W}{P} \rightarrow \uparrow N_t \wedge \downarrow C_t$$



$$K_{t+1}: \beta^t \lambda_t = E_t \left(\left(\frac{Z}{P}\right)_{t+1} + 1 - \delta\right) \beta^{t+1} \lambda_{t+1}$$

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \underbrace{\left(\left(\frac{Z}{P}\right)_{t+1} + 1 - \delta\right)}_{R_{t+1}} \right) \qquad (2)$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (R_{t+1}) \right]$$

Remember that $R_t = (1 - \delta) + f_{k_t}$ (7)

FIRMS

Capital demand $\alpha A_t \left(\frac{N}{K}\right)_t^{1-\alpha} = \left(\frac{Z}{P}\right)_t$ (3)

Labor demand $(1-\alpha)A_t \left(\frac{K}{N}\right)^{\alpha} = \left(\frac{W}{P}\right)_t$ (4)

Equilibrium:

$C_t + I_t = Y_t$ (5)

$K_{t+1} = (1-\delta)K_t + I_t$ (6)

The model

Variables $C, N, K, \frac{W}{P}, \frac{Z}{P}, Y, I$

Need extra equations of R_t (interest rate)

Finding steady state:

Euler → $\frac{1}{C} = \frac{1}{C}\beta R \quad \Rightarrow \quad \boldsymbol{\beta = \frac{1}{R}}$

Real interest rate

$$R = (1-\delta) + \frac{Z}{P}$$

$$R = (1-\delta) + \alpha\left(\frac{N}{K}\right)^{1-\alpha}$$

$$1 + r = 1 - \delta + \alpha\left(\frac{N}{K}\right)^{1-\alpha}$$

$$\frac{r+\delta}{\alpha} = \left(\frac{N}{K}\right)^{1-\alpha} \Rightarrow \left(\frac{K}{N}\right)^{\alpha-1} = \left(\frac{\alpha}{r+\delta}\right)$$

$$\frac{K}{N} = \left(\frac{\alpha}{r+\delta}\right)^{1/\alpha-1}$$

Capital law of motion

$K = (1-\delta)K + I$

$I = \delta K \Rightarrow \frac{I}{K} = \delta$

Equilibrium:

$Y = C + \delta K$

$1 = \frac{C}{Y} + \delta \frac{K}{Y}$

Labor supply

$$\frac{\theta C}{1-N} = \frac{W}{P}$$

$\frac{Y}{Y}\frac{N}{N} \cdot \frac{\theta C}{1-N} = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \Rightarrow \frac{N}{Y} \cdot \frac{\theta C}{1-N} = (1-\alpha)\left(\frac{K}{N}\right)^{d} \frac{N}{Y}$

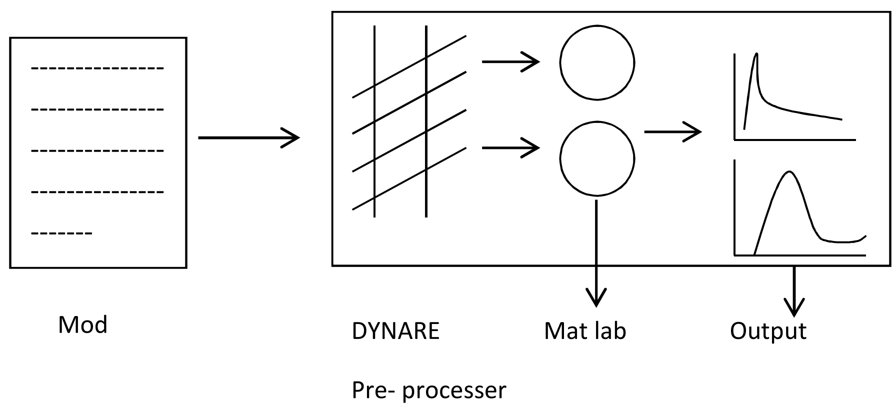
$\frac{N}{Y} = (1-\alpha)\frac{K^{\alpha}N^{1-\alpha}}{Y} \cdot \frac{1-N}{\theta C} \Rightarrow \frac{N}{Y} = (1-\alpha)\frac{(1-N)}{\theta C}$

Log-Lin around S.S.

- ✓ Euler
- ✓ Labor demand
- ✓ Capital demand
- ✓ Equilibrium
- ✓ Production function
- ✓ Capital law of motion
- ✓ Labor supply consider first $\hat{L}_t$
- ✓ Restriction $L_t + N_t = 1$

5. DYNARE

- ✓ Solve simulate and estimate DSGE models
- ✓ Facility for imputing model



Mod file, we declare the structure of the model

Pre processer translate into mat lab routine to solve or estimate the model

What kind of work does DYNARE?

- Compute SS of a model
- Compute the solution of determined models
- Compute 1^{st} and 2^{nd} order approximation to solve stochastic models
- Estimate parameters using Maximum Likelihood or Bayesian estimation
- Compute optimal polices in linear quadratic models
- We are interested in two things:
- Compute solution functions to a set of first order conditions

How the model response to shocks?

- Temporary
- Permanent

How the system come back to SS or finds a new SS

Keep in mind what kind of model you are treating:

- Stochastic: distribution of future shocks

- Deterministic: Occurrences of shocks are known when we are doing the model solution

For instance:

Technology shock:

- Deterministic: agents know what is gone happen so this innovation will be zero
- Stochastic: agents only know that it is random and will have zero mean

Stochastic models solution.

Agents made its decisions about policy or feedback rule for future and it will be contingent with the realization of the shocks.

What we look for is a solution that satisfies the first order condition of the model

Solution of deterministic models

- Numeral methods: series of number that match the equations.

Characteristic

DETERMINISTIC

- ✓ Introduce the impact of a change of regime for instance introduction of a new tax.
- ✓ Assume full information, no uncertainty a zero shocks, expectative rationales (perfect foresight).
- ✓ Shock is known and can hit for 1 or reserved periods.
- ✓ Solution not require linearization
- ✓ Is useful when linearization offer poor approximation around SS.

STOCHASTIC

- ✓ Popular in RBC model or new Keynesian models.
- ✓ Shock hit with a surprise today and after this $E(S_{t+i}) = 0$ this is because of Taylor approximation.
- ✓ Linearized the model permit agents behave as it future shocks where equal to zero: called certainty equivalence.
  It's doesn't permit the model be deterministic.

Work with DYNARE

Write the mod file

As DYNARE calls Mat Lab routines; DYNARE produces m-file

Solves non linear models with forward looking variables

Steps

- Declaration of variables
- Declaration of parameters
- Equations of the model
- SS values of the model if….
- Definition of the properties of the shocks

DYNARE is designed to simulate efficiently non linear models with forward looking consistent expectations.

DYNARE facilitates building macro models without knowing much of Mat Lab.

DYNARE is overfed toward consistent forward today expectations; means that we have perfect information about future evolution of the system so we solve simultaneously and theoretical infinite number of periods.

Have to add transversality conditions.

In practice DYNARE simulate a finite large numbers of periods with evolutions imposed at horizon, and we will approx these last by the long run equilibrium of the system.

A practical feature; DYNARE simulates a nonlinear dynamic behavior of the system around a given SS invariant trough time.

In the model shocks are all expected at period 1 and unexpected before.

Example:

$$E_t \sum_{t=0}^{\infty} \beta^t [\lg C + \Psi \lg 1 - L_t]$$

St. $$C_t + K_{t+1} = \left(\frac{W}{P}\right)_t N_t + \left(\frac{Z}{P}\right)_t K_t + (1-\delta)K_t$$

We can see as accounting identity

LHS: expenditures
RHS: revenues

a) Can be interpreted as capital accumulates noting that $\left(\frac{W}{P}\right)_t N_t + \left(\frac{Z}{P}\right)_t K_t$ are total payments of factors = aggregate output Y_t imposing zero profits
So $I_t = Y_t - C_t \Rightarrow$ law of notion $i_t = K_{t+1} - (1-\delta)K_t$
That show that investment diminish the effects of δ
$\therefore$ the consumers faces a trade off consuming and investing in order to increase the capital stock and assuming more in following periods.

FOC

Euler equation:

$$\frac{1}{C_t} = \beta E_t\left(\frac{1}{C_{t+1}}\left(1-\delta+\left(\frac{Z}{P}\right)_{t+1}\right)\right)$$

Labor supply

$$\Psi\frac{C_t}{1-L_t} = W_t$$

Firm is involved in a competitive market and has

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

Capital demand $\quad \alpha A_t \left(\frac{N}{K}\right)_t^{1-\alpha} = \left(\frac{Z}{P}\right)_t \quad || \frac{K}{K} \Rightarrow \alpha\frac{Y_t}{K_t} = \left(\frac{Z}{P}\right)_t$

Labor demand $\quad (1-\alpha)A_t\left(\frac{K}{N}\right)^{\alpha} = \left(\frac{W}{P}\right)_t \Rightarrow (1-\alpha)\left(\frac{Y_t}{N_t}\right) = \left(\frac{W}{P}\right)_t$

+ Shocks $K^{\alpha-1}$

No matrix representation is necessary

Variable in t just $x_t \rightarrow x$

$$\left.\begin{array}{r} t-n \rightarrow x_{t-n} \rightarrow x(-n) \\ t+n \rightarrow x_{t+n} \rightarrow x(+n)\ or\ x(n) \end{array}\right\}$$ Take care of backward "n" forward today

K_{t-1} Because is a predetermined variable

Conventions:

$A(+n)$ Indicates that variable should jump, is a forward variable or called non- predetermined variable.

Blanchard – Kahn condition is met when the number of non predetermined variables equals the numbers of eigenvalues qre greater than one.

Specify initial values

A stochastic model needs to have SS values

SS values are the reference points to simulations and impulse response functions

6. RULE OF THUMB CONSUMERS

Introduction of rule of thumb consumers change dramatically the response of consumption to shocks, in principle to monetary shocks.

Non Ricardians consumers alter the effects of monetary shocks. They don't borrow, nor save in order to smooth consumption and each period they consume their current labor income

Presence of rule of thumb can capture important aspects of actual economies which are missing in conventional models.

Support of the presence for industrialized economies can be found in Campell and Mankiw(1989)

Consumption, income and interest rates: reinterpreting the times series evidence.

No single representative consumer but by two groups

Half consumers are forward – looking and consume their permanent income, reluctant to substitute intertemporal consumption in response to interest rate move, rule – of – thumb of consuming their current income.

The presence of rule – of – thumb housholds rejects the permanent income hypothesis on the basic of aggregate data.

Rule – of – thumb households have important consequences for fiscal policy and its effect on the economy.

Interpretations includes myopia, lack of access to capital markets, fear of saving, ignorance of intertemporal trading opportunities.

Ricardian household

$$Max\ E_t \sum_{0}^{\infty} \beta^t \mu(C_t^0, L_t^0)$$

$$N_t^0 + L_t^0 = 1$$

$$P_t C_t^0 + P_t I_t^0 + R_t^{-1} B_{t+1}^0 = W_t N_t^0 + Z_t^0 K_t^0 + B_t^0 + \pi_t$$

To form the Lagrangian made the budget constraint in real terms

Assume that optimizer have the following utility functions

King Plosser and Rebelo (1988)

Basic Neoclassical model of growth in business cycle $\rightarrow U(\cdot) = \frac{1}{1-\sigma}\left(C^0 v(L^0)\right)^{1-\sigma} \rightarrow$ Production, growth and Business Cycle

FOC C_t^0

The basic neoclassic model

$v(\ )$ Twice differentiable and concave

$G = 1 \rightarrow$ The concavity requires $\log(v)$ que sea creciente y convexa

With this utility function

- ✓ Consumers must be willing to expand their consumption at a constant rate when real interest rate is constant:

$$C_t = C_{t+1}$$

- ✓ Optimal to supply a constant number of hours when the real interest rate is constant and wage rate grows at a constant rate:

$v(\ )$ Is concave if $\sigma < 1$

$v(\ \ )$ Is convex if $\sigma > 1$

To ensure the $v(\ \ )$ concavity:

$$-\sigma v(L)v''(L) > (1 - 2\sigma)[v'(L)]^2$$

Find labor supply

Capital supply $\rightarrow$ $d/dK_{t+1} : 1 = E_0\left(R_t^{-1}\left(\left(\frac{Z}{P}\right)_{t+1}\right) + 1 - \delta\right)$

In the equation real interest rate and the return should be equal to 1

From ${}^{d}\!/_{dB^0_{t+1}}$ ⇒ an optimal C, L, N, K plans follow the sequences:
$\{C_t\}_{t=0}^{\infty}\{K_t\}_{t=0}^{\infty}\{N_t\}_{t=0}^{\infty}\ and\ \{K_t\}_{t=0}^{\infty}$ that satisfies the FOC conditions and the transversality requirement $K_t \to \lim_{t\to\infty} \beta^t \lambda_t K_{t+1} = 0$

Rule Of Thumb Consumers

$$U(C^r, L^r)$$

$$\text{st.} \qquad P_t C_t^R = W_t N_t^R$$

But they can choose optimally the hours worked

⇒ Labor supply is = to the optimizer

$$\frac{1}{L_t^R} = \frac{1}{C_t^R}\left(\frac{W}{P}\right)_t$$

In the case of the elasticity σ is high⇒ $\sigma = 1$

$$U(\quad) = \ln C_t^{0,R} + \ln L_t^{0,R}$$

In the restring labor supply

$$\left.\begin{array}{r} \dfrac{N_t^R}{L_t^R} = \dfrac{N_t^R}{C_t^R}\left(\dfrac{W}{P}\right)_t \\ L_t^R = 1 - N_t^R \\ \text{Ex.}\quad N_t^R = {}^1\!/_2 \end{array}\right\} \begin{array}{l} C_t^R = \left(\dfrac{W}{P}\right)_t N_t^R \\ C_t^R = \left(\dfrac{W}{P}\right)_t {}^1\!/_2 \end{array}$$

Aggregation

$$C_t = (1-\lambda)C_t^0 + \lambda C_t^R$$

$$N_t = (1-\lambda)N_t^0 + \lambda N_t^R$$

$$C_t = (1-\lambda)C_t^0 + \lambda\left[\left(\frac{W}{P}\right)N_t^R\right]$$

$$C_t = (1-\lambda)\left(\frac{W}{P}\right)L_t^0 + \lambda\left[\left(\frac{W}{P}\right)N_t^R\right]$$

$$C_t = \left((1-\lambda)(1-N_t^0) + \lambda N_t^R\right)\left(\frac{W}{P}\right)$$

$$C_t = \left[(1-\lambda) - (1-\lambda)N_t^0 + \underbrace{\lambda N_t^R + (1-\lambda)N_t^0}_{N_t} - (1-\lambda)N_t^0\right]\left(\frac{W}{P}\right)_t$$

$$C_t = \left[N_t + 1 \underbrace{-\lambda - 2(1-\lambda)N_t^0}_{-2N_t} \right] \left(\frac{W}{P} \right)$$

$$N_t = (1-\lambda)N_t^0 + \lambda \frac{1}{2} \quad ||2$$

$$2N_t = 2N_t^0(1-\lambda) + \lambda$$

$$C_t = (N_t + 1 - 2N_t) \left(\frac{W}{P} \right)$$

$C_t = (1 - N_t) \left(\frac{W}{P} \right)$ Agregate Supply (AS) of labor

In general the AS doesn't change with the different types of agents

Consumption faces no liquidity restrictions in the long run

$$\frac{\bar{C}^R}{C} = \frac{\bar{C}^0}{C} = 1$$

The effect is on aggregation of consumption

$$C_t = \lambda C_t^R + (1-\lambda) C_t^0$$

$$\bar{C}(1 + \hat{C}_t) + \lambda \bar{C}^R (1 + \hat{C}_t^R) + (1-\lambda)\bar{C}^0 (1 + \hat{C}_t^0)$$

$$\hat{C}_t = \lambda \frac{\bar{C}^R}{\bar{C}} \hat{C}_t^R + (1-\lambda) \frac{\bar{C}^0}{\bar{C}} \hat{C}_t^0$$

$$\hat{C}_t = \lambda \hat{C}_t^R + (1-\lambda) \hat{C}_t^0$$

But it change because of the euler equation

$$\hat{C}_t^0 = E_t(\hat{C}_{t+1}) - \hat{r}_t$$

$$\hat{C}_t^0 = \hat{C}_{t+1}^0 - \hat{r}_t + w_{t+1}^R \rightarrow E_t(\hat{C}_{t+1}) - \hat{C}_{t+1}$$

$$\hat{C}_t = \lambda \hat{C}_t^R + (1-\lambda)\hat{C}_{t+1}^0 - (1-\lambda)\hat{r}_t + \lambda \hat{C}_{t+1}^R - \lambda \hat{C}_{t+1}^r$$

$$\hat{C}_t = \underbrace{(1-\lambda)\hat{C}_{t+1}^0 + \lambda \hat{C}_t^R}_{\hat{C}_{t+1}} - (1-\lambda)\hat{R}_t + \lambda \hat{C}_{t+1}^R - \lambda \hat{C}_{t+1}^r$$

$$\hat{C}_t = \hat{C}_{t+1} - (1-\lambda)\hat{R}_t + \lambda \hat{C}_{t+1}^R - \lambda \hat{C}_{t+1}^r$$

$$C_t^R = \frac{1}{2} \left(\frac{W}{P} \right) \Rightarrow \hat{C}_t^R = \left(\frac{\widehat{W}}{P} \right)$$

$$\hat{C}_t = \hat{C}_{t+1} - (1-\lambda)\hat{R}_t + \lambda\left(\Delta\left(\frac{\widehat{W}}{P}\right)_{t+1}\right)$$

$\hat{R}_t$: Intertemporal effect

$\left(\frac{\widehat{W}}{P}\right)_{t+1}$: Credit restrictions

∴ Not only consumption depends on interest rate, but also on the intertemporal effect of $\left(\frac{W}{P}\right)$

Example:

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma}e^{(\sigma-1)v(N)}$$

T.A.

$$g(x) = g(\bar{x})\left[1 + \frac{g'(\bar{x})}{g(\bar{x})}\bar{x}\hat{x}_t\right]$$

$$g(x) - g(\bar{x}) = g'$$

7. LONG RUN LABOR SUPPLY AND THE ELASTICITY OF INTERTEMPORAL SUBSTITUTION FOR CONSUMPTION

Three contradictions

- Consumption and labor are additively but in utility separable function
  - Euler equation are not influenced by labor
- The elasticity of intertemporal substitution is below < 1
- Labor supply is not totally inelastic in the long run

$\uparrow \left(\frac{W}{P}\right)$ Have little effect on labor supply that relies on labor income

Besides, as Euler equations reflects also permanent income hypothesis, getting out of labor from the analysis of income is by a time separable utility function

Hall (1988) "intertemporal substitution in consumption" in Journal of Political Economy

ISE: is measured by a response of the change of consumption to changes in the expected real interest rate

1. $\uparrow R_t \rightarrow \downarrow C_t \wedge \uparrow C_{t+1}$ whenever a $\uparrow AD$
2. DWL is important
3. Reduction of natural debt or unfunded soul security is relatively unimportant
4. Consumption moves for changes in interest rate over the cycle

Let consider

$$\Delta \ln(C_t) = s(r_t - \rho) + \epsilon_t + \theta\epsilon_{t-1}$$

$$\ln(C_t) = \ln(C_{t+1}) + s(r_t - \rho) + \underbrace{\epsilon_t + \theta\epsilon_{t-1}}_{AR(1)}$$

ρ: utility discount rate

s: elasticity of intemporal substitution consumption

Let take $s = 0.2 \leftarrow$ Hall (1988)

Kimbal et. al. (1995)

And suppose a time separability utility function

$$s = \frac{1}{\theta} \Rightarrow 0.2 = \frac{1}{\theta} = \frac{1}{5} \Rightarrow \theta = 5$$

θ: Aversion coefficient

K-P-R

$$U(\quad) = \frac{C^{1-\theta}}{1-\theta} - V(N)$$

$$U(\quad) = -\frac{1}{4C^4} - V(N) \qquad \rightarrow \frac{1}{C^5} = \lambda$$

Labor supply $\rightarrow -\frac{U_N(C,N)}{U_C(C,N)} = \frac{W}{P_C} \Rightarrow C^5 V'(N) = \frac{W}{P}$

K-B in USA consumption in 35 years has doubled $\approx 2\%$ per years $\uparrow C \approx 2\%$ per years, while hours worked is stable $N = {}^1\!/_3 \quad N$ ≅increase in a small proportion cause of $V(N)$

Number of work hours P/person

Taking $\underset{\substack{\downarrow \\ \cong 1}}{C^5 V'(N)} = \frac{W}{P} \;\wedge\; 2^5 = \frac{W}{P} \Rightarrow \underset{\downarrow}{\left(32 = \frac{W}{P}\right)} \longrightarrow \text{Not¡¡¡}$

$\frac{W}{P}$ didn't increase in that period, just doubled

Even

$0.333 = \frac{1}{\theta} \Rightarrow \frac{1}{3} = \frac{1}{\theta} \Rightarrow 2^3 = \frac{W}{P} = 8 \rightarrow \text{Not ¡¡¡}$

An alternative way $\uparrow C \; and \; \uparrow \frac{W}{P} \Rightarrow$ as we have intratemporal substitution effects between $C \; and \; N \Rightarrow \downarrow N_t \leftarrow$ this falling is explained by household satisfied consumption so, turned to additional leisure.

The income effect through wages

∴ Maintaining a separable function ⇒ leads to an IES in consumption reinforce the income effect of permanent wage increase stronger than the substitution effect of a permanent wage

Micro founding:

- ✓ If we have 100% of household surveys on average 75 of percentile work as much as of 25 percentile
- ✓ Wage shocks not affect much to an individual on its labor supply ← may be explained By law restrictions ← wage rigidities

In wage increase → income effect is larger than substitution effect it violates the evidence on long run labor supply.

So non separability make sense → using K-P-R utility function *the* elasticity of substitution $IES = 0.6$

$$\frac{10}{6}=\frac{5}{3}=\frac{1}{\theta}\Rightarrow\theta=\frac{3}{5}$$

$$2^{0.5}=\frac{W}{P}\Rightarrow\frac{W}{P}=1.41 \text{ with } V'(N)\cong 1$$

$$\frac{W}{P}\cong 2$$

Including evolution of labor in the Euler equation, as Campell and Mankiw (89,91) helps us to finding that predictable movements in disposable income are too predictable movement in consumption.

The long –run labor supply is not inelastic but it increases slightly over the time

The separable utility function

$$U(\quad)=\frac{C_t^{1-\theta}}{1-\theta}-V(N_t)$$

We have just an Euler equation

$$C_t^{-\theta}=\beta E_t\left(C_{t+1}^{-\theta}\cdot R_t\right)$$

Log – lin

$$C^{-\theta}\left(1-\theta\hat{C}_t\right)=\beta C^{-\theta}R\left(1-\theta\hat{C}_{t+1}+\hat{r}_t\right)$$

$$\hat{C}_t=\hat{C}_{t+1}-\frac{1}{\theta}\hat{r}_t$$

So the ISE

$$\frac{d\left(\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)}{d\hat{r}_t}=\frac{1}{\theta}$$

In the long –run income effect and substitution effect are kindly the same, so the whole effect disappears.

A reasonable assumption is a non separable utility function as King – Plosser- Rebelo

$$U(C,N)=\frac{C^{1-\theta}}{1-\theta}e^{(\theta-1)v(N)}$$

And

$$s=\frac{1}{\theta}$$

s: is labor –held- constant elasticity of intertemporal substitution in consumption.

So

FOC

$C_t{:}\,\beta^t C_t^{-\theta}e^{(\theta-1)v(N_t)}=\beta^\lambda\lambda_t$

$N_t{:}\,\beta^t(-)\frac{C^{1-\theta}}{1-\theta}e^{(\theta-1)v(N_t)}(\theta-1)v'(N)=\left(\frac{W}{P}\right)_t\beta^t\lambda_t$

$\frac{C^{1-\theta}e^{(\theta-1)v(N_t)}v'(N_t)}{C^{-\theta}e^{(\theta-1)v(N_t)}}=\left(\frac{W}{P}\right)_t$

$C_t v'(N_t)=\left(\frac{W}{P}\right)_t$

It establish his worked

Are stables through a roughly double consume and wages

$\uparrow C\wedge\uparrow W$ in the same proportion $\Rightarrow\overline{N}_t$ stable

$\Rightarrow$ Income effect = substitution effect

In SS $C\cdot v'(N)=\frac{W}{P}\Rightarrow\frac{W}{P}\cdot\frac{N}{C}=v'(N)C$ =ctte: stable

Again the macro implications is through the Euler equation

Euler:

$C_t^{-\theta}e^{(\theta-1)V(N_t)}=\beta E_t\left(C_{t+1}^{-\theta}e^{(\theta-1)v(N_{t+1})}R_t\right)$

Log lin

$$C^{-\theta}e^{(\theta-1)v(N)}\left(1-\theta\hat{C}_t+\frac{(\theta-1)e^{(\theta-1)v(N_t)}V'(N_t)}{e^{(\theta-1)v(N_t)}}\cdot\overline{N}\widehat{N}_t\right)$$

$$= \beta C^{-\theta} e^{(\theta-1)v(N)} \left(1 - \theta \hat{C}_{t+1} + \frac{(\theta-1)e^{(\theta-1)v(N_{t+1})} V'\left(\widehat{N}_{t+1}\right)}{e^{(\theta-1)v(N_{t+1})}} \cdot \overline{N}\widehat{N}_t + \hat{r}_t\right) + w^c_{t+1}$$

$$-\theta \hat{C}_t + (\theta-1)V'(N_t)\overline{N}\widehat{N}_t = -\theta \hat{C}_{t+1} + (\theta-1)V'(N_{t+1})\overline{N}\widehat{N}_{t+1} + \hat{r}_t$$

$V'(N_t) = V'(N_{t+1})$ cause of SS and N_t is stable

$$\hat{C}_t = \hat{C}_{t+1} + \left(\frac{1-\theta}{\theta}\right) V'(N) N \Delta \widehat{N}_{t+1} - \underbrace{\frac{1}{\theta}}_{ISE \to 0.6} \hat{r}_t$$

Macro implications

Labor and consume are complementary if $\theta = 5$ intratemporal

$$\hat{C}_t = \hat{C}_{t+1} + \left(-\frac{4}{5}\right) V'(N) N \Delta \widehat{N}_{t+1} - \frac{1}{5}\hat{r}_t$$

$$\uparrow \hat{C}_t = \hat{C}_{t+1} - \frac{4}{5} V'(N) N \widehat{N}_{t+1} + \frac{4}{5} V'(N_t) N \widehat{N}_t - \frac{1}{5}\hat{r}_t$$

Note T.A.

$$f(x) = f(\bar{x}) + f'(x)(x - \bar{x})$$

$$\frac{f(x) - f(\bar{x})}{f(\bar{x})} = \frac{f'(x)(x - \bar{x})}{f(\bar{x})} \cdot \frac{\bar{x}}{\bar{x}}$$

$$f(\hat{x}) = \frac{f'(x)}{f(\bar{x})} \bar{x}\hat{x}$$

If $\theta = {}^{3}/_{5}$ $\quad C \ and \ N$ are substitutes

As King and Rebelo, used by Gali, Lopez-Salido, Valles (2005)

$$U(C_t, L_t) = \frac{1}{1-\theta}\left\{C_t v(L_t)^{1-\theta} - 1\right\}$$

Sustitution effect are = income effect

Most used $\theta \to 1 \to \ln C_t + \ln v(L_t) \ \wedge v(L_t) = L_t$

Labor supply $\quad \frac{1}{L_t} = \frac{1}{C_t}\left(\frac{W}{P}\right)_t$

Log – lin $\quad \hat{C}_t - \frac{\overline{N}}{\overline{N}-1}\widehat{N}_t = \left(\frac{\widehat{W}}{P}\right)_t$

$$\hat{C}_t + \frac{\overline{N}}{1-\overline{N}}\widehat{N}_t = \left(\frac{\widehat{W}}{P}\right)_t \hat{L}_t = \frac{\overline{N}}{\overline{N}-1}\widehat{N}_t$$

But if $N = 0.2 \; \Rightarrow$ labor elasticity $\uparrow \frac{1-\bar{N}}{\bar{N}}$: e. g. $\frac{1-0.2}{0.2} = 4$

We need the labor supply more elastic, Smets and Wouters:

$$U(\quad) = \frac{1}{1-\theta}\left[C_t^{\left(\frac{\Psi(1-N_t)^{1-v_g}-1}{1-v_g}\right)}\right]^{1-\theta} - \frac{1}{1-\theta}$$

$$C_t v'(L) = \left(\frac{W}{P}\right)_t$$

$$v'(L) = \Psi L_t^{-v_g} \Rightarrow \; C_t \Psi L_t^{-v_g} = \left(\frac{W}{P}\right)_t$$

$$\bar{C}\Psi L^{-v_g}\left(1+\hat{C}_t - v_g\hat{L}_t\right) = \left(\frac{\bar{W}}{P}\right)\left(1+\left(\frac{\widehat{W}}{P}\right)_t\right)$$

The elasticity respect to $\frac{W}{P}$ in the long run is 0 because hour worked not change $\Rightarrow$ income effect = sust effect.

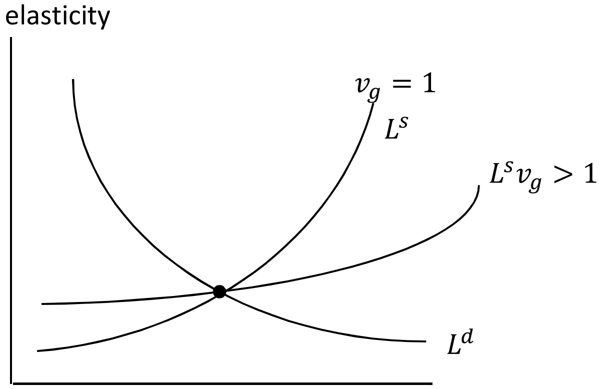
Log – lin

$$\hat{C}_t - v_g\hat{L}_t = \left(\frac{\widehat{W}}{P}\right) \quad \wedge \; \hat{L}_t = \frac{\bar{N}}{\bar{N}-1}\widehat{N}_t$$

$$\hat{C}_t + \frac{\bar{N}}{1-\bar{N}}v_g\widehat{N}_t = \left(\frac{\widehat{W}}{P}\right)_t$$

$$\widehat{N}_t = \frac{1-\bar{N}}{\bar{N}}\frac{1}{v_g}\left(\frac{\widehat{W}}{P}\right)_t - \left(\frac{1-\bar{N}}{\bar{N}}\frac{1}{v_g}\right)\hat{C}_t$$

So if $\uparrow v_g \;\Rightarrow \downarrow$ elasticity



$\uparrow v_g \rightarrow$it is useful to approximate micro data

If we like to expose the response to shock we must play with v_g

8. LABOR SUPPLY AND INDIVISIBLE LABOR

Most of RBC models that includes separable utility functions predicts very high elasticity of leisure across time periods for household, which is inconsistent with panel data, e.g. if ESI is $\theta = 5 \Rightarrow$ elasticity in respect to real wages is 32.

So modeling non separability utility functions and indivisible labor → meaning that labor includes in the Euler equation and permits us to get low elasticity of substitution →elasticity of the labor supply is nearly. 2 as we can see it in micro data Ej. Kimbal and Basu

Let consider a K-P-R utility function

$$U(C_t N_t) = \frac{1}{1-\sigma}\left\{\left(C_\gamma(L)\right)^{1-\sigma} - 1\right\}$$

$$V(L) = e^{V(L)}$$

$$\sigma \to 1 \Rightarrow \ln C + \ln v(L_t)$$

$$\ln(C_t) + \ln(1 - N_t)$$

Labor supply is

$$\frac{1}{L_t} = \frac{1}{C_t}\left(\frac{W}{P}\right)_t \Rightarrow \hat{L}_t = \hat{C}_t - \left(\frac{\widehat{W}}{P}\right)_t$$

$$\hat{L}_t = \frac{N}{N-1}\widehat{N}_t$$

The labor supply will be

$$\frac{N}{N-1}\widehat{N}_t = \hat{C}_t - \left(\frac{\widehat{W}}{P}\right)_t \Rightarrow N_t = \frac{N-1}{N}\hat{C} - \overbrace{\underbrace{\frac{N-1}{N}}_{\substack{\text{elasticity of} \\ \text{the labor supply}}}}^{-\rho}\left(\frac{\widehat{W}}{P}\right)_t$$

Microeconomic data says elasticity is nearly 1

But we have $N = {}^{1}/_{3} \approx 8\ hrs$

$$\rho = \frac{1-N}{N} = \frac{1-{}^{1}/_{3}}{{}^{1}/_{3}} = \frac{{}^{2}/_{3}}{{}^{1}/_{3}} = 2$$

So it’s necessary to introduce indivisible labor because movements or fluctuations in aggregate hours worked arise due to

- Changes in both number of hours people choose to work (intensive margin)

- The number of people entering and leaving the work force (extensive margin)

Hansen's Lottery (1985)

- Each individual in the economy has to choose between working o fixed shift of numbers of hours and not working at all
- Random Lottery

Two kinds $var \lg H = var\ (\lg h_t) + var\ (\lg n_t) + 2cov(\lg h_t\ , \lg n_t)$

$H \rightarrow$ Total hours worked

$n_t =$ Number of people at work

$h_t =$Average hours worked

Since agent chooses we have

$$p\ U(C_1, H) + (1 - \rho)(C_2, l)$$

If $\sigma = 1$

$$U(C, L) = \underbrace{p \ln C_1 + (1 - \rho) \ln C_2}_{\underbrace{\ln \mathrm{C}}_{\text{Percapita consumption}}} + \underbrace{p \ln(1 - H)}_{(1-\rho)\ln(L)}$$

Hansen (1985) finds that: with quarterly data for U.S. 55% of $var \lg H$ is in function of variation in the number of people at work and 20% of the $var\ (\lg h_t)$

$$var \lg H = 20\% + 55\% + 2cov\ (h_t n_t)$$

Now most of the variation of total hours worked is due to individuals either working or not working.

So this supports using indivisible labor in the utility function.

Besides indivisible labor displays larger fluctuations than the divisible labor in the economy

- Indivisible labor increases the volatility of the stochastic model given a shock of technology.
- Indivisible labor generates standard deviation that is closer to the observed values.

What does Hansen proposed?

Another way to reduce the income effect is through Hansen's lotteries

We can maximize

$$E_t \sum_{0}^{\infty} \alpha_t \left(\lg c_t + \lg\left(1-\bar{h}\right) + (1-\alpha_t)(\lg c_t - \lg(l))\right)$$

St. Restrictions

α_t probability of work

$$A = \frac{\lg\left(1-\bar{h}\right)}{F} \leftarrow labor\ work\ force$$

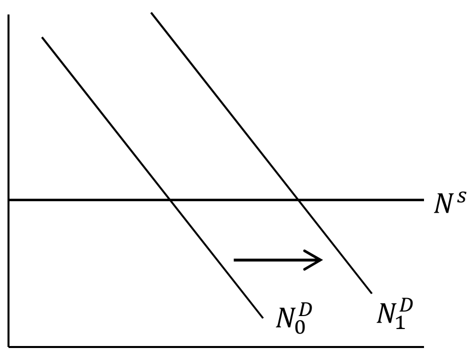
$$\Rightarrow Max\ E_0 \sum_{0}^{\infty} \lg C_t + AN_t$$

FOC

$$C_t : \frac{1}{C_t} = \lambda_t$$

$$N_t : A - \lambda_t \left(\frac{W}{P}\right)_t = 0$$

Labor supply $C_t : A = \left(\frac{w}{P}\right)_t$ ← labor supply is elastic



If we have a technological shock (behaving that consume is stable) →↑labor demand, it produces that only labor varies and the variation of real wages not

Remember that through K-P-R non separable utility function

$$U(C,N) = \frac{C^{1-\theta}}{1-\theta} e^{(\theta-1)V(N)}$$

And Smets and Wouter

$$U(\quad) = \frac{1}{1-\theta}\left[C_t \quad e^{\frac{\Psi(1-N_t)^{1-v}g_{-1}}{1-\mathrm{v}\mathrm{g}}}\right]^{1-\theta} - \frac{1}{1-\theta}$$

We can have close income effects nearly (≅) substitution effect

Let's form our system

$$U(C,N) = \frac{C^{1-\theta}}{1-\theta} e^{(\theta-1)\gamma(N)}$$

We have

Euler:

$$\hat{C}_t = \hat{C}_{t+1} - \frac{1}{\theta}\hat{r}_t + \left(\frac{1-\theta}{\theta}\right) V'(N)N(\hat{n}_{t+1} - \hat{n}_t)$$

$V'^{(N)} \cong 1$

$N = {}^{1}/_{3}$

Labor supply $C_t V'(N_t) = \left(\frac{w}{P}\right)_t$

$$C\ \ V'(N)\left(1 + \hat{C}_t + \frac{V''(N)}{V'(N)} N N_t\right) = \frac{W}{P}\left(1 + \left(\widehat{\frac{W}{P}}\right)_t\right)$$

$$\hat{C}_t + N\widehat{N}_t = \left(\widehat{\frac{W}{P}}\right)_t$$

$$\hat{n}_t = \frac{1}{N}\left(\widehat{\frac{W}{P}}\right)_t - \frac{1}{N}\hat{C}_t$$

Labor demand and Capital demand

$$\alpha \frac{Y_t}{K_t} = \left(\frac{Z}{P}\right)_t \leftarrow \alpha A_t N^{1-\alpha} K^{\alpha-1} = \frac{z}{p}$$

$$(1-\alpha)\frac{Y_t}{N_t} = \left(\frac{W}{P}\right)_t$$

Log – lin

$$\hat{y}_t - \hat{n}_t = \left(\widehat{\frac{W}{P}}\right)_t$$

$$\hat{y}_t - \hat{k}_t = \left(\widehat{\frac{z}{P}}\right)_t$$

$\widehat{K}_{t+1} = (1-\delta)\widehat{K}_t - \delta i_t$

$\hat{r}_{t+1} = (1-\delta)\widehat{K}_t + \left(\widehat{\frac{z}{P}}\right)_{t+1} \rightarrow \hat{r}_{t+1} = \left(\frac{\delta + r}{1+r}\right)\left(\widehat{\frac{z}{P}}\right)_{t+1}$

$y_t = a_t + \alpha\widehat{K}_t + (1-\alpha)\hat{n}_t$

Daney Valdivia ®

$c_t + i_t = y_t \Rightarrow c(1+\hat{c}_t) + i(1+\hat{\imath}_t) = y(1+\hat{y}_t)$

$\frac{C}{y}\hat{c}_t + \frac{I}{y}i = \hat{y}_t$

$a_t = \rho a_{t-1} + \varepsilon_t$

With Hansen's specification we have the following labor supply

$$\sum \beta^t \lg C_t - AN_t$$

FOC

$$\left.\begin{aligned} C_t&: \frac{1}{C_t} = \lambda_t \\ N_t&: A = \left(\frac{W}{P}\right)_t \end{aligned}\right\} C_t A = \left(\frac{W}{P}\right)_t$$

$\hat{C}_t = \left(\frac{\widehat{W}}{P}\right)_t \leftarrow$ Labor supply

But if we consider the type of $\theta \neq 1$

$$Max\ E_0 \sum_{t=1}^{\infty} \beta^t \left(\frac{C^{1-\theta} - 1}{1-\theta} - AN_t\right)$$

FOC

$C_t = C^{-\theta} = \lambda_t$

Labor supply:

$C^{\theta} A = \left(\frac{W}{P}\right)_t$

$\theta \hat{C}_t = \left(\frac{\widehat{W}}{P}\right)_t$

$Var\ \ c, \quad r, \quad n, \quad \frac{w}{P}, \quad \frac{z}{P}, \quad y, \quad k, \quad i, \quad a$

$Var\ exo\ \varepsilon_t$

Parameters $\theta\ Nv'(N)\ \delta\ \alpha\ {}^{C}\!/_{y}\ {}^{I}\!/_{y}\ \rho$

$\theta = {}^{3}\!/_{5}$

$N = {}^{1}\!/_{3}$

Daney Valdivia ®

$V'(N) = 0.9999$

$\delta = 0.2$

$\alpha = 0.44$

$\frac{C}{Y} = 0.7$

$\frac{I}{Y} = 0.5$

$\rho = 0.8$

$K = 9$

$C = 0.6$

$n = 0.3$

$w_p = 2$

$z_p = 0$

$h = 0$

$V = 0$

$r = 0.03$

9. THE PROBLEM OF THE FIRM

Firm seeks to maximize the value of shareholders.

The Tobin's Q will be the value of one partner claim to the firm and is what the firm is going to maximize.

The firm only produce Capital goods and has the following profit function:

$$Max\ E_t \sum_{0}^{\infty} \frac{z_{t+i}K_{t+i} - P_{t+i}I_{t+1}}{\pi^i}$$

π^i ←Factor discount

St.

$$K_{t+1} = (1-\delta)K_t + \underbrace{\phi\left(\frac{I_t}{K_t}\right)K_t}_{\substack{\text{adjustment}\\ \text{cost}}}$$

According to Correia, Neves and Rebelo (1995) and Getler $\phi(\ )$ is increasing and convex, depends on the scale of the firm and is convex in the absolute value of ${}^{I_t}/_{K_t}$.

The presence of adjustment cost, for example installing new capital cost, turns the investment problem into a dynamic problem.

$\phi(\ )$ is what makes the decision of installing new capital different from the employment decision.

Let assume $\phi\left(\frac{I_t}{K_t}\right)K_t = I_t - \frac{1}{2}\chi\left(\frac{I_t}{K_t} - \mu\right)^2 K_t$

μ is the steady state of ${}^{I}/_{K}$ stock associated with no adjustment cost μK_t. The level of investment necessary to maintain the plant.

$$\phi\left(\frac{I_t}{K_t}\right)K_t = I_t\frac{K_t}{K_t} + \frac{1}{2}\chi\left(\frac{I_t}{K_t} - \mu\right)^2 K_t$$

$$\phi\left(\frac{I_t}{K_t}\right)K_t = \underbrace{\left[\frac{I_t}{K_t} + \frac{1}{2}\chi\left(\frac{I_t}{K_t} - \mu\right)^2\right]}_{\phi\left(\frac{I_t}{K_t}\right)}K_t$$

If it will not exist adjustment cost $\frac{d\phi K_t}{dI_t} = \phi'(\ \ )\frac{1}{K_t}K_t$

In S.S. $\phi = \frac{I}{K}$

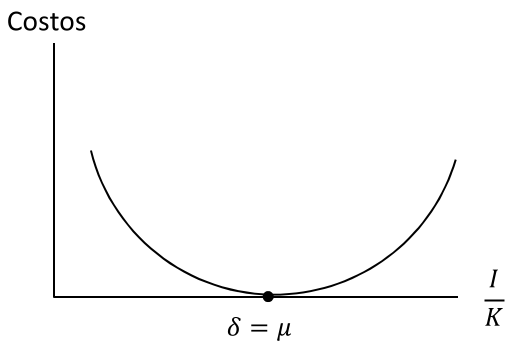
Why? $\phi\ \left(\frac{I}{K}\right) = \frac{I}{K} + \frac{1}{2}\chi\left(\frac{I}{K} - \frac{I}{K}\right)^2$

$$\phi = \frac{I}{K}$$

So in the law of motion $\quad K = (1-\delta)K + \frac{I}{K}K$

$$1 = 1 - \delta + \frac{I}{K} \Rightarrow \frac{I}{K} = \delta$$

$\phi' = 1 \leftarrow$ No adjustment cost



Bellman equation

Necessary condition for optimality associated with the mathematical method knows as dynamic programming

Firm Maximization

$$\underset{I_t, K_t}{\text{Max}}\, v^t(K_t) = Z_t K_t - P_t I_t + \frac{E_t v^{t+1}(K_{t+1})}{1 + i_{t+1}} \Big\} \begin{array}{l}\text{the firm decides} \\ \text{how much invest}\end{array}$$

St. $\quad K_{t+1} = (1-\delta)K_t + \phi\left({}^{I_t}/_{K_t}\right) K_t$

$$Max\ v^t(K_t) \equiv Z_t K_t - P_t I_t + \frac{E_t v^{t+1}\left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t\right)}{1 + i_{t+1}}$$

FOC

$$I_t{:} - P_t + \frac{E_t v^{t+1}\left((1-\delta)K_t + \phi'(\quad)\frac{1}{K_t} K_t\right)}{1 + i_{t+1}} = 0$$

$$-P_t + E_t\left[\frac{v^{t+1}\phi'(\quad)}{1 + i_{t+1}} \cdot \frac{P_t}{P_t} \cdot \frac{P_{t+1}}{P_{t+1}}\right] = 0$$

We need this because investment is in function of real interest rate and

$$1+\pi_{t+1}=\frac{P_{t+1}}{P_t}\wedge 1+r_{t+1}=R_{t+1}=\frac{1+i_{t+1}}{1+\pi_{t+1}}$$

$$-P_t+E_t\left[v^{t+1}\phi'(\quad)\frac{P_t}{P_{t+1}}\underbrace{\frac{1+\pi_{t+1}}{1+i_{t+1}}}_{\frac{1}{R_{t+1}}}\right]$$

$$Q_t=\frac{v^{t+1}}{R_{t+1}P_{t+1}}$$

$$-P_t+E_tQ_t\phi'(\quad)\cdot P_t=0$$

$-1+E_t[Q_t\phi'(\quad)]=0$

Since Q_t is the future flow, and it will tell me if incentives to invest

Interpretation $\Rightarrow E_t[Q_t\phi'(\quad)]>1$

$\phi'(\quad)$ Marginal cost of an extra unit of Capital

Q_t Marginal benefit

In equilibrium we can expect $E_t[Q_t\phi'(\quad)]=1$

Q_t How much is my marginal benefit when I produce one unit of Capital

FOC

$$K_t\colon Z_t+E_tv_K^{t+1}\left[\frac{1-\delta+\phi(\quad)_t\frac{I_t}{K_t}}{1+i_{t+1}}\right]$$

$1+i_{t+1}$ it discount the future flow of the benefits.

v_K^{t+1} is in function of the future flow and the interest rate.

Forward one period:

$$E_t\left(\frac{v^{t+1}(K_{t+1})}{1+i_{t+1}}\right)=E_t\left[\frac{1}{1+i_{t+1}}\left(\frac{Z_{t+1}+v^{t+2}\left(1-\delta+\phi(\quad)_{t+1}-\phi'(\quad)_{t+1}\frac{I_{t+1}}{K_{t+1}}\right)}{1+i_{t+2}}\right)\right]$$

$$E_t\left(\frac{v^{t+1}(K_{t+1})}{1+i_{t+1}}\frac{P_t}{P_t}\cdot\frac{P_{t+1}}{P_{t+1}}\right)=E_t\left[\frac{1}{1+i_{t+1}}\frac{P_t}{P_t}\cdot\frac{P_{t+1}}{P_{t+1}}\left(\frac{Z_{t+1}+v^{t+2}\left(1-\delta+\phi(\quad)_{t+1}-\phi'(\quad)_{t+1}\frac{I_{t+1}}{K_{t+1}}\right)}{1+i_{t+2}}\right)\right]$$

$$\frac{v^{t+1}(\;\;)}{R_{t+1}P_{t+1}} = E_t\left[\frac{1}{R_{t+1}}\frac{1}{P_{t+1}}\left[Z_{t+1} + \frac{v^{t+2}\left(1-\delta+\phi\,(\;\;)_{t+1} - \phi'(\;\;)_{t+1}\left(\frac{I}{K}\right)_{t+1}\right)}{1+i_{t+2}}\right]\right]$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left[\left(\frac{Z}{P}\right)_{t+1} + \frac{v^{t+2}}{(1+i_{t+2})P_{t+1}}\frac{P_{t+2}}{P_{t+2}}\left(1-\delta+\phi_{t+1} - \phi'_{t+1}\left(\frac{I}{K}\right)_{t+1}\right)\right]\right]$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left[\left(\frac{Z}{P}\right)_{t+1} + \frac{v^{t+2}}{R_{t+2}P_{t+2}}\left(1-\delta+\phi_{t+1} - \phi'_{t+1}\left(\frac{I}{K}\right)_{t+1}\right)\right]\right]$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left[\left(\frac{Z}{P}\right)_{t+1} + Q_{t+1}\left(1-\delta+\phi_{t+1} - \phi'_{t+1}\left(\frac{I}{K}\right)_{t+1}\right)\right]\right] \; (2)$$

Log - lin

$$(1)E_0(Q_t) = \frac{1}{\phi'\left(\frac{I}{K}\right)_t} \Rightarrow Q_t\phi'_t = 1$$

$$Q\phi'\left(1+\hat{q}_t + \frac{\phi''}{\phi'}\cdot\frac{I}{K}\left(\hat{I}_t - \widehat{K}_t\right)\right) = 1(1+0)$$

$$\hat{q}_t + \underbrace{\frac{\phi''}{\phi'}\cdot\frac{I}{K}}_{-\xi}\left(\hat{I}_t - \widehat{K}_t\right) = 0$$

$$-\frac{\phi''}{\phi'}\cdot\frac{I}{K} = \xi$$

$$\xi\hat{q}_t = \hat{I}_t - \widehat{K}_t$$

ξ Resumes adjustment cost

ξ Investment elasticity of the Tobin's Q

Log – lin (2)

$$Q_t = E_t\left(\frac{1}{R_{t+1}}\left[\left(\frac{Z}{P}\right)_{t+1} + Q_{t+1}\left((1-\delta)+\phi_{t+1} - \phi'_{t+1}\left(\frac{I}{K}\right)_{t+1}\right)\right]\right)$$

$$Q_t = \frac{1}{R_{t+1}}\left(\frac{Z}{P}\right)_{t+1} + \frac{Q_{t+1}}{R_{t+1}}(1-\delta) + \frac{Q_{t+1}}{R_{t+1}}\phi_{t+1} - \frac{Q_{t+1}}{R_{t+1}}\phi'(\;)_{t+1}\left(\frac{I}{K}\right)_{t+1}$$

$$Q\left(1+\hat{Q}_t\right)=R^{-1}\left(\frac{Z}{P}\right)\left(1-\hat{R}_{t+1}+\left(\frac{\hat{z}}{P}\right)_{t+1}\right)+\frac{Q}{R}(1-\delta)\left(1+\hat{q}_{t+1}-\hat{R}_{t+1}\right)$$
$$+\frac{Q}{R}\phi\left(1+\hat{q}_{t+1}-\hat{R}_{t+1}+\frac{\phi'}{\phi}\cdot\frac{I}{K}\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)\right)$$
$$-\phi'\frac{Q}{R}\frac{I}{K}\left(1+\hat{q}_{t+1}-\hat{R}_{t+1}+\frac{\phi''}{\phi'}\cdot\frac{I}{K}\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)+\hat{I}_{t+1}-\hat{K}_{t+1}\right)$$

$$Q=1$$

$$\phi'=1$$

$$\phi=\frac{I}{K}=\delta$$

$$R=\frac{Z}{P}+1-\delta\Rightarrow 1+r=\frac{Z}{P}+1-\delta\Rightarrow r+\delta=\frac{Z}{P}$$

$$\hat{Q}_t=\frac{r+\delta}{1+r}\left(\left(\frac{\hat{z}}{P}\right)_{t+1}-\hat{R}_{t+1}\right)+\frac{1-\delta}{1+r}\left(\hat{q}_{t+1}-\hat{R}_{t+1}\right)+\frac{\delta}{1+r}\left(\hat{q}_{t+1}-\hat{R}_{t+1}+\hat{I}_{t+1}-\hat{K}_{t+1}\right)$$
$$-\frac{\delta}{1+r}\left(\hat{q}_{t+1}-\hat{R}_{t+1}-\xi\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)+\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)\right)$$

Collecting in term of commons:

$$\hat{Q}_t=\frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1}-\left(\frac{r+\delta}{1+r}+\frac{1-\delta}{1+r}-\frac{\delta}{1+r}+\frac{\delta}{1+r}\right)\hat{R}_{t+1}+\left(\frac{1-\delta}{1+r}+\frac{\delta}{1+r}-\frac{\delta}{1+r}\right)\hat{q}_{t+1}$$
$$+\left(\frac{\delta}{1+r}+\frac{\delta\xi}{1+r}-\frac{\delta}{1+r}\right)\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)$$

$$\hat{Q}_t=\frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1}-\left(\frac{r}{1+r}\right)\hat{R}_{t+1}+\left(\frac{1-\delta}{1+r}\right)\hat{q}_{t+1}+\left(\frac{\delta\xi}{1+r}\right)\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)$$

We know that:

$$\hat{q}_t=\xi\left(\hat{I}_t-\hat{K}_t\right)$$

$$\Rightarrow\frac{\hat{q}_{t+1}}{\xi}=\hat{I}_{t+1}-\hat{K}_{t+1}$$

$$\hat{q}_t=\frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1}-\left(\frac{r}{1+r}\right)\hat{R}_{t+1}+\left(\frac{1-\delta}{1+r}\right)\hat{q}_{t+1}+\left(\frac{\delta\xi}{1+r}\right)\xi^{-1}\hat{q}_{t+1}$$

$$\hat{q}_t=\left(\frac{r+\delta}{1+r}\right)\left(\frac{\hat{z}}{P}\right)_{t+1}-\left(\frac{r}{1+r}\right)\hat{R}_{t+1}+\left(\frac{1-\delta+\delta\xi^2}{1+r}\right)\hat{q}_{t+1}$$

$$\hat{q}_t=\left(\frac{r+\delta}{1+r}\right)\left(\frac{\hat{z}}{P}\right)_{t+1}-\left(\frac{r}{1+r}\right)\hat{R}_{t+1}+\left(\frac{1}{1+r}\right)\hat{q}_{t+1}$$

∴ Tobin's Q depends on the future path of price shadow ($\hat{q}_{t+1}$) and the interest rate

When we use capacity installed $\delta_t = \delta U_t^n; \quad n > 1$ we suppose rate of utilization, U_t

This modification reduces the variance of productivity shock

E.g.

$$U(H, N) = \frac{H^{1-\theta} e^{\theta-1\, v(N)} - 1}{1-\theta}$$

$$H_t = C_t - \gamma C_{t-1}$$

S.t. restriction

10. INVESTMENT

$$Max\ E_t \sum_{i=0}^{\infty} \frac{Z_{t+i}K_{t+i} + P_{t+i}I_{t+i}}{\pi^i}$$

St

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t$$

$$\phi\left(\frac{I_t}{K_t}\right) = I_t - \frac{1}{2}\chi\left(\frac{I_t}{K_t} - \mu\right)^2 K_t$$

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{1}{2}\chi\left(\frac{I_t}{K_t} - \mu\right)^2 \frac{K_t}{K_t}$$

$$Max\ v^t(K_t) = (Z_tK_t - P_tI_t) + \frac{E_t v^{t+1}(K_{t+1})}{1 + i_{t+1}}$$

St

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t$$

FOC

$$I_t: -P_t + \frac{E_t v_k^{t+1}\phi'}{1 + i_{t+1}} = 0$$

$$-P_t + E_t v_k^{t+1}\phi' \cdot \frac{1}{1 + i_{t+1}} \cdot \frac{P_t}{P_t} \cdot \frac{P_{t+1}}{P_{t+1}} = 0$$

By Fisher

$$i = r + \pi \ \Rightarrow r = i - \pi$$

$$-P_t + \frac{E_t v_k^{t+1}\phi' P_t}{P_{t+1}} \cdot \frac{1}{1 + i_{t+1}} \cdot \frac{P_{t+1}}{P_t} = 0$$

$$-P_t + \frac{E_t v_k^{t+1}\phi' P_t}{P_{t+1}R_{t+1}} = 0$$

$$Q_t = \frac{E_t v_k^{t+1}}{P_{t+1}R_{t+1}}$$

$$-P_t + E_t\big(Q_t\phi'(\quad)\big)P_t = 0$$

$$-1 + E_t\big(Q_t \phi'(\quad)\big) = 0$$

$$Q_t = \frac{1}{\phi'(\quad)}$$

$$\phi_I\left(\frac{I_t}{K_t}\right) = I_t - \frac{1}{2}\chi\left[\left(\frac{I_t}{K_t}\right) - 2\frac{I_t}{K_t}\mu + \mu^2\right]K_t$$

$$\phi'_{I_t} = 1 - \frac{1}{2}\chi\left[2\frac{1}{{K_t}^2}I_t - \frac{2}{K_t}\mu\right]$$

$$\bar{Q}_t = 1$$

$$\phi'(\quad) = 1$$

$$\Rightarrow \bar{Q}_t = 1$$

FOC

$$K_t\colon\ v_K^t(K_t) = Z_t + \frac{E_t v^{t+1}\left[1 - \delta + \phi\left(\frac{I_t}{K_t}\right) - \phi'(\quad)\frac{I_t}{{K_t}^2}K_t\right]}{1 + i_{t+1}}$$

Forward one period:

$$v_K^{t+1}(K_{t+1}) = E_t\left[Z_{t+1} + v^{t+2}\frac{\left[1 - \delta + \phi_{t+1} - {\phi'}_{t+1}\frac{I_{t+1}}{K_{t+1}}\right]}{1 + i_{t+2}}\right] \ || \frac{1}{1 + i_{t+1}}$$

$$\frac{v^{t+1}(K_{t+1})}{1 + i_{t+1}} = E_t\left[\frac{1}{1 + i_{t+1}}\left(Z_{t+1} + v^{t+2}\frac{\left[1 - \delta + \phi_{t+1} - {\phi'}_{t+1}\frac{I_{t+1}}{K_{t+1}}\right]}{1 + i_{t+2}}\right)\right]$$

$$\frac{v^{t+1}(K_{t+1})}{1 + i_{t+1}}\frac{P_{t+1}}{P_t}\cdot\frac{P_t}{P_{t+1}} = E_t\left[\frac{1}{1 + i_{t+1}}\frac{P_{t+1}}{P_t}\cdot\frac{P_t}{P_{t+1}}\left(Z_{t+1} + \frac{v^{t+2}\left[1 - \delta + \phi_{t+1} - {\phi'}_{t+1}\frac{I_{t+1}}{K_{t+1}}\right]}{1 + i_{t+2}}\right)\right]$$

$$Q_t P_t = E_t\left[\frac{1}{R_{t+1}}\frac{P_t}{P_{t+1}}\left(Z_{t+1} + \frac{v^{t+2}\left[1 - \delta + \phi_{t+1} - {\phi'}_{t+1}\frac{I_{t+1}}{K_{t+1}}\right]}{1 + i_{t+2}}\right)\right]$$

$$Q_t P_t = E_t\left[\frac{1}{R_{t+1}}\frac{P_t}{P_{t+1}}\left(Z_{t+1} + \frac{v^{t+2}\left[1 - \delta + \phi_{t+1} - {\phi'}_{t+1}\frac{I_{t+1}}{K_{t+1}}\right]}{1 + i_{t+2}}\right)\right]P$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left(\frac{Z_{t+1}}{P_{t+1}} + \frac{v^{t+2}}{1+i_{t+2}}\frac{1}{P_{t+1}}\cdot\frac{P_{t+2}}{P_{t+2}}\left(1-\delta+\phi_{t+1}-\phi'_{t+1}\frac{I_{t+1}}{K_{t+1}}\right)\right)\right]$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left(\frac{Z_{t+1}}{P_{t+1}} + \underbrace{\frac{v^{t+2}}{R_{t+2}P_{t+2}}}_{Q_{t+1}}\left(1-\delta+\phi_{t+1}-\phi'_{t+1}\frac{I_{t+1}}{K_{t+1}}\right)\right)\right]$$

$$Q_t = E_t\left[\frac{1}{R_{t+1}}\left(\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1}\left(1-\delta+\phi_{t+1}-\phi'_{t+1}\frac{I_{t+1}}{K_{t+1}}\right)\right)\right]$$

$$Q_t = \frac{Z_{t+1}}{P_{t+1}}\frac{1}{R_{t+1}} + \frac{Q_{t+1}}{R_{t+1}}(1-\delta) + \phi_{t+1}\frac{Q_{t+1}}{R_{t+1}} - \frac{Q_{t+1}}{R_{t+1}}\phi'_{t+1}\frac{I_{t+1}}{K_{t+1}}$$

$$\begin{aligned}Q(1+\hat{q}_t) = {} & \frac{Z}{P}\frac{1}{R}\left(1+\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\right) + \frac{Q}{R}(1-\delta)(1+\hat{q}_{t+1}-\hat{r}_{t+1}) \\ & + \phi\frac{Q}{R}\left(1+\hat{q}_{t+1}-\hat{r}_{t+1}+\frac{\phi'}{\phi}\frac{\bar{I}}{\bar{K}}(\hat{I}_{t+1}-\hat{K}_{t+1})\right) \\ & - \frac{Q}{R}\phi'(\quad)\frac{I}{K}\left(1+\hat{q}_{t+1}-\hat{r}_{t+1}+(\hat{I}_{t+1}-\hat{K}_{t+1}) + \underbrace{\frac{\phi''}{\phi'}\frac{\bar{I}}{\bar{K}}}_{-\xi}(\hat{I}_{t+1}-\hat{K}_{t+1})\right)\end{aligned}$$

$$R = \frac{Z}{P} + 1 - \delta \Rightarrow 1 + r = \frac{Z}{P} + 1 - \delta \Rightarrow r + \delta = \frac{Z}{P}$$

$\bar{\phi} = \bar{\phi}' = \frac{I}{K} = \delta$por $\qquad K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t$

$$K = (1-\delta)K + \phi(\quad)K$$

$$1 = 1 - \delta + \phi(\quad)$$

$$\phi = \delta = \frac{I}{K}$$

$$\begin{aligned}\hat{q}_t = {} & \frac{r+\xi}{1+r}\left(\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\right) + \left(\frac{1-\delta}{1+r}\right)(\hat{q}_{t+1}-\hat{r}_{t+1}) + \frac{\delta}{1+r}(\hat{q}_{t+1}-\hat{r}_{t+1}+\hat{I}_{t+1}-\hat{K}_{t+1}) \\ & - \frac{\delta}{1+r}\left(\hat{q}_{t+1}-\hat{r}_{t+1}+\hat{I}_{t+1}-\hat{K}_{t+1}+(-\xi)(\hat{I}_{t+1}-\hat{K}_{t+1})\right)\end{aligned}$$

We know that:

$$-\hat{Q} = -\xi(\hat{I}_t - \hat{K}_t) \Rightarrow \hat{q}_t = \xi(\hat{I}_t - \hat{K}_t)$$

$$\hat{q}_t = \frac{r+\delta}{1+r}\left(\left(\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\right) + \left(\frac{1-\delta}{1+r}\right)(\hat{q}_{t+1} - \hat{r}_{t+1}) + \frac{\delta}{1+r}\left(\hat{q}_{t+1} - \hat{r}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \frac{\delta}{1+r}\left(\hat{q}_{t+1} - \hat{r}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1} + (-\hat{q}_{t+1})\right)\right)$$

$$\hat{q}_t = \frac{r+\delta}{1+r}\left(\left(\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\right) + \left(\frac{1-\delta}{1+r}\right)(\hat{q}_{t+1} - \hat{r}_{t+1}) + \frac{\delta}{1+r}\left(\hat{q}_{t+1} - \hat{r}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \frac{\delta}{1+r}\left(+\hat{I}_{t+1} - \widehat{K}_{t+1} - \hat{r}_{t+1}\right)\right)$$

$$\hat{q}_t = \frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\left[\left(\frac{r+\delta}{1+r}\right) + \frac{1-\delta}{1+r} + \frac{\delta}{1+r} - \frac{\delta}{1+r}\right] + \hat{q}_{t+1}\left(\frac{1-\delta}{1+r} + \frac{\delta}{1+r}\right) + (I_{t+1} - K_{t+1})\frac{\delta}{1+r} - \frac{\delta}{1+r}\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)$$

$$\hat{q}_t = \frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1} - \hat{r}_{t+1}\left[\frac{r+1}{1+r}\right] + \hat{q}_{t+1}\left(\frac{1}{1+r}\right)$$

$$\hat{q}_t = \frac{1}{R}\hat{q}_{t+1} - \hat{r}_{t+1} + \frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1} \; (a)$$

We insert in a:

$$\hat{q}_t = \xi\left(\hat{I}_t - \widehat{K}_t\right)$$

$$\xi\left(\hat{I}_t - \widehat{K}_t\right) = \frac{1}{R}\left(\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)\right) - \hat{r}_{t+1} + \frac{r+\delta}{1+r}\left(\frac{\hat{z}}{P}\right)_{t+1}$$

By Euler equation we also know:

$$U = \sum \beta^t \log C^0 - \frac{N^{1+\varphi}}{1+\varphi}$$

The nominal interest rate is:

$$\frac{\pi_{t+1}}{R_t} = r_{t+1}$$

$$C^0 + Q_t K_{t+1} - (1-\delta)Q_t K_t + \phi\,(\quad)K_t Q_t + R_t^{-1}\frac{B_{t+1}}{P_t} = \frac{W_t}{P_t}N_t^0 + \frac{R_t^k}{P_t}K_t^0 + \frac{B^t}{P_t} - \frac{T_t^0}{P_t}$$

$$\frac{1}{R_t}b_{t+1}\frac{P_{t+1}}{P_t}$$

$$\frac{d\,U}{b_{t+1}}: -\beta^{t+1}\lambda_{t+1}\frac{\pi_{t+1}}{R_t} + \beta^{t+1}\lambda_{t+1} = 0$$

$$\frac{d}{dC}: \frac{1}{C_t} = \lambda_t$$

$$\frac{C_{t+1}}{C_t} = \beta R_t \pi_{t+1}$$

$$\frac{d}{dK_{t+1}}: -\beta^t \lambda_t Q_t + \beta^{t+1}\lambda_{t+1}(1-\delta)Q_{t+1} - Q_{t+1}\phi_{t+1}(\quad)\beta^{t+1}\lambda_{t+1}$$
$$+\beta^{t+1}\lambda_{t+1}\phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}}Q_{t+1} + \beta^{t+1}\lambda_{t+1}\left(\frac{R^k_{t+1}}{P_{t+1}}\right)$$

$$\frac{d}{dK_{t+1}}: \beta^t \lambda_t Q_t = \beta^{t+1}\lambda_{t+1}Q_{t+1}\left(1-\delta-\phi_{t+1}\left(\frac{I_{t+1}}{K_{t+1}}\right)+\phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}}\right) + -\beta^{t+1}\lambda_{t+1}$$
$$+ \beta^{t+1}\lambda_{t+1}\left(\frac{Z_{t+1}}{P_{t+1}}\right)$$

Euler consumption

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta\frac{1}{Q_t}\left(\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1}(1-\delta) - \phi_{t+1}(\quad) + \phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}}\right)$$

The adjustment cost of the interest rate is:

$$R_t = \frac{1}{Q_t}\left(\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1}(1-\delta) - \phi_{t+1}(\quad) + \phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}}\right)$$

$$Q_t R_t = \frac{Z_{t+1}}{P_{t+1}} + Q_{t+1}\left(1-\delta-\phi_{t+1}(\quad) + \phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}}\right)$$

From the Euler equation

$$Q_t\frac{C_{t+1}}{C_t} = \beta\frac{Z_{t+1}}{P_{t+1}} + \beta Q_{t+1}(1-\delta) - \beta Q_{t+1}\phi_{t+1}(\quad) + \beta Q_{t+1}\phi'_{t+1}(\quad)\frac{I_{t+1}}{K_{t+1}} + \pi_{t+1}$$

Log – lin

$$Q\frac{C}{C}(1+\hat{q}_{t+1}+\hat{c}_{t+1}-\hat{c}_t)$$
$$= \beta\frac{Z}{P}\left(1+\left(\frac{\hat{Z}}{P}\right)_{t+1}\right) + \beta(1-\delta)Q(1+\hat{q}_{t+1})$$
$$-\beta Q\phi\left(1+\hat{q}_{t+1}+\frac{\phi'}{\phi}\frac{I}{K}(\hat{I}_{t+1}-\widehat{K}_{t+1})\right)$$
$$+\beta Q\phi'\frac{I}{K}\left(1+\hat{q}_{t+1}+\hat{I}_{t+1}-\widehat{K}_{t+1}+\frac{\phi''}{\phi'}\frac{I}{K}(\hat{I}_{t+1}-\widehat{K}_{t+1})\right) + \pi\left(1+\frac{1}{\pi_{t+1}}\right)$$

$$\hat{q}_{t+1} + \hat{c}_{t+1} - \hat{c}_t$$
$$= \beta \frac{Z}{P}\frac{\hat{Z}_{t+1}}{P_{t+1}} + \beta(1-\delta)\hat{q}_{t+1} - \beta\delta\left(\hat{q}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1}\right)$$
$$+ \beta\delta\left(\hat{q}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1} + \underbrace{(-\xi)\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)}_{-q_{t+1}}\right) + \pi_{t+1}$$

$$\hat{q}_{t+1} + \hat{c}_{t+1} - \hat{c}_t$$
$$= \beta \frac{Z}{P}\left(\frac{\hat{Z}_{t+1}}{P_{t+1}}\right) + \beta(1-\delta)\hat{q}_{t+1} - \beta\delta\left(\hat{q}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1}\right)$$
$$+ \beta\delta\left(\hat{q}_{t+1} + \hat{I}_{t+1} - \widehat{K}_{t+1} + -q_{t+1}\right) + \pi_{t+1}$$

$$\hat{q}_{t+1} + \hat{c}_{t+1} - \hat{c}_t$$
$$= \beta \frac{Z}{P}\left(\frac{\hat{Z}_{t+1}}{P_{t+1}}\right) + [\,\beta(1-\delta) - \beta\delta]\hat{q}_{t+1} - \beta\delta\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) + \beta\delta\left(\hat{I}_{t+1} - \widehat{K}_{t+1} + \right)$$
$$+ \pi_{t+1}$$

$$\hat{q}_{t+1} + \hat{c}_{t+1} - \hat{c}_t = \frac{\beta}{R}\left(\frac{\hat{Z}_{t+1}}{P_{t+1}}\right) + \left(\frac{1-2\delta}{R}\right)\hat{q}_{t+1} + \pi_{t+1}$$

$$\xi\left(\hat{I}_t - \widehat{K}_t\right) + \hat{c}_{t+1} - \hat{c}_t = \frac{\beta}{R}\left(\frac{\hat{Z}_{t+1}}{P_{t+1}}\right) + \left(\frac{1-2\delta}{R}\right)\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) + \pi_{t+1}$$

$$\left(\frac{\hat{z}}{P}\right)_{t+1} = \frac{R}{B}\left[\hat{c}_{t+1} - \hat{c}_t + \xi\left(\hat{I}_t - \widehat{K}_t\right) - \frac{1-2\delta}{R}\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)\right] - \hat{\pi}_{t+1}$$

$$(b)\left(\frac{\hat{z}}{P}\right)_{t+1} = \frac{R}{B}\left[\hat{c}_{t+1} - \hat{c}_t + \xi\left(\hat{I}_t - \widehat{K}_t\right) - \frac{1-2\delta}{R}\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{\pi}_{t+1}\right]$$

$$(b) \rightarrow (a)$$

$$q_t = \frac{1}{R}q_{t+1} - \hat{r}_{t+1} + \frac{r+\delta}{R}\left(\frac{R}{\beta}\left[\hat{c}_{t+1} - \hat{c}_t + \xi\left(\hat{I}_t - \widehat{K}_t\right) - \frac{1-2\delta}{R}\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{\pi}_{t+1}\right]\right)$$

$$\xi\left(\hat{I}_t - \widehat{K}_t\right) = \beta\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{r}_{t+1}$$
$$+ \frac{r+\delta}{\beta}\left(\hat{c}_{t+1} - \hat{c}_t + \xi\left(\hat{I}_t - \widehat{K}_t\right) - \frac{1-2\delta}{R}\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{\pi}_{t+1}\right)$$

$$\xi\left(\hat{I}_t - \widehat{K}_t\right) = \beta\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{r}_{t+1}$$
$$+ \frac{r+\delta}{\beta}\left(\hat{c}_{t+1} - \hat{c}_t + \xi\left(\hat{I}_t - \widehat{K}_t\right) - \frac{1-2\delta}{R}\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{\pi}_{t+1}\right)$$

$$\hat{I}_t - \widehat{K}_t \left(\xi + \left(\frac{r+\delta}{\beta}\right)\xi\right)$$
$$= \beta\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \hat{r}_{t+1} + \frac{r+\delta}{\beta}(\hat{c}_{t+1} - \hat{c}_t) - \frac{r+\delta}{\beta}(1-2\delta)\beta\xi\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)$$
$$- \frac{r+\delta}{\beta}\hat{\pi}_{t+1}$$

$$\hat{I}_t - \widehat{K}_t \left(\frac{\beta\xi + (r+\delta)\xi}{\beta}\right)$$
$$= \left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right)(\beta\xi - (r+\delta)(1-2\delta)\xi) - \hat{r}_{t+1} + \frac{r+\delta}{\beta}(\hat{c}_{t+1} - \hat{c}_t)$$
$$- \frac{r+\delta}{\beta}\hat{\pi}_{t+1}$$

$$\hat{I}_t - \widehat{K}_t = \left(\frac{\beta\xi + (r+\delta)(1-2\delta)\xi}{\beta\xi + (r+\delta)\xi}\right)\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \frac{\beta}{\beta\xi + (r+\delta)\xi}\hat{r}_{t+1}$$
$$+ \frac{r+\delta}{\beta\xi + (r+\delta)\xi}(\hat{c}_{t+1} - \hat{c}_t) - \frac{r+\delta}{\beta}\hat{\pi}_{t+1}\frac{\beta}{\beta\xi + (r+\delta)}$$

$$\hat{I}_t - \widehat{K}_t = \left(\frac{\beta + (1-2\delta)(r+\delta)}{\beta + r + \delta}\right)\beta\left(\hat{I}_{t+1} - \widehat{K}_{t+1}\right) - \frac{\beta}{\beta\xi + (r+\delta)\xi}\hat{r}_{t+1}$$
$$+ \frac{r+\delta}{\xi\beta + (r+\delta)\xi}(\hat{c}_{t+1} - \hat{c}_t) - \frac{r+\delta}{\beta\xi + (r+\delta)\xi}\hat{\pi}_{t+1}$$

11. ADVANCED PICKS IN DYNARE

We are going to introduce cash in advance (CIA) model as Shorffeide (2000)

Households

$$\max_{C,H,M_{t+1},D_t} E_0 \sum_{0}^{\infty} \beta^t (1-\phi) C_t$$

$$P_t C_t \leq M_t - D_t + W_t H_t$$

$$0 \leq D_t$$

$$M_{t+1} = (M_t - D_t + W_t H_t - P_t C_t) + R_{H,t+1} D_{t+1} + F_t B_t$$

Firms: maximize the present value of future dividends (discounted at a marginal utility of consumption of they are owned by households) by choosing dividends next periods capital stock K_{t+1}, labor demand, N_t, and loans.

$$\sum_{0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1} P_{t+1}}$$

s.t.

$$F_t \leq L_t + P_t [K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t] - W_t N_t$$

$-L_t R_{F,t}$ Summarize the use of production function

$W_t N_t \leq L_t$ Bank loans are used to pay for wage cost

In eqq $H_t = N_t$

$P_t C_t = Y_t + X_t$

$$R_{H,t} = R_{F,t}$$

Technologies (Shock is anAR (1)) two sources of perturbation

$$\ln A_t = \gamma + \ln A_{t-1} + \varepsilon_{A,t}$$

$$\ln m_t = (1-\rho) \ln m + \ln m_{t-1} + \varepsilon_{\mu,t}$$

Growth rate of money

$$m_t = \frac{M_{t+1}}{M_t}$$

The system will be

$$E_t\left\{-\frac{\hat{P}_t}{\hat{C}_{t+1}\hat{P}_{t+1}m_t}\right\}=\frac{\beta e^{-\alpha(\gamma+\varepsilon_{A,t+1})}P_{t+1}}{\hat{C}_{t+2}\hat{P}_{t+2}m_{t+1}}\left\{\alpha\hat{K}_t^{\alpha-1}\hat{N}_{t+1}^{1-\alpha}+1-\delta\right\}$$

$$\hat{W}_t=\frac{\hat{L}_t}{N_t}$$

$$\frac{\phi}{1-\phi}\left(\frac{\hat{C}_t\hat{P}_t}{1-N_t}\right)=\frac{\hat{L}_t}{N_t}$$

$$R_t=\frac{(1-\alpha)\hat{P}_t e^{-\alpha(\gamma+\varepsilon_{A,t+1})}K_{t-1}^{\alpha}N_t^{-\alpha}}{\hat{W}_t}$$

$$\frac{1}{\hat{P}_t\hat{C}_t}=\frac{\beta}{\hat{L}_t m_t\hat{C}_{t+1}\hat{P}_{t+1}}\left((1-\alpha)\hat{P}_t e^{-\alpha(\gamma+\varepsilon_{A,t+1})}\hat{K}_{t-1}^{\alpha}\hat{N}_t^{1-\alpha}\right)$$

$$\hat{C}_t+\hat{K}_t=e^{-\alpha(\gamma+\varepsilon_{A,t})}\hat{K}_{t-1}^{\alpha}\hat{N}_t^{1-\alpha}+(1-\delta)e^{-(\gamma+\varepsilon_{A,t})}\hat{K}_{t-1}$$

$$\hat{P}_t\hat{C}_t=m_t$$

$$m_{t-1}+\hat{D}_t=\hat{L}_t$$

$$\hat{Y}_t=\hat{K}_{t-1}^{\alpha}N^{1-\alpha}e^{-\alpha(\gamma+\varepsilon_{A,t})}$$

$$\ln m_t=(1-\rho)\ln m_t+\rho\ln m_{t-1}+\varepsilon_{M,t}$$

$$\frac{A_t}{A_{t-1}}\equiv dA_t=\exp(\gamma+\varepsilon_{A,t})$$

$$\left.\begin{array}{l}\dfrac{Y_t}{Y_{t-1}}=e^{\gamma+\varepsilon_{A,t}}\dfrac{\hat{Y}_t}{\hat{Y}_{t-1}}\\[2ex]\dfrac{P_t}{P_{t+1}}=\dfrac{\hat{P}_t}{\hat{P}_{t-1}}\dfrac{m_{t-1}}{e^{\gamma+\varepsilon_{A,t}}}\end{array}\right\}\text{this doesn't have a SS}$$

We have stochastic trends in technology and money

We have to declare observables

12. STICKY PRICE MODEL

Taylor (1990), Calvo (1985) emphasize in staggered wages and sticky prices in an forward looking manner.

So there is New Keynesian Phillips Curve, that G.G. (1999) and G.G.L.S. (2001).

Difficult to detect y^* Potential

HYBRID NEW KEYNESIAN PHILIPS CURVE

Calvo price setting $\quad P_t = \theta \hat{P}_{t-1} + (1-\theta)\hat{P}_t^* \quad$ (1)

Two firms $\quad P_t^* = (1-w)P_t^f + wP_t^b \quad$ (2)

Backward looking $\quad P^b = P_{t-1}^* + \pi_{t-1} \quad$ (3)

Forward looking sets in an optimization manner

$$\underset{P_t^*}{\mathrm{Max}} \sum_0^\infty \theta^K E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \psi_{t+k}\left(Y_{t+k|t}\right)\right)\right\}$$

s.t

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k} \quad or \;\; Y_{t+k}$$

On the other hand

$$\sum_0^\infty \beta^t U(C,N)_t$$

s.t. $\quad P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + \Pi_t$

So from FOC conditions of consumption we know that:

$$Q_{t,t+k} = \beta^k \left(\frac{U_{C_{t+k}}}{U_{C_t}} \frac{P_t}{P_{t+k}} \right)$$

Posing the problem

$$\underset{P_t^*}{\mathrm{Max}} \sum_0^\infty (\beta\theta)^k E_t \left[P_t^* \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k} - \psi_{t+k}\left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}\right)\right]$$

FOC

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left[Y_{t+k|t}+P_t^*(-\varepsilon)Y_{t+k|t}\frac{1}{P_t^*}-(-\varepsilon)\frac{1}{P_t^*}\rho_{t+k|t}\right]=0$$

$$\rho_{t+k|t}=\psi'_{t+k}$$

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left[(1-\varepsilon)Y_{t+k|t}+\frac{\varepsilon}{P_t^*}\rho_{t+k|t}\right]=0$$

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left[Y_{t+k|t}\left[(1-\varepsilon)+\frac{\varepsilon}{P_t^*}\rho_{t+k|t}\right]\right]=0$$

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left(P_t^*-\frac{\varepsilon}{\varepsilon-1}\rho_{t+k|t}\right)=0$$

$\frac{\varepsilon}{\varepsilon-1}$ is the gross function price mark up and the one prevailing when we have zero inflation in SS

Define real marginal cost

$$MC_{t+k|t}=\frac{\rho_{t+k|t}}{P_{t+k}} \qquad \wedge \quad \pi_{t+k,t}=\frac{P_{t+k,t}}{P_t}$$

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left(P_t^*-\mu\frac{\rho_{t+k|t}}{P_{t+k}}P_{t+k}\frac{P_{t-1}}{P_{t-1}}\right)=0$$

$$\sum_{0}^{\infty}(\beta\theta)^k E_t\left(P_t^*-\mu MC_{t+k|t}\pi_{t+k,t-1}P_{t-1}\right)=0$$

$$\frac{P_t^*}{P_{t-1}}=(1-\beta\theta)\sum_{0}^{\infty}(\beta\theta)^k\left(MC_{t+k|t}\frac{P_{t+k}}{P_{t-1}}\right) \quad (4)$$

from (1)

$$(1-\theta)\hat{p}_t^*-\hat{p}_t+\theta\hat{p}_t-\theta p_t=-\theta p_{t-1}$$

$$(1-\theta)\hat{p}_t^*-(1-\theta)\hat{p}_t=\theta(p_t-p_{t-1})$$

$$(\hat{p}_t^*-\hat{p}_t)=\frac{\theta}{1-\theta}\pi_t \;\Rightarrow \pi_t=\left(\frac{1-\theta}{\theta}\right)(\hat{p}_t^*-\hat{p}_t) \quad (5)$$

to (2)

$$P_t^*=(1-w)\hat{P}_t^f+w\hat{P}_t^b+\hat{P}_t-\hat{P}_t+w\hat{P}_t-w\hat{P}_t$$

$$\hat{P}_t^*-\hat{P}_t=(1-w)\hat{P}_t^f-(1-w)\hat{P}_t+w\hat{P}_t^b-w\hat{P}_t$$

$$\hat{P}_t^* - \hat{P}_t = (1-w)\left(\hat{P}_t^f - \hat{P}_t\right) + w\left(\hat{P}_t^b - \hat{P}_t\right) \qquad (6)$$

Introduce 6 in 5

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left[(1-w)\left(\hat{P}_t^f - \hat{P}_t\right) + w\left(\hat{P}_t^b - \hat{P}_t\right)\right](6.1)$$

Let's define

$$\hat{\pi}_{t+k,t} = \hat{P}_{t+k,t} - \hat{P}_t$$

in 3

$$\hat{P}_t^b = P_{t-1}^* + \hat{\pi}_{t-1}$$

$$\hat{P}_t^b - \hat{P}_t = P_{t-1}^* - \hat{\pi}_{t-1} - (\theta P_{t-1} + (1-\theta)P_t^*)$$

$$\hat{P}_{t-1}^* = \hat{P}_t^* \quad \wedge \quad P_{t-1} = P_{t-1}^*$$

$$\hat{P}_t^b - \hat{P}_t = \hat{\pi}_{t-1} - (1-\theta)\hat{\pi}_t$$

$$\hat{P}_t^b - \hat{P}_t = \frac{\hat{\pi}_{t-1}}{(1-\theta)} - \hat{\pi}_t(7)$$

Log – lin

Modify (4), and eliminate P_{t-1}

$$P_t^f = (1-\beta\theta)\sum_0^\infty (\beta\theta)^k E_t\left(MC_{t+k|t}P_{t+k}\right)$$

Develop

$$\frac{P^f}{P}\left(1 + \hat{P}_t^f - P_{t-1}\right) = (1-\beta\theta)\sum_0^\infty (\beta\theta)^k MC\frac{P_t}{P}\left(1 + \widehat{mc}_{t+k|t} + \hat{P}_{t+k} - \hat{P}_{t-1}\right)$$

$$\hat{P}_t^f = (1-\beta\theta)\sum_0^\infty (\beta\theta)^k\left(1 + \widehat{mc}_{t+k|t} + \hat{P}_{t+k}\right)$$

$k = 0,1,2$

$$\hat{P}_t^f = \widehat{mc}_t + \hat{P}_t - \beta\theta\left(\widehat{mc}_t + \hat{P}_t\right) + \beta\theta\left(\widehat{mc}_{t+1} + \hat{P}_{t+1}\right) - (\beta\theta)^2\left(\widehat{mc}_{t+1} + \hat{P}_{t+1}\right) + (\beta\theta)^2\left(\widehat{mc}_{t+2} + \hat{P}_{t+2}\right) + (\beta\theta)^3\left(\widehat{mc}_{t+2} + \hat{P}_{t+2}\right)$$

$$\hat{P}_t^f - \hat{P}_t = mc_t + \sum_1^\infty (\beta\theta)^k(mc_{t+k} - mc_{t+k-1} + \pi_{t+k}) \qquad (8)$$

7, 8 en 6.1

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left[(1-w)\left(mc_t + \sum_1^\infty (\beta\theta)^k (mc_{t+k} - mc_{t+k-1} + \pi_{t+k})\right) + w\left(\frac{\pi_{t-1}}{1-\theta} - \pi_t\right)\right]$$

$$\pi_t = \frac{w}{\theta}\pi_{t-1} - \left(\frac{1-\theta}{\theta}\right)w\pi_t + \left(\frac{1-\theta}{\theta}\right)(1-w)\left(mc_t + \sum_1^\infty (\beta\theta)^k (mc_{t+k} - mc_{t+k-1} + \pi_{t+k})\right)$$

$$\pi_t\left(1 + \frac{(1-\theta)w}{\theta}\right) = \frac{w}{\theta}\pi_{t-1} + \frac{(1-\theta)}{\theta}(1-w)\left(mc_t + \sum_1^\infty (\beta\theta)^k (mc_{t+k} - mc_{t+k-1} + \pi_{t+k})\right)$$

$$\pi_t\left(\frac{\theta + (1-\theta)w}{\theta}\right) = \frac{w}{\theta}\pi_{t-1} + \frac{(1-\theta)(1-w)}{\theta}\left(mc_t + \sum_1^\infty (\beta\theta)^k (mc_{t+k} - mc_{t+k-1} + \pi_{t+k})\right)$$

$$\pi_t = \underbrace{\frac{(1-\theta)(1-w)}{\theta + (1-\theta)w}}_{a}\left(mc_t + \sum_1^\infty (\beta\theta)^k (mc_{t+k} - mc_{t+k-1} + \pi_{t+k})\right) + \underbrace{\frac{w}{\theta + (1-\theta)w}}_{b}\pi_{t-1}$$

Develop in$k = 1,2$

$$\pi_t = a[mc_t + \beta\theta(mc_{t+1} - mc_t + \pi_{t+1})] + (\beta\theta)^2(mc_{t+2} - mc_{t+1} + \pi_{t+2}) + \frac{w}{\theta + (1-\theta)w}\pi_{t-1}$$

$$\begin{aligned}\pi_t - \beta\theta\pi_{t+1} = {} & a[mc_t + \beta\theta(mc_{t+1} - mc_t + \pi_{t+1})] + (\beta\theta)^2(mc_{t+2} - mc_{t+1} + \pi_{t+2}) \\ & + \frac{w}{\theta + (1-\theta)w}\pi_{t-1} - \beta\theta[mc_{t+1} + \beta\theta(mc_{t+2} - mc_{t+1} + \pi_{t+2})] \\ & - \frac{w}{\theta + (1-\theta)w}\pi_t\end{aligned}$$

$$\pi_t - \beta\theta\pi_{t+1} = a[mc_t + \beta\theta mc_t + \beta\theta\pi_{t+1}] + \frac{w}{\theta + (1-\theta)w}\pi_{t-1} - \frac{\beta\theta w}{\theta + (1-\theta)w}\pi_t$$

$$\pi_t\left(1 + \frac{\beta\theta w}{\theta + (1-\theta)w}\right) = a(1 - \beta\theta mc_t) + a\beta\theta\pi_{t+1} + \frac{w}{\theta + (1-\theta)w}\pi_{t-1} + \beta\theta\pi_{t+1}$$

$$\pi_t\left(1 + \frac{\beta\theta w}{\theta + (1-\theta)w}\right) = a(1 - \beta\theta mc_t) + \beta\theta\pi_{t+1}(1 + a) + \frac{w}{\theta + (1-\theta)w}\pi_{t-1}$$

$$\pi_t\left(1 + \frac{\beta\theta w}{\theta + (1-\theta)w}\right) = a(1 - \beta\theta mc_t) + \beta\theta\pi_{t+1}\left(1 + \frac{(1-\theta)(1-w)}{\theta + (1-\theta)w}\right) + \frac{w}{\theta + (1-\theta)w}\pi_{t-1}$$

$$\pi_t\left(1+\frac{\beta\theta w}{\theta+(1-\theta)w}\right)$$
$$= a(1-\beta\theta mc_t)+\beta\theta\pi_{t+1}\left(\frac{\theta+(1-\theta)w+(1-\theta)(1-w)}{1+(1-\theta)w}\right)$$
$$+\frac{w}{\theta+(1-\theta)w}\pi_{t-1}$$

$$\pi_t\left(1+\frac{\beta\theta w}{\theta+(1-\theta)w}\right)$$
$$= a(1-\beta\theta mc_t)+\beta\theta\pi_{t+1}\left(\frac{\theta+(1-\theta)w+(1-\theta)-(1-\theta)w}{1+(1-\theta)w}\right)$$
$$+\frac{w}{\theta+(1-\theta)w}\pi_{t-1}$$

$$\pi_t\left(\frac{\theta+(1-\theta)w+\beta\theta w}{\theta+(1-\theta)w}\right)$$
$$=\frac{(1-\theta)(1-w)(1-\beta\theta)}{\theta+(1-\theta)w}mc_t+\frac{w}{\theta+(1-\theta)w}\pi_{t-1}+\frac{\beta\theta}{\theta+(1-\theta)w}\pi_{t+1}$$

$$\pi_t\big(\theta+w(1-\theta+\beta\theta)\big)=(1-\theta)(1-w)(1-\beta\theta)mc_t+w\pi_{t-1}+\beta\theta\pi_{t+1}$$

$$\pi_t\Big(\theta+w\big(1-\theta(1-\beta)\big)\Big)=(1-\theta)(1-w)(1-\beta\theta)mc_t+w\pi_{t-1}+\beta\theta\pi_{t+1}$$

$$\pi_t=\frac{(1-\theta)(1-w)(1-\beta\theta)}{\big(\theta+w(1-\theta+\beta\theta)\big)}mc_t+\frac{w}{\big(\theta+w(1-\theta+\beta\theta)\big)}\pi_{t-1}+\frac{\beta\theta}{\big(\theta+w(1-\theta+\beta\theta)\big)}\pi_{t+1}$$

The hybrid HNKPC will be

$$\pi_t=\lambda mc_t+\gamma^f E(\pi_{t+1})+\gamma^b\pi_{t-1}$$

$$\lambda=(1-\theta)(1-w)(1-\beta\theta)\phi^{-1}$$

$$\phi^{-1}=\big(\theta+w(1-\theta+\beta\theta)\big)$$

$$\gamma^f=\beta\theta\phi^{-1}$$

$$\gamma^b=w\phi^{-1}$$

13. FLEXIBLE VS STICKY ORICES

Let consider a non separable utility function

$$U(\quad) = \sum_{0}^{\infty} \beta^t \left(\frac{C^{1-\theta}}{1-\theta} e^{(\theta-1)V(L)} \right)$$

Household

$$\hat{C}_t = \hat{C}_{t+1} + \left(\frac{1-\theta}{\theta} \right) V'^{(L)} N \Delta \widehat{N}_{t+1} - \frac{1}{\theta} \hat{r}_t$$

$$\hat{n}_t = \frac{N-1}{N(1-N)} \left(\frac{\widehat{W}}{P} \right)_t - \frac{N-1}{N(1-N)} \hat{C}_t$$

Firms

$$Y = A_t K_t^{\alpha} N_t^{1-\alpha}$$

$$\hat{a}_t + (1-\alpha)\left(\widehat{N}_t - \widehat{K}_t\right) = \left(\frac{\hat{Z}}{P} \right)_t$$

$$\hat{a}_t + \alpha\left(\widehat{K}_t - \widehat{N}_t\right) = \left(\frac{\widehat{W}}{P} \right)_t$$

$$Y_t = \hat{a}_t + \alpha \widehat{K}_t + (1-\alpha) \widehat{N}_t$$

Investment

$$\widehat{R}_t = \left(\frac{\delta + r}{1+r} \right) \left(\frac{\hat{Z}}{P} \right)_t + w_{t+1}^{R} ☺$$

Equilibrium

$$\hat{Y}_t = \left(\frac{C}{Y} \right) \hat{C}_t + \left(\frac{I}{Y} \right) \hat{I}_t$$

Shock

$$\hat{a}_t = \rho \hat{a}_{t-1} + \varepsilon_t^{a}$$

Variables

$$C, n, a, -\frac{Z}{P}, \frac{W}{P}, Y_t, R, K, I = 9 \; variables, 9 \; equations$$

Model with sticky prices

Firms does not max profits, just min cost

Household

$$\hat{C}_t = \hat{C}_{t+1} + \left(\frac{1-\theta}{\theta}\right) V'^{(L)} N \Delta \widehat{N}_{t+1} - \frac{1}{\theta} \hat{r}_t \qquad (1)$$

$$\hat{n}_t = \frac{N-1}{N(1-N)} \left(\frac{\widehat{W}}{P}\right)_t - \frac{N-1}{N(1-N)} \hat{C}_t \qquad (2)$$

Investment

$$\hat{R}_t = \left(\frac{\delta + r}{1+r}\right) \left(\frac{\hat{Z}}{P}\right)_t + w_{t+1}^R \qquad (3)$$

$$\hat{R}_t = (1-\delta)\widehat{K}_t + \delta \hat{I}_t \qquad (4)$$

Firms

$$\left(\frac{\hat{Z}}{P}\right) - \left(\frac{\widehat{W}}{P}\right) = \widehat{N}_t - \widehat{K}_t \qquad (5)$$

$$\pi_t = \beta \hat{\pi}_{t+1} + \lambda \hat{s}_t \qquad (6)$$

$$\hat{s}_t = (1-\alpha) \left(\frac{\widehat{W}}{P}\right)_t - \alpha \left(\frac{\hat{Z}}{P}\right)_t - \hat{a}_t \qquad (7)$$

$$Y_t = \hat{a}_t + \alpha \widehat{K}_t + (1-\alpha)\widehat{N}_t \qquad (8)$$

Equilibrium

$$\hat{Y}_t = \left(\frac{C}{Y}\right) \hat{C}_t + \left(\frac{I}{Y}\right) \hat{I}_t \qquad (9)$$

Shock

$$\hat{a}_t = \rho \hat{a}_{t-1} + \varepsilon_t^a \qquad (10)$$

Variables

$$C, n, a, \left(\frac{Z}{P}\right), \left(\frac{W}{P}\right), R, K, I, \pi, s, y = 11 \; variables$$

- ✓ I have only 10 equations for 11 variables
- ✓ Since here is a non competitive market, we need to specify the monetary policy because monetary policy is not neutral with sticky prices and operates through nominal interest rate

Taylor rule

$$i = i_{t-1} + \phi\hat{\pi}_t + \varphi\hat{y}_t + \varepsilon_t \qquad (11)$$

$$\hat{R}_t = i_t - \pi_{t+1} \qquad (12)$$

So we close the model with this

How operated

$\uparrow i_t \;\rightarrow \uparrow R_t \;\Rightarrow \downarrow C_t$ in Euler's equation as prices are sticky $\Rightarrow \downarrow Y_t$, capital and labor falls

$$\left(\frac{Z}{P}\right) \downarrow \left(\frac{W}{P}\right) \downarrow \Rightarrow \downarrow s_t \;\Rightarrow\; \hat{\pi}_t \downarrow$$

$$\phi\,\pi = 0.015$$

$\uparrow a_t$ and what happens with $\hat{N}$

$$\varepsilon = 0{,}6 \qquad \beta = 0{,}9 \qquad \delta = 0{,}02 \qquad \alpha = 0{,}33 \qquad N = {}^{1}\!/_{3}$$

$$\phi = 1{,}5 \qquad \phi\pi = 0{,}001$$

14. INVIDIVUAL MAXIMIZATION IN AND MONETARY MODEL

We introduce Money in the utility function → real balances $\left(\frac{M}{P}\right)$ enter in the U ( ) and allow agents to reduce times in transactions.

Individual can accumulate 2 assets: $B_{t+1}\ and\ M_t$ financed by real constraint

$$U() = \lg C_t + \xi \lg L_t + (1-\xi-\alpha)\lg m_t$$

$$m_t = \frac{M_t}{P_t}$$

St.

$$\frac{B_{t+1}}{1+i_t} + M_t + PC_t + P_tT_t = B_t + M_{t-1} + w_t(1-L_t) + P_t\pi_t$$

$$\frac{1}{1+i_t}\frac{B_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t} + m_t + C_t + T_t = b_t + \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \left(\frac{w}{P}\right)_t(1-L_t) + \pi_t$$

$$\frac{1+\pi_{t+1}}{1+i_t}b_{t+1} + m_t + C_t + T_t = b_t + m_t\frac{1}{\underbrace{1+\pi_t}_{\pi_t}} + \left(\frac{w}{P}\right)_t(1-L_t) + \pi_t$$

Let define the real interest rate as:

$$1 + r_t = \frac{1+i_t}{1+\pi_{t+1}} = R_t$$

$$\frac{b_{t+1}}{R_t} + m_t + T_t = b_t + \frac{m_{t+1}}{1+\pi_{t+1}} + \left(\frac{w}{P}\right)_t(1-L_t) + \pi_t$$

FOC

$$C_t:\ \frac{\alpha}{C_t} = \lambda_t$$

$$L_t:\ \frac{\xi}{L_t} = \lambda_t\left(\frac{w}{P}\right)_t$$

$$b_{t+1}:-\frac{\beta^t\lambda_t}{R_t} + E_t\beta^{t+1}\lambda_{t+1} = 0 \Rightarrow \lambda_t = \beta E_t\lambda_{t+1}R_t$$

$$m_t\ :\beta^t\frac{(1-\xi-\alpha)}{m_t} - \lambda_t\beta^t + \beta^{t+1}\frac{E_t\lambda_{t+1}}{1+\pi_t} = 0$$

$$\frac{(1-\xi-\alpha)}{m_t}\frac{1}{\lambda_t} - 1 + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{1+\pi_t}$$

$$\frac{(1-\xi-\alpha)}{m_t}\frac{C_t}{\alpha} - 1 + \frac{1}{(1+r_t)(1+\pi_t)} = 0$$

Furthermore we know that:

$(1+i_t) = (1+r_t)(1+\pi_{t+1})$

$\frac{1-\xi-\alpha}{m_t} \cdot \frac{C_t}{\alpha} + \frac{1}{1+i_t} - 1 = 0$

$\frac{1-\xi-\alpha}{m_t} \cdot \frac{C_t}{\alpha} + \frac{1-1-i_t}{1+i_t} = 0 \ \Rightarrow$

$m_t = \left(\frac{1-\xi-\alpha}{\alpha}\right) C_t \left(\frac{1+i_t}{i_t}\right)$ $demand\ of\ real\ balances$

$\uparrow C \rightarrow \uparrow m$

$\uparrow i \ \rightarrow \downarrow m_t$

$\frac{d\left(\frac{1+i_t}{i_t}\right)}{di_t} = \frac{i_t - (1+i_t)}{i_t^2} = -\frac{1}{i_t^2} < 0$

$\frac{x}{y} = \frac{dx \cdot y - dyx}{dy}$

Money supply

$$M_t = M_{t-1} + \varepsilon_t$$

$m_t = \frac{m_{t-1}}{1+\pi_t} + \varepsilon_t$ But in practice we define i rather than m_t, not in Bolivia

Option (De Gregorio)

$$i_t = \bar{\imath} + \left(1 + \frac{\theta}{\phi(\theta^2+\lambda)}\right)(\pi^e - \bar{\pi}) + \frac{\theta}{\phi(\theta^2+\lambda)}\xi$$

The loss function was $\min \lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2$

St. $\pi_t = \pi^e + \theta(y_t - \bar{y}) + \varepsilon$

θ: Parameter output deviation from potential output

λ: Parameter of loss function

ϕ: Investment sensibility to real interest in the equation

$$y_t - \bar{\bar{y}} = A - \phi(i_t - \pi^e) + \mu$$

15. FISCAL POLICY

Implications of fiscal policy differ from some models.

Daney Valdivia ®

Calvo and Vegh (2005) find that in developing countries fiscal policy is prociclycal. This leads us to make a question:

1. Is the fiscal policy a mechanism that helps the economy against the business cycle? Or it harms the economy or push up?
2. With a positive co-movement over the cycle?

Gali , Lopez - Salido and Valles (2007)

1. What are the effects in government purchases on the aggregate activity?
2. How are those transmitted

Most models $\underbrace{\uparrow G}_{\substack{purchases \\ taxes}} \rightarrow \uparrow Y$ but ¿? C

Standard RBC $\uparrow G \rightarrow \downarrow C$ (ricardian) because households behave in a Ricardian manner

IS – LM $\uparrow G \rightarrow \uparrow C$

Aiyami, Christiano and Eichembaun (1999)

Fatas and Mihov (2001)

$\uparrow G \rightarrow \downarrow I$ private investment falls $\rightarrow \downarrow$wealth$\rightarrow \downarrow C_t$

On the other hand$\uparrow G$ (financed by lump sum taxes) $\rightarrow \downarrow wealth \rightarrow \downarrow C_t$

$\uparrow N^s$ at any wage $\Rightarrow \downarrow \left(\frac{w}{P}\right)_t \rightarrow \uparrow N \rightarrow \uparrow Y$

$\uparrow \underbrace{N}_{\text{persistent}} \rightarrow \uparrow \frac{R}{b} \rightarrow \uparrow I_t$ The multiplier is greater or less than one depending on the parameters of the value.

Blanchard (2003)

$\uparrow G \rightarrow$ effects on output depend on the investment response

If $\left(\frac{\bar{M}}{P}\right) \Rightarrow \uparrow C \rightarrow \downarrow I$ (resulting from $\uparrow R$)

If Central Bank in response to $\uparrow G$ maintain $\bar{R} \Rightarrow$ effects in investment is zero)

Empiric studies find: that in response to a positive government spending shock consumption drops and there is a fall in I or at least it doesn't move (null effect)

So, two contributions:

Developed a DSGE model that incorporates:

- ✓ Sticky price models (Woodford 1999,2003)
- ✓ Presence of rule – of – thumb consumer (Campell and Mankiw 1989)

Blanchard and Peroty (2002)

1. $\uparrow G$ is persistence
2. $\uparrow G \rightarrow\uparrow Y$
3. $\uparrow G \rightarrow\uparrow C$ large and significant
   $\uparrow G \rightarrow\downarrow I$ significant

$F \wedge M$ $\uparrow G \rightarrow\uparrow I$ insignificant

The model $U(C, L)$

Government budget constraint

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t$$

All the variables we can express in real terms or as in Gali as deviation from its natural level and respect to out put.

So

$$P_t G_t = P_t T_t + \frac{B_{t+1}}{R_t} - B_t$$

Nominal return or nominal pay

$$P_t G_t = P_t T_t + \frac{B_{t+1}}{1 + i_t} - B_t$$

And let assume we hold a constant level of debt

$$P_t G_t = P_t T_t + B_{t+1}\left(\frac{1}{1 + i_t} - 1\right)$$

$$P_t G_t = P_t T_t + B_{t+1}\left(\frac{1 - i - 1}{1 + i_t}\right)$$

$$P_t G_t = P_t T_t - \frac{i}{1 + i_t} B_{t+1}$$

$$P_t T_t - \frac{i}{1 + i_t} B_{t+1} = P_t T_t + \frac{B_{t+1}}{1 + i_t} - B_t$$

$$B_t = \left(\frac{1}{1 + i_t} + \frac{i_t}{1 + i_t}\right) B_{t+1}$$

$B_t = B_{t+1}$ just pay the interest and debt is sustainability

FISCAL RULE ad – hoc

Daney Valdivia ®

$$T_t = \phi_b b_t + \phi_g g_t$$

Gov purchases are AR(1) $g_t = \rho g_{t-1} + \varepsilon_t$

From government constraint

$$P_t T_t + \frac{B_{t+1}}{R_t} = B_t + P_t G_t$$

And in real terms

$$t_t + b_t \frac{1 + \pi_{t+1}}{1 + i_t} = b_t + g_t$$

$$t_t + \frac{b_{t+1}}{\underbrace{1 + r_t}_{R_t}} = b_t + g_t$$

$$b_{t+1} = (1 + r_t)(b_t - g_t - t_t)$$

Since $\beta = \frac{1}{R_t} \wedge \frac{1}{1+\rho}$ $\quad 1 + r = 1 + \rho$

$$b_{t+1} = (1 + \rho)(b_t - g_t) - (1 + \rho)(\phi_b b_t + \phi_g g_t)$$

$$b_{t+1} = (1 + \rho)(1 - \phi_b) b_t + (1 + \rho)(1 - \phi_g) g_t$$

Under this rule necessary condition for not to be explosive

$$(1 + \rho)(1 - \phi_b) < 1$$

16. OPTIMAL MONETARY POLICY

What the general course of monetary policy should be?

- Taylor (1993), the well know example → the principle Taylor

Bernanke and Frederic Mishkin (1997) endorsement of inflation targeting

- Choose how to conduct monetary policy has important consequences on aggregate activity.
- Now we have techniques of dynamic equilibrium theory pioneered in RBC analysis → the so called DSGE models.
- Incorporation of market frictions.
- More knowledge about how works macroeconomics and the monetary policy improvements→ determinants of inflation.
  Output / inflation trade off is sensitive to the degree and nature of persistence in inflation ⇐ It`s the speed at which monetary policy should try to reach optimal inflation rate.
  As Gali and Gertler (1999), persistence in inflation may be related to sluggish adjustment of unit labor cost vis –a –vis movements in output that has important repercussions for monetary policy.
- Introduce an open eco framework are likely to provide alternative monetary policy rules.
- Choice of exchange rate regime ⇒optimal response to shock originated abroad.

⇒ Understand why central banks smooth interest rate adjustment?

⇒ How Central Bank deal with financial stability, policy rules discussed in the literature do include contingences for financial crises

Woodford

Inflation forecast targeting was developed at central banks like the reserve Bank of New Zealand, bank of Canada, Bank of England, and Sweden.

Inflation targeting literature finds that optimal monetary policy might be implemented through procedures that share important features of the inflation – forecast targeting that is currently practiced at central Banks like those just mentioned.

Inflation targeting safeguard CB against the trap of discretionary policy monetary and help to private sector anticipate future policy, increasing effectiveness.

Batini and Laxton (2006)

- Inflation targeting in emerging market countries have important effects rather than adopt money or exchange rate targeting.
- Shows that inflation and inflation expectations improve with no adverse effects on output
- Under inflation targeting volatility of interest rate, exchange rates, and international reserves are less.

- Inflation targeting can help build credibility and anchor inflation expectation more rapidly and durably.
- It provides more flexibility.
- Involves a lower economic cost in the face of monetary policy failure.
    - But there`s disadvantages
- It offers to little discretion and this unnecessarily restrains growth → this is because CB acts consistently and convincing to attain the inflation target.
- It offer too much discretion (may only worry about IT) and can't anchor expectations.

IT implies high exchange rate volatility → it could have negative implication on exchange rate.

Chile → to control inflation they push up interest rate, as a consequence the economy reserves more dollars so exchange rate falls.

Condition → technical capability of CB to implement IT, **ABSENCE THE FISCAL DOMINANCE**, good financial markets and efficient institutional support to motivate the commitment to low inflation

Preconditions:

Institutional independence → fall legal autonomy and be free from fiscal and political pressure

Well – developed technical infrastructure → must have inflation forecasting and modeling capabilities and the data needed to implement this.

Economic structure → prices deregulated

→ the economy should not overly sensitive to commodity prices and exchange rate and dollarization should be criminal

Healthy financial system → to guarantee effective monetary transmission

→ Capital markets will developed

Instead → adopt inflation targeting depend on the commitment and ability to plan and drive institutional change after introducing targeting

Despite, we have to study also how this fiscal regime can affect inflation targeting

See fiscal consequences of monetary policy:

Non - distorting sources of government renew exit

Fiscal policy can be rise to ensure intertemporal government solvency

Benigno – Woodford (2006) found that fiscal regime has important consequences for the optimal conduct of monetary policy.

An optimal target rule involves commitment to an explicit target for an output gap adjusted price level.

Optimal policy could allow departures from long – run target of growth in the gap adjusted price level in response to disturbances that affect the government's budget, but it involve commitment to restore variables to the normal level.

In the medium term inflation expectation should remain firmly anchored despite the occurrence of fiscal shocks.

Monetary policy has consequences for intertemporal solvency at government – under a given fiscal policy ⇒ Δ in monetary policy require changes in fiscal policy ⇒ welfare consequences.

Fiscal policy affects supply – side that affects the available trade – off between inflation stabilization and the central bank's ability to stabilize the welfare – relevant output gap.

THE MODEL

Credibility → Blanchard and Fisher (1989)

$$L = w\pi^2 + \left(y - \underbrace{k\bar{y}}_{\exists\ distartions}\right)^2 \qquad k > 1$$

St Phillips curve ⇒ $y = \bar{y} + \beta(\pi - \pi^e)$

1^{st} Alternative

- ✓ Have low – zero inflation ← compromise
- ✓ $PC \rightarrow y = \bar{y}$
- ✓ Loss function $L = (K-1)^2\bar{y}^2$

Inflationary bias ⇒ CB looks for boot the economy with a inflationary shock → ↑ y to converge $k\bar{y}$.

2^{nd} Discretion

$$L = w\pi^2 + (\bar{y} + \beta(\pi - \pi^e) - k\bar{y})^2$$

$$\frac{dL}{d\pi}$$

$$\pi: 2\pi w + 2(\bar{y} + \beta(\pi - \pi^e) - k\bar{y})\beta = 0$$

$$2\pi w + (2\bar{y} + 2\beta(\pi - \pi^e) - 2k\bar{y})\beta = 0$$

$$\pi(w+\beta^2)+\beta(\bar{y}+\beta(\pi^e)-1k\bar{y})=0$$

$$\pi_t=(w+\beta^2)^{-1}\beta[(k-1)\bar{y}+\beta\pi^e]$$

If we have rational expectations $\pi=\pi^e$

$$\pi_t(w+\beta^2)-\beta^2\pi_t=\beta(k-1)\bar{y}$$

$$\pi_t(w+\beta^2-\beta^2)=\beta(k-1)\bar{y}$$

$$\pi=w^{-1}\beta(k-1)\bar{y}$$

$$\Rightarrow L=(k-1)^2\bar{y}^2(1=w^{-1}\beta^2)>L_c(k-1)^2\bar{y}^2$$

∴ Discretion is worse than compromise

And we must deal with low, conservative and reputation.

Pool analysis → min cyclical fluctuation of the product.

17. IS MONETARY POLICY A SCIENCE?

It also depend on individual judgment

1^{st} Focus on output gap → but how to measure output gap?

Despite it stabilizing inflation around an inflation target

2^{nd} Follow the Taylor principle ← ensure policy reaction in response to high inflation.

Through moves in nominal interest rate we can stimulate private spending + or –

But, how can we estimate a Taylor principle for the economy? **IT'S DEPEND NO THE ECONOMY STRUCTURE!!!**

3^{rd} Be forward looking actions affect economy with lags E.g. Interest rate cut: as Walsh (2008) pointed out: it has impact on real output after twelve or even eighteen months.

This is explained by the presence of price setting and non-competitive market.

E.g. $\theta = 75$ ⇒implis the adjustment cause of inercy is within 3 – 4 quarters.

Lags mean that CB must be forward looking to stabilize possible effects of adverse shocks

Is monetary policy an art?

Request fine touch of policy maker

⇒ Two principles

1^{st} How can we focus the output gap when we don't know what it is?

It has important impact because authority must know if we are over or below the potential

2^{nd}Implement Taylor principle

Does CB respond to inflation changes with > 0,1 point 1,5 point, 2 refered to nominal interest rate?

Responding strongly will help to keep π more stable around low average level, but it will result in larger fluctuations in output and employment. Hence there is a trade – off between inflation stability and employment stability this trade – off require good judgment

Chile ↑ i to ↓ π but ↓ exchange rate ⇒↓ XN

Valdivia (2008) shows that we have a weak tradeoff between inflation and employment because we have a high frequency price setting

We must have

Art of forecasting →forecast future economic conditions

Not only based on good data or good models, but also on good judgment

So conducting policy is for from routine

Daney Valdivia ®

Bibliography

Acemoglu D. (2008). Introduction to Modern Economic Growth. Princeton University Press.

Andrés, J. and R. Domenéch (2005). Automatic Stabilizers, Fiscal Rules and Macroecomomic Stability. Universidad de Valencia.

Armas, A., Ize, A., and Yeyati, E. L. (2006). Financial Dollarizarion: An Overview. International Monetary Fund.

Basu, S., J. G. Fernald, J. Fisher, and M. Kimball (2011). Sector-Specific Technical Change. Unpublished Manuscript, Federal Reserve Bank of San Francisco.

Basu, S., J. G. Fernald, and M. S. Kimball (2006). Are Technology Improvements Contractionary?. American Economic Review, 96(5), 1418-1448.

Basu, Ferland and Liu (2012). Technology Shocks in a Two-Sector DSGE Model. 2012 Annual Meeting of the Society for Economic Dynamics.

Batini, N., Breuer,P., and Kochlar, K.(2006). Inflation Targeting and the IMF. International Monetary Fund Board Paper.

Baxter, M., y R. G. King (1993). Fiscal Policy in General Equilibrium. American Economic Review, 83(3), 315–334.

Bejarano J. A. (2004). Estimación estructural y análisis de la curva de Phillips Neo – Keynasiana para Colombia. Facultad de Economía de la Universidad del Rosario.

Bénassy, J-P. (2002). The Macroeconomics of Imperfect Competition and Nonclearing Markets. MIT Press.

Bogdanski, J., Tombini, A. and Werlang, S. Implementing Inflation Targeting in Brazil. BCB Working Paper Series no 1, july, 2000.

Blejer, M. (1998). Macroeconomic Policy Coordination in a More Integrated World. X Seminario Regional de Política Fiscal, Compendio de Documentos, 1998, CEPAL.

Buiter, W. (1997). Aspects of Fiscal Performance in some TransitionEconomies under Fund Supported Programs. IMF Working Paper 97/31.

Blanchard, O. and R. Perotti (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. Quarterly Journal of Economics, Nº117, pg 1329–1368.

Canzoneri, M.B. and H. Edison (1990). A new interpretation of the coordination problem and its empirical significance. Monetary Aggregates and Financial Sector Behaviour in Interdependent Economies, P. Hooper and others, Washington D. C., Federal Reserve Board.

Caputo, R., Liendo, F and J.P. Medina (2006). New Keynesian Models for Chile in the Inflation – Targeting Period: A Structural Investigation. Central Bank of Chile, WP 402.

Canova F. (2007). Methods for Applied Macroeconomic Research. Princeton University Press.

Carneiro, D. and Wu, T. Y. H (2005). Dominância fiscal e desgaste do instrumento único de política monetária no Brasil. Texto para Discussão IEPE / CdG, Rio de Janeiro, n. 7, maio 2005.

Céspedes L., Ochoa M. and C. Soto (2005). An Estimated New Keynesian Phillips Curve for Chile.Banco Central de Chile. DTBC 355.

Dave and De Jong (2007). Structural macroeconometrics. Princeton University Press.

Christiano, L. and J., Fitzgerald, T. J (2000).Understanding the Fiscal Theory of the Price Level.NBER Working Paper, No. 7668, Apr. 2000.

Chung, H., T. Davig, and E. M. Leeper (2007). Monetary and Fiscal Policy Switching. Journal of Money, Credit and Banking, 39(4), 809–842.

Cagliarini, A. and W. McKibbin (2009). Global Relative Price Shocks: The Role ofMacroeconomic Policies. Reserve Bank of Australia, Research Discussion Papers 2009/10.

Clarida, R., J. Gali, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. Journal of Economic Literature, 37(4), 1661–1707.

Collard, F. and Juillard, M. (2001). A Higher-Order Taylor Expansion Approach to Simulation of Stochastic Forward-Looking Models with an Application to a Non-Linear Phillips Curve. Computational Economics, 17, 125-139, 2001.

Davig, T. and E. M. Leeper (2010). Monetary-fiscal policy interactions and fiscal stimulus.European Economic Review In Press, Corrected Proof.

De Castro M., Gouvea S., Minella A. Santos R. and F. Souza-Sobrinho (2011). SAMBA: Stochastic Analytical with a Bayesian Approach. Banco Central Do Brasil. WP 239.

Eichembaum C. (2005), A short course of DSGE models. *http://faculty.wcas.northwestern.edu/~lchrist/course/syllabus.htm*

Eggertsson, G. B. and M. Woodford (2003). The zero bound on interest rates and optimal monetary policy. Brookings Papers on Economic Activity 34 (2003-1), 139-235.

Florian D. and C. Montoro (2009). Development of MEGA-D: A DSGE Model for Policy Analysis. Banco Central de la Reserva del Perú.

Fatás, A. and I. Mihov (2001). The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence. INSEAD, Working paper.

Favero, C. (2001): Applied Macroeconomitrics, Oxford University Press.

Favero C. and Monacelli, T (2003). Monetary - Fiscal Mix and Inflation Performance Evidence fromthe US. CEPER Working Paper, n. 234, Apr. 2003.

Fialho, M. M. and Portugal, M. S (2005). Monetary and Fiscal Policy Interactions in Brazil: AnApplication of the Fiscal Theory of the Price Level. Estudos Econômicos, V.35, n. 4 p: 657-685, Out/Dez 2005.

Fisher, J. D. M. (2006). The Dynamic Effects of Neutral and Investment – Specific Technology Shocks. Journal of Political Economy, 114(3), 413-451.

Francies, N., and V. A. Ramey (2005). Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. Journal of Monetary Economics, 52(8), 1379-1399.

Gali (2008). Monetary Policy, Inflation, and the Business Cycle. An introduction to the New Keynesian Framework. Princeton University Press.

Galí, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?. American Economic Review, 89(1), 249-271.

Galí, J. and M. Echeinbaum (2004). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies. Nº 72, pg 707 – 734

Galí, J., López – Salido, D. and J. Vallés (2007). Understanding the Effects of Government Spending on Consumption. European Economic Association, Vol 5, Nº 1, pgh 227 – 270.

Galí, J. and M. Gertler (1999). Inflation dynamics: a structural econometric approach. Journal of Monetary Economics, Vol. 44, Nº 2, pg. 195–222.

García, C. and J. Restrepo (2006). The Case of Countercyclical Rule – Based Fiscal Regime. Central Bank of Chile.

García, C. and J. Restrepo (2007).How Effective is Government Spending in a Small Open Economy with Distortionary Taxes. Central Bank of Chile.

Hamann F., Julián P. and D. Rodríguez (2006). Bringing a DSGE model into policy environment in Colombia. Banco de la República de Colombia.

Kocherlacota N. (2010), The New Dynamic Public Finance.Princeton University Press.

Kongsamut, P., Rebelo, S. and D. Xie (1997). Beyond Balanced Growth. National Bureau of Economic Research, NBER Working Papers Nº 6159.

Lacunza, H. and J. Carrera (2003). Cooperación macroeconómica en el Mercosur. Un análisis de la interdependencia y una propuesta de coordinación. Serie Estudios del CEI, Nº4, Buenos Aires, abril.

Leeper, E. M. (1991). Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies. Journal of Monetary Economics, 27(1), 129-147, Feb. 1991.

Leeper, E. M (2005). A simple model of the fiscal theory of the price level. Bloomington: Indiana University, mimeo, 2005.

Leith, C. and Wren-Lewis, S. (2007). Counter Cyclical Fiscal Policy: Which Instrument is Best? Mimeo.

López- Salido D. and R. Blakrishnan (2002).Understanding UK inflation: the role of openness. Bank of England, WP 164.

Mancini T. (2007), Dynare User Guide. http://www.dynare.org/documentation-and-support/user-guide

Mazali A. A. and J. A. Divino (2010). Real Wage Rigidity and the New Phillips Curve: the Brazilian Case.

Medina, J.P. and Soto, C. (2007). Copper Price, Fiscal Policy and Business Cycle in Chile. Central Bank of Chile, Working Paper N° 458.

Michalak, T., J. Engwerda, and J. Plasmans (2009). Strategic Interactions between Fiscal and Monetary Authorities in a Multi-Country New-Keynesian Model of a Monetary Union. CESifo Working Paper Series 2534, CESifo Group Munich.

Monacelli, T., and R. Perotti (2008). Fiscal Policy, Wealth Effects, and Markups. NBER Working Paper No. 14584.

Ruge - Murcia (2006), Methods to estimate dynamic stochastic general equilibrium models. http://www.econ.yale.edu/~gjh9/econ526b/Methods3.pdf

Persson, T. and G. Tabellini (1990). Macroeconomic Policy, Credibility and Politics. Harwood Academic Publishers, Suiza.

Rodríguez G. (2009). Using a Forward-Looking Phillips Curve to Estimate the Output Gap in Peru. Banco Central de la Reserva del Perú. Serie de Documentos de Trabajo DT. N° 2009-010.

Sargent T. J. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review, 5(3), 1-17, Winter 1981.

Sargent T. et. Al. (2008). Practicing Dynare. https://files.nyu.edu/ts43/public/research/AP_tom16.pdf

Schmitt - Grohe, S. and M. Uribe (2005). Optimal Fiscal and Monetary Policy in a Medium - Scale Macroeconomic Model: Expanded Version. NBER Working Papers 11417.

Schmidt-Hebbel, K. and M. Tapia (2002): “Inflation Targeting in Chile”, North American Journal of Economics and Finance Nº 13, pg 125 – 146.

Smets, F. and R. Wouters (2002). An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. European Central Bank.

Smets, F. and Wouters, R. (2003). An estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. Journal of the European Economic Association, 1(5), 1123–1175.

Smets, F., and R. Wouters(2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97, 586-606.

Uhlig, H. (2003). One Money, but Many Fiscal Policies in Europe: What Are the Consequences?. In M. Buti (Ed.), Monetary and Fiscal Policies in EMU: Interactions and Coordination, pp. 29-56. Cambridge University Press.

Valdivia, D. (2008). ¿Es importante la fijación de precios para entender la dinámica de la inflación en Bolivia?. INESAD, WP Nº 02/2008.

Wallace, N. (1981). A Modigliani-Miller Theorem for Open-Market Operations. American Economic Review, 71(3), 267–274.

Warr, P. (2009). Aggregate and sectorial productivity growth in Thailand and Indonesia. Research School of Pacific and Asian Studies, WP 2009/10.

Woodford, M. (2003). Foundations of a Theory of Monetary Policy. Princeton University Press.

Woodford, M. (2010). Simple analytics of the government expenditure multiplier. Working Paper 15714, National Bureau of Economic Research.