A School Choice Compromise: Between Immediate and Deferred Acceptance

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Abstract

School assignment procedures aim to improve student welfare, but must balance efficiency and equity goals and provide incentives for students to report their preferences truthfully. Debate centers largely on two rules: immediate acceptance (IA), the so-called Boston mechanism, and deferred acceptance (DA). IA's strength is efficiency, while DA is touted for its superior strategic properties. Thinking of these as extremes, we advocate a compromise rule, immediate-acceptance-with-skips (IA^+), which slightly modifies IA to achieve better strategic properties while retaining efficiency. IA^+ proceeds in rounds of applications and, like IA, finalizes assignments in each round. However, unlike IA or DA, IA^+ allows students to "skip" applications to schools with no remaining capacity. We show that IA^+ is efficient and less manipulable than IA^+. Unfortunately, IA^+ violates solidarity properties that both IA and DA satisfy. Considering robustness, we find that each of the three rules satisfies a different set of three natural invariance properties.

Keywords: School choice; deferred acceptance; immediate acceptance; immediate-acceptance-with-skips; Boston mechanism.

JEL Classification Numbers: C78, D63, H75, I28.

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1 Introduction

With careful theoretical, experimental, and empirical work, economists have won the ear of school choice policy makers[1]. By drawing attention to the incentives rules provide for students to truthfully report their preferences, economic analysis has shifted the debate and spurred school districts to adopt new assignment procedures across the United States and beyond[2]. To minimize opportunities to manipulate, districts have turned largely to rules based on the deferred acceptance algorithm (Gale and Shapley, 1962).

Yet no assignment procedure meets all social objectives. Insisting that truth-telling be a dominant strategy for students, the formal requirement of *strategy-proofness*, comes at a cost. With deferred acceptance[3] (DA), the most popular *strategy-proof* rule, the cost is in terms of student welfare and may be substantial (Kesten, 2006). As in Boston, DA often replaces an easily manipulable but *efficient* rule. With these trade-offs in mind, we ask whether there is a middle ground with superior properties overall.

School choice rules typically simulate rounds of applications and rejections. While other methods are possible, districts develop school priorities with this interpretation in mind. Application-rejection algorithms differ primarily along two dimensions: When acceptances are finalized and where rejected students apply next. At one extreme, DA makes acceptances tentative until the very end. At the other extreme is Boston's original rule, immediate acceptance[4] (IA), which finalizes acceptances in each round. Further refinement along this dimension leads to a family of rules (Chen and Kesten, 2013).

While DA and IA finalize assignments at different points, both ask students to apply to schools in the order listed in their preference rankings. As obvious as this approach seems, it nevertheless leads to paradoxical results. With DA for example, a student's application and tentative acceptance may set off a chain of rejections that leads to her eventual rejection. Her futile application only makes other students worse off. Similarly, a student may apply to a school with no available seats. While inconsequential for DA, this leads to the most serious cases of manipulability for IA. What if students "skipped" such applications? Beginning

[1]Abdulkadiroğlu and Sönmez (2003) kick-started the theoretical literature on school choice; Chen and Sönmez (2006) and Pais and Pintér (2008) contributed early experimental evidence.

[2]See, for example, case studies of New York (Abdulkadiroğlu et al., 2005a) and Boston (Abdulkadiroğlu et al., 2005b, 2006) as well as discussion of assignment procedures used in Great Britain (Pathak and Sönmez, 2008) and China (He, 2012; Chen and Kesten, 2013).

[3]Another *strategy-proof* proposal adapts Gale's Top Trading Cycles and is also efficient (Abdulkadiroğlu and Sönmez, 2003). It has gained little traction, however, because many object to the idea of "trading" priorities (Abdulkadiroğlu et al., 2006).

[4]This rule is also called the "Boston mechanism".

with DA, eliminating futile applications[5] restores *efficiency* but sacrifices *strategy-proofness* (Kesten, 2010). We begin instead with IA and modify the algorithm so that students skip applications to schools that have reached capacity. Our paper systematically analyzes this new rule[6], which we call immediate-acceptance-with-skips (IA^+), and argues that it may be a reasonable compromise between IA and DA.

To assess IA^+, we recall the desirable properties motivating IA and DA. IA derives much of its appeal from its efficiency properties[7]. Moreover, IA maximizes the number of students assigned to their top-ranked schools. News reports often emphasize this easy-to-interpret statistic, underscoring its salience and importance to the public. Like IA, IA^+ always selects efficient assignments (Proposition 1), and for problems in which all schools begin with available seats, it also assigns as many student as possible to their top-ranked schools. Both properties are advantages over DA. To further compare IA and IA^+, we can refine our notion of efficiency by counting students assigned to their first choice, second choice, and so on[8]. Neither rule, however, performs uniformly better on this dimension.

DA's primary strength is *strategy-proofness*. In school choice, this is valuable for two reasons. The first is practical: If a rule is manipulable, the properties it satisfies with respect to reported preferences may no longer apply. For example, when students manipulate IA or IA^+, the outcome need not be efficient with respect to their true preferences (Abdulkadiroğlu and Sönmez, 2003). Worse, since the Nash equilibrium outcomes under IA and IA^+ include the entire stable set (Alcalde, 1996; Ergin and Sönmez, 2006), the DA assignment Pareto dominates the equilibrium assignments of IA and IA^+. However, optimal manipulation requires knowledge of priorities which may be unavailable to students. Moreover, in some low information settings with cardinal preferences, manipulation may actually enhance efficiency (Abdulkadiroğlu et al., 2011; Miralles, 2008; Featherstone and Niederle, 2008). While we do not model incomplete information or include cardinal preferences, these examples show that the cost of manipulability may be less than originally presumed. Furthermore, this criticism may apply to DA as well: In experiments, large fractions of participants students misrepresent their preferences even though truth-telling is a dominant strategy (Chen and Sönmez, 2006; Pais and Pintér, 2008; Chen and Kesten, 2013).

[5] In fact, deciding which applications to skip is a delicate procedure. Kesten (2010)'s "efficiency adjusted" DA does so in a way that minimizes priority violations. Doğan (2013) modifies DA in a similar way to accommodate affirmative action goals.

[6] The definition is not entirely new. Alcalde (1996) first formulated IA^+ as a means to implement the stable correspondence in two-sided matching. Miralles (2008) also suggests this modification.

[7] Indeed, efficiency has motivated renewed interest in IA. See, for example, Miralles (2008), Featherstone and Niederle (2008), and Abdulkadiroğlu et al. (2011).

[8] Featherstone (2013) suggests this approach.

A second rationale for *strategy-proofness* invokes equity. If some students are sincere or naive, we may worry that these students will be harmed when strategic students manipulate. By making truth-telling a dominant strategy for all students, *strategy-proofness* "levels the playing field" (Pathak and Sönmez, 2008). Indeed, this is true for IA: In data from Boston, students often listed two over-demanded schools as their top choices, thereby losing priority to savvier students (Abdulkadiroğlu et al., 2006). Experimental data also show that many students fail to "skip the middle" and list an over-demanded school second (Chen and Sönmez, 2006). While this criticism applies to IA, it has much less force against IA^+. By design, IA^+ proactively "skips the middle", thereby carrying out the most natural manipulation for sincere students. This protective measure makes IA^+ less manipulable than IA in a formal sense[9]: Each problem in which IA^+ is manipulable, IA is manipulable as well (Proposition 5).

In addition to being *strategy-proof*, DA respects students' priorities. More precisely, DA is *stable*: If a student prefers another school to her own, then each student assigned to that school has higher priority than her. Neither IA nor IA^+ meets this requirement, and neither can be called "more stable" than the other (Proposition 2). While this comparison favors DA, we argue that the normative advantage is less than imagined.

As with *strategy-proofness*, our interest in *stability* depends on its interpretation. In two-sided matching, *stability* is compelling. When matching residents to hospitals or pairing men and women, the preferences on both sides count and *stability* is essential to prevent unraveling[10]. Yet for school choice, where school priorities are administratively assigned, *stability* is not obviously desirable. Fundamentally, we aim to answer the objections of dissatisfied students and families.[11] Interpreted by *stability*, justifications refer exclusively to administratively determined priorities. But why ignore preferences? Instead, we may evaluate objections with respect to augmented priorities. One example is rank-adjustment: Assign highest priority to students who rank a school first, then those students who rank the school second, and so on, using the administrative priorities to order students within each class. If objections are evaluated against these rank-adjusted priorities, then *stability* actually favors IA: IA is stable with respect to rank-adjusted priorities whereas DA is not[12].

[9] This notion of relative manipulability is due to Pathak and Sönmez (2013).

[10] See Roth (2008) for further discussion and examples.

[11] Balinski and Sönmez (1999) propose this interpretation of *stability*, and this reasoning motivates the term "justified envy".

[12] The force of the justified envy argument weights most heavily against assignment procedures that allow "trading" of priorities. There, it is difficult to provide a compelling reply to student objections which may have been a decisive factor leading Boston to choose DA over TTC (Abdulkadiroğlu et al., 2006). No similar objections were leveled against IA.

Moving beyond *stability*, we turn to notions of equity and robustness based on changes in the environment. School assignments are not made in a static environment. For example, current students may depart or new students arrive. Similarly, school capacities may change as hiring decisions are finalized or renovation projects begun or completed. Even the set of schools may change as new schools may open or existing schools close. We would like to understand how these changes affect student welfare. First consider the departure of a student. To the extent that school seats are scarce, this reduces competition. In this case, it is natural to ask that remaining students be at least as well off as initially. This is the formal requirement of *population monotonicity*[13]. Analogously, competition also decreases when new seats become available. In this case, *capacity monotonicity* asks that all students be at least as well off as before. These natural solidarity properties are common to DA and IA. Unfortunately, IA^+ satisfies neither (Proposition 3).

Some changes in the environment are arguably irrelevant to the assignment problem. We consider three situations: (i) removal of an unavailable school from the list of schools; (ii) removal of a student ultimately unassigned by a rule; and (iii) pre-assignment of a student to the school the rule would have her attend. In each case, we require invariance. Surprisingly, none of our rules is robust to all three changes (Proposition 4). IA^+ satisfies (i), an advantage over DA, and (ii), an advantage over IA. Only IA satisfies (iii), however[14].

Since IA^+ does not satisfy all desirable properties on the full domain of assignment problems, we ask whether it does on a smaller domain. In particular, we identify a maximal domain formed by restricting priority-capacity structures. The conditions are stringent: The combined capacity of even the smallest two schools with available seats must be sufficient to accommodate the entire student population (Proposition 6). This is slightly less demanding than the corresponding condition for IA (Chen, 2014) and somewhat more demanding than a related condition sufficient to ensure that DA is *efficient* (Ergin, 2002; Klaus and Klijn, 2013).

Related literature

Our paper contributes to three strands of the school choice literature. First, several recent studies have reassessed IA. Miralles (2008) makes an efficiency-based case for IA, while Abdulkadiroğlu et al. (2011) reach similar conclusions in incomplete information environments. Ongoing experimental work has yielded mixed evaluations of the rules, suggesting

[13]For original references, see Thomson (1983a) and Thomson (1983b).

[14]In fact, the same conclusion obtains when we weaken (iii) to *non-bossiness*, which requires invariance when a student's preferences change but her school does not change.

opportunities to improve on them in practice[15]. Also, Kojima and Ünver (2014) characterize IA using several of the properties we consider here.

Second, intending to improve on the baseline rules, numerous papers recommend compromises. We have already mentioned one important example, Kesten (2010)'s "efficiency adjusted" DA, which modifies DA to attain *efficiency*. Doğan (2013) adapts this procedure to improve the welfare of minority students in a controlled choice setting. In similar spirit, Abdulkadiroğlu et al. (2014) propose "choice augmented" DA, which enriches the choice framework so students influence tie-breaking by "targeting" schools. Morrill (2013d) and Morrill (2013c) take another approach, beginning instead with top trading cycles and modifying it to reduce priority violations. This tact preserves both *efficiency* and *strategy-proofness*. More closely related to our approach, Mennle and Seuken (2014) study incentive properties of a randomized version of IA^+ with uniform priorities across schools. Also, concurrent with our work, Dur (2013)[16] defines a version of IA^+ and independently proves part of our Proposition 5.

Finally, our technical results for restricted domains add to a growing literature on the role of priorities. Ergin (2002) shows that the priority-capacity structure must satisfy a stringent "acyclicity" condition to ensure that DA is *efficient* or *group strategy-proof*. Klaus and Klijn (2013) extends this result to general domains and Chen (2014) develops analogous conditions under which IA is *stable* or *strategy-proof*. Relatedly, Erdil and Ergin (2011) and Abdulkadiroğlu et al. (2009) investigate the role of indifferences in school priorities.

The remainder of the paper comprises four sections. Section 2 introduces the formal model and our leading rules. We discuss efficiency and equity properties in Section 3 and turn to incentives in Section 4, and conclude in Section 5. We relegate all proofs to the appendix.

2 Model

There is a set of potential students $\boldsymbol{\mathcal{N}}$ and a set of potential schools $\boldsymbol{\mathcal{A}}$. A school choice problem consists of a population of students and list of schools. Students are defined by their preferences over schools, and schools are defined by their capacities and priorities over students. Because we will be interested in a variety of relational properties, we specify the

[15]The most relevant experimental studies are Chen and Sönmez (2006), Pais and Pintér (2008), Featherstone and Niederle (2008), and Chen and Kesten (2013). Additionally, He (2012) and Abdulkadiroğlu et al. (2006) report empirical preference data from Boston and Beijing, and Miralles (2008) compares IA and DA with simulated data.

[16]I thank Battal Doğan for bringing this paper to my attention.

model with sufficient generality to allow all aspects to vary. To emphasize these properties, we retain the standard assumption of complete information.

Formally[17], let $\boldsymbol{\mathcal{R}}$ be the set of linear orders over $\mathcal{A}$ and $\boldsymbol{\Pi}$ the set of linear orders over $\mathcal{N}$. A **problem** is now a list $\boldsymbol{e \equiv (N, A, q, R, \succ)}$ such that $N \subseteq \mathcal{N}$, $\emptyset \in A \subseteq \mathcal{A}$, $q \equiv (q_a)_{a \in A} \in \mathbf{Z}_+^A$, $R \equiv (R_i)_{i \in N} \in \mathcal{R}^N$, and $\succ \equiv (\succ_a)_{a \in A} \in \Pi^A$. By assumption, a null school, $\boldsymbol{\emptyset}$, is always available ($q_\emptyset = \infty$). The set of problems is $\boldsymbol{\mathcal{E}}$. An **assignment** (or match) for e is a function $\mu\colon N \to A$ such that for each $a \in A$, $|\{i \in N : \mu_i = a\}| \leq q_a$ and the set of assignments is $\boldsymbol{\mathcal{M}}$. A **rule** is a function $\boldsymbol{\varphi}\colon \mathcal{E} \to \mathcal{M}$.

We study three rules defined by closely related algorithms. Each algorithm proceeds in rounds with students applying to schools according to their preferences.

- **Immediate acceptance**

  First round: Each student applies to her most preferred school. Each school accepts students from among its applicants in order of priority up to its capacity and rejects the others. These acceptances are final.

  Subsequent rounds: Each unassigned student applies to her most preferred school to which she has not previously applied. Each school accepts students from among its new applicants in order of priority up to its remaining capacity and rejects the others.

- **Immediate-acceptance-with-skips**

  First round: Each student applies to her most preferred school *among those with available seats*. Each school accepts students from among its applicants in order of priority up to its capacity and rejects the others. These acceptances are final.

  Subsequent rounds: Each unassigned student applies to her most preferred school *among those with available seats* to which she has not previously applied. Each school accepts students from among its new applicants in order of priority up to its remaining capacity and rejects the others.

- **Deferred acceptance**

  First round: Each student applies to her most preferred school. Each school accepts students from among its applicants in order of priority up to its capacity and rejects the others. These acceptances are *tentative*.

[17] We define problems with preferences over the set of potential schools and priorities over the set of potential students for notational convenience. The rules we study depend only on the restriction of preferences and priorities to the sets of schools and students named in the problem, and our results apply if preferences and priorities are replaced with these restrictions.

Subsequent rounds: Each unassigned student applies to her most preferred school to which she has not previously applied. Each school accepts students from among its new applicants *and tentatively accepted students* in order of priority up to its remaining capacity and rejects the others.

The **immediate acceptance rule**, $\boldsymbol{IA}$, the **immediate-acceptance-with-skips rule**, $\boldsymbol{IA^+}$, and **deferred acceptance rule**, $\boldsymbol{DA}$, apply the corresponding algorithms in each problem.

The three algorithms differ in two ways: (i) when acceptances are finalized and (ii) where rejected students apply next. According to the immediate acceptance algorithm, students may apply to schools with no remaining capacity. The immediate-acceptance-with-skips algorithm modifies the immediate acceptance algorithm by allowing students to "skip" these applications. We can think of tentative acceptances as a further change along the same lines: Effectively, the deferred acceptance algorithm allows students to "skip" applications to schools where they will be rejected. Carried out in a forward-looking manner, "rejection skips" lead to the same outcome as tentative acceptances. In this way, the immediate-acceptance-with-skips rule moves "part way" toward deferred acceptance[18]. While intuitive to call these improvements, the transitions entail costs as well as benefits. The remainder of the paper explores these trade-offs. Before turning to properties, we illustrate differences among the three rules in an example.

Example 1. Comparing $\boldsymbol{IA}$, $\boldsymbol{IA^+}$, and $\boldsymbol{DA}$. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4, 5\}$, $A \equiv \{a, b, c, d\}$, and $(q, R, \succ)$ as specified in the table:

					(2)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
a	b	b	c	c	1	4	2	$\vdots$
$\vdots$	c	c	b	a	$\vdots$	2	3	
	$\vdots$	a	$\vdots$	d		$\vdots$	4	
		d		b			$\vdots$	

The table below computes the assignments made by each rule. Boxes represent final accep-

[18] In addition to immediate and (fully) tentative acceptances, intermediate notions are possible. Varying the conditions under which tentative acceptances become final leads to a family of rules which includes IA and DA (Chen and Kesten, 2013). However, IA^+ is not a member of the family.

tances and asterisks represent tentative acceptances. We have:

$$
\begin{aligned}
DA(e) &= \big((1,a),(2,c),(3,a),(4,b),(5,d)\big),\\
IA(e) &= \big((1,a),(2,b),(3,d),(4,c),(5,a)\big), \text{ and}\\
IA^{+}(e) &= \big((1,a),(2,b),(3,a),(4,c),(5,d)\big).
\end{aligned}
$$

Both IA and IA^+ assign student 4 to school c in the first round of applications. In contrast, DA makes only a tentative assignment which is later overturned. IA and IA^+ differ in their treatment of student 3's preferences. In the second round, c is unavailable and so IA^+ skips this school and has student 3 apply directly to her third choice, a, where she is accepted. Under IA, student 3 nevertheless applies to c in the second round and consequently loses her priority at a to student 5. Both IA and IA^+ select efficient assignments. However, the DA assignment is inefficient because exchanging the assignments of students 2 and 4 would make both better off. On the other hand, the DA assignment is *stable* and cannot be manipulated. Since student 3 has higher priority than student 4 at c, the IA and IA^+ assignments are unstable. Moreover, student 3 can manipulate both assignments by reporting preferences with c at the top.

$DA(e)$

Round	a	b	c	d
1	1^*	2^*, 3	4^*, 5	
2	1^*, 5^*	2^*	3^*, 4	
3	1^*, 5^*	2, 4^*	3^*	
4	1^*, 5^*	4^*	2^*, 3	
5	1^*, 3^*, 5	4^*	2^*	
6	$\boxed{1}$, $\boxed{3}$	$\boxed{4}$	$\boxed{2}$	$\boxed{5}$

$IA(e)$

Round	a	b	c	d
1	$\boxed{1}$	$\boxed{2}$, 3	$\boxed{4}$, 5	
2	$\boxed{1}$, $\boxed{5}$	$\boxed{2}$, 3	$\boxed{4}$	
3	$\boxed{1}$, 3, $\boxed{5}$	$\boxed{2}$	$\boxed{4}$	
4	$\boxed{1}$, $\boxed{5}$	$\boxed{2}$	$\boxed{4}$	$\boxed{3}$

$IA^+(e)$

Round	a	b	c	d
1	$\boxed{1}$	$\boxed{2}$, 3	$\boxed{4}$, 5	
2	$\boxed{1}$, $\boxed{3}$, 5	$\boxed{2}$	$\boxed{4}$	
3	$\boxed{1}$, $\boxed{3}$	$\boxed{2}$	$\boxed{4}$	$\boxed{5}$

In the example, student 3's application to her first choice school is ineffectual, and under either IA or IA^+ she can achieve a better assignment by skipping this application. However, applying to b may be reasonable if she is uncertain about her priority at the school. In this case, her choice whether to list b or c first reflects her willingness to roll the dice. Now her decision reflects her preference intensity, rather than a naive failure to strategize. On the other hand, both b and c are oversubscribed so no degree of optimism about her priorities justifies listing *both* of these schools at the top. Here, we have legitimate concern that sincere reporting disadvantages student 3. IA^+ corrects this.

3 Efficiency, equity, and robustness

School choice aims to improve student welfare. Of course, seats are scarce, so not all students can be assigned to their most preferred schools. We also expect rules to resolve conflicts of preferences fairly. Similarly, we expect rules to respond reasonably to changes in the environment. To achieve these additional objectives, we may even trade-off efficiency[19]. In this section, we evaluate our rules according to these goals, beginning with efficiency.

3.1 Efficiency

An assignment is **(Pareto) efficient** if there is no other assignment that all students find at least as good and one student prefers, and a rule is **efficient** if it always selects efficient assignments. IA, of course, is *efficient*, and we show that IA^+ is as well. In contrast, DA can be highly inefficient[20]. Moreover, IA and IA^+ may select assignments that Pareto dominate the DA assignment. Restrictions on the priorities can ensure *efficiency*, but priorities must be nearly identical across schools (Ergin, 2002). However, sibling attendance, proximity, and other commonly considered factors introduce too much diversity across schools to satisfy these conditions.

To further compare IA and IA^+, we consider a refinement of *efficiency* based on rank distributions.[21] The rank distribution of an assignment counts the number of students assigned their first choice, second choice, and so on. Formally, given an assignment μ, for each $k = 1, \ldots, |A|$, let $\boldsymbol{N(k, \mu)} \equiv \left|\left\{i \in N : |\{a \in A : a \; R_i \; \varphi_i(e)\}| = k\right\}\right|$. The **rank distribution** of μ is $\boldsymbol{N(\mu)} \equiv \left(N(k, \mu)\right)_{k=1}^{|A|}$ and μ is **rank efficient** if there is no other match whose

[19] For example, auctioning school seats would lead to a (fully) efficient assignment. That no serious proposal recommends an auction shows that other social objectives weigh heavily in school choice.

[20] In fact, DA may assign nearly all students to their *least* preferred schools. See Kesten (2010) for examples.

[21] Featherstone (2013) introduces the following notion and shows that it implies *efficiency*.

rank distribution first-order stochastically dominates $N(\mu)$.[22] A rule is **rank efficient** if it always selects *rank efficient* assignments.

When all schools have available seats, IA and IA^+ make the same assignments in the first round and maximize the number of students assigned their first choice schools. However, neither rule is *rank efficient*. IA^+ "skips" some applications and so moves more quickly to lower ranks than does IA. Consequently, we may expect IA to yield superior rank distributions. For example, if a student's first choice school is unavailable, she applies directly to her second choice school under IA^+ and may displace a student who ranks that school first. In fact, even the DA assignment may rank dominate the IA^+ assignment. Somewhat surprisingly, as we illustrate in the appendix, the comparison of rank distributions can also favor IA^+. Proposition 1 summarizes our conclusions.

Proposition 1. *(i) Both IA and IA^+ are* efficient *while DA is not. (ii) Moreover, there exist problems in which IA and IA^+ may Pareto dominate DA. (iii) None of the rules is* rank efficient*. (iv) Furthermore, there exist problems in which IA rank dominates IA^+ as well as problems in which IA^+ rank dominates IA. (v) IA rank dominates DA whenever their rank distributions are comparable by first-order stochastic dominance.*

3.2 Equity

When choosing among school choice rules, efficiency is not the only concern. As DA derives much of its appeal from equity properties, it is important to understand how IA^+ compares according to these criteria. We begin with *stability* and then turn to solidarity properties.

Given $e \in \mathcal{E}$, μ is **individually rational** if for each $i \in N$, we have $\mu_i \; R_i \; \emptyset$ and is **non-wasteful** if for each $i \in N$ and each $a \in A$, $a \; P_i \; \varphi(e)$ implies $|\varphi_a(e)| = q_a$. The assignment **violates student *i*'s priority at *a*** if $a \; P_i \; \mu_i$ and there is $j \in N$ such that $i \succ_a j$ and $\mu_j = a$. The assignment is **stable** if it is *individually rational*, *non-wasteful*, and eliminates priority violations. A rule is *individually rational*, *non-wasteful*, or *stable* if it always selects assignments with the corresponding properties. *Stability* implies both *individual rationality* and *non-wastefulness*[23]. It is immediate that IA, IA^+, and DA are *individually rational* and *non-wasteful*. However, only DA is *stable*.

Just as DA sometimes selects an efficient assignment, both IA and IA^+ sometimes select stable assignments. In particular, this occurs whenever either rule select the same assignment

[22]The rank distribution μ first-order stochastically dominates, abbreviated **sd-dominates**, μ' if $\mu \neq \mu'$ and for each $l = 1, \ldots, |A|$, $\sum_{k=1}^{l} |N(k,\mu)| \geq \sum_{k=1}^{l} |N(k,\mu')|$.

[23]*Efficiency* also implies these properties.

as DA. At first glance, we might expect IA^+ to be "more stable" than IA and to coincide with DA whenever IA does, but this is not the case. More precisely, there is no relationship between the sets of problems for which IA and IA^+ select stable assignments. Proposition 2 summarizes.

Proposition 2. *(i) DA, IA, and IA^+ are* individually rational *and* non-wasteful. *(ii) DA is* stable *while IA and IA^+ are not. (iii) IA^+ may select a stable match when IA does not and, conversely, IA may select a stable match when IA^+ does not.*

In our context, *individual rationality* and *non-wastefulness* are unobjectionable; violation of either property would exclude a rule from serious consideration. In contrast, concern about priority violations is less well founded. As argued earlier, the typical motivation as "justified envy" gives undue attention to administrative priorities. Again, however, our criticism is model-specific: If schools are active agents with preferences rather than priorities, then *stability* retains its full appeal and deserves top billing.

Our properties so far apply to static environments. Yet students arrive and depart, schools open and close, and capacities change, and rule should respond appropriately to these changes. When the environment changes, solidarity is the appropriate equity notion: Students should be affected in the same direction so all gain or all lose. First consider the departure of a student. In this case, there is less competition for existing seats and it is natural to expect the remaining students to be better off. Reasoning similarly, if new seats become available at existing schools, we may expect that students will be better off. Formally, we ask that a rule satisfy[24]:

Population monotonicity: For each $e \in \mathcal{E}$, each $i \in N$, and each $j \in N \backslash \{i\}$, $\varphi_j(N \backslash \{i\}, A, q, R_{-i}, \succ) \; R_j \; \varphi_j(e)$.

Capacity monotonicity: For each $e \in \mathcal{E}$, each $q' \in \mathbf{Z}^A_+$, and each $i \in N$, if $q' \leq q$, then $\varphi_i(e) \; R_i \; \varphi_i(N, A, q', R, \succ)$.

Importantly, *capacity monotonicity* considers only changes in the capacity vector and not the set of schools[25]. Unfortunately, unlike our other rules, IA^+ satisfies neither of these

[24] *Capacity monotonicity* is a form of *resource monotonicity* and sometimes introduced by this name. Alternate versions of these axioms model "departure" by raising the empty-set to the top of a students' preference ranking (e.g., Kojima and Manea (2010); Afacan (2013)). These formulations are connected to ours by *uninterested student invariance*, which we introduce in the next section.

[25] *Capacity monotonicity* can be given a strategic interpretation (Kesten, 2012): If schools have (unmodeled) preferences over students and report capacities, violations of *capacity monotonicity* translate into opportunities for schools to manipulate by misreporting their capacities. In this richer setting, IA is immune to such manipulation while both DA and IA^+ are manipulable via capacities (Kesten, 2012). The manipulability of DA shows that *capacity monotonicity* is not sufficient for non-manipulability.

properties.

Proposition 3. *Both DA and IA are* population monotonic *and* capacity monotonic *while* IA^+ *satisfies neither property.*

The reason IA^+ may violate these properties is that schools may reach their capacities at later steps in the algorithm after students leave or capacities increase. Consequently, students "skip" fewer applications. Additional applications in the new problem may delay the student application to her original school, at which point it may already be filled.

3.3 Robustness

When students depart or new seats become available at schools, we expect assignments to change. In contrast, some changes to the environment are arguably irrelevant to the assignment problem, and a desirable rule should be robust to these changes.

One possibility is to "pre-assign" some students to the schools they will ultimately attend. Intuitively, doing so should not affect the assignments of other students. This motivates *pre-assignment invariance.*[26] An even milder requirement, *uninterested student invariance*, limits the conclusion to situations when the pre-assigned students are unassigned in the original problem. Similar reasoning applies to schools. If a school has no available seats, *unavailable school invariance* requires that the assignments be unchanged when that school is removed[27]. Another common invariance property applies when a student's preferences change: If that student's assignment is unchanged, *non-bossiness* requires that no assignment change. Formally,

Pre-assignment invariance: For each $e \in \mathcal{E}$, each $i \in N$, and each $j \in N \backslash \{i\}$, if $a = \varphi_i(e)$, then $\varphi(N\backslash\{i\}, A, (q_a - 1, q_{-a}), R_{-i}, \succ) = \varphi_{N\backslash\{i\}}(e)$.

Unassigned student invariance: For each $e \in \mathcal{E}$ and each $i \in N$, if $\varphi_i(e) = \emptyset$, then $\varphi(N\backslash\{i\}, A, q, R_{-i}, \succ) = \varphi_{N\backslash\{i\}}(e)$.

Unavailable school invariance: For each $e \in \mathcal{E}$ and each $a \in A$, if $q_a = 0$, then $\varphi(N, A\backslash\{a\}, q_{-a}, R, \succ_{-a}) = \varphi(e)$.

[26]This property adapts *consistency* to our environment. Kojima and Ünver (2014)) propose a related notation applicable to fixed populations: For each $j \in N\backslash\{i\}$, $\varphi_j(N, A, (q_a - 1, q_{-a}), (R^{\emptyset}, R_{-i}), \succ) = \varphi_j(e)$. *Consistency* implies this property. For a comprehensive treatment of *consistency*, see Thomson (2009).

[27]Ehlers and Klaus (2012) formulate a version of this axiom which requires invariance when student preferences are restricted to schools with available seats.

Non-bossiness: For each $e \in \mathcal{E}$, each $i \in N$, and each $R'_i \in \mathcal{R}$, if $\varphi_i(e) = \varphi_i(N, A, q, (R'_i, R_{-i}), \succ)$, then $\varphi(e) = \varphi(N, A, q, (R'_i, R_{-i}), \succ)$.

Unfortunately, none of our rules satisfies all of these properties. IA^+ compares favorably on two counts: First, it satisfies *unassigned student invariance* while DA does not; and second, it satisfies *unavailable school invariance* while IA does not. On the other hand, among our rules, only IA is *pre-assignment invariant* or *non-bossy*.

Proposition 4. *(i) Both IA and IA^+ satisfy* unassigned student invariance *while DA does not. (ii) Both IA^+ and DA satisfy* unavailable school invariance *while IA does not. (iii) IA satisfies* pre-assignment invariance *while IA^+ and DA do not. (iv) IA is* non-bossy *while IA^+ and DA are* bossy.

In a sense, IA^+ modifies IA precisely to achieve *unavailable school invariance*. Of course, this improvement comes at the cost of other invariance properties. Although the examples differ, IA^+ is *bossy* and sensitive to pre-assignment for essentially the same reason that it violates our solidarity axioms. For example, if a student moves her assigned school to the top of her ranking, that school may reach capacity in an earlier round and cause other students to "skip" their applications to that school, leading to different assignments for those students.

4 Incentives

4.1 Manipulation outcomes and relative manipulability

Since rules rely on preferences reported by students, we must consider their incentives to report truthfully. A rule is *strategy-proof* if no student can benefit by misrepresenting her preferences and is *group strategy-proof* if no group of students can so benefit. Formally,

Strategy-proofness: For each $e \in \mathcal{E}$, each $i \in N$, and each $R'_i \in \mathcal{R}$, $\varphi_i(e) \; R_i \; \varphi_i(N, A, q, (R'_i, R_{-i}), \succ)$.

Group strategy-proofness: For each $e \in \mathcal{E}$, each $S \subseteq N$, and each $R'_S \in \mathcal{R}^S$, if there is $i \in S$ such that $\varphi_i(N, A, q, (R'_S, R_{-S}), \succ) \; P_i \; \varphi_i(e)$, then there is $j \in S$ such that $\varphi_j \; P_j \; \varphi_j(N, A, q, (R'_S, R_{-S}), \succ)$.

DA is *strategy-proof* (Dubins and Freedman, 1981; Roth, 1982) whereas IA and IA^+ are not (Roth and Sotomayor, 1990; Abdulkadiroğlu and Sönmez, 2003). None is *group strategy-proof*.

What outcomes can we expect when students optimally manipulate? Since DA is *strategy-proof*, it has a unique dominant strategy Nash equilibrium whose outcome is the student-optimal stable assignment (Gale and Shapley, 1962). Interestingly, the DA assignment is always a Nash equilibrium outcome under both IA and IA^+. In fact, the full range of possibilities is easy to describe: For each rule, the set of Nash equilibrium outcomes is precisely the set of stable assignments (Alcalde, 1996; Ergin and Sönmez, 2006).

Although IA and IA^+ are both manipulable, we can nevertheless compare them. According to the IA^+ algorithm, students "skip" unavailable schools. In a sense, the IA^+ algorithm modifies the IA algorithm by carrying out some manipulations for students. As a result, we expect that IA^+ will be less manipulable that IA. This intuition is correct. Formally, φ is **more manipulable** than φ' if φ is manipulable whenever φ' is manipulable and is also manipulable in at least one problem where φ' is not manipulable[28]. We might further expect a stronger comparison to hold: If a student is able to manipulate IA^+, then the same student is able to manipulate IA. Surprisingly, this intuition is incorrect[29].

Proposition 5. *(i) IA is more manipulable than IA^+, which is more manipulable than DA. (ii) There exist problems in which a student can manipulate IA^+ but cannot manipulate IA.*

4.2 Restricted domains

To better understand our rules, we look for conditions on capacities and priorities under which they are *strategy-proof* or *group strategy-proof*. First, for IA and IA^+, *group strategy-proofness* is no more restrictive than *strategy-proofness*.

Lemma 1. *For a given problem, IA or IA^+ is manipulable by a group if and only if is manipulable by an individual.*

Since IA is *non-bossy*, the result is immediate because *group strategy-proofness* is equivalent to *strategy-proofness* and *non-bossiness*[30]. For IA^+, which is *bossy*, the result is more surprising and requires proof. In contrast, for DA, the gap between *strategy-proof* and *group strategy-proofness* is quite large. For example, DA is manipulable by a group whenever it selects an inefficient assignment.

Second, for both IA and IA^+, *stability* is a prerequisite for non-manipulability: If either rule selects an unstable assignment, then it is manipulable by each student who is part of

[28]Pathak and Sönmez (2013) introduce this concept of relative manipulability.

[29]In the language of Pathak and Sönmez (2013), IA is not **more intensely and strongly manipulable** than IA^+.

[30]This equivalence has been observed in other contexts as well (Barberà and Jackson, 1995; Pápai, 2000).

a blocking pair. Since IA and IA^+ are *efficient* and DA selects the student-optimal stable assignment, these rules are non-manipulable only when they select the DA assignment. However, the converse is false: There exist problems in which IA and IA^+ coincide with DA yet are manipulable (see Example 2).

Building on these observations, we can describe domains on which our rules have desirable incentive properties. Some conditions are known. First, DA is *group strategy-proof* on the domain of problems with "acyclic" priority-capacity structures (Ergin, 2002). Second, IA is *strategy-proof* on the domain of problems in which the combined capacity of the smallest two schools can accommodate all students (Kumano, 2013; Chen, 2014). The condition for IA^+ is similar, but slightly less rigid since it cannot be violated by unavailable schools. Although stated for *strategy-proofness*, by Lemma 1, the results for IA and IA^+ immediately extend to *group strategy-proofness.* Moreover, these domains are maximal: Each result fails on any enlargement of the domain.

Proposition 6. *Each restriction specifies a maximal domain on which the named rule is* group strategy-proof*:*
(i) (Ergin, 2002) DA: Acyclic priority-capacity structures.
(ii) (Chen, 2014) IA: The number of students does not exceed the combined capacity of the smallest two schools.
(iii) IA^+: The number of students does not exceed the combined capacity of the smallest two schools with available seats.

Both conditions (ii) and (iii) imply acyclicity, so the conditions for IA and IA^+ are more restrictive than for DA. The difference between conditions (ii) and (iii) concerns unavailable schools. Under IA, students apply to these schools, so these schools must be considered when comparing capacities of pairs of schools. Since IA^+ ignores unavailable schools, the capacity restrictions apply only to schools with available seats. The difference is small, but may be relevant in practice. For example, if the opening of a new school is delayed, IA may penalize students who rank the new school, thereby encouraging misrepresentation. This possibility does not arise with IA^+ or DA.

5 Conclusion

Complementing recent studies that aim to improve school choice rules, we provide a systematic analysis of immediate-acceptance-with-skips. By modifying the immediate acceptance algorithm, IA^+ moves part way from IA toward DA, and the properties of the rule reflect

	IA	IA^+	DA	TTC	Priority family
non-wastefulness	+	+	+	+	+
efficiency	+	+	−	+	+
rank efficiency	−	−	−	−	−
indiv. rationality	+	+	+	+	+
stability	−	−	+	−	−
mutual best	+	+	+	+	−
favor higher ranks	+	−	−	−	−
unavail. school inv.	−	+	+	+	+
capacity monotonicity	+	−	+	−	+
uninterest. student inv.	+	+	+	+	+
unassign. student inv.	+	+	−	+	+
population monotonicity	+	−	+	−	+
consistency	+	−	−	−	+
improvement inv.	+	+	+	+	+
weak monotonicity	+	+	+	+	+
non-bossiness	+	−	−	+	+
strategy-proof.	−	−	+	+	+
group strategy-proof.	−	−	−	+	+

Table 1: **Properties satisfied by leading rules.** We include top-trading cycles (TTC) and additional axioms for completeness.

this compromise. Table 5 summarizes our results.[31] Moving from IA to IA^+ leads to better incentive properties, both when considering manipulations by individuals and manipulations by groups. It also responds more reasonably to the removal of an unavailable school. Moving from DA to IA^+ improves efficiency and eliminates undesirable sensitivity to the removal of unassigned students. However, these benefits come at a cost: Unlike IA, IA^+ violates natural solidarity and invariance properties; unlike DA, IA^+ is manipulable and allows justified envy. Our analysis demonstrates once again the inevitability of difficult trade-offs. Although the properties of benchmark rules are well understood, much work remains to delineate the properties of compromise methods.

[31]For comparison, we also include properties of other rules discussed in recent characterizations. For DA, see Kojima and Manea (2010), Morrill (2013a), and Ehlers and Klaus (2012). For TTC, see Abdulkadiroğlu and Che (2010), Morrill (2013b), and Dur (2012). For IA, see Kojima and Ünver (2014), Afacan (2013), and Chen (2012).

A Omitted proofs

This appendix collects proofs and examples omitted from the text. For ease of presentation, we organize the proofs according to the section in which they appear.

A.1 Efficiency, equity, and robustness

A.1.1 Proof of Proposition 1

Proof. (i) It is well-known that IA is *efficient* and that DA is not, so we consider IA^+. When applied to a given problem, in each round of the algorithm unassigned students apply to their most preferred school among those with remaining capacity. Therefore, to make a student better off, she must be assigned to a school whose capacity is exhausted in a round earlier than the round in which she is assigned. Since students are only rejected when a school reaches its capacity, such an assignment is possible only if another student is assigned to a school in a later round than initially. But then this student is worse off. Thus, no Pareto improvement is possible.

(ii) We construct a problem in which both IA and IA^+ select a match that Pareto dominates the DA match. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3\}$, $A \equiv \{a, b, c\}$, and $(q, R, \succ)$ as specified in the table:

			(1)	(1)	(1)
R_1	R_2	R_3	$\succ_a$	$\succ_b$	$\succ_c$
b	a	a	1	2	3
a	b	b	2	3	1
c	c	c	3	1	2

The assignments are

$$\begin{aligned} IA(e) &= \big((1, b), (2, a), (3, c)\big), \\ IA^+(e) &= \big((1, b), (2, a), (3, c)\big), \text{ and} \\ DA(e) &= \big((1, a), (2, b), (3, c)\big). \end{aligned}$$

Students 1 and 2 prefer their assignments under $IA(e) = IA^+(e)$ while student 3 receives the same assignment, so $IA(e) = IA^+(e)$ Pareto dominates $DA(e)$.

(iii) *Rank efficiency* implies *efficiency* (Featherstone, 2013) and DA is not *efficient*, so it is not *rank efficient*. To show that neither of the other rules are *rank efficient*, we construct a problem in which both IA and IA^{+} select rank dominated matches. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3,4\}$, $A \equiv \{a,b,c,d\}$, and $(q, R, \succ)$ as specified in the table:

				(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
a	a	c	c	1	1	1	1
b	c	d	b	2	2	2	2
$\vdots$	b	$\vdots$	d	3	3	3	3
	d		a	4	4	4	4

The assignments are

$$\begin{aligned} IA(e) &= \big((1,a),(2,d),(3,c),(4,b)\big) \text{ and} \\ IA^{+}(e) &= \big((1,a),(2,b),(3,c),(4,d)\big). \end{aligned}$$

However, both matches are rank dominated by $\mu \equiv \big((1,b),(2,a),(3,d),(4,c)\big)$: The rank distributions are $N(IA(e)) = (2,1,0,1)$, $N(IA^{+}(e)) = (2,0,2,0)$, and $N(\mu) = (2,2,0,0)$, so $N(\mu)$ sd-dominates both $N(IA(e))$ and $N(IA^{+}(e))$.

(iv) First we show that IA and DA may rank dominate IA^{+}. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3,4,5\}$, $A \equiv \{a,b,c,d,e\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
a	a	a	e	e	1	2	3	4	5
$\vdots$	b	b	a	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	c	c						
		e	d						
		d	b						

The assignments are

$$\begin{aligned} IA(e) &= \big((1,a),(2,b),(3,c),(4,d),(5,e)\big) \text{ and} \\ IA^{+}(e) &= \big((1,a),(2,b),(3,d),(4,c),(5,e)\big). \end{aligned}$$

The rank distributions are $N(IA(e)) = N(DA(e)) = (2,1,1,1,0)$ and $N(IA^{+}(e)) = (2,1,1,0,1)$, so $N(IA(e))$ and $N(DA(e))$ sd-dominate $N(IA^{+}(e))$.

Next we show that IA^+ may rank dominate IA. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4, 5\}$, $A \equiv \{a, b, c, d, e\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
a	a	a	d	d	1	2	3	4	5
$\vdots$	b	b	$\vdots$	a	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	c		c					
		e		b					
		d		e					

The assignments are

$$IA(e) = \big((1, a), (2, b), (3, c), (4, d), (5, e)\big) \text{ and}$$
$$IA^+(e) = \big((1, a), (2, b), (3, e), (4, d), (5, c)\big).$$

The rank distributions are $N(IA(e)) = (2, 1, 1, 0, 1)$ and $N(IA(e)) = (2, 1, 1, 1, 0)$, so $N(IA^+(e))$ sd-dominates $N(IA(e))$.

(v) Let $e \in \mathcal{E}$ and suppose that $N(DA(e))$ sd-dominates $N(IA(e))$. We show that $DA(e) = IA(e)$. In the first round of each algorithm, each student applies to her first choice school. Consequently, the same applications and rejections occur in the first round. Each student accepted in the first round under IA is permanently assigned to her first choice and the set of students finally assigned to their first choice schools is a subset of these students. By assumption, $N_1(DA(e)) \geq N_1(IA(e))$, so these students are never rejected in later rounds under DA. That is, IA and DA assign the same set of students to their first choice schools. Moreover, the set of students rejected in the first round is the same for both algorithms.

In the second round of each algorithm, each rejected student applies to her second choice school. Since these are the same sets of students, the same applications occur. Also, since no student tentatively assigned by DA in the first round is subsequently rejected, the same rejections occur. Reasoning as before, $N_2(DA(e)) = N_2(IA(e))$ and the same set of students are IA and DA assign the same set of students to their second choice schools. Repeating this argument for each $k = 2, \ldots, |A|$, we conclude that $DA(e) = IA(e)$. $\square$

A.1.2 Proof of Proposition 2

Proof. It is well known that DA is *stable* (Roth and Sotomayor, 1990). To complete the proof, we provide examples showing that IA^+ may select a *stable* assignment when IA does not and vice-versa.

IA^+ selects a stable assignment and IA does not. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3,4\}$, $A \equiv \{a,b,c,d\}$, and $(q, R, \succ)$ as specified in the table:

				(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
a	b	a	b	1	2	3	4
$\vdots$	$\vdots$	b	c	$\vdots$	$\vdots$	$\vdots$	$\vdots$
		c	d				
		d	a				

The assignments are

$$IA(e) = \big((1,a),(2,b),(3,d),(4,c)\big) \text{ and}$$
$$IA^+(e) = \big((1,a),(2,b),(3,c),(4,d)\big).$$

In this problem, $IA^+(e)$ is *stable* because it matches each student with a school at which she has first priority. However, $IA(e)$ is not *stable* because $(3,c)$ is a blocking pair: $c\ P_3\ d$ and $3 \succ_c 4$ while $IA_3(e) = d$ and $IA_4(e) = c$.

IA selects a stable assignment and IA^+ does not. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3,4,5\}$, $A \equiv \{a,b,c,d,e\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
a	a	a	e	e	1	2	3	4	5
$\vdots$	b	b	a	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	c	c						
		d	d						
		e	b						

The assignments are

$$IA(e) = \big((1,a),(2,b),(3,c),(4,d),(5,e)\big) \text{ and } IA^+(e) \ = \big((1,a),(2,b),(3,d),(4,c),(5,e)\big).$$

In this problem, $IA(e)$ is *stable* because it matches each student with a school at which she has first priority. However, $IA^+(e)$ is not *stable* because $(3, c)$ is a blocking pair: $c\ P_3\ d$ and $3 \succ_c 4$ while $IA_3(e) = d$ and $IA_4(e) = c$. $\square$

A.1.3 Proof of Proposition 3

Proof. Both DA and IA satisfy a version of *population monotonicity* in which departures are modeled by moving the emptyset to the top of a student's preference relation (Kojima and Manea, 2010; Kojima and Ünver, 2014), and together with *uninterested student invariance*, this version implies *population monotonicity*. Since DA and IA also satisfy *uninterested student invariance*, they are *population monotonic*. Furthermore, both rules are known to satisfy *capacity monotonicity* (Kojima and Manea, 2010; Kojima and Ünver, 2014). We provide examples showing that IA^+ satisfies neither property.

***IA*⁺ is not population monotonic.** Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4, 5\}$, $A \equiv \{a, b, c, d, e\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
a	a	a	a	b	1	2	3	4	$\vdots$
$\vdots$	b	b	c	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	c	c	d						
	d	d	b						
	e	e	e						

Then $IA^+(e) = \big((1, a), (2, e), (3, c), (4, d), (5, b)\big)$. Now suppose student 5 leaves. Let $e' \equiv (N \backslash \{5\}, A, q, R_{-5}, \succ)$. Then $IA^+(e') = \big((1, a), (2, b), (3, d), (4, c)\big)$. Since $IA^+_3(e) = c\ P_3\ d = IA^+_3(e')$, this violates *population monotonicity*.

***IA*⁺ is not capacity monotonic.** Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4, 5\}$, $A \equiv \{a, b, c, d\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(2)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
a	b	a	a	a	1	2	3	$\vdots$
$\vdots$	$\vdots$	b	c	b	$\vdots$	5	4	
		c	d	d		3	$\vdots$	
		d	b	c		$\vdots$		

Then $IA^+(e) = \big((1,a),(2,b),(3,c),(4,d),(5,d)\big)$. Now suppose an extra seat becomes available at b so $q' \equiv (1,2,1,2)$ and $e' \equiv (N, A, q', R, \succ)$. In the new economy, $IA^+(e') = \big((1,a),(2,b),(3,d),(4,c),(5,b)\big)$. Since $IA_3^+(e) = c \; P_3 \; d = IA_3^+(e')$ while $IA_4^+(e') = c \; P_4 \; d = IA_4^+(e)$, this violates *resource monotonicity*. □

A.1.4 Proof of Proposition 4

Proof. (i) **Pre-assignment invariance**

IA satisfies a limited version of *pre-assignment invariance* in which instead of removing pre-assigned students their preferences are replaced with the emptyset moved to the top (Kojima and Ünver, 2014). Since IA is invariant when such students are removed from the problem, it satisfies *pre-assignment invariance*.

We show by example in (ii) below that DA may violate *unassigned student invariance*. Since *pre-assignment invariance* implies *unassigned student invariance*, DA also violates *pre-assignment invariance*.

We show by example that IA^+ may violate the property. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3\}$, $A \equiv \{a,b,c\}$, and $(q, R, \succ)$ as specified in the table:

			(1)	(1)	(1)
R_1	R_2	R_3	$\succ_a$	$\succ_b$	$\succ_c$
a	a	b	1	2	$\vdots$
$\vdots$	b	c	$\vdots$	$\vdots$	
	c	a			

Then $IA^+(e) = \big((1,a),(2,c),(3,b)\big)$. Now fix student 1's assignment. Let $q' \equiv (0,1,1)$ and $e' \equiv (N \backslash \{1\}, A, q', R_{-1}, \succ)$. Then $IA^+(e') = \big((2,b),(3,c)\big)$. Since $IA^+(e') \neq IA^+_{-1}(e)$, this violates *pre-assignment invariance*.

(ii) **Unassigned student invariance**

According to the IA and IA^+ algorithms, an unassigned student is rejected by each school to which she applies. Therefore, removing the student does not affect the pattern of applications or acceptances and rejections for other students, so IA and IA^+ satisfy *unassigned student invariance*.

We show by example that DA may violate the property. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2,3\}$, $A \equiv \{a,b\}$, and $(q, R, \succ)$ as specified in the table:

			(1)	(1)
R_1	R_2	R_3	$\succ_a$	$\succ_b$
a	a	b	3	2
$\emptyset$	b	a	1	$\vdots$
			2	

Then $DA(e) = \big((1,\emptyset),(2,b),(3,a)\big)$. Now remove student 1 who is unassigned. Let $e' \equiv (N\backslash\{1\}, A, q, R_{-1}, \succ)$. Then $DA(e') = \big((2,a),(3,b)\big)$. Since $DA(e') \neq DA_{-1}(e)$, this violates *unassigned student invariance.*

(iii) **Unavailable school invariance**

According to the IA^+ algorithm, no student ever applies to an unavailable school. Removing the school does not affect the pattern of applications or acceptances and rejections, so IA^+ satisfies *unavailable school invariance.* According to the DA algorithm, an unavailable school never tentatively accepts any students. Removing the school does not change the pattern of tentative acceptances or final assignments, so DA also satisfies *unavailable school invariance.*

We show by example that IA may violate the property. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1,2\}$, $A \equiv \{a,b\}$, and $(q, R, \succ)$ as specified in the table:

		(0)	(1)
R_1	R_2	$\succ_a$	$\succ_b$
a	b	$\vdots$	1
b	a		2

Then $IA(e) = \big((1,\emptyset),(2,a)\big)$. Now remove the unavailable school a. Let $e' \equiv (N, A\backslash\{a\}, q_{-a}, R|_{A\backslash\{a\}}, \succ_{-a})$. In the new problem, $IA(e') = \big((1,a),(2,\emptyset)\big)$. Since $IA^+(e') \neq IA^+(e)$, this violates *unavailable school invariance.*

(iv) **Non-bossiness**

To see that IA is *non-bossy,* let $e \in \mathcal{E}$, $i \in N$, $R'_i \in \mathcal{R}$, and $e' \equiv (N, A, q, (R'_i, R_{-i}), \succ)$. If $IA(e) = IA(e')$, there is nothing to show, so suppose not and let k be the first round in which the assignments made by the IA algorithm in e and e' differ. Suppose there is $j \in N\backslash\{i\}$ whose assignment changes in this round and let a be the school student j applies to in round k. Then either $IA_j(e) = a \neq IA_j(e')$ or $IA_j(e) \neq a = IA_j(e')$. Without loss of generality, suppose $IA_j(e) = a \neq IA_j(e')$. This requires that student i applies to and is accepted by a in round k in e' and that student i does not apply to a in round k in e.

Furthermore, a reaches capacity in round k in both e and e'. Therefore, $IA_i(e') = a \neq IA_i(e)$ and student i's assignment also changes.

It is well-known that DA is *bossy* (Ergin, 2002). We show by example that IA^+ is *bossy* as well. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4, 5\}$, $A \equiv \{a, b, c, d, e\}$, and $(q, R, \succ)$ as specified in the table:

					(1)	(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	R_5	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
a	a	e	e	e	1	2	3	4	5
$\vdots$	b	b	c	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	c	$\vdots$						
		$\vdots$							

Then $IA^+(e) = \big((1, a), (2, b), (3, d), (4, c), (5, e)\big)$. Now suppose student 2 reports R'_2 which ranks b first. Let $e' \equiv (N, A, q, (R'_2, R_{-2}), \succ)$. In the new problem, $IA^+(e') = \big((1, a), (2, b), (3, c), (4, d), (5, e)\big)$. Since $IA^+_2(e') = IA^+_2(e)$ but $IA^+(e') \neq IA^+(e)$, this violates *non-bossiness.* $\square$

A.2 Incentives

A.2.1 Proof of Proposition 5

Proof. (i) Since DA is *strategy-proof*, both IA and IA^+ are more manipulable than DA. It remains to compare IA and IA^+.

Suppose that IA^+ is manipulable at $e \equiv (N, A, q, R, \succ)$. Then there is $i \in N$ and $R'_i \in \mathcal{R}$ such that $IA^+_i(N, A, q, (R'_i, R_{-i}), \succ)\ P_i\ IA^+_i(e)$. Let $e' \equiv (N, A, q, (R'_i, R_{-i}), \succ)$, $a \equiv IA^+_i(e')$, and $b \equiv IA^+_i(e)$ so $a\ P_i\ b$. Without loss of generality, we may assume that R'_i ranks a first. Also, we may assume that a is the best attainable school for student i given R_{-i}: For each $\tilde{R}_i \in \mathcal{R}$, $a\ R_i\ IA(N, A, q, (\tilde{R}_i, R_{-i}), \succ)$. Let $c \equiv IA_i(e)$, $R''_i \in \mathcal{R}$ rank c first, $R'' \equiv (R''_i, R_{-i})$, and $e'' \equiv (N, A, q, R'', \succ)$. Then $IA_i(e'') = c$ as well. Moreover, since IA^+ and IA form the same matches in the first step,

$$
\begin{aligned}
a &= IA^+_i(e) = IA^+_i(e') = IA_i(e') \text{ and} \\
c &= IA_i(e) = IA_i(e'') = IA^+_i(e'').
\end{aligned}
$$

By the choice of a, since student i can obtain c under IA^+ given R_{-i}, $a\ R_i\ c$. If $a\ P_i\ c$, then student i can can manipulate IA at e, so suppose $a = c$. Then there is $j \in N \backslash \{i\}$ such that

$IA_j^+(e) = a$ and $IA_j(e) \neq a$. Let $a' \equiv IA_j(e)$. If $a \; P_j \; a'$, then student j can manipulate IA at e. Suppose instead that $a' \; P_j \; a$. Let $R'_j \in \mathcal{R}$ rank a' first and $e''' \equiv (N, A, q, (R'_j, R_{-j}), \succ)$. Then

$$a' = IA_j(e) = IA_j(e''') = IA_j^+(e'''),$$

so student j can manipulate IA^+ at e. Then there is $k \in N \backslash \{i, j\}$ such that $IA_k^+(e) = a'$ and $IA_k(e) \neq a'$. Applying the same arguments again, if student k cannot manipulate IA at e, then student k can manipulate IA^+ at e. Continuing in this fashion, we obtain a sequence of students, each of whom can manipulate IA^+ at e. By finiteness, either at least one student can manipulate IA at e or the sequence contains all students. However, IA^+ is *efficient*, so not all students can manipulate IA^+ at e. Instead, IA is manipulable at e.

Finally, we show by example that IA may be manipulable when IA^+ is not. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3, 4\}$, $A \equiv \{a, b, c, d\}$, and $(q, R, \succ)$ as specified in the table:

				(1)	(1)	(1)	(1)
R_1	R_2	R_3	R_4	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
a	b	a	a	1	2	3	4
$\vdots$	$\vdots$	b	c	$\vdots$	$\vdots$	$\vdots$	$\vdots$
		c	$\vdots$				
		d					

The assignments are

$$IA(e) = \big((1, a), (2, b), (3, d), (4, c)\big) \text{ and } IA^+(e) \qquad = \big((1, a), (2, b), (3, c), (4, d)\big).$$

Student 3 can manipulate IA at $IA(e)$: If student 3 instead reports R'_3 with c ranked first, then $IA_3(N, A, q, (R'_3, R_{-3}), \succ) = c$ and $c \; P_3 \; d$. On the other hand, $IA^+(e)$ is not manipulable. Students 1 and 2 are matched with their most preferred schools and neither student 3 nor 4 can obtain a better school by reporting different preferences.

(ii) We show by example that a student may be able to manipulate IA^+ but unable to manipulate IA. Let $N \equiv \{1, 2, 3, 4, 5, 6\}$, $A \equiv \{a, b, c, d, e, f\}$, and $(q, R, \succ)$ as specified in the table:

R_0	R'_0	(1) $\succ_a$	(1) $\succ_b$	(1) $\succ_c$	(1) $\succ_d$	(1) $\succ_e$	(1) $\succ_f$
a	e	1	6	2	3	5	6
b	a	$\vdots$	2	3	4	$\vdots$	$\vdots$
c	b		$\vdots$	$\vdots$	$\vdots$		
d	f						
e	c						
f	d						

Let $R \equiv (R_0, R_0, R_0, R_0, R'_0, R'_0)$ and $e \equiv (N, A, q, R, \succ)$. The assignments are

$$IA(e) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix} \quad \text{and} \quad IA^+(e) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & c & d & f & e & b \end{pmatrix}.$$

Student 2 can manipulate IA^+ at e: If student 2 reports R'_2 with b ranked first, then $IA_2^+(N, A, q, (R'_2, R_{-2}), \succ) = b$ and $b\ P_2\ c$. However, $IA_2(e) = b$ and student 2 cannot obtain a preferred assignment under IA by reporting different preferences. Thus, while student 2 can manipulate IA^+ at e, the same student cannot manipulate IA at e. □

Example 2. A problem in which IA^+ and IA coincide with DA but are manipulable. Let $e \equiv (N, A, q, R, \succ)$ with $N \equiv \{1, 2, 3\}$, $A \equiv \{a, b, c\}$, and $(q, R, \succ)$ as specified in the table:

R_1	R_2	R_3	(1) $\succ_a$	(1) $\succ_b$	(1) $\succ_c$
a	a	a	1	2	3
b	b	b	$\vdots$	$\vdots$	$\vdots$
c	c	c			

Then $IA(e) = IA^+(e) = DA(e) = \big((1, a), (2, b), (3, c)\big)$. However, student 3 can manipulate both IA and IA^+ at e. Let $R_3 \in \mathcal{R}$ rank b first and $e' \equiv (N, A, q, (R'_3, R_{-3}), \succ)$. As computed in the table below, $IA(e) = IA^+(e) = \big((1, a), (2, c), (3, b)\big)$. Since $b\ P_3\ c$, student 3 benefits from the misrepresentation. In contrast, $DA(e') = DA(e)$.

A.2.2 Proof of Lemma 1

Proof. In our model, *group strategy-proofness* is equivalent to *strategy-proofness* and *non-bossiness*[32]. Since IA is *non-bossy*, it is manipulable by an individual whenever it is manip-

[32] This well-known fact is proven in many contexts, e.g. Pápai (2000).

ulable by a group.

Now consider IA^+. Let $e \in \mathcal{E}$ and suppose $S \subseteq N$ can manipulate IA^+ at e by reporting $R'_S \in \mathcal{R}^S$. Let $e' \equiv (N, A, q, (R'_S, R_{-S}), \succ)$. Then there is $i \in S$ such that $IA^+_i(e')$ P_i $IA^+_i(e)$. Let $a \equiv IA^+_i(e')$ and $R''_i \in \mathcal{R}$ be formed from R'_i by moving a to the top. Also let $\hat{e} \equiv (N, A, q, (R''_i, R_{-i}), \succ)$ and $\hat{e}' \equiv (N, A, q, (R''_i, R'_{S\setminus\{i\}}, R_{-S}), \succ)$. Then $IA^+_i(\hat{e}') = a$. If $IA^+_i(\hat{e}) = a$, then student i can manipulate IA^+ at e.

Suppose instead that $IA^+_i(\hat{e}) \neq a$. When IA^+ is applied to $\hat{e}$, a rejects student i in the first round. Therefore, a is filled to capacity in the first round and for each $j \in N$ such that $IA^+_j(\hat{e}) = a$, $j \succ_a i$. If $j \notin S$, then j also applies to a in the first round when IA^+ is applied to $\hat{e}'$. Since $IA^+_i(\hat{e}') = a$ and $j \succ_a i$, $IA^+_j(\hat{e}') = a$ and $IA^+_j(e') = a$ as well. Instead, there is $i' \in S$ such that $IA^+_{i'}(\hat{e}) = a \neq IA^+_{i'}(\hat{e}')$. But then $R_{i'}$ ranks a first and so i' is worse off at e' than at e which contradicts the assumption that R'_S is a profitable deviation for S. □

A.2.3 Proof of Proposition 6

To formally describe the restricted domains, we introduce additional notation. Let $Q\colon \mathcal{N} \times \mathcal{A} \to \mathbf{Z}^{\mathcal{A}}_+$ and $\pi\colon \mathcal{N} \times \mathcal{A} \to \Pi$ be such that for each $(N, A) \subseteq \mathcal{N} \times \mathcal{A}$, $Q(N, A)|_A \subseteq \mathbf{Z}^A_+$ and $\pi(N, A) \subseteq \Pi|_N$. Let $\boldsymbol{\mathcal{E}(Q, \pi)}$ be the set of problems with capacities and priorities restricted to (Q, π): $\mathcal{E}(Q, \pi) \equiv \{e \in \mathcal{E} : q \in Q(N, A) \text{ and } \succ \in \pi(N, A)\}$. For each $(N, A) \subseteq \mathcal{N} \times \mathcal{A}$, define $\boldsymbol{Q^*}$ and $\boldsymbol{Q^{**}}$ by

$$Q^*(N, A) \equiv \{q \in \mathbf{Z}^{\mathcal{A}}_+ : \forall a, b \in A,\ q_a + q_b \geq |N|\}$$

$$Q^{**}(N, A) \equiv \{q \in \mathbf{Z}^{\mathcal{A}}_+ : \forall a, b \in A,\ q_a, q_b > 0 \Rightarrow q_a + q_b \geq |N|\}.$$

Then $\mathcal{E}(Q^*, \Pi) \subsetneq \mathcal{E}(Q^{**}, \Pi)$. In words, $\mathcal{E}(Q^*, \Pi)$ is the set of problems in which no two schools have combined capacity less than the total number of students and $\mathcal{E}(Q^{**}, \Pi)$ enlarges this set by limiting the restriction to schools with available seats. We now prove Proposition 6.

Proof. (i-ii) DA is known to be *group strategy-proof* on the domain of problems with acyclic priority-capacity structures and is not *group strategy-proof* on any domain that strictly includes this domain (Ergin, 2002). Similarly, IA is *strategy-proof* on $\mathcal{E}(Q^*, \Pi)$ and is not *strategy-proof* on any domain that strictly includes this domain (Chen, 2014). By Lemma 1, this conclusion applies to *group strategy-proofness* as well.

(iii) We show that IA^+ is *group strategy-proof* on $\mathcal{E}(Q^{**}, \Pi)$. Let $e \in \mathcal{E}(Q^{**}, \Pi)$ and $A^* \equiv \{a \in A : q_a > 0\}$. According to the IA^+ algorithm, each student applies to a school in A^* in the first round. Let $i \in N$ and let $a, b \in A^*$ be the schools ranked first and second in R_i. By assumption, $q_a + q_b \geq |N|$. Therefore, if student i is rejected by a in the first round, then she will be accepted by b in the second round. Consequently, student i cannot benefit by misreporting. Similarly, no student accepted in the first round has incentive to misreport, so $IA(e)$ is not manipulable. That is, IA^+ is *strategy-proof* on $\mathcal{E}(Q^{**}, \Pi)$. By Lemma 1, IA^+ is also *group strategy-proof* on $\mathcal{E}(Q^{**}, \Pi)$.

(ii) We show that IA^+ is not *group strategy-proof* on any domain that strictly includes $\mathcal{E}(Q^{**}, \Pi)$. Let (Q, π) be such that $\mathcal{E}(Q, \pi) \nsubseteq \mathcal{E}(Q^{**}, \pi)$ and let $e \in \mathcal{E}(Q, \pi) \backslash \mathcal{E}(Q^{*}, \pi)$. Then there is a pair $a, b \in A$ such that $q_a, q_b > 0$ and $q_a + q_b < |N|$. Let R^{ab} rank a first and b second and let R^{ba} rank b first and a second. Let $i \in N$ be the student with the lowest priority at b. Let $R'_i \equiv R^{ba}$ and for each $j \in N \backslash \{i\}$, let $R'_j = R^{ab}$. Then $e' \equiv (N, A, q, R', \succ) \in \mathcal{E}(Q, \pi)$.

We claim that $IA(e')$ is manipulable. Under IA^+, $|N| - 1$ students apply to a in the first round and $|N| - 1 - q_a$ are rejected. By assumption, $q_a < |N| - q_b$, so $|N| - 1 - q_a \geq q_b \geq 1$. Since i applies to b in the first round, $IA_i(e') = b$. In the second round, the $|N| - 1 - q_a$ unassigned students apply to b. However, only $q_b - 1 < |N| - 1 - q_a$ seats remain so at least one student, say j, is rejected by b. By choice of student i, $j \succ_b i$, so $IA^+(e')$ is not *stable*. Moreover, it is manipulable by student j: If student j reports R^{ba} instead, then student j will be assigned to b in the first round. Therefore, IA^+ is not *strategy-proof* on $\mathcal{E}(Q, \pi)$. $\square$

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