Incentives and Risks in Relationships Between the Principal and the Agent

Minasyan, Vigen

Russian Presidential Academy

May 2014

International Journal of Advanced Multidisciplinary Research and Review
ISSN 2330-1201

International Journal of Advanced Multidisciplinary Research and Review
Volume 2, No.:3, 2014 Summer
Pages: 159 – 181

Incentives and Risks in Relationships Between the Principal and the Agent

Vigen Minasyan[1]

The paper addresses a basic model of moral hazard (risk) [Gibbons, 2010; Gibbons, 2005] and suggests some of its modifications. In the basic model of moral risk, questions are put and examined that have not been considered in the previous researches. In particular, it is proved that the level of agent's efforts that maximizes its expected utility coincides with the level of efforts that minimize the risk of obtaining this maximum utility. Modifications of the moral risk model are considered where the optimal behavior of the principal and the agent considerably differ from the respective behavior in the moral risk model.
The paper introduces moral risk measures VaR for the principal and VaR for the agent that specify the qualitative assessments of risk on the part of the principal and the agent in their relationships.

Keywords: model of moral hazard (risk), expected utility, VaR for the principal, VaR for the agent, measure of the utility risk, lognormally distributed random variable.

1. Basic model of moral risk

With the agent not inclined to risk, the principal's (employer's) choice of the incentives' force is defined by a tradeoff between the incentives and the insurance.

[1] Professor, PhD, Head of Corporate Finance, Investment Decisions and Valuation Chair Russian Presidential Academy and Public Administration, Certified International Investment Analyst (CIIA) / Russian Federation - E-mail: minasyanvb@yandex.ru

The basic model of moral risk considers interaction between the principal and the agent (employee). The agent makes unobservable (hence uncontrollable) by the principal effort a aimed at obtaining result y (which is usually considered as gain). Obtaining of this result depends not only on the agent's efforts, but also on the influence of random factors leading to uncertainty of the result. The realized value y is the value observable by the principal and is a basis for building an incentive contract from the principal to the agent.

Relationships between the principal and the agent are built in the following sequence (Gibbons R., 2010; Gibbons R., 2005).

1. The principal and the agent conclude a contract $w(y)$ that fixes the pattern and value of remuneration.
2. The agent chooses an action, with real influence on the result of size a, but the principal has no information about the choice made by the agent (i.e. he "observes" neither the actual choice of the agent nor its result a).
3. Some random events take place that lead to a random contribution to the result, of value ε, not controllable by the agent.
4. As a result of the agent's actions a and a random contribution to result ε, the value of result (production function) y is defined.
5. The agent receives a remuneration stipulated in the contract.

The basic model of moral risk also makes the following additional suggestions (Gibbons R., 2010; Gibbons R., 2005).

- The production function is linear: $y = a + \varepsilon$, where ε - normally distributed random value with a zero mean and variance σ^2.
- The incentive contract is also linear: $w(y) = s + by$, where s corresponds to the value of the fixed remuneration, and coefficient b corresponds to the force of the set incentives.
- The agent has a constant absolute disinclination to risk, i.e. his utility function looks like $u_A(x) = -e^{-R_A x}$, where x is a value of the agent's net gain, $R_A > 0$ is a constant coefficient of the agent's absolute disinclination to risk.

- Net (monetary) gain of the agent is equal to the difference between the obtained remuneration and the subjective monetary valuation (on the part of the agent) of the costs of making efforts $x = w - c(a)$, where $c(a)$ is a convex function.
- The principal is neutral to the risk and hence seeks only maximization of the expected value of his own return, $E\Pi = E(y - w)$.

The agent can maximize the expected utility for himself with the help of choosing effort a. I.e. his choice corresponds to the solution of the following optimization problem: to determine value $a = a^*$, at which $\max_a E(u_A(a))$ is reached.

As is known (Gibbons R., 2010; Gibbons R., 2005), the optimal level of the agent's efforts represented as $a^*(b)$ is the solution of equation $c'(a) = b$, and the certainty equivalent (agent's gain) CE with efforts' level $a^*(b)$ is:

$$CE(s,b) = s + ba^*(b) - c[a^*(b)] - \frac{1}{2} R_A b^2 \sigma^2.$$

The expected benefit of the principal with such a choice of the agent is:

$$E\Pi(s,b) = (1-b)a^*(b) - s.$$

The basic model of moral risk suggests that thought the principal's aim is the maximization of his expected benefit, the company considers the aim of maximizing the total gain of the principal and the agent, defined in the form of a sum of a certainty equivalent of the agent and the expected benefit of the principal:

$$CE(s,b) + E\Pi(s,b) = a^*(b) - c[a^*(b)] - \frac{1}{2} R_A b^2 \sigma^2.$$

The company has an opportunity to solve this optimization problem with the help of choosing a parameter of incentives force in contract b. As is known (Gibbons R., 2010; Gibbons R., 2005), the optimal value of the incentives force is determined by formula:

$$b^* = \frac{1}{1 + R_A \sigma^2 c''}.$$

Since R_A, σ^2 and c'' are positive, value b^* will lie between a zero (full insurance for the agent) and a one (the agent receives the entire earning).

Moreover, value b^* is the lesser, the…:

(1) higher is the agent's degree of disinclination to risk R_A;

(2) higher is the degree of uncertainty σ^2;

(3) faster grow the marginal costs of making effort c''.

It is suggested that the principal may be unaware of these values.

It is interesting to note that if the subjective monetary valuation (on the part of the agent) of the cost of making efforts linearly depended on the made effort, i.e. $c(a) = c_0 + ka$, then, first of all, from the condition $c'(a) = b$ it would follow that $k = b$, i.e. $c(a) = c_0 + ba$. Moreover, it turns out that $b^* = 1$, i.e. it is optimal to transfer the entire result to the agent (selling the business to the agent).

2. Additional research of the basic model of moral risk

A) Minimization of the utility risk for the agent

The agent, apart from the intention to maximize the expected utility for himself, may also set other aims. Let us suppose that the agent's disinclination to risk is reflected in the fact that he chooses such efforts that minimize the risk of his utility. As a measure of the utility risk for the agent may be used the variance of his utility:

$$\sigma^2(u_A(x)) = E((u_A(x))^2) - (E(u_A(x)))^2,$$

where $u_A(x) = -e^{-R_A x}$.

The net monetary gain of the agent is

$$x = w - c(a) = s + b(a + \varepsilon) - c(a) = s + ba - c(a) + b\varepsilon,$$

where ε is a normally distributed value with mean value 0 and variance σ^2 (which is usually written as $\varepsilon \in N(0, \sigma^2)$).

This is why the expected value of the agent's net monetary gain is equal to $E(x) = s + ba - c(a)$, and its standard deviation is $\sigma(x) = b\sigma$. From the above and from the form of the utility function for the agent it follows that variable $-u_A(x)$ is a lognormally distributed random value. It follows from the fact that

$$-\frac{\ln(-u_A(x))}{R_A} = x \in N(s+ba-c(a), b\sigma).$$

But then $\ln(-u_A(x)) \in N(-R_A(s+ba-c(a)), R_A b\sigma)$.

However, for any lognormally distributed random value X are known (see, for example, (Ayvazyan S.A., MkhitaryanV.S. 2001)) the formulas for its expected value and variance:

$$E(X) = e^{\mu+\frac{\sigma^2}{2}},\ \sigma^2(X) = (e^{\sigma^2}-1)e^{2\mu+\sigma^2},$$

where μ - mean value of the respective normally distributed value (i.e. ln(X)), and σ^2 - its variance.

Applying these formulas in our case to random variable $-u_A(x)$, we obtain:

$$E(-u_A(x)) = e^{-R_A(s+ba-c(a))+\frac{R_A^2 b^2\sigma^2}{2}},$$

$$\sigma^2(-u_A(x)) = (e^{R_A^2 b^2\sigma^2}-1)e^{-2R_A(s+ba-c(a))+R_A^2 b^2\sigma^2},$$

or

$$E(u_A(x)) = -e^{-R_A[s+ba-c(a))-\frac{R_A b^2\sigma^2}{2}]}, \tag{1}$$

$$\sigma^2(u_A(x)) = e^{-2R_A(s+ba-c(a))}(1-e^{-R_A^2 b^2\sigma^2})e^{2R_A^2 b^2\sigma^2}. \tag{2}$$

Applying the necessary minimum condition to (2), we obtain that an optimal level of the agent's efforts, represented as $a^*(b)$, is the solution of equation $c'(a) = b$, which coincides with the equation that determines the optimal level of the agent's efforts maximizing its expected utility level.

Thus, the following statement is proved.

Statement 1

The level of the agent's efforts that maximizes his expected utility coincides with the level of efforts that minimize the risk of obtaining this maximum utility. And this level of efforts $a^*(b)$ is a solution of equation $c'(a) = b$.

Thus, the agent, maximizing his expected utility, automatically minimizes the risk of failure to achieve this utility.

B) Maximization of the utility of the total result of the principal and the agent and minimization of the risk of this utility

In the basic model of moral risk, the company only considers the aim to maximize the total gain of the principal and the agent presented as a sum of the certainty equivalent of the agent and the expected benefit of the principal.

Let us assume that the agent's interests coincide with the principal and company's interests.

In this case both the company and the agent are interested in the total result of the agent and the principal.

Since for the agent the result is $x = s + by - c(a)$, and for the principal it is $\Pi = y - s - by$, then the total result is

$z = x + \Pi = y - c(a) = a - c(a) + \varepsilon.$

Obviously, without any assumptions of disinclination to risk, the expected total result will be equal to $E(z) = a - c(a)$, and its dispersion will be σ^2.

Thus, the expected total result turns out to be independent of coefficient b – the force of the set incentives.

If the interests of the agent and the company coincide, he will choose the effort that maximizes the total result. It is obvious that in this case the level of the agent's efforts presented as a^* is a solution of equation $c'(a) = 1$ and the risk of result doesn't depend on efforts of the agent.

Now, let us assume for the company some absolute disinclination to risk with a utility function looking like $u_c(z) = -e^{-R_c z}$, where z – the value of net total gain of the company, $R_c > 0$ - the constant coefficient of the company's absolute disinclination to risk.

From the above and from the form of the utility function it follows that variable $-u_c(z)$ is a lognormally distributed random variable. It follows from the fact that

$-\dfrac{\ln(-u_c(z))}{R_c} = z \in N(a - c(a), \sigma^2)$.

But then $\ln(-u_c(z)) \in N(-R_A(a - c(a)), R_c^2\sigma^2)$.

However, for any lognormally distributed value X are known (see, for example, (Ayvazyan S.A., Mkhitaryan V.S. 2001)) the formulas for its expected value and variance:

$$E(X) = e^{\mu + \frac{\sigma^2}{2}},\ \sigma^2(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2},$$

where μ - mean value of the respective normally distributed value (i.e. ln(X)), and σ^2 - its variance.

Applying these formulas in our case to random value $-u_c(z)$, we obtain:

$$E(-u_c(z)) = e^{-R_A(a - c(a)) + \frac{R_c^2\sigma^2}{2}},$$

$$\sigma^2(-u_c(z)) = (e^{R_c^2\sigma^2} - 1)e^{-2R_A(a - c(a)) + R_A^2\sigma^2},$$

or

$$E(u_c(z)) = -e^{-R_c[a - c(a)) - \frac{R_c\sigma^2}{2}]}, \tag{3}$$

$$\sigma^2(u_c(z)) = (e^{R_c^2\sigma^2} - 1)e^{-2R_c(a - c(a)) + R_c^2\sigma^2} \tag{4}$$

If the aim is set to maximize the expected utility for the company, then, again, the required condition for achieving this optimal expected utility is fulfillment of equation $c'(a) = 1$.

Disinclination of the agent (and hence, in this case, of the company as well) to risk is reflected in the fact that he chooses such efforts that minimize the risk of his utility. As a measure of the utility risk for the agent may be used the variance of his utility $\sigma^2(u_c(z))$.

Applying the necessary minimum condition, we obtain that an optimal level of the agent's efforts, represented as a^*, is the solution of equation $c'(a) = 1$, which coincides with the equation that determines the optimal level of the agent's efforts maximizing its expected utility level.

Thus, the following statement is proved.

Statement 2

In case of coincidence of the interests of the agent and the company, the level of the agent's efforts maximizing the expected utility of the total result coincides with the level of efforts minimizing the risk of obtaining this maximum utility. And this level of efforts a^* is a solution of equation $c'(a) = 1$.

Thus, the agent, maximizing his expected utility, automatically minimizes the risk of failure to achieve this utility.

C) The principal and the agent maximize the utility for themselves having agreed upon the monetary valuation of the efforts on the part of the agent

Now let us consider the case when both the principal and the agent, each attempts to maximize the expected utility for himself, having agreed upon the monetary valuation of the efforts on the part of the agent, i.e. in the form of function c(a).

As we have already ascertained, when attempting to maximize the expected utility for himself, the maximally disinclined to risk agent will make effort $a^*(b)$ satisfying equation $c'(a^*(b)) = b$. At the same time, as it was shown above, the agent automatically minimizes the utility risk for himself.

The gain for the principal is $\Pi = y - s - by = a(1-b) - s + \varepsilon(1-b)$,

The expected gain of the principal is equal to $E\Pi(s,b) = (1-b)a^*(b) - s$, and the variance of this gain is equal to $\sigma^2(\Pi) = (1-b)^2\sigma^2$.

If in this case the principal wishes to maximize the expected utility for himself, he will choose an optimal value of incentive force b^* from condition

$(E\Pi)' = -a^*(b) + (1-b)(a^*(b))' = 0.$

However, from $c'(a^*(b)) = b$ it follows that $\mathrm{c}''(\mathrm{a}^*)(a^*)' = 1$, hence $(a^*(b))' = \dfrac{1}{c''}$.

Therefore, the condition of optimality of the incentive force is presented in the form of equation for determining b^*:

$$a^*(b) = (1-b)\frac{1}{c''(b)}.$$

By the way, from this it is seen that if the principal wishes, in the given conditions, to minimize the risk for himself, choosing as a value risk the variance of his gain, he should choose the value of set incentives $b = 1$.

I.e. he should transfer the entire earning to the agent or, in other words, it is more profitable for the principal to sell the company to the agent.

Now, let us assume for the principal some absolute disinclination to risk with a utility function looking like $u_\Pi(\Pi) = -e^{-R_\Pi \Pi}$, where Π - the value of net total gain of the principal, $R_\Pi > 0$ - the constant coefficient of the principal's absolute disinclination to risk.

From the above and from the form of the utility function for the principal it follows that variable $-u_\Pi(\Pi)$ is a lognormally distributed value. It follows from the fact that

$$-\frac{\ln(-u_\Pi(\Pi))}{R_\Pi} = \Pi \in N(a^*(1-b) - s, (1-b)^2\sigma^2).$$

But then $\ln(-u_\Pi(\Pi)) \in N(-R_A(a^*(1-b) - s), R_\Pi^2(1-b)^2\sigma^2)$.

However, for any lognormally distributed value X are known (see, for example, (Ayvazyan S.A., Mkhitaryan V.S. 2001)) the formulas for its expected value and dispersion:

$$E(X) = e^{\mu + \frac{\sigma^2}{2}},\ \sigma^2(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2},$$

where μ - mean value of the respective normally distributed value (i.e. ln(X)), and σ^2 - its dispersion.

Applying these formulas in our case to random value $-u_\Pi(\Pi)$, we obtain:

$$E(-u_\Pi(\Pi)) = e^{-R_\Pi(a^*(1-b)-s) + \frac{R_\Pi^2(1-b)^2\sigma^2}{2}},$$

$$\sigma^2(-u_\Pi(\Pi)) = (e^{R_\Pi^2(1-b)^2\sigma^2} - 1)e^{-2R_\Pi(a^*(1-b)-s) + R_\Pi^2(1-b)^2\sigma^2},$$

or

$$E(u_\Pi(\Pi)) = -e^{-R_\Pi[a^*(1-b) - s - \frac{R_\Pi(1-b)^2\sigma^2}{2}]}, \tag{5}$$

$$\sigma^2(u_\Pi(\Pi)) = (e^{R_\Pi^2(1-b)^2\sigma^2} - 1)e^{-R_\Pi[2(a^*(1-b)-s) - R_\Pi(1-b)^2\sigma^2]} \tag{6}$$

If the aim is set to maximize the expected utility for the principal, then the required condition for achieving this optimal expected return is fulfillment of equation

$$-a^*(b)+(a^*)'(1-b)+R_\Pi(1-b)\sigma^2=0.$$

Since in our case

$(a^*(b))'=\dfrac{1}{c''(b)}$ we receive that

$$a^*(b)=\frac{1}{c''(b)}(1-b)+R_\Pi(1-b)\sigma^2.$$

The solution of this equation b^* will in these conditions be an optimal for the principal, from the point of view of maximizing the expected utility for himself, value of the incentive force.

D) Determining the optimal level of the agent's efforts for particular kinds of subjective monetary valuation of the costs of making efforts

1. Let us suggest that the function of subjective monetary valuation of the costs of making efforts is linear: $c(a)=c_0+c_1a$.

A) If the interests of the agent and the principal do not coincide, the necessary condition of optimality of the agent's actions looks like $c'(a)=b$, from which it follows that $c_1=b$ and the function of subjective monetary valuation of making efforts looks like $c(a)=c_0+ba$.

With such a function of subjective monetary valuation of making efforts and the agent's absolute disinclination to risk, any effort of agent a maximizes the expected utility for himself and minimizes the risk of utility for himself.

B) If the interests of the agent and the principal coincide, the necessary condition of optimality of the agent's actions looks like $c'(a)=1$, from which it follows that $c_1=1$ and the function of subjective monetary valuation of making efforts looks like $c(a)=c_0+a$.

With such a function of subjective monetary valuation of making efforts and the agent's absolute disinclination to risk, any effort of agent a maximizes the expected utility of the total result and minimizes the risk of obtaining this maximum utility.

C) If the principal and the agent, independently, attempt to maximize the expected utility for himself, then from the condition of maximizing utility for the agent $c'(a) = b$ we obtain that $c_1 = b$ and the function of subjective monetary valuation of making efforts looks like $c(a) = c_0 + ba$. And the condition of optimality of incentive force b^* for the principal, written as $a^*(b)c''(b) = 1 - b + R_\Pi(1-b)\sigma^2 c''(b)$, leads to the conclusion that it would be optimal for the principal to choose incentive force $b^* = 1$, i.e. it is optimal to transfer the entire result to the agent (selling the business to the agent).

2. Let us suggest that the function of subjective monetary valuation of the costs of making efforts is quadratic: $c(a) = c_0 + c_1 a + c_2 a^2$, where $c_0 \geq 0, c_1 > 0, c_2 > 0$.

A) If the interests of the agent and the principal do not coincide, the necessary condition of optimality of the agent's actions for maximization of the expected utility for himself and minimization of the risk of this utility looks like $c'(a) = b$, from which it follows that $c_1 + 2c_2 a^* = b$, and hence the value of the agent's optimal effort is $a^* = \dfrac{b - c_1}{2c_2}$. This optimal solution exists when $b \geq c_1$ and does not exist otherwise. As we know, the optimal value of the incentive force on the part of the company maximizing the total gain of the principal and the agent is determined by the formula:

$b^* = \dfrac{1}{1 + R_A \sigma^2 c''}$. In our case, we obtain the following expression $b^* = \dfrac{1}{1 + 2R_A \sigma^2 c_2}$.

B) If the interests of the agent and the principal coincide, the necessary condition of optimality of the agent's actions looks like $c'(a) = 1$, from which it follows that $c_1 + 2c_2 a^* = 1$, and hence the value of the agent's optimal effort is $a^* = \dfrac{1 - c_1}{2c_2}$. This optimal solution exists when $0 < c_1 \leq 1$ and does not exist otherwise. This effort simultaneously maximizes the value of the utility of the total gain and its risk.

C) If the principal and the agent, independently, attempt to maximize the expected utility for himself, then from the condition of maximizing utility for the agent $c'(a) = b$ we obtain that $c_1 + 2c_2 a^* = b$ and hence the value of the agent's optimal effort is $a^* = \frac{b - c_1}{2c_2}$. This optimal solution exists when $b \geq c_1$ and does not exist otherwise. And the condition of optimality of incentive force b^* for the principal $a^*(b) = (1-b)\frac{1}{c''(b)}$ leads to $\frac{b - c_1}{2c_2} = \frac{1-b}{2c_2}$, from which it follows that $b^* = \frac{1 + c_1}{2}$. Thus, if the company knows the method of monetary valuation by the agent of his efforts, such a choice of the incentive force is optimal for the principal. Let us note that from condition $b \geq c_1$ it follows that $\frac{1 + c_1}{2} \geq c_1$, i.e. $0 \leq c_1 \leq 1$. Only with such values of c_1 the described optimizations of the interests of the agent and the principal is possible.

If the principal shows an absolute disinclination to risk a condition of an optimality of power of incentives b^* for the principal

$a^*(b) = \frac{1}{c''(b)}(1-b) + R_\Pi (1-b)\sigma^2$ results in equality

$\frac{b - c_1}{2c_2} = \frac{1-b}{2c_2} + R_\Pi (1-b)\sigma^2$, from where follows that

$$b = \frac{1 + c_1 + 2c_2 R_\Pi \sigma^2}{2(1 + c_2 R_\Pi \sigma^2)} .$$

We will notice that from a condition $b \geq c_1$, follows that $\frac{1 + c_1 + 2c_2 R_\Pi \sigma^2}{2(1 + c_2 R_\Pi \sigma^2)} \geq c_1$ i.e.

$$(1 - c_1)(1 + 2c_2 R_\Pi \sigma^2) \geq 0 .$$

This inequality is equivalent to simultaneous performance of two conditions. Or $0 \leq c_1 \leq 1$ and $0 \leq c_2 \leq \frac{1}{2R_\Pi \sigma^2}$ or $c_1 \geq 1$ and $c_2 \geq \frac{1}{2R_\Pi \sigma^2}$.

Only at such values c_1 and c_2 the described optimization of interests of the agent and the principal is possible.

3. Let us suggest that the function of subjective monetary valuation of the costs of making efforts is exponential: $c(a) = \alpha e^{\lambda a}$, where $\alpha > 0, \lambda > 0$.

A) If the interests of the agent and the principal do not coincide, the necessary condition of optimality of the agent's actions for maximization of the expected utility for himself and minimization of the risk of this utility looks like $c'(a) = b$, from which it follows that $\alpha\lambda e^{\lambda a^*} = b$, and hence the value of the agent's optimal effort is $a^* = \frac{1}{\lambda}\ln(\frac{b}{\alpha\lambda})$. This optimal solution exists when $b \geq \alpha\lambda$ and does not exist otherwise. As we know, the optimal value of the incentive force on the part of the company maximizing the total gain of the principal and the agent is determined by the formula:

$b^* = \frac{1}{1 + R_A\sigma^2 c''}$. Т.к. $c'' = \alpha\lambda^2 e^{\lambda a}$, то $c''(a^*) = \alpha\lambda^2 e^{\ln(\frac{b}{\alpha\lambda})} = \lambda b$ In our case we obtain the following equation $b^* = \frac{1}{1 + R_A\sigma^2\lambda b^*}$ for determining optimal value of the incentive force.

This equation is a quadratic equation of form:

$R_A\sigma^2\lambda b^2 + b - 1 = 0$. Positive and making sense solution of this equation has the form:

$$b^* = \frac{-1 + \sqrt{1 + 4R_A\sigma^2\lambda}}{2R_A\sigma^2\lambda}.$$

It is easy to verify that this value satisfies the natural conditions: $0 \leq b^* \leq 1$.

B) If the interests of the agent and the principal coincide, the necessary condition of optimality of the agent's actions looks like $c'(a) = 1$, from which it follows that $\alpha\lambda e^{\lambda a^*} = 1$, and hence the value of the agent's optimal effort is $a^* = -\frac{1}{\lambda}\ln(\alpha\lambda)$. This optimal solution exists when $\alpha\lambda \geq 1$ and does not exist otherwise. This effort simultaneously maximizes the value of the utility of the total gain and its risk.

C) If the principal and the agent, independently, attempt to maximize the expected utility for himself, then from the condition of maximizing utility for the agent $c'(a) = b$ we obtain that the value of the agent's optimal effort is $a^* = \frac{1}{\lambda}\ln(\frac{b}{\alpha\lambda})$. This optimal solution exists when $b \geq \alpha\lambda$ and does not exist otherwise. And the condition of optimality of incentive force

b^* for the principal is $a^*(b) = (1-b)\frac{1}{c''(b)}$, and since $c''(a^*) = \lambda b$, we obtain $\frac{1}{\lambda}\ln(\frac{b}{\alpha\lambda}) = (1-b)\frac{1}{\lambda b}$, from which it follows that $\ln(\frac{b}{\alpha\lambda}) = \frac{1-b}{b}$. Thus, if the company knows the method of monetary valuation by the agent of his efforts, then, solving this transcendental equation numerically, the principal finds out the optimal incentive force for himself.

If the principal shows an absolute disinclination to risk a condition of an optimality of power of incentives b^* for the principal

$a^*(b) = \frac{1}{c''(b)}(1-b) + R_\Pi(1-b)\sigma^2$ results in equality

$\frac{1}{\lambda}\ln(\frac{b}{\alpha\lambda}) = (1-b)\frac{1}{\lambda b} + R_\Pi(1-b)\sigma^2$. Thus, if the company knows the method of monetary valuation by the agent of his efforts, then, solving this transcendental equation numerically, the principal finds out the optimal incentive force for himself.

4. Let us suggest that that the function of subjective monetary valuation of the costs of making efforts is power function : $c(a) = \alpha + \beta a^k$, where $\alpha > 0, \beta > 0, k > 0$.

A) If the interests of the agent and the principal do not coincide, the necessary condition of optimality of the agent's actions for maximization of the expected utility for himself and minimization of the risk of this utility looks like $c'(a) = b$, from which it follows that $\beta k(a^*)^{k-1} = b$, and hence the value of the agent's optimal effort is $a^* = \sqrt[k-1]{\frac{b}{\beta k}}$. As we know, the optimal value of the incentive force on the part of the company maximizing the total gain of the principal and the agent is determined by the formula:

$b^* = \frac{1}{1 + R_A\sigma^2 c''}$. Since $c'' = \beta k(k-1)a^{k-2}$, then $c''(a^*) = (\frac{b}{\beta k})^{\frac{k-2}{k-1}}\beta k(k-1)$. In our case we obtain the following equation $b^* = \frac{1}{1 + R_A\sigma^2(\frac{b^*}{\beta k})^{\frac{k-2}{k-1}}\beta k(k-1)}$ for determining the optimal

International Journal of Advanced Multidisciplinary Research and Review
Volume 2, No.:3, 2014 Summer
Pages: 159 - 181

value of the incentive force. This equation is a complicated irrational equation, which can be solved numerically.

B) If the interests of the agent and the principal coincide, the necessary condition of optimality of the agent's actions looks like $c'(a)=1$, from which it follows that $\beta k(a^*)^{k-1}=1$, and hence the value of the agent's optimal effort is $a^*=\sqrt[k-1]{\frac{1}{\beta k}}$.

This effort simultaneously maximizes the value of the utility of the total gain and its risk.

C) If the principal and the agent, independently, attempt to maximize the expected utility for himself, then from the condition of maximizing utility for the agent $c'(a)=b$ we obtain that the value of the agent's optimal effort is $a^*=\sqrt[k-1]{\frac{b}{\beta k}}$. And the condition of optimality of incentive force b^* for the principal is $a^*(b)=(1-b)\frac{1}{c''(b)}$, and since $c''(a^*)=\beta k(k-1)(\frac{b}{\beta k})^{\frac{k-2}{k-1}}$ then we obtain $(\frac{b}{\beta k})^{\frac{1}{k-1}}=(1-b)\frac{1}{\beta k(k-1)(\frac{b}{\beta k})^{\frac{k-2}{k-1}}}$, from which, with the help of simple reductions, it follows that $b^*=\frac{1}{k}$. Since b^* should satisfy natural condition $b^*\le 1$, then if the company knows the method of monetary valuation by the agent of his efforts, this optimal for the principal choice of the incentive force is only possible of condition $k\ge 1$is fulfilled.

If the principal shows an absolute disinclination to risk a condition of an optimality of power of incentives b^* for the principal

$a^*(b)=\frac{1}{c''(b)}(1-b)+R_\Pi(1-b)\sigma^2$ results in equality

$$(\frac{b}{\beta k})^{\frac{1}{k-1}}=(1-b)\frac{1}{\beta k(k-1)(\frac{b}{\beta k})^{\frac{k-2}{k-1}}}+R_\Pi(1-b)\sigma^2 \text{ or}$$

$$bk=1+R_\Pi(1-b)\sigma^2(\frac{b}{\beta k})^{\frac{k-2}{k-1}}.$$

Thus, if the company knows the method of monetary valuation by the agent of his efforts, then, solving this transcendental equation numerically, the principal finds out the optimal incentive force for himself.

4. Risks for certain players with various relationships between the agent and the principal expressed using the risk measures VaR and ES

We have already considered the utility risks for the agent, the principal and the company (agent + principal). But of interest is considering the risk measures similar to such risk measures as VaR and ES, existing in the risk management (see, for example, (Crouhy M., Galai D., Mark R. 2011), (Hull J.C. 2007) and (Jorion P. 2007)) for assessment of asset risks, that have already found use in assessment of risks in other spheres (for example, see application of similar risk measures for project risk assessment (Limitovsky M.A., Minasyan V.B. 2011)).

Let us first consider these notions for determining the risk for the agent.

It will be recalled that a random value of the agent's gain in our model is expressed by the formula: $x = s + ba - c(a) + b\varepsilon.$

The value at risk with confidence probability p for the agent will be a value expressed as VaR_p^x, such that the probability that the agent's gain will be greater than this value is equal to p. I.e. it is the worst of all possible values of the agent's gain that may occur with probability p. I.e. $P\{x > VaR_p^x\} = p$.

As is known (see, for example, (Crouhy M., Galai D., Mark R. 2011), (Hull J.C. 2007) and (Jorion P. 2007)), in our suppositions, the risk measure VaR for the agent will be expressed by the formula:

$$VaR_p^x = E(x) - k_p^{0,1}\sigma(x),$$

where E(x) – expected value of random result x for the agent, $\sigma(x)$ - standard deviation of value x, and $k_p^{0,1}$ - quantile of standard normal distribution.

It will be recalled that $x = s + ba - c(a) + b\varepsilon.$ Therefore, $E(x) = s + ba - c(a)$, and $\sigma(x) = b\sigma.$

Hence, we obtain this expression of risk measure VaR for the agent:

$$VaR_p^x = s + ba - c(a) - k_p^{0,1} b\sigma \qquad (7)$$

Having an idea of the form of function c(a), depending on the relationships between the agent and the principal, we can substitute into this expression the optimal values of a^* and b^* and calculate the value of risk measure VaR for the agent.

Cases are possible when there is a significant probability of stress (catastrophic) scenarios when the results may be considerably lower than the VaR. For such situation, measure VaR is not always effective for measuring risks. In this case, the risk may be determined by measure ES. (About risk measure ES for assets see, for example, (Crouhy M., Galai D., Mark R. 2011), (Hull J.C. 2007) and (Jorion P. 2007)).

Conditional value at risk (expected shortfall) with confidence probability p.

ES_p – the mean resultant value which may be predicted in (1- p)% of the worst scenarios.

As is known (see, for example, (Crouhy M., Galai D., Mark R. 2011), (Hull J.C. 2007) and (Jorion P. 2007)), if the resultant value is normally distributed with standard deviation σ, then ES_p is calculated by the formula:

$$ES_p = \frac{\exp(-0{,}5(k_p^{0,1})^2)}{\sqrt{2\pi}(1-p)}\sigma.$$

It will be recalled that $\sigma(x) = b\sigma.$

Hence, we obtain this expression of risk measure for the agent:

$$ES_p^x = \frac{\exp(-0{,}5(k_p^{0,1})^2)}{\sqrt{2\pi}(1-p)} b\sigma. \qquad (8)$$

In case of absolute disinclination of the agent to risk with utility function $u_A(x) = -e^{-R_A x}$, of interest is risk measure VaR of utility for the agent.

The value at risk with confidence probability p for the agent will be a value expressed as $VaR_p^{u(x)}$, such that the probability that the utility for the agent will be greater than this value is equal to p. I.e. it is the worst of all possible values of utility for the agent that may occur with probability p. I.e. $P\{u(x) > VaR_p^{u(x)}\} = p$.

It would be desirable to express $VaR_p^{u(x)}$ through VaR_p^x. Truth of the following absolutely general statement can be proved.

Statement 3.

For any continuously distributed random variable x and increasing function u(x) holds the formula:

$VaR_p^{u(x)} = u(VaR_p^x).$

Proof.

By definition of value $VaR_p^{u(x)}$, true is the equation

$P\{u(x) > VaR_p^{u(x)}\} = p$.

Since function u(x) is a increasing one, there is an inverse function expressed as $u^{-1}(y)$. Then it is obvious that the last equation is equivalent to the following:

$P\{x > u^{-1}(VaR_p^{u(x)})\} = p$. Hence, by definition of VaR_p^x, we obtain

$VaR_p^x = u^{-1}(VaR_p^{u(x)})$. Then

$VaR_p^{u(x)} = u(VaR_p^x).$

Since the agent's utility function $u_A(x) = -e^{-R_A x}$ is a increasing one, then, applying to it Statement 3 and keeping in mind formula (7), we obtain the following formula for $VaR_p^{u_A(x)}$:

$$VaR_p^{u_A(x)} = -e^{-R_A[s+ba-c(a)-k_p^{0,1}b\sigma]}. \qquad (9)$$

Let us consider risk measure $ES_p^{u_A(x)}$ of utility for the agent.

Using definition of $ES_p^{u_A(x)}$, since the agent's utility function $u_A(x) = -e^{-R_A x}$ is a increasing one, applying to it Statement 3 and keeping in mind formula (7), we obtain

$ES_p^{u_A(x)} = E(u_A(x) \,|\, u_A(x) < VaR_p^{u_A(x)}) = E(u_A(x) \,|\, u_A(x) < u_A(VaR_p^x)) = E(u_A(x) \,|\, x < VaR_p^x).$

Note that condition $x < VaR_p^x$ is equivalent to condition $s + ba - c(a) + b\varepsilon < s + ba - c(a) - k_p^{0,1}b\sigma$, hence $\varepsilon < -k_p^{0,1}\sigma$. Then we have:

$$ES_p^{u_A(x)} = -\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_A(s+ba-c(a)+b\varepsilon)}e^{-\frac{\varepsilon^2}{2\sigma^2}}d\varepsilon = -\frac{1}{\sqrt{2\pi}\sigma}e^{-R_A(s+ba-c(a))}\int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_A b\varepsilon}e^{-\frac{\varepsilon^2}{2\sigma^2}}d\varepsilon =$$

$$= -\frac{1}{\sqrt{2\pi}\sigma}e^{-R_A(s+ba-c(a))}e^{\frac{1}{2}R_A^2b^2\sigma^2}\int_{-\infty}^{-k_p^{0,1}\sigma} e^{-\frac{1}{2\sigma^2}(\varepsilon+R_Ab\sigma^2)^2}d\varepsilon = = -e^{-R_A(s+ba-c(a)-\frac{1}{2}R_Ab^2\sigma^2)}N(R_Ab\sigma - k_p^{0,1}).$$

Therefore

$$ES_p^{u_A(x)} = -e^{-R_A(s+ba-c(a)-\frac{1}{2}R_Ab^2\sigma^2)}N(R_Ab\sigma - k_p^{0,1}). \tag{10}$$

Here, N(x) is a function of standard normal distribution.

Thus, if the function of monetary evaluation of the agent's efforts is known, risk measure $ES_p^{u_A(x)}$ can be calculated.

Let us deal with risk measure VaR for the principal.

The value at risk with confidence probability p for the principal will be a value expressed as VaR_p^{Π}, such that the probability that the principal's gain will be greater than this value is equal to p. I.e. it is the worst of all possible values of the principal's gain that may occur with probability p. I.e. $P\{\Pi > VaR_p^{\Pi}\} = p$

It will be recalled that the gain for the principal is $\Pi = y - s - by = a(1-b) - s + \varepsilon(1-b)$,

Expected gain of the principal is $E\Pi(s,b) = (1-b)a^*(b) - s$, and dispersion of this gain is $\sigma^2(\Pi) = (1-b)^2\sigma^2$.

Hence, we obtain this expression of risk measure VaR for the principal:

$$VaR_p^{\Pi} = a(1-b) - s - k_p^{0,1}(1-b)\sigma \tag{11}$$

Having an idea of the form of function c(a), depending on the relationships between the agent and the principal, we can substitute into this expression the optimal values of a^* and b^* and calculate the value of risk measure VaR for the principal.

Let us consider risk measure ES_p^{Π} for the principal. It is obvious that the expression for this risk measure for the agent looks like:

$$ES_p^{\Pi} = \frac{\exp(-0{,}5(k_p^{0,1})^2)}{\sqrt{2\pi}(1-p)}(1-b)\sigma. \tag{12}$$

In case of absolute disinclination of the principal to risk with utility function $u_{\Pi}(\Pi) = -e^{-R_{\Pi}\Pi}$, of interest is risk measure VaR of utility for the principal.

The value at risk with confidence probability p for the principal will be a value expressed as $VaR_p^{u_{\Pi}(\Pi)}$, such that the probability that the utility for the principal will be greater than this value is equal to p. I.e. it is the worst of all possible values of utility for the principal that may occur with probability p. I.e. $P\{u_{\Pi}(\Pi) > VaR_p^{u_{\Pi}(\Pi)}\} = p$.

Since the principal's utility function $u_{\Pi}(\Pi) = -e^{-R_{\Pi}\Pi}$ is increasing one, then, applying to it Statement 3 and keeping in mind formula (9), we obtain the following formula for $VaR_p^{u_{\Pi}(x)}$:

$$VaR_p^{u_{\Pi}(\Pi)} = -e^{-R_A[a(1-b)-s-k_p^{0,1}(1-b)\sigma]} \tag{13}$$

Let us consider risk measure $ES_p^{u_{\Pi}(\Pi)}$ of utility for the principal.

Using definition of $ES_p^{u_{\Pi}(\Pi)}$, since the principal's utility function $u_{\Pi}(\Pi) = -e^{-R_A\Pi}$ is a increasing one, applying to it Statement 3 and keeping in mind formula (7), we obtain

$$ES_p^{u_{\Pi}(\Pi)} = E(u_{\Pi}(\Pi) \,|\, u_{\Pi}(\Pi) < VaR_p^{u_{\Pi}(\Pi)}) = E(u_{\Pi}(\Pi) \,|\, u_{\Pi}(\Pi) < u_{\Pi}(VaR_p^{\Pi})) =$$
$$E(u_{\Pi}(\Pi) \,|\, \Pi < VaR_p^{\Pi}).$$

Note that condition $\Pi < VaR_p^{\Pi}$ is equivalent to condition $a(1-b) - s + \varepsilon(1-b) < a(1-b) - s - k_p^{0,1}(1-b)\sigma,$ hence $\varepsilon < -k_p^{0,1}\sigma.$ Then we have:

$$ES_p^{u_{\Pi}(\Pi)} = -\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_{\Pi}(a(1-b)-s+\varepsilon(1-b))} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon = -\frac{1}{\sqrt{2\pi}\sigma} e^{-R_{\Pi}(a(1-b)-s)} \int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_{\Pi}(1-b)\varepsilon} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon =$$

$$= -\frac{1}{\sqrt{2\pi}\sigma} e^{-R_{\Pi}(a(1-b)-s)} e^{\frac{1}{2}R_{\Pi}^2(1-b)^2\sigma^2} \int_{-\infty}^{-k_p^{0,1}\sigma} e^{-\frac{1}{2\sigma^2}(\varepsilon + R_{\Pi}(1-b)\sigma^2)^2} d\varepsilon =$$

$$= -e^{-R_{\Pi}(a(1-b)-s-\frac{1}{2}R_{\Pi}(1-b)^2\sigma^2)} N(R_{\Pi}(1-b)\sigma - k_p^{0,1}).$$

Therefore

$$ES_p^{u_{\Pi}(\Pi)} = -e^{-R_{\Pi}(a(1-b)-s-\frac{1}{2}R_{\Pi}(1-b)^2\sigma^2)} N(R_{\Pi}(1-b)\sigma - k_p^{0,1}). \tag{14}$$

Thus, if the function of monetary evaluation of the agent's efforts is known, risk measure $ES_p^{u_{\Pi}(\Pi)}$ can be calculated.

Let us consider risk measure VaR for the company.

The value at risk with confidence probability p for the company will be a value expressed as VaR_p^c, such that the probability that the company's gain will be greater than this value is equal to p. I.e. it is the worst of all possible values of the company's gain that may occur with probability p. I.e.

$P\{z > VaR_p^c\} = p$.

It will be recalled that the gain for the company is $z = a - c(a) + \varepsilon$,

Expected gain of the company is $Ez = a - c(a)$, and dispersion of this gain is $\sigma^2(z) = \sigma^2$.

Hence, we obtain this expression of risk measure VaR for the company:

$$VaR_p^c = a - c(a) - k_p^{0,1}\sigma \qquad (15)$$

Having an idea of the form of function c(a), depending on the relationships between the agent and the principal, we can substitute into this expression the optimal values of a^* and b^* and calculate the value of risk measure VaR for the company.

Let us consider risk measure ES_p^c for the company. It is obvious that the expression for this risk measure for the company looks like:

$$ES_p^c = \frac{\exp(-0{,}5(k_p^{0,1})^2)}{\sqrt{2\pi}(1-p)}\sigma.$$

In case of absolute disinclination of the company to risk with utility function $u_c(z) = -e^{-R_c z}$, of interest is risk measure VaR of utility for the company.

The value at risk with confidence probability p for the company will be a value expressed as $VaR_p^{u_c(z)}$, such that the probability that the utility for the company will be greater than this value is equal to p. I.e. it is the worst of all possible values of utility for the company that may occur with probability p. I.e. $P\{u_c(z) > VaR_p^{u_c(z)}\} = p$.

Since the company's utility function $u_c(z) = -e^{-R_c z}$ is a increasing one, then, applying to it Statement 3 and keeping in mind formula (15), we obtain the following formula for $VaR_p^{u_c(z)}$:

$$VaR_p^{u_c(z)} = -e^{-R_c[a - c(a) - k_p^{0,1}\sigma]} \qquad (16)$$

Let us consider risk measure $ES_p^{u_c(z)}$ of utility for the company.

Using definition of $ES_p^{u_c(z)}$, since the company's utility function $u_c(z) = -e^{-R_c z}$ is a increasing one, applying to it Statement 3 and keeping in mind formula (7), we obtain

$$ES_p^{u_c(z)} = E(u_c(z) \,|\, u_c(z) < VaR_p^{u_c(z)}) = E(u_c(z) \,|\, u_c(z) < u_c(VaR_p^z)) = E(u_c(z) \,|\, z < VaR_p^z).$$

Note that condition $z < VaR_p^z$ is equivalent to condition $a - c(a) + \varepsilon < a - c(a) - k_p^{0,1}\sigma$, hence $\varepsilon < -k_p^{0,1}\sigma.$ Then we have:

$$ES_p^{u_c(z)} = -\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_c(a-c(a)+\varepsilon)} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon = -\frac{1}{\sqrt{2\pi}\sigma} e^{-R_c(a-c(a))} \int_{-\infty}^{-k_p^{0,1}\sigma} e^{-R_c\varepsilon} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon =$$

$$= -\frac{1}{\sqrt{2\pi}\sigma} e^{-R_c(a-c(a))} e^{\frac{1}{2}R_c^2\sigma^2} \int_{-\infty}^{-k_p^{0,1}\sigma} e^{-\frac{1}{2\sigma^2}(\varepsilon+R_c\sigma^2)^2} d\varepsilon = = -e^{-R_c(a-c(a)-\frac{1}{2}R_c\sigma^2)} N(R_c\sigma - k_p^{0,1}).$$

Therefore

$$ES_p^{u_c(z)} = -e^{-R_c(a-c(a)-\frac{1}{2}R_c\sigma^2)} N(R_c\sigma - k_p^{0,1}). \qquad (17)$$

Thus, if the function of monetary evaluation of the agent's efforts is known, risk measure $ES_p^{u_c(z)}$ can be calculated using formula (17).

Conclusion

In contractual relationships between any two or more persons, of importance is the specification of private rights that determines how costs and remunerations will be distributed among the participants of these relationships. The role of contracts as a vehicle for voluntary exchange is brought out in paper (Alchian A. A. and Demsetz H. 1972). We, following (Jensen M.C., Meckling W.H. 1976) and (Jensen M.C. 1998), consider the agent relationships as a contract, which, on the part of one or more persons (principal(s)), is concluded with the other person (agent) for rendering some service on their behalf, which includes delegation of some decision making authorities to the agent. If both the parties in relationships maximize the utility for themselves, then the agent will not always act to the best interests of the principal. The monetary equivalent of reduction of the principal's well-being from this divergence is the cost of the agent relationships. The principal may limit the divergence of the agent's actions from his interests setting respective incentives through concluding additional contracts with the agent. An example of modeling the agent relationships is the moral risk model this research is based on. The paper considers relationships between the principal and the agent of various degrees of closeness and studies the possibility to optimize the expected utility and risk for each party.

For various kinds of relationships between the principal and the agent there were obtained computational formulas for introduced risk measures VaR and ES both for the principal and the agent.

REFERENCES

Alchian A. A. and Demsetz H. (1972), «Production, Information Costs, and Economic Organization», American Economic Review, 5, pp. 777-795.

Ayvazyan S.A., Mkhitaryan V.S. (2001), Probability theory and applied statistics, UNITY, Moscow, Russia.

Crouhy M., Galai D., Mark R. (2006), The essentials of risk management, McGraw-Hill.

Gibbons R. (2005), «Incentives between firms (and within)», Management Science, 1, pp. 2-17.

Gibbons R. (2010), «Incentives between firms (and within)», Russian magazine of management 2, pp. 77-106.

Hull J.C. (2007), Risk Management and Financial Institutions, Pearson Education International.

Jensen M.C., Meckling W.H. (1976), «Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure», Journal of Financial Economics, 4, pp. 305-360.

Jensen M.C. (1998), Foundations of Organizational Strategy, Harvard University Press

Jorion P. (2007), Value at Risk, McGraw-Hill.

Limitovsky M.A., Minasyan V.B. (2011), «Analysis of the investment project risks» Financial risk management. 2 : 132- 150.