Military Expenditures of Dictatorial Regimes: A Strategic Theory

Saito Tetsuya

7. December 2007
Military Expenditures of Dictatorial Regimes: A

Strategic Theory*

Tetsuya Saito

Department of Economics, SUNY at Buffalo

415 Fronczak Hall, Buffalo, NY 14260

tsaito5@buffalo.edu

Abstract

Unlike usual approaches to military expenditures that concentrate on
foreign affairs, this paper analyzes a strategic structure for a dictatorial
government to plan military expenditures concentrating on domestic af-
fairs. Then we find two dilemmas: The dictatorial government may spend
less on military equipment if they have some destructive devices and then
citizens may have relatively larger disposable incomes in exchange for mil-
itary oppressions; and the dictatorial government increases military ex-
penditures as their economy grows to sap revolutionary interests. Based
on these results, I also make some closing discussions.

*I am grateful to Peter Morgan for insightful comments through discussions.
1 Introduction

Recent development in Economic History enables us to deal with various issues of “institutions.” A major thought among them is that institutions are formed by conflicts among elites and non-elites—for example, Acemoglu and Robinson (2006), Acemoglu and Robinson (2000) and Engerman and Sokoloff (2005). This paper extends the model of Acemoglu and Robinson (2006) to examine how dictatorial institutions persist facing pressures of revolutions by introducing influences of military expenses. A notion such that some groups exchange concessions to keep peace is not quite novel—for example, Azam (1995) and Roe-mer (1985). The most advantageous feature of Acemoglu and Robinson is that their model deals with a wide range of topics and very simple to easily extend to obtain other implications as well as their efforts of empirical verifications.

In this paper, I mention two factors influence on military expenditures; inexpensive mass destruction weapons and national income. It is not mentioned a lot in Economics how mass destruction weapons influence on economy. However, some countries utilized cheap weapons such as biochemicals and actually used against their opponents. For example, Iraq against Kurd and it is also warned a fear of terrs by dirty bombs which uses coarse radioactive materials. Those weapons are inexpensive and easily obtained by dictatorial countries. In
practice, those weapons also bring fears to suppress opponents to make revolutions. In this study, a parameter that represents destructions during violent actions represents mass destructive strategies. Then we find such abilities reduce military spendings followed by lower tax rate arises as an equilibrium, which indicates disposable income rises in exchange for suffering by those weapons.

Relations among military expenditures and economic performances are frequently studies topic—for example, see Cuaresma and Reitschuler (2006) and Yakovlev (2007). This paper, however, studies a static model of incentives and income levels are regarded as a parameter eligible for comparative statics. In this sense, economy determines military spendings and then higher income raises military expenses in order to sap incentives of revolts, which go up in accordance with the accordingly higher income after the revolt succeeds, followed by higher tax rate.

This paper develops as follows. I briefly review the model of Acemoglu and Robinson (2006) with some modifications in Section 2 and Section 3 to extend in Section 4. Then I make some discussions with regard to implications given by this study in Section 5 as the conclusion.

2 The Base Model

With some modifications, following two sections review the model of institutional changes suggested by Acemoglu and Robinson (2006) to extend as the base model of this study. They consider institutional changes as results of so-
cial conflicts between elites, say rich, and non-elites, say poor. In this model, institutions change either by violent events as such revolutions and coups or concessions. The concessions, in their sense, are taken place by redistributing incomes from the rich to the poor.

For more detail, let $\delta$ and $1 - \delta$ respectively be the fractions of the rich and the poor such that $0 < \delta < 1/2$ (i.e., the poor is the majority of this economy). Incomes of those two classes are respectively given by $y^r$ and $y^p$ such that $y^r > y^p > 0$. Suppose policy packages are represented by tax rates $\tau \in [0, 1]$ and the fraction of the cost of collecting tax revenues are represented by its cost function $C(\tau)$, which is at least twice differentiable, such that $C''(\tau), C'''(\tau) > 0$ and $C'(0) = 0$. In addition, I assume $C'(1) \geq 1$ as well. This cost function represents losses from collecting taxes; for example, opportunity losses from economic activities while tax-payments and opportunity losses from assignments of tax collectors who could produce economic goods—Okun (1975) and Acemoglu and Robinson (2006). Then the subsequent lump-sum redistributions are represented by $T_r \geq 0$ to the rich and $T_p \geq 0$ to the poor. In this sense, redistributions must satisfy

$$\delta T_r + (1 - \delta) T_p = (\tau - C'(\tau)) \bar{y},$$

(1)

where $\bar{y}$ is the average income defined as

$$\bar{y} = \delta y^r + (1 - \delta) y^p.$$  

(2)
Let sub- and super-scripts $i = \{r, p\}$ represent variables for each social class.

Then *money metric* utility for $i$ is given by

$$V^i = (1 - \tau)y^i + T_i.$$  \hfill (3)

By a technical reason avoiding indeterminacy in collective actions followed by Arrow (1950), the government determines the tax rate at first and then the lump-sum redistributions.

**Remark 1** If the government is dictatorial, $T_p > T_r = 0$ followed by $\tau > 0$ realizes as the equilibrium.

**Proof:** If the government is dictatorial, their social planner maximizes only the utility of the rich subject to Eq. (1), so that the problem is

$$\max_{\tau, T_p} (1 - \tau)y^r + T_r \quad \text{s.t.} \quad T_r = \frac{1}{\delta} \left[ (\tau - C(\tau))\hat{y} - (1 - \delta)T_p \right],$$

which is simplified to the problem

$$\max_{\tau, T_p} (1 - \tau)y^r + \frac{1}{\delta} \left[ (\tau - C(\tau))\hat{y} - (1 - \delta)T_p \right].$$

Hence the first order condition with respect to $\tau$ is given by

$$-y^r + \frac{1}{\delta} \left(1 - C'(\tau)\right)\hat{y} = 0,$$  \hfill (4)
which is arranged to get

$$C'(\tau) = 1 - \theta,$$  \hspace{1cm} (5)

where \(\theta = \delta y^\tau / \bar{y}\) is the fraction of income accrues to the rich, so that, \(1 - \theta\) represents that of the poor. By \(C'(0) = 0\) and \(C''(\tau) > 0, \tau > 0\) at the optimum. Because the first order condition with respect to \(T_p\) is negative, \(T_p = 0\) holds and then \(T_r > 0\). □

3 Revolutionary Constraint

Consider a possibility of revolution that succeeds with some cost represented by the fraction lost in the violence \(\mu \in (0, 1)\). For simplicity, the rich is completely purged after the revolution. Then the poor brings about the revolt if

$$\frac{(1 - \mu)\bar{y}}{1 - \delta} > (1 - \tau)y^p + T_p.$$  \hspace{1cm} (6)

A similar notion such that economic gains from revelions attract participants is also argued by Weinstein (2005).

**Remark 2** Under the revolutionary pressure, \(T_r = T_p = 0\) followed by \(\tau = 0\) holds at the equilibrium.

Then the rich solves the problem to maximize their utility Eq. (3) subject to Eq. (1) and Eq. (6). In particular, they solves the reduced form problem such
that
\[
\max_{\tau}(1 - \tau)y^r + T_r \quad \text{s.t.} \quad T_r = \frac{1}{\delta} \left[ (\tau - C(\tau))\bar{y} - (1 - \delta)T_p \right] \\
T_p = \frac{(1 - \mu)\bar{y}}{1 - \delta} - (1 - \tau)y^p,
\]
which is simplified to
\[
\max_{\tau}(1 - \tau)y^r + \frac{1}{\delta} \cdot \left[ (\tau - C(\tau))\bar{y} - (1 - \mu)\bar{y} + (1 - \tau)(1 - \delta)y^p \right]. \tag{7}
\]
The first order derivative is then given by
\[
-y^r + \frac{1}{\delta} \cdot \left[ (1 - C'(\tau))\bar{y} - (1 - \delta)y^p \right], \tag{8}
\]
which is arranged to get the first order condition
\[
\frac{1}{\delta} \cdot \left[ (1 - C'(\tau))\bar{y} - (\delta y^r + (1 - \delta)y^p) \right] = -\frac{C'(\tau)}{\delta} = 0. \tag{9}
\]
Therefore, \(\tau = 0\) is the equilibrium of this model. 

Note, at the optimum, the revolutionary constraint is given by
\[
\theta > \mu. \tag{10}
\]
4 Revolutionary Constraint, Continued

This section extends the original setting of Acemoglu and Robinson (2006) to introduce the probability of revolution settled \( p \) which is influenced by military expenditures \( M \). Let \( p = \phi(M) \) such that \( \phi(0) = 1, \phi(\bar{M}) = 0 \) and \( \phi'(M) > 0 \) for \( 0 < M < \bar{M} \). In addition, I assume \( \phi'(0) = \infty \) and \( \phi'(M) = 0 \) for \( M \geq \bar{M} \).

With the military expenditures, the budget constraint Eq. (1) is rewritten as

\[
\delta T_r + (1 - \delta)T_p + M = (\tau - C(\tau))\bar{y}. \tag{11}
\]

In this case, we have the probability of revolution settled but do not know when it ends. So that the expected payoff to the poor after the revolution is given by

\[
\frac{\sum_{t=1}^{\infty} (1 - \mu)^t (1 - p)^{t-1} p}{1 - \delta} \cdot \bar{y} = \frac{(1 - \mu)p}{1 - \delta} \cdot \bar{y} \cdot \frac{\sum_{t=1}^{\infty} (1 - \mu)^{t-1} (1 - p)^{t-1}}{1 - \delta} = \frac{(1 - \mu)\bar{y}}{1 - \delta} \cdot \frac{1}{(1 - \mu) + \mu / \bar{p}}. \tag{12}
\]

Therefore the revolutionary constraint is now rewritten as

\[
\frac{(1 - \mu)\bar{y}}{1 - \delta} \cdot \frac{1}{(1 - \mu) + \mu / \phi(\bar{M})} = \frac{(1 - \mu)\bar{y}}{1 - \delta} \cdot \frac{1}{(1 - \mu) + \mu / \bar{p}} > (1 - \tau)\bar{y}p + T_p. \tag{13}
\]

Because there is no incentive to invest on arms if there is no incentive for the poor to revolt, I consider the situation such that

\[
1 - \mu > (1 - \tau)(1 - \theta), \tag{14}
\]
which indicates it is better to bring revolts if it succeeds in the next period.\footnote{If \( \tau = 0 \), Eq. (14) coincides with Eq. (10). Technically, this constraint ensures \( p \in (0, 1) \) at the optimum. If Eq. (14) is not satisfied, the revolutionary constraint is slack and then \( M = 0 \) is the optimum spending on arms, so that, the problem is identical to the one studied in Section 2.}

Under Eq. (14), the dictatorial government has no incentive to transfer concessions more than just. Then the problem for the dictatorial government is given by

\[
\max_{\tau} (1 - \tau)y' + T_r \quad \text{s.t.} \quad T_r = \frac{1}{\delta} \left[ (\tau - C(\tau))\bar{y} - (1 - \delta)T_p - M \right] \\
T_p = \frac{(1 - \mu)\bar{y}}{1 - \delta} \cdot F(M) - (1 - \tau)y^p.
\]

**Lemma 1** There exists an optimum tax rate \( \tau = \tau^* \) such that \( 0 < \tau^* < 1 \) and it is unique.

**Proof:** The optimization problem of the dictatorial government is simplified to the problem such that

\[
\max_{\tau} (1 - \tau)y' \\
+ \frac{1}{\delta} \cdot \left[ (\tau - C(\tau))\bar{y} - (1 - \mu)\bar{y}F(M) + (1 - \tau)(1 - \delta)y^p - M \right]. \tag{15}
\]

Note, we have

\[
\frac{\partial F}{\partial M} = \frac{\mu \phi'(M)}{((1 - \mu)\phi(M) + \mu)^2} < 0, \tag{16}
\]

\[
\frac{\partial M}{\partial \tau} = (1 - C'(\tau))\bar{y}. \tag{17}
\]
Then the first order condition is given by

$$-y^r + \frac{1}{\delta} \left[ -(1 - \mu)\dot{y} \cdot \frac{\partial M}{\partial \tau} \cdot \frac{\partial F}{\partial M} - (1 - \delta)y^h \right] = 0, \quad (18)$$

which is rearranged to get

$$-\frac{\partial M}{\partial \tau} \cdot \frac{\partial F}{\partial M} = \frac{1}{1 - \mu}. \quad (19)$$

By the assumptions, the left hand side Eq. (19) is infinity at $\tau = 0$ and attains zero at some value $\tau = a < 1$. For example, Figure 1 depicts the case in which there is some value $\tau = b > a$ such that $\phi'(b) = 0$. Then we can find $\tau = \tau^*$ as the equilibrium such that $0 < \tau^* < a(< 1)$. ■

**Theorem 1** Under the revolutionary pressure, $M > T_p = T_r = 0$ followed by $\tau > 0$ holds as the equilibrium.

**Proof:** Let $V$ be the value function for Eq. (15) to apply the envelope theorem to get

$$\frac{\partial V}{\partial M} = \frac{1}{\delta} \left[ -(1 - \mu)\dot{y} \cdot \frac{\partial F}{\partial M} - 1 \right]. \quad (20)$$

At the optimum, by Eq. (16) and Eq. (17), Eq. (20) is given by

$$\frac{1}{\delta} \left[ (1 - \mu)\dot{y} \cdot \frac{1}{(1 - \mu)\dot{y}(1 - C'(\tau))} - 1 \right] = \frac{1}{\delta} \cdot \frac{C''(\tau)}{1 - C'(\tau)} > 0, \quad (21)$$

where the last inequility follows from $C'(\tau) > 0$ and $1 - C'(\tau) > 0$ by $\partial M/\partial \tau > 0$ at the optimum (see Figure 1). Therefore the rich wishes to spend all tax rev-
Theorem 1 tells us the dictatorial government facing pressures of revolutions levies tax to finance military expenditures to enforce their regime that contrasts to the case discussed in Section 3—the government cannot control the probability of revolution succeeds.

**Theorem 2** If $p/(1-p) \leq \mu$, then $\tau^*$ is decreasing in $\mu$ and so does $M$.

**Proof:** Both sides of Eq. (19) are increasing in $\mu$ because it is obvious that $1/(1-\mu)$ is increasing in $\mu$, $\partial M/\partial \tau$ is irrelevant to $\mu$ and

$$
\frac{\partial^2 F}{\partial M \partial \mu} = \frac{\phi'(M) \cdot (\phi(M) - \mu \cdot \phi(M))}{((1-\mu)\phi(M) + \mu)^2},
$$

(22)

The numerator of Eq. (22) is rewritten as

$$
\phi'(M) \cdot (p - \mu \cdot (1-p)),
$$

(23)

which is strictly negative if $p/(1-p) > \mu$ and weakly positive if $p/(1-p) \leq \mu$ because $\phi'(M) < 0$. For $p/(1-p) \leq \mu$, it reveals

$$
\frac{\partial}{\partial \mu} \cdot \left( -\frac{\partial M}{\partial \tau} \cdot \frac{\partial F}{\partial M} \right) < 0.
$$

(24)

Those arguments imply the downward shift of the left hand side and the upward shift of the other side of Eq. (19), hence, the optimum tax rate goes down—
example, Figure 2 depicts a case which brings a change in tax rates from $\tau^*$ to $\tau^{**}$ such that $\tau^* > \tau^{**}$. ■

Theorem 2 provides a result of comparative statics about the destruction rate $\mu$. Given the likelihood ratio of the probability of revolution succeeds not to be larger than the destruction rate, an increasing in the destruction rate brings lower tax rate and following smaller military expenses. In other words, the dictatorial government that successfully suppress the possibility of regime changes at sufficiently low level reduces the tax rate to reduce their military expenses if revolutions are getting destructive. In particular, the destruction rate will rise if arms improve at the same cost or if they introduce cheap but effective mass destruction devices such as biochemical weapons or nuclear weapons as an extreme. In this sense, the fear of those weapons reduces the pressure of revolutions to reduce tax rate while it reinforces the dictatorial regime.

**Theorem 3** $\tau^*$ is increasing in $\bar{y}$ and so does $M$.

**Proof:** We have $\frac{\partial^2 M}{\partial \tau \partial \bar{y}} = 1 - C'(\tau)$ and that is positive around the equilibrium, so that, it is true that

$$\frac{\partial}{\partial \bar{y}} \left( -\frac{\partial M}{\partial \tau} \cdot \frac{\partial F}{\partial M} \right) > 0. \tag{25}$$

Because the right hand side of Eq. (19) is irrelevant to $\bar{y}$, it implies only the upward shift of the left hand side, so that, the optimum tax rate goes up and so does $M$. ■
Theorem 3 exhibits an interesting feature of the dictatorship. An increase in their national income raises the pressure of revolutions and then the dictatorial government spend more on military expenditures.

**Remark 3** We obtain following implications from the model:

1. Mass destruction devices may increase disposable income of the dictatorial nation while it brings large fears to the inside of the country.

2. Economic growth in a dictatorial country brings more military expenditures followed by larger tax rate to suppress higher incentives to make revolutions.

### 5 Closing Discussions

Despite usual analysis on military expenditures which consider in terms of neighbor countries’ military affairs, this paper analyzed the relation among an dictatorial government and citizens applying a simple model of revolutionary threat. Then some new characters are found: Introducing destructive devices may increase disposable incomes of citizens because of increased threat of tragic consequences for revolutionists (Remark 3-1); and economic growth will increase military expenditures because of increasing rewards after revolts for citizens (Remark 3-2). Although appropriate empirical data is not available, those findings are intuitively plausible in the real world. For example, behind the genocide applying chemical gases against Kurdish and the military oppression against Sunna Muslims, Iraqi enjoyed relatively higher living standards under the reign
of Sadam Hussain at least until the economic embargo by the United Nations. Although
majorities are not directly oppressed, demonstrations against minorities and politi-
cally opposite groups have some influences on majorities. Inclusive such military
actions, giving presentations for citizens about the attitude of the government has been important.

Now our world faces a dilemma, whether or nor to bring a regime change. That is, everyone agrees physical oppressions shall be eliminated from the said country but some may claim the country has been stable until the regime change. That is really observed in the aftermath of the Iraq War. With regard to this point, I believe any dictatorial governments that suppress human rights shall be removed and then the world community consider reformations.

The second implication is that the dictatorial government need to prepare for revolts when the economy goes well in order to sap any incentives to challenge against the government. Then military expenditures relative to their national income necessarily increase. If oppressive demonstrations are necessary to show the power of the dictatorial government, we have again a dilemma, whether or not to support economic progress of such countries. The western countries have kept providing supports expecting economic growth brings democratization. However, the fastest growing country is Communist China and they continue oppressing Tibet and people inside the country behind their economic growth on the shore. In addition, the international community cannot control Chinese military expansion. With this point, I also believe democratization shall be the first and then economic supports. Otherwise human rights,
which have been the most important consensus among the western countries after the World War II, are outraged by dictators and accepting those regimes imply we accept the end of our common ideology “Democracy.”

References


![Figure 1: Optimum Tax Rate](image-url)
Figure 2: Comparative Statics when $p/(1 - p) \leq \mu$