Retail Sector and Economic Development

Ngotran, Duong and Ngo, Tien

State University of New York, University at Albany, US, Banking Academy, Vietnam

July 2013

Retail Sector and Economic Development*

Duong Ngotran†
SUNY Albany, US

Tien Ngo‡
Banking Academy, Vietnam

Abstract

We build a model for emerging economies where households could search goods through two retail platforms: the legal organized (supermarket) and the informal unorganized (mom-and-pop store). We highlight the role of the retail sector as a special two-sided platform in goods market. A positive shock on the productivity of supermarket pulls both consumers (demand) and firms (supply) to make transactions there. This effect is amplified through the interaction between demand and supply, motivating manufacturing firms to become formal to supply goods through this channel. We do the quantitative exercise for the Indian economy; a 1 percent increase in the productivity of organized retailers leads to an increase in the aggregate productivity of 0.3 percent, 80 percent of which is contributed indirectly by the shift in the manufacturing sector.

Keywords: Retail sector; Platform; Manufacturing sector; Informal firms; India; Tax

*We wishes to thank Adrian Master, Betty Daniel and John Jones for helpful comments and discussions. We also thank Garima Siwach, Savita Ramaprasad, Minhee Kim, Sun Qi and other participants in workshops.
†Email: dngo@albany.edu
‡Email: tienngo1896@gmail.com

1 Introduction

As one of the sectors with the largest employment, the retail sector has a special role in the economy, playing as the platform to match demand and supply in goods market. This functionality, which is often ignored in models with frictionless goods market, rarely attracts the economists' attention. The lack of interest is partly originated from the fact that the structures of the retail industry in developed countries are stable and well organized, having few implications for the structure of the supply side.

This is not the case with developing countries. The retail sector in developing countries has two distinctive retail platforms: the legal organized retailing (big boxes, supermarkets) and the informal unorganized retailing (mom and pop stores, street vendors); where the latter is usually a dominant force in the market (Figure 1). There is an important link between the informal retail platform and the informality in the the manufacturing sector as informal firms could only supply goods to consumers through this platform. Therefore, this paper shows that an increase in the productivity of the organized retailing does not end at itself, but has a large transmission mechanism to the manufacturing sector due to the platform effect. This makes the role of the retail sector in developing countries much more important than the traditional view, which only highlights the big employment size of this sector.

This paper builds a dynamic general equilibrium model with directed search to clarify the link between the retail sector, the manufacturing sector, the government budget and the aggregate productivity in developing countries. The most significant result shows that the development of the organized retail sector has a huge impact on the government budget and the aggregate productivity, mainly through giving the incentives for informal firms to become formal in the manufacturing sector. The mechanism comes from two channels: the platform effect and the tax effect.

The platform effect happens when both demand and supply respond to the change in the efficiency of the organized retailing. The lower retailing cost of supermarkets makes them the more attractive platforms as compared to mom-and-pop stores, inducing firms to become formal to supply goods there. As households search goods in a place with cheap prices and many brands, the transition of some firms to supermarkets makes shopping there become more convenient

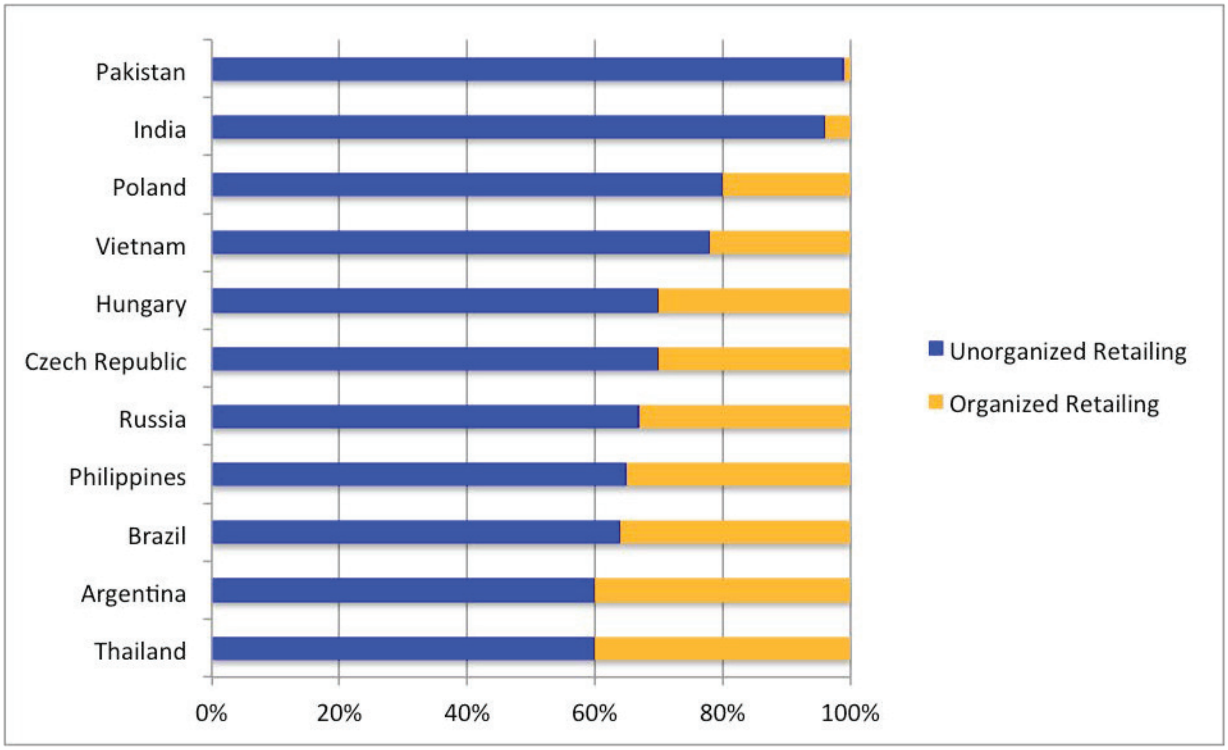


Figure 1: Retail sector in emerging economies 2006 (Data from Sharma (2011))

than before. Because firms want to use a platform where they can approach most consumers, the change in demand, in turn, has a feedback effect on the firms' decision, and therefore the supply side. The interaction between demand and supply could amplify the platform effect. When more transactions take place in supermarkets, government's sales tax revenue improves, triggering the tax effect.

Governments in developing countries often lose a big share of sales tax when transactions take place in the informal platform (see International Tax Compact (2010)).The shift of transactions to the organized platform improves the government budget, allowing the government to cut the tax rate on formal firms. The reduction in the tax distortion makes the firm's option of becoming informal less attractive. When there is a large number of firms become formal, the burden of tax on each formal firm will decline substantially.

Both the platform effect and the tax effect shift the paradigm of the economy to the formal side, increasing the efficiency in the resource allocation process. Informal firms, although gain the benefit of tax avoidance, often have the lower productivity than their formal counterparts.

Therefore, the increase in the efficiency of the organized retailing not only affects the productivity of the retail sector itself but also raises the productivity of the manufacturing sector and the aggregate productivity of the economy.

We also calibrate model for the Indian economy, where the retail sector has been the hottest topic in the policy reform agenda for recent years. Before December 2012, the Indian government forbade all foreign groups from any ownership in supermarkets, convenience stores or any retail outlets. We find that, if India successfully reforms the retail sector, increasing the market share of the organized retailers from 8 percent (2012) to the level of China 20 percent, then its outputs could increase by 0.82 percent. That effect is extremely huge if we consider the number of workers in the organized retail sector itself only accounts 1 percent of the total labor force in India. The reason for the strong response of the economy lies in the link between the retail sector and the manufacturing sector. On average, a 1 percent increase in the productivity of the organized retailers could result in 0.3 percent increase in the aggregate productivity of the economy, 80 percent of which is contributed indirectly by the shift in the manufacturing sector.

There are two key contributions in our paper. First, we highlight the role of the retail sector as a special two-sided platform in the economy. In the existence of frictions in goods market, the productivity of supermarkets has a huge impact on the manufacturing firms' choice of becoming formal. In models without frictions in goods market, the supermarkets' productivity only shifts the supply side where more firms want to become formal to enjoy the lower retailing cost. In our model, both demand and supply respond to the change in the supermarkets' productivity. Moreover, this effect is amplified by the interaction between demand and supply. Second, by calibrating our model for the Indian economy, we raise a different view on the debate on Indian retail reforms. The previous quantitative exercises highly underestimate the role of the retail sector as they ignore its feedback effect on the manufacturing sector and the government budget.

Our competitive goods search model is built based on Bai, Rios-Rull and Storesletten (2011) (here after BRS), where they give insight about the role of demand in business cycles by incorporating Moen (1997)'s directed search into DSGE framework. Matha and Pierrard (2011), Petrosky-Nadeau and Wasmer (2011), Gourio and Rudanko (2011) also incorporate the friction in goods market to investigate business cycles. Our model is more about the persistence change in total factor productivity (TFP), in this sense it is related to Lagos (2006), though Lagos' paper is about the labor market.

Our paper is also related to the literature about the retail sector in developing countries. There are two different views explaining why the unorganized retailing is a dominant force in the market. Lewis (2004) considers the policy distortion as the main cause. The burden of tax on the organized retailers makes them less competitive than the informal retailers. On the other side, Lagakos (2009) gives insight that developing countries rationally choose the unorganized retailing. The lack of car ownership makes search cost in supermarkets higher than in mom-and-pop stores. This view comes from the appropriate technology literature in Basu and Weil (1998), Acemoglu and Zilibotti (2001).

Although the main purpose of our model is to find the link between the retail sector and the manufacturing sector, both above insights exist in our model. The policy distortion is explained through the tax effect, in which supermarkets must pay the corporate tax and transactions in supermarkets are imposed the sales tax. The search cost effect is abstractly captured by the shopping habit persistence in households' utility function. In our model, the search cost is lower in mom-and-pop stores as households' member are more familiar with that kind of retailing. If the organized retailing develops, the advantage of the lower searching cost in mom and pop stores will be vanished over time[1].

Our paper is also related to research on the informal economy and the tax distortion. Erasmo and Moscoso Boedo (2012) show a low degree of debt enforcement and high cost of formality lead to the large share of output is produced by low productivity informal firms. The relationship between tax, the informal sector and the aggregate productivity could also be found in Gollin (1995), De Paula and Scheinkman (2010) and Ordonez (2014). However, none of the above papers are about the retail sector. In our model, the aggregate productivity goes up as a result of the resource reallocation. In this sense, it is also related to Restuccia and Rogerson (2008), Hsieh and Klenow (2009).

The paper is organized as follows. Section 2 lays out the model. Section 3 gives the analyt-

[1] See Reardon, Timmer and Berdegue (2004) where they provide evidences for the transition from the traditional retailing to supermarkets in developing countries.

ical results about the platform effect and the tax effect. In section 4, we calibrate the model for the Indian economy and do some experiments to gauge the impact of the retail sector reforms on the aggregate output. Finally, section 5 gives the conclusion.

2 The model

Time is discrete and continues forever. The economy consists of four types of agents: households, retailers, firms and the government. The detail description about each agent will be discussed later, but the main interaction between four agents could be described as follows.

In order to buy consumption goods, every period a household must spend time on shopping (or send their members to go shopping). The household could search goods through two retail platforms: the organized retail platform (the supermarkets' platform - denoted by "s") and the unorganized one (the mom and pop stores' platform- denoted by "m"). The good search process is characterized by Moen (1997)'s directed search. There is a continuum of submarkets in each platform. We follow BRS when a submarket belonging to the retail platform j is indexed by a triplet $(\theta_j,\, p_j,\, f_j)$ of the market tightness, the unit price and the quantity of goods each seller has. One important assumption here is the household can only send all their members into one single submarket in each platform.

There is a continuum [0,1] of firms. These firms either supply for the platform "s" or "m" but not both. A firm, who supplies goods for the platform j, is called firm j. Firm j will use the retailing service of the retailers in the channel j $(j = s, m)$. After deciding the retail channel, firm j chooses the single submarket $(\theta_j,\, p_j,\, f_j)$ to enter. It means that this firm will produce exactly the quantity f_j, sell goods with the unit price p_j and know the ratio of firms to shoppers in this submarket as θ_j. The expected firm's revenue could be deducted alone from the information of the submarket he enters. Similar to households, we assume that firms can only enter one submarket.

In equilibrium, there is only one unique submarket in each retail platform where firms and households could meet each other [2]. Let D_j be the aggregate measure of shoppers in the plat-

[2]There is an assumption here to ensure the unique equilibrium. When choosing the submarket in the retail channel j, both households and firms could enter only a single submarket. If the household can send members to

form j, E_j be the measure of firms j ($E_s + E_m = 1$). Following BRS, the matching process between the buyers and sellers in the platform j is taken by:

$$M_j = A_j D_j^{\varphi} E_j^{1-\varphi}, \qquad j = s, m$$

The market tightness, defined as the ratio of number of firms to shoppers, is $\theta_j = E_j / D_j$. In a successful meeting, a buyer will buy all f_j units of goods from a seller with price p_j per unit.

Therefore, when entering the submarket (θ_j, p_j, f_j), the number of goods a household can find when sending d_j shoppers is:

$$c_j = d_j \Delta_j(\theta_j) f_j = d_j A_j \theta_j^{1-\varphi} f_j, \qquad j = s, m$$

Similarly, choosing the submarket (θ_j, p_j, f_j), a supplier j expects to get the revenue:

$$R(\theta_j, p_j, f_j) = p_j \Gamma_j(\theta_j) f_j = p_j A_j \theta_j^{-\varphi} f_j, \qquad j = s, m$$

At the end of the period, we assume all the unsold goods will be perished [3].

About the cost side, a firm incurs two kinds of cost: the production cost and the retailing cost; in which, the latter is paid to the retailers. In our model, all the retailers in the same channel share the unique platform in their channel, where the transactions between firms and households are taken place. Supermarkets are the retailers for the platform "s" and mom-and-pop stores are the retailers for the platform "m". The retailers transport goods to a particular submarket in the shared platform under the firms' direction and sell goods for firms. Firms pay the retailing fees for these services. The ownership of goods will be transferred directly from firms to households.

Although the introduction of the friction in goods market is abstract, it captures some important features to match the reality. First, shoppers prefer the retail platform with low price, a lot of brands without too many other shoppers. Second, firms prefer the retail channel where

the different submarkets in each channel, then there might be multiple price dispersion equilibria.

[3]The model can be extended to allow the inventory. However, adding the inventory will make the model much more complex without changing the main insight of the paper.

the retailing cost is low and households often go shopping. Third, the number of successful transactions is increasing with the measure of shoppers and firms. Fourth, not all goods are sold at the end of the period. These four features are important to characterize the main insight of our paper. Next, we describe the link between the retail sector and the manufacturing sector.

The most important assumption in our paper mentions that all goods supplied through the supermarket system must come from formal firms. Laws in many countries require supermarkets to ensure the origin of goods. As the products of informal firms are not registered with the government, it is likely that they do not meet the standards to be sold in supermarkets. Bohme and Thiele (2012) find the strong empirical evidence in Africa that informal goods are hardly bought through the formal distribution channels. By using a survey of 48000+ small firms in Brazil, De Paula and Scheinkman (2010) show the formality of suppliers and purchasers is highly correlated.

Therefore, we want to model firms in the channel "s" must be formal; while the channel "m" includes both formal firms and informal firms.[4] However, the problem in the directed search will be much more complicated with two kinds of firms in the channel "m".[5] We deal with this problem by directly assuming there is only one kind of firm in the channel "m", and firm "m" will produce from two plants: one formal plant and one secret informal plant. This assumption still allows us to see the resource reallocation between the formal plant and the informal plant within the channel "m". To ensure the number of total plants in the economy is unchanged with the firms' platform choice, we also assume each formal firm "s" will produce from two formal plants.

The government collects taxes from firm "s", the formal plant of firm "m" and supermarkets. Another source of the government's revenue comes from the sales tax; which is assumed to be collected fully in the platform "s", but only partially in the platform "m". The relationship between four agents could be described by the figure 2.

[4]In developing countries, goods displayed in informal mom-and-pop stores come from both formal firms and informal firms.

[5]The unique submarket in the platform "m" in equilibrium might no longer be true and the household's problem could not be broken down into two subproblems. See section 2.1 for details.

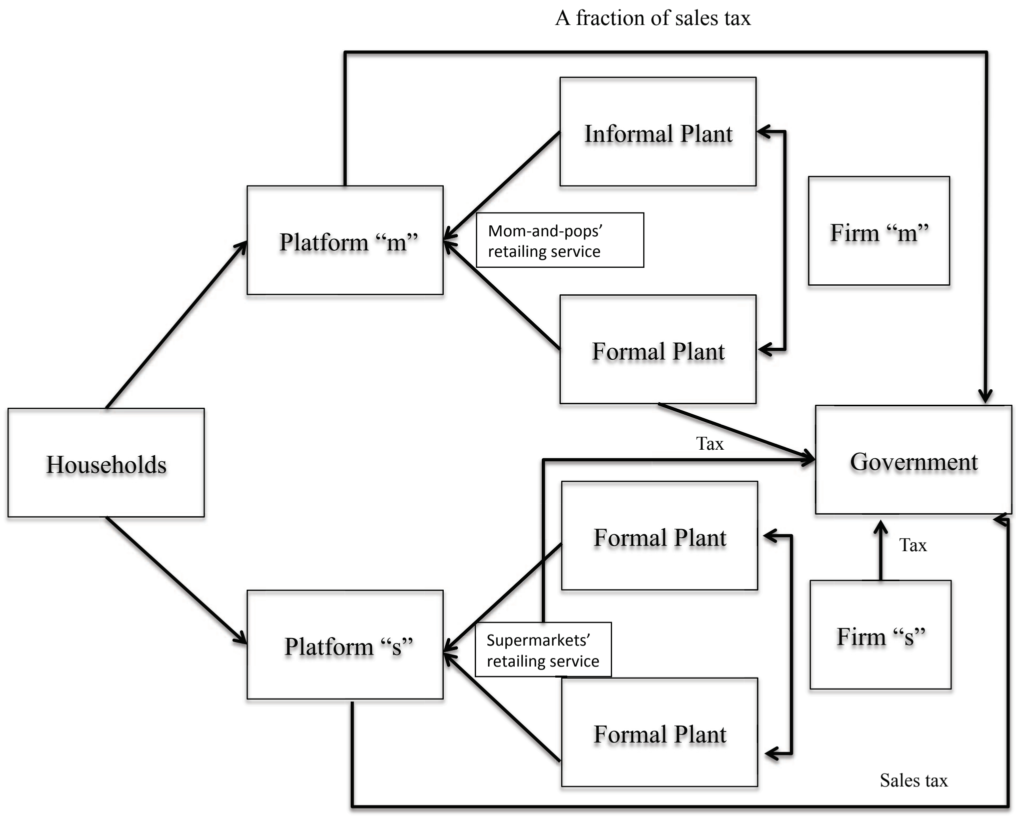


Figure 2: The relationship between agents

2.1 Households

There is a measure one of identical households in the economy. Each household is endowed with one unit of labor, which they supply inelastically in the labor market every period. Their preference includes the goods they consume (c) and the disutility when they sends their members to go shopping in the platform j (d_j). The household's utility in one period has the form:

$$U(c, d_s, d_m) = u(c) - h(d_s - \gamma D_s^-) - h(d_m - \gamma D_m^-)$$

$$\text{with } u'(.) > 0, u''(.) < 0, h'(.) > 0, h''(.) > 0$$

We incorporate the external shopping habit persistence D_j^- into the household's utility function[6]. The literature of the external habit persistence in DSGE model was first introduced by

[6]The notation D_j^- shows one-period-behind value of D_j, which is the aggregate number of shoppers in the platform j. It is also the aggregate counterpart of d_j.

Abel (1990), then Campbell and Cochrane (1999), Smets and Wouters (2007), where the slow adjustment in consumption is explained by the agent's habit. The same idea applies in our model, but for the shopping habit. When most members of households are familiar with the mom-and-pop platform, it becomes more convenient for them to search goods there. Even when the supermarkets' platform becomes more attractive, it needs a transition time to adjust the household' behavior.[7] The speed of the adjustment in the shopping activities is governed by the parameter γ. The bigger it is, the more persistent is the shopping habit.

It might be more appropriate to also include the internal habit persistence (d_j^- itself) into the utility function. However, the household's problem in the directed search becomes too complicated with the appearance of d_j. The transition of the model is also not much different with the appearance of d_j as the state variable, so we only include the external habit in our model.

Each household makes their decision based on the state of economy $\Omega = (z_s, D_s^-, D_m^-)$[8] where z_s is the productivity of supermarkets[9]; their own state b, which is the number of firms' shares they hold. We normalize the price of a share to unity. We follow BRS in formulating the household's problem. Here the household treats a number of the aggregate variables as given: the market tightness θ_j, price p_j and quantity f_j in each retail channel as well as the wage w and the law motion of the aggregate state $\mathscr{G}(.)$.

It is important to note that we separate the household's problem into two sub-problems: i) Given the identified single submarket (θ_j, p_j, f_j) in each platform, what are the household's choices on d_j and c (which we introduce in this section); and ii) How do households and firms choose the submarket (θ_j, p_j, f_j) in the competitive search environment (which we introduce in the section 2.4)[10]. A household's problem could be written as:

$$V(b,\Omega) = \max_{c,d_s,d_m,b'} U(c,d_s,d_m) + \beta V(b',\Omega') \tag{1}$$

[7] See Khalifa, Limayem and Liu (2002) for the empirical evidences on the shopping habit persistence.

[8] The fact of including D_j^- into the state of economy is very reasonable if we think about the supply side. When choosing the retail channel, firms, in reality, must do the data analysis on households' shopping activities in the past as they expect the persistences on households' behaviors.

[9] The target of our model is to examine the response of households, firms and economy when there is a jump in the supermarkets' productivity.

[10] Similar to BRS, the unique submarket in each retail platform "j" in equilibrium allows us to break down the households' decision into two parts.

subject to

$$c_j = d_j \Delta_j[\theta_j(\Omega)] f_j(\Omega), \qquad j = s, m \tag{2}$$

$$c = c_s + c_m, \tag{3}$$

$$\Omega' = \mathscr{G}(\Omega), \tag{4}$$

$$b' + (1+\tau_s) p_s(\Omega) c_s + (1+\delta\tau_s) p_m(\Omega) c_m = (1+\pi(\Omega)) b + w(\Omega) \tag{5}$$

The first equation shows that the number of goods households could find in each channel depends on the number of shoppers the household sends to each platform as well as the market tightness and number of goods displayed in each platform. The second equation simply states that total consumption is equal to the sum of goods bought in each channel. The third equation is the motion of the state variables while the last one is typically a budget constraint. In the budget constraint, τ_s is the sales tax which the government collects fully in the platform "s". In the platform "m", the government could only collect a fraction δ of total sales tax.[11] The variable π is the profit of firms, and w is the wage rate. The solution to this problem is a set of individual decision rules $c(b,\Omega)$, $d_j(b,\Omega)$. In equilibrium, the individual decision is consistent with the aggregate part, so we introduce the aggregate counterparts of these functions:

$$b'(1,\Omega) = 1$$

$$C(\Omega) = c(1,\Omega)$$

$$D_j(\Omega) = d_j(1,\Omega)$$

We will characterize some important households' decisions under the aggregate form. The first one shows how the household allocates the shoppers between two retail channels:

$$\frac{-h'\left[D_s(\Omega) - \gamma D_s^-\right]}{I_s p_s(\Omega)\Delta_s[\theta_s(\Omega)] f_s(\Omega)} + \frac{u'(C(\Omega))}{I_s p_s(\Omega)} = \frac{-h'\left[D_m(\Omega) - \gamma D_m^-\right]}{I_m p_m(\Omega)\Delta_m[\theta_m(\Omega)] f_m(\Omega)} + \frac{u'(C(\Omega))}{I_m p_m(\Omega)} \tag{6}$$

[11] As the platform "m" belongs to the informal economy, the government could not control all the activities there. We could think δ as the fraction of transactions the government detects and forces the households to pay the sales tax.

$$\text{where } I_j = \begin{cases} 1+\tau_s & \text{if } j=s \\ \\ 1+\delta\tau_s & \text{if } j=m \end{cases}$$

The second one is the standard intertemporal condition:

$$u'(C(\Omega)) - \frac{h'\left[D_j(\Omega)-\gamma D_j^-\right]}{\Delta_j[\theta_j(\Omega)]f_j(\Omega)} = \beta\frac{p_j(\Omega)(1+\pi(\Omega'))}{p_j(\Omega')}\left(u'(C(\Omega')) - \frac{h'\left[D_j(\Omega')-\gamma D_j(\Omega)\right]}{\Delta_j[\theta_j(\Omega')]f_j(\Omega')}\right) \tag{7}$$

Let $m(b,\Omega)$ denote the discounted value in terms of marginal utility of an additional unit of savings and let $M(\Omega)$ be its aggregate part, we have:

$$\begin{aligned} m(b,\Omega) &= \beta\left(\frac{\partial V(b',\Omega')}{\partial b'}\right), \\ M(\Omega) &= \beta\frac{1+\pi(\Omega')}{I_j\, p_j(\Omega')}\left(u'(C(\Omega')) - \frac{h'\left[D_j(\Omega')-\gamma D_j(\Omega)\right]}{\Delta_j[\theta_j(\Omega')]f_j(\Omega')}\right) \end{aligned} \tag{8}$$

2.2 The retailers

There are two types of retailers in the economy: supermarkets (in the channel "s") and mom-and-pop stores (in the channel "m"). The retailing market in each channel is perfectly competitive. We assume all the retailers in the channel j share the single platform j, where takes place goods transactions between firms and households. Firms pay the retailers for transporting their goods to a particular submarket in this shared platform. The retailers also sell goods for firms. We assume retailers only conduct the retailing service, they never own the goods in the whole process.

The matching ability of the shared platform j is shown by the parameter A_j. These platforms incur no cost in the matching process between households and firms.

A firm's goods must be transported and sold by a single retailer. To transport and sell f units of goods in the shared platform j, a retailer in the channel j needs to hire :

$$l_j^R(f) = \frac{f^{1/\alpha}}{z_j}; \qquad \alpha < 1 \tag{9}$$

We could interpret z_j as the productivity of the retailers in the channel j. We assume firms are in the different locations; therefore, a retailer could not bundle the services for different firms together.[12]

We assume all supermarkets in the channel "s" are formal while all mom-and-pop stores in the channel "m" are informal. There are two significant characteristics to differentiate the formal sector from the informal one: the former pays the tax with rate τ_c to the government; the latter could avoid the tax but it has the lower productivity $z_m < z_s$.[13] We assume τ_c is imposed on the total cost, rather than profit, to reflect all the distortions on the formal sector (corporate tax, compensation for workers, red tapes, fees).

If we account on tax, the total costs for retailing f units of goods in the retail channel "s" and "m", respectively, are :

$$x_s^R(f,\Omega) = \frac{(1+\tau_c)}{z_s} f^{1/\alpha} w(\Omega) \tag{10}$$

$$x_m^R(f,\Omega) = \frac{1}{z_m} f^{1/\alpha} w(\Omega) \tag{11}$$

As the retail market is perfectly competitive, the minimum cost of retailing is also the price firms must pay for the retailers for the services of transporting and selling goods .

2.3 Firms

There is a measure one of firms, who at the beginning of period, based on the household's shopping pattern in the previous period and the productivity in supermarkets $\Omega = (D_s^-, D_m^-, z_s)$, must make two important decisions in order: (i) first, whether to use the retail channel "s" or "m" as the platform to approach consumers, (ii) second, the specific submarket in each channel to get into. If they choose supermarkets as their retailers, they will produce all their products from two formal plants. If they choose the mom and pop stores' platform, they will produce

[12]This assumption is equivalent to a retailer must use the different workers when conducting the services for different firms.

[13]In this paper, we assume directly the informal firm is less efficient than the formal firm. We do not model the micro-foundation for this phenomenon. However, this fact is well recognized in the literature, see Basu (2003), Porta and Shleifer (2008), De Paula and Scheinkman (2010), Ordonez (2014).

from two plants: one formal plant and one informal plant.[14]

The production function of the formal plant and informal plant are respectively as:

$$f^f = l^\alpha, \qquad \alpha < 1$$
$$f^m = \left(\frac{l}{1+\varepsilon}\right)^\alpha, \qquad \alpha < 1, \varepsilon > 0$$

The difference in the cost of production reflects the same thing in the retail sector. The formal plant suffers the tax τ_c imposed on the production cost while the informal plant, though less efficient, could avoid the tax. When entering the submarket, firms know explicitly their expected revenue.

2.3.1 Firms "s"

As firm "s" owns two identical formal plants exhibiting the decreasing return to scale, the optimal allocation for firm "s" to produce f_s units of goods is that each plant produces $f_s/2$ units.

By using (10), the total cost of producing and retailing f_s units of goods for the platform "s" is:

$$\begin{aligned} x_s(f_s,\Omega) = & \quad \underbrace{2(1+\tau_c)(f_s/2)^{1/\alpha} w(\Omega)}_{\text{Production Cost}} \quad + \quad \underbrace{(1+\tau_c)(z_s)^{-1} f_s^{1/\alpha} w(\Omega)}_{\text{Retailing Cost}} \\ = & \quad \Theta_s f_s^{1/\alpha} w(\Omega) \end{aligned}$$

where

$$\Theta_s = (1+\tau_c)\left(2^{1-1/\alpha} + \frac{1}{z_s}\right) \tag{12}$$

Let $\Pi^j(\theta_j, p_j, f_j)$ be the expected profit a firm could earn when entering the submarket (θ_j, p_j, f_j). It is clear that:

$$\Pi^s(\theta_s, p_s, f_s, \Omega) = \quad \underbrace{p_s \Gamma_s(\theta_s) f_s}_{\text{Expected Revenue}} \quad - \quad \underbrace{\Theta_s f_s^{1/\alpha} w(\Omega)}_{\text{Cost}}$$

[14] The reason for the existence of two kinds of plants is explained at the beginning of the model. The firms' choice of becoming formal is implicitly given by the choice of the retail platform.

2.3.2 Firms "m"

The problem with firms in the channel "m" is more complicated. To minimized the cost of producing f_m units, firstly the owner must allocate the production between the formal plant (f_m^f) and the informal plant (f_m^i). His problem is

$$\min_{f_m^f f_m^i} \quad (1+\tau_c)(f_m^f)^{1/\alpha} w(\Omega) + (1+\varepsilon)(f_m^i)^{1/\alpha} w(\Omega)$$

$$\text{subject to } f_u^f + f_u^i = f_m$$

The solution for the above problem is:

$$\begin{cases} f_m^f = \dfrac{1}{\sigma+1} f_m & \text{where } \sigma = \left(\dfrac{1+\tau_c}{(1+\varepsilon)}\right)^{\frac{\alpha}{1-\alpha}} \\ f_m^i = \dfrac{\sigma}{\sigma+1} f_m \end{cases}$$

By using (11), the total cost of producing and retailing f_m units of goods for the shared platform "m" is:

$$\begin{aligned} x_m(f_m, \Omega) &= \underbrace{\frac{1+\tau_c}{(\sigma+1)^{1/\alpha}} f_m^{1/\alpha} w(\Omega)}_{\text{Formal plant production cost}} + \underbrace{(1+\varepsilon)\left(\frac{\sigma f_m}{\sigma+1}\right)^{1/\alpha} w(\Omega)}_{\text{Informal plant production cost}} + \underbrace{\frac{1}{z_m} f_m^{1/\alpha} w(\Omega)}_{\text{Informal retailing cost}} \\ &= \Theta_m f_m^{1/\alpha} w(\Omega) \end{aligned}$$

where

$$\Theta_m = \frac{1+\tau_c}{(\sigma+1)^{1/\alpha}} + (1+\varepsilon)\left(\frac{\sigma}{\sigma+1}\right)^{1/\alpha} + \frac{1+\varepsilon}{z_m} \tag{13}$$

So when entering the submarket (θ_m, p_m, f_m),the expected profit a firm "m" can earn:

$$\Pi(\theta_m, p_m, f_m, \Omega) = \underbrace{p_m \Gamma_m(\theta_m) f_m}_{\text{Expected Revenue}} - \underbrace{\Theta_m f_m^{1/\alpha} w(\Omega)}_{\text{Cost}}$$

Let $\Upsilon^j(\Omega)$ be the maximum expected profit a firm in the channel "j" can earn over all the available submarkets $(\theta_j,\, p_j,\, f_j)$, it means:

$$\Upsilon^j(\Omega) = \underset{(\theta_j, p_j, f_j)}{max} \Pi(\theta_j, p_j, f_j, \Omega)$$

Then at the equilibrium:

$$\Upsilon^s(\Omega) = \Upsilon^m(\Omega) \tag{14}$$

Otherwise, firms will flock into the channel with the higher expected profit until the expected profits of two channels are equal.

2.4 Competitive search for goods

In the section 2.1, we examined the household's choice given the information of submarkets in equilibrium are identified. In this section, we set up the condition for the submarket $(\theta_j,\, p_j,\, f_j)$ in the platform j in equilibrium.

Given the state (Ω, b) and the value function $V()$; let ξ be an object showing the household's value when they enter an arbitrary submarket $(\theta_s,\, p_s,\, f_s)$ in the platform "s" and an arbitrary submarket $(\theta_m,\, p_m,\, f_m)$ in the platform "m". We define:

$$\begin{aligned}
&\xi\left(\theta_s,\, p_s,\, f_s, \theta_m,\, p_m,\, f_m, \Omega, b\right) \\
&= \max_{d_s, d_m} \quad u[d_s A_s \theta_s^{1-\varphi} f_s + d_m A_m \theta_m^{1-\varphi} f_m] - h[d_s - \gamma D_s^-] - h[d_m - \gamma D_m^-] + \\
&\qquad\qquad \beta V[(1+\pi(\Omega))b + w(\Omega) - I_s p_s d_s A_s \theta_s^{1-\varphi} f_s - I_m p_m d_m A_m \theta_m^{1-\varphi} f_m\, , \Omega']
\end{aligned} \tag{15}$$

This is the household's value function with the last term is simply taken by using the household's budget constraint (5) to replace b'. The competitive goods search problem could be characterized as follows. The household will search and choose one submarket in each retail platform to maximize their value; however, the household could only meet firms j in the submarket $(\theta_j,\, p_j,\, f_j)$ if that submarket offers firms j the expected profit at least equal to $\Upsilon^j(\Omega)$ (the firm's participation constraint).

The directed search problem could be written as:

$$\max_{\theta_s, p_s, f_s, \theta_m, p_m, f_m} \xi\left(\theta_s, p_s, f_s, \theta_m, p_m, f_m, \Omega, b\right) \tag{16}$$

subject to

$$p_s \Gamma_s(\theta_s) f_s - \Theta_s f_s^{1/\alpha} w(\Omega) \geq \Upsilon^s(\Omega) \tag{17}$$

$$p_m \Gamma_m(\theta_m) f_m - \Theta_m f_m^{1/\alpha} w(\Omega) \geq \Upsilon^m(\Omega) \tag{18}$$

Let λ_j be the Lagrange multiplier of the firm's participation constraint in the channel j ($j = s, m$). We define:

$$\begin{aligned}\mathscr{L} =& \xi\left(\theta_s, p_s, f_s, \theta_m, p_m, f_m, \Omega, b\right) + \lambda_s \left\{ p_s \Gamma_s(\theta_s) f_s - \Theta_s f_s^{1/\alpha} w(\Omega) - \Upsilon^s(\Omega) \right\} + \\ & \lambda_m \left\{ p_m \Gamma_m(\theta_m) f_m - \Theta_m f_m^{1/\alpha} w(\Omega) - \Upsilon^m(\Omega) \right\}\end{aligned}$$

Take the first order condition of $\mathscr{L}$ with respect to θ_j, p_j and f_j ($j = s, m$), then impose the equilibrium condition, we could get two following important equations for each channel: (the detail solution in the appendix):

$$p_j(\Omega) = \frac{(1-\varphi) u(C(\Omega))}{I_j M(\Omega)} \tag{19}$$

$$\alpha \underbrace{p_j(\Omega) A_j \theta_j(\Omega)^{-\varphi} f_j}_{\text{Expected Revenue}} = (1-\varphi) \underbrace{\Theta_j f_j(\Omega)^{1/\alpha} w(\Omega)}_{\text{Cost}} \tag{20}$$

The equation (19) shows the condition for the prices at two retail channels in equilibrium. Although we set up the problem differently from BRS, the equation (19) has the similar form to their result. The difference between the price in the retail platform "s" and the one in the retail platform "m" is the sales tax wedge that is uncollected in the platform "m".

The equation (20) shows the relation between the firm j's expected revenue and its cost in equilibrium. To ensure firms to get the positive expected profit, we need to make the following assumption.

Assumption A1: $\alpha < 1 - \varphi$

Does this condition make sense? First, we must understand the role of φ. In the economy without any frictions in goods market $\varphi = 0$, total goods consumers buy $D_j(E_j/D_j)f_j$ equal the total goods supply in the market E_jf_j . As we expect our model is near with the frictionless benchmark model, the value of φ is very small near 0. So the above condition is not an unreasonable assumption.

2.5 Government

The government collects the sales tax and corporation tax to serve for its purpose of producing the public goods. The production function of public goods is:

$$y_g = l_g$$

Assume the government will produce exactly $\bar{g}$ unit of public goods in every period, the government expenditure will be: $G = \bar{g}w$. This means $\bar{g}$ workers are in the public sector every period. We assume the sales tax rate τ_s and the level $\bar{g}$ are fixed.[15] The government will adjust the rate τ_c each period to keep the government budget balanced. [16]

Let T_s and T_c be, respectively, the tax revenue from the sales tax and from the formal sector. The budget balance condition is:

$$T_s + T_c = G \tag{21}$$

2.6 Equilibrium

Let $l_j^R(\Omega)$ and $l_j^F(\Omega)$ be the measure of workers in the retail sector and manufacturing sector in the channel j. We define the equilibrium as followings:

Definition: Equilibrium is a set of decision rules and values for the household c, d_j, b', V as a function of its state (b, Ω); the retailers and firms' decision rules (l_j^R, l_j^F) as a function of the

[15] In most countries, the sales tax (or VAT) rate is much more stable than the corporation tax rate.

[16] After a jump in z_s, there might be multiple new steady states with different values of τ_c satisfying the budget balance condition. We assume the government will choose the path of τ_c such that the value of τ_c at the new steady state will be the minimum value satisfying the budget balance condition (21).

general state Ω, and the aggregate measure of shoppers in the supermarkets D_s, shoppers in the mom and pop's stores D_m, consumption C, profit Υ^j, the measure of firms in the channel "s" E_s, the measure of firms in the channel "m" E_m, wages w, tax τ_c, goods' prices p_s and p_m, market tightness θ_s and θ_m, production of firms f_s and f_m as functions of the aggregate state Ω that satisfy the following conditions:

1. Households' choices and value functions satisfy (1-5) and (6-7).

2. Firms' decision of hiring labor is to minimize the cost of production.

3. Competitive search conditions: Households and firms go to the submarkets satisfied (14) and (19-20).

4. Individual household decisions are consistent with aggregate variables.

5. Government budget satisfies (21).

6. Market clearing conditions:

$$b'(1,\Omega) = 1 \tag{22}$$

$$C_j(\Omega) = D_j(\Omega)\,\Delta_j[\theta_j(\Omega)]\,f_j(\Omega), \qquad (j = s, m) \tag{23}$$

$$\sum_{j=s,m} \left(l_j^R(\Omega) + l_j^F(\Omega)\right) = 1 - \bar{g} \tag{24}$$

$$\theta_s(\Omega) = \frac{E_s(\Omega)}{D_s(\Omega)} \tag{25}$$

$$\theta_m(\Omega) = \frac{1 - E_s(\Omega)}{D_m(\Omega)} \tag{26}$$

3 Analytical results

In this section, we will find how a permanent rise in the productivity of the organized retail sector z_s could affect the allocation of resources between the formal sector and the informal one. The shock on the retail productivity z_s transmits to the manufacturing sector through two channels: (i) (platform effect) by making the supermarkets' platform better for both households and firms making transactions, it induces firms to become formal to supply goods through the supermarkets' system; (ii) (tax effect) by allowing the government to lower the tax rate τ_c, it increases the firms' benefits of becoming formal.

Both effects lead to the result that more firms will choose to become formal and supply goods through the platform "s". Consequently, the aggregate productivity will increase, as a formal plant is more efficient than an informal one. We will discuss in detail two mechanisms, then illustrate why the shock on the retail sector has a huge amplified effect on the aggregate productivity in the economy. We start this section by making some assumptions on the form of households' utility function.

Assumption A2: *The individual household's utility in one period has a particular form:*

$$U(c,d_s,d_m) = \frac{c^{1-\zeta}}{1-\zeta} - \left(\frac{(d_s - \gamma D_s^-)^{1+\nu}}{1+\nu} + \frac{(d_m - \gamma D_m^-)^{1+\nu}}{1+\nu}\right), \quad \nu > 0$$

The parameter γ controls how persistent the shopping habit is. The bigger it is, the more unlikely households will switch to the unfamiliar shopping channel. The parameter ν controls how intensively a household can shop in one platform. Let think that a household includes a positive measure of members working in the different locations. Some job locations might be near the platform "m" and very far from the supermarkets' platform "s". Forcing these members to switch to the supermarkets' platform will increase substantially the search cost. Therefore, it is reasonable to assume the parameter ν is positive.

Assumption A3: $\gamma = 0$

In most parts of this section, we temporarily omit the shopping habit persistence. Although the shopping habit is an important parameter to reflect the transition of the model, it might be more useful to leave it aside to discover the main mechanism of the platform effect and the tax effect. When $\gamma = 0$, the model becomes static, making it more convenient to do the comparative exercises. We also omit the state variable Ω as $\gamma = 0$. We will be back to the full dynamic model in the quantitative exercise.

3.1 The Platform Effect

We first investigate the effect of z_s on the demand side. Then we show the interaction between the demand side and the supply side in choosing the platform to make transactions. Finally, we characterize how the shock in z_s transmits to the manufacturing sector.

Proposition 1: *Under the assumptions (A1)-(A3), the number of shoppers between two platforms in equilibrium:*

$$\frac{D_s}{D_m} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{\frac{1-\varphi}{\varphi\nu}} \left(\frac{A_s}{A_m}\right)^{\frac{1}{\varphi\nu}} \left(\frac{\Theta_m}{\Theta_s}\right)^{\frac{\alpha}{\varphi\nu}} \tag{27}$$

Under the additional assumption τ_c is fixed, then:

$$\frac{\partial(D_s/D_m)}{\partial z_s} = \left[\frac{\alpha(1+\tau_c)}{\varphi\nu z_s^2 \Theta_s}\right] \frac{D_s}{D_m} > 0 \tag{28}$$

where Θ_s and Θ_m are from (12) and (13).

The number of shoppers in the platform "s" depends on how much this platform is better than its rivalry in terms of the matching technology and the offered price. As the change in the supermarkets' productivity z_s triggers the change in the government's tax policy, we temporarily assume τ_c is fixed to separate the platform effect from the tax effect. The first remark is the response of (D_s/D_m) to the change in z_s could be huge, depending on two parameters φ and ν. The smaller those parameters are, the bigger is the platform effect on the demand side. What is the mechanism behind this?

An increase in the supermarkets' productivity will lower the cost of goods sold, and therefore the unit price, in the platform "s". Responding to the more attractive price, more households will switch to the supermarkets' platform for shopping. However, when many shoppers flock in the platform "s", the probability of finding goods, controlled by the parameter φ, might be declined. As we explained in the assumption (A1), the congestion of finding goods is extremely small in reality; we expect the parameter φ near 0. This implies that the relative demand could jump strongly in response to the change in z_s.

The other parameter, ν, controls how intensively households could go shopping in one platform. The lower in the unit price in the channel "s" will only pull a large number of shoppers to itself if the cost of switching the platforms is low for households. That is how the parameter ν goes into the equation (28). After fully understanding the effect of z_s on the relative demand (D_s/D_m), we move on investigating its effect on the supply side. We characterize an important equation (which remains valid in the dynamic setting) showing the interaction between the demand side and the supply side:

Proposition 2: *Under the assumption (A1)-(A2), in equilibrium, the ratio between measure of firms using the retail channel "s" and the ones using the retail channel "m" will satisfy:*

$$\underbrace{\frac{E_s}{E_m}}_{Supply} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{1/\varphi} \underbrace{\left(\frac{D_s}{D_m}\right)}_{Demand} \left(\frac{A_s}{A_m}\right)^{1/\varphi} \left(\frac{\Theta_m}{\Theta_s}\right)^{\alpha/\varphi} \tag{29}$$

Equation (29) clearly shows that a firm's decision of entering a specific retail channel depends on both the relative cost structure (Θ_m/Θ_s) and the demand side (D_s/D_m). A rise in the supermarket's productivity z_s simultaneously decreases the cost of goods sold Θ_s in the channel "s" and increases the relative demand in channel "s" (the proposition 1). The jumps in two important components on the right hand side in equation (29) implies that the change in z_s triggers the big movement of firms in the manufacturing sector from the channel "m" to the channel "s". To see it more clearly, we have:

Corollary 1: *Under the assumption (A1)-(A3), in equilibrium:*

$$\frac{E_s}{E_m} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{\frac{1-\varphi+\nu}{\varphi\nu}} \left(\frac{A_s}{A_m}\right)^{\frac{1+\nu}{\varphi\nu}} \left(\frac{\Theta_m}{\Theta_s}\right)^{\frac{\alpha(1+\nu)}{\varphi\nu}} \tag{30}$$

Assume the tax τ_c is fixed:

$$\frac{\partial(E_s/E_m)}{\partial z_s} = \left[\frac{\alpha(1+\nu)(1+\tau_c)}{\varphi\nu z_s^2\Theta_s}\right]\frac{E_s}{E_m} > 0$$

The equation (30) is just the result derived from (28) and (29), but it is important in the sense of showing the aggregate effect of the retail sector's shock z_s on the manufacturing sector. The supply side, like the demand side, is very sensitive to the small change in A_s or z_s, given φ is near 0. That happens due to the propagation effect when the retail sector plays the role as a two-sided platform. The lower retailing cost makes more firms choose the platform "s" to approach households. When households observe more brands now in the supermarkets' platform, they switch to the platform "s" as they can buy more goods there. The change in demand, in turn, has a feedback effect on the supply side. The interaction between the demand side and the supply side is propagated as there is only small friction in the goods market. (Figure 3).

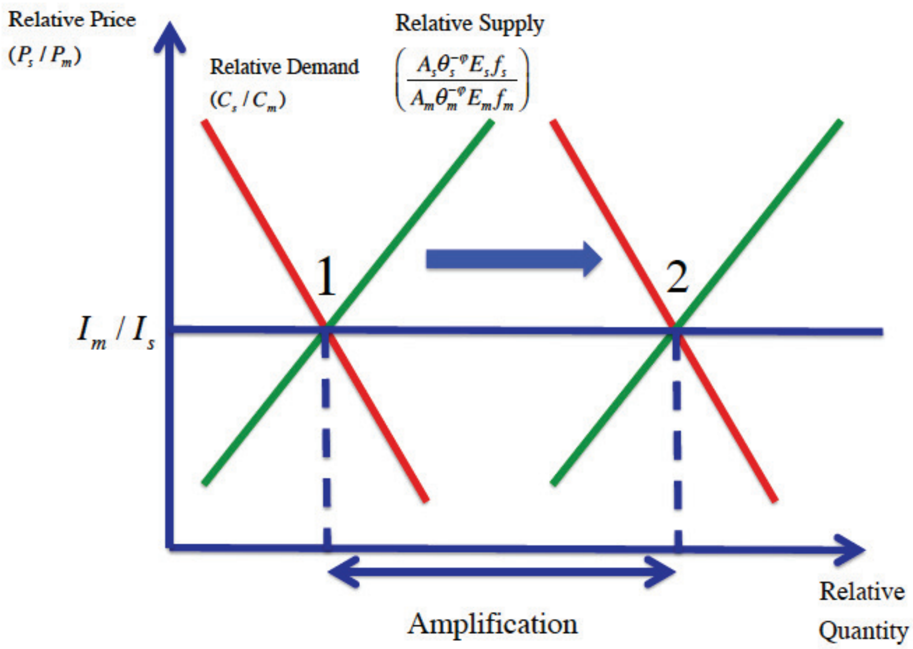


Figure 3: The effect of z_s on economy with searching

Discuss the role of search friction in our model

By introducing the frictions in goods market, we highlight the role of the retail sector as the two-sided platform. In the searching environment, a small positive productivity shock on supermarkets can be amplified through the interaction between demand and supply. The increase in z_s shifts both demand (consumers) and supply (firms) to the platform s, increasing transaction in supermarkets.

In the environment without searching, the literature normally assumes the consumer will consume a basket of goods (C) from both goods: the one bought from supermarkets (C_s) and the other from mom-and-pop stores C_m:

$$C = (\omega C_m^{\alpha} + (1-\omega) C_s^{\alpha})^{1/\alpha}$$

In this case, the role of the retail sector is much smaller. An increase in z_s only shifts the relative supply side to the right. The supermarkets can only attract consumers by lowering the price of goods (the movement along the demand curve). In our model, supermarkets can attract customers not only through the price but also by having more firms supplying goods there, making the purchasing process more convenient for consumer. The distance of the shift in the supply side in the model without goods searching is also smaller as the interaction between demand and supply is lost. Figure 4 illustrates our argument.

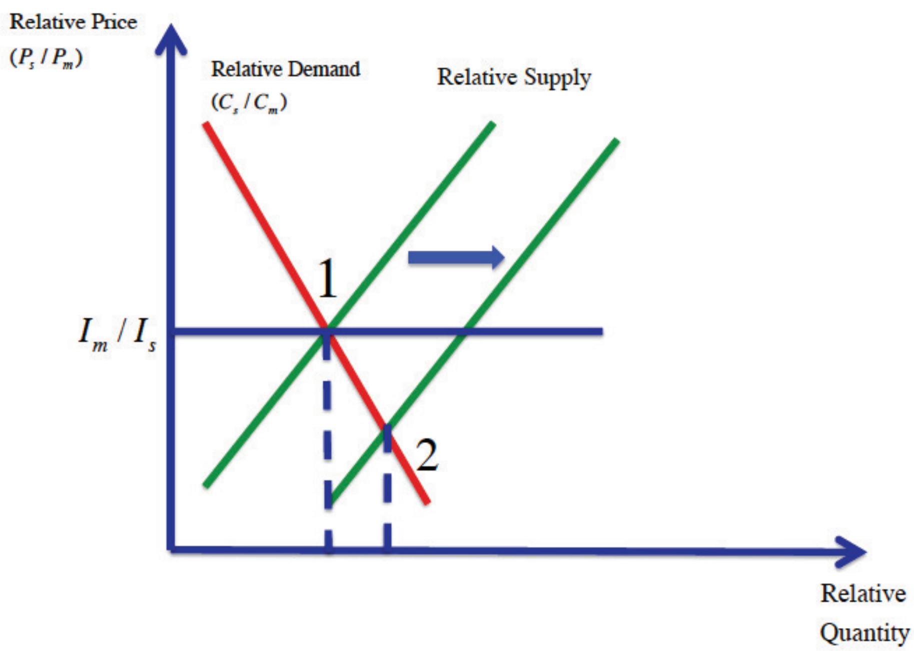


Figure 4: The effect of z_s on economy without searching

Recalling that all the suppliers in the channel "s" are formal (supermarkets only sign contract with formal firms), a change in the productivity of the organized retailers could hugely motivate informal firms to become formal. There is an implication in two equations (27) and (29) that the retail sector might be the key sector for the transformation process from informality to formality in emerging economies. Instead of inefficiently trying detecting all informal firms, the government could use the resources to promote the organized retail platform.

It is also worth noting that households will only choose one channel for shopping in the case of no friction in the goods market ($\varphi = 0$) or the marginal disutility from sending a shopper to a platform is constant ($\nu = 0$). In the former case, there is no role for demand, so firms just choose the best channel for their cost structures. In the latter case, households do not incur any extra costs when intensively shopping in one platform. They just choose the best platform to find goods. This forces all firms to flock to one channel to adapt the demand side.

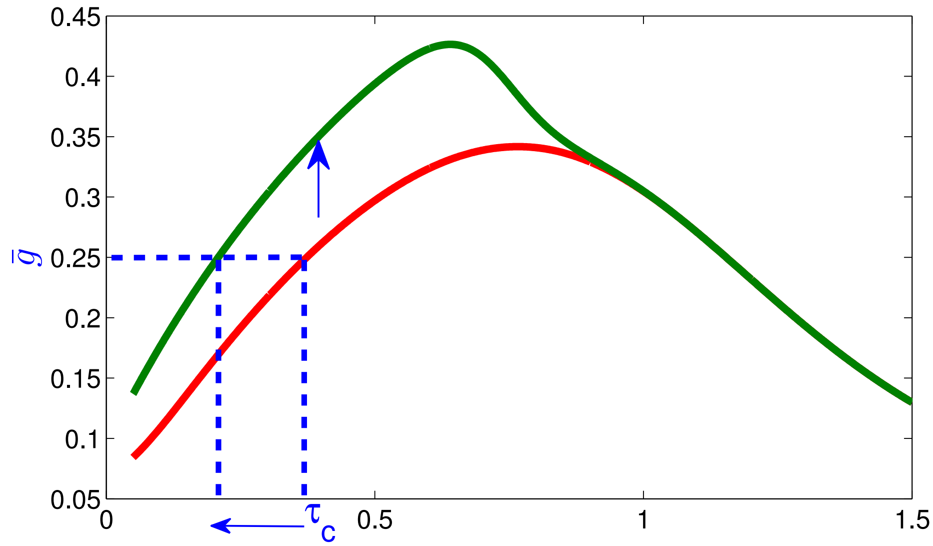


Figure 5: Laffer curve responds to the rise in z_s

3.2 The Tax Effect

If more transactions take place in the supermarkets' platform, the government tax revenue will increase. This allows the government to cut the tax rate τ_c while their tax receipts still meet the spending needs of producing public goods. As the formal sector tax rate τ_c affects the firms' decision whether to become formal, the lower tax rate has a feedback effect on the aggregate economy. Figure 5 shows the change in the Laffer curve when z_s goes up.

As z_s goes up, both households and firms prefer the supermarkets' platform to make transactions, increasing the percentage of goods transacted in supermarkets. Therefore, the tax rate τ_c could be reduced firstly due to the higher contribution of the sales tax. Second, more firms become formal to sign contracts with supermarkets, making the burden of tax on each individual formal firm and supermarket also reduce.

The effect of reducing the tax rate τ_c will affect the whole allocation in the economy, not only between two channels in the retail sector and the manufacturing sector, but also within the channel "m" itself. The result is summarized in the following proposition.

Proposition 3: *Under (A1)-(A3), when there is a reduction in τ_c :*

(i) Firm "m" will reallocate the production from the informal plant toward the formal plant (within channel reallocation).

(ii) There is also a firms' movement from the channel "m" to the channel "s" (between channel reallocation).

Intuitively, when τ_c reduces, the option of using the informal plant becomes less attractive (σ decreases), firms in the channel "m" produce more outputs by the formal plant, that explains for the allocation within the channel "m". For the reallocation between the two channels (detail proof in the appendix), the basic idea is both channels enjoy the reduction in τ_c. However, the relative cost of goods sold becomes more favorable to the channel "s" as τ_c affects both the retailers and the manufacturers in this channel:

$$\frac{\partial\left(\dfrac{\Theta_m}{\Theta_s}\right)}{\partial\tau_c} = \frac{-\left[2^{1-1/\alpha}+\dfrac{1}{z_s}\right]\left[(1+\varepsilon)\left(\dfrac{\sigma}{\sigma+1}\right)^{1/\alpha}+\dfrac{1}{z_m}\right]}{\Theta_s^2} < 0$$

This leads to the reallocation from the channel "m" to the channel "s". Both reallocations substantially gain the higher productivity for the economy.

3.3 The Aggregate Productivity

The change in the productivity of supermarkets z_s, through the combination of the platform effect and the tax effect, could raise significantly the total outputs and the aggregate productivity of the economy. Mom-and-pop stores in the retail sector and informal plants in the manufacturing sector are not as efficient as supermarkets and formal plants. When z_s goes up, in the retail sector, workers will move from mom and pop stores to supermarkets. In the manufacturing sector, workers will move from informal plants to formal plants. If we consider manufacturing and retailing as the one whole process with the production function $f = (Zl)^\alpha$, we can divide goods into three groups, based on how they are produced and retailed. We can determined the productivity of each group as follows:

$$
\begin{aligned}
Z_{FS} &= \left(1+\frac{1}{z_s}\right)^{-1} && \text{(Formal plant, supermarket's retailing)}\\
Z_{FM} &= \left(1+\frac{1}{z_m}\right)^{-1} && \text{(Formal plant, mom and pop's retailing)}\\
Z_{IM} &= \left(1+\varepsilon+\frac{1}{z_m}\right)^{-1} && \text{(Informal plant, mom and pop's retailing)}
\end{aligned}
$$

Under the condition $z_s > z_m$ and $\varepsilon > 0$, we have $Z_{FS} > Z_{FM} > Z_{IM}$. When z_s goes up, through the combination of the platform effect and the tax effect, goods will be reallocated from the IM process to the production process FM, and especially FS. The transmission of shocks from the retail sector to the manufacturing sector significantly improves the productivity of the whole economy.

4 Experiment

We use the above model to do some experiments, gauging the impact of the transformation of the retail sector on the aggregate productivity in India. Although India is one of the biggest retail markets in the world, the organized retailers (supermarkets, modern stores) only account for about 10 percent of the market share. Market is dominated by informal, unorganized, small mom-and-pop stores. Before 2012, the Indian government forbade all foreign groups from any ownership in supermarkets, convenience stores or any retail outlets. This made the productivity of the Indian retail industry much lower in comparison to other countries allowing FDI in this sector. However, in Dec 2012, India significantly changed the policy, allowing multi-brand retailers with 51 percent ownership from foreign groups to enter the local market. The appearances of the global retailers are expected to boost the productivity of the organized retail sector.

To estimate the impact of this structural change, we calibrate the initial steady state of the model to the Indian economy in 2012.[17] Then we make a simulation of the model to examine

[17] There are some indicators about informal firms in India in which we could only find data from the survey in 2006.

Table 1: Parameter values

Param.	Definition	Value	Source
ζ	Risk aversion	2	Set Exogenously
β	Discount factor	0.99	Set Exogenously
γ	Habit persistence parameter	0.3	Set Exogenously
α	Span of control parameter	0.9	Set Exogenously
ε	Labor productivity in informal plants	1.13	Match targets
τ_s	The VAT rate on consumption goods	0.1	Match targets
A	The matching of the retail platform	0.95	Match targets
φ	Elasticity of the matching function	0.07	Match targets
$\bar{g}$	Government's public goods produced in each period	0.11	Match targets
ν	The elasticity of shopping	0.1	Match targets
z_s	Productivity of supermarkets	4.31	Match targets
z_m	Productivity of mom-and-pop stores	4.15	Match targets
δ	The fraction of VAT collected in the informal platform	0.3	Match targets

the transition path when the productivity of supermarkets z_s jumps to a new level.[18] We fix the productivity of the manufacturing sector and mom-and-pop stores, as we want to focus on the effect of this structural change alone on the aggregate productivity.

4.1 Calibration

The model at the initial steady state is calibrated in order to match the Indian economy in 2012. The form of the household's utility function is the same as the one in the assumption (A2). We also assume the matching technology in two platforms are identical, so $A_s = A_m = A$. We divide parameters into two groups: (i) a set of parameters are set exogenously or estimated outside the model, (ii) the other ones are calibrated within the model to match some particular targets. Table 1 summarizes the parameter values.

To estimate the difference between the productivity of formal plants and informal plants in the manufacturing sector in India, we use two datasets to estimate the difference in value-added per worker. The first dataset is "Informal (Micro) Enterprises Survey in India" (IES) conducted by The World Bank (2006). The second dataset is the Annual Survey of Industries (ASI), which

[18]We could think the appearance of the foreign retailers as the shock pushing up z_s to a new level. We assume the foreign retailers still use all the local labors for their business activity.

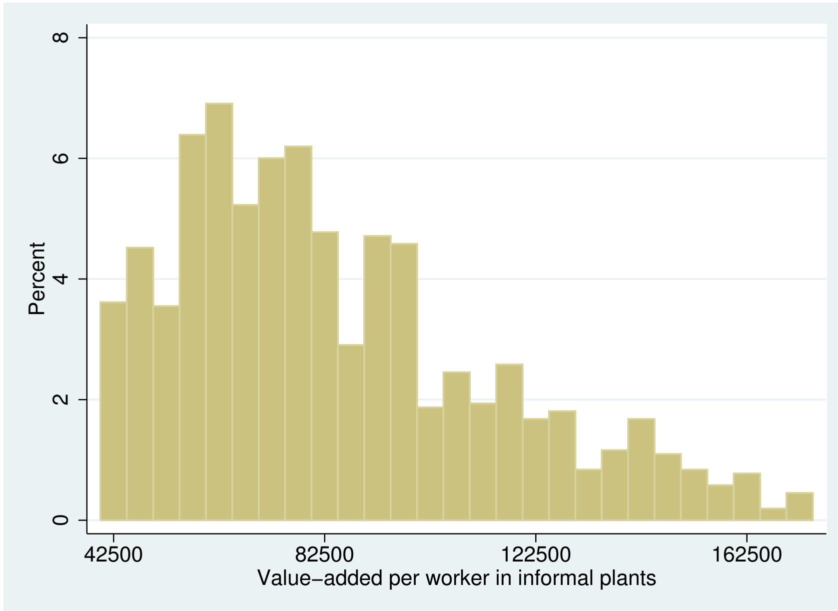


Figure 6: Indian Informal Plant's Labor Productivity

covers the registered manufacturing plants in India. Our method to estimate the value-added per worker is similar to Lagakos (2009). Data and method is recorded in our data appendix.

The average value-added per worker of a formal plant estimated from the annual report of ASI (2006) is (Rs 214,256). The value-added per worker of an informal plant, mainly ranging from Rs 42500 to Rs 100000, is much lower than this number. The Figure 6 shows the distribution of the value-added per worker calculated for informal plants from the "Informal Enterprise Survey" .

It can be clearly seen that the average productivity of an informal plant is only as half as the one of the formal plant in the Indian manufacturing sector. To pin down into one point data serving for the calibration purpose, we calculate the average labor's productivity of all informal plants, which is (Rs 95,801). The result is an informal plant's productivity is only equal to 47% of the formal plant's productivity. We use this information to calibrate the value of ε:

$$\frac{1}{1+\varepsilon} = 0.47 \quad \leftrightarrow \quad \varepsilon = 1.13$$

Since 2005, India has replaced the sales tax system by VAT (Value-added tax). Our model has only one stage of production, implying no differences between the effect of sales tax and

VAT. Although VAT system is different across states, there are two common VAT rates in India: 4 percent (for food) and 12.5 percent (for majority of goods). We calculate the weighted tax rate based on the breakdown revenue in organized retailing sales (IBEF, 2013). The result is $\tau_s = 0.1$ reported in Table 1.

The remaining parameters are calibrated to match the statistics in the initial steady state with the Indian economy (Table 2). The ratio between the government's final consumption expenditure (excluding government investment) and Indian GDP is from World Bank 2012; the contribution of the retail sector in GDP is from Ablett et al. (2007); the market share of the organized retailers is from IBEF (2013), and the share of informal worker in retail sector is from Naik (2009). For the ratio between VAT revenue T_s and the revenue T_c, the ratio is calculated from two reports. The total VAT revenue (217,000 in crores of Rupees) across 28 states in the fiscal year in 2011-2012 is from Santra and Hati (2014). The total tax revenue from the formal sector[19] (493,158 in crores of Rupees) is from Ministry of Finance (2013).

The calibration for two parameters A and φ is specific to this kind of model. We match this two parameters with the indicator of inventory-to-sales ratio (unsold-to-sold goods in the model) in the organized manufacturing sector in India and the ratio between household's time spent on shopping and working. The first indicator is calculated from the ASI report as the ratio between the physical working capital (1666.2 in crores of Rupees) in census to the total output (9379.55 in crores of Rupees). The second indicator could not be found in the Indian survey of households. We must borrow this target from the American's time use survey (2013). On average, an American spends 0.37 hours/day on purchasing the consumer goods and 3.14 hours/day on working. The ratio between shopping time and working time is roughly 0.12 .

4.2 Steady State

We compare the Indian economy at 2012 (the initial steady state) to the two hypothetical steady states where the organized retailers in India capture the market share identical to the level in

[19] We calculate this tax revenue by adding the total corporation tax revenue and the income tax revenue. As the model has only labor as the factor of production and tax is imposed on total cost, it is reasonable to think T_c as the sum of income tax and corporation tax. See Ordonez (2014) for the similar calibration.

Table 2: Calibration Target

Statistics	Target	Model
Government's final consumption expenditure - GDP ratio	0.11	0.10
The ratio of tax revenue T_c to VAT revenue	2.2	2.1
Contribution of the retail sector to GDP	0.16	0.17
Organized retailers' market share	0.08	0.08
Share of informal worker in retail sector	0.94	0.95
Inventory-to-sales ratio in the manufacturing sector	0.18	0.20
Shopping time- working time ratio	0.12	0.14

China (20 percent in 2006) and the one in US (73.51 percent in 2007).[20] We assume that the Indian economy reaches to the new steady state by the change in the supermarkets' productivity alone z_s. Table 3 below compares the key economic indicators in these steady states.

Table 3: Steady States Comparison

	Current India (8%)	Organized retailing same as China (20%)	Organized retailing same as US (73.51%)
z_s	4.310	4.425	4.62
C	0.6186	0.6242	0.6495
Y	0.7450	0.7511	0.7832
E_s	0.0764	0.1900	0.7284
τ_c	0.1027	0.0898	0.0389
$\mathscr{Z}$	0.8371	0.8439	0.8800
$\mathscr{Z}_M$	1.0394	1.0478	1.0874
$\mathscr{Z}_R$	4.3007	4.3377	4.6127

We define $\mathscr{Z}$, $\mathscr{Z}_M$, $\mathscr{Z}_R$ respectively as the aggregate productivity of the private sector, the

[20]We use the US Economic Census 2007 to calculate the the market share of the organized retailers in US. We define the organized retailers as the one has more than 100 employees. We calculate the total sales of this group (exclude the sales on "automotive dealers and service station" (code 441 on NAICS 2002), then compare it to the total sales of all retail industry to get this ratio.

manufacturing sector and the retail sector:

$$\begin{aligned}
\mathscr{Z} = \quad & \frac{\text{Total goods produced (Y)}}{\text{Labor in the private sector}} & = \frac{E_s f_s + E_m f_m}{1-\bar{g}} \\
\mathscr{Z}_M = \quad & \frac{\text{Total goods produced (Y)}}{\text{Labor in the manufacturing sector}} & = \frac{E_s f_s + E_m f_m}{l_s^F + l_m^F} \\
\mathscr{Z}_M = \quad & \frac{\text{Total goods produced (Y)}}{\text{Labor in the retail sector sector}} & = \frac{E_s f_s + E_m f_m}{l_s^R + l_m^R}
\end{aligned}$$

The relationship between $\mathscr{Z}$, $\mathscr{Z}_M$, $\mathscr{Z}_R$ is:

$$\frac{1}{\mathscr{Z}} = \frac{1}{\mathscr{Z}_M} + \frac{1}{\mathscr{Z}_R} \tag{31}$$

Recalling that the total labor supply in the private sector in these steady states are identical, the increase in outputs comes both from the resources reallocation in the manufacturing sector and the retail sector. A 2.6 percent increase in the productivity of the organized retailers leads to an increase in total outputs of 0.82 percent. It also results in an increase in the aggregate productivity of 0.81 percent. On average, a 1 percent increase in z_s in India leads to an increase in the aggregate productivity of 0.31 percent. Recall that in 2012 the number of workers in supermarkets only accounts for 1 percent of the total labor force in India. How does a sector with only 1 percent of total labor force could trigger this huge effect on the whole economy?

We decompose the total effect into parts to understand the mechanism behind the aggregate number. By the equation (31), we could break down the increase of the aggregate productivity into the contribution of the manufacturing sector and the retail sector. When z_s goes up by 2.6 percent, $\mathscr{Z}_M$ goes up by 0.80 percent, $\mathscr{Z}_R$ rises up by 0.86 percent. We could rewrite (31) under the form:

$$\Delta\left(\frac{1}{\mathscr{Z}}\right) = \underbrace{\Delta\left(\frac{1}{\mathscr{Z}_M}\right)}_{\text{Contribution of manufacturing sector}} + \underbrace{\Delta\left(\frac{1}{\mathscr{Z}_R}\right)}_{\text{Contribution of retail sector}}$$

The result is that 80 percent of the change in the aggregate productivity is caused by the change in the structure of the manufacturing sector, while the remaining 20 percent is due to the change in the retail sector. The key result here is the retail productivity shock, through the platform effect and the tax effect, has a spillover effect on the manufacturing sector. The biggest benefit of reforming the retail sector in India is the shift of the manufacturing sector to the formality.

To see the transformation of the economy, the next section will discuss about the transition between these steady states. Analyzing the transition also gives us the image of the resource reallocation process as well as the pace of transformation, which could not be observed through the comparison of the steady states.

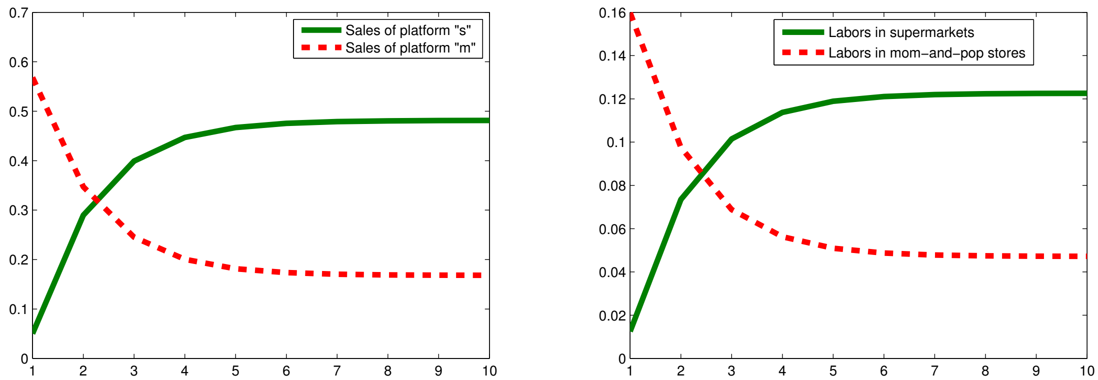
4.3 The Transition Experiment

We simulate the transition by allowing z_s to jump from 4.31 to 4.62 at the period 1, then it stays at the new level forever. In this section, we illustrate the resource reallocation within the retail sector as well as the manufacturing sector by graphs.

The Retail Sector: Even at the initial steady state, the productivity of supermarkets is higher than the one of small informal mom-and-pop stores. However, 96 percent of workers in the retail sector stuck in the latter, leading to a massive inefficiency for the Indian economy. When the productivity of supermarkets z_s goes up, the aggregate productivity of the private sector first gains from the reallocation of labor from mom-and-pop stores to supermarkets. Figure 7(a) shows how the market share of supermarkets changes and Figure 7(b) illustrates the labor trend in the retail sector when z_s goes up.

The Manufacturing Sector: The increase in the productivity of supermarkets also leads to the reallocations of firms in the manufacturing sector: firms move from the channel "m" to the channel "s" to supply goods for the supermarkets' platform. Figure 8(a) illustrates the transition of firms between two channels and Figure 8(b) shows the rise in the aggregate productivity of the manufacturing sector with this transition.

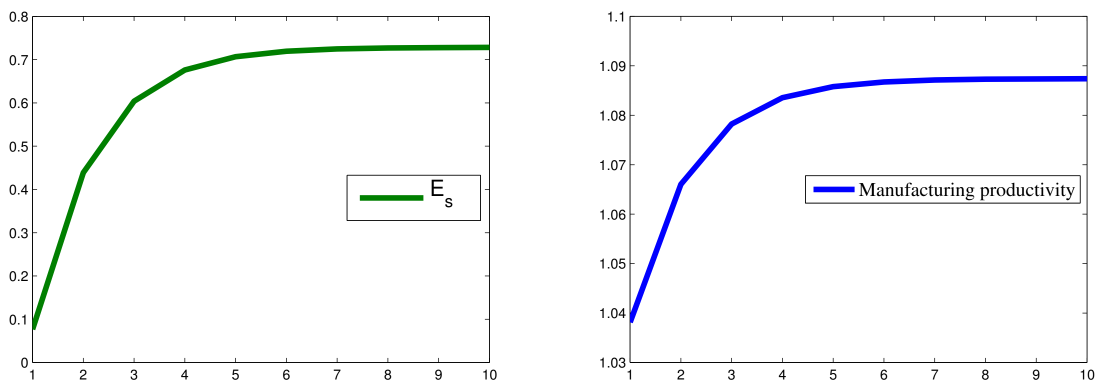
From two datasets IES and ASI, we did estimate that the formal plant' s productivity nearly doubles the informal plant's productivity. The transition of the manufacturing sector from the channel "m" (mixed between formal plants and informal plants) to the channel "s" (all formal plants) significantly raises the aggregate productivity of the manufacturing sector as the inefficient informal plants are replaced by the formal plants. And this transition is also the most



(a) Sales in two retail platforms

(b) Distribution of workers in retail sector

Figure 7: The reallocation within the retail sector



(a) The transition of firms

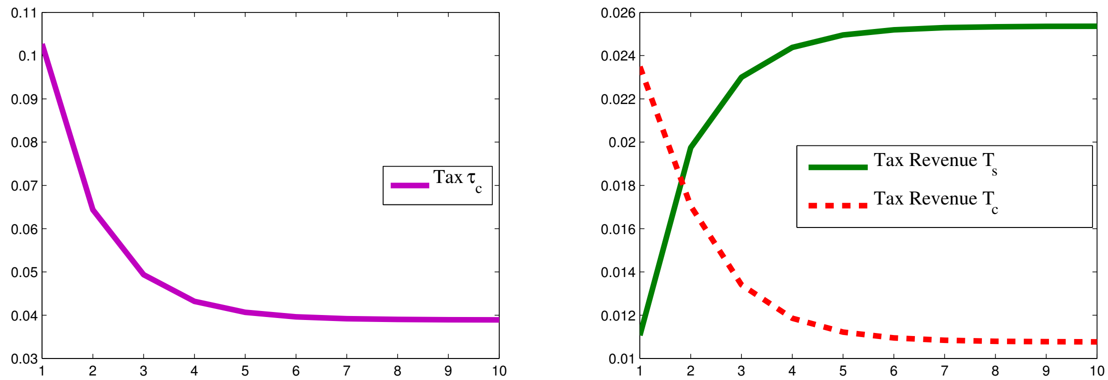
(b) The productivity $\mathscr{Z}_M$

Figure 8: The reallocation in the manufacturing sector

important one to push up the total outputs.

The Government Budget: The transformation also improves the government budget, leaving room for the possibility of cutting tax rate τ_c on the formal sector. As analyzing in the section 3, the reallocation between two channels is a consequence of both the platform effect and the tax effect. The lower in the tax rate τ_c mitigates the distortion favoring informal plants. The simulation also lets us see the cut in formal sector tax rate could be large.

The rise in the productivity of supermarkets also changes the structure of the government's tax revenue. In the developed economy like UK, the contribution of VAT doubles the contribution of the corporation tax. In the fiscal year 2011-2012, the VAT revenue in UK was £100



(a) Tax rate τ_c

(b) The government tax revenues

Figure 9: Government Sector

billions while the corporation tax revenue was only £48 billions[21]. In the case of India, the government could not collect VAT from the informal economy, shifting the burden of revenue on the corporation tax. In the fiscal year 2011-2012, the contribution of VAT was 217,000 in crores of Rupees, only equal two-third the contribution of the corporation tax (322,816 in crores of Rupees)[22]. The increase in z_s changes this poor pattern, improving the revenue from VAT. The formalization process and the rise in the contribution of VAT revenue (Figure 9) are the main reasons for the less burden of tax on each formal firm.

5 Conclusion

This paper examines the link between the retail sector, the manufacturing sector, the tax policy and the informality problem of emerging economies in the dynamic framework. With the special role as the platform, the transformation of the retail structure toward the big-box retailing could be the best remedy to formalize the informal economy. The spread of big-box retailing gives firms the big incentive to become formal. Transactions mainly taking place in the organized platform also help the government budget and reduce the tax burden for formal firms. The increase in the productivity of the organized retailers could propagate the aggregate productivity

[21] Data Source: Office for Budget Responsibility

[22] Data Source: Santra and Hati (2014) and Ministry of Finance (2013).

of the whole economy.

We also use the model to shed lights for the Indian economy, where there are hot debates on the government's policies of protecting small mom-and-pop stores. We find out that, the role of the organized retailing is extremely underestimated in the Indian case. On average, a 1 percent increase in the productivity of the organized retailers leads to an increase in the aggregate productivity of the Indian economy of 0.3 percent; in which, the contribution of the change in the retail sector itself is only 20 percent. The remaining 80 percent is contributed indirectly by the change in the structure of the manufacturing sector.

The significance of the retail sector might still be underestimated. The role of providing information as well as managing the inventories for firms to mitigate the demand uncertainty is worth considering in future research.

References

Abel, Andrew B. 1990. "Asset Prices under Habit Formation and Catching Up with the Joneses." *American Economic Review*, 80(2): 38–42.

Ablett, Jonathan, Aadarsh Baijal, Eric Beinhocker, Anupam Bose, Diana Farrell, Ulrich Gersch, Ezra Greenberg, Shishir Gupta, and Sumit Gupta. 2007. "The 'bird of gold': The rise of India's consumer market." McKinsey Global Institute.

Acemoglu, Daron, and Fabrizio Zilibotti. 2001. "Productivity Differences." *The Quarterly Journal of Economics*, 116(2): 563–606.

Bai, Yan, Jose-Victor Rios-Rull, and Kjetil Storesletten. 2011. "Demand Shocks that Look Like Productivity Shocks." Society for Economic Dynamics 2011 Meeting Papers 99.

Basu, Kaushik. 2003. *Analytical Development Economics: The Less Developed Economy Revisited.* Vol. 1. 1 ed., The MIT Press.

Basu, Susanto, and David N. Weil. 1998. "Appropriate Technology And Growth." *The Quarterly Journal of Economics*, 113(4): 1025–1054.

Bohme, Marcus, and Rainer Thiele. 2012. "Is the Informal Sector Constrained from the Demand Side? Evidence for Six West African Capitals." *World Development*, 40(7): 1369–1381.

Campbell, John Y., and John Cochrane. 1999. "Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107(2): 205–251.

De Paula, Aureo, and Jose A. Scheinkman. 2010. "Value-Added Taxes, Chain Effects, and Informality." *American Economic Journal: Macroeconomics*, 2(4): 195 – 221.

Erasmo, Pablo N., and Hernan J. Moscoso Boedo. 2012. "Financial Structure, Informality and Development." *Journal of Monetary Economics*, 59(3): 286 – 302.

Gollin, Douglas. 1995. "Do Taxes on Large Firms Impede Growth? Evidence from Ghana." University of Minnesota, Economic Development Center Bulletins 7488.

Gourio, Francois, and Leena Rudanko. 2011. "Customer Capital." National Bureau of Economic Research, Inc NBER Working Papers 17191.

Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *The Quarterly Journal of Economics*, 124(4): 1403–1448.

IBEF. 2013. "India: Retail Sector Report." India Brand Equity Foundation.

International Tax Compact. 2010. "Addressing tax evasion and tax avoidance in developing countries." Deutsche Gesellschaft fur.

Khalifa, Mohamed, Moez Limayem, and Vanessa Liu. 2002. "Online Customer Stickiness: A Longitudinal Study." *Journal of Global Information Management (JGIM)*, 10(3): 1–14.

Lagakos, David. 2009. "Superstores or mom and pops? Technolgy adoption and productivity differences in retail trade." Federal Reserve Bank of Minneapolis Staff Report 428.

Lagos, Ricardo. 2006. "A Model of TFP." *Review of Economic Studies*, 73(4): 983 – 1007.

Lewis, William W. 2004. *The power of productivity: Wealth, poverty, and the threat to global stability.* Chicago and London:.

Matha, Thomas Y., and Olivier Pierrard. 2011. "Search in the product market and the real business cycle." *Journal of Economic Dynamics and Control*, 35(8): 1172–1191.

Ministry of Finance. 2013. "Receipt Budget 2013-2014." *http://www.indiabudget.nic.in/ub2013-14/rec/tr.pdf*.

Moen, Espen R. 1997. "Competitive Search Equilibrium." *Journal of Political Economy*, 105(2): 385–411.

Naik, Ajaya. 2009. "Informal Sector and Informal Workers in India." IARIW-SAIM Conference 2.

Ordonez, Julio Cesar Leal. 2014. "Tax collection, the informal sector, and productivity." *Review of Economic Dynamics*, 17(2): 262–286.

Petrosky-Nadeau, Nicolas, and Etienne Wasmer. 2011. "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions." Institute for the Study of Labor (IZA) IZA Discussion Papers 5763.

Porta, Rafael La, and Andrei Shleifer. 2008. "The Unofficial Economy and Economic Development." National Bureau of Economic Research, Inc NBER Working Papers 14520.

Reardon, Thomas, Peter Timmer, and Julio Berdegue. 2004. "The Rapid Rise of Supermarkets in Developing Countries: Induced Organizational, Institutional, and Technological Change in Agrifood Systems." *The Electronic Journal of Agricultural and Development Economics*, 1(2): 168–183.

Restuccia, Diego, and Richard Rogerson. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants." *Review of Economic Dynamics*, 11(4): 707–720.

Santra, Sattwik, and Koushik Kumar Hati. 2014. "India's Move from Sales Tax to VAT: A Hit or Miss?" Centre for Studies in Social Sciences, Calcutta.

Sharma, Pankaj. 2011. "Retail-Organized vs Unorganized or coexistence of Organized and Unorganized." Zee Research Group.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

The World Bank. 2006. "Enterprise Surveys." *http://www.enterprisesurveys.org*.

A Data Appendix

To estimate the difference between the productivity of formal plants and informal plants in the manufacturing sector in India, we use two datasets. The first one is "Informal (Micro) Enterprises Survey in India" (IES) conducted by The World Bank (2006). It provides the detail performance of 1549 informal plants in the manufacturing sector in the fiscal year 2005-2006. The second dataset is the Annual Survey of Industries (ASI), which covers the registered manufacturing plants in India. It is conducted by the Indian Ministry of Statistics and Programme Implementation. For ASI, we only use the annual report instead of the whole dataset.

Follow Lagakos (2009), the method we use to estimate the labor productivity is the value-added per worker method. In both datasets, we define value-added as the value of goods sold minus the cost of raw materials and intermediate goods used in production. The electricity cost is also included in the cost of intermediate goods. The total number of workers in a plant is defined as the total number of paid and unpaid employees plus the owner.When calculating a total number of workers in a plant, we let a part-time worker be equivalent to a half of full-time worker.

B Mathematical Appendix

Competitive goods search:

First, we start from the object ξ. Let d_m^*, d_s^* be the values that maximize (15) when the household enter an arbitrary submarket (θ_s, p_s, f_s) and an arbitrary (θ_m, p_m, f_m). Then:

$$\begin{aligned}
&\xi((\theta_s, p_s, f_s), (\theta_m, p_m, f_m), \Omega, b) \\
&= u[d_s^* A_s \theta_s^{1-\varphi} f_s + d_m^* A_m \theta_m^{1-\varphi} f_m] - h[d_s^* - \gamma D_s^-] - h[d_m^* - \gamma D_m^-] + \\
&\quad \beta V[(1+\pi(\Omega))b + w(\Omega) - I_s p_s d_s^* A_s \theta_s^{1-\varphi} f_s - I_m p_m d_m^* A_m \theta_m^{1-\varphi} f_m, \Omega']
\end{aligned}$$

We use the short notation:

$$V_b = \left.\frac{\partial V(b', \Omega')}{\partial b'}\right|_{d_s = d_s^*, d_m = d_m^*} \qquad \text{and} \qquad u_c = u'[d_s^* A_s \theta_s^{1-\varphi} f_s + d_m^* A_m \theta_m^{1-\varphi} f_m]$$

Apply the envelope theorem, we have for $j = s,m$:

$$\frac{\partial \xi((\theta_s, p_s, f_s),(\theta_m, p_m, f_m),\Omega,b)}{\partial p_j} = -I_j d_j^* A_j \theta_j^{1-\varphi} f_j \beta V_b \tag{32}$$

$$\frac{\partial \xi((\theta_s, p_s, f_s),(\theta_m, p_m, f_m),\Omega,b)}{\partial \theta_j} = (1-\varphi) d_j^* A_j \theta_j^{-\varphi} f_j \big(u_c - I_j p_j \beta V_b\big) \tag{33}$$

$$\frac{\partial \xi((\theta_s, p_s, f_s),(\theta_m, p_m, f_m),\Omega,b)}{\partial f_j} = d_j^* A_j \theta_j^{1-\varphi} \big(u_c - I_j p_j \beta V_b\big) \tag{34}$$

We come back to the household's search problem with the Lagrangian:

$$\begin{aligned}\mathscr{L} =& \xi((\theta_s, p_s, f_s),(\theta_m, p_m, f_m),\Omega,b) + \lambda_s \Big\{ p_s \Gamma_s(\theta_s) f_s - \Theta_s f_s^{1/\alpha} w(\Omega) - \Upsilon^s(\Omega) \Big\} + \\ & \lambda_m \Big\{ p_m \Gamma_m(\theta_m) f_m - \Theta_m f_m^{1/\alpha} w(\Omega) - \Upsilon^m(\Omega) \Big\}\end{aligned}$$

Take the first order condition of $\mathscr{L}$, respectively, with respect to p_j, θ_j, f_j ($j = s,m$) and using (32), (33), (34). We have:

$$\frac{\partial \mathscr{L}}{\partial p_j} = -I_j d_j^* A_j \theta_j^{1-\varphi} f_j \beta V_b + \lambda_j A_j \theta_j^{-\varphi} f_j = 0 \tag{35}$$

$$\frac{\partial \mathscr{L}}{\partial \theta_j} = (1-\varphi) d_j^* A_j \theta_j^{-\varphi} f_j \{u_c - I_j p_j \beta V_b\} - \lambda_j \varphi p_j A_j \theta_j^{-\varphi-1} f_j = 0 \tag{36}$$

$$\frac{\partial \mathscr{L}}{\partial f_j} = d_j^* A_j \theta_j^{1-\varphi} \big(u_c - I_j p_j \beta V_b\big) + \lambda_j \Big\{ p_j A_j \theta_j^{-\varphi} - \alpha^{-1} \Theta_j f_j^{1/\alpha - 1} w(\Omega) \Big\} = 0 \tag{37}$$

From (35), we get:

$$\lambda_j = I_j d_j^* \theta_j \beta V_b \tag{38}$$

Substitute the result from (38) into (36), we get:

$$p_j = \frac{(1-\varphi) u_c}{I_j \beta V_b} \tag{39}$$

Divide both sides of (37) by λ_j, then substitute the result from (38) into (37), we get:

$$\begin{aligned} & \frac{A_j \theta_j^{-\varphi} u_c}{I_j \beta V_b} - p_j A_j \theta_j^{-\varphi} = -p_j A_j \theta_j^{-\varphi} + \alpha^{-1} \Theta_j f_j^{1/\alpha - 1} w(\Omega) \\ \leftrightarrow \quad & \frac{A_j \theta_j^{-\varphi} p_j}{1-\varphi} - p_j A_j \theta_j^{-\varphi} = -p_j A_j \theta_j^{-\varphi} + \alpha^{-1} \Theta_j f_j^{1/\alpha - 1} w(\Omega) \quad (\text{ Use } 39) \\ \leftrightarrow \quad & \alpha p_j A_j \theta_j^{-\varphi} f_j = (1-\varphi) \Theta_j f_j^{1/\alpha} w(\Omega) \end{aligned} \tag{40}$$

Impose equilibrium condition and using our definition $M(\Omega) = \beta V_b$. We can rewrite (39) and (40) at equilibrium for each channel:

$$p_j(\Omega) = \frac{(1-\varphi)u(C(\Omega))}{I_j M(\Omega)}$$
$$\alpha p_j(\Omega) A_j \theta_j(\Omega)^{-\varphi} f_j = (1-\varphi)\Theta_j f_j(\Omega)^{1/\alpha} w(\Omega)$$

Proof for proposition 1:
From the competitive goods search condition (20) and the equal profit condition between two types of firms (14), in equilibrium:

$$\begin{aligned} \Theta_s f_s^{1/\alpha} &= \Theta_m f_m^{1/\alpha} \\ \Leftrightarrow \quad \frac{f_s}{f_m} &= \left(\frac{\Theta_m}{\Theta_s}\right)^{\alpha} \end{aligned} \tag{41}$$

The price condition at competitive goods search (19) give us:

$$(1+\delta\tau_s)\, p_m = (1+\tau_s)\, p_s \tag{42}$$

The two equations (20) and (14) give us the equal revenue condition:

$$p_s A_s \theta_s^{-\varphi} f_s = p_m A_m \theta_m^{-\varphi} f_m$$

Substitute (41) and (42) into the above equation, we have the condition for market tightness:

$$\left(\frac{\theta_s}{\theta_m}\right)^{\varphi} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right) \frac{A_s}{A_m} \left(\frac{\Theta_m}{\Theta_s}\right)^{\alpha} \tag{43}$$

Back to the household's allocation shopping time between the two channels (6) and use (41), (42), (43) and assumption A3 ($\gamma = 0$):

$$\begin{aligned} \frac{h'(D_s)}{h'(D_m)} &= \frac{A_s}{A_m} \left(\frac{\theta_s}{\theta_m}\right)^{1-\varphi} \frac{f_s}{f_m} \\ \Leftrightarrow \quad \frac{D_s}{D_m} &= \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{\frac{1-\varphi}{\varphi\nu}} \left(\frac{A_s}{A_m}\right)^{\frac{1}{\varphi\nu}} \left(\frac{\Theta_m}{\Theta_s}\right)^{\frac{\alpha}{\varphi\nu}} \end{aligned} \tag{44}$$

We finish the first part of proposition 1. The second part is simply take the partial derivate of (44) with respect to z_s.

$$\frac{\partial(D_s/D_m)}{\partial z_s} = \left[\frac{\alpha(1+\tau_c)}{\varphi\nu z_s^2\Theta_s}\right]\frac{D_s}{D_m} > 0$$

Proof for Proposition 2:

Substitute θ_j=E_j/D_j into the market tightness condition (43), we have the proof:

$$\frac{E_s}{E_m} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{1/\varphi}\frac{D_s}{D_m}\left(\frac{A_s}{A_m}\right)^{1/\varphi}\left(\frac{\Theta_m}{\Theta_s}\right)^{\alpha/\varphi}$$

Proof for Corollary 1:

Substitute the result of (44) into the result of the proposition 2, we have the equation showing the ratio between measure of firms in two channels:

$$\frac{E_s}{E_m} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{\frac{1-\varphi+\nu}{\varphi\nu}}\left(\frac{A_s}{A_m}\right)^{\frac{1+\nu}{\varphi\nu}}\left(\frac{\Theta_m}{\Theta_s}\right)^{\frac{\alpha(1+\nu)}{\varphi\nu}}$$

Assume the tax τ_c is fixed, take the partial derivate:

$$\frac{\partial(E_s/E_m)}{\partial z_s} = \left[\frac{\alpha(1+\nu)(1+\tau_c)}{\varphi\nu z_s^2\Theta_s}\right]\frac{E_s}{E_m} > 0$$

Proof for Proposition 3:

The allocation production within channel "u" is decided by σ. We have:

$$\frac{\partial\sigma}{\partial\tau_c} = \frac{\alpha}{1-\alpha}\left(\frac{1+\tau_c}{1+\varepsilon}\right)^{\frac{2\alpha-1}{1-\alpha}}\frac{1}{1+\varepsilon} > 0$$

So when there is a reduction in formal sector tax, σ reduces, implying that more goods will be produced in the formal plant.

For the second part of the proof, we first rewrite the result of the corollary 1:

$$\frac{E_s}{E_m} = \left(\frac{1+\delta\tau_s}{1+\tau_s}\right)^{\frac{1-\varphi+\nu}{\varphi\nu}}\left(\frac{A_s}{A_m}\right)^{\frac{1+\nu}{\varphi\nu}}\left(\frac{\Theta_m}{\Theta_s}\right)^{\frac{\alpha(1+\nu)}{\varphi\nu}}$$

We have

$$
\begin{aligned}
\frac{\partial \Theta_m}{\partial \tau_c} &= \frac{1-\frac{1}{\alpha}(\sigma+1)^{-1}(1+\tau_c)\frac{\partial\sigma}{\partial\tau_c}}{(\sigma+1)^{1/\alpha}} + \frac{(1+\varepsilon)\sigma^{1/\alpha-1}(\sigma+1)^{-1}\frac{\partial\sigma}{\partial\tau_c}}{\alpha(\sigma+1)^{1/\alpha}} \\
&= \frac{\alpha(\sigma+1)-(1+\tau_c)\frac{\partial\sigma}{\partial\tau_c}+(1+\varepsilon)\sigma^{1/\alpha-1}\frac{\partial\sigma}{\partial\tau_c}}{\alpha(\sigma+1)^{1/\alpha+1}} \\
&= \frac{1}{(\sigma+1)^{1/\alpha}} \quad \text{(The last two terms in numerator cancels out)} \qquad (45) \\
\frac{\partial \Theta_s}{\partial \tau_c} &= 2^{1-1/\alpha} + \frac{1}{z_s} \qquad (46)
\end{aligned}
$$

From the above results, we have:

$$
\frac{\partial\left(\frac{\Theta_m}{\Theta_s}\right)}{\partial \tau_c} = \frac{-\left[2^{1-1/\alpha}+\frac{1}{z_s}\right]\left[(1+\varepsilon)\left(\frac{\sigma}{\sigma+1}\right)^{1/\alpha}+\frac{1}{z_m}\right]}{\Theta_s^2} < 0 \tag{47}
$$

Use (47), we have final result:

$$
\frac{\partial(E_s/E_m)}{\partial \tau_c} = \left(\frac{\alpha(1+\nu)\,\Theta_s}{\varphi\nu\,\Theta_m}\right)\frac{E_s}{E_m}\frac{\partial(\Theta_m/\Theta_s)}{\partial \tau_c} < 0
$$

So, when τ_c reduces, there is a reallocation to channel "s".