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Abstract

We analise emission permit auctions under leader-follower competition when the leader bids strategically and the follower acts as price-taker both at the auction and the secondary market. We obtain linear equilibrium bidding strategies for both firms and a unique equilibrium of the auction, which is optimal ex-post for the leader. Under specific distributional assumptions we conclude that the auction always awards less permits to the leader than the cost-effective amount. Our central result is a cautionary note on the properties of auctioning under market power. Under interior solution the auction allocation is dominated by grandfathering in terms of aggregated cost with probability one. As a policy implication, the specific design of the auction turns out to be crucial for cost-effectiveness. The chances of the auction to outperform grandfathering require that the former is capable of diluting the market power that is present in the secondary market.

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1 Introduction

Emission trading has become an increasingly popular policy instrument to regulate polluting emissions. Important applications include the US SO2 trading system under the framework of the Acid Rain Program of the 1990 Clean Air Act and, more recently, the European Union Emission Trading System (EU ETS). This policy approach is particularly popular among economists as it theoretically allows emissions to be reduced in a cost-effective way by means of a price system. As long as marginal abatement costs differ across firms, trading among them can help to achieve a pre-specified environmental target at a minimum cost.[1] One of the main decisions to be made when implementing a so-called cap and trade (CAP) system is the rule to do the initial distribution of permits among firms. The most popular methods are auctioning and grandfathering. The latter, which consists of distributing the permits for free based on past emissions, has traditionally been the most widespread method although we are currently witnessing a tendency to revert this situation.

The Regional Greenhouse Gas Initiative (RGGI), which began in 2009 and includes 9 northeastern US states, represents a substantial break with the past since, instead of giving the permits away for free, the participants decided to auction close to 90 percent of their permit budgets (see Burtraw *et al.* 2009). In the European Union Emission Trading System (EU ETS), considered to be the largest environmental market in the world, the role of auctioning is becoming increasingly relevant as for the third trading period, starting in 2013, auctioning of allowances will be the default allocation method. The arguments posed by the European Commission (EC) to support the introduction of auctions in the third period are that auctioning "best ensures the efficiency, transparency and simplicity of the system, creates the greatest incentives for investment in a low-carbon economy and eliminates windfall profits".[2] Permit auctioning is also typically seen by economists as an allocation mechanism with rather desirable properties. See e.g. Cramton and Kerr (2002) and Hepburn *et al.* (2006).

This paper studies auctions of permits in markets involving leader-follower competition. We analyse the bidding behaviour of the firms, the equilibrium of the auction and its efficiency. We take grandfathering as a comparison benchmark to determine if moving from a grandfathering scheme to auctioning can deliver any efficiency improvements. Our results raise a cautionary note about the properties of the auction in those frameworks in which there is some big firm that can exert market power both in the auction and the secondary market. We conclude that, in these conditions, the auction outcome can be really unfavourable in terms of overall cost.

We adopt an auction-theory approach by modelling a Bayesian game of incomplete information. The

[1] As an additional argument, Antoniou *et al.* (2014) show in a model of strategic environmental policy that permits trading is a dominant strategy and it ensures that welfare is strictly higher than in a situation where permits are non-tradable. When the permit market is efficient, it create incentives for exporting countries to tighten regulation in order to enhance their firms' competitiveness.

[2] See http://ec.europa.eu/clima/policies/ets/cap/auctioning/faq_en.htm, section "Why are allowances being auctioned?".

firms are characterized by idiosyncratic types whose realizations are privately observed while their distribution is common knowledge. After the initial allocation is made in the auction, the firms engage in a secondary market in which one firm acts as a leader and the other one as a follower. Consistent with this assumption, we consider that the firm that is a leader in the secondary market is also so in the auction in the sense that it bids strategically anticipating the effect of its own bid on the equilibrium price, while the firm that behaves as a follower in the secondary market bids non-strategically in the auction by taking the price as given. In this way, we try to represent market situations in which big influential firms coexist with small powerless firms. An example of this situation can be the power sector in the European Union (see e.g. Hinterman 2011). In such a framework, our claim is that the fact of auctioning the permits does not necessarily remove market power and thus the bidding behaviour of both types of firms will have a different impact on the equilibrium of the auction.

Under these conditions, and assuming that he conditional expectation of the rival's type is linear, we endogenously obtain linear optimal strategies for both firms and we characterize a unique equilibrium of the auction. A remarkable property of the auction equilibrium is that it is optimal ex-post for the leader in the sense that he can play in such a way that he enjoys the same profit that he would get if he could observe, not only his own type, but also his rival's type at the time of bidding. The interpretation of this result is that the auction reinforces the leadership position and endows the leader with the capacity to drive the market equilibrium to the most favourable position for him. This seems to suggest that, as far as the leader-follower relationship of the secondary market has a reflection in the auction, this allocation mechanism is likely to suffer from the type of inefficiencies that are typically associated to market power.

Once we get this general result, in order to get more accurate insights we introduce specific distributional assumptions on the types. We assume the join probability distribution is uniform and we consider three supports, all of which lie within a bounded square. The first support is the whole square, so that the types are mutually independent. The other two supports are upper and lower triangles within the square, respectively, such that types are positively correlated in both cases and either the leader or the follower has a lower type with probability 1 (w.p.1), which in our framework has the interpretation that it has lower marginal abatement costs than its rival.

Under these assumptions we conclude that only in the case of independent distribution the allocation of permits under grandfathering is symmetric in the sense that it under- and overstates the cost-effective one with the same probability. In the other two cases it displays some kind of bias in the sense that it tends to over-allocate the follower with a probability higher than one half in case 2 and the opposite happens in case 3. The auction, in turn, introduces a systematic bias in the sense of assigning to the leader less permits than the cost-effective amount w.p.1 in all three cases, and accordingly the leader will always act as a monopsonist in the secondary market. This result is in the same line as the one by Antelo and Bru (2009) and Montero (2009), who show in a deterministic framework that it is optimal for the leader not to take part in the auction and buy all the permits in the secondary market instead. Under an incomplete information scenario we obtain a less extreme and more realistic result in the sense that the leader's allocation is lower than the optimal amount, but not necessarily zero.

Importantly, in any of our distributional scenarios and assuming interior solution we conclude that the auction allocation is dominated by grandfathering in terms of aggregated cost. It deserves to be stressed that this dominance is not only in terms of expected costs, but in a remarkably stronger manner: for each possible realization of the types within the relevant range the auction leads to larger realized cost than grandfathering, which implies first-order stochastic dominance of grandfathering with respect to auctioning.

As a central policy implication we conclude, not only that the mere fact of auctioning the permits does not ensure a gain in terms of efficiency, but rather, on the contrary, if the auction inherits the leader and follower roles from the secondary market, if we move from grandfathering to auctioning the results are prone to be worse rather than better.

This paper is related to two strands of literature. The first strand has to do with CAP systems under market power. It is well documented that under perfect competition and complete information, the secondary market equilibrium is cost-effective for any initial allocation of the permits (see Montgomery, 1972). Unfortunately, this desirable property fails to hold if the secondary market is not perfectly competitive. As Hahn (1984) first showed, the efficiency loss due to market power depends on the initial allocation of permits. The dominant firm will manipulate the price (upwards if he is a seller and downwards if he is a buyer) in the secondary market, unless the initial allocation equals the cost-effective one, which requires a perfectly informed regulator. Hagen & Westskog (1998) extended the Hahn setting in a dynamic two-period model. An overview of this literature can be found in Montero (2009). To what extent market power is a real problem in practice has to be determined for each market separately (see, e.g., Sturn, 2008). In the case of the UE ETS, some authors have reported the presence of market imperfections and evidence of price manipulation. For example, Ehrhart *et al.* (2008) claim that there are loopholes in EU emissions trading law that foster tacit collusion and impacts oligopolistic product markets (p. 347). Hinterman (2011) concludes that the largest electricity producers in Germany, the UK and the Nordpool market could have found it profitable to manipulate the permit price upwards and he claims that this could explain the elevated allowance price level during the first 18 months of the EU ETS.

The second strand of literature is auction theory and, more specifically, multi-unit multi-bid auctions, in which more than one unit is being auctioned (multi-unit) and bidders can bid for more than one unit (multi-bid). Our framework is characterized by non-common value and the existence of a secondary market, which is not typically the case in most papers in this literature (see, for example, Wang and Zender (2002) or Alvarez and Mazón (2012)). More general settings, as the one by de Castro and Riascos (2009) do not fully characterize the equilibrium as we do, but just optimal responses. In our framework, the non-common-value nature of the auction is not an ad-hoc assumption but the natural consequence of how we model the secondary market. Specifically, the common-value assumption fails to hold for two reasons. First, the value of an additional permit is not constant across firms due to their different role as a leader and a follower respectively. Second, the value of an additional permit is not constant for a specific firm either, because marginal abatement cost is not constant and depends on the total amount of permits held by each firm.

Under common value, efficiency is not a relevant question since any allocation of units across bidders is efficient. In much simpler settings, such as single-unit auctions, the resulting allocation is also efficient. For a large range of auction formats it is always the case that the bidder with the highest valuation is the one who gets the unit on sale. But in our multi-unit multi-bid non-common-value framework efficiency of the auction allocation turns out to be a relevant question as results that holds true for simpler auctions do not apply to this setting. As efficiency is one of the main arguments of the EC to support the introduction of auctions in the EU ETS, we consider relevant to pay particular attention to this issue.

The increasing interest in permit auctioning has given rise to some related empirical and experimental studies (see e.g., Ledyard and Szakaly-Moore (1994), Godby (1999, 2000), Muller *et al.* (2002) Burtraw *et al.* (2009), Reeson *et al.* (2011), Mougeot *et al.* (2011), Cong and Wei (2012), Grimm and Ilieva (2013)), but there is still a lack of theoretical developments and, as far as we know, the properties of auctions of permits have not been thoroughly studied. In one of the few related theoretical papers, Antelo and Bru (2009) compare auctioning and grandfathering in a permit market with a dominant firm when the government is concerned both about cost-effectiveness and public revenue, but one of our main building blocks, incomplete information, is absent in their work. As the planner is assumed to be perfectly informed, the cost-effective solution can be trivially achieved as it can be readily implemented by means of grandfathering. This is not our case as we assume incomplete information. The closest paper to ours is Alvarez and André (2014), which compares auctioning and grandfathering, but is restricted to a setting in which both firms are assumed to act non-strategically at the auction. This assumption seems a-priori favourable for the auction by depriving the firm which enjoys market power in the secondary market of the possibility to act strategically and reinforce his market power. Even in this favourable environment, Alvarez and André (2014) conclude that the auction equilibrium is never cost-effective, because the leader will either over-bid or under-bid to place him in the most advantageous position for the secondary market. In general, auctioning is not very successful in the comparison versus grandfathering performed in that paper. In some of the cases they consider, auctioning results in higher expected costs while in other cases the comparison is inconclusive as it depends on the parameter values.

This paper complements Alvarez and André (2014) by considering a situation in which the leader acts strategically in the auction while the follower still acts as a price taker. This assumption is based on the belief that, if one firm has market power in the secondary market, due to its size or dominant position, it is likely that this power has a reflection in the auction. Our results reinforce the ones by Alvarez and André (2014) in a particularly strong way as we conclude that, under rather reasonable conditions, auctioning is always dominated by grandfathering in terms of cost-effectiveness. Moreover, the dominance is not only in terms of expected cost but with probability one.

The reminder has the following structure. In Section 2 we present the main elements of the model, the technical assumptions and the structure of the secondary market. Section 3 carries out a general treatment of the auction in the sense that no specific distributional assumption is imposed. Section 4 introduces specific distributional scenarios and compares auctioning and grandfathering in terms of cost. Section 5 concludes. Appendix 1 summarizes the notation and all the proofs are gathered in Appendix 2.

2 The model

2.1 Basic setting

Consider two polluting firms. One of them (labelled as L) acts as a leader and the other one (F) is a follower. We use the index i to denote an arbitrary firm. Both firms have the ability to do some abatement effort to reduce their emissions. The marginal abatement cost function (MC) of both firms is linearly decreasing in the amount of emissions:

$$MC_i\left(e_i\right) = -TC_i' = \alpha_i - \beta e_i, \qquad \text{with } \alpha_i,\, \beta \geq 0, \tag{1}$$

where e_i represents the effective emissions of firm i and β is the slope of the marginal cost function, which is assumed to be deterministic and constant across firms. Finally, the intercept α_i is a firm-specific random variable.[3] We call α_i the *type* of firm i. For simplicity, we do not explicitly consider output production.

For notational convenience, denote the vector of types as $\boldsymbol{\alpha} = (\alpha_F, \alpha_L)$. As is usual in auction theory, we assume that there exists incomplete information and thus, at the beginning of the game each firm observes its own type but only knows the distribution of the rival's type, which is common knowledge. This assumption is consistent with the fact that, in reality, a firm itself is the one that has more accurate information about its own technology and the effect of random shocks on the firm's results.

Total abatement cost (TC) associated to an amount of emissions e_i is computed as the area below the marginal cost curve:

[3] With a slight abuse of notation, throuthoug the paper we denote indistinctly a random variable and an arbitrary realization of it.

$$TC_i\left(e_i\right) = \int_{e_i}^{e_i^{BAU}} MC_i\left(t\right) dt = -\alpha_i e_i + \frac{\beta}{2} e_i^2, \tag{2}$$

where e_i^{BAU} is the laissez-faire or BAU emissions, i.e., the amount that firm i would emit if it were not subject to any legal obligation to curb emissions. Such a value is computed as the amount of emissions that would minimise TC_i. By making the marginal abatement cost equal to zero, we obtain

$$e_i^{BAU} = \frac{\alpha_i}{\beta}. \tag{3}$$

The environmental authority sets a quantitative objective of $\bar{Q}$ units of emissions and implements a CAP system by issuing $\bar{Q}$ emission permits and auctioning them between the firms. Both firms bid with incomplete information as explained above and, in the auction equilibrium, each firm i receives q_{i0} as an initial allocation, with $q_{L0} + q_{F0} = \bar{Q}$. After the initial allocation is made, the types are publicly revealed and the firms engage in the secondary market and trade allowances with full information. There are two arguments to back the assumption that the types are known in the secondary market but not in the auction. The first is that transactions in the secondary market are more frequent than auctions, and thus, it is realistic to assume that in the secondary market the agents handle more information. The second is that the auction regulations typically ensure the anonymity of the participants, while this is not necessarily the case in the secondary market.[4] Denote as q_{i1} the amount of permits that firm i holds after trading in the secondary market, which has to coincide with its realized emissions ($q_{i1} = e_i$). Therefore, the amount of permits sold (if $q_{i0} > q_{i1}$) or bought (if $q_{i1} > q_{i0}$) in the secondary market by firm i is $|q_{i0} - q_{i1}|$. Denote as p_0 the price of the permits in the auction p_1 is the price in the secondary market. For the sake of analytical tractability, the permits are assumed to be perfectly divisible and hence we take the number of permits as a continuous variable.[5]

The cost-effective allocation $\left(e_L^{CE}, e_F^{CE}\right)$ can be computed by equating marginal abatement costs across firms and imposing the market-clearing condition $e_L + e_F = \bar{Q}$:

$$e_L^{CE} = \tfrac{\alpha_L - \alpha_F + \beta\bar{Q}}{2\beta}, \qquad e_F^{CE} = \tfrac{\alpha_F - \alpha_L + \beta\bar{Q}}{2\beta}, \tag{4}$$

where CE stands for "cost-effective". As we know from the previous literature (see Montgomery, 1972), under any initial distribution the cost-effective allocation would be reached as a market equilibrium if the secondary market were perfectly competitive. In that case, the equilibrium price p^C, where C stands for

[4] Consider over the counter (OTC) transactions, which have been rather important in the EU ETS, specially in the initial phases. See Ellerman *et al.* (2010).

[5] See Wilson (1979) or, more recently, Wang and Zender (2002) or Alvarez and Mazon (2012) for auctions of perfectly divisible goods.

"competitive", would be given by

$$p^C = MC_1(e_1) = MC_2(e_2) = \frac{\alpha_L + \alpha_F - \beta\bar{Q}}{2}. \tag{5}$$

For notational convenience we define $\delta := q_{F0}/\bar{Q}$ as the proportion of permits held by the follower, so that the leader holds a proportion $1-\delta$. In the cost-effective allocation this proportion is given by

$$\delta^{CE} = \frac{\alpha_F - \alpha_L + \beta\bar{Q}}{2\beta\bar{Q}} = \frac{1}{2} + \frac{\alpha_F - \alpha_L}{2\beta\bar{Q}}, \tag{6}$$

from which it is straightforward to conclude that when both firms are identical ($\alpha_F = \alpha_L$), it is optimal that each firm gets one half of the permits and if $\alpha_i > \alpha_j$ it is optimal that firm i receives more permits than j.

For some results we need to assume that conditional expectations of the types are linear. We present this linearity assumption next, together with some notation.

Assumption 1 *Firm i's expectation of the rival's type conditional on the own type is a linear function, denoting its coefficients as follows:*

$$E\{\alpha_i \mid \alpha_j\} = \mu_j + \lambda_j\alpha_j, \qquad i,\ j \in \{L, F\},\ i \neq j. \tag{7}$$

Throughout the paper we deal with interior solutions in the sense that both firms receive a non-negative amount of permits both in the primary and the secondary market and the prices of both markets are also non-negative. In section 3 we take interiority as an assumption without questioning the conditions under which this is endogenously true. In section 4 we identify conditions under which the solution is indeed interior.

2.2 The secondary market

In the secondary market we assume that firm L chooses its required amount or permits, or alternatively the price, anticipating the follower's reaction. The latter moves second and decides its demand for permits acting as a price-taker. The firms aim at maximizing their profits, given by

$$\Pi_i(q_{i1}) := p_1(q_{i0} - q_{i1}) - TC_i(q_{i1}) = p_1(q_{i0} - q_{i1}) - \int_{q_{i1}}^{e_i^{BAU}} MC_i(e)\,de, \tag{8}$$

where the fist term is the revenue or the expenses due to permits trading and the second is abatement cost. The difference between both firms' behaviour is that the follower chooses its net demand while taking p_1 as given, whereas the leader takes into account the effect of his own behaviour as well as the follower's on the market price. As proven by Alvarez and André (2014, Proposition 1) the number of permits hold by each firm in an interior secondary-market equilibrium is given by

$$q_{L1}^{*} = \frac{\alpha_L - \alpha_F}{3\beta} + \frac{\bar{Q} + q_{L0}}{3}, \tag{9}$$

$$q_{F1}^{*} = \frac{\alpha_F - \alpha_L + \beta\left(2\bar{Q} - q_{L0}\right)}{3\beta}, \tag{10}$$

and the associated equilibrium price is

$$p_1^{*} = \frac{\alpha_L + 2\alpha_F - \beta\left(2\bar{Q} - q_{L0}\right)}{3}. \tag{11}$$

The resulting firms' profits are given by

$$\pi_L\left(q_{L0}, \boldsymbol{\alpha}\right) = \Theta_L + \frac{\alpha_L + 2\alpha_F - 2\beta\bar{Q}}{3} q_{L0} + \frac{\beta}{6} q_{L0}^2, \tag{12}$$

$$\pi_F\left(q_{F0}, \boldsymbol{\alpha}\right) = \Theta_F + \frac{2\alpha_L + 7\alpha_F - 2\beta\bar{Q}}{9} q_{F0} - \frac{5\beta}{18} q_{F0}^2, \tag{13}$$

where Θ_L and Θ_F are two terms that depend on the parameters of the model as well as the types but are independent of the initial allocation. As a consequence of market power, the equilibrium depends on the initial allocation and thus the secondary market renders the cost-effective solution only if the leader initially receives exactly the amount of permits that corresponds to the cost-effective allocation. Moreover, using (4) in (9) it can be shown that, if $q_{L0} < e_L^{CE}$, then $e_L^{CE} > q_{L1} > q_{L0}$. Symmetrically, if $q_{L0} > e_L^{CE}$ we conclude $e_L^{CE} < q_{L1} < q_{L0}$. Thus, if the auction assigns to the leader less permits than in the cost-effective allocation, in the secondary market he will act as a monopsonist and will buy less permits than what would be cost-effective. If, on the contrary, he receives more than the cost-effective amount, he will act as a monopolist and will sell less permits than required to reach cost-effectiveness.

Summing up, as far as the initial allocation is not cost-effective, the secondary market generates a movement from the initial allocation towards the cost-effective one without reaching it. A technical implication of this fact is that a sufficient condition for the secondary market equilibrium to be interior is that the initial allocation and the cost-effective one are both interior.

Using (2), (9) and (10), bearing in mind that $q_{L0} + q_{F0} = \bar{Q}$ and summing across firms we can express the aggregate abatement cost as a function of the follower's initial allocation, q_{F0} (see Alvarez and André, 2014, Corollary 1):

$$TC\left(q_{F0}, \boldsymbol{\alpha}\right) := TC_L + TC_F = \Theta + \frac{\alpha_L - \alpha_F - \beta\bar{Q}}{9} q_{F0} + \frac{\beta}{9} q_{F0}^2, \tag{14}$$

where Θ is a term that depends on the parameters and the types but not on the initial allocation. An implication of (14) is that the total cost entailed by an allocation system can be assessed just by checking how close the initial allocation is to the cost-effective one. Using (14) allows us to skip computing the secondary-market allocation to evaluate an allocation system, since such effect is already incorporated in (14). By applying a simple linear transformation, we define the following auxiliary function in terms of δ:

$$h(\delta, \boldsymbol{\alpha}) := \frac{9}{\bar{Q}} \left[TC\left(q_{F0}, \boldsymbol{\alpha}\right) - \Theta \right] = \delta^2 \beta \bar{Q} + (\alpha_L - \alpha_F - \beta \bar{Q})\delta. \tag{15}$$

For simplicity, and with a slight abuse of terminology, sometimes we will refer to h as "cost", though it actually is a linearly increasing transformation of it. This is innocuous since we are using costs just to determine which allocation method, auctioning or grandfathering, is more cost-effective and the relative position is not altered by a monotone transformation.[6]

3 Auctioning: the general case

Assume now that the permits are auctioned by means of a uniform auction, which means that all the awarded units pay the stop-out price.[7] Consistent with our assumption of a leader-follower relationship, we consider that, if one firm has market power in the secondary market, due to its size or dominant position, it is likely that this power has a reflection in the auction.[8] To capture this notion, we assume that the follower neglects the effects of its own bid on the auction price and thus acts as a price-taker, whereas the leader bids strategically in the sense that he is capable of predicting the follower's strategy and taking into account the effect his own and the follower's bid on the clearing price.[9] We are faced with a multi-unit multi-bid auction as there are multiple units to be auctioned and every bidder can request more than one unit. Moreover, due to the asymmetry between firms, and to the fact that MC_i is not constant, it is a non-common value auction.

A strategy for firm i is a mapping from the support of its type into the set of feasible bids. Specifically, a strategy is a demand function of the form $q_i^b(\alpha_i, p_0)$, where the requested amount q_i^b depends on the price p_0 and firm i's type, but not on the rival's type, and b stands for "bid". For the sake of tractability, we restrict ourselves to continuous strategies and interior solutions, which implies that, in equilibrium, both the leader and the follower require an amount of permits between 0 and $\bar{Q}$, in such a way that $q_F^b + q_L^b = \bar{Q}$ and rationing is ruled out. Denote as $v_i\left(q_{i0}, \alpha_i\right) := E\{\pi_i\left(q_{i0}, \boldsymbol{\alpha}\right) \mid \alpha_i, q_{i0}\}$ the value function of firm i at

[6] Note that $h\left(\delta, \boldsymbol{\alpha}\right)$ is proportional to the difference between TC and Θ. From (14) we can interpret Θ as the total cost associated to a situation in which the follower receives no permits ($q_{F0} = 0$), which we can take as a comparison benchmark. So, a positive (negative) value of h means that the realized cost is higher (lower) than that in the benchmark situation.

[7] An alternative common format is the discriminatory auction, in which every awarded unit pays its bid. See, e.g., chapter 12 of Krishna (2002) for a detailed explanation.

[8] Another way to justify this setting is to interpret that "the follower" is not a single firm but a fringe of firms, each of whom considers that it is too small to influence the auction price.

[9] In fact, in many -and very important- real life auctions, such as Treasury auctions, there are both price-taker and price-setter players acting simultaneously.

the auction, i.e., the expected profit of firm i, provided that it gets q_{i0} permits in the auction conditional on his own type, where π_i is given by (12) or (13). Note that a firm's value function depends on its beliefs about the rival's type.

Both firms are assumed to be risk-neutral and thus aim at maximizing their expected profits. An equilibrium in the auction is a pair of best strategies, q_F^{b*}, q_L^{b*}, and a stop-out price p_0^* that clears the market. Firm i's best strategy is the demand function that maximises its value function, as defined above, minus the cost incurred to get q_{i0} permits in the auction, $p_0 q_{i0}$. As both players behave in a different way, we have to address their strategies separately.

Regarding the follower, as it acts non-strategically its problem is

$$q_i^{b*}(\alpha_i, p_0) := \arg\max_{q_{io}} \{v_i(q_{i0}, \alpha_i) - p_0 q_{i0}\}, \tag{16}$$

where the price is taken as given. Using the definition of the value function, together with (13), and manipulating the first order condition of problem (16), we get the following expression for the follower's best strategy:

$$q_F^{b*}(\alpha_F, p_0) = \frac{2E\{\alpha_L \mid \alpha_F\} + 7\alpha_F - 2\beta\bar{Q} - 9p_0}{5\beta}, \tag{17}$$

which is continuously decreasing in the auction price, p_0. Moreover, it is increasing both in α_F and the expected value of α_L. As α_F shifts the follower's abatement cost up, the higher α_F the more F is willing to pay for the permits. Regarding α_L, although it affects L's, and not F's cost, the follower forecasts that a higher value of α_L will make the permits more valuable in the secondary market, which would also make F more willing to pay in the auction. Finally, the bid depends negatively on the total amount of issued permits, $\bar{Q}$, because the firm forecasts that it will be easier to obtain cheaper permits in the secondary market and thus is less willing to pay.

The leader's best strategy is a bid function $q_L^{b*}(\alpha_L, p_0)$ that represents a best response to F's strategy. To model the leader's problem, define total demand in the auction as $\Phi(\boldsymbol{\alpha}, p_0) := q_L^b(\alpha_L, p_0) + q_F^b(\alpha_F, p_0)$, from which the opt-out price can be obtained by imposing the market-clearing condition $\Phi(\boldsymbol{\alpha}, p_0^*) = \bar{Q}$ and solving for p_0^* we get $p_0^* = \Phi^{-1}(\boldsymbol{\alpha}, \bar{Q})$.[10] Therefore, the leader's problem is

$$q_L^{b*}(\alpha_L, p_0) = \arg\max_{\{q_{L0}\}} v_L(q_{L0}, \alpha_L) - E\left\{\Phi^{-1}\left(\boldsymbol{\alpha}, \bar{Q}\right) \times q_{L0} \mid \alpha_L\right\}. \tag{18}$$

Note that L's problem depends on F's strategy through $\Phi(\boldsymbol{\alpha}, p_0)$. Moreover, unlike the follower, L does not take the price as given, but he plays by predicting the equilibrium price that will result from his own and F's demand.

[10] (17) reveals that q_F^b the former is decreasing and Proposition 2 shows that q_L^b also is, which ensures that Φ is invertible in p_0.

One important property of this problem is that there exists an strategy for L that is optimal ex-post in the sense that it guarantees the L's profit maximizing price in the auction w.p.1. As a first step to prove this result consider the following artificial auxiliary problem:

$$\max_{\{p_0\}} \quad \pi_L\left(\bar{Q} - q_F^b\left(\alpha_F, p_0\right), \boldsymbol{\alpha}\right) - p_0 \times \left[\bar{Q} - q_F^b\left(\alpha_F, p_0\right)\right], \tag{19}$$

$q_F^b\left(\alpha_F, p_0\right)$ being given by (17). This problem is artificial in the sense that it is written as if the leader could observe the real value of α_F and pick the value of the equilibrium price of the auction that maximises his profit. The solution of this problem is given in the following lemma:

Lemma 1 *If the solution of problem (19) is interior, it is given by*

$$p_0^* = \frac{44\alpha_F + 15\alpha_L + 4E\left\{\alpha_L \mid \alpha_F\right\} - 44\beta\bar{Q}}{63} \qquad \blacksquare \tag{20}$$

Lemma 1 shows the expression of the clearing price that maximises the leader's realized profit. The essential characteristic is that, as expected, L's profit maximizing price depends positively on both F's and L's type, which determine how strongly each firm needs the permits to reduce its abatement costs. It also depends on the follower's expectation about the leader's type because such expectation affect the follower's prediction about the scarcity of permits and thus, its willingness to pay for them. The leader's expectation about the follower's type is not involved because α_F is taken as known in problem (20). Note also that the coefficient of α_F in (20) is larger than that of α_F, which implies that, due to the leader's strategic behaviour , his optimal price is more responsive to the follower's than to his own type. Finally, p_0^* decreases as the amount of issued permits increases, as less scarcity entails a lower market value.

Proposition 2 shows that, under the assumption of linear expectation (7), the equilibrium strategy of the leader is such that the equilibrium price of the auction is given exactly by (20).

Proposition 2 *Under Assumption (7), and considering interior solution, there is a unique equilibrium of the auction in which the leader's strategy has the form*

$$q_L^{b*}\left(\alpha_L, p_0\right) = m_0 + m_1\bar{Q} + m_2\alpha_L + m_3 p_0, \tag{21}$$

where $m_0 \leq 0$, $m_1 \leq 0$, $m_2 \geq 0$ and $m_3 \leq 0$ depend only on β and λ_L. Moreover, the equilibrium price of the auction for any $\boldsymbol{\alpha} = (\alpha_L, \alpha_F)$ is the solution to (19) $\blacksquare$

To understand this proposition, it is important to underline that (19) selects the leader's optimal price in the auction for any given pair (α_L, α_F), and so it represents an ideal situation for the leader since he would not be affected by incomplete information. Proposition 2 states that there exists a demand function such that the equilibrium price of the auction replicates the solution to problem (19) and such a function depends only on known information for the leader (α_F is absent). We say that the auction allocation is ex-post optimal for the leader in the sense that the leader can play in such a way that the ultimate equilibrium will be the one that he would choose if he had full information. In other words, if the leader happened to know the value of α_F, he would not change his strategy.[11] Note also that, although this is a remarkably strong result for the leader, the bidding function used to get that result is rather standard as it is a linear function with intuitively reasonable properties: it is decreasing in the price and the total amount of issued permits and increasing in L's type.

Little more can be said at this level of generality. In order to get more accurate insights about the properties of the auction allocation and its comparison to grandfathering we need some distributional assumption about the types, which we introduce in Section 4. Nevertheless, even without such assumptions we have the important insight that the leader is in a privileged situation to exert market power, which is expected to be negative for the efficiency of the equilibrium. This intuition is confirmed by the analysis that we carry out in the next section.

4 Auctioning vs grandfathering under specific distributional assumptions

4.1 Modeling grandfathering

In practice, grandfathering is usually applied by allocating permits proportionally to observed past emissions as a proxy for BAU emissions. The idea is to take, as a benchmark, what firms would do if they were not subject to the CAP system. Nevertheless, past emissions data are not always available or, even if they exist, they are not always reliable enough, as it was the case, for example, in the first stage of the EU ETS.[12] As is done in Alvarez and André (2014), we capture this idea by assuming that under grandfathering the permits are allocated based on expected BAU emissions, which, using (3), can be expressed as

$$E\left\{e_i^{BAU}\right\} = E\left\{\frac{\alpha_i}{\beta}\right\} = \frac{E\left\{\alpha_i\right\}}{\beta}.$$

According to this assumption, the follower receives $q_{F0}^G = \delta^G \bar{Q}$ permits and the leader receives $q_{L0}^G =$

[11] Wang and Zender (2002) obtain a similar result on the ex-post optimality for the strategic bidders under a uniform auction format, though they consider quite a different setting, with common value and across-bidder symmetry assumptions.

[12] For example, Ellerman *et al.* (2010) claim that, during the first period of the EU ETS, "the task of setting a cap that was at or close to business-as-usual (BAU) emissions was made enormously more difficult by poor data. The problem was that no member state government had a good idea of the exact emissions within the ETS sectors ... and the data problem was even worse in the new member states of the eastern Europe". Moreover, "the problems created by poor data were not limited to cap-setting; they extended into the allocation of allowances to installations, which required installation-level emissions data ... Not surprisingly, since allocations to these installations depended on the data submitted, industrial firms were forthcoming, although there has always been a suspicion that the intended use of the data imparted an upward bias to these data" (p. 37-38).

$\left(1-\delta^G\right)\bar{Q}$, where G stands for grandfathering and

$$\delta^G = \frac{E\{\alpha_F\}}{E\{\alpha_L\}+E\{\alpha_F\}}. \tag{22}$$

Consider, as a particular case, that $E\{\alpha_L\} = E\{\alpha_F\}$, which implies $\delta^G = \frac{1}{2}$ and thus the permits are split evenly between both firms. Using (6) it is immediate to conclude that, in this particular case, the grandfathering allocation coincides with the cost-effective one if and only if the realization of both firms' types are the same, it overstates the cost-effective allocation if $\alpha_F < \alpha_L$ and it understates it if $\alpha_L < \alpha_F$. Using (15) and taking expected values, we get total cost under grandfathering and its (unconditional) expected value in this case:

$$h(\delta^G, \boldsymbol{\alpha}) = \tfrac{2(\alpha_L-\alpha_F)-\beta\bar{Q}}{4}, \qquad E\{h(\delta^G, \boldsymbol{\alpha})\} = -\tfrac{\beta\bar{Q}}{4}. \tag{23}$$

To analyse different cases we need to make some assumptions about the distribution of the types.

4.2 Distributional assumptions

Consider the $\alpha_F\alpha_L$ plane, in which $\boldsymbol{\alpha} := (\alpha_F, \alpha_L)$ represents an arbitrary point. In that plane, we assume that the support of the pair $\boldsymbol{\alpha}$ is contained in a square Ω, with lower-left and upper-right corners denoted as (θ, θ) and $(\theta+\sigma, \theta+\sigma)$ respectively, where θ, $\sigma > 0$. Formally, $\Omega := [\theta, \theta+\sigma] \times [\theta, \theta+\sigma]$. Thus, θ captures the size whereas σ accounts for variability in the types. Within this square, we consider three possibilities, all of which are consistent with our linear expectation assumption (7).

Assumption 2 *The support of the pair (α_F, α_L) is a square in the $\alpha_F\alpha_L$ plane whose lower-left and upper-right corners are (θ, θ) and $(\theta+\sigma, \theta+\sigma)$ respectively. Within that square, we consider three different probability distributions for $(\alpha_F,\, \alpha_L)$:*

- *Case 1 ("Independent types"): uniform on the whole square Ω. In this case the coefficients of the linear conditional expectation as defined in (7) are given by*

$$\mu_F = \mu_L = \theta + \frac{\sigma}{2}, \qquad \lambda_F = \lambda_L = 0, \tag{24}$$

  and the (unconditional) expectation of both types is given by $E\{\alpha_F\} = E\{\alpha_L\} = \theta + \frac{\sigma}{2}$

- *Case 2 ("L-ex ante efficient"): uniform on the lower diagonal triangle of Ω, that is, uniform on all* *points satisfying $\alpha_L \leq \alpha_F$. Therefore,*[13]

$$\mu_F = \frac{\theta}{2}, \qquad \mu_L = \frac{\theta+\sigma}{2}, \qquad \lambda_F = \lambda_L = \frac{1}{2}, \tag{25}$$

$$E\{\alpha_F\} = \theta + \frac{2\sigma}{3}, \qquad E\{\alpha_L\} = \theta + \frac{\sigma}{3}. \tag{26}$$

- *Case 3 ("F-ex ante efficient"): uniform on the upper diagonal triangle of Ω, that is, uniform on all points satisfying $\alpha_F \leq \alpha_L$. Thus,*

$$\mu_F = \frac{\theta+\sigma}{2}, \qquad \mu_L = \frac{\theta}{2}, \qquad \lambda_F = \lambda_L = \frac{1}{2}, \tag{27}$$

$$E\{\alpha_F\} = \theta + \frac{\sigma}{3}, \qquad E\{\alpha_L\} = \theta + \frac{2\sigma}{3}. \tag{28}$$

In case 1 we assume that the types are independently distributed (which implies $E\{\alpha_L\} = E\{\alpha_F\}$). In cases 2 and 3 the types are positively correlated. In case 2 we say that L is ex-ante (more) efficient (than F) in the sense that the abatement cost function of L is below that of F w.p.1. In case 3 $\alpha_F \leq \alpha_L$ holds w.p.1 and we say that F is ex-ante efficient. If $\alpha_i \geq \alpha_j$ holds, the marginal abatement cost function of firm i is higher than that of firm j, i.e., $MC_i(e) \geq MC_j(e)$ for any amount of emissions.[14]

Up to now we have assumed interior solution without checking under which conditions this is true. Under our specific distributional assumptions, we are in the position to determine the conditions under which our model renders an interior solution. For that purpose, we include the following technical assumption, which allows us to introduce Corollary 1.

Assumption 3 *The parameters of the model satisfy the following condition:*

$$\frac{63}{44}\theta \geq \beta\bar{Q} \geq 2\sigma \tag{29}$$

Corollary 1 *If Assumption 3 holds, under any of the three distributional scenarios considered in Assumption 2, both the equilibrium allocation under the auction as described in Proposition 2 and the resulting allocation of the secondary market are interior w.p.1* ■

Assumption 3 ensures that in all three cases included in Assumption 2, and for any realizations of the types, the equilibrium of the auction and the secondary market is interior in the sense that, in equilibrium, both firms demand a quantity of permits that is non-negative and is smaller than its BAU emissions. To

[13] In case 2 the unconditional expectations are given by solving the integrals $E\{\alpha_F\} = \frac{2}{\sigma^2}\int_\theta^{\theta+\sigma}\int_\theta^{\alpha_F}\alpha_F d\alpha_L d\alpha_F$ and $E\{\alpha_L\} = \frac{2}{\sigma^2}\int_\theta^{\theta+\sigma}\int_\theta^{\alpha_F}\alpha_L d\alpha_L d\alpha_F$ respectively. The results in case follow from the same calculations just by swapping the firm subscripts.

[14] This difference in marginal cost can be due to technological reasons in the sense that a firm may have access to a more efficient abatement technology than the other. But there are other possible interpretations, for example, in terms of size: for a bigger firm it may be harder to cut down emissions.

guarantee that this is true, two conditions need to hold: First, the dispersion of the types' distribution cannot be too large with respect to the bottom value and, second, the total amount of permits, $\bar{Q}$, cannot be too large or too small to prevent, on the one hand, that it is profitable for any firm to buy more permits than the BAU emissions and, on the other hand, that the permits are so scarce that any firm prefers to keep a negative amount of them. Under grandfathering, the initial allocation is trivially interior as far as the expected value of both types is positive, which does not require any additional assumption.

4.3 Efficiency analysis

As we have discussed in Subsection 2.2, the efficiency of an allocation method can be determined just by studying which proportion of the permits that method assigns to each firm and how close that allocation is to the cost-effective solution. We are now in the position to compare auctioning and grandfathering.

Firstly, consider grandfathering under each of the three distributional scenarios introduced above. In case 1 we know $E\{\alpha_L\} = E\{\alpha_F\}$ and thus the ratio of permits received by the follower is $\delta^G = \frac{1}{2}$. Since the distribution is symmetric it is straightforward to conclude that $\Pr\left\{\delta^G > \delta^{CE}\right\} = \Pr\left\{\delta^G < \delta^{CE}\right\} = 0.5$, i.e., grandfathering overstates the cost-effective allocation with probability 0.5 and understates it with exactly the same probability.

In case 2 ($\alpha_L \leq \alpha_F$), using (26) we have $\delta^G = \frac{3\theta+2\sigma}{6\theta+3\sigma}$. In case 3 ($\alpha_F \leq \alpha_L$), using (28) we have $\delta^G = \frac{3\theta+\sigma}{6\theta+3\sigma}$. There is an obvious symmetry between cases 2 and 3 due to the fact that the domains are symmetric and the leader-follower relationship is irrelevant both for grandfathering and the cost-effective allocation. In both cases, only the relative values of the realizations of the types are taken into account. So, if we compare grandfathering with the cost-effective allocation, it follows that the probability that δ^G is smaller than δ^{CE} in case 2 is the same that δ^G is larger than δ^{CE} in case 3. Proposition 3 bounds this probability.

Proposition 3 *Consider the semispace of parameter values satisfying Assumption 3. The following results hold under cases 2 and 3 of Assumption 2, respectively:*
i) In case 2, $0.5 < \Pr\left(\delta^G \leq \delta^{CE}\right) \leq 1$ *and this probability is arbitrarily close to 1 for some parameter values within the semispace.*
ii) In case 3, $0.5 < \Pr\left(\delta^G \geq \delta^{CE}\right) \leq 1$ *and this probability is arbitrarily close to 1 for some parameter values within the semispace* ∎

Note that there is a difference between case 1 on the one hand, and cases 2 and 3 on the other hand. In case 1 grandfathering is symmetric in the sense that it overstates or understates the cost-effective allocation with equal probability. In cases 2 and 3, on the contrary, there is certain bias in the sense that both events do not happen with the same probability. Specifically, in case 2 (3) the event that the follower receives an amount of permits smaller (larger) than the cost-effective amount happens with a larger probability than the opposite event. Actually, as it is shown in the proof of Proposition 3, the infimum for the probability of this event is around 0.7. This value is the probability that $\delta^G \leq \delta^{CE}$ holds in case 2 for the most adverse parameter combination within the range defined by (29). In the most favourable extreme case, we obtain that $\Pr\left(\delta^G \leq \delta^{CE}\right)$ happens with probability 1. Although the latter is an uninteresting degenerate case that involves $\beta\bar{Q} = \sigma = 0$, it serves to show that grandfathering tends to over-allocate the leader in case 2 with a high probability that can become arbitrarily close to one and exactly the opposite happens in case 3.

In the auction, cases 2 and 3 are not symmetric, because the different roles played by each of the firms has an impact on the equilibrium allocation. Actually, Proposition 4 shows that the auction allocation is always biased in the same direction and, more strikingly, this happens with probability one.

Proposition 4 *Under Assumption (3), in cases 1, 2 and 3 of Assumption 2, the proportion of permits awarded to the follower in the auction is larger than its cost-effective allocation and larger than the amount that it would receive under grandfathering, i.e.,* $\delta^A > \delta^{CE}$ *,* $\delta^A > \delta^G$*, w.p.1* ■

According to Proposition 4, the leader will always receive less permits from the auction than the cost-effective amount, e_L^{CE} , and, therefore, will act as a monopsonist in the secondary market. According to the discussion presented in subsection 2.2, we also know that $e_L^{CE} > q_{L1}^A > q_{L0}^A$, i.e., the leader will buy less permits than what would be required to achieve the cost-effective allocation. As a consequence of this result, it can also be proved that

$$p_0 < p_1 < p^C.$$

The intuition behind this result is that leader understates his demand in the auction to keep the price p_0 low. When he goes to the secondary market to buy more permits, the resulting equilibrium price p_1 increases to some extent with respect to the one in the auction, but it will still be lower than the price that would prevail under perfect competition, p^C. This result is somewhat related to the one by Antelo and Bru (2009). In a framework of complete information they conclude that it is optimal for the dominant firm to abstain from the auction and buy all the permits in the secondary market instead (Prop 1, p. 325). Under incomplete information, we get a softer result in the sense that the leader tends to demand less than socially optimal, but the demanded amount is no necessarily zero.

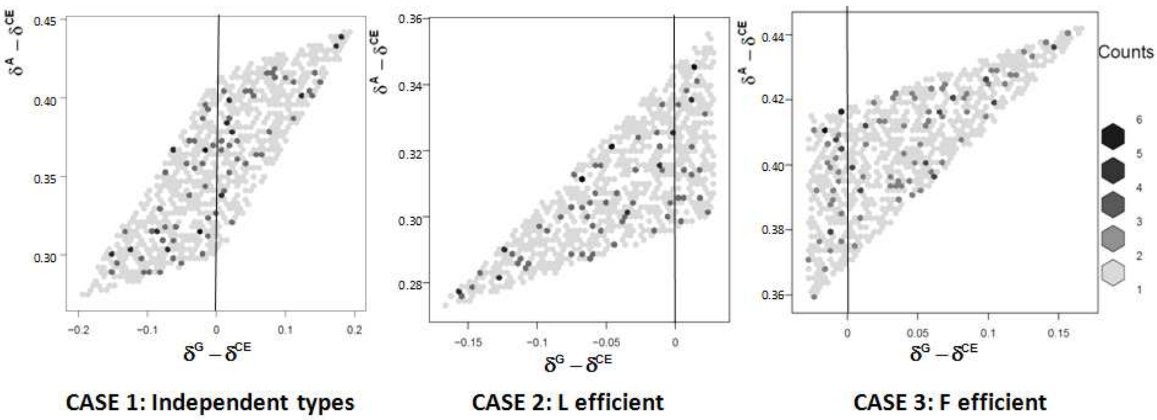

Figure 1: Deviation of the auction (vertical axis) and grandfathering (horizontal) w.r.t. δ^G.
1,000 realizations of $\boldsymbol{\alpha}$ with $\theta = 10$, $\beta\bar{Q} = 10$, $\sigma = 4$. Colors indicate number of realizations in each spot.

To determine which of the two allocation methods is more cost-effective, we first present an intuitive graphical discussion based on a numerical example and then we confirm that intuition with a very conclusive analytical result in Proposition 5. Figure 1 compares both allocation methods under each of the three distributional scenarios considered above. The plots represent 1,000 random realizations of $\boldsymbol{\alpha}$. For each realization we compute the allocation under auctioning and the cost-effective one, δ^A and δ^{CE}. In each of the three cases δ^G has a constant value because the grandfathering allocation is driven by the unconditional mean of the types rather than the realized values of the types. The horizontal axis displays the difference between δ^G and δ^{CE} and the vertical one the difference between δ^A and δ^{CE}.

The main messages that we can extract from the figure are the following. As predicted by Proposition 4, the auction systematically assigns to the follower more permits than what would cost-effective and, as a consequence, the vertical coordinate of all the dots in all three graphs is positive. On the contrary, as we know from Proposition 3, the follower might receive more or less permits than the cost-effective amount, depending on the realizations of the types and, therefore, the horizontal coordinates can be positive or negative. Note also that the horizontal coordinates are always lower than the vertical ones, consistent with the result that $\delta^A > \delta^G$, as stated in the Proposition 4.

From the analysis conducted in Section 2.2 we know that the efficiency of an allocation can be assessed by the proximity of that allocation to the cost effective one. The graphical results, together with Proposition 4, allow us to derive straightforward conclusions for some of the cases. Specifically, when δ^G is above δ^{CE} we must have $\delta^A > \delta^G > \delta^{CE}$ and thus we know for sure that the cost associated to the auction allocation is larger than that under grandfathering because it is further away from the cost-effective one. The comparison is not so straightforward if grandfathering understates the cost-effective solution, i.e.,

$\delta^A > \delta^{CE} > \delta^G$.

Consider first the left panel of Figure 1, which corresponds to case 1 and thus $\delta^G = 0.5$. The right part of the graph, i.e., to the right of the vertical axis, displays realizations in which $\alpha_F < \alpha_L$ and thus $\delta^{CE} < 0.5$, which happens with probability 0.5. In this part, both the auction and grandfathering overstate the follower's allocation and Proposition 4 allows us to conclude that for each of those realizations $h\left(\delta^A, \boldsymbol{\alpha}\right) > h\left(\delta^G, \boldsymbol{\alpha}\right)$, i.e., auctioning is always dominated by grandfathering in terms of costs. The left part of the graph corresponds to $\alpha_F > \alpha_L$ and thus $\delta^{CE} > 0.5$. In this case we have $\delta^{CE} > \delta^G$ and the ranking is not so straightforward. Nevertheless, on the vertical axis we have $\alpha_F = \alpha_L$ and therefore $\delta^G = \delta^{CE}$. Using a continuity argument, if we consider those realizations that are close enough to the vertical axis, where α_F and α_L are roughly equal, we know from (6) that δ^{CE} is close to 0.5 and thus, close to the grandfathering allocation. So, to the left of, but close to the vertical axis, the comparison also favours grandfathering. As we move further to the left, the comparison is not so straightforward and, in principle, the ranking could get reversed. But under uniform distribution we know that the mass of probability decreases when we move to the corners and therefore, it seems that the event $h\left(\delta^A, \boldsymbol{\alpha}\right) < h\left(\delta^G, \boldsymbol{\alpha}\right)$ can only happen with a small probability. As a general assessment of this case, one can conclude that grandfathering tends to beat auctioning in the majority of cases.

Consider now the right panel, which corresponds to case 3. Following the same reasoning as above we conclude that grandfathering beats the auction for all the realizations to the right of the zero line, which in this case are the vast majority and, using the continuity argument, the same can be said in an interval to the left of the zero line. Therefore, this case seems even less favourable for the auction.

The less clear case is the one in the center, as most realizations lie to the left of the zero line. Anyway, the comparison is still clearly favourable to grandfathering for all the realizations to the right of the zero line and those in an interval around this line. For the rest of the realizations, the only thing we can say is that the results are not conclusive. As al overall assessment, the balance does not seem particularly favourable to the auction either.

In general terms, this analysis suggests that, on average, the auction tends to be worse than grandfathering in terms of costs. Although this comparison has been made just for a numerical example, the conclusions are very robust within the parameter range defined by condition (29) and for the three distributional scenarios. This qualitative comparison is confirmed and even reinforced in Proposition 5, which shows that in the conditions described above, the auctioning solution is less cost-effective than the grandfathering solution w.p.1. As it is shown in Corollary 2, this implies first-order stochastic dominance (FOSD) of grandfathering over auctioning.

Proposition 5 *Under Assumption (3), in cases 1, 2 and 3 of Assumption 2, the total cost of abatement* *when the permits are auctioned is higher than under grandfathering, i.e.,*

$$h(\delta^A, \boldsymbol{\alpha}) > h(\delta^G, \boldsymbol{\alpha}) \qquad \forall \boldsymbol{\alpha}$$

Corollary 2 *Under the conditions described in Proposition 5, the cumulative probability distribution (CPD) of TC under the grandfathering allocation first-order-stochastically dominates the corresponding CPD under the auction* ■

Proposition 5 is remarkably strong since it states that, in any event, auctioning will always be beaten by grandfathering from the point of view of cost. It deserves to be stressed that this does not only hold in expected terms, but it holds true for every feasible realization of the types, as far as interior solution is guaranteed. The main message of this result is that introducing an auction such that the leader in the secondary market is also so in the auction can only worsen the results in terms of costs as compared to grandfathering. Moreover, under interior solution, this result holds with certainty. The reason is that the leader will have strong incentives to use his leadership to distort the market to own advantage and such distortion will result in a cost increase. Corollary 2 translate this result in the standard concept of FOSD.[15]

A clear-cut policy implication is that, with imperfect competition in the secondary market, switching from grandfathering to auctioning is likely to worsen the situation (in terms of cost-effectiveness) if it cannot be avoided that the market power spills over to the auction.

5 Conclusions

There seems to be an increasing interest in auctioning as an alternative to grandfathering in order to allocate emission permits. The EU ETS is a notable example of this tendency, as auctioning is being introduced as the default allocation method based on the arguments that it ensures the efficiency, transparency and simplicity of the system, creates incentives for investment and eliminates windfall profits. This paper offers a cautionary note on the efficiency argument by presenting a model in which two firms, one being a leader and the other a follower, bid for permits in a uniform auction and then trade in a secondary market, as is the case in the EU ETS.

As stated in Alvarez and André (2014), the chances of auctioning to outperform grandfathering come essentially from the fact that, by means of the bids, the auction incorporates information that are only privately observable and, hence, could not be used by the planner to implement a centralized allocation. This is what we can call the "information effect". On the other hand, the risk of the auction is that a firm

[15] As a matter of fact, the result in Proposition 5 is even stronger than FOSD since the latter does not preclude the possibility that the cost under auctioning is lower than under grandfathering for some (small enough) range of the parameter values, what is discarded in Proposition 5.

with market power can use such power to distort the equilibrium in his own benefit, which we can call "market power effect". We have shown that the leaders' ability to bid strategically while the follower is a price taker exacerbates the market power effect to the extent that it always outperforms the information effect and makes the auction be dominated with probability one. We have shown that this situation enables the leader to obtain an ex-post optimal result in the sense that he can bid in such a way that he gets rid of the consequences of incomplete information.

Since we have developed a highly simplified model, we should not take Proposition 5 as a quantitatively exact forecast for a real life policy framework, but the essence of the message is still valid as a warning about what we can expect from auctioning permits. Our central conclusion is that we cannot expect that an auction "per se" has the property of providing more cost-effective allocations of permits. Indeed, the design of the auction and, specifically, its ability to preserve or dilute market power is a crucial element for the comparison. As it is shown in Alvarez and André (2004), if the auction is capable of removing market power, then there is a chance that the results under the auction will be more cost-effective than under grandfathering. If, on the contrary, the auction reproduces the leader and follower roles, our analysis has demonstrated that one can only expect that this situation results in more costly outcomes than grandfathering.

As a policy implication, the environmental authorities should pay attention to the auction design and be cautious with those situations in which one big firms or a little amount of them could have enough power to distort the auction result to reinforce their leadership position.

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6 Appendix 1: notation

– Fundamentals –	
$\bar{Q}$	Total amount of permits to be shared, exogenous.
$i, j \in \{F, L\}$	Firms: F and L denote Follower and Leader, resp.
$MC_i(e) = \alpha_i - \beta e$	Marginal abatement cost of firm i, where e is the emission level.
TC_i	Total abatement cost of firm i
$TC = TC_L + TC_F$	Total abatement cost of the industry
α_i, $\boldsymbol{\alpha} = (\alpha_F, \alpha_L)$	Firms' types, random. The support of $\boldsymbol{\alpha}$ belongs to $\Omega := [\theta, \theta + \sigma]^2$.
$E\{\alpha_i \mid \alpha_j\} = \mu_j + \lambda_j \alpha_j$	Expectation of α_i conditional on α_j, assumed linear with coeffs. μ_j, λ_j.
e_i^{BAU}, e_i^{CE}	Business as usual and cost-effective emissions of i, resp.
$\Theta, \Theta_L, \Theta_F$	Auxiliary coefficients
– Secondary market –	
q_{i1}, p_1	Post-market holdings of permits for firm i and market price, resp.
$\Pi_i(q_{i1}, p_1, \boldsymbol{\alpha})$	Realized profit of firm i.
$\pi_i(q_{i0}, \boldsymbol{\alpha}) := \Pi_i(q_{i1}^*, p_1^*, \boldsymbol{\alpha})$	Realized profit of firm i (under eq. in the secondary market).
	(the asterisk denotes equilibrium value).
$v_i(q_{i0}, \alpha_i)$	Value function for firm i (valuation of initially getting q_{i0} permits).
– Primary market (auctioning or grandfathering) –	
q_{i0}	Initial allocation of permits for firm i (under grand. or auctioning).
p_0	Price of permits at the auction.
$\delta := \frac{q_{F0}}{\bar{Q}}$	F's share of permits (δ^A, δ^G under auctioning or grandf., resp.).
$h(\delta, \boldsymbol{\alpha})$	Monotone transformation of total abatement cost.
	(incorporates secondary-market behaviour).
$q_i^b(\alpha_i, p_0)$	Firm i's bid (mapping from support of α_i to set of demand funct.)
m_0, m_1, m_2, m_3	Auxiliary coefficients of the leader's optimal demand

Firm i's bid (mapping from support of α_i to set of demand funct.)

D Diagonal of the support of types.

Ω_l Range of types under which a type-l equilibrium takes place in the non-strategic auction ($l = 1, 2, 3$).

$E\{\cdot\}$ Expectation operator.

7 Appendix 2: proofs

7.1 Proof of Lemma 1

The first-order condition (FOC) of problem (19) is

$$-\frac{d\pi_L}{dq_{L0}} \cdot \frac{\partial q_F^{b*}}{\partial p_0} - \left[\bar{Q} - q_F^{b*}(p_0, \alpha_F)\right] + p_0 \frac{\partial q_F^{b*}}{\partial p_0} = 0,$$

where $q_F^{b*}(\alpha_F, p_0)$ is given by (17). Using the expressions for π_L given in (12) together with the market-clearing and rearranging, the FOC can be written as

$$\frac{2}{5} q_F^{b*} - \frac{8\bar{Q}}{5} + \frac{3(\alpha_L + 2\alpha_F)}{5\beta} - \frac{9}{5\beta} p_0 = 0.$$

Using (17) and rearranging we get (20).

7.2 Proof of Proposition 2

Assume initially that the solution is interior (which we check below). Using (7) in (20) we get the following value for the optimal price:

$$p_0^* = \frac{4\mu_F + 15\alpha_L + 4(11 + \lambda_F)\alpha_F - 44\beta\bar{Q}}{63}. \tag{30}$$

Assume now the leader's strategy is linear (we argue below that this is necessarily the case) and takes the form $q_L^{b*}(p_0, \alpha_L) = m_0 + m_1\bar{Q} + m_2\alpha_L + m_3 p_0$. We now show that there exist values of the coefficients m_0, m_1, m_2, m_3 such that the market clearing condition $q_F^{b*}(p_0, \alpha_F) + q_L^{b*}(p_0, \alpha_L) = \bar{Q}$ has p_0^* as a solution. This means that, when L and F bid $q_F^{b*}(p_0, \alpha_F)$ and $q_L^{b*}(p_0, \alpha_L)$ respectively, p_0^* arises as a clearing price for the auction. Using the expressions for $q_F^{b*}(p_0, \alpha_F)$ and $q_L^{b*}(p_0, \alpha_L)$ in the market clearing condition, substituting (30) for p_0 and collecting terms, we get the following equation:

$$\frac{\mu_L(4\beta m_3 + 18)}{63\beta} + m_0 + \frac{18\lambda_L + \beta m_3(5\lambda_L + 44) + 9}{63\beta}\alpha_F + \frac{\beta(7m_2 + m_3) - 3}{7\beta}\alpha_L + \frac{63m_1 - 44\beta m_3 + 54}{63}\bar{Q} = \bar{Q}.$$

For this equation to hold for any arbitrary pair (α_L, α_F), we need that the, in the left-hand side, the constant term and the coefficients associated to α_F and α_F are equal to zero and the coefficient associated to $\bar{Q}$ is equal to one. From those conditions we get a system of four equations which has, as a unique solution, $m_0 = \frac{-3\lambda_L}{\beta(\lambda_L+11)} \leq 0$, $m_1 = \frac{-3\lambda_L}{\lambda_L+11} \leq 0$, $m_2 = \frac{6\lambda_L+21}{4\beta(\lambda_L+11)} \geq 0$, $m_3 = \frac{-(18\lambda_L+9)}{4\beta(\lambda_L+11)} \leq 0$, where the signs follow from the fact that, under Assumption 2, λ_L only can be equal to 0 (in case 1) or 1/2 (in cases 2 and 3). The fact that $q_L^{b*}(p_0, \alpha_L)$ is necessarily linear comes from the fact that both $q_F^{b*}(p_0, \alpha_F)$ and p_0^* are linear and hence, in the systems of equations set above, any non-linear term must be zero to ensure that the market-clearing condition holds for any pair (α_L, α_F).

Plugging (30) in the bidding function, we find the equilibrium allocation in the auction:

$$q_{L0}^{A*} = \frac{\beta\bar{Q} - 2\mu_L + 3\alpha_L - (2\lambda_L + 1)\alpha_F}{7\beta}, \qquad q_{F0}^{A*} = \frac{6\beta\bar{Q} + 2\mu_L + (2\lambda_L + 1)\alpha_F - 3\alpha_L}{7\beta}. \tag{31}$$

The third step is to show that $q_F^{b*}(p_0, \alpha_F)$ and $q_L^{b*}(p_0, \alpha_L)$ constitute an equilibrium of the game and it is unique. By construction, p_0^* is an equilibrium price and q_F^{b*} is the best strategy for F for the same arguments used in Section 3. So, we only have to show that $q_L^{b*}(p_0, \alpha_L)$ is the best strategy for L, which requires to prove that the value of q_{L0}^{A*} given in (31) solves (18). To prove this, note that, for a specific value of α_F, solving (19) in terms of p_0 is totally equivalent to solving it in terms of q_{L0}. Therefore, we conclude that (21) provides an allocation for L that maximises π_L for any possible value of α_F, which implies that it also maximises it on average and, therefore, he have solved (18). The equilibrium is unique since both $q_F^{b*}(p_0, \alpha_F)$ and $q_L^{b*}(p_0, \alpha_L)$ are unique.

7.3 Proof of Corollary 1

Using (9), (10), (11), (30) and (31), and rearranging, we conclude that the occurrence of an interior solution w.p.1 requires that the following conditions hold:

$$\Pr\left(-13\beta\bar{Q} \leq 2\mu_F + (2\lambda_F + 8)\alpha_F - 10\alpha_L \leq 8\beta\bar{Q}\right) = 1, \tag{32}$$

$$\Pr\left(-6\beta\bar{Q} \leq 2\mu_F + (2\lambda_F + 1)\alpha_F - 3\alpha_L \leq \beta\bar{Q}\right) = 1, \tag{33}$$

$$\Pr\left(13\beta\bar{Q} \leq 10\alpha_L + (13 - 2\lambda_F)\alpha_F - 2\mu_F\right) = 1, \tag{34}$$

$$\Pr\left(44\beta\bar{Q} \leq 4(\mu_F + (\lambda_F + 11)\alpha_F) + 15\alpha_L\right) = 1. \tag{35}$$

The first condition guarantees interior solution for quantities in the secondary market[16] $(0 \leq q_{i1} \leq \bar{Q})$ and the second does the same for the auction $(0 \leq q_{i0} \leq \bar{Q})$. The third and fourth conditions ensure that the price is non-negative in the secondary market and the auction respectively ($p_1 \geq 0$, $p_0 \geq 0$).

To ensure that these inequalities hold w.p.1 we need to check that each of them holds true for the most adverse realizations of the types. Under the three cases considered in Assumption 2, using the relevant expressions for μ_L and λ_L conditions (32) to (35), collapse to the following conditions, all of which hold under Assumption 3:

Case 1 (independent types): $2\sigma \leq \beta\bar{Q} \leq \frac{63\theta}{44} + \frac{\sigma}{22}$;

Case 2 (L efficient): $2\sigma \leq \beta\bar{Q} \leq \frac{63\theta}{44}$;

[16] Notice that condition (29) guarantees interior solution in the secondary market for any initial allocation of permits, whereas (32) guarantees it only for the auction allocation considered in this section. Furthermore, we write the conditions in terms of probabilities and not in terms of parameters, as in (29). In order to present these conditions in terms of parameters, it suffices to consider the most adverse realizations for each inequality.

Case 3 (F efficient): $\sigma \leq \beta\bar{Q} \leq \frac{63\theta}{44} + \frac{\sigma}{22}$.

7.4 Proof of Proposition 3

Given the symmetry of cases 2 and 3, it is sufficient to prove the result for case 2. Using the expressions for δ^G under case 2 and δ^{CE} we have

$$\delta^G - \delta^{CE} = \frac{\beta\bar{Q}\sigma - (6\theta + 3\sigma)(\alpha_F - \alpha_L)}{2\beta\bar{Q}(6\theta + 3\sigma)},$$

from which we obtain

$$\delta^G \leq \delta^{CE} \Longleftrightarrow \alpha_F \geq \alpha_L + \chi$$

where $\chi := \beta\bar{Q}\sigma(6\theta + 3\sigma)^{-1}$ and the latter condition is true within the triangle delimited by the points $(\theta + \chi, \theta)$, $(\theta + \sigma, \theta)$, $(\theta + \sigma, \theta + \sigma - \chi)$. The area of this triangle is $\frac{(\sigma-\chi)^2}{2}$ and dividing by the area of the relevant domain, which in this case is the triangle delimited by the points (θ, θ), $(\theta + \sigma, \theta)$ and $(\theta + \sigma, \theta + \sigma)$, we obtain the probability of the event $\delta^G \leq \delta^{CE}$:

$$\Pr\left(\delta^G \leq \delta^{CE}\right) = \frac{2(\sigma - \chi)^2}{\sigma^2 2} = \left(\frac{6\theta + 3\sigma - \beta\bar{Q}}{6\theta + 3\sigma}\right)^2. \tag{36}$$

Since the latter expression is decreasing in σ and $\beta\bar{Q}$, we can find the infimum value by setting them a their highest possible values compatible with 29 , i.e., $2\sigma = \beta\bar{Q} = \frac{63}{44}\theta$. Plugging these values in (36) we get $\left(\frac{197}{239}\right)^2 \simeq 0.68 > 1$. By setting $\sigma = \beta\bar{Q} = 0$ we obtain 1 as the supremum value.

7.5 Proof of Proposition 4

In case 1, using the definition of δ, (24), (31) and (6) we obtain a lower bound for the difference between δ^A and δ^{CE}:

$$\begin{aligned}
\delta^A - \delta^{CE} &= \frac{2\theta + \sigma + \alpha_F + 6\beta\bar{Q} - 3\alpha_L}{7\beta\bar{Q}} - \frac{1}{2} - \frac{\alpha_F - \alpha_L}{2\beta\bar{Q}} \\
&= \frac{4\theta + 2\sigma + \alpha_L - 5\alpha_F + 5\beta\bar{Q}}{14\beta\bar{Q}} \geq \frac{5\beta\bar{Q} - 3\sigma}{14\beta\bar{Q}}
\end{aligned}$$

where the last inequality follows by setting α_L at its lowest possible value, θ, and α_F at its highest possible value, $\theta + \sigma$. Using this expression for the lower bound and condition (29) we conclude $\delta^A > \delta^{CE}$. Using (24), (31) and noting that $\delta^G = 0.5$, we conclude

$$\delta^A - \delta^{CE} = \frac{4\theta + 2\sigma + 2\alpha_F - 6\alpha_L + 5\beta\bar{Q}}{14\beta\bar{Q}} \geq \frac{5\beta\bar{Q} - 4\sigma}{14\beta\bar{Q}} \geq 0,$$

where the first inequality follows by setting $\alpha_L = \theta$, $\alpha_F = \theta + \sigma$ and second inequality from (29).

In cases 2 and 3 the procedure is similar, using the adequate values for μ_L and λ_L as given in (25) and (27) together with the relevant values of δ^G according to (26), (28) and (22).

7.6 Proof of Proposition 5

The strategy of the proof is to show that $h(\delta^A, \boldsymbol{\alpha}) > h(\delta^G, \boldsymbol{\alpha})$ for all the relevant values of the parameters. In turn, this is done by minimizing $h(\delta^A, \boldsymbol{\alpha}) - h(\delta^G, \boldsymbol{\alpha})$ in terms of α_L and α_F and showing that the minimum value is positive, which implies that it is positive for any combination of α_L and α_F. First, note that in the three cases included in Assumption 2 $h(\delta^A, \boldsymbol{\alpha}) - h(\delta^G, \boldsymbol{\alpha})$ is a continuous and bounded function defined on a compact set and, therefore, we can use the Weierstrass theorem to state that there exists a minimum in the relevant interval. Although the strategy of the proof is the same for all three cases, the development is slightly different for each case and so we consider them separately.

CASE 1: α_L and α_F are not correlated. Using (15), (23), (24) and (31), the difference between total cost under auctioning and grandfathering can be written as

$$h(\delta^A, \boldsymbol{\alpha}) - h(\delta^G, \boldsymbol{\alpha}) = \frac{1}{196\beta\bar{Q}}\left[-24\alpha_F^2 - 48\alpha_L^2 + 88\alpha_L\alpha_F - 2\alpha_F\left(20\theta + 10\sigma + 25\beta\bar{Q}\right)\right.$$
$$\left.+2\alpha_L\left(4\theta + 2\sigma + 5\beta\bar{Q}\right) + 4\left(2\theta + \sigma\right)^2 + 25\beta^2\bar{Q}^2 + 20\beta\bar{Q}\left(2\theta + \sigma\right)\right],$$

the sign of which is determined by the term in square brackets, which we denote as $\Delta_1(\alpha_L, \alpha_F)$. For the relevant values of the parameters we have $\frac{\partial \Delta}{\partial \alpha_L} > 0$ and $\frac{\partial \Delta}{\partial \alpha_F} < 0$, which implies that $\Delta(\alpha_L, \alpha_F)$ reaches a minimum at $(\alpha_L, \alpha_F) = (\theta, \theta + \sigma)$. Using these values, we get $\Delta_1(\theta, \theta + \sigma) = -40\sigma^2 - 30\sigma\beta\bar{Q} + 25\beta^2\bar{Q}^2$, which is always positive under the interior solution condition $2\sigma < \beta\bar{Q}$.

CASE 2: $\alpha_L \leq \alpha_F$. Using $\delta^G = \frac{3\theta + 2\sigma}{6\theta + 3\sigma}$ together with (15), (23), (25) and (31), the difference of total cost between both systems can be written as

$$h(\delta^A, \boldsymbol{\alpha}) - h(\delta^G, \boldsymbol{\alpha}) = \frac{1}{49\beta\bar{Q}X^2}\left\{X^2\left(-10\alpha_F^2 - 12\alpha_L^2 + 23\alpha_L\alpha_F\right) + \alpha_F\left[-X^2\left(3\theta + 32\beta\bar{Q}\right) + 49\beta\bar{Q}XY\right]\right.$$
$$\left.+\alpha_L\left[X^2\left(\theta + 27\beta\bar{Q}\right) - 49\beta\bar{Q}XY\right] + \beta^2\bar{Q}^2\left[225\theta\left(\theta + \sigma\right) + 44\sigma^2\right] + X^2\left(\theta^2 + 5\theta\beta\bar{Q}\right)\right\}$$

where $X := (6\theta + 3\sigma)$, $Y = (3\theta + 2\sigma)$. Denote as $\Delta_2(\alpha_L, \alpha_F)$ the term in curly brackets, which determines the sign of the whole expression. Now we solve the problem of minimizing $\Delta_2(\alpha_L, \alpha_F)$ subject to $\alpha_L \geq \theta$, $\alpha_F \leq \theta + \sigma$ and $\alpha_L \leq \alpha_F$. We conclude that there are two candidates that satisfy the first-order Kuhn-Tucker conditions. The first candidate is $\alpha_L = \theta$, $\alpha_F = \theta + \sigma$ and the second one is $\alpha_L = \alpha_F = \theta + \sigma$.[17] For the first candidate we have

$$\Delta_2(\theta, \theta + \sigma) = \sigma X\left[\beta\bar{Q}\left(-45\theta + 2\sigma\right) - 10\sigma X\right] + \beta^2\bar{Q}^2\left(225\theta\left(\theta + \sigma\right) + 44\sigma^2\right) > 0,$$

[17] The second is a candidate only if σ and $\beta\bar{Q}$ are high enough as compared to θ.

where the inequality comes from the fact that $\Delta_2(\theta, \theta+\sigma)$ is increasing in $\beta\bar{Q}$ for any $\beta\bar{Q} > 2\sigma$ (which is a required condition to guarantee interior solution) and replacing $\beta\bar{Q}$ by 2σ we get $\Delta_2(\theta, \theta+\sigma) > \sigma^2\left(334\theta\sigma + 98\sigma^2\right) > 0$. Analogously, for the second candidate we have

$$\Delta_2(\theta+\sigma, \theta+\sigma) = \sigma X^2\left[\sigma - 5\beta\bar{Q}\right] + \beta^2\bar{Q}^2\left(225\theta(\theta+\sigma) + 44\sigma^2\right) > 0$$

where, again, the inequality comes from the fact that $\Delta_2(\theta+\sigma, \theta+\sigma)$ is increasing in $\beta\bar{Q}$ for any $\beta\bar{Q} > 2\sigma$ and replacing $\beta\bar{Q}$ by 2σ we get $\Delta_2(\theta+\sigma, \theta+\sigma) > \sigma^2\left(576\theta^2 + 95\sigma^2 + 414\theta\sigma\right) > 0$.

CASE 3: $\alpha_F \leq \alpha_L$. Using $\delta^G = \frac{3\theta+\sigma}{6\theta+3\sigma}$ together with (15), (23), (27) and (31), the difference of total cost between auctioning and grandfathering can be written as

$$\begin{aligned} h(\delta^A, \boldsymbol{\alpha}) - h(\delta^G, \boldsymbol{\alpha}) &= \tfrac{1}{49\beta\bar{Q}X^2}\left\{X^2\left(-10\alpha_F^2 - 12\alpha_L^2 + 23\alpha_L\alpha_F\right)\right. \\ &+\alpha_F\left[49\beta\bar{Q}XZ - X^2\left(3\theta + 3\sigma + 32\beta\bar{Q}\right)\right] + \alpha_L\left[X^2\left(\theta+\sigma+27\beta\bar{Q}\right) - 49\beta\bar{Q}XZ\right] \\ &\left.+(\theta+\sigma)\left(5\beta\bar{Q} + (\theta+\sigma)\right) + \beta^2\bar{Q}^2\left[49Z(X-Z) - 6\right]\right\} \end{aligned}$$

where $X := (6\theta + 3\sigma)$, $Z = (3\theta + \sigma)$. Denote as $\Delta_3(\alpha_L, \alpha_F)$ the term in curly brackets, which determines the sign of the whole expression. We conclude that the only candidate that satisfies the Kuhn-Tucker first-order conditions to minimise $\Delta_3(\alpha_L, \alpha_F)$ subject to $\alpha_F \geq \theta$, $\alpha_L \leq \theta+\sigma$ and $\alpha_F \leq \alpha_L$ is $\alpha_L = \theta+\sigma$, $\alpha_F = \theta$. Evaluating $\Delta_3(\alpha_L, \alpha_F)$ for this candidate we get

$$\begin{aligned} \Delta_3(\theta+\sigma, \theta) &= -10\sigma^2X^2 + \beta\bar{Q}\sigma X(45\theta + 47\sigma) + \beta^2\bar{Q}^2\left(225\theta(\theta+\sigma) + 44\sigma^2\right) \\ &> \sigma^2\left[X(45\theta + 47\sigma - 10X) + (225\theta(\theta+\sigma) + 44)\right] > 0, \end{aligned}$$

$$\begin{aligned} \Delta_3(\theta+\sigma, \theta) &= -10\sigma^2X^2 + \beta\bar{Q}\sigma X(45\theta + 47\sigma) + \beta^2\bar{Q}^2\left(225\theta(\theta+\sigma) + 44\sigma^2\right) \\ &\geq -10\sigma^2X^2 + 2\sigma^2X(45\theta + 47\sigma) + 4\sigma^2\left(225\theta(\theta+\sigma) + 44\sigma^2\right) > 0 \end{aligned}$$

where the first inequality comes from the fact that $\Delta_3(\theta, \theta+\sigma)$ is increasing in $\beta\bar{Q}$ and then we can use $\beta\bar{Q} = 2\sigma$ to obtain a lower bound and the second inequality follows simply by using the expression for X and rearranging.

7.7 Proof of Corollary 2

Consider an arbitrary value of the (monotone transformation of) total cost h, say $\tilde{h}$. For grandfathering, denote as $\Phi^G\left(\tilde{h}\right) := \left\{\boldsymbol{\alpha} \;/\; h(\delta^G, \boldsymbol{\alpha}) \leq \tilde{h}\right\}$ the set of values of the types, (α_L, α_F), such that the cost under grandfathering is not larger than $\tilde{h}$. Similarly, define $\Phi^A\left(\tilde{h}\right) := \left\{\boldsymbol{\alpha} \;/\; h(\delta^A, \boldsymbol{\alpha}) \leq \tilde{h}\right\}$ for the auction. From

Proposition 5 we know that, for any realization of the types, we have $h(\delta^G, \boldsymbol{\alpha}) \leq h(\delta^A, \boldsymbol{\alpha})$. In particular, this will be the case for those types contained in $\Phi^A\left(\tilde{h}\right)$. Then, we conclude that $\Phi^A\left(\tilde{h}\right)$ is included in $\Phi^G\left(\tilde{h}\right)$ or, in other words, $F^A\left(\tilde{h}\right) \leq F^G\left(\tilde{h}\right)$ for any value of $\tilde{h}$, where F^A and F^G are the distribution functions of h under auctioning and grandfathering respectively, which implies FOSD of G over A.