Elementary game theory

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ELEMENTARY GAME THEORY

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THEORY OF GAMES:

- The theory of games (or game theory) is a mathematical theory that deals with the general features of competitive situations. It was developed by John Von Neumann (called father of the game theory) in 1928, but it was only after 1944 when he and Morgenstern published their work named 'Theory of Games and Economic Behavior'.
- Game theory can be termed as strategic interactions for self-interested agents. It is used by governments, economists, and political scientists and in the fields of computer science.

SELF INTERESTED AGENTS:

- It doesn't mean that agents are adversarial or want to harm others or only care about themselves.
- It means that agents have opinions, preferences and so there is some description of the world, how the world could be and in different descriptions, agents have different preferences and utility.

UTILITY FUNCTION:

- It is a mathematical measure that tells us how much an agent likes or dislikes the situation.
- It captures not only their attitude towards definitive events, it also describes the preferences towards the distribution of such outcomes, in other words it captures their attitude towards uncertainty about events.

Decision theoretic rationality: it means that agents will always act in a way to maximize their expected or average utility.

DEFINING GAMES:

G = F (N, A, U). It means that, game is a function of players, actions, and utility.

PLAYERS: They are the central decision makers in the game .for ex: people, government, companies etc.

ACTIONS: It refers to the decision taken by the players. For ex while investing, deciding how much stock to buy, when to buy is an action by the investor.

PAYOFFS: It motivates players.

GAME MODELS:

There are various types of game models, which are based on factors, like the number of players participating, the sum of gains or losses and the number of strategies available.

1. *Number of persons:* If a game involves only two players, it is called two persons game. If there are more than two players, it is named as n-persons game. This means that the participants can be divided into *n* mutually exclusive groups, with all the members in the group having identical interests.

2. *Sum of payoffs:* If the sum of payoffs to the players is zero, then it is known as zero sum game or constant sum game, otherwise non-zero sum game.

3. *Number of strategies:* If the number of strategies is finite, the game is called a finite game else it is known as an infinite game.

STANDARD FORMS OF THE GAME

Normal   Extensive

Normal Form Game: Both players move simultaneously, in this form.

Extensive Form Game: The other players' moves is a reaction to the former players' action i.e. players do not move simultaneously in an extensive form game. For example, chess.

NORMAL FORM'S FEATURES:

- The number of players is N: {1, 2, 3,…, N}.
- A_i depicts the action set of player.
- $A = A_1*A_2*....*A_N$
- Utility function(Payoffs): $U = U_1*U_2*...*U_N$
- $A \rightarrow R$(Real numbers)

<u>TYPES OF GAMES:</u>

<u>MATCHING PENNIES:</u>

This is a 2x2 matrix game, where two players toss a coin each. If both players get a head or tail (same face of the coin), then player 1 gets a utility of 1 and player 2 gets -1. If both the players get dissimilar faces, then player 2 gets a utility of 1 and player 1 gets -1.

Player 1 →

Player 2 ↓

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

COORDINATION GAME:

Let there be two drivers, driving on a road. If both drivers drive in the same direction, then there would be no collision and both will secure a payoff of 2. However, if both choose opposite sides of the road, then the two will collide (hypothetically, always) and the utility will be -1. Hence it would be both players interest to coordinate with each other.

L: left side, R: right side

Driver 1 →

Driver 2 ↓

	L	R
L	2,2	-1,-1
	-1,-1	2,2

PRISONER'S DILEMMA:

Prisoner 1 → Prisoner 2 ↓	C	D
C	-2,-2	-6,0
D	0,-6	-3,-3

This is a 2x2 matrix. Prisoner 1 is the row player and prisoner 2 is the column player. C stands for co-operation between the two prisoners, and D for defect. Here, if the two prisoners co-operate with each other, then both of them receive payoffs of -2; and if both are defective then payoffs for both are -3. However, if one is defective and the other not, then the payoff is 0 for the former and -6 for the latter.

BATTLE OF THE SEXES:

Male → Female ↓	A	C	
A	2,1	0,0	
C	0,0	1,2	C

In this example, we assume that the male is interested in watching an action movie, namely 'A' while the female prefers watching a comedy, viz. 'C'. But, both primarily prefer watching a movie together too. So the payoffs are zero, if both go to watch different movies. And the payoffs for watching the movie together are positive and slightly higher for the one whose initial preference it was.

NASH EQUILIBRIUM

According to strategic reasoning, we can say that the phenomenon where each player responds best to the other is Nash Equilibrium. It is a solution concept of a non-cooperative game comprising of two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by only changing their own strategy. Nash equilibrium is best for both players, if all players abide by it.

ADVANTAGES OF NASH EQUILIBRIUM:

- It is self-consistent.
- Players use it to maximize their own payoffs.
- There's no incentive for a player to deviate from equilibrium.
- Nash equilibrium is stable.

BEST RESPONSE

$A_{-i} = (A_1, A_2, \ldots., A_{i-1}, A_i, \ldots., A_n)$: Other players' response

$A = (A_{-i}, A_i)$: Player's response.

Then, Best Response:

$$\boxed{A_i < BR\,(A_{-I}) \text{ iff all } a_i\ \varepsilon\ A_i\ ,\ U_i\,(a_i^*\ ,\ a_{-I}\,) \geq U_i\,(a_i\ ,\ a_{-I}\,)}$$

A^*_I is the best response only if an agent chooses A^*_I instead of any other action profile, say $A_{i,}$ when the other agents simultaneously choose A_{-I} ; and the player is always at least as better off after choosing A^*_I , as he would've been by choosing some other action.

<u>PRISONER'S DILEMMA REVISITED:</u>

Prisoner 1 →

Prisoner 2 ↓	C	D
C	-2,-2	-6,0
D	0,-6	-3,-3

We've discussed the features of this game initially. Now, we recall this game to understand Nash equilibrium cognitively. If we say that prisoner 1 chooses to cooperate (row 1, column 1) and prisoner 2 (row 1, column 2) knows about this strategy, then there is surely an incentive for prisoner 2 to defect. If the tables turn and prisoner 1 is now aware that prisoner 2 would defect then, he would defect too to reduce his negative payoff (row 2, column 2). And this cycle would continue perennially. As both the players are uninformed about the opponent's strategy, choosing to defect would be the least risky choice for them (-3,-3). This is how, we find the Nash equilibrium. Conclusively, (-3,-3) is the Nash equilibrium.

DOMINANT STRATEGIES

A strategy is said to be dominant if it dominates all the other strategies of the game.

A strategy is said to be **strongly** dominant, if U_i (S_i, S_{-i}) > Ui (S'_i , S_{-i}). Strategy is said to be **weak** if U_i $(S_i, S_{-i}) \geq U_i$ (S'_i , S_{-i}).

Equilibria and Dominance:

- Equilibrium in strictly dominant strategies must be unique.
- If one S_i is dominant to all the other, it is dominant throughout.

All players are assumed to be rational, so they make choices which result in the outcome they prefer most, given what their opponents do. In the extreme case, a player may have two strategies A and B so that, given any combination of strategies of the other players, the outcome resulting from A is better than the outcome resulting from B. Then strategy A is said to dominate strategy B. A rational player will never choose to play a dominated strategy. In some games, examination of which strategies are dominated results in the conclusion that rational players could only ever choose one of their strategies.

An example, which can best explain this, is prisoner's dilemma.

PRISONER'S DILEMMA

Prisoner 1 →

Prisoner 2 ↓	C	D
c	-2,-2	-6,0
d	0,-6	-3,-3

In this game, each strategy combination defines a payoff pair, for example (-6, 0) for (D, c), which is given in the respective table entry. Each cell of the table gives the payoff value of player I and player 2. Note that in the strategic form, there is no order between player I and 2 since they act simultaneously (that is, without knowing each other's action), which makes the symmetry possible. Symmetry means that the game stays the same when the players are exchanged.

In the Prisoner's Dilemma game, "defect" is a strategy that dominates "cooperate." Strategy D of player 1 dominates C because if player 2 chooses c, then player 1's payoff is -6 while choosing D and -2 while choosing C; if player 2 chooses d, then player 1 receives -3 for D as opposed to 0 for C. These preferences of player I are indicated by the downward pointing arrows in Figure 2. Hence, D is indeed always better and dominates C. In the same way, strategy d dominates c for player 2.

No rational player will choose a dominated strategy since the player will always be better off when changing to the strategy that dominates it. The unique outcome in this game, as recommended to utility-maximizing players, is therefore (D, d) with payoffs (-3, -3). Somewhat paradoxically, this is less than the payoff (-2,-2) that would be achieved when the players chose (C, c).

"Prisoner's Dilemma" is actually a story of two prisoners who are suspects of a serious crime. There is no judicial evidence for this crime except if one of the prisoners testifies against the other. If one of them testifies then, he will become immune from prosecution (payoff 0), whereas the other will be sentenced for a long term (payoff -6). If both testify, their punishment will be less severe (payoff -3 for each). However, if they both "cooperate" with each other by not testifying at all, they will be imprisoned for a short duration, for example only for possessing illegal weapons (payoff -2 for each). The "defection" from that mutually beneficial outcome is to testify, which gives a higher payoff no matter what the other prisoner does, with resulting lower payoff to both. This sums up their "dilemma."

Prisoner's Dilemma games arise in various contexts where individual "defections" at the expense of others lead to overall less desirable outcomes. Examples include arms races, litigation instead of settlement, environmental pollution, or cut-price marketing, where the resulting outcome is detrimental for the players. Its justification on the lines of game theory on individual grounds is taken as a case for treaties and laws, which enforce cooperation.

Game theorists have tackled the "inefficiency" of the result of the Prisoner's Dilemma game. For example, playing it repeatedly basically changes the game. In such a repeated game, patterns of cooperation can be established as rational behavior when players' fear of punishment in the future outweighs their gain from defecting today.

MIXED STRATEGY AND NASH EQUILIBRIUM

In game theory, the **Nash equilibrium** is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by only changing their own strategy. If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

For example, let us say that the Indian cops are to set up checkpoints at various national borders by randomly choosing them. If a potential attacker seeks to attack through any border having a checkpoint then we assume that the cops shoot him without fail. The only way in which the attacker can succeed is when he attacks through a border without a checkpoint. Since, the randomization in selecting borders with checkpoints was done very secretively; the attackers have absolutely no idea about the positioning of checkpoints but they are well aware that checkpoints are stationed at some borders. This lack of information of the attackers limits their attacks.

If the cops shoot the attacker, on the fulfilment of the above conditions then the attackers get huge negative payoffs and the cops get positive payoffs. But, if the cops are unable to defend then the attackers receive positive payoffs and cops get 0 utility.

Under such situations, Nash equilibrium involves defenders defending such kinds of random attacks, which are known as mixed strategy.

Matching Pennies

This is a 2x2 matrix game, where two players toss a coin each. If both players get a head or tail (same face of the coin), then player 1 gets a utility of 1 and player 2 gets -1. If both the players get dissimilar faces, then player 2 gets a utility of 1 and player 1 gets -1.

Player 2 / Player 1	H	T
H	1,-1	-1,1
T	-1,1	1,-1

In games like matching pennies it is a bad idea to play a deterministic strategy. This is so because, if we assume that player 1 tosses the coin first and gets head (row 1, column 1) , then player 2 will always opt for tails (row 1, column 2) in order to maximize his payoffs and lower that of the opponent. Now, when player 2 has chosen tails then player 1 again chooses head (row 2, column 2) to maximize his payoffs and lower that of the opponent. And the cycle continues. This ambiguity now brings us to briefly understand the:

UTILITY UNDER MIXED STRATEGY

When we are unable to compute the payoffs by the matrix method, then we use mixed strategy. When there can be more than one action with a positive probability, we call it a mixed strategy. If all mixed strategies can be represented by S_i , then mixed Strategy profile is given by $S_1*S_2*....*S_n$ .The idea behind playing a mixed strategy is to randomly confuse the player without playing any deterministic strategy.

According to decision theory, expected utility function is as follows:

→ $U_i\ (s) = \sum_{a \varepsilon A} U_i\ (a)\ Pr\ (a|s)$

→ $Pr\ (a|s) = \prod s_j\ (a_j)$ where $j\ \varepsilon\ N$. (Product of the probability of each player playing his action profile.)

Here, U_i (s) is the payoff of the strategy profile and U_i (a) is the payoff of the action chosen. Pr (a|s) is the probability of action 'a' given by the strategy 's'. Therefore, we can say, that the sum of all action profiles multiplied by the corresponding probabilities of profile 's' gives us the desired utility.

This can be explained by the given matrix:

Player 1 →

Player 2 ↓

	0.5	0.5
0.5	0.25	0.25
0.5	0.25	0.25

Now, we can again discuss the matching pennies game, this time by using mixed strategy. Since, this is a zero sum game, the players would be interested in randomizing by playing 50-50. So no matter what action is chosen by either player, probability distribution remains uniform throughout. Here, 0.25 is Pr (a|s) for each cell. It is obtained by multiplying the probability of each row with the corresponding column.

- Best Response:
  $S*_I\ \varepsilon\ BR\ S_{-I}$ iff for all $S_i\ \varepsilon\ S_i$, $U_i\ (S*_I, S_{-i}) \geq U_i\ (S_I, S_{-i})$.
  $S*_I$ is the best response only if an agent chooses $S*_I$ instead of any other strategy profile, say $S_{i,}$ when the other agents choose S_{-I} ; and the player is always at least as better off after choosing $S*_I$, as he would've been by choosing some other strategy.
- Nash Equilibrium :
  $S = (S_1, S_2, \ldots , S_n)$ for all i, $S_i\ \varepsilon\ BR\ (S_{-i})$.

If S is the best response among all the strategy profiles, then it is said to be the Nash equilibrium as well.

Every finite game played using a mixed strategy has a Nash equilibrium. This is so because, Nash equilibrium always assures everyone with a stable thing no matter what the game is. A player will play a mixed strategy only when he/she is indifferent between more than one actions else not.

BATTLE OF THE SEXES

Male → / Female ↓	A (p)	C (1-p)
A (q)	2,1	0,0
C (1-q)	0,0	1,2

C

In this example, we assume that the male is interested in watching an action movie, namely 'A' while the female prefers watching a comedy, viz. 'C'. But, both primarily prefer watching a movie together too. So the payoffs are zero, if both go to watch different movies. And the payoffs for watching the movie together are positive and slightly higher for the one whose initial preference it was.

It's neither always easy to randomize the game 50-50 nor justified. So, we need to calculate the probabilities of each player and the actions. The funny thing about Nash equilibria is that, though it tells us about the existence of equilibrium, it doesn't teach us how to compute it.

For such cases, we need to guess the Nash equilibria. We do so by guessing the support (set of pure strategies that have positive probabilities under mixed strategy). While computing the mixed Nash equilibrium, the agent is indifferent between the two actions and hence, by equating the utilities one can calculate the probability. p and q denote the probabilities associated with the actions.

$U_1 (A) = U_1 (C)$

$2p + 0(1-p) = 0(p) + 1(1-p)$ …Multiplying the grey shaded figures from the matrix with the associated probabilities.

- ➔ $P = \frac{1}{3}$

Similarly, $U_2 (A) = U_2 (C)$.

$q + 0(1-q) = 0(q) + 2(1-q)$ …Multiplying the yellow shaded figures from the matrix with the associated probabilities.

- ➔ $Q = \frac{2}{3}$

Therefore, mixed strategies are $(\frac{2}{3}, \frac{1}{3})$ and $(\frac{1}{3}, \frac{2}{3})$, and they are in Nash equilibrium.

Example: Compliance inspections

A consumer buys a software from a company that sells it along with a set of rules and regulations. The consumer has an incentive to violate these rules. The seller can however, inspect about the violation of rules but this would require a heavy cost. If the seller inspects and catches the consumer cheating, then the consumer is liable of paying a hefty amount for noncompliance.

Player 1 / Player 2	Comply	Cheat
Don't inspect	0,0	-5,5
Inspect	-1,0	-2,20

The figure drawn above depicts the possible payoffs for such a game. The standard result is that the seller chooses not to inspect and the consumer chooses to comply. Without inspection, the consumer prefers to cheat since that gives her payoff 5, and the seller gets a negative payoff of -5. The seller can also decide to inspect. If the consumer complies, inspection leaves her payoff 0 unchanged, while the seller incurs a cost resulting in a negative payoff −1. If the consumer cheats, inspection will result in a heavy loss (payoff −20 for player 2) but will still create a little trouble for player I (payoff −2).

If player 1 knows beforehand that player 2 would choose to comply then he would surely choose to not inspect. However, this knowledge is beyond player 1's reach. And, vice versa the seller would prefer to always inspect if he knew that the customer is going to cheat (since −2 is better than −5). If the seller always preferred not to inspect, then this would be a dominating strategy (we will study about dominant strategy later in the text) and be part of a (unique) equilibrium where the consumer cheats.

This game has no equilibrium in pure strategies. If any of the players settles on a deterministic choice (like no inspection by player I), the best response of the other player would be to cheat which will be a unique response, to which the original choice would not be a best response. The strategies in a Nash equilibrium must be best responses to each other, so in this game this fails to hold for any pure strategy combination.

Maxmin Strategy

What should the players do in the game described above? One possibility is that they choose a max-min strategy. A **max-min strategy** maximizes the player's own worst case payoffs against all possible choices of the opponent. The max-min strategy for player I is to Inspect (where the seller guarantees himself payoff of −2), and for player 2 it is to comply (which guarantees her payoff 0). Since this is not a Nash equilibrium; it is not a stable recommendation to the two players, since player I could switch his strategy and improve his payoff.

A mixed strategy of player I in this game is to Inspect only with a certain probability. While inspecting, a seller can also think of randomizing as it will reduce cost. Even if an inspection is uncertain, the fear of being caught should stray the consumer away from cheating.

If the probability of inspection is very low, for example 0.01, then player 2 receives (irrespective of that probability) payoff 0 for comply, and payoff $0.99\times10+0.01\times(-90) = 9$, which is bigger than zero, for cheat. Hence, player 2 will still cheat. If the probability of inspection is much higher, for example 0.2, then the expected payoff for cheat is $0.8 \times 10 + 0.2 \times (-90) = -10$, which is less than zero, so that player 2 prefers to comply. If the inspection probability is either too low or too high, then player 2 has a unique best response. As shown above, such a pure strategy cannot be part of equilibrium.

Hence, the only case where player 2 herself could possibly randomize between her strategies is if both strategies give her the same payoff, that is, if she is indifferent. It is never optimal for a player to assign a positive probability to playing a strategy that is inferior, given what the other players are doing. It is not hard to see that player 2 is indifferent if and only if player I inspects with probability 0.1, since then the expected payoff for cheat is $0.9 \times 10 + 0.1 \times (-90) = 0$, which is then the same as the payoff for comply.

With this mixed strategy of player I (Don't inspect with probability 0.9 and Inspect with probability 0.1), player 2 is indifferent between her strategies. Hence, she can mix them (that is, play them randomly) without losing payoff. The only case where, in turn, the original mixed strategy of player I is a best response is if player I is indifferent. According to the payoffs in this example, player 2 should choose to comply with probability 0.75 and cheat with probability 0.25. The expected payoffs to player I are then for no inspection $0.75 \times 0 + 0.25 \times (-5) = -1.25$, and for Inspect $0.75 \times (-1) + 0.25 \times (-2) = -1.25$, so that player I is indeed indifferent, and his mixed strategy is a best response to the mixed strategy of player 2.

This explains the only Nash equilibrium of the game. Since, it uses mixed strategies, it is called a mixed equilibrium. The resulting expected payoffs are −1.25 for player I and 0 for player 2.

MAXMIN AND MINMAX STRATEGY: A BRIEF DESCRIPTION

Maxmin value or the safety level is the minimum payoff guaranteed by a maxmin strategy. The maxmin strategy maximizes agent's own worst case payoffs.

Maxmin strategy is given by max S_i min S_i U_i (S_1 , S_2). All maxmin strategy profiles constitute a Nash equilibrium.

Minmax strategy minimizes other agents' payoffs on the assumption that they were trying to maximize it. In a zero sum game, each player receives a payoff equivalent to both maxmin and minmax value.

- Why is Maxmin and Minmax Strategy used?
  I. It is used by conservative agents who want to maximize their worst case payoffs.

II. It is also used by paranoid agents who believe everyone is out to get him.

For two players, minmax strategy is always solvable.

Computing Minmax

Minimize U^*_1 (payoff in equilibrium)

Subjected to: 1. $\sum_{k \varepsilon A_2} U_1 (a_1^j , a_2^k). S_2^k \leq U_1^*$ for all $j \varepsilon A_1$.

2. $\sum_{k \varepsilon A_2} S_2^k = 1$ for $S_2^k \geq 0$.

INTERPRETATIONS OF MIXED STRATEGY

- Randomize your actions when:
  1. Uncertain about other agents' actions
  2. To confuse the opponent

- Mixed strategy gives a count of pure strategies in a limit.
- It describes population dynamics. Mixed strategy gives a probability of getting each pure strategy.

PARETO OPTIMALITY

We generally use pareto optimality in situations, where one agent's interests are more important than the other. If outcome O ≥ O', then we can say that O pareto dominates O' and if the agent also prefers O to O'. An outcome, O* is pareto optimal if there's no other outcome that pareto dominates it. There is always at least one pareto optimality, it can even be more or all. Nash equilibrium is the only non-pareto optimal outcome.

EXAMPLES: 1. Matching Pennies

Player 1

Player 2

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

In this form of the game, all the strategies are pareto optimal as it is a zero sum game.

2. Battle of the Sexes

Male →

Female ↓

	A	C
A	3,1	0,0
C	0,0	1,3

In this game, both the shaded payoffs are pareto optimal. Since, the other two cells sum up to a zero; the remaining two cells with similar sums win.

3. Lane Driving

Driver 1 →

Driver 2 ↓

	L	R
L	2,2	-1,-1
R	-1,-1	2,2

In this game, both the shaded utilities are pareto optimal. The unshaded cells sum up to be negative while the shaded ones are positive and similar.

4. Prisoner's Dilemma (Two Player Game)

Prisoner 1 →

Prisoner 2 ↓

	C	D
C	-2,-2	-6,0
D	0,-6	-3,-3

In prisoner's dilemma, all the outcomes are pareto optimal except (-3,-3). (-3,-3) is dominated by (-2,-2). Moreover, (-3,-3) gives the nash equilibrium.

LINEAR COMPLEMENTARITY FORMULATION

The formula used over here to calculate the nash equilibria best response beyond 2x2 games is as follows:

$$\sum_{k \varepsilon A_2} U_1 (a^j_1, a^k_2). S^k_2 + r^j_1 = U^*_1 \text{ for all } j \varepsilon A_1.$$

→The shaded grey part of the formula is the expected payoff of player 1 while playing the strategy. s is the mixed strategy played by two players. r^j_1 is called the slack variable or the magnitude by which player 1 is missing from the best response. U^*_1 is the best response.

$$\sum_{j \varepsilon A_1} U_2 (a^j_1, a^k_2). S^j_1 + r^k_2 = U^*_2 \text{ for all } k \varepsilon A_2.$$

→This formula can also be explained on similar lines, as above for player 2.

$S^j_1 \geq 0$, $S^k_2 \geq 0$, $r^k_2 \geq 0$, $r^j_1 \geq 0$ for all $j\varepsilon A_1$ and $k \varepsilon A_2$:Pre-requisites for the above formulae. In Nash equilibrium, slack variable's value is 0.

EXTENSIVE GAMES WITH PERFECT INFORMATION

The normal form game (strategic form) does not incorporate any notion of sequence or time of the action of the players. In a normal form game, both players choose their strategy together without knowing the strategies of other players in the game. While the extensive form game is a game, which makes the temporal structure explicit i.e. it allows us to think more naturally about factors such as time. In an extensive game with perfect information there are no simultaneous moves and every player at any point of time is made aware of all the previous choices of all other players.

EXTENSIVE FORM'S FEATURES:

- It depends on timing, i.e. the order of things or actions.
- The information of players plays a huge role.

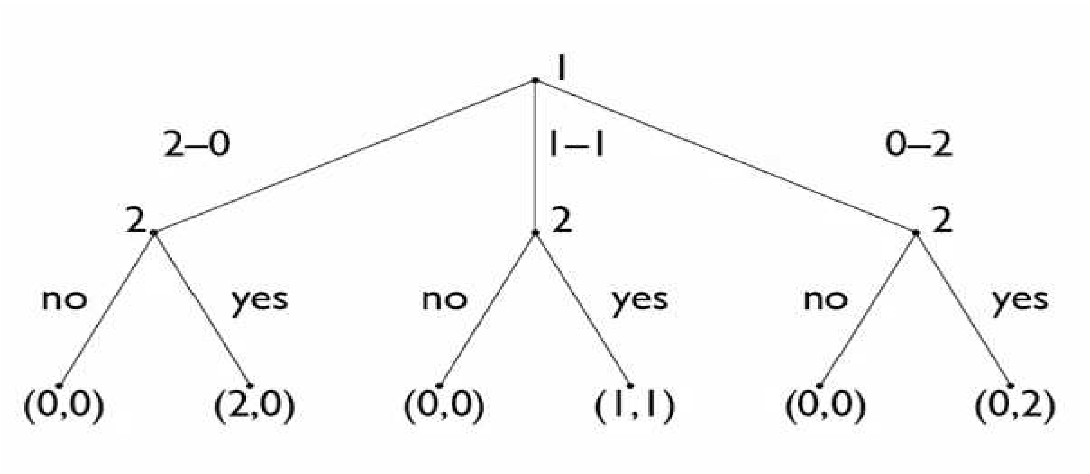
A Finite perfect information game is defined by the tuple (N, A, H, Z ,x, p ,u σ) where:

- Players: N is a set of finite n players
- Actions: A is a single set of actions

Choice nodes and label for these nodes:

- Choice nodes: H is a set of non terminal choice nodes
- Action function: x: H⟶ 2^A assigns to each choice node a set of possible actions
- Player function: p: H⟶ N. It tells us for every choice node who is it that gets to make a choice in that node.
- Terminal nodes: Z. It is disjoint from H.
- Successor function: σ: H X A⟶H⋃Z

Choice nodes form a tree: notes encode history. for example:



The above example is a sharing game where a brother and a sister want to decide between each other how they want to share $2. It begins with the brother stating how he proposes to divide the money. He starts out at 'choice node 1'. He has 3 actions which he can take, the first one being 2-0 where he takes $2 and his sister gets $0. The other one being 1-1 where they both share the money equally and the third one being 0-2 where he gives the entire amount to his sister.

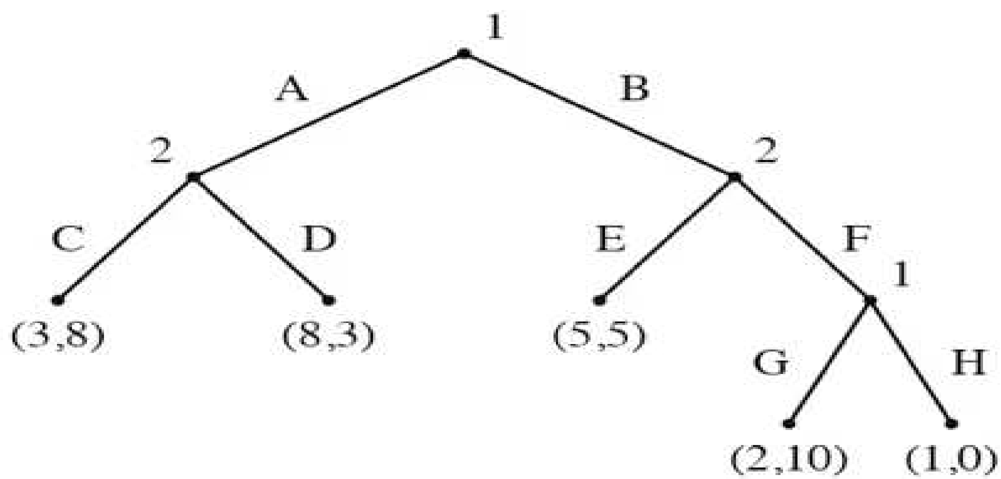
In each of these cases when brother takes his action, we transition to a new choice node where the sister gets to act. In each case sister gets 2 actions, she gets to accept the offer he makes or reject it. If she rejects the 1st offer , both of them get a payoff of $0 whereas if she accepts the offer both of them gets a payoff corresponding to what the brother proposes.

STRATEGIES, BEST RESPONSE, NASH EQUILIBRIUM IN CASE OF PERFECT INFORMATION EXTENSIVE FORM GAME :

- A pure strategy for a player in a perfect information game is a complete specification of which action to take at each node belonging to that player.

- The pure strategy for a player in given perfect information extensive form game is the cross product of the action set of that player.

EXAMPLE



The pure strategies for player 2 in the above example will be the cross products of the action sets at each of their different choice nodes.

So the pure strategies for player 2 is {(C, E), (C, F), (D, E), (D, F)}.

For example the pure strategy (C, F) means that at one choice node player 2 will play C while at other choice node player 2 will play F. Since there are 2sets of side 2, there is a total of 4 pure strategies.

The pure strategy for player 1 in the above example will be the cross product of the action set at different choice nodes. Hence the pure strategy for player 1 is {(A,G),(A,H),(B,G),(B,H)}.This is true even though , conditional on taking A, the choice between G and H will never have to be made.

Since we have given a new definition for pure strategy in case of a extensive form game we can leverage it in order to use all of the old definition of all kinds other concepts.

For example: In normal form games we define mixed strategies as probability distribution over pure strategies and the same definition can be used for mixed strategies in normal form game, all that changes is the underlying pure strategies is different.

Also the best response and Nash equilibrium in case of extensive form game is same as in a normal form game.

We can also convert an extensive form game in a normal form game but the reverse is not possible.

Player 1/player 2	CE	CF	DE	DF
AG	3,8	3,8	8,3	8,3
AH	3,8	3,8	8,3	8,3
BG	5,5	2,10	5,5	2,10
BH	5,5	1,0	5,5	1,0

The above table is the representation of the above extensive form game in normal form game.in the above table we listed the pure strategy of each agent as the action in the normal form game.

In order to fill the payoff value (lets say in cell BG,CF) we follow the tree and the payoff value at its terminal node(in this case 2,10) is the value of the cell.

However in real practice it is very difficult to write an extensive form game in normal form game because games aren't always this small, even here we write down 16 pairs instead of 5.

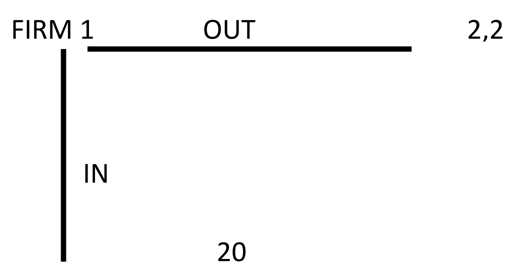
SUBGAME PERFECT EQUILIBRIUM:

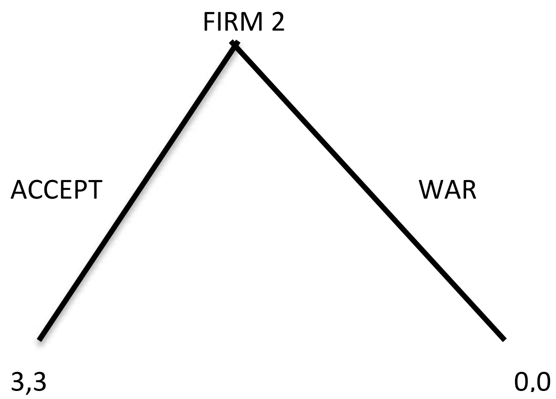
A strategy profile is a sub game perfect equilibrium if it represents Nash equilibrium of every sub game of the original game. Informally, this means that if (1) the players played any smaller game, which is a part of larger game and (2) their behavior represents Nash equilibrium of the smaller game, then their behavior is a sub game perfect equilibrium of the larger game.

A subgame perfect equilibrium is a complete and contingent plan of action. It must state what happens on and off the equilibrium path of play

FOR EXAMPLE: THE GAME

- A firm is deciding to enter the market, which another currently has monopoly over.
- If the firm enters, the monopolist chooses whether to accept or declare a price war.
  - -The firm only wants to enter if the monopolist does not engage in price war.
  - -A price war is unprofitable for the monopolist.





IF firm 1 decides against entering the market they both get a payoff of (2,2), if firm 1 enters the market, then firm 2 gets to decide whether to accept the price or wage a price war. If it accepts firm 1 will get a payoff of 3 since they would have better have better production techniques and hence profit margin would be higher, while if it decides for a price war, then both the firms get a payoff of (0,0).

In the following game there are two equilibriums.

The first equilibrium is (IN, ACCEPT) where firm 1 decides to enter and firm 2 decide to accept. They both get a payoff of (3,1). The firm 1 gets a payoff of 3 which is greater than 2 which he would get if he choose not to enter and firm 2 will get a payoff of 1 which is greater than payoff of 0 which he would get if he decides to engage in a price war. Hence (IN, ACCEPT) IS NASH equilibrium.

The second equilibrium is (OUT, WAR) where firm 1 decides to stay out while firm 2 decides to wage a price war. In this situation they get a payoff of 2,2. Here the choice of firm 2 does not matter, as he does not get to decide.

However if firm 1 actually decides to enter, one must observe that it is never in firm 2 best interests to engage in a price war. Firm 2 would always want to accept rather than engage in a price war. Hence this equilibrium does not make any sense as firm 1 will eventually realize that it is better for him to enter the market rather than staying out of the market.

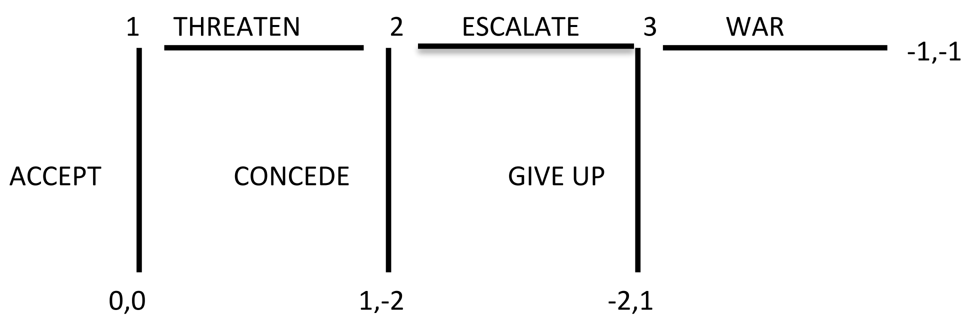
Hence the idea of sub game equilibrium is that to make sure that every sort of threat made by a player is credible and the way we can do this is by chopping things into individual sub games. For example if we look what action firm 2 will take, we can find out whether his threat to engage in a price war is credible or not. Hence the whole idea of sub game equilibrium is to make sure whether the threat made by a player is credible or not.

BACKWARD INDUCTION:

The process of deducing backward from the end of a problem or scenario to infer a sequence of optimal actions is known as backward induction. Backward induction starts at final step of the game, and by anticipating what the last player in a two-player game will do at that point, determines what moves likely to lead it.

Backward induction was first mentioned by game theory inventor's john von Neumann and Oskar Morgenstern in 1994.

FOR EXAMPLE:



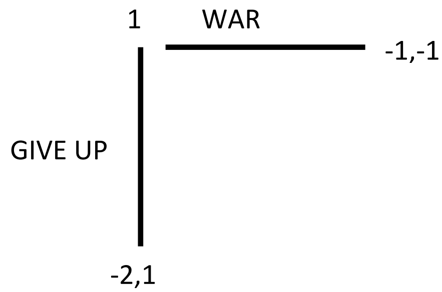
THE above example represents a war situation growing between 2 states, state 1 and state 2.state 1 can begin by just accepting the situation and getting a payoff of zero. Alternatively player 1 can threaten state 2.

State 2 can respond either by conceding in which case state 1 gets a bit of a gain i.e. payoff for player 1 is 1 and state 2 losses something and looks very week and hence get a payoff of -2.alternatively state 2 can escalate the situation in which case state 1 again gets to make a choice.

State 1 can know respond either by giving up in which case state 1 gets a payoff of -2 and state 2 gets a payoff of 1 as it looks very strong or state 1 can declare war in which case both of them gets a negative payoff of -1.

We can solve this game by starting at the end and work our way towards the beginning , because lets think from the point of view of state 1, state 1 can't just threaten and hope for the best. It only makes sense to threaten if what's goanna happen after threatening is better for you if you just accept and the only way to know what will happen is if we look ahead in the future and work our way backward from there. Likewise it does not makes sense for player 2 to escalate, until he is sure that he would get a better payoff by escalating rather than conceding.but the only way to know what is better for her is to see what player 1 is going to do and work our way backward from there.

Hence we must ignore the first two situations in the beginning and we are left with player 1 final decision.



IF player 1 is ever in this situation, it is better for player 1 to engage in war rather than giving up because the payoff that player 1 gets after engaging in war is greater than the payoff he gets when he quits. Hence player 1 will choose WAR in this situation.

NOW player 2 knows that if escalates, player1 will declare war. In this case

If player 2 concedes he gets a payoff of -2 which is less than the payoff he would get if he escalates and player 2 then declares war. Hence player 2 in this case would choose to ESCALATE rather than CONCEDES.

Now we come back to our first decision i.e. player 1's decision whether to accept or threaten. Player 1 knows that if he threatens, player 2 would escalate, and then he would choose war and get a payoff of -1 but if he accepts in the beginning, player 1 would get a payoff of 0, which is greater than payoff of -1. Hence he would choose to accept in the beginning, which is his optimal strategy. Hence the sub game equilibrium in this game is <ACCEPT, WAR>; <ESCALATE>.

EXTENSIVE GAMES WITH IMPERFECT INFORMATION:

An imperfect information game is an extensive form game in which each agent's choice nodes are partitioned into information sets.

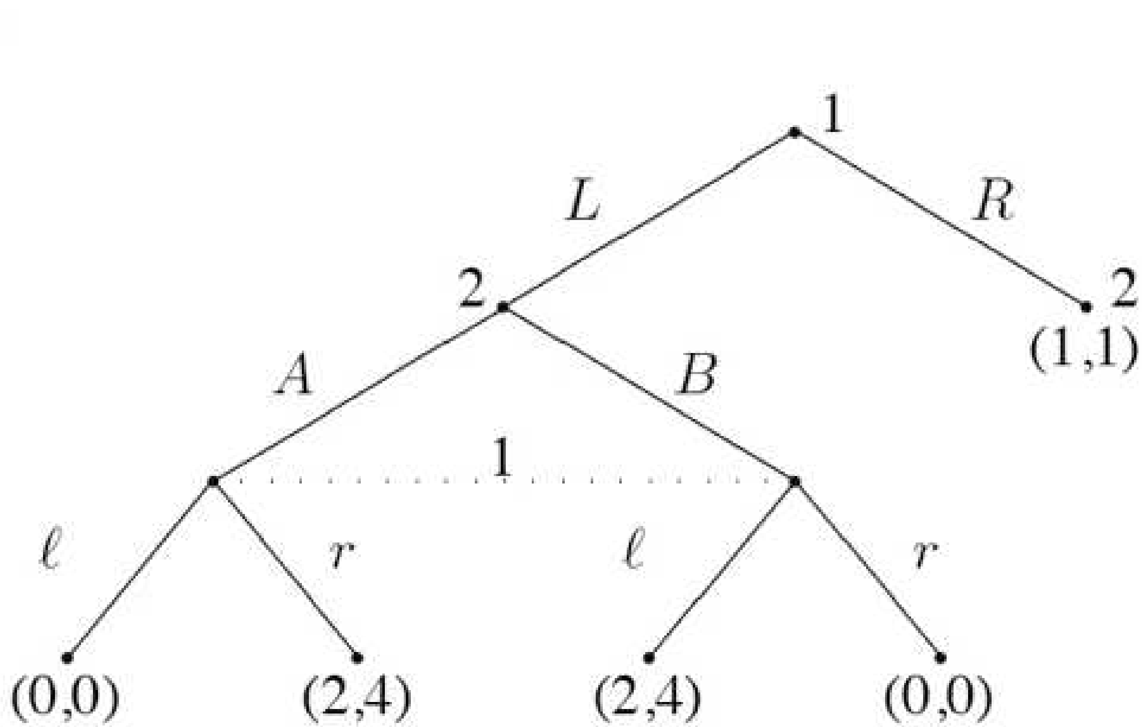
- An information set = {all the nodes you might be at}.
- The nodes in information set are indistinguishable to the agent.
- So all have the same set of actions.

- An imperfect information extensive form game is a tuple (N, A, H, Z ,x, p ,u σ,I) ,where (N, A, H, Z ,x, p ,u σ) is an extensive form game and

- I = ($I_{1,.}$, . , . $I_{N)}$ where $I_i = (I_{i,1}$, . . . $I_{i,ki})$ is an equivalence relation on (that is , a partition of) { h ∈ H : p(h) = I} with the following exceptions that x(h) = x($h^{/}$) and p(h) = p($h^{/}$)

- Here I is the set of equivalence classes which a player is not able to differentiate. If each of these equivalence classes contain one choice nodes then we are back to the perfect information extensive form game , if they contain more than one choice nodes then we have a imperfect information extensive form game.

FOR EXAMPLE:



In the following example player 1 has 2 equivalence classes. We use a dotted line to connect together choice nodes that belong to the same equivalence class. On the other hand player 2 has only 1 choice node and hence has only one equivalence class.

In the following game if player 1 moves right the game end with each of them getting a payoff of (1,1) but if he goes left then player 2 gets to make a choice.

Player 2 will choose either A or B and then player 2 will choose for the second time but player 1 is not able to observe the action of player 2.

If player 1 chooses left, then he would have to go left from both choice nodes.

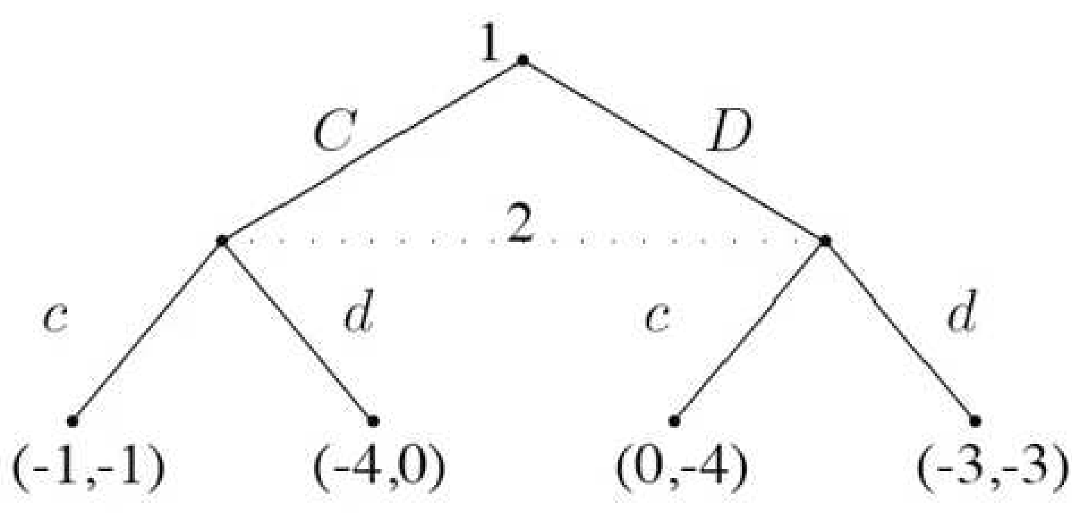
PURE STRATEGIES IN IMPERFECT INFORMATION GAME:

The pure strategies for any player in an imperfect information game are the cross product of the action sets in every different equivalence class of a player.

Therefore the pure strategy for player 1 in the above example is the cross product of player's 1 equivalence class i.e. the cross product of (L, R) and (l, r), i.e. {Ll, Rl, Lr, Rr}.

INDUCED NORMAL FORM:

We can represent any imperfect information extensive form game in a normal form game, which we could not do with perfect information game.



The above diagram is an example of prisoner's dilemma in imperfect information extensive form game. Here player 1 gets to decide whether to cooperate or defect and then player 2 gets to decide whether to cooperate or defect, but player 2 isn't able to tell which action player 1 took , so it does not matter whether he goes 2nd or 1st because he is not informed about what action player 1 opted for.

Hence it would be same if we put 2 as the root node as time isn't playing a role in the following game.

Now we can proceed as we did in perfect information game i.e. we can take all the pure strategies of player 1 and make them into rows and we can take all the pure strategies of player 2 and can make them into columns. Hence the induced normal form game would like:

Player 1 /player 2	Cooperate	Defect
Cooperate	-1, -1	0, -4
Defect	-4, 0	-3, -3

We fill in the payoff value in each cell the same way we filled it in perfect information game.

Once we have an induced normal form game all the information about mixed strategies, best response and Nash equilibrium of the induced normal form game can be carried forward to imperfect information extensive form game. For example: from Nash's theorem, we know that Nash equilibrium always exist in a imperfect information extensive form game because we can make a finite game from it.

MIXED AND BEHAVIORAL STRATEGIES:

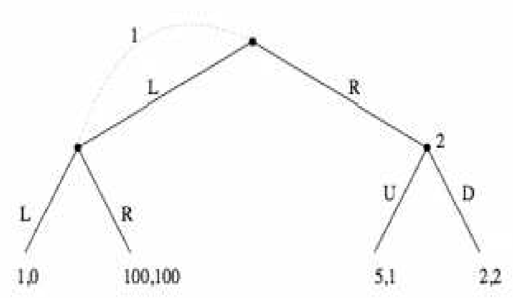
There are two meaningful different kinds of randomized strategies in imperfect information extensive form game.

- MIXED STRATEGY: randomize over pure strategy.
- BEHAVIORAL STRATEGY: independent coin toss every time an information set is encountered.

In games of perfect information mixed strategies and behavioral strategies can emulate each other i.e. the equilibria in mixed strategy is equal to the outcome in behavioral strategy.

The same is true in case of imperfect information so long as game has a perfect recall. A game has perfect recall if the agents have full recollection of their experience in the game i.e. they know all the information set they have visited before and all the action they have taken.

FOR EXAMPLE:



In the above example the pure strategy for player 1 is (L, R) and player 2 is (U, D) We observe that player 2 has a dominant strategy D i.e. no matter what action player 1 takes player 2 will always choose D.

Hence the best response for player 1 in response to player 2 choosing D is playing R because player 1 would get a payoff of 2 instead of 1 which he would get if he plays strategy 1. Hence (R, D) is a mixed equilibrium.

However we have a very high payoff that is not accessible (100,100) under mixed strategies.

WQUILIBRIUM UNDER BEHAVIOURAL STRATEGY:

Again D is strictly dominant for player 2. If player 1 uses a behavioral strategy i.e. he chooses action L with probability p and action R with probability 1-p,

- Then his expected utility will be $P^2 + 100p(1-p) + 2(1-p)$
- Simplifies to $-99p^2 +98p +2$
- Maximum at p=98/198
- Thus equilibrium is (98/198, 100/198) for player 1 and (0,1) for player 2

Thus in imperfect information extensive form game, we can have an equilibria in behavioral strategy that is different from equilibria in mixed strategy.

- N is set of agents
- G is a set of games with N agents each such that if $g, g^{/} \in G$ then for each agent $I \in N$ the strategy space in g is identical to the strategy space in $g^{/}$.
- $P \in \Pi$ (G) is a common prior over games, where Π (G) is the set of all probability distribution over G, and
- I = (I, ... I) Is a set of partitions of G, for each agent.

COALITIONAL GAME THEORY:

In coalitional games, our focus is on what group of agents, rather than individual agents can achieve.

A sub coalitional group is written as S, real valued payoff is denoted by v(S) and every S is a subset of N (finite set of players).

- Transferable Utility Assumption:
  - Payoffs maybe distributed among a coalition's members.
  - Members are always satisfied whenever payoffs are dispensed in a universal currency.
  - Each coalition can be assigned a single value as its payoff.

N is a finite set of players; indexed by i, and v: $2^N \rightarrow R$ associates with each coalition S (which is a subset of N), a real valued payoff v(S) that the coalition's members can redistribute among themselves.

The questions that now arise are:

1. Which coalition will form?
2. How should the coalition divide its payoff among its members?

"Grand coalition" of all agents in N is the solution for question 1, but it also depends on the right choice made while answering question 2.

SUPERADDITIVITY

G is said to be superadditive, where G = (N, v) for all S, T (subcoalitional groups) belonging to N and $S \cap T = \Phi$. Then, we can also say that $v(S \cup T) \geq v(S) + v(T)$.

G is convex if $v(S \cup T) \geq v(S) + v(T) - v(S \cap T)$ where $S \cap T = \Phi$. Convex means being non empty inclusive of Shapley value.

Superadditivity is justified when coalitions can work without interfering with one another.

- ❖ How should coalition divide payoffs?

Fairness Stability

SHAPLEY VALUE: It is a fair way of dividing the coalition's payment.

- ➢ Conditions For Shapley Value:
    - $v(N) = 1$ and $v(S) = 0$ if $N \neq S$.
    - Marginal contribution i.e. $v(N) - v(N|\{i\}) = 1$.
    - Everyone can't be paid their marginal contribution

→ Shapley's Axioms:

1. $v(S \cup \{i\}) = v(S \cup \{j\})$ if i and j are interchangeable.
2. i is dummy if $v(S) = v(S| \cup \{i\})$ and in this case, i should receive nothing.
3. If $v = v_1 + v_2$, then payments should be decomposed.

<u>Shapley Value Payoff</u>:

$$\Phi_i (N, v) = 1/N! \sum_{S \sqsubseteq N} |S|! (|N| - |S| - 1)! [v(S \cup \{i\}) - v(S)] \text{ for each } i, \text{ where } x(v) = \Phi(N, v).$$

x is the payoff vector.

- **CORE:** It shows the willingness to form smaller coalitions rather than grand coalitions. This is so because mixed strategy equilibrium always exists, however same can't be said about pure strategy.
  A player is said to be a veto player if $v(N|\{i\}) = 0$.

Is it always unique? Answer: Not necessarily always.

Core

Is it always empty? Answer: Can be.

x is the core if $\sum x_i \geq v(S)$ where $i \varepsilon S$.

Example: Voting Game

- Say, there are four political parties A, B, C & D with 45, 25, 15 and 15 representatives respectively. $100 million spending bill is to be discussed on should it be passed or not, and how much of this amount should be controlled by each of the parties. A minimum of 51 votes is required out of 100; and if the bill isn't passed then each party gets $0 for spending. In such a situation, can a subcoalition gain by defecting?

Solution: We can calculate the Shapley's value by using the above mentioned formula. It comes out to be 50, 16.67, 16.67 and 16.67 for parties A, B, C & D respectively.

As we can see that, A and B have an incentive to defect by forming a subcoalition as then they can divide $100 million between them, say in the ration 3:1. Therefore, yes, in a situation like this, a subcoalition can gain by defecting.

Conclusion: Shapley value doesn't give the right incentive in such cases, though it is fair.

- **Features of Coalitional Game Theory:**
  - Analogous to Nash equilibrium, it allows deviations by groups of agents.
  - Sum of payoffs to agents in any subcoalition S is at least as much as they could earn on their own.

REAL WORLD APPLICATION

INTERVENTION OF RBI IN FOREIGN EXCHANGE MARKET

The Reserve Bank of India (RBI) seeks to coordinate the exchange rate by intervention in the foreign exchange market with other central banks in order to reduce exchange rate volatility. The present study found that the coordinated exchange rate intervention had reduced exchange rate volatility relative to the unilateral exchange rate intervention level.

Even though the central banks intervene in the exchange rate but they cannot completely stabilize the exchange rate because of external factors (namely, cross border capital flows). However flexible exchange rate system can be sustained indefinitely in the absence of a speculative attack which can succumb to adverse market sentiments. It has been observed that when the central bank intervenes in the foreign exchange market (after the fixed exchange rate system gave way to the flexible exchange rate system), the volatility of exchange rate reduced to an extent.

This study looks deeper into the coordination of game theory in the foreign exchange markets, which explains tradeoff between exchange rate and intervention. The country's central bank has been actively traded to reduce exchange rate volatility as well as to ensure orderly condition of exchange rate. In the context of coordination, intervention in the foreign exchange markets achieves payoff matrix. However, coordination of exchange rate intervention of the central bankers is actively engaged on selling and buying foreign currency assets. They met coordination pay off between traders since the domestic currency appreciation (depreciation). The global game theory was developed by Carlsson and van Damme.

NASH EQUILIBRIUM IN RBI'S INTERVENTION

The exchange rate fluctuations that occur on the coordination of central bank intervention and speculators' trades (in the foreign exchange market) are seen as a tradeoff between the central bank and traders. However, the exchange rate is highly volatile and hence, the monetary authority (RBI) intervenes to stabilize the foreign exchange rate. The major factor for this political instability affects the nominal exchange rate. We can address the central bank utility of intervention actual exchange rate and expected exchange rate as:

$$\lambda_t = \frac{1}{\nu + \chi^2}\left[\frac{\partial U_t}{\partial \lambda_t} - \tau(E_s - E_s^T)\frac{\partial E_t}{\partial \lambda_t} + \nu\lambda_t^e\right]$$

where $\tau \in (0,\infty)$ weighs the central bank aversion to deviation from its exchange rate target; $\lambda_s^e \in (-\infty,\infty)$ is the intervention expected by speculator at time *s;* $\nu \in (0,\infty)$ *is the* cost of expectation of intervention; E_t is the nominal exchange rate; E_s^T is the target exchange rate; U_t is a generic utility function at time t. The reaction function λ_t affects both, the utility and the nominal exchange rate, the left portion, $\partial U_t / \partial \lambda_t \quad and \quad \partial E_t / \partial \lambda_t$ are the marginal changes in the generic utility function and the nominal exchange rate with respect to time respectively.

Intervention is effective during the market based exchange rate regime.

The speculator's loss function defined below.

$$L_t^s = \sum_{s=t}^{\infty} \sigma^{s-t} \left[\frac{(\lambda_s^e - \lambda_s)^2}{2} \right] - U_t$$

Where $\sigma \in (0,1)$ is a discount factor relative to the loss of the speculator, the coordination of central bank intervention seeks to minimize his cost to reduce exchange rate volatility. Sometimes, the central banks announce to intervene in the foreign exchange market signaling the appreciation/depreciation of the exchange rate due to no buying/selling operation. This can be represented as follows, as then the speculators' intervention is equivalent to that of the central bank:

$$\lambda_t^e = \lambda_t \tag{3}$$

The Nash equilibrium is obtained by substituting (3) into (1) :

$$\lambda_t = \frac{1}{\chi^2} \left[\frac{\partial U_t}{\partial \lambda_t} - \tau(E_s - E_s^T) \frac{\partial E_t}{\partial \lambda_t} \right]$$

Equation (4) gives the cost of intervention with the central bank.

Further central bank intervenes to minimize exchange rate volatility without loss of foreign exchange reserves. Since foreign exchange reserves have volatile effect on current account deficit, as a result the central bank would not achieve his goal. It needs to repay more foreign exchange reserves to manage exchange rate stability $\left[\tau(E_t - E_t^T) \partial E_t / \partial \lambda_t \right]$, so it gives more importance to it than the utility function $(\partial U_t / \partial \lambda_t)$. The RBI's intervention policy is effective during market based exchange rate system. Though the intention of such interventions is to stabilize exchange rate, it yields a return to the intervening authority because of various difficulties encountered in measuring the profitability of intervention.

PURE STRATEGY EQUILIBRIUM

The exchange rate is destabilizing in nature, since it is dependent on various external factors such as international crude oil price, gold price, interest rate and instability cross border of capital flows; demand and supply of money and instability food and non-food product. Though, the central bankers must control the price of food and non-food articles but it should mainly focus on influencing the exchange rate fluctuation. It follows pure strategies and mixed strategies to intervene in the foreign exchange markets. We explain pure-strategy is an n-player game where y_i is the set of strategy player i, $y = y_1 * y_2 * y_3 *, \ldots, * y_n$ and $x = x_1 * x_2 * x_3 *, \ldots, * x_n$ are the payoff functions; such that $x \in y$.

According to Nash equilibrium strategy for each player, if one player sells his currency; then at the same time someone else holds his currency, as we can see in the given matrix.

Central Bank →
Speculators ↓

	Enter	Stay out
Enter	-2, 2	1, 0
Stay out	1, 0	0, 0

The matrix shows the rivalry between player one (central bank) and player two (speculator). When both the speculator and the RBI enter the market, then the speculator always faces a loss of -2 and RBI receives an incentive of +2. If either of them however chooses to stay out and only one enters the market then the entrant gets a positive payoff of 1 and the other gets nothing. But usually, when the central bank intervenes, the others stay out. During depreciation of the domestic currency the monetary authority intervenes in the foreign exchange markets to correct the exchange rate volatility for its own currency. They release the dollar instead of rupee to volatility Re/USD.

If both stay out, then none of them receive any payoffs.

MIXED STRATEGY EQUILIBRIUM

In the mixed strategy game; we can check for four possibilities to intervene in the foreign exchange market such as $(U,L),(D,L),(U,R), and\ (D,R)$ in Nash equilibrium. Each player is chooses a strategy out of four strategies which are called *pure strategies*. We'll have to study another matrix given below to understand mixed strategies.

If player 1 chooses the mixed strategy $\sigma^1 \in \Delta(S^1)$, player 2 chooses the mixed strategy $\sigma^2 \in \Delta(S^2)$ and n players choose $\sigma^n \in \Delta(S^n)$, expected pay off to i^{th} player $u^i(\sigma^1, \sigma^2, \sigma^3, \dots, \sigma^n)$

$$u^i(\sigma^1, \dots, \sigma^n) = \sum \sigma^1(s^1) * \dots * \sigma^n(s^n)\pi^i(s^1, \dots, s^n)$$

.

The mixed strategy among the central banks is that of intervention with mutual interests to reduce volatility. They minimize their own cost of intervention by coordinating. As Domniguez had revealed that the mixed strategy exchange rate volatility surrounds the exchange rate market.

Player 1 $\longrightarrow$

Player 2 $\downarrow$

	L	R
U	2, 0	0, 1
D	0, 2	1, 0

This matrix shows the payoffs in case, the two players do not coordinate in accordance with the game theory. If player 1 moves only left/right and player 2 only up/down, without coordinating beforehand then one's gain will always be the other's loss and there won't be any stability. As we can see from the matrix, if player 1 chooses to move left (buys foreign assets) and player 2 chooses to move downwards (sells foreign assets), then the payoff would be 0 and 2 respectively, and so on.

Conclusively, this kind of distribution of payoffs proves the instability of the exchange rate at very high levels in the absence of intervention by coordination.

CENTRAL BANK'S COORDINATION INTERVENTION

The central bank coordination intervention agrees with other central banks to intervene in the foreign exchange market because without multiple coordination of exchange rate intervention, the cost of intervention would increase. They are holding foreign currency assets which are spent without profit while losing of exchange rate appreciation (depreciation). The central bankers have to choose tradeoff (loss/profit) therefore, carefully. The mixed strategy plays a prime role in reducing exchange rate volatility. It must give proper signals to the market participants otherwise speculator would maximize his own profit. If the authority agrees with coordination of intervention or on the other hand if the signal is true; expectation is that of inflation to fall. The future's monetary policy is implied coordination of intervention operations have several central banks who do not behave homogeneously. For example, India is involved in purchasing and selling of foreign currency assets with negligible respect for exchange rate volatility. In practice, over the three year period the G-3 central banks agree to coordination intervention operations on 81 out of 760 trading days.

CONCLUSION

Basically this study addressed application of game theory to imply that the RBI intervenes to reduce volatility with other central banks. If it coordinates, then it reduces exchange rate volatility when other central banks remain constant. Further the central bankers have been regularly intervening to reduce the exchange rate fluctuations but they are not; basically the developed countries have coordinated intervention whereas the developing countries don't have coordination operation facility. This study suggests to the Reserve Bank of India to go for the coordination intervention with developing countries in order to reduce the exchange rate volatility without reducing the foreign exchange reserves. Moreover, the RBI reduces the cost of holding the foreign exchange reserves. So that it will enhance the exchange rate stability and enhance the export competitiveness.