Differences in monetary policies between two hypothetical closed economies:one which is concerned with avoiding a large negative output gap and the other which is not

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Differences in monetary policies between two hypothetical closed economies: one which is concerned with avoiding a large negative output gap and the other which is not. *

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Abstract

This research is focused on the effect of varying output gap target bounds on monetary policy. Here, a mathematical theory known as the 'Viability Theory' is employed to approach this problem in the context of satisficing policies, as discussed by Nobel Prize winning Herbert Simon, [see Simon (1955)]. A closed economy's monetary policy problem of controlling inflation is considered to be this sort of satisficing policy problem. The viability theory is used to form viability kernels (using VIKAASA), which are a collection of points from which evolutions can start and remain within a certain constraint set K given some restricted controls, [see Krawczyk and Kim

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(2009)]. Using VIKAASA one can build such kernels for various exogenously defined constraint sets K and policy instruments. This study contributes in filling a gap of knowledge about what the viable economic states are if the output gap is targeted. The main results of this research show that, when smaller than historically acceptable output gaps are targeted, the central banks should avoid high level inflation at extreme positive output gaps, while at lower output gap limits very small inflation should also be avoided. The former prescription should be realised by having higher level interest rates and the latter by having lower level interest rates. Early interest rates adjustments are always recommended for central banks to avoid extreme situations.

Keywords: Viability theory, Viability kernels, Monetary policy problem

Contents

1 Introduction

This research will help macro-economists and central bankers to look at the monetary policy problem in a new context. Most of the time, macro-economists have optimised the monetary policy problem using central bankers' loss functions, or Taylor's rule by observing the inflation and output gap. Here, however, a different method known as "Viability Theory" is employed. This method is used to look at the output gap target's variation. So, as opposed to a less liberal banker who considers output gap as less important (than inflation), I will investigate how targeting different output gaps affects monetary policy.

In the past, research has been conducted to investigate the effect of output gap on monetary policy problem. Rudebusch and Svensson (1999), Galí (2009) and Taylor and Williams (2010) are examples of this. They have approached the problem using central bankers' loss function and Taylor's rule. Krawczyk and Kim (2009) and Krawczyk and Kim (2014) have used viability theory in order to look at the impact of interest rates on kernel size and the resulting monetary policy. Among other conclusions, they mention that high level interest rates should not be used at very low output gaps and level inflations and if used, evolutions would violate K in both closed and open economy context, [see Krawczyk and Kim (2009) and Krawczyk and Kim (2014)]. This research will be developed upon the knowledge gathered from these papers using viability theory and viability kernels to look at the central banker's problem.

The main outcomes of this research show that central banks should avoid high level inflation at positive output gaps by having higher level interest rates, while at very negative output gaps low level inflations should be avoided by lowering interest rates. The further the economy is away from the steady state the faster the system dynamics move. Therefore early adjustment of interest rates are recommended to avoid the economic states from leaving the boundaries of constraint set K.

A less liberal central banker, that is, someone who worries about both negative output gaps and high level inflation levels, will try to keep the economy in tighter boundaries. That is, this central banker will try to avoid negative output gap as much as they try to avoid high level inflation.

This paper will be structured as follows, in section 2, the monetary policy problem is explained, in section 3 viability theory has been considered, and in section 4 and 5 modeling results will be discussed. In section 4, a base case scenario of fixed output gap between -0.04 to 0.04 is considered and section 5 will concentrate on reducing the output gap target range to -0.02 to 0.04. Section 6 presents the concluding remarks of this research. Section 7, some future research directions are provided whereas with section 8, this research will be concluded while providing a useful appendix on how to use VIKAASA.

2 The Monetary Policy Problem

2.1 Economic Dynamics and Targets

The central concern of this research is the central banker's problem. The central banker uses interest rate (i) increments u as the instrument to control inflation rate (π) and output gap (y). The following (1)-(2) dynamic equations and (3) differential inclusion will be used to describe the economy's reaction to adjustments $u \in U$ of interest rate i (U-set of constraints that the control needs to be satisfied). They will then be used to develop the viability kernel by adjusting the output gap target bounds, [see Aubin (1997) and Krawczyk and Kim (2009)]. This model is a simplification of the models used in Batini and Haldane (1999), Rudebusch and Svensson (1999) and Svensson (2000).

$$\frac{dy}{dt} = -\alpha y - \xi(i - \pi) \quad (1)$$

$$\frac{d\pi}{dt} = \zeta y \quad (2)$$

$$\frac{di}{dt} \in [-\underline{u}, \overline{u}] \quad (3)$$

All variables here y, π, i are the expected deviation values. The precise meaning of y is the output gap, i.e., deviation of output from the equilibrium level of output (obtained from aggregate demand equation). π is the deviation from some equilibrium inflation rate which is obtained from Phillip's equation. i is the deviation of nominal interest rate from a historical mean interest rate while u is the speed of interest rate adjustment which is the "control" variable.

Therefore, the constraint set is set as below:

$$R^3 \supset K \equiv \{(y(t), \pi(t), i(t)) : -\underline{y} \leq y(t) \leq \overline{y},\ \underline{\pi} \leq \pi(t) + \Pi(t) \leq \overline{\pi},\ 0 \leq i(t) + I(t) \leq \bar{i}\} \quad (4)$$

2.2 Model Calibrations

The unknown parameter values in-front of each variable in dynamic equations (1)-(2) are then replaced with some calibrated parameters to get equations (5)-(6) which were obtained from previous literature particularly, [see Walsh (2003) and Fuhrer (1994)]. The usual changes to i are normally made in every quarter and the changes are typically about 0.25 percent. Observing this differential inclusion range in (3) is set equal to the values defined in equation (7) below. This means that the i can range between 0 to 1/2 percent per quarter, [see Krawczyk and Kim (2009)].

$$\frac{dy}{dt} = -0.02y - 0.35(i - \pi) \quad (5)$$

$$\frac{d\pi}{dt} = 0.002y \quad (6)$$

$$\frac{di}{dt} \in [-0.005, 0.005] \quad (7)$$

Then the level values would need to be constrained to particular values. Level inflation according to inflation band in NZ should be set between 0.01 to 0.03. In previous studies output gap target zone has been fixed between -0.04 and 0.04. Here, this output gap target zone is what needs to be varied to obtain the final result of the overall research question discussed in this paper. First, just like in previous studies, a base case is considered here with a fixed output gap target bounds of -0.04 to 0.04. Then, this target would be adjusted to -0.02 and 0.04 to observe what would happen under such a central banker, who considers that a very negative output gap and a very high level inflation should be avoided at all times. Level interest rate is assumed to lie between 0 to 0.07 and normally depends on the economic and political state of the country, [see Krawczyk and Kim (2009)]. It is important to note that when we communicate π and i terms, they are considered as deviations from an average equilibrium level of 0.02 and 0.04 respectively. Hence this is the reason to have plus $\Pi = 0.02$ and $I = 0.04$ added to π and i respectively in (8) below. Therefore, the calibrated constraint set is set as in (8) below:

$$R^3 \supset K \equiv \{(y(t), \pi(t), i(t)) \,:\, -0.04 \leq y(t) \leq 0.04,\ 0.01 \leq \pi(t) + 0.02 \leq 0.03,$$
$$0.0 \leq i(t) + 0.04 \leq 0.07\} \quad (8)$$

Steady state solution for the above two dynamic equations can be seen in the diagram below where $y = 0$ crosses $i = \pi$ (Figure 1). Figure 2 further explains what can happen on the plane of y and π. That is, if $y < 0$ then π decreases (arrow pointing toward A) and if $y > 0$ then π increases (arrow pointing toward C). When looking at figure 1 again, above $i = \pi$, $i > \pi$ which decreases y (arrows pointing left) and below $i = \pi$, $i < \pi$ which increases y (arrows pointing to right), [see Krawczyk and Kim (2009) and Krawczyk and Sethi (2007)].

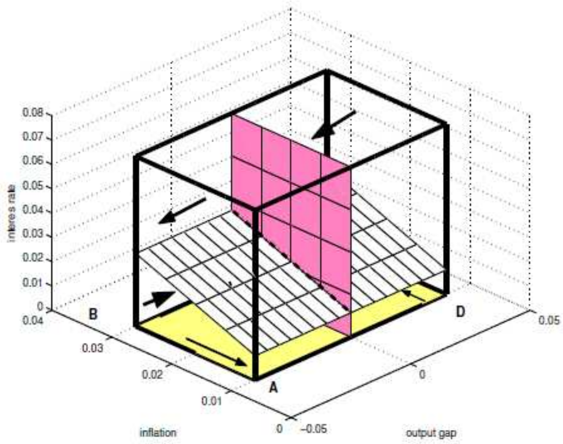

Figure 1: Shows the steady state solution for above (1)-(3) dynamic equations, [see Krawczyk and Kim (2009), p.63 and Krawczyk and Sethi (2007), p.22].

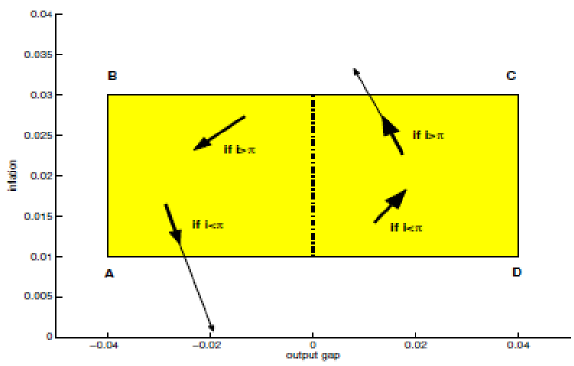

Figure 2: Shows system's dynamics on y and π plane, [see Krawczyk and Kim (2009), p.63 and Krawczyk and Sethi (2007), p.22].

3 Viability Theory for Monetary Policy

The method for determining the viability kernel is through viability theory. There are detailed introductions to viability theory available in Aubin (1991), Aubin et al. (2011), Veliov (1993) and Quincampoix and Veliov (1998).

Basically, there is a closed set K which represents some normative constraints. What viability theory tries to solve here is whether, for a given initial condition, a control strategy exists that keeps the system within the normative bounds of the constraint set. Viability kernel for close set K is the largest subset of K which contains all initial conditions for such a strategy, [see Krawczyk and Kim (2009) and Krawczyk and Sethi (2007)].

Viability kernels are more formally defined as follows:

Definition 3.1 *The viability kernel of the constraint in K for dynamic $f(.)$ and the control set U is the set of initial conditions $x_0 \in K$ denoted as V,*

$$V \equiv \{\, x_0 \in\ K : \exists\, x(t)\, solution\, to \quad \dot{x}(t) \ =\ f\,(x(t)\,, u(t)),$$
$$with\, x(0) \ =\ x_0\, s.t.\, x(t) \in\ K, \forall_{\,t} \in \Theta\} \tag{9}$$

When $V \neq \emptyset$ viability problem has a solution.

Viability Kernel can be explained using the diagram below (figure 3). The yellow circle K is the constraint set while pink area V is the viable region. Trajectories starting inside this viability kernel (pink section) can be kept inside the viable region under current control. But any trajectories starting outside (yellow area) this area cannot be controlled to remain inside the viable area under any assumed available control and will escape the constraint set. Anything outside the constraint set is obviously considered to be non-viable region anyway (white area, X), [see Krawczyk and Kim (2009) and Krawczyk and Sethi (2007)].

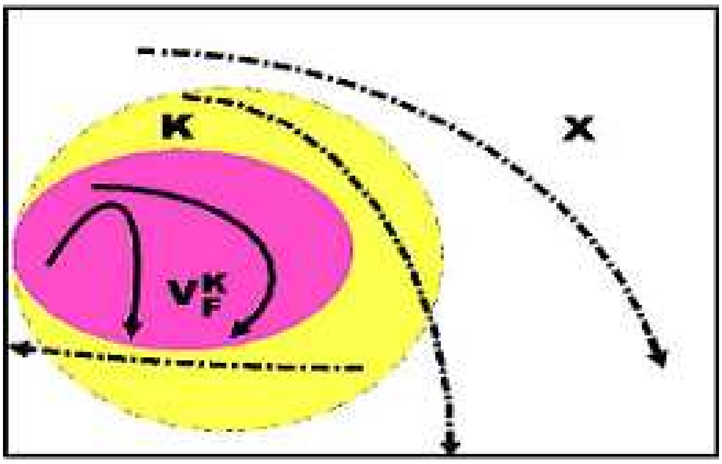

Figure 3: Viable (solid lines) and non-viable trajectories (dash lines), [see Krawczyk and Kim (2009), p.50 and Krawczyk and Sethi (2007), p.4].

To understand viability theory[1] better, assume that you are rowing through a river. The boundaries of the river include a waterfall which you must avoid. These constitute the normative bounds of the constraint set of the monetary policy problem. To make sure that you are not going to fall down the waterfall you would have to paddle with some strength, where the paddling strength is like the current control used by the central banker to keep the system within its bounds. If you manage to escape the waterfall this is like a viable trajectory in figure 3, on the pink area, where the economy moves smoothly under current controls used by the central banker. But, if you do fall down the waterfall, this is like a non-viable trajectory on figure 3 shown in the yellow area, thus meaning that the current controls used by the central banker are not tight enough to keep the economy under its normative bounds.

The size of the pink area, or how closely you can approach the waterfall, is

[1]As I understood at the first symposium when I presented in May 2014 at VUW, viability theory is a new field and currently it's not been exposed to many. Therefore, a real life paradigm which my supervisor is fond of will surely help new readers to understand viability theory better.

proportional to your strength. Or, in the banker's case, how large an interest rate adjustment can be done. The further you are away from the steady state the faster the system dynamics (5)-(6) move. This is illustrated by the small red arrows on the figure 4. Therefore, the further we move from the steady state, the tighter the controls will be needed to keep the system viable. If we have reached the boundaries of the constraint set, it is already too late to turn the economy back into the constraint set (shown in point B on the left panel of figure 4). But for the same system dynamics, if the banker reacts early enough, they can stop the economy from leaving the boundaries of the constraint set (shown by point A on right panel of figure 4), [see Krawczyk and Kim (2009)].

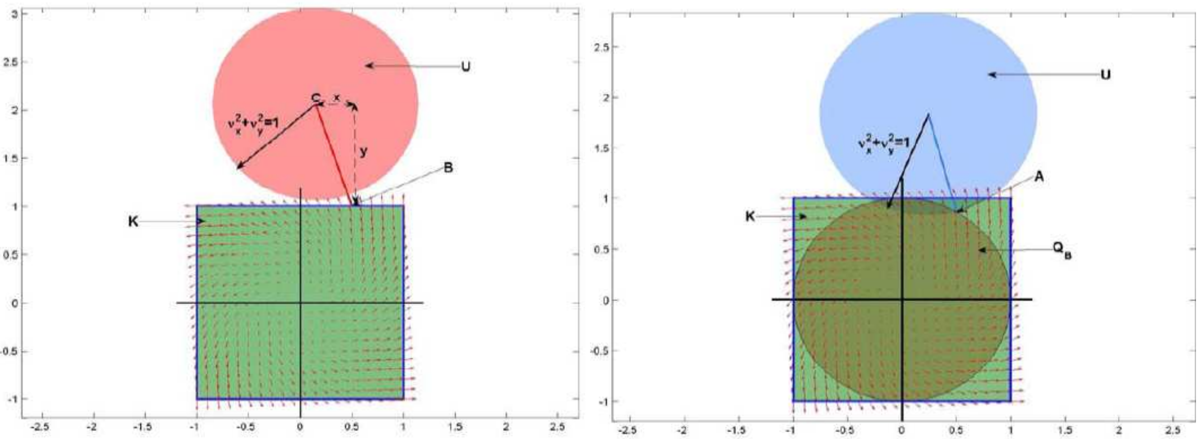

Figure 4: Geometric characterisations of viable and non-viable points, [see Krawczyk and Sethi (2007), p.8-9].

4 Results for the Base Case

4.1 Viability Analysis

When a 3D-closed economy described by dynamics (5)-(7) and constraints (8) is modeled using VIKAASA [2] with a fixed output gap target between -0.04 and 0.04, the resulting kernel looks like figure 5 (in red). Where, the box represents the constraint set K.

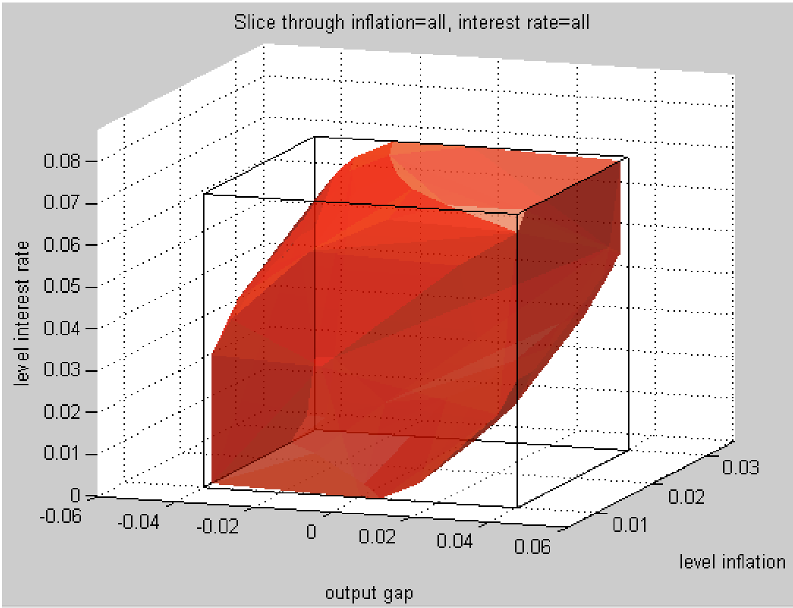

Figure 5: The 3D kernel for fixed output gap target zone of -0.04 to 0.04.

The obvious conclusion that can be made is that the kernel touches upon the boundaries of output gap target variations. Furthermore, at positive output gap, central banks should avoid low interest rates. Similarly, at negative output gap, central banks should avoid very high interest rates.

[2]VIKAASA, Krawczyk and Pharo (2011) is a piece of special software dedicated specially for viability analysis which has been briefly described in Appendix A.

This is in agreement with the common knowledge about monetary policy. That is, when the economy is booming above its maximum output target, central banks should increase (level) interest rates to make sure the economy slows down. Otherwise inflation will start increasing above the level which is desirable. Alternatively, when the economy is in a recession, (when $y < 0$) the central bank should make sure that interest rates are low enough so that people will start consuming more instead of saving in order to speed up the economy.

To have a better understanding of the kernel above, it is necessary to take a closer look at several cuts of the kernel.

Firstly, the kernel is sliced through the lower level inflation, $\pi = 0.0157$ on the left and higher level inflation, $\pi = 0.0243$ on the right, see figure 6. Again, the 'box' corresponds to the constraint set K.

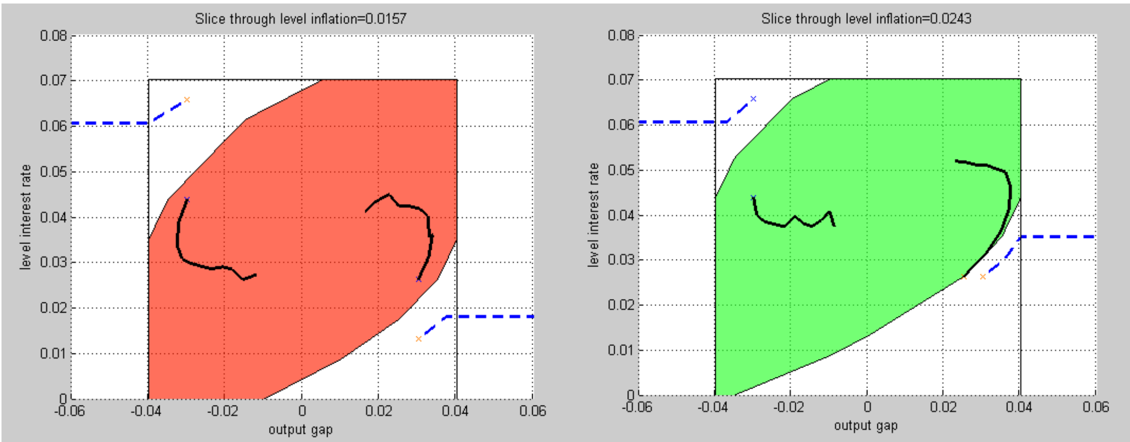

Figure 6: The plane of interest rate and output gap for fixed output range -0.04 to 0.04 sliced at $\pi = 0.0157$ (left in red) and $\pi = 0.0243$ (right in green).

What can be deduced from such slices are that, with high positive output gaps, low interest rates are not possible for both low and high level inflations. Also, with negative output gap, very high interest rates should be avoided for both high and low level inflations. This makes sense as if there is very negative output gap, we would want to increase our output somewhat closer to what we call the optimal level of output, or natural level of output level. In such a case, if we have very high interest rates it would be very hard or impossible to stimulate growth. It is also true at high positive output gaps where we would want to make sure that output will decrease eventually to the natural level. And in order to do this interest rates need to rise. If we do not eventually do this, we will have high inflation running in the future, and it would be very costly for the economy. Avoiding high inflation is much easier than trying to reduce inflation when it starts spiraling out of control. This is exactly what we observed when looking at the overall kernel in figure 5 above.

Also, the slices in figure 6 display viable trajectories (solid black lines) starting within the kernels. Obviously (i.e. by the kernel definition) they stay within the kernels. For both low and high level inflation and for positive output gaps, when interest rates are low, output grows. Then as the boundary is almost reached, the evolutions bounce back with high interest rates. The opposite is also true for low output gaps and for both high and low level inflation. That is, under such a situation where interest rates are high, output will start decreasing and once the boundary is almost reached as interest rates become low, output start rising again. So at the end, these trajectories make sure to stay within the kernel by adjusting the interest rate and hence the output at right speed. Whereas, the non-viable trajectories (dashed blue lines) starting outside the kernels violates the constraint set at both the negative and positive boundaries of output gap. This makes sense because if the economy is having a negative output gap, and if high interest rates are applied, then output will decrease further and make sure to leave the constraint set (shown by dashed blue line in the top left corner of both slices). The opposite is also true for very positive output gap when low interest rates are applied; output will increase further and further, which will also make the trajectory violate the constraint set. In each case, the economy is too close to the boundary for the limited interest rate adjustments.

Secondly, the interest rates and level inflation slices are shown in figure 7 below, sliced at both negative output gap, $y = -0.03$ (on the left in red) and positive output, $y = 0.03$ (on the right in green).

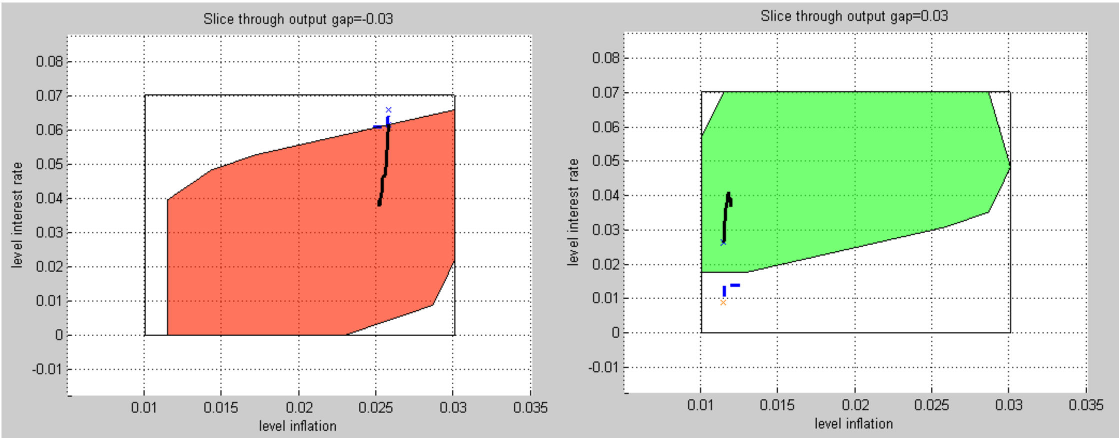

Figure 7: The plane of interest rate and level inflation for fixed output of -0.04 to 0.04, sliced at $y = -0.03$ (left in red) and $y = 0.03$ (right in green).

It can be seen that at very negative output gap, central banks should avoid high interest rates at all levels of inflation. This also makes sense theoretically. If we have very low level inflation and a negative output gap, it is unnecessary and unwise for central banks to set high interest rates. If high interest rates are used in such a situation, only thing it will do is to slow down economy further in an unnecessary way. This is because high interest rates will make people save more to gain advantage of greater returns which will further dampen investment and private consumption levels. Therefore, output will also be reduced as both consumption and investment are a large part of y. At the same time when level inflation is high, very low interest rates should be avoided. This makes sense because otherwise inflation will increase further. This is shown in left slice in figure 7. Whereas, at a very positive output gap central banks should avoid very low interest rates at all level of inflation. This is because it would create more inflation if central banks do not have a contractionary monetary policy when high level inflation and positive output gap are possible. Therefore, when level inflation is high, lower interest rates should be avoided. And, at the same time when level inflation is both very low and very high, extreme positive interest rates should be avoided. This makes sense because at very high interest rates, it will either reduce level inflation or positive output gap in unnecessary ways when they are not needed to be adjusted.

Two viable trajectories are shown in black solid lines and non-viable trajectories are shown in blue dash lines in figure 7. When $y = -0.03$ and when level inflation is high, viable trajectory moves towards lower level inflation as high interest rates are used. When $y = 0.03$ and at low level inflation, as lower interest rates are used viable trajectory moves toward slightly higher level inflation as the interest rates start rising.

Finally, a 2D cut of output gap and level inflation is shown in figure 8 below, sliced at both low interest rate, $i = 0.0087$ (on the left) and high level interest rate, $i = 0.06$ (on the right).

This shows that for very low interest rates at very positive output gaps, central bank should avoid any level inflation. At the same time, at very low output gaps, very low level inflation should be avoided. It makes sense instantly to see why central banks should avoid high level inflation when there is a positive output gap in such a situation. This is because positive output gap causes high level inflation, so if interest rates are low, very high level inflation cannot be reduced therefore these should be avoided. It is not immediately visible why central bank should avoid very low level inflation rates at very negative output gaps. But, the reason for it can be argued as follows. That is, at very low output gaps central banks

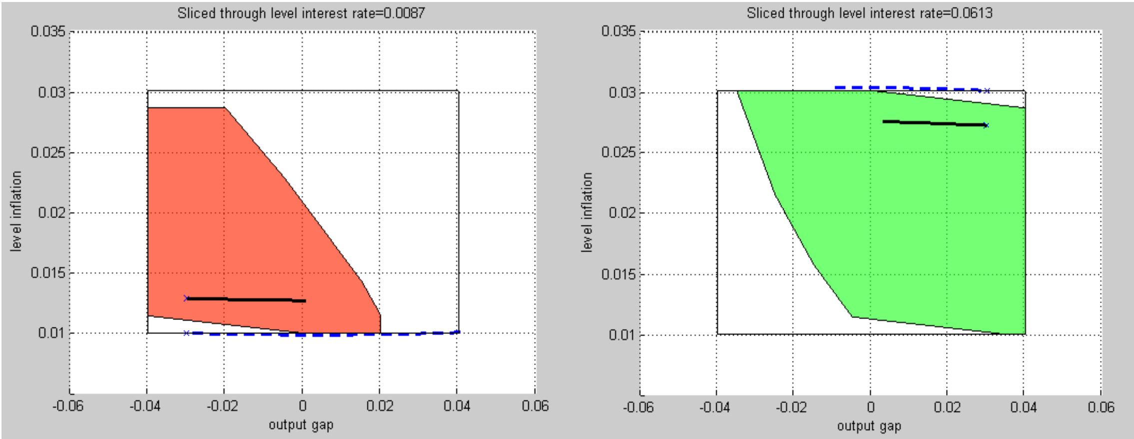

Figure 8: Cut of level inflation and output gap for fixed output range -0.04 to 0.04 sliced at $i = 0.0087$ (on the left) and $i = 0.0613$ (on the right) respectively.

would want to reduce interest rate by a considerable amount to make sure that the economy recovers from the current recession. To do this, it is easier when inflation is somewhat positive since interest rates are already quite small in a recession. Therefore, by having somewhat positive inflation according to the real interest rate fisher equation $r = i - \pi$, a negative real interest rate can be achieved in order to get the economy to grow eventually. Such a situation which is avoided is known as a liquidity trap. This situation is shown on the bottom left corner in the left slice in figure 8, which the kernel tends to avoid. For a detailed analysis on how to avoid a liquidity trap, follow McCallum (2006). In the right slice it shows that, when interest rates are high at negative output gaps, any inflation should be avoided. This makes sense since if interest rates are increased, it will dampen y further. Also, at very high output gaps very high inflation should be avoided, otherwise one would have to increase interest rates further to reduce higher inflation that will dampen positive output growth, which is not desirable. Such a situation is often referred to as a hot economy in some economic literature (which our model tends to avoid).

The viable trajectory (solid black line) in left slice, both level inflation and interest rates are low and output is therefore increasing. The viable trajectory (solid black line) on right slice at high level inflation and high interest rates therefore output is decreasing. The non-viable trajectories (blue dashed lines) violate the boundaries of the constraint set. For these slices, non-viable trajectories are leaving the constraint set because the speed of adjustment of interest rates change is not fast enough for making the initial y viable.

4.2 Why Some States are Non-Viable

In figure 9 below, the black solid line shows a viable trajectory as it stays within the kernel. The dashed blue line, shows a non-viable trajectory since it leaves the constraint set K through boundary of positive output gap.

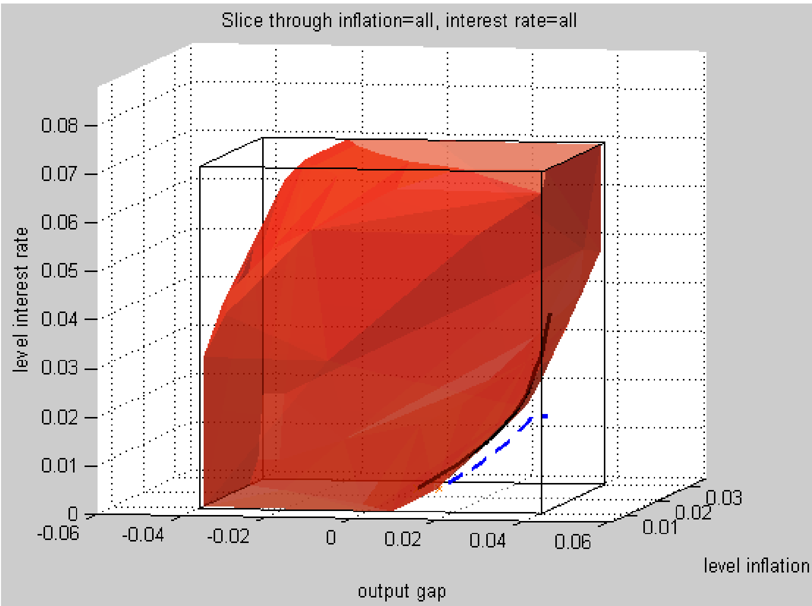

Figure 9: Showing the trajectories starting from a viable (solid black line) and non-viable (dashed blue line) points in a kernel which has fixed output gap zone of -0.04 to 0.04.

The viable trajectory starting at such a position where $y = 0.01$, $\pi = -0.0086$ and $i = -0.0356$ i.e., $\pi = 0.0114$ and i=0.0043 in level terms. The non-viable trajectory starts at the same initial values of level inflation and level interest rates but at a positive output gap of 0.015. According to the output dynamic equation, speed of adjustment of initial y is greater for such viable point than compared to such a non-viable point. Then once the trajectory almost touched the boundary of the kernel, adjustment of speed of interest rates change is sufficient to keep the initial y viable for such a viable state. So, the speed of adjustment of interest rates change is not fast enough for making the initial output gap viable for such a non-viable state.

4.3 Time Profiles

One advantage of VIKAASA is that it can be used to create time profiles for both viable and non-viable trajectories in order to understand the economic situation to a better extend. Figure 9 and figure 10 below show time profiles for trajectories for each particular point respectively. What time profiles basically show is how each variable y, π and i reacts to u over the time horizon.

In figure 10 below, the black solid line for the state where $y = 0.01$, level $\pi = 0.0114$ and level $i = 0.0044$ speed of adjustments of interest rates changes are sufficient for keeping output gap within the kernel as it reaches closer to the boundary. With an initial increasing output gap, level inflation also starts increasing. As level inflation increases, we should increase interest rates as that is how a central banker would react to such a situation. The velocity of interest rate adjustment is high at the beginning and eventually becomes very small since the economy is heading to a steady state. This means that there is at least one strategy that can be employed to keep the economy under control in such a situation described as starting from such a viable state. Wiith reference to the dashed blue lines, for non-viable state, where $y = 0.015$, level $\pi = 0.0114$ and level $i = 0.0044$, the same incrementation of speed of interest rate adjustment is insufficient to keep the economy within the constraint set. We can see that the blue dashed line is leaving the constraint set in the upper boundary of output gap, velocity has not reached the zero and economy is unstable.

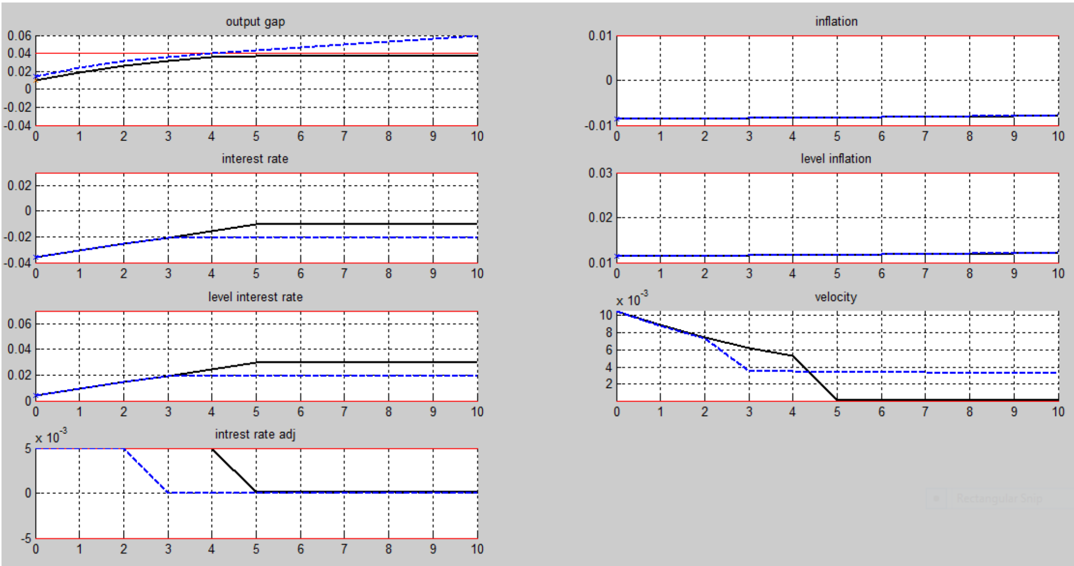

Figure 10: Time profiles of trajectories starting from a viable and a non-viable point for fixed output gap target zone of -0.04 to 0.04.

In figure 11, for the state where $y = 0.015$, level $\pi = 0.0114$ and level $i = 0.0044$, the black solid line shows up to four periods of output gap that keeps increasing above the upper boundary of 0.04. The inflation is constant or somewhat increasing toward the end. Although with the effort to reduce inflation by increasing interest rate by adjusting it by half percent for two consecutive quarters, inflation does not seem to be decreasing. The velocity of interest rate adjustment is very high at the beginning and then decreasing, but after a while, it stays at a certain level and not decreasing any further to take the economy to a steady state (i.e. at such a situation, central banks would find it hard to find a strategy which helps to keep the economy within its normative bounds under given controls). The blue dashed lines, show what happens if one tries to keep adjust interest rates at half percent for another two consecutive quarters. In fact, the economy will reach a steady state in 5 quarters, but the upper boundary of y will be temporarily breached. That is, if the same controls are used for a longer time horizon or if more strength controls are applied earlier, the economy will return to the constraint set.

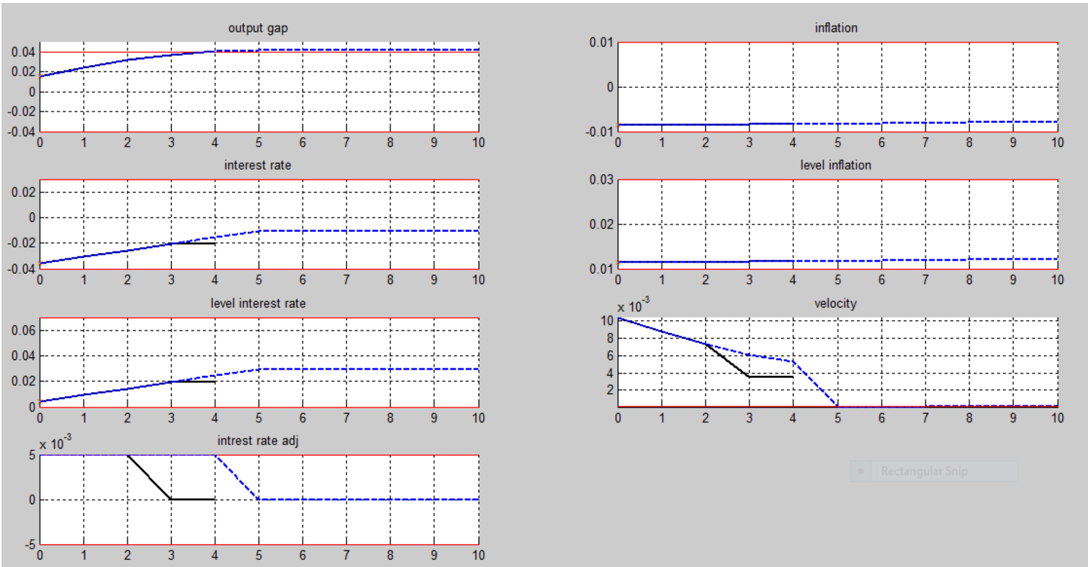

Figure 11: Time profiles of trajectories starting from a non-viable point for fixed output gap target zone of -0.04 to 0.04.

5 Reduced Output Gap Target Range

5.1 Viability Analysis

To answer the main research question that we are concentrating on, that is how reducing the output gap target variations influence monetary policy using viability approach, we will have to variate the output gap target from the fixed situation of -0.04 to 0.04 above to something such as -0.02 to 0.04. This will help us to see how this would affect the kernels, and therefore monetary policy.

We have created a kernel with output gap target between -0.02 to 0.04 and have plotted this kernel with the previous kernel of fixed output gap target between -0.04 to 0.04 to make things look clearer (shown in figure 12 below). The orange kernel is the original kernel with fixed output gap of -0.04 to 0.04 and in pink is the new kernel with the reduced output gap target of -0.02 to 0.04. The main impression we can obtain from these two kernels is that by changing the output gap target zone,we find high interest levels must be avoided for much larger (less negative) output values than before. Everything else seems to have stayed constant just as with the fixed output gap target variation case of -0.04 to 0.04 which was shown above.

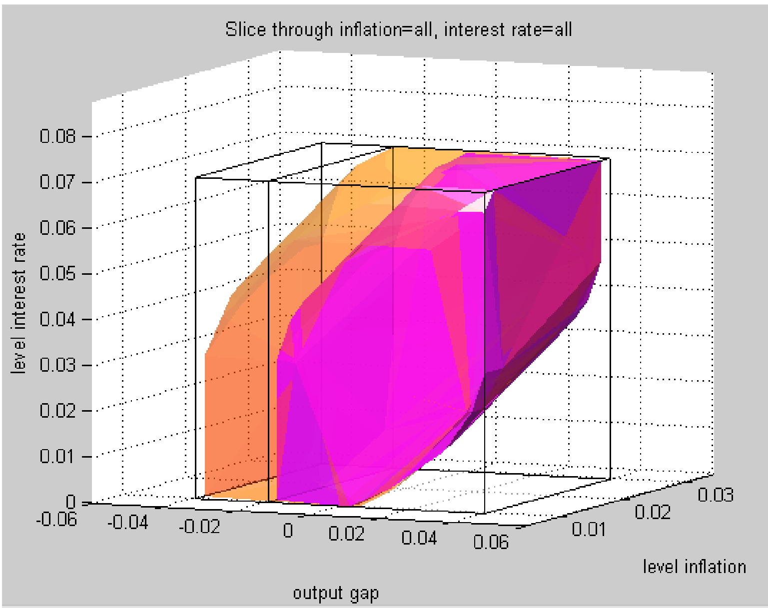

Figure 12: The kernels with fixed output gap target variation of -0.04 to 0.04 (in orange) and deviated output gap target variation of -0.02 to 0.02 (in pink).

To take a closer look at what is happening, it is again necessary to look at several slices of the new kernel, as with the case of fixed output gap target between -0.04 to 0.04.

Firstly, in figure 13, the output gap and interest rate sliced through low level inflation of $\pi = 0.0157$ (on the left) and high level inflation of $\pi = 0.0243$ (on the right). It can be seen again as the same case which was seen above with fixed output gap of -0.04 to 0.04.

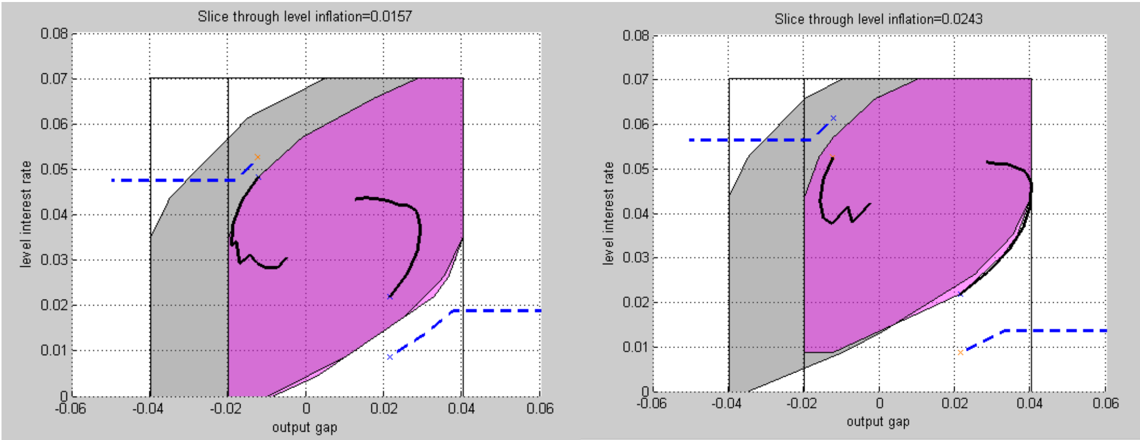

Figure 13: The interest rate and output gap slice for both kernels with output gap of -0.04 to 0.04 (in grey) and -0.02 to 0.04 (in pink).

That is, at very positive output gap, it is important to avoid low interest rates. At the same time, at even marginally negative output gap levels, very high interest rates should be avoided. This seems to be true for both low level inflation (left panel) as well as for high level inflation (right panel). If this is not followed, the economy will not run smoothly as central bank would expect. That is, when the economy is in a boom, and if low interest rates are used, it would be very hard to reduce inflationary pressure on the economy. In a recession, where unemployment is already high, if high interest rates are used, it would be hard to increase output as well as decrease unemployment to its natural rate.

One obvious difference that can be observed is that at negative output gap values, the kernel tends to avoid high level interest rates by more than under the base case scenario. This is represented by the grey area on the top left corner which is not overlapping with the pink kernel within new boundary of output gap. This makes sense because under the reduced output gap target range, such a central banker will be more averse to having a negative output gap. Therefore, when there is a recession, they will try harder to get out of it by starting to lower interest rates earlier than compared to a more liberal central banker.

The trajectories are also similar to what we saw with the base case. Again, solid black lines are trajectories starting at viable points while in dashed blue lines are trajectories starting at non-viable points. The difference is, the dashed blue line trajectories are now starting from the grey area, which under the original base case would have been claimed as a viable point and a viable trajectory. But, under the new kernel it will be considered as a non-viable trajectory since it violates the new negative output gap boundary. That is, according to the dynamic output equation, at such a non-viable state speed of adjustment of interest rate is not sufficient to decrease initial output at a faster rate than compared to similar viable state which is within the kernel. We can see from the black solid lines within the kernels the further it is closer to the boundary of the kernels the faster they move. Therefore, we need more strength on controls to stay viable. This is why it is important for central bankers to adjust interest rates early enough. This will help to keep the economy under control given limited strengths of interest rates adjustments they allow.

Now looking at the second slice of interest rates and level inflation cut through negative output gap target of $y = -0.02$ (on the left) and positive output gap target of $y = 0.0212$ (on the right), generally again we can see the same story that we saw with the original output gap of -0.04 to 0.04, which is shown on figure 14.

That is, for negative output gaps, very high interest rates should be avoided for all level inflation (top of left slice), and at very positive output gaps very low interest rates should be avoided for all level inflation (bottom of right slice). This is sensible since the first case that was explained is a recession and at such a point we should avoid further recessions by avoiding high interest rates. Also when having a boom, that is when the output gap is positive, low interest rates should be avoided since this will increase output further and therefore increase level inflation too. Just as before, the kernels tend to avoid extreme high interest rates when output gap is positive and level inflation is low (as shown in the top left corner of the right slice). Because under such a situation level inflation is already low, there is no need to use extreme high interest rates to reduce level inflation any further. The opposite is also true in a case where level inflation is high but output gaps are extremely low, as shown in the bottom right corner of the left slice. Now, the level inflation is already high although output gaps are negative, so it would be unwise to have extreme low interest rates since that will increase level inflation to a greater level.

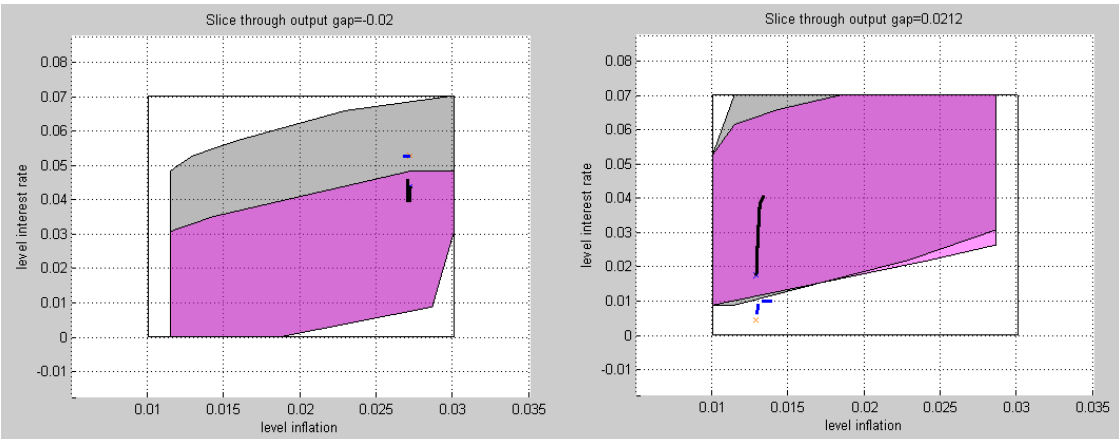

Figure 14: The level interest rate and level inflation slice for both kernels with output gap of -0.04 to 0.04 (in grey) and -0.02 to 0.02 (in pink).

The difference observed here with the base case is that now the new kernel tends to avoid even medium range level interest rates for all level inflation when output gap is negative. Also, when output gap is positive and level inflation is low, kernel tends to avoid higher interest rates more than in the base case. These are shown with the grey area on both planes which does not overlap with the pink area on the top of both slices. The original point of avoiding negative output gap than the base case would mean that for all level inflation, lower interest rates should be used to help the economy recover in a recession (shown in the left slice avoiding grey area). Also, in a boom it is unnecessary to have extreme high interest rates if the economy is having lower level inflation (shown on the right slice avoiding grey area).

Lastly, looking at the cuts in figure 15 of output gap and level inflation; sliced through low interest rate of $i = 0.0087$ (on the left) and high interest rate of $i = 0.0613$ (on right), same scenario is observable as with the base case above.

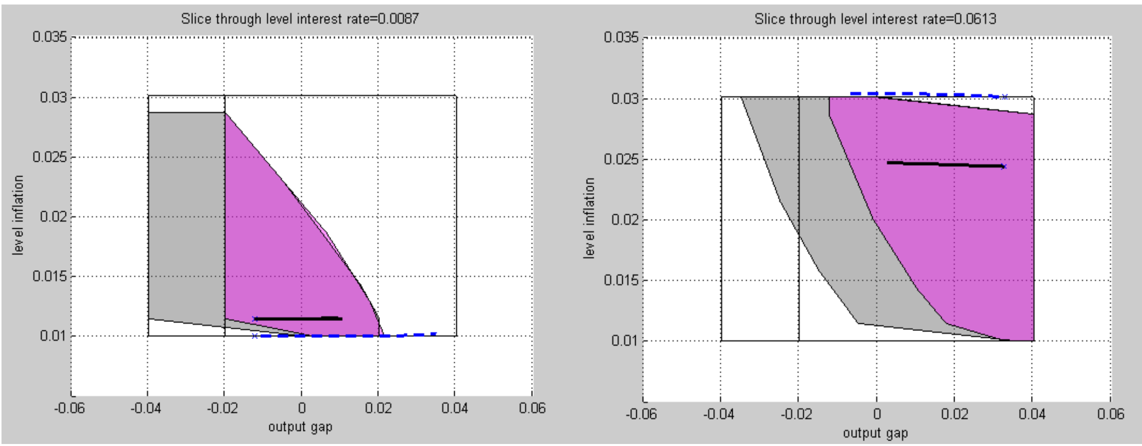

Figure 15: The level interest rate and level inflation slice for both kernels with output gap of -0.04 to 0.04 (in grey) and -0.02 to 0.02 (in pink).

At low interest rates, when output gaps are positive, all level inflations should be avoided by the central bank as shown in the right hand side of left slice. Because at very positive output gaps when interest rates are low, any level inflation would be bad since low interest rates cannot be used to reduce level inflation. The opposite is also true; for very high interest rates when output gaps are negative as shown by the left hand side of right slice. At such a situation, all level inflation should be avoided by the central banks. Because when the interest rates are already high, any inflation would be bad when trying to get out of a recession. This is since any inflation would mean having to increase interest rates by further amounts.

Again, same as with the base case, bottom left corner of left slice represents a liquidity trap which both kernels tend to avoid. It seems that the new kernel tends to avoid such a situation more than the base case although it does not seem to vary by much. The liquidity trap happens when output gap is negative and at the same time level inflation is very low. In such a situation, the central bank will face trouble reducing the interest rates by enough to increase output gap in order to get out of the recession. In such a situation, second best option that the central bank could follow is to achieve a real interest rate of zero percent. With such an interest rate, when level inflation is small positive, negative real interest rate can be achieved. Therefore, with such an interest rate, the central bank would be able to get rid of recessions eventually and head toward positive output growth, [see McCallum (2006)]. Also, similar to the base case, top right corner of the right slice represents a hot economy which both new and old kernels tend to avoid by the same amount i.e., both kernels have overlapped with each other perfectly at such a position. The overheating economy happens when output gaps are extremely positive; at the same time level inflation is very high. At such a situation central bank would need to have a more contractionary monetary policy. That is, to increase interest rates to reduce the extra inflationary pressure in the economy. But, when deciding on by how much to increase the interest rates, the central bank has to make sure to change the interest rate by the perfect amount without pushing the economy into a greater recession. The best way to do this is to target an interest rate and output gap of zero percent. This will make sure to avoid such situation such as hot economies which the kernels have avoided.

The difference now with the base case is that we tend to avoid any level inflation more when interest rates are high and output gaps are negative (which is represented by the grey area on the right slice). This make sense since we are trying to avoid negative output gaps by more than in the base case, because any level inflation will affect our objective of getting out of recession adversely. With the left slice there is not much difference because grey area is obviously avoided under the new boundaries of the output gap targets. The only minor difference is in the case of the liquidity trap in the bottom left corner of the left slice which was explained above.

Trajectories in solid black lines represent viable trajectories, and in dashed blue lines are non-viable trajectories. In the left slice, current speed of interest rate adjustment is not sufficient for increasing initial y fast enough to be claimed viable for dashed blue line comparison to the black solid line. Again, in the right slice under the current speed of interest rate adjustment, the non-viable state according to y dynamic equation is not reducing initial y by fast enough to be claimed viable in comparison to the viable state shown within the kernel.

5.2 Why Some States are Non-viable

In the figure 16 below, in solid black lines show viable trajectories staying within the kernel and dashed blue lines show non-viable trajectories violating the boundaries of output gap. On the bottom right corner of the figure, the viable trajectory starts from a state where $y = 0.01$, $\pi = -0.0086$ and $i = -0.0356$, with level values of $\pi = 0.0114$ and $i = 0.0044$ and the non-viable trajectory start at the same values of inflation and interest rate but at a higher output gap of 0.015. According to the output dynamic equation 5 the bottom right corner viable trajectory is initially increasing output at a faster rate than that of non-viable trajectory just below it. For a viable state, as the trajectory gets closer to the boundary of the kernel and if the given speed of interest rate adjustment are sufficient enough for it to bounce back and start reducing y, this trajectory will stay within the kernel. However, the speed of adjustment of interest rates are not sufficient for non-viable trajectory to be claimed viable as it violates the constraint set.

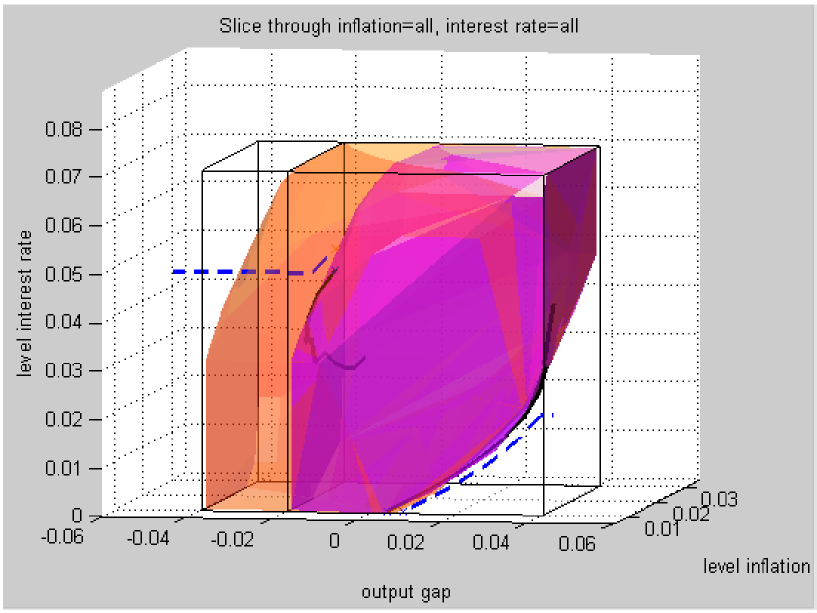

Figure 16: The kernels with output gap target zone of -0.04 to 0.04 (in orange) and -0.02 to 0.04 (in pink) with simulations. The solid black lines represent the viable trajectories and the dashed blue lines represent non-viable trajectories.

In the top left corner of the figure 16, for the viable states $y = -0.0125$, $\pi = -0.0043$ and $i = 0.0081$ where level variables of, $\pi = 0.0157$ and $i = 0.0481$ and the non-viable trajectory start at the same values of level inflation and output gap but at a greater $i = 0.0125$ where the level $i = 0.0525$. According to the output dynamic equation (5), for such viable state, y is initially decreasing at faster rate than that of the non-viable point. As the viable trajectory reaches near the boundary of the kernel the speed of interest rate adjustments is enough for the trajectory to bounce back, and to increasing y. For non-viable state the speed of adjustment of interest rate changes are not adequate for making the initial output gap viable.

5.3 Time Profiles

Let's take a closer look at time profiles again, which shows what happens to y and level variables π and i over the time horizon with respect to u. The figure 17 below shows the time profiles for viable trajectory while figure 18 shows the time profiles for a non-viable trajectory (shown on the bottom right corner of figure 16). Whereas the figure 19, shows the time profiles for viable trajectory while figure 20 shows the time profiles for a non-viable trajectory (shown on the top left corner of figure 16).

Figure 17 displays the states where initially output gap is increasing and therefore level interest rates should be increased by the central bank under such a situation. This will push output gap to a lower and stable level. The level inflation initially increases by a small amount, and with higher interest rates level inflation is becoming stable. The velocity is becoming very small over time as the economy heads toward the steady state. This shows that, the adjustment of the speed of interest rates are sufficient for the central banker to keep the economy within the constraint set in such a situation.

Figure 18, in solid black lines show where output gap is increasing and has crossed the upper limit of output gap. At the same time level inflation is also increasing. If we follow what the central bank would do in such a situation, then we should start increasing interest rates. The velocity is decreasing with the effort of increasing level interest rates yet the velocity is not small enough to reach the

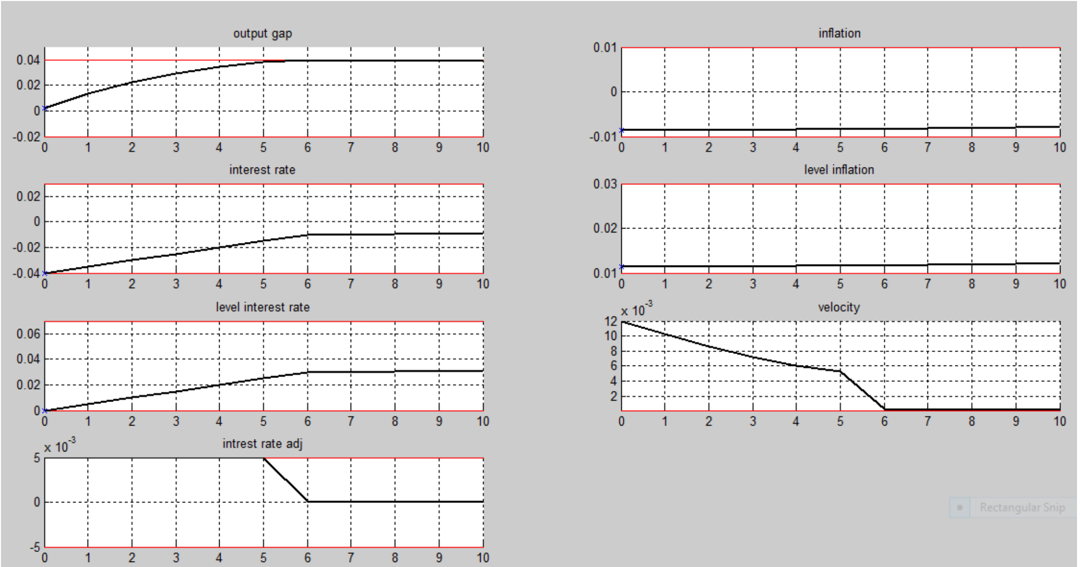

Figure 17: The time profiles for a trajectory starting from a viable point for fixed output gap of -0.02 to 0.04; which was shown at bottom right corner of figure 15.

steady state. In this sort of a situation, central banks would certainly be in trouble given the limits to adjustment of speed of interest rate. The central bank would have to make sure earlier on, that the economy would not lead to such a situation, where there is no strategy available to keep economy within its constraint set.

In figure 18, in dashed blue lines show what happens if we keep adjusting interest rates at half percent for another two quarters. Initially output is increasing and at the same time, level inflation is also increasing. With the effort of increasing level interest rates at half percent for another two quarters; all y, π and i have become somewhat stable. We can also see that the velocity has reached zero after about six quarters. The economy has reached a steady state. But, under our defined boundaries of the output gap target zone, this will be still claimed non-viable since y has left the upper boundary of the output gap.

In figure 19, it shows that y is decreasing and once the lower boundary of y

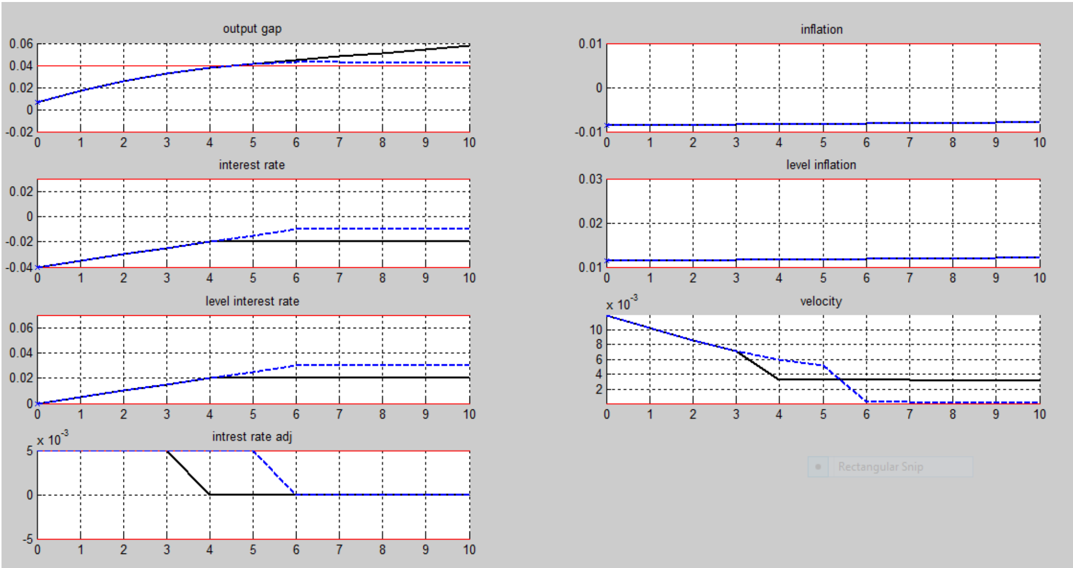

Figure 18: The time profiles of trajectories starting from a non-viable point for fixed output gap range of -0.02 to 0.04; which was shown in bottom right corner of figure 15.

is almost touched, it starts increasing and eventually becoming stable. The level interest rates are been adjusted at negative half percent for two quarter, and at some lower and higher rates for the next quarters till inflation becomes stable. The velocity is decreasing and also becoming stable in 18 quarters. Therefore in such a situation, there is a strategy which is available to keep the economy under control in one and a half year time period.i.e., adjustment speed of interest rates changes are adequate to keep the initial state viable.

In figure 20, solid black lines show that y is proportionally decreasing and hitting the lower boundary of output gap in ten quarters. The interest rates are been adjusted at negative half percent only at period zero, then it is increasing for a period and becoming stable. Under given interest rates, inflation is decreasing at some level. The velocity is decreasing but not by enough to reach the steady

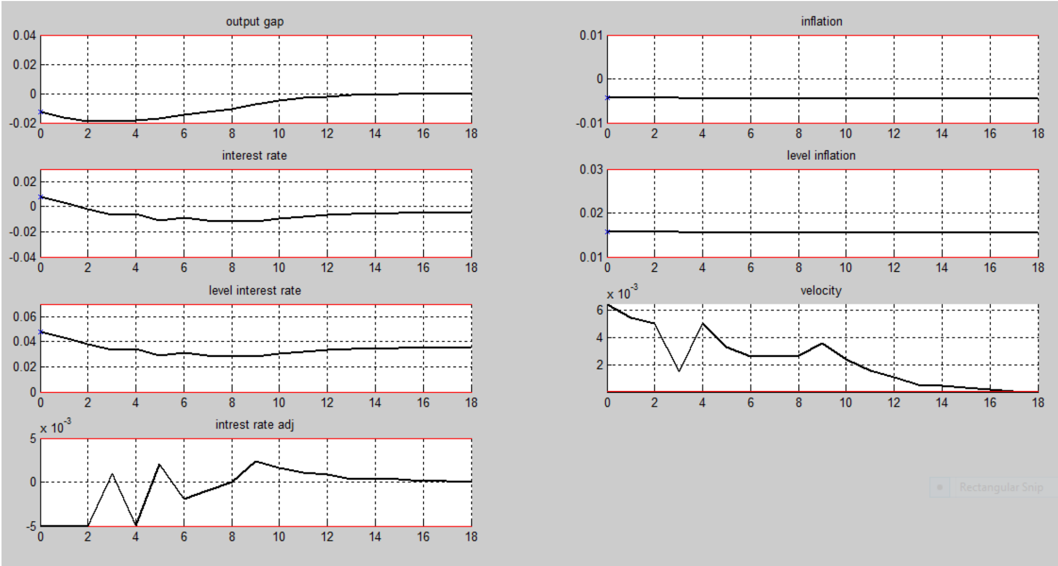

Figure 19: The time profiles for a trajectory starting from a viable point for fixed output gap between -0.02 to 0.04; which was shown in top left corner of figure 15.

state. In here, adjustment speed of interest rate are not appropriate to make the initial state viable.

In figure 20, dashed blue lines show what happens if we keep adjust interest rate at negative half percent for another two quarters. In such a situation, all y, π and i are decreasing and becoming stable. Velocity is also decreasing and becoming stable. Yet y has breach the lower boundary of output gap, therefore this is still non-viable under given boundaries of our problem.

6 Concluding Remarks

In this research, I have looked at a simple monetary policy model with varying output gap target intervals. The based case which was considered has a fixed output gap target zone of -0.04 to 0.04. Then, this was reduced to an output gap of -0.02 to 0.04. The method which was employed to solve this problem

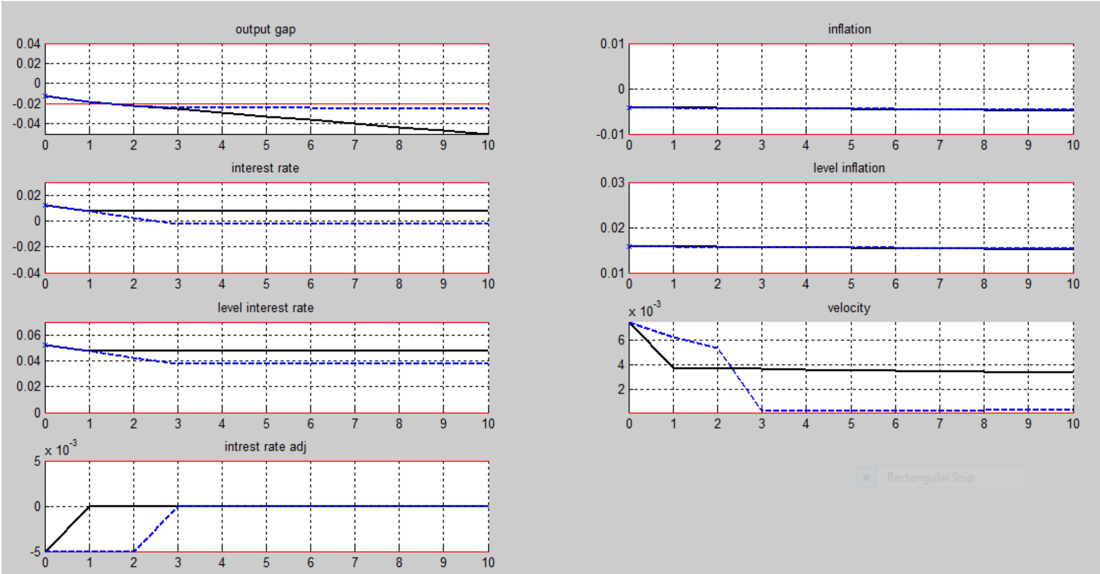

Figure 20: The time profiles for a trajectory starting from a non-viable point for fixed output gap of -0.02 to 0.04; which was shown in top left corner of figure 15.

used viability theory to form viability kernels using a computer software called VIKAASA which runs on MATLAB [see Krawczyk and Pharo (2011), Krawczyk et al. (2011) and Krawczyk and Pharo (2012) for details]. The viability kernel basically defined: given control constraints whether there are strategies which can keep the system within its normative bounds [see Krawczyk and Kim (2009)]. This research is important to central bankers and macro-economists who want to look at/study the output gap target variations therefore the monetary policy problem using a new approach. In essence, we suggest that central bankers' should try to keep the economy within the viability kernel. The closer it is to the boundaries of the kernel, the faster the system dynamics move therefore would need a faster adjustment of interest rates to keep them viable. But, preemption is the best practice we should follow here, i.e, to make sure that the economy do stay within the kernel, it is important for the central bankers to adjust level interest rates early enough to avoid economy from leaving the boundaries of constraint set K.

The main observation from this small model is that the kernels touch upon the boundaries of y. Furthermore, at extreme positive output gap target variations, central banks should try to avoid extremely low interest rates. Because, at such a situation where we are in a boom, level inflation is likely to rise. If low interest rates are used then level inflation will eventually be running out of control. While at extreme negative output gaps, central banks should try to avoid very high interest rates. This is because if we are in a recession and if we use higher interest rates, it would be harder to escape from said recession by growing output levels.

The kernels also show how central banks should try to avoid liquidity traps and overheating economies. A liquidity trap happens when level inflation is low and there is a negative output gap. The central bank has lowered the interest rates later than they should have, and negative output gap will further reduce level inflation to zero. At such a situation you cannot reduce interest rates any further. This explains why central banks should always target small positive level inflation. When they target such inflations, they will be able to get a negative real interest rates even when real interest rates equal to zero, and therefore be able to avoid liquidity trap situations, [see McCallum (2006)]. Heating economy happens when level inflation is high and output gap target variations are positive. In such a situation, it would be hard for the central banks to avoid economy from leaving the upper boundary of output gap targets variations. Therefore, it is important for central banks to avoid such a situation by increasing i early enough. When doing so it is important to make sure they increase i by the perfect amount without falling into a greater recession. The best way to do this is to target y and π equal to 0.

Given the final solution to this research, and looking at the literature on this topic, in conjunction with the time profiles, it turns out that targeting output gap does not have a big impact on level inflation. The coefficient on the dynamic inflation equation is small as 0.002 in the literature, [see Walsh (2003)]. Also, on the time profiles when output is increasing by large amounts level inflation is increasing only by minute amounts. This may explain why the central bankers' should be more conservative and be concerned about avoiding higher levels of inflation. Therefore, to target lower levels of inflation by having an inflation band, rather than targeting output gap target levels by legislations.

7 Future Research

This monetary policy problem can be simply extended to a small open economy, as done by [see Krawczyk and Kim (2009)], where then the exchange rate comes into play. In prospect, a completely new research area may well be the use of the viability theory to model the combined effect of monetary policy problem in conjunction with the fiscal policy problem to analyse the impact on a closed economy and a small open economy.

8 Appendix

8.1 A VIKAASA

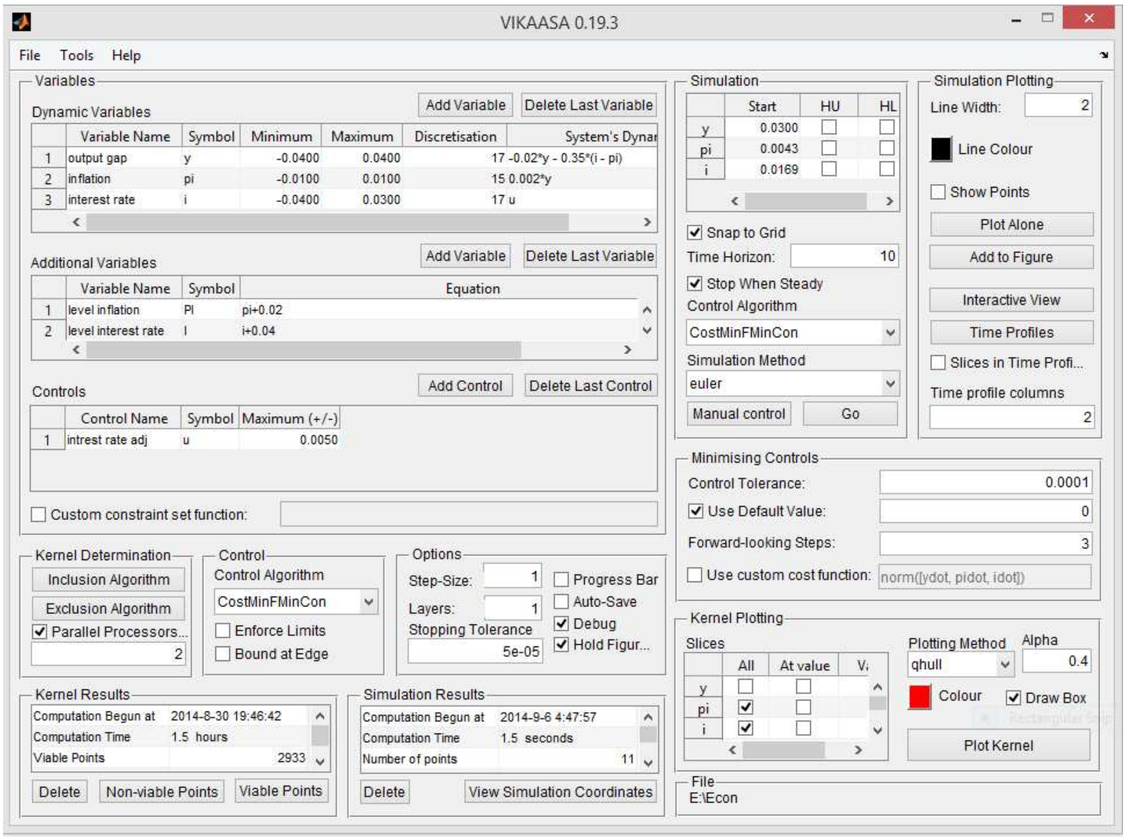

Figure 21: Main window of VIKAASA

VIKAASA is a specialised piece of software that runs on MATLAB. To run VIKAASA can enter 'vikaasa' on the main window of MATLAB. In figure 21 shows, the main window of VIKAASA as you run it on MATLAB. Specifically it shows the details of kernel of the based case scenario that was considered in this research. On the top left corner it shows where dynamic variables, system's dynamics and their boundaries can be entered which includes y, π and i in our problem set. Just below that we can define additional variables if there are any.

In this research we use this to define level inflation and level interest rates. Below that we can enter the control variable we want to use with its differential inclusion values (which was u speed of interest rates adjustment in our problem which was set as 0.005). Below that kernel determination can be used to define the algorithm we use to calculate kernels (inclusion algorithm in our case). The control algorithm just beside kernel is a time consistent feedback rule which can be used to slow down system velocities (CostMinFMinCon for this problem set). On the top right corner simulation and simulation plotting can be used to create time profiles for specified values of dynamic variable and simulations methods (Euler in our case). Just below that we have to select how many forward looking steps are reasonable to use. Three forward looking steps were sufficient for this research. Kernel plotting can be used to plot 3-dimensional kernels and these can also sliced at defined values of each variable. Just beside kernel plotting stopping tolerance can be entered, i.e., which is the criteria for near steadiness, [see Krawczyk and Pharo (2011)].

For further details follow the Manual of Vikaasa available to download from http://researcharchive.vuw.ac.nz/handle/10063/3432, [seeKrawczyk and Pharo (2011)].

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