



Munich Personal RePEc Archive

Dynamic Oligopolistic Pricing with Endogenous Change in Market Structure and Market Potential in an Epidemic Diffusion Model

Zieseimer, Thomas

U Maastricht

1993

Online at <https://mpra.ub.uni-muenchen.de/61831/>

MPRA Paper No. 61831, posted 05 Feb 2015 08:56 UTC

Dynamic Oligopolistic Pricing with Endogenous Change in Market Structure and Market Potential in an Epidemic Diffusion Model

Thomas Ziesemer¹, Maastricht University, Department of Economics and MERIT, 1993

1. Introduction

In Dosi *et al.* (1988) there is a repeated claim that economic models should be imperfectly competitive, dynamic and of the disequilibrium type. At first sight this seems to be demanding too much from one paper. However, in this paper it will be shown that adding an epidemic diffusion model to a simple static monopoly model and to duopoly models generates all three properties.

After having mainly been used as an empirical tool without explicit supply considerations, the epidemic diffusion model has begun a second career as an information technology constraint imposed on monopoly models quite common in economic theory (see Glaister 1974, Metcalfe 1981, Amable 1992, Stoneman 1983, ch. 9, and Mahajan, Muller and Bass 1990 for an introduction to the literature on "diffusion and supply"). Glaister (1974), starting from an epidemic information diffusion assumption with an exogenous long-run equilibrium value of demand, derived a non-logistic diffusion curve skewed to higher growth rates in earlier product phases and continuous intertemporal price differentiation. Since that time the marketing literature has used it in the framework of dynamic optimization and differential games. Metcalfe (1981) assumed an epidemic demand curve and derived a sigmoid diffusion curve using a monopoly mark-up assumption on the supply side and the assumption that diffusion is proportional to profits. Amable (1992) extended this analysis to allow for increasing returns and competing technologies.

This literature has been developed with the intention of improving the theoretical explanation of the observed sigmoid diffusion curves. Very often these curves show a logistic or skewed diffusion pattern. Several non-epidemic approaches to explain these patterns have been tried in the literature. The introduction of learning economies is used in the models by Bass (see Mahajan, Muller and Bass 1990) and Stoneman and Ireland (1983). However, in the former the diffusion curve derived depends on the specification of an exogenously imposed shift function, which bears great similarity with the epidemic functions used in the literature mentioned above, and in the latter on a special relation between price and threshold adopter derived from the imposition of a distribution of firm size that again bears great similarity with the types of functions used in epidemic models. Soete and Turner (1984) use the assumption of N techniques in a classical growth model. At the macro level there is a classical investment function from which sectoral investment deviates

¹ I would like to thank Bruno Amable for correspondence, Paul Diederer, Theon van Dijk, René Kemp, Bart Verspagen and Adriaan van Zon for helpful discussions, Marc van Wegberg and Arjen van Witteloostuijn for their detailed comments on the conference version, Gene Grossman for providing me with the written version of his conference remarks and very helpful comments on the second but last version and Reinoud Joosten, Hans Peters and Jerry Silverberg for commenting on an earlier version of the paper. Responsibility remains entirely mine.

positively (negatively) if the sector in question has a higher (lower) rate of profit than the average. Asymptotically, the best-practice technique approaches a 100% share of capital, K_N in total capital, K . K_N/K follows a sigmoid diffusion curve. The way any technique α gains capital from inferior and loses capital to superior techniques bears great formal similarity with the way a product loses demand to a new product and gains from older products in the purely empirical "law of capture" of Norton and Bass (1992). Vintages of capital with a finite horizon optimization model to investigate intra-firm diffusion are used in Felmingham (1988). Investments made during the early periods have a correspondingly longer productive lifetime. The lower the discount rate and the higher the expected prices, the larger investment will be. Therefore investment is first increasing, then decreasing and the vintage capital stock has a sigmoid form.

The examples given above are on the borderline between evolutionary and (neo)classical economics and the marketing hybrid of the two. A survey of the microeconomic literature can be found in Reinganum (1989). Silverberg, Dosi and Orsenigo (1988) broadly introduce the evolutionary literature and Mahajan, Muller and Bass (1990) the marketing literature. Chatterjee and Eliashberg (1990) connect the marketing literature to the microeconomic literature, thereby giving the epidemic diffusion model and the marketing literature the status of microfounded aggregates.

This paper uses Amable's version of the model (consisting of a linear estimate of a demand curve, a linear quadratic cost curve and a logistic diffusion curve) and four types of imperfectly competitive behaviour — monopolistic intertemporal profit maximization, a dynamic Bertrand oligopoly, and a duopolistic differential game with endogenous market potential, which leads to a Stackelberg-leadership situation or a phase of pure monopoly.

Neither endogenous market potential nor endogenous change in market structure have been treated in the differential games literature using the epidemic diffusion model or similar constructs (see Mahajan, Muller and Bass 1990 and Dockner and Jorgensen 1988).

The structure of the paper is as follows. In Section 2 the basic elements of Amable's model are presented. Intertemporal profit maximization results for the case of a slightly extended version of Jorgensen's (1983) pure monopoly model are derived in Section 3 to facilitate the understanding of the differential game presented later. Section 4 looks at the results of an oligopolistic market structure where n firms producing homogeneous goods in an otherwise unchanged model. In Section 5 it is assumed that consumers associate the product with the name of one of two firms; a differential game of profit maximizing duopolists leads to either monopoly profits for one of the firms and for the other to first positive, then zero profits and finally exit, or a Stackelberg leader/follower constellation. The Stackelberg leadership-follower constellation is analyzed in Section 6. In all cases variants of sigmoid diffusion curves are obtained. In Section 7 the results are summarized. After introducing the models in each section I will discuss the literature to which they are related.

2. Basic Elements of the Models

The model consists of a linear-quadratic total cost function in output y :

$$C = c_0y + c_1y^2 \quad (1)$$

(where $c_0 > 0, c_1 \geq 0$ or $c_1 \leq 0$ indicates whether unit costs are rising or falling) and a possibly correct estimate of potential market demand

$$D = a_0 - a_1p, a_0, a_1 > 0 \quad (2)$$

where D is the long run demand potential for given p . Moreover, the choice of D and y is restricted by the epidemic (information) diffusion curve:

$$\dot{y} \equiv \frac{dy}{dt} = \beta D y \left(1 - \frac{y}{D}\right), \beta > 0 \quad (3)$$

Eq. (3) mirrors the assumption that consumers get to know or forget the product only slowly, depending on sales already made or the number of persons who have bought the product, y , multiplied by the probability, $1 - y/D$, that they meet somebody who is part of the market potential and does not know the product. The specification used in eq. (3) is known as the Chow logistic (see Stoneman 1983, p. 70). Glaister used the model for the analysis of the diffusion of consumer products, and Metcalfe used it for the analysis of diffusion of production technologies. In both contributions, the quantity sold and the number of adopters are identical, implying that each individual buys or hires one unit per period, an assumption also widespread in the marketing literature (see Mahajan, Muller and Bass 1990). In a later section we consider a model with two such information diffusion technologies for one firm as initiated by Amable (1992). y , D and p depend on time t .

The epidemic model considers a purely random process of diffusion of knowledge about the existence of a product or technology and the desire to buy it. The existence of two classes of individuals, one of which knows and buys the product and one that does not, is analogous to two classes of people with respect to a disease, one of which is randomly infected and the other is not. Once the product is known there is no uncertainty with respect to quality or price. The assumption of a purely random process is of course an exaggeration that emphasizes the costlessness of information diffusion. In the variants that consider diffusion and supply together, labour investment of household into the search for new products, or advertisement costs of sellers are neglected (for an introduction to the epidemic diffusion models see Thirtle and Ruttan 1987, ch. 3). Efforts to analyze price policy and advertising together yield untractable differential games if goodwill dynamics are added. Therefore advertising is not considered here. Instead, the focus will be on price policy and market structure. Before one proceeds to draw managerial consequences, the above-mentioned simplifications should be taken into consideration. However, complexity sometimes requires us to treat things separately.

In Glaister (1974) β is a function of p , and D is fixed. The difference is that $\beta(p)$ allows a direct choice of the growth rate of y at each value of y , whereas in this specification the growth rate can only be indirectly influenced through the impact of the control variable p on D . In Metcalfe (1981), Batten (1987) and Amable (1992) the growth of supply depends

on profits. Amable (1992) assumes that the parameters of the model are constant over time and known to the firm at each moment but not for the future. We make the same assumption in Section 4, but in Sections 3, 5 and 6 we add the assumption that the firm knows the constancy of parameters over time.

In the next section we shall investigate the model under the assumption of monopolistic behaviour. This is based upon S. Jorgensen's (1983) contribution, which serves as a reference model for the later oligopolistic models in this paper. We do not consider the microfoundation of eqs. (2) and (3) (see Oren and Schwartz 1988 and Chatterjee and Eliashberg 1990 on this point) or the marketing instruments that provide the firm with estimates of them.

The logistic diffusion curve $y(t)$ derived for constant D from eq. (3) is an empirically well-established relation (see Rosegger 1986, ch. 9 and Coombs, Saviotti and Walsh 1987, ch. 5.6 for descriptive realism, and Mahajan, Muller and Bass 1990 and Karshenas and Stoneman 1992 for econometric estimations and tests of models) and will turn out to be the result of the monopolistic (Section 3) and oligopolistic (Section 4) considerations and of the change in market structure (Sections 5 and 6) here. Therefore we try to contribute to the literature on diffusion and supply with special emphasis on market structure.

3. Intertemporal Profit Maximization with a Single Diffusing Technology

If the monopolist producing y maximizes intertemporal profits for an infinite horizon subject to eq. (3) because no (potential) competitor is threatening his position, the Hamiltonian for his program is

$$H = (p - c_0 - c_1 y) y + \lambda [\beta y (a_0 - a_1 p) - \beta y^2].$$

The first order conditions for a singular solution are eqs. (4), (5) and $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) = 0$:

$$\delta H / \delta p = y + \lambda \beta y (-a_1) = 0. \quad (4)$$

The economic problem here is that high prices increase temporary profits but decrease future profits by reducing the speed of diffusion. As the Hamiltonian is linear in p a singular solution is optimal for some time phase if it exists. During such a phase a singular solution requires a constant λ , following from eq. (4):

$$\lambda = 1 / (\beta a_1).$$

The shadow price of y is the inverse of the impact of p on the growth rate of y , βa_1 . The first order condition for y is (with discount rate ρ)

$$-\partial H / \partial Y = -p + c_0 + c_1 2y - \lambda [\beta (a_0 - a_1 p) - 2\beta y] = \dot{\lambda} - \rho \lambda \quad (5)$$

From the solution for λ and its constancy we obtain

$$-p + c_0 + c_1 2y - (\beta a_1)^{-1} [\beta(a_0 - a_1 p) - 2\beta y] = -\rho(\beta a_1)^{-1}. \quad (5')$$

The condition from static theory that marginal revenue should equal marginal cost is modified by two dynamic terms here, one due to diffusion and one due to discounting. Terms containing ρ are identical up to their sign and thus can be dropped. Then solving for y yields:

$$y^* = (-\rho/\beta - c_0 a_1 + a_0) / [2(c_1 a_1 + 1)]. \quad (6)$$

y^* is a constant value here. This solution differs from the solution of the static monopoly model only by the term ρ/β . The monopoly solution eq. (6) with diffusion is lower than the static monopoly solution because of the impatience expressed by the positive discount rate. Here the change of y is zero and the price is determined by $a_0 - a_1 p^* = y^*$. As a consequence in this phase the last term subtracted on the left hand side of eq. (5') is negative. If ρ is close to zero, price then has to be higher than marginal cost. This is all the more the case if $\rho \gg 0$. For profits to be positive for all ρ we must have $a_0 - c_0 a_1 > 0$. For a static monopoly solution to exist ($\rho \rightarrow 0$) and y^* to be positive for small ρ , eq. (6) requires that $c_1 a_1 + 1 > 0$, which we shall assume henceforth unless noted otherwise.

However, for $y(0) < y^*$ (where $y(0)$ may be the sales to workers producing the product who are the most natural candidates to know first about the existence of the product) there must be a phase where this value is approached. This requires that the change of y is zero and therefore some $p' \leq p^*$ is sufficient to reach the singular solution. However, Jorgensen (1983) shows that for $c_1 = 0$ a most rapid approach to a singular solution reached by prices as low as possible is the optimal solution. These are introductory prices to be defined below. During such a phase y is lower than D and supply is therefore lower than estimated potential demand. In this sense there is a temporary disequilibrium due to slow diffusion. Of course, "one can turn any disequilibrium model into an equilibrium equivalent [if an equilibrium exists, *T.Z.*] and *vice versa* by a suitable definition of the information sets and perceptions of adopting agents" (Metcalfe 1988, p. 561).

Due to the linearity of H in p a lower bound for p, p' , has to be imposed at which the diffusion of knowledge takes place. Some examples of such bounds can be discussed for the purpose of illustration. If the chosen p' goes to minus infinity one obtains jumps (impulse control) in the stock variable y . As this is at variance with the idea of slow diffusion we do not consider this possibility. Free sample copies, hence $p' = 0$, are a possibility as well. However, the accompanying losses during a phase with prices lower than costs require credit that can be repaid only in the singular solution phase with positive profits, and therefore is an unnecessary complication. $p' = \max(c_0 - c_0 + c_1 y^*)$ ensures non-negative profits in the introduction phase. Even $p' = p^*$ would be a candidate because y^* would be approached from any value $y(0) < y^*$. For any such value we have a diffusion process that stops increasing at y^* . For a graphical summary of the solution see Figures 1 and 2. During the introductory phase with the lower price p' , the firm moves along the (higher) diffusion curve going through D' . Once $y^* = D^*$ is reached this curve is no longer relevant because the change of y becomes zero as the price jumps to the higher p^* . In spite of a jump of the price there is no corresponding jump in the quantity because the new price implies constant y . The product is no longer made better known because one prefers to have high profits now, if $\rho > 0$. The equilibrium value $y^* = D^*$ belongs to a diffusion curve that has not been followed, because there was a jump from $D'(p')$ to $D^*(p^*)$. As ρ approaches zero the solution approaches that of the static Cournot monopoly. The comparison with Cournot monopoly is illustrated in Figure 3 below for Jorgensen's case with $c_1 = 0$ and $p' = c_0$, where MR is marginal revenue, and y_m, p_m is the static monopoly solution. Starting from $y(0)$, y grows at $p' = c_0$ until y^* where p jumps to $p^* > p_m$.

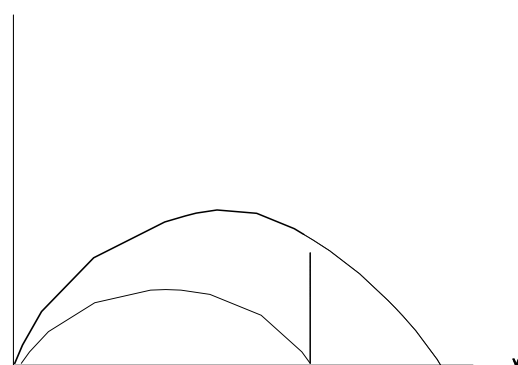
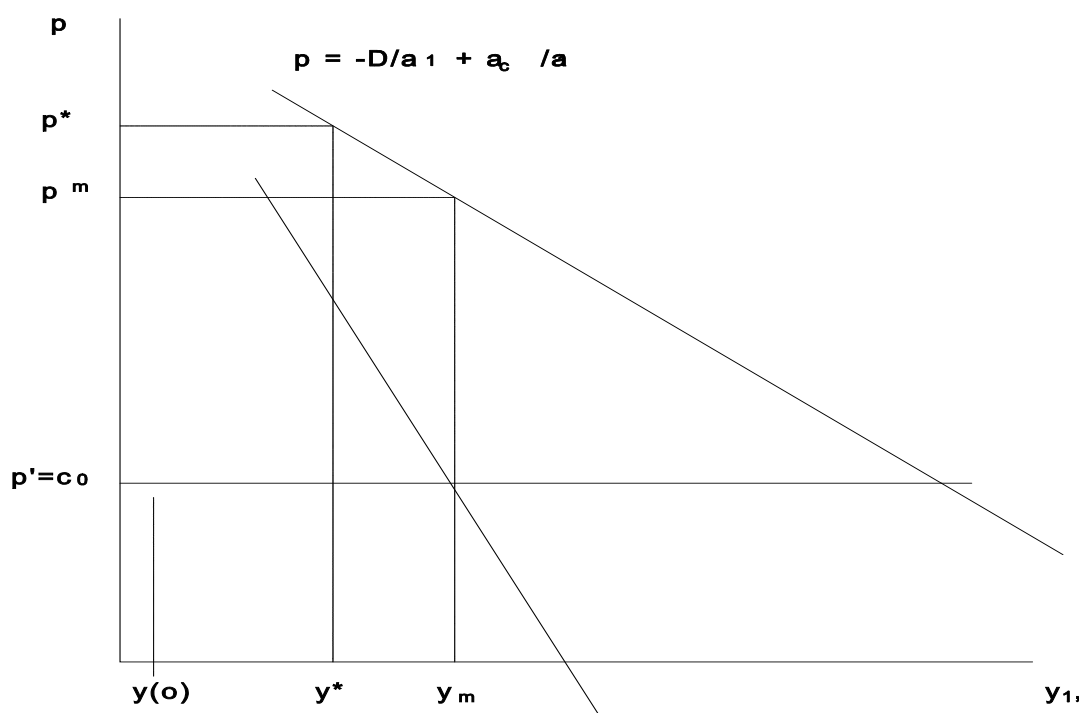
Figure 1: p on the vertical, t on the horizontal axisFigure 2: dy/dt on the vertical axis

Figure 3



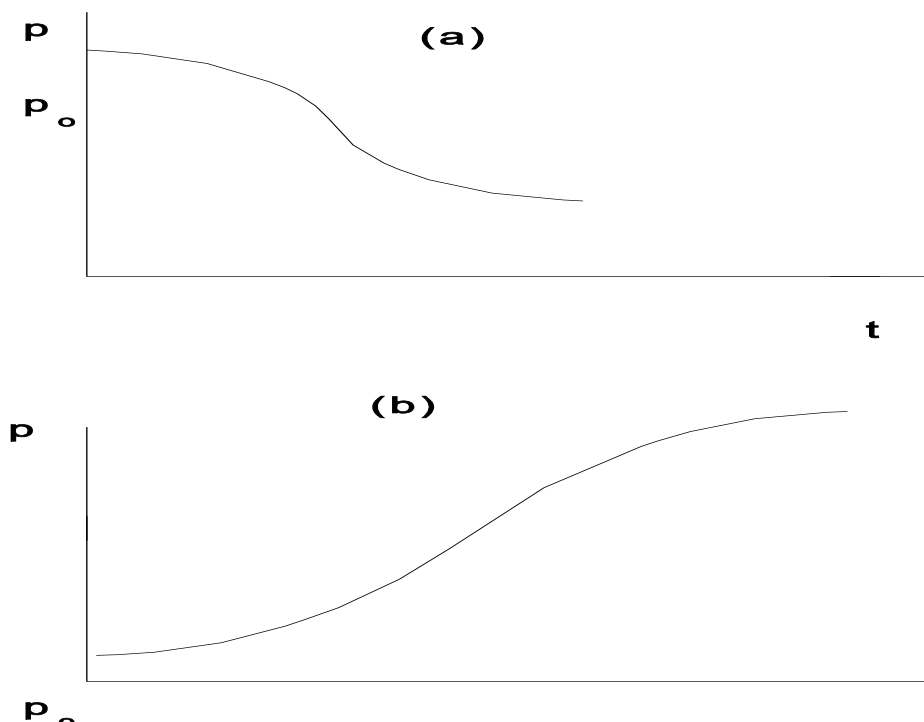
Recall that Glaister (1974) regarded two prices as the more realistic case compared to the permanent intertemporal price differentiation obtained in his model. The linearity of the Hamiltonian which generates this solution with two prices is due to the insertion of a linear demand function (see Feichtinger 1982, p. 240) into the Chow logistic, which is the simplest special case of the Bass model, which in turn is the preferred one of several possible variants (see Stoneman 1983) in the marketing literature and generates a variant of discontinuous price policy in models of diffusion and supply. If one gives up one of these specifications the price paths will be either that

i) of Glaister (1974) (see Figure 4 below) with continuous price differentiation from low to high prices (called market

penetration in the marketing literature) or

ii) of Robinson and Lakhani (1975) for optimizing and Metcalfe (1981) for non-optimizing behaviour under supply

Figure 4



constraints, both with continuous price differentiation from high to low prices (market skimming).

In Metcalfe's case diffusion may depend strongly on making profits, which requires high introductory prices, whereas in the Robinson and Lakhani case the fall in prices is due to falling costs and positive discount rates. The cases of continuously falling or increasing prices as well as a combination of both were contained in Spremann (1975). The specifications for the case of discontinuous price policy emphasized here will turn out below to be a starting point for tractable differential games because these are the simplest possible of all specifications used in the literature.

4. Price Policy in a Dynamic Bertrand Duopoly

In this section we use the same model as above but assume that there are n firms introducing the new product at the same time. Patenting is excluded. Knowledge about the existence of the product again works as in eq. (3). However, the oligopolists now compete for the buyers, y , who know of the product:

$$y = y^1 + y^2 + \dots + y^n.$$

The upper indices of the supply terms on the right-hand side of the equation are the indices of the firms. Buyers are assumed to be perfectly informed about the prices of all firms such that eq. (2) holds with $p = \min [p^1, p^2, \dots, p^n]$. We assume that all firms have identical cost functions. As outputs y^i and prices p^i are fast variables, given the value of the slow variable y , the model is symmetric with respect to the firms and therefore they are assumed to produce the same quantities $y^i = y/n$. These assumptions are used in the following differential game:

$$\begin{aligned} & \text{Max}_{p_i} p_i y_i - (c_0 + c_1 y_i) y_i \\ & \text{s.t. (2), (3) and } y_i = \frac{y}{n}, p_i \in \left[c_0 + c_1 y_i, \frac{a_1}{a_0} \right] \end{aligned}$$

with $c_0 + c_1 y_i$ as a lower bound for p_i as in section 3. For $n=1$ this game yields the same results as section 3. For $n>1$ the rationing rule is $y_i = y/n$, all firms choose identical quantities and prices because they are identical in all respects. Instead of (5') and (6) one gets for a singular solution:

$$p_i \left(1 - \frac{1}{n} \right) = c_0 + 2c_1 \frac{y}{n} - \frac{a_0}{na_1} + \frac{\rho}{n\beta a_1} + \frac{2\beta y}{n\beta a_1}$$

which is identical to (5') for $n=1$, and

$$\frac{y^*}{n} = \frac{a_0 - a_1 - \frac{\rho}{\beta}}{n-1+2(a_1 c_1 + 1)} \quad (7)$$

which is identical to (6) for $n=1$. In (6) p dropped out because $n=1$. Therefore a solution for y was obtained which had to be approached gradually in a transition to the singular solution. For $n>1$, p does not drop out now. Given the initial value for y/n , p is determined in the equation that is similar to (5') and the shadow price can be determined such that a singular solution is valid from the beginning. No transition is necessary. Insertion of the price equation into the differential equation for y would allow to analyse the dynamics of y again. Under the assumption that the numerator and the denominator of the stationary solution of y are both positive one gets the same type of a curve as in Figure 2. In this model profits will in general be positive as in the model of section 3.

A rather similar model is that of Rao and Bass (1985), where products are also homogeneous and therefore prices have also to be identical across firms, but costs are dependent on cumulative output. The model by Eliashberg and Jeuland (1986) differs from that of Rao and Bass (1985) in that they consider differentiated products with homogeneous products as the special case of a huge impact of price differences and no learning effects in the cost functions. For that limiting case their (very complex) model is one with an exogenous market potential.

In summary, the introduction of competitors has made a continuous price policy out of what was a discontinuous price policy in the previous, monopolistic model.

5. A Duopolistic Differential Game with Endogenous Market Potential and Transition to Monopoly or Stackelberg Leadership

In this section we investigate a differential game with two firms that differ in initial market share and have to make their

own reputations. The products of the two firms considered are imperfect substitutes. The difference between the products is indicated by the name or reputation of the firms producing with identical unit cost functions

$$c = c_0 + c_1 Y.$$

Consumers knowing about the product of a firm infect those consumers who do not know either of the two firms or their product. The epidemic process is then (see Amable 1992)

$$\dot{y}_i = \beta y_i (D - y_i - y_j) \quad i=1, 2; j \neq i$$

These are two diffusion processes — one for each firm's reputation or number of clients — with the same growth rates. This dynamic process imposes identical growth rates on the reputations of the two firms, which is a rather strong dynamic rigidity. However, it simplifies the analysis considerably because the difference in levels of the y_i will be determined by their initial values and the common growth rate (on the tractability of differential games with respect to the problem at hand see Dockner and Jorgensen 1988). In the y_1 - y_2 plane the duopoly then moves along a ray through the origin determined by the initial values. If a differential equation drives y_i upwards (downwards) then the firm's quantity cannot grow faster (decline slower) and by assumption it can never jump. The firm can sell less but not more than indicated by the differential equation.

Overall potential demand D is assumed to be

$$D = a_0 - a_1 p_1 / 2 - a_2 p_2 / 2.$$

This demand function can be derived as a special case of the demand functions for imperfect substitutes:

$$D_1 = e_1 - b_1 p_1 + d_1 p_2 \quad D_2 = e_2 + b_2 p_1 - d_2 p_2$$

Summing left and right hand sides yields

$$D = D_1 + D_2 = e_1 - b_1 p_1 + d_1 p_2 + e_2 + b_2 p_1 - d_2 p_2 = e_1 + e_2 - (b_1 - b_2) p_1 - (d_2 - d_1) p_2.$$

Assuming $b_1 - b_2 = d_2 - d_1$ and defining $e_1 + e_2 \equiv a_0$ and $b_1 - b_2 = d_2 - d_1 \equiv a_1/2$, the above demand function is obtained. $a_1/2$ is positive if own-price effects are stronger than cross-price effects. For the case $p_1 = p_2$ this demand function reduces to the demand function in the monopolistic part of the paper. The assumption $b_1 - b_2 = d_2 - d_1 \equiv a_1/2$ is a strong simplification which helps to focus on differences in the initial values of market shares as the main point of interest concerning differences between firms. Which firm becomes known to the buyer is again determined solely by the epidemic random process operating on a common market potential D for both firms. Both prices have an impact on the speed of diffusion of the knowledge about the existence of the product. High prices increase current profits given a vertical short-run demand

curve y_i because y_i is a slow variable, and low prices contribute to the diffusion. Therefore individual firms' intertemporal profit maximization subject to the differential equations takes the form of a duopolistic differential game where each firm bases its own decision on expectations about the other firm's behaviour concerning price and quantity:

$$\begin{aligned} & \text{Max}_{p_i, t_1} \int_0^{t_1} [p_i y_i - (c_0 + c_1 y_i)] e^{-\rho t} dt, i = 1, 2 \\ & \text{s.t.} \\ & \dot{y}_1 = \beta y_i (D - y_i - y_j), i = 1, 2; j \neq i, \\ & D = a_0 - a_1 p_1 / 2 - a_1 p_2 / 2 \quad \text{and } y_1(0) = y_{10}, y_2(0) = y_{20}, t_i \geq 0. \end{aligned}$$

Up until now in the literature such games have only been considered for constant values of D , although endogenous D is held to be desirable (see Dockner and Jorgensen 1988). In that literature prices $p_{1,2}$ are introduced in a manner that is more reminiscent of Glaister's $\beta(p)$ function. We consider the Nash equilibrium for open-loop strategies. Whereas the marketing literature focuses on price policy and profits, we also look at diffusion and change in market structure in connection with leadership in terms of market shares.

Insertion of D into the differential equation yields the generalized Hamiltonian for firm 1:

$$H_1 = p_1 y_1 - (c_0 + c_1 y_1) y_1 + \lambda_1 \beta y_1 (a_0 - a_1 p_1 / 2 - a_1 p_2 / 2 - y_1 - y_2) + \mu_1 \beta y_2 (a_0 + a_1 p_1 / 2 - a_1 p_2 / 2 - y_2 - y_1).$$

The first order conditions for a singular solution phase are the differential equations (3'), $\lambda_1(t_1) = \mu_1(t_1) = 0$, $H(t_1) = 0$, implying zero profits if finite t_1 exists, and

$$\begin{aligned} \delta H_1 / \delta p_1 &= y_1 + \lambda_1 \beta y_1 (-a_1 / 2) + \mu_1 \beta y_2 (-a_1 / 2) = 0 \\ -\delta H_1 / \delta y_1 &= [p_1 - c_0 - 2c_1 y_1 + \lambda_1 \beta (a_0 - a_1 p_1 / 2 - a_1 p_2 / 2 - y_2 - 2y_1) - \mu_1 \beta y_2] = \lambda_1 - \rho \lambda_1 \end{aligned}$$

(where ρ is the discount rate as in the monopolistic model) and

$$-\delta H_1 / \delta y_2 = \lambda_1 \beta y_1 - \mu_1 \beta (a_0 - a_1 p_1 / 2 - a_1 p_2 / 2 - y_1) + \mu_1 \beta 2y_2 = \mu_1 - \rho \mu_1.$$

From these first order conditions one can solve for p_1 and p_2 . The results are:

$$-c_0 - (2c_1 + 4/a_1)y_1 - \rho 2/(a_1 \beta) + (2/a_1)a_0 - 4y_2/a_1 = p_2$$

$$-c_0 - (2c_1 + 4/a_1)y_2 + (2/a_1)a_0 - 4y_1/a_1 - \rho 2/(a_1\beta) = p_1.$$

In eqs. (11'') and (12'') y_1 and y_2 are slow state variables. Therefore they determine those (expected) values of prices which can be equilibrium prices of a singular solution. From the monopoly model one might have expected that there would again be a transitional phase where imposition of values for $p_{1,2}$ would lead to such a singular solution. However, there is no reason not to be in a singular solution from the beginning because the equations for a singular solution do not yield results such as y^* in the monopoly model that must be approached slowly. The most rapid approach in this case is to begin with (correctly expected) prices as determined by eqs. (11'') and (12''). Insertion of D and the solutions for $p_{1,2}$ into the differential equations using $y_1 + y_2 = y$ yields after some manipulations

$$\dot{y} = \beta y [a_0 + a_1 c_0 - 2a_0 + 2\rho/\beta + y(3 + a_1 c_1)].$$

Eq. (13) summarizes the development of the market quantity y . y_1 and y_2 differ only by their initial values and have growth rates identical to that of y . eq. (13) is graphed in Figure 5. It has an unstable threshold value at

$$y^* = (a_0 - a_1 c_0 - 2\rho/\beta)/(3 + a_1 c_1).$$

Again we assume that the discount rate is sufficiently low to ensure a positive solution y^* with positive numerator and denominator. As a consequence of eq. (14) one can draw a no-growth line $y_1 = -y_2 + y^*$ in the $y_1 - y_2$ -plane (see Figures 6-8). If the game starts below that line both quantities will be decreasing, and if it starts above it both quantities will be increasing until a zero profit line is reached. To be able to derive the zero-profit line and to understand the movement to that line we have to consider the price equations again. Price equations can be rewritten as functions of y and the initial values of y and $y_{1,2}$ in the following form:

$$p_2 = -c_0 - \rho 2/(a_1\beta) + (2/a_1)a_0 - y[(2c_1 + 4/a_1)y_{10}/y_0 + 4y_{20}/(y_0 a_1)]$$

$$p_1 = -c_0 + (2/a_1)a_0 - \rho 2/(a_1\beta) - y[4y_{10}/(y_0 a_1) + (2c_1 + 4/a_1)y_{20}/y_0].$$

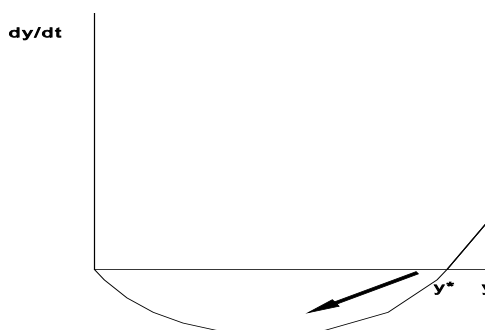


Figure 5

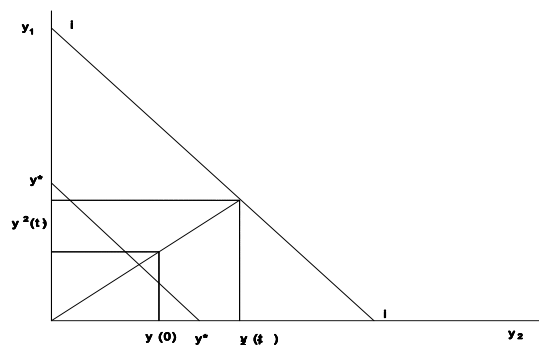


Figure 6

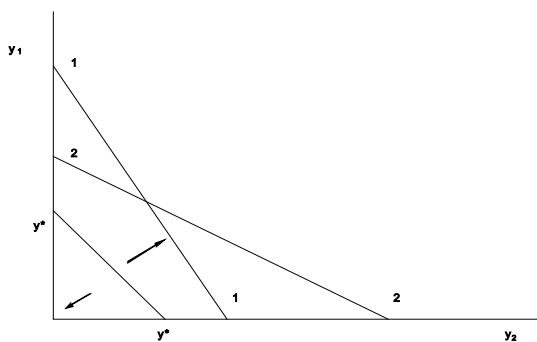


Figure 7

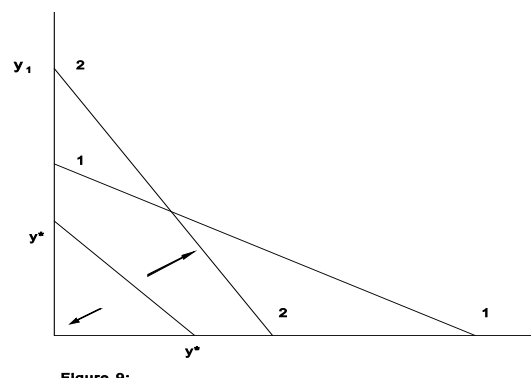


Figure 8

Prices are negatively related to the quantity y . If the initial values were below the threshold value both firms could become smaller and smaller at positive profits with no finite value of t_f existing, because — as will be shown below — zero-profit lines will be to the right of the no-growth line. Firms would converge to atomistically small suppliers. To understand the movement to the zero-profit line it is important to recognize that in spite of the assumption that in the static demand curves a firm's own price has a stronger impact than the other firm's price, $b_1 - b_2 = d_2 - d_1 = a_1/2 > 0$, eqs. (11'') and (12'') imply that a price is more (less) strongly affected by the other firms' quantity than by its own quantity if :

$$|\delta p_i / \delta y_j| > < |\delta p_i / \delta y_i| \quad \text{as} \quad c_1 > < 0.$$

Inserting the price equations into the zero-profit condition $p_i - c_0 - c_1 y_i = 0$ yields the zero-profit lines. To illustrate three possible outcomes we distinguish three cases.

Case 1. For $c_1 = 0$ (see Figure 6) the no-growth line has intercepts

$$y^* = (a_0 - a_1 c_0 - 2\rho/\beta)/3$$

and the zero-profit lines (indicated as ii in Figure 6) becomes

$$y_i = -y_j + (a_0 - a_1 c_0 - \rho/\beta)/2.$$

Obviously, zero-growth lines are identical for both firms in this case, have the same slope as the zero-growth line and their intercepts are larger than y^* , the intercept of the no-growth line. On whichever ray the firms move to the zero-profit line, they both reach it at the same time t_f . From that point in time onwards they may have a weak preference to stay in the market, which leads to the same results as the Bertrand model of the previous section. Alternatively one or both exit.

Case 2. For $c_1 > 0$ we find that

$$y^* = (a_0 - a_1 c_0 - 2\rho/\beta)/(3 + a_1 c_1)$$

Therefore horizontal intercepts are clearly ranked and 11, the zero-profit line of firm 1 is steeper than 22, the zero-profit line of firm 2 (see Figure 7). As the zero-profit lines intersect at the 45 degree line, firm 1 will reach its zero profit line first if firm 2 has the higher initial value, and the firms therefore move along a ray that is flatter than the 45 degree line. The reason is that for $c_1 > 0$ the negative effects of one firm's quantity on the other firm's prices indicated in eq. (15) are stronger than the negative effects of quantities on a firm's own price. When reaching the zero-profit line firm 1 may either exit and leave a monopoly position for firm 2 or it may weakly prefer a zero profit strategy leaving a Stackelberg leadership position to firm 2, which we analyze below.

Case 3. For $c_1 < 0$ for small $|c_1|$ one can show that

$$y^* < Y_{2|\pi_2=0, y_1=0} < Y_{2|\pi_1=0, y_1=0}$$

and 11 is now flatter than 22, as shown in Figure 8. As a consequence, for $y_{20} > y_{10}$ firm 2 will come first to its zero-profit line because its quantity is depressing its own price more than that of firm 1. Therefore it is either optimal to exit in spite of having the larger market share or to weakly prefer a zero profit strategy, thus leaving a Stackelberg leader position to firm 1.

Other possible cases might exist because zero-profit lines may be below the no-growth line for $c_1 \ll 0$. Initial values above the zero-profit line but below the no-growth line lead to negative growth rates driving the process to the origin. There will be a phase with negative profits followed by one with positive profits. The question then is whether overall profits are positive. If a process begins above both lines it will not be started at all because profits become increasingly more negative. If the zero-profit line lies above the no-growth line an alternative to the singular solution phase might be that firms price at the corner $p_1 = e_1/d_1, p_2 = e_2/d_2$. Then they move back to the no-growth line with maximum speed. Arriving there they could price at the no-growth maximum profit price of the singular solution, which may yield higher discounted long-run profits. However, then the firms no longer are confronted with a diffusion problem.

6. The Stackelberg Leadership Phase

Suppose that firm 2 has gained a Stackelberg Leadership position either because $c_1 > 0$ has brought firm 1 to its zero-profit line because $y_{20} > y_{10}$ or $c_1 < 0$ has brought firm 1 to its zero-profit line because $y_{20} < y_{10}$ and it weakly prefers to stay in the market. Firm 1 may then try to produce $y_1(t_1)$, the quantity reached during the phase of the duopolistic game, provided that the differential equation does not produce negative growth rates such that consumers forget the firm. This will turn out to be no problem, because it will be shown that firm 2 goes to a higher quantity.

The problem for firm 2 from t_1 onwards then is

$$\begin{aligned} & \text{Max}_{p_2} \int_{t_1}^{\infty} [p_2 y_2 - (c_0 + c_1 y_2) y_2] dt \\ & \text{s.t. } \dot{y}_2 = \beta y_2 \left[a_0 - \frac{a_1 (c_0 + c_1 y_{1t_1})}{2} - \frac{a_1 p_2}{2} - y_2 - y_{1t_1} \right] \end{aligned}$$

given t_1 and $y_{1,2}(t_1)$ from the previous phase. The current value Hamiltonian can be written as

$$H = p_2 y_2 - (c_0 + c_1 y_2) y_2 + \lambda_2 y_2 \left[a_0 - a_1 p_2 / 2 - a_1 (c_0 + c_1 y_{1t_1}) / 2 - y_{1t_1} - y_2 \right].$$

First-order conditions for a singular solution phase are: the differential equation, $\lim_{t \rightarrow \infty} e^{-\rho t} = 0$ and

$$\delta H / \delta p_2 = y_2 + \lambda_2 \beta y_2 (-a_1 / 2) = 0 \tag{16}$$

and

$$-\delta H / \delta y_2 = -\{p_2 - c_0 - c_1 2y_2 + \lambda_2 \beta [a_0 - a_1 p_2 / 2 - a_1 (c_0 + c_{sub1} y_{1t_1}) / 2 - y_{1t_1} - 2y_2]\} = \lambda_2 - \rho \lambda_2. \tag{17}$$

From these first order conditions one can derive the level of the market quantity y :

$$y_2 = \frac{a_0 - a_1 c_0 - \rho / \beta}{2(\frac{a_1 c_1}{2} + 1)} - \frac{y_{1t_1}}{2} \tag{18}$$

The first term is the value of y_2 at the end of the zero-profit line of firm 1 (of the differential game phase), i.e. for $\pi_1 = 0$ and $y_1 = 0$ in the y_1 - y_2 -plane (see Figure 9). In that plane eq. (18) describes a straight line with slope minus 2 through $y_{2|_{\pi_1=0, y_1=0}}$. Therefore it is steeper than all zero-profit lines of the differential game phase. Since y_{1t_1} is a constant and the line determined by eq. (18) lies to the right of the zero-profit lines of the previous phase, firm 2 moves from the zero profit line of firm 1 parallel to the horizontal axis to the line of eq. (18), thus extending its market share in the Stackelberg phase because firm 1 sells less than it's market reputation would allow it to, because it has changed its strategy in order to avoid running into negative profits, whereas the behaviour of firm 2 must have made both products better known during the transition to eq. (18), to which we turn now.

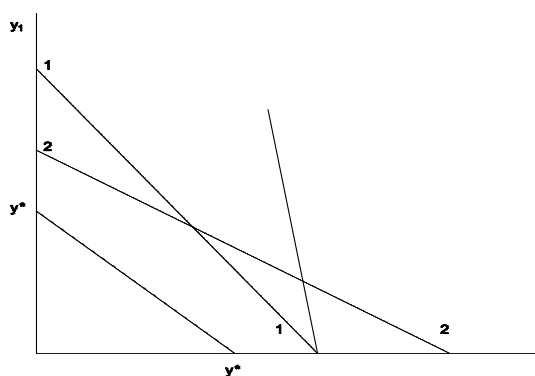


Figure 9

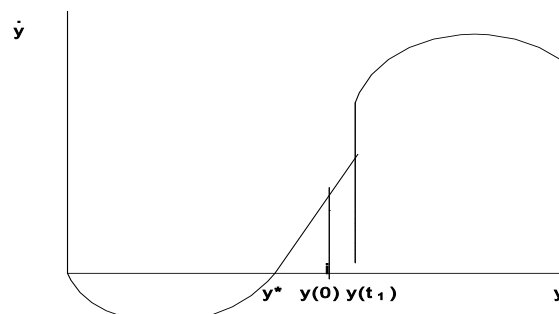


Figure 10

For a transition to this singular solution phase of the monopoly for the whole market one would have to define lower and upper bounds for prices of firm 2 again. Here we assume that the lower bound is the zero-profit condition. However, when the price jumps down at t_1 from its positive profit level for one of the oligopolists (firm 2 in our example) there must be a jump in dy/dt . The curve of Figure 5 stops at $y(t_1)$. From that time onwards the curve of Figure 2 is valid. Its value of $dy/dt(t_1^+)$ - where t_1^+ is the first moment of the Stackelberg phase - must be higher than the value $dy/dt(t_1)$

of the differential game phase. There will be a jump to a different diffusion curve when p_2 is raised such that the growth stops at a value of D belonging to the monopoly price. The complete diffusion curve of the whole game is drawn in Figure 10. A similar curve could be derived for the case that firm 1 exits and firm 2 has a monopoly position instead of one of a Stackelberg leadership.

Whereas in Fershtman, Mahajan and Muller (1990) pioneering advantage is defined with respect to market shares of a pioneer and a competitor that turn out to be equal in the long run in their model, here market shares are constant during the differential game phase due to the dynamic rigidity. Pioneering advantage leads to exit or weak preference for staying in with zero profits of the leader if $c_1 < 0$, allowing for Stackelberg-leadership or monopoly profits in a later phase for the one who was lagging behind in terms of market share in the beginning. Only in the case of an increasing unit cost curve will the market share "advantage" of the leader not be lost, because in the equations for prices (i.e., eqs. (11") and (12")) and in contrast to the properties of the demand functions D_1 and D_2) cross effects of quantities on competitors prices are larger in that case than those on a firm's own prices. If unit cost curves are falling the quantities of a firm have a higher influence on its own price. Then higher initial quantities (pioneer advantage) lead the pioneer to his zero-profit line, Stackelberg-followership or exit (optimal timing of withdrawing a product from the market). The Stackelberg leader then extends his market share. This phase is initiated by a downward jump of the price. Another example of a downward jump of the price can be found in Eliashberg and Jeuland (1986), where a monopolist decreases his price at the moment of entry of a competitor.

7. Conclusions

In this paper the model used by Metcalfe, Batten and Amable and in the marketing literature has been used to derive the dynamics of output and price under different behavioral assumptions than in their papers. It seems to me that the imposition of the epidemic diffusion assumption as a technological constraint on information flows is an interesting approach to dynamic imperfectly competitive disequilibrium behaviour, although I would not go as far as Batten (1987), who views this model as competing with the neoclassical growth model. The latter is a general equilibrium model whereas the Metcalfe and marketing models are partial (dis)equilibrium models. In the following we summarize the results obtained with this approach.

This paper has started from Jorgensen's (1983) version of the monopoly model. In that version there is an optimal introductory price and a jump to a long run price that is higher than that of a static monopoly solution under a positive discount rate. This result differs from the permanent, continuous price differentiation obtained by Glaister (1974), Metcalfe (1981) and others; disequilibrium growth is found in the introduction phase of the product but not in the phase of a singular solution.

Allowing for competition one has to make an assumption on whether or not the dynamics of the product becoming known is associated with the name of the firm. If it is not, firms compete under perfect information for the number of consumers knowing the product. Under mutually perfect information a zero-profit strategy is the only equilibrium strategy because for higher prices a competitor can always take over the whole market. As a consequence the two-prices strategy of the monopoly version of the model is replaced by a continuous price strategy. Before the diffusion process has proceeded sufficiently the firm will not be on the estimated and possibly true demand curve, and therefore the economy will be in disequilibrium.

If the product becomes known and the consumers associate the name of the firm with the product and do not know of the other firm's supply, then substitutes are imperfect. In a differential game with dynamic rigidity higher prices lead to higher instantaneous profits but slow down the diffusion. If the initial values are below a critical minimum level the quantities of both firms will decline towards zero and prices will rise. If the initial values are above a critical minimum level the quantities will grow and prices will decline. Under decreasing (increasing) cost curves the leader (follower) arrives at his zero-profit line when the other firm still makes positive profits. If it switches to a zero profit strategy the other firm becomes Stackelberg leader, and if it exits the other firm gets a monopoly position.

In the latter case we are back in the slightly extended Jorgensen model. In short, epidemic diffusion dynamics combined with a demand and a cost function can be used in models that are imperfectly competitive, and can also be used to justify dynamic monopolistic (dis)equilibrium or Stackelberg leadership, both possibly resulting from a duopolistic differential game.

An alternative version that we shall investigate in a further paper will use individual market potentials $D_i = a_i - b_i p_i + b_j p_j$ with $b_{ij} > 0$ and individual diffusion curves $\dot{y}_i = \beta y_i (D_i - y_i)$. In that case the dynamic rigidity will vanish and there will be competition for market potential. The main problem will presumably be the tractability of the differential game. Advertising may be added to the model by making β dependent on the expenditures for advertising. This would avoid adding additional differential equations for goodwill.

The purely empirical literature as far as it is based on the epidemic model considers the sigmoid or similar diffusion patterns which can be generated formally by the models mentioned above. In the models, however, diffusion is driven by firms' price policy, investment or output decisions. The models using price policy exhibit a great variety of possible policies. They are all based on continuous intertemporal price differentiation. To get a better impression of how all of this works it is necessary to enlarge empirical research to include the price policy and changes in market structure accompanying the diffusion process. A crucial empirical question will be how the price paths are correlated with the diffusion paths and perhaps those of other variables.

This paper has looked with somewhat neoclassical eyes at the literature connecting the epidemic diffusion model with dynamic firm behaviour and therefore ended up in marketing science, where this model is most prominent. It is clear that evolutionary economists will find this effort to be too neoclassical and will doubt the availability of the necessary information, and that neoclassical economists will shift emphasis to even better information emphasizing advertising and search costs of customers. However, I share the marketing literature's view that all of these measures may improve information but will not change the character of the basic problem because advertising and search costs will increase β and will therefore increase the term in our model, but will not necessarily change results radically. However, the models might become more complicated. We hope that there are more economists outside the marketing field who like parts of all the three directions of research and are convinced that the epidemic diffusion model has something to offer to neoclassical economics and deserves the effort of extension to general equilibrium modelling which might make clear what are the exact differences between the two directions of research.

References

- Amable, B. (1992), "Competition among Techniques in the Presence of Increasing Returns to Scale", *Journal of Evolutionary Economics*, 2, 147-158.
- Batten, D. (1987), "The Balanced Path of Economic Development: a Fable for Growth Merchants", in Batten, D., J. Casti and B. Johansson (eds.), *Economic Evolution and Economic Adjustment*, Heidelberg: Springer-Verlag.
- Chatterjee, R. and Eliashberg, J. (1990), "The Innovation Diffusion Process in a Heterogeneous Population: A Micromodelling Approach", *Management Science*, 36, 1057-1079.
- Coombs, R., Saviotti, P. and Walsh, V. (1987), *Economics and Technical Change*, London: MacMillan.
- Dockner, E. and Jorgensen, S. (1988), "Optimal Pricing Strategies for New Products in Dynamic Oligopolies", *Marketing Science*, 7, 315-334.
- Dosi, G., Freeman, C., Nelson, R.R., Silverberg, G. and Soete, L. (1988), *Technical Change and Economic Theory*, London: Pinter.
- Eliashberg, J. and Jeuland, A.P. (1986), "The Impact of Competitive Entry in a Developing Market upon Dynamic pricing Strategies", *Marketing Science*, 5, 20-36.
- Feichtinger, G. (1982), "Optimal Pricing in a Diffusion Model with Concave Price-Dependent Market Potential", *Operations Research Letters*, 1, 236-240.
- Felmingham, B.S. (1988), "Intra-firm Diffusion and the Wage Bargain", *Economics Letters*, 26, 89-93.
- Fershtman, C., Mahajan, V. and Muller, E. (1990), "Market Share Pioneering Advantage: A Theoretical Approach", *Management Science*, 36, 900-918.
- Glaister, S. (1974), "Advertising Policy and Returns to Scale in Markets where Information is passed between Individuals", *Economica*, 41, 139-156.
- Jorgensen, S. (1983), "Optimal Control of a Diffusion Model of New Product Acceptance with Price-Dependent Total Market Potential", *Optimal Control Applications & Methods*, 4, 269-276.
- Karshenas, M. and Stoneman, P. (1992), "A Flexible Model of Technology Diffusion Incorporating Economic Factors with an Application to the Spread of Colour Television Ownership in the UK", *Journal of Forecasting*, 11, 577-601.
- Mahajan, V., Muller, E. and Bass, F.M. (1990), "New Product Diffusion Models in Marketing: A Review and Directions for Research", *Journal of Marketing*, 54, 1-26. Reprinted in N. Nakicenovic and A. Grübler (eds.) (1991), *Diffusion of Technologies and Social Behaviour*, Heidelberg: Springer-Verlag.
- Metcalfe, J.S. (1981), "Impulse and Diffusion in the Study of Technical Change", *Futures*, 13, 347-359.
- Metcalfe, J.S. (1988), "The Diffusion of Innovations: an Interpretative Survey", in Dosi, G., Freeman, C., Nelson, R.R., Silverberg, G. and L. Soete (eds.), *Technical Change and Economic Theory*, London: Pinter.
- Norton, J.A. and Bass, F.M. (1992), "Evolution of Technological Generations: The Law of Capture", *Sloan Management Review*, 66-77.
- Oren, S.S. and Schwartz, R.G. (1988), "Diffusion of New Products in Risk-sensitive Markets", *Journal of Forecasting*, 7, 273-287.
- Rao, C.R. and Bass, F.M. (1985), "Competition, Strategy, and Price Dynamics: A Theoretical and Empirical Investigation", *Journal of Marketing Research*, XXII, 283-96.
- Reinganum, J.F. (1989), "The Timing of Innovation: Research, Development, and Diffusion", in R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organization*, 1, Amsterdam: Elsevier.

- Robinson, B. and Lakhani, C. (1975), "Dynamic Price Models for New-Product Planning", *Management Science*, 21, 1113-1122.
- Rosegger, G. (1986), *The Economics of Production and Innovation*, London: Pergamon Press.
- Silverberg, G., Dosi, G. and Orsenigo, L. (1988), "Innovation, Diversity and Diffusion: A Self-Organisation Model", *The Economic Journal*, 98, 1032-1054.
- Soete, L. and Turner, R. (1984), "Technology Diffusion and the Rate of Technical Change", *The Economic Journal*, 94, 612-23.
- Spremann, K. (1975), "Optimale Preispolitik bei dynamischen deterministischen Absatzmodellen", *Zeitschrift für Nationalökonomie*, 35, 63-76.
- Stoneman, P. (1983), *The Economic Analysis of Technical Change*, Oxford: Oxford University Press.
- Stoneman, P. and Ireland, N.J. (1983), "The Role of Supply Factors in the Diffusion of New Process Technology", *The Economic Journal*, 66-78.
- Thirtle, C.G. and Ruttan, V.W. (1987), *The Role of Supply and Demand in the Generation and Diffusion of Technical Change*, London: Harwood Academic Publishers.