Offshoring of Medium-skill Jobs, Polarization, and Productivity Effect: Implications for Wages and Low-skill Unemployment

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Offshoring of Medium-skill Jobs, Polarization, and Productivity Effect: Implications for Wages and Low-skill Unemployment*

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Abstract

We examine the effects of endogenous offshoring on cost-efficiency, wages and unemployment in a task-assignment model with skill heterogeneity. Exact conditions for the following insights are derived. The distributional effect of offshoring (high-) low-skill-intensive tasks is similar to (unskilled-) skill-biased technology changes, while offshoring medium-skill-intensive tasks induces wage polarization. Offshoring improves cost-efficiency through international task reallocation and puts a downward pressure on all wages through domestic skill-task reallocation. If elasticities of task substitution are low (high), the downward pressure on wages in neighboring skill segments is low (high) with a net effect of higher (lower) wages and employment.

Keywords Task Assignment · Offshoring · Skills · Cost-efficiency Effect · Equilibrium Unemployment

JEL F16 · F66 · J21 · J24 · J64

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1 Introduction

One key feature of recent globalization trends is the growing phenomenon of international reorganization of production and work processes, or, put differently, offshoring of jobs, which has heightened concerns regarding job and wage cuts in many advanced countries (cf. Bhagwati et al., 2004; Snower et al., 2009).[1] While earlier studies have highlighted the labor market impact of international fragmentation of the value added chain, captured by the increasing penetration of intermediate goods (Feenstra and Hanson, 1996, 1999; Jones and Kierzkowski, 1990, 2001; Kohler, 2004a,b), recent observations accentuate the important role of job characteristics and task content of occupations (cf. Blinder, 2009a,b). To put it in the words of Blinder (2009b, p.54), "...this time it's not the British who are coming, but the Indians...neither by land nor by sea, but electronically".

More precisely, the recent development in employment and wages depicts a polarizing trend in many advanced countries, indicating a deflection of global competition towards the medium-skilled workforce (Acemoglu and Autor, 2011).[2] The rationale for this new trend is, on the one hand, advances in information and communication technology (ICT), as well as declines in trade transaction and transportation costs of goods and services that have accelerated the integration process of national markets into a global market. On the other hand, an important factor is rapid growth in major emerging countries, such as Brazil, Russia, India, and China (BRIC). These countries are characterized by high accumulation of human-capital and advanced technologies as well as improvements in the economic and business infrastructure that make them highly competitive in areas such as information technology services in which the advanced countries have been dominant (Bhagwati et al., 2004; Snower et al., 2009; Spence, 2011).[3] Both developments have reduced the locational viability of some occupations. That is, jobs that have a high content of routine, non-interactive, and non-cognitive tasks are most likely at peril. The rationale is that these routine-intensive job tasks can be easily codified, enabling firms in many advanced countries to reorganize production and work processes by decomposing the various stages of production geographically into clusters of tasks and locating each task cluster in the countries where it is most profitable (Snower et al., 2009). Therefore the comparative advantage of performing a specific type of job tasks has become important.

In the recent literature on offshoring job tasks, two main forces have received particular attention, introduced by Grossman and Rossi-Hansberg's (2008) trade-in-tasks approach.[4] While allocating jobs abroad (offshoring) induces a direct *displacement effect* of domestic workers, leading to lower wages, offshoring activities may generate a *productivity effect* similar to technology improvement by lowering a firm's production cost. This productivity effect, in turn, will lead to an expansion of output and thus raise employment and wages. The balance between these two forces will determine the direction of the wage and employment ef-

[1] Blinder (2009a) estimates that 30 million to 40 million jobs in the USA are potentially offshorable, while job tasks that require face-to-face contact as well as abstract and cognitive skills are protected. See also the studies by Jensen and Kletzer (2010) and Moncarz et al. (2008) regarding offshorability of service occupations. For example, Moncarz et al. (2008) identify the offshorability of 160 service occupations, where the range of occupations includes scientists, mathematicians, radiologists and editors at the high end of the market as well as those of telephone operators, clerks and typists at the low end.

[2] For recent empirical evidence regarding the polarization effect in the US labor market see Autor et al. (2003); Autor and Dorn (2009, 2013); Autor et al. (2006, 2008); Firpo et al. (2011); Michaels et al. (2014); and in the European labor markets Baumgarten et al. (2013); Dustmann et al. (2009); Goos and Manning (2007); Goos et al. (2009, 2014); Spitz-Oener (2006).

[3] More specifically, Bhagwati et al. (2004) highlight that the stock of skilled workers in India and China will reach about 300 million in a few decades. Spence (2011) provides a critical discussion on globalization and labor market effects for the USA.

[4] A third channel, as put forward in Grossman and Rossi-Hansberg (2008), is via the terms-of-trade effect that may wipe out the productivity effect. However, see Bhagwati et al. (2004) for a discussion regarding the empirical insignificance of terms-of-trade effects of offshoring.

fect of offshoring (Ebenstein et al., 2014; Becker et al., 2013; Baumgarten et al., 2013; Harrison and McMillan, 2011; Hummels et al., 2014; Olney, 2012; Ottaviano et al., 2013; Wright, 2014).

However, there exists another important mechanism that shapes substantially the labor market outcomes of offshoring: the spillover effects on other skill groups induced by job tasks mobility of offshoring-induced displaced workers. This channel is omitted in the existing theoretical literature on offshoring and task allocation. Our first contribution is, therefore, to address the two hypotheses regarding polarization and productivity effects in a more general framework that accounts explicitly for several important features: task-skill heterogeneity, endogenous offshoring, and spillover effects due to mobility across job tasks. In doing so, we build on recent important contributions that incorporate the explicit assignment of skills to tasks.[5] In particular, we provide a theoretical framework that augments recent contributions by Acemoglu and Autor (2011), Grossman and Rossi-Hansberg (2008), and Ottaviano et al. (2013) in two ways. First, we allow for endogenous specialization of different skill groups into a continuum of tasks. The implied matching between tasks and skills allows for task competition among different skill groups and thus enables us to jointly investigate changes in the task composition performed by different skills in the economy due to exogenous offshoring shocks, a feature that is absent in Grossman and Rossi-Hansberg (2008) and Ottaviano et al. (2013).[6] Second, and contrary to Acemoglu and Autor (2011), who consider offshoring exogenously, we analyze the offshoring decision by domestic firms as an endogenous process, capturing changes at both the intensive margin, i.e. the range of offshorable tasks, as well as the extensive margin, i.e. the offshorability of a marginal domestic task.

Thus our theoretical framework accounts for all these different mechanisms, which have been addressed separately in the previous literature. Particularly, several important insights can be gained from our approach. First, we show that any offshoring scenario of domestic job tasks can be described by a non-monotonic, U-shaped task productivity schedule. This permits us to capture various phases of international competition and to address their distributional effects for the domestic workforce. As we elaborate below, if offshoring activities are limited to low-skilled job tasks – depicting globalization trends in the past (cf. Snower et al., 2009) – then lower offshoring frictions generate a distributional effect similar to *skill*-biased technology changes. If, on the other hand, offshoring is limited to high-skilled job tasks, then easier offshoring induces wage impacts similar to *unskill*-biased technology changes.

Second, we capture three main channels characterizing the recent phase of globalization: i) accumulation of advanced technologies abroad, i.e. a *Ricardian effect*, ii) accumulation of human capital abroad, i.e. a *Heckscher–Ohlin effect*, and iii) decline in transport barriers, i.e. a *trade cost effect*. Although qualitatively they generate similar effects, accounting explicitly for them permits us to address not only changes in the nature of North–South trade, i.e. trade in goods and services between “rich” and “poor” countries, where trade barriers are still substantial, but also the implications of market integration, such as in the context of the enlargement of the European Union towards Central, Eastern, and South-Eastern European countries since 2004, where trade costs are effectively null, but changes in comparative advantages are characterized by a rapid accumulation of advanced technologies and human capital in these regions.

[5]See Acemoglu and Autor (2011) for an elaborate discussion on the limitation of the standard nested CES (so-called “canonical”) model compared to the “task-assignment” approach. For the alternative task-skill-assignment approach, see also Autor and Dorn (2013); Costinot and Vogel (2010); Dupuy (2012); Sly (2012).

[6]Notice the important difference to Ottaviano et al. (2013), who assess the task allocation between immigrant, offshore and native workers, though each factor is homogeneous in terms of skills. Our framework, instead, could be easily extended to incorporate task competition between immigrants and natives, e.g. by a CES decomposition of factor labor per task.

Another new insight, gained from the general equilibrium analysis, is that the cost-efficiency effect induced by easier offshoring depends now on the magnitude of the spillover effects. On the one hand, easier offshoring induces a reallocation of tasks performed by medium-skilled workers to offshore workers. This *external* reallocation is the main source of the productivity effect due to lower offshoring cost. There is, on the other hand, a offshoring-induced spillover effect on other domestic skill groups, which we refer to as the domestic reallocation of workers, i.e. from medium-skilled to low- and high-skilled job tasks. This *internal* reallocation countervails the cost-efficiency effect induced by the external reallocation. Moreover, our analysis reveals that the difference between these two forces depends crucially on the elasticity of substitution between skill groups at the respective extensive task margins.

The importance of this internal reallocation for the labor market impact of offshoring has been put forward in several recent empirical studies (Baumgarten et al., 2013; Ebenstein et al., 2014; Hummels et al., 2014). In a nutshell, the empirical evidence suggests that switching occupation is costly for offshoring-induced displaced workers. Using matched worker-firm data from Denmark, Hummels et al. (2014) find that offshoring increases the skill premium within firms, i.e. the relative wage of skilled workers, and that the downward wage pressure is more pronounced in occupations that involve routine tasks. However, by allowing for labor mobility across occupations, they find that the cohort-average wage loss (i.e. of workers who leave the firm, and those who stay) is exacerbated for both low- and high-skilled workers. The authors relate the latter outcome to losses in specific human capital and search cost that considerably hinder the reattachment to the labor market for the offshoring-induced displaced workers. Ebenstein et al. (2014) investigate the impact of trade and offshoring on wages for the USA. The empirical findings show that import penetration and offshoring induce a downward pressure for workers performing routine intensive occupations, while export activities have a positive impact. Moreover, the empirical evidence emphasizes that the negative wage effect becomes substantial once occupation-sector mobility of workers is taken into account, suggesting the important role of occupation-specific human capital. Using data for Germany, Baumgarten et al. (2013) find a substantial negative cross-industry wage pattern due to offshoring in occupations with a high routine task content relative to interactivity and non-routine content of occupational tasks, for both low-skilled and high-skilled workers. Our framework contributes also to the empirical literature by providing structural guidance on the occupational mobility of displaced workers. As we discuss below, the degree of task substitutability between different skill groups is the critical parameter that accounts for the magnitude of internal reallocation.

Another particular debate on offshoring is the displacement effect of the least skilled workers from the labor market. We address this concern by assuming equilibrium unemployment of low-skilled workers in the economy. To keep the framework tractable, we allow for two types of labor market friction. The first source is given by a minimum wage scheme that is set above the market-clearing wage rate and thus leads to unemployment. In a second step, we allow for endogenous supply of low-skilled labor services. While the former – reflecting a perfectly elastic labor supply curve – corresponds to the mirror image of the full employment case, the latter facilitates a more general notion of how the labor market adjusts to offshoring shocks by allowing an elastic labor supply curve. In this second step we derive clear conditions under which the offshoring-induced spillover effects translate into higher low-skilled employment. This is the second contribution to the literature on offshoring tasks.[7]

[7] It is worth mentioning the study by Kohler and Wrona (2011), who introduce equilibrium unemployment following the search and

The rest of the paper is organized as follows. In the next section, we present the task assignment model. In section 3, we describe the production technology and derive the solution to the firm's optimization problem. Section 4 provides the general equilibrium solution followed by the comparative statics in section 5. The implications of offshoring and spillover effects on low-skill unemployment are discussed in section 6. Finally, section 7 offers concluding remarks.

2 Task Assignment

In this section, we discuss the main properties of the task-based framework introduced by Acemoglu and Autor (2011) and Grossman and Rossi-Hansberg (2008). As will be seen below, the factor labor loses its static property and is now assigned to a continuum of tasks based on a Ricardian type of comparative advantage.

Consider an economy where different groups of workers perform a job task. The range of the existing tasks is defined over a unit interval. More specifically, a task i can be performed in different modes: by domestic low- (L), medium- (M), and high-skilled (H) labor, as well as by foreign, offshored workers (O). Formally, the production function for a task is written as

$$t(i) = A_L a_L(i) l_L(i) + A_M a_M(i) l_M(i) + A_H a_H(i) l_H(i) + A_O a_O(i) l_O(i)/\tau, \;\; i \in [0,1], \tag{1}$$

where A denotes the factor-augmenting technology and $a(i)$ indicates the job-task productivity schedule of the respective labor type. The "iceberg" type offshoring friction is captured by the parameter $\tau \geq 1$. The number of low-, medium-, high-skilled and offshore workers performing a job-task i is denoted by $l_k(i),\ k = \{L, M, H, O\}$.

Below we derive the optimal allocation between the three domestic skill groups as well as the offshore workers. While the single good producer solves simultaneously the optimal assignment of tasks to different labor groups, we demonstrate this procedure sequentially to make the optimal decision behavior better understandable.

2.1 Allocation of Domestic Tasks

We assume that, over the unit interval, tasks are ordered according to the skill requirement in a monotonic way. For example, one can think of this order as manual (e.g. restoring houses and servicing), routine (e.g. bookkeeping and running a machine), and cognitive (e.g. research and management activities) tasks.

Assumption 1. *(Domestic Task Productivity)*
For all $i \in [0,1]$, $\frac{a_L(i)}{a_M(i)}$ and $\frac{a_M(i)}{a_H(i)}$ are strict monotonically decreasing in i, and by transitivity property $\frac{a_L(i)}{a_H(i)}$ is also strict monotonically decreasing in i.

Since the firm allocates tasks to the group that is the most cost-efficient in performing those tasks, this assumption allows us to determine the equilibrium allocation of tasks among the domestic workers. Let the unit cost of producing task i domestically be $c_k(i) = \frac{w_k}{A_k a_k(i)}, k = \{L, M, H\}$ where w_k denotes the wage costs,

matching theory and the efficiency wage theory into the original task framework of Grossman and Rossi-Hansberg (2008), characterized by a single sector and homogeneous factor labor. However, our objective is neither to compare different paradigms of labor market friction nor to discuss their different adjustment mechanisms. Thus we deliberately avoid any unnecessary complexity of the model.

and $c_L(0) < c_M(0) < c_H(0)$. Denoting relative task-productivity by $\beta_L(i) = \frac{a_L(i)}{a_M(i)}$ and $\beta_H(i) = \frac{a_M(i)}{a_H(i)}$, then, in equilibrium, it follows

Lemma 1 (Domestic Task Allocation)**.** *By Assumption 1, there exist task margins I_L, I_H, and $\tilde{I}$, respectively, where a representative domestic firm is indifferent between*

- *Low- and medium-skilled workers*
$$\frac{w_L}{A_L}\big(\beta_L(I_L)\big)^{-1} = \frac{w_M}{A_M} \tag{2}$$
- *Medium- and high-skilled workers*
$$\frac{w_H}{A_H}\beta_H(I_H) = \frac{w_M}{A_M} \tag{3}$$
- *Low- and high-skilled workers*
$$\frac{w_L}{A_L}\big(\beta_L(\tilde{I})\big)^{-1} = \beta_H(\tilde{I})\frac{w_H}{A_H}. \tag{4}$$

Proof. See Appendix C.1.

Thus, by Lemma 1, the domestic allocation of tasks to skill groups is characterized by the two endogenous thresholds, I_L and I_H. Moreover, Eqs. (2) and (3) determine the degree of substitutability between the domestic skill groups. Put differently, the reallocation of medium-skilled workers to low- and high-skilled intensive job tasks depends on their comparative advantages in the neighborhood of I_L and I_H relative to low-skilled and high-skilled workers, which is characterized by $\beta_L(i)$ and $\beta_H(i)$. More formally,

$$\frac{\mathrm{d}I_L}{\mathrm{d}\ln(w_M/w_L)} = \frac{1}{\varepsilon_L} \tag{2$'$}$$

$$\frac{\mathrm{d}I_H}{\mathrm{d}\ln(w_H/w_M)} = \frac{1}{\varepsilon_H}, \tag{3$'$}$$

where $\varepsilon_L = -\frac{\partial \ln \beta_L(I_L)}{\partial I_L} > 0$ and $\varepsilon_H = -\frac{\partial \ln \beta_H(I_H)}{\partial I_H} > 0$ denote the semi-elasticities at the respective equilibrium extensive margins. Thus, higher values of ε_L and ε_H denote a relative high comparative advantage of medium-skilled workers at the respective task margin, which in turn implies a low substitutability. As pointed our by Acemoglu and Autor (2011), this indicates an additional source of substitution effect, i.e. the substitutability of skills across tasks.

A corollary follows from Lemma 1

Corollary 1. *For all $\frac{w_L}{A_L}\big(\beta_L(\tilde{I})\big)^{-1} = \frac{w_H}{A_H}\beta_H(\tilde{I}) > \frac{w_M}{A_M} > \max\{\frac{w_L}{A_L}\beta_L(0)^{-1}, \frac{w_H}{A_H}\beta_H(1)\}$, it follows that $0 < I_L < \tilde{I} < I_H < 1$.*

This is a necessary and sufficient condition permitting the existence of all three skill groups in the economy. The lower boundary indicates that low-skilled workers are the most efficient ones at the least skill-intensive task $i = 0$ and high-skilled workers are the most efficient ones at the most skill-intensive task $i = 1$. In addition, the upper boundary ensures that medium-skilled workers have comparative advantages in the middle range of the task distribution. For example, if $\frac{w_L}{A_L}\beta_L(\tilde{I})^{-1} = \frac{w_H}{A_H}\beta_H(\tilde{I}) \leq \frac{w_M}{A_M}$, then employment consists of only low- and high-skilled workers.[8] Medium-skilled workers have then no comparative advantage

[8] Notice that at strict equality the employer is indifferent between all three skill groups at margin $\tilde{I}$.

in performing any task relative to low- and high-skilled workers. To sum up, by Lemma 1 and Corollary 1, the domestic labor force is allocated over the unit interval as follows: low-skilled workers are employed in the interval $i \in [0, I_L]$, medium-skilled in $i \in (I_L, I_H)$, and high-skilled in $i \in [I_H, 1]$. We depict the equilibrium task allocation in Figure 1.

2.2 Allocation of Offshored Tasks

As discussed in the introduction, the empirical evidence shows that over recent decades there has been a strong tendency to reallocate domestic tasks abroad. This process particularly applies to jobs intensive in routine tasks and concentrated in the middle range of the skill distribution. This in turn may explain the recent trends in wage polarization in many advanced countries. Based on this pattern, Acemoglu and Autor (2011) investigate the distributional effect of offshoring by allowing a sub-range of domestic tasks, which were previously performed by medium-skilled workers, to be exogenously offshored.

However, this ad-hoc introduction of offshoring neglects to address another important aspect discussed in the literature: the offshoring-induced productivity effect. In order to address this effect, we need to account not only for changes at the extensive margin, as in Acemoglu and Autor (2011), but also for changes at the intensive margin. The intuition is that easier offshoring, e.g. due to lower trade cost, will not only induce more domestic job tasks to be offshored (extensive margin), but will additionally have implications for all job tasks that have already been offshored (intensive margin). Therefore it may affect overall production costs.

We augment the general task-based framework of Acemoglu and Autor (2011) by allowing domestic firms to choose endogenously the cost-optimal range of tasks to be offshored. As we discuss below, this in turn requires a non-monotonic comparative advantage schedule between domestic medium-skilled and offshore workers. More precisely, the relative task-productivity schedule becomes a U- (or V-)shaped relationship. Moreover, as we discuss below, any scenario of offshoring can be described by this U-shaped functional form. Formally, we impose the following assumption

Assumption 2. *(Offshoring Task Productivity)*

Defining the job-task productivity between medium-skilled and offshore workers by $\zeta(i) = \frac{a_M(i)}{a_O(i)}$, there exists a threshold $\check{I}$ such that for all $i \in [0, \check{I})$, $\zeta(i)$ is (strict) monotonically decreasing, and for all $i \in (\check{I}, 1]$, $\zeta(i)$ is (strict) monotonically increasing.

Let the offshoring wage rate be given by w_O. Then, a domestic firm engaged in offshoring chooses the optimal amount of offshore workers, given the wage rate w_O, according to the following unit cost of producing task i abroad: $c_O(i) = \tau \frac{w_O}{A_O a_O(i)}$. The term $\frac{\tau w_O}{A_O}$ is exogenous to the domestic firm and comprises the aforementioned three channels of globalization process, a feature that is missing in Acemoglu and Autor (2011). For example, a decline in w_O is associated with skill accumulation abroad, indicating the *Heckscher–Ohlin effect*, an increase in A_O denotes advances in foreign technology, the *Ricardian effect*, and a decline in τ indicates lower transportation barriers, the *trade cost effect*. Summarizing all these exogenous channels by $\omega \equiv \frac{A_O}{\tau w_O}$, lower offshoring friction is now captured by $\mathrm{d}\omega > 0$. Although each channel generates a similar qualitative effect, accounting explicitly for each of them has the following advantages. First, it allows us to address the implication of various forms of globalization, e.g. North–South vs. East–West type of trade and offshoring activities. Second, it enables us to provide clear policy guidance regarding how the source country of offshoring could react to the increasing global competition on domestic jobs, e.g. if lower offshoring friction is due to $\mathrm{d}w_O < 0$, then one possible response could be to increase the trading costs $\mathrm{d}\tau > 0$; if it is due to $\mathrm{d}A_O > 0$, then the source economy could possibly invest more in R &D to increase its technology frontier on those job tasks (i.e. $\mathrm{d}A_M > 0$). The optimal task allocation between foreign and domestic medium-skilled workers is summarized as follows.

Lemma 2. *By Assumption 2, there exist two task margins where a firm is indifferent between the domestic medium-skilled and offshore workers, i.e.*

$$\frac{\zeta(I_1)}{\omega} = \frac{w_M}{A_M} \tag{5}$$

$$\frac{\zeta(I_2)}{\omega} = \frac{w_M}{A_M}. \tag{6}$$

Proof. See Appendix C.2.

Thus, by Lemma 2 the effect of offshoring is captured by the two endogenous offshoring boundaries, I_1 and I_2. They capture both the extensive margin, i.e. the offshorability of the marginal domestic task, as well as the intensive margin, i.e. the amount of tasks that have been already offshored. However, it is useful and sufficient to consider changes to the length of offshoring interval, i.e.

$$I_O = I_2 - I_1,$$

which accounts implicitly for two conditions, (5) and (6). This has analytical convenience. Let the semi-elasticities at the two offshoring margins be constant and defined as $\varepsilon_1 = -\frac{\partial \ln \zeta_1(I_1)}{\partial I_1} > 0$ and $\varepsilon_2 = \frac{\partial \ln \zeta_2(I_2)}{\partial I_2} > 0$. Then, by a simple positive monotone transformation, we obtain

Lemma 3. *If Lemma 2 holds, then the two no-arbitrage conditions determining the task reallocation between medium-skilled and offshore workers can be expressed as one monotonically increasing function in the length of the offshorable task interval*

$$\frac{w_M}{A_M} = \frac{\beta_O(I_O)}{\omega}, \tag{7}$$

with $\beta_O(I_O) = \exp[\mu I_O]$ *and* $\mu = \frac{\varepsilon_2 \varepsilon_1}{\varepsilon_2 + \varepsilon_1} > 0$.

Proof. See Appendix C.3.

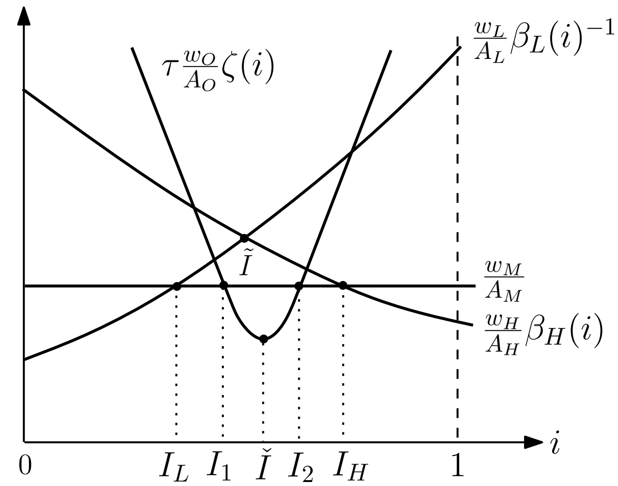
From Lemma 2 and Lemma 3 it follows

Corollary 2. *For all* ω *in the interval* $\frac{A_M}{w_M}\beta_O(0) < \omega < \{\frac{A_M}{w_M}\zeta(I_L)), \frac{A_M}{w_M}\zeta(I_H))\}$, *and* $I_L < \breve{I}|_{\frac{\partial \ln \zeta(\cdot)}{\partial i}=0} < I_H$, *it follows that* $I_O > 0$ *and* $I_O \in (I_L, I_H)$.

Thus Corollary 2 ensures that offshoring activities are permitted, but limited to a sub-range of medium-skilled job tasks. Put differently, if ω is sufficiently low such that $\omega = \frac{A_M}{w_M}\zeta(I_L)$ (or $\omega = \frac{A_M}{w_M}\zeta(I_H)$), then there is direct competition between foreign and domestic low-skilled (or high-skilled) workers.[9] Thus, for all four types of workers to exist in the economy, we assume that both Corollaries 1 and 2 hold. These conditions, as well as the equilibrium task allocations, are sketched in Figure 1.

[9] Notice that such an outcome – as pointed out by Blinder (2009a,b) – might not be unrealistic. However, the polarization effect would become ambiguous due to the direct competition of offshore workers with low-skilled (or high-skilled) workers next to medium-skilled workers.

Figure 1: Equilibrium task allocation



Summarizing the results above, the task production function emerges as

$$t(i) = \begin{cases} A_L a_L(i) l(i) & \text{for all } i \in [0, I_L] \\ A_M a_M(i) m(i) & \text{for all } i \in (I_L, I_H) \setminus I_O \\ A_O a_O(i) o(i)/\tau & \text{for all } i \in I_O \\ A_H a_H(i) h(i) & \text{for all } i \in [I_H, 1]. \end{cases} \tag{8}$$

Equation (8), therefore, reveals the new feature of our model. First, the range of offshorable tasks is determined endogenously. Second, the reallocation of domestic medium-skilled tasks and the size of the expansion of offshorable tasks is crucially determined by the relative productivity schedules in the neighborhood of I_1 and I_2, i.e. by the parameters ε_1 and ε_2. Below, we elaborate analytically on these features.

3 Production Technology and Labor Demand

The economy produces a final (consumption) good under perfect competition. The output is generated by using a labor composite, E, according to the following Cobb–Douglas technology function

$$Y = BE^{1-\alpha}, \;\; \alpha \in (0, 1), \tag{9}$$

where B is a positive parameter that may contain exogenous variables such as total factor productivity (TFP) and physical capital, and α denotes the standard share of physical capital. Thus the production function (9) has diminishing returns in labor input.[10] We consider Y as the numeraire, i.e. $P_Y = 1$. The labor composite input is, in turn, described by differentiated tasks, over a unit interval, according to the following Cobb–

[10] Notice that when $B = 1$ and $\alpha = 0$, equation (9) reduces to the one used by Acemoglu and Autor (2011).

Douglas technology:[11]

$$E = \exp\left(\int_0^1 \ln(t(i))\mathrm{d}i\right). \tag{10}$$

This specification allows us to deliver new insights regarding the offshoring-induced cost-efficiency effect.

3.1 Optimal Labor Demand

The firm's optimization problem is as follows. Taking the output price as given, the representative firm maximizes its profit by choosing the optimal amount of the labor composite factor. Formally, this is given by

$$\max_E \pi = Y - P_E E, \text{ s.t. } (9).$$

The solution yields the optimal total employment

$$E = P_E^{-\frac{1}{\alpha}} \mathcal{B}, \tag{11}$$

where $\mathcal{B} = ((1-\alpha)B)^{1/\alpha}$ and P_E denotes the price index of the labor composite, which will be defined below. The optimal amount of labor input per job task is determined by means of cost minimization. Formally, we write the optimal programming of the firm as follows:

$$\begin{aligned} C_E^*(w_L, w_M, w_H, w_O) \quad \equiv \quad & \min_{l_L(i), l_M(i), l_H(i), l_O(i), E} \left[w_L \int_0^{I_L} l_L(i)\mathrm{d}i + w_M \int_{i\in\mathcal{S_M}} l_M(i)\mathrm{d}i \right. \\ & \left. + w_O \int_{i\in I_O} l_O(i)\mathrm{d}i + w_H \int_{I_H}^1 l_H(i)\mathrm{d}i \right] \quad \text{s.t. } (8),\ (10), \end{aligned} \tag{12}$$

where the corresponding set of domestic medium-skill-intensive job tasks is denoted by $\mathcal{S_M} = (I_H - I_L - I_O)$.

The corresponding minimized cost function of the labor composite is given by:

$$C_E^*(\cdot) = \exp\left[\int_0^{I_L} \ln\left(\frac{w_L}{A_L a_L(i)}\right)\mathrm{d}i + \int_{i\in\mathcal{S_M}} \ln\left(\frac{w_M}{A_M a_M(i)}\right)\mathrm{d}i + \int_{i\in I_O} \ln\left(\frac{\tau w_O}{A_O a_O(i)}\right)\mathrm{d}i + \int_{I_H}^1 \ln\left(\frac{w_H}{A_H a_H(i)}\right)\mathrm{d}i\right] E. \tag{13}$$

Then, by Shepard's lemma, we obtain the optimal labor demand per task:[12]

$$\frac{\partial C_E^*(\cdot)}{\partial w_L} = \left(\frac{w_L}{P_E}\right)^{-1} E = l_L \tag{14a}$$

$$\frac{\partial C_E^*(\cdot)}{\partial w_M} = \left(\frac{w_M}{P_E}\right)^{-1} E = l_M \tag{14b}$$

$$\frac{\partial C_E^*(\cdot)}{\partial w_H} = \left(\frac{w_H}{P_E}\right)^{-1} E = l_H \tag{14c}$$

$$\frac{\partial C_E^*(\cdot)}{\partial w_O} = \left(\frac{w_O}{P_E}\right)^{-1} E = l_O, \tag{14d}$$

[11] Grossman and Rossi-Hansberg (2008) assume perfect complementarity, i.e. a Leontief production function. Ottaviano et al. (2013) assume a more general functional form by using a CES production technology.

[12] In Appendix D, we derive the minimized cost function $C^*(\cdot)$ and show that at the optimum the number of workers per task is constant across the respective range of tasks. This is particularly due to the Cobb–Douglas assumption of the production technology as well as the law of one price within each skill group. This implies that the marginal productivity within each skill-task group is constant, i.e. $l(i) = l$, $m(i) = m$, and so on. See Firpo et al. (2011) for a critical discussion regarding the assumption of law of one price.

where we used the fact that in perfect competitive equilibrium price equals marginal costs, i.e.

$$P_E = \exp\left[\int_0^{I_L} \ln\left(\frac{w_L}{A_L a_L(i)}\right) \mathrm{d}i + \int_{i\in\mathcal{S}_\mathcal{M}} \ln\left(\frac{w_M}{A_M a_M(i)}\right) \mathrm{d}i + \int_{i\in I_O} \ln\left(\frac{\tau w_O}{A_O a_O(i)}\right) \mathrm{d}i + \int_{I_H}^{1} \ln\left(\frac{w_H}{A_H a_H(i)}\right) \mathrm{d}i\right]. \tag{15}$$

The price index contains all three exogenous variables leading to changes in the offshoring friction: foreign technology (A_O), foreign wages (w_O), offshoring trade cost (τ). Recall that $\omega = \frac{A_O}{w_O \tau}$, then it can be shown that the partial effect of easier offshoring ($\mathrm{d}\omega > 0$) causes the marginal (average) cost of labor to decrease. That is,

$$\frac{\partial P_E}{\partial \omega} < 0. \tag{15$'$}$$

The impact of lower offshoring friction refers to what is called the offshoring-induced *cost-efficiency improvement* in the literature.[13] However, the general equilibrium implications (i.e. taking into account changes in the task margins) reveal a countervailing effect due to an internal (domestic) reallocation of workers, which is generally ignored in the literature. To account for this, Eq. (15) needs further manipulation. Utilize the cut-off conditions (2) and (3) from Lemma 1 to substitute for $\frac{w_L}{A_L}$ and $\frac{w_H}{A_H}$, respectively. Analogously, utilize the no-arbitrage condition (7) from Lemma 3 to substitute for $\frac{\tau w_O}{A_O}$. Then, P_E reduces to

$$P_E = \frac{w_M}{A_M} \exp\left[\int_0^{I_L} \ln\left(\frac{\beta_L(I_L)}{a_L(i)}\right) \mathrm{d}i - \int_{i\in\mathcal{S}_\mathcal{M}} \ln(a_M(i))\mathrm{d}i + \int_{i\in I_O} \ln\left(\frac{1}{\beta_O(I_O) a_O(i)}\right) \mathrm{d}i + \int_{I_H}^{1} \ln\left(\frac{1}{\beta_H(I_H) a_H(i)}\right) \mathrm{d}i\right].$$

Decompose the term $\int_{i\in\mathcal{S}_\mathcal{M}} \ln(a_M(i))\mathrm{d}i$ to combine it with the terms $\ln a_L(i)$, $\ln a_O(i)$, and $\ln a_H(i)$. Then, after some rearranging and manipulation, we obtain

$$P_E = \frac{w_M}{A_M}\Omega(I_L, I_H, I_O), \tag{16}$$

where[14]

$$\Omega(\cdot) = \exp\left[\left(\int_0^{I_L} \ln\left(\frac{\beta_L(I_L)}{\beta_L(i)}\right) \mathrm{d}i + \int_{I_H}^{1} \ln\left(\frac{\beta_H(i)}{\beta_H(I_H)}\right) \mathrm{d}i + \int_{i\in I_O} \ln\left(\frac{\zeta(i)}{\beta_O(I_O)}\right) \mathrm{d}i - \int_0^1 \ln(a_M(i))\mathrm{d}i\right)\right] \tag{17}$$

denotes the *generalized common part* of the marginal task cost. Thus this common part Ω accounts for the general equilibrium effect due to changes in the offshorability of domestic tasks. Contrary to the original version of Grossman and Rossi-Hansberg (2008), where Ω is simply defined as a function of one offshoring margin, easier offshoring does not necessarily induce a *cost-efficiency* effect. As we elaborate below, it de-

[13]It is worth mentioning that this effect is omitted in Acemoglu and Autor (2011) as the price index P_E is set to unity.

[14]More precisely, after substitution, the term within the square brackets becomes

$$\left[\int_0^{I_L} \ln\left(\frac{\beta_L(I_L)}{a_L(i)}\right) \mathrm{d}i + \int_0^{I_L} \ln(a_M(i))\mathrm{d}i + \int_{I_H}^{1} \ln(a_M(i))\mathrm{d}i + \int_{i\in I_O} \ln(a_M(i))\mathrm{d} - \int_0^1 \ln(a_M(i))\mathrm{d}i \right.$$
$$\left. + \int_{i\in I_O} \ln\left(\frac{1}{\beta_O(I_O) a_O(i)}\right) \mathrm{d}i + \int_{I_H}^{1} \ln\left(\frac{1}{\beta_H(I_H) a_H(i)}\right) \mathrm{d}i\right].$$

As is readily seen, the expression can be reduced further. For example,

$$\int_0^{I_L} \ln(a_M(i))\mathrm{d}i - \int_0^{I_L} \ln(a_L(i))\mathrm{d}i = \int_0^{I_L} \ln(\beta_L(i))\mathrm{d}i,$$

and so on. Following these steps, we obtain Eq. (17).

pends on the interaction between internal reallocation, i.e. medium-skilled workers to low- and high-skilled job tasks, and external reallocation, i.e. moving domestic jobs abroad.

4 General Equilibrium Solution

The general equilibrium closed solution to the equilibrium task margins is characterized by the cut-off conditions (2), (3) and (7) derived respectively in Lemma 1 and Lemma 3, and the optimal labor demand functions, (14a)–(14d). From these conditions, we obtain a system of three equations determining simultaneously the implicit solution to the task margins, as we elaborate in this section.

From the optimal task-skill allocation (8) we obtain the labor-market-clearing condition

$$\int_0^{I_L} l_L(i)\mathrm{d}i = N_L,\ \int_{i\in\mathcal{S_M}} l_M(i)\mathrm{d}i = N_M,\ \int_{I_H}^1 l_H(i)\mathrm{d}i = N_H,\ \int_{i\in I_O} l_O(i)\mathrm{d}i = n_O, \tag{18}$$

where $N_k, k \in \{L, M, H\}$ denotes the total (exogenously given) mass of domestic labor supply by skill and n_O indicates the total (endogenous) mass of offshore employment. The labor-market-clearing condition (18) can be solved for l_L, l_M, l_H and l_O. Utilizing the resulting expressions, respectively, in the demand functions (14a)–(14d) and solving for relative medium-skilled wages, we obtain

$$\frac{w_L}{w_M} = \frac{N_M}{N_L}\frac{I_L}{I_H - I_L - I_O} \tag{19a}$$

$$\frac{w_M}{w_O} = \frac{n_O}{N_M}\frac{I_H - I_L - I_O}{I_O} \tag{19b}$$

$$\frac{w_M}{w_H} = \frac{N_H}{N_M}\frac{I_H - I_L - I_O}{1 - I_H}. \tag{19c}$$

It is readily seen from Eqs. (19a), (19b) and (19c) that, *ceteris paribus*, the relative (inverse) demand functions are decreasing in the labor supply and increasing in the respective equilibrium task margins. Note, however, that Eq. (19b) contains on the right-hand side an additional endogenous variable, n_O, which is defined by the FOC (14d). Since the offshoring wage rate, w_O, is exogenously given for the domestic firm, any changes in the interval of offshorable tasks I_O will affect n_O. To adjust for this effect, substitute for n_O from the optimal demand equation (14d) and condition (18) to obtain

$$\frac{n_O}{I_O} = \frac{P_E E}{w_O}. \tag{14d$'$}$$

Furthermore, from (11) we get $P_E E = P_E^{-\frac{1-\alpha}{\alpha}}\mathcal{B}$. Substituting it back into (14d′) yields

$$\frac{n_O}{I_O} = \frac{P_E^{-\frac{1-\alpha}{\alpha}}\mathcal{B}}{w_O}. \tag{14d$''$}$$

However, the price index is a function of task margins, thus endogenous too. To account for this, utilize the cut-off conditions (7) from Lemma 3 for $\frac{w_M}{A_M}$ in (16), and rearrange to obtain

$$P_E = \frac{\beta_O(I_O)}{\omega}\Omega(I_L, I_H, I_O).$$

Substituting this for P_E in (14d$''$), and combining the outcome with (19b), we obtain the relative demand function

$$\frac{w_M}{w_O} = \frac{\mathcal{B}}{\left(\frac{\beta_O(I_O)}{\omega}\Omega(I_L, I_H, I_O)\right)^{\frac{1-\alpha}{\alpha}} w_O} \frac{I_H - I_L - I_O}{N_M}. \tag{20}$$

Combining now the Eqs. in (19a) and (19c) , respectively, with the no-arbitrage conditions (2) and (3) as well as (20) with the no-arbitrage condition (7), after rearranging slightly, yields

$$\frac{N_L A_L}{N_M A_M} = \frac{I_L}{(I_H - I_L - I_O)\beta_L(I_L)} \tag{21a}$$

$$\frac{\mathcal{B}}{N_M A_M}\omega^{\frac{1}{\alpha}} = \frac{\beta_O(I_O)^{\frac{1}{\alpha}}\Omega(\cdot)^{\frac{1-\alpha}{\alpha}}}{I_H - I_L - I_O} \tag{21b}$$

$$\frac{N_M A_M}{N_H A_H} = \frac{I_H - I_L - I_O}{(1 - I_H)\beta_H(I_H)}, \tag{21c}$$

where again $\omega \equiv \frac{A_O}{\tau w_O}$ captures exogenous changes in offshorability. We summarize the equilibrium characteristics in the following proposition.

Proposition 1 (Unique Equilibrium)**.** *By Lemmata 1, 2, and 3, together with the Corollaries 1 and 2, the system of equations* (21a)–(21c) *determines the unique equilibrium values for all endogenous task margins* $\{I_L, I_H, I_O\}$ *as a function of the exogenous variables and parameters.*

Proof. A rigorous formal discussion is provided in the supplementary mathematical Appendix F.

Notice that the left-hand side of the Eqs. in (21) consists only of exogenous variables and parameters of the model. The right-hand sides denote the medium-skilled labor demand relative to other type of workers at the equilibrium set of tasks. For example, in Eq. (21a) the right-hand side can be seen as the relative demand for low-skill-intensive tasks, in (21c) the relative demand for high-skill-intensive tasks is denoted, and so on. Therefore the 3×3 system of equations in (21) determines the general equilibrium closed solution of the endogenous task margins.

5 Comparative Statics

Utilizing the system (21a)–(21c), we compute in this section the general equilibrium effects of easier offshoring on the endogenous equilibrium margins. Particularly, as mentioned earlier, easier offshoring is associated with $\mathrm{d}\omega > 0$ induced either by i) a *Heckscher–Ohlin effect*, lower foreign wage costs ($\mathrm{d}w_O < 0$) due to accumulation of human capital abroad; ii) a *Ricardian effect*, advances in technology ($\mathrm{d}A_O > 0$), e.g. owing to utilization of more advanced machines abroad; and/or iii) a *trade cost effect*, lower offshoring cost ($\mathrm{d}\tau < 0$), e.g. because of abolition of transportation barriers.

Therefore, next to trade cost, we explicitly account for two additional channels of increasing globalization of the production process. This feature is missing from both the original framework of Grossman and Rossi-Hansberg (2008) and the generalized one of Acemoglu and Autor (2011).

5.1 Increasing Offshorability and Task Reallocation

Taking logs in the equations derived in (21) and rearranging, we obtain

$$-\ln\left(\frac{N_L A_L}{N_M A_M}\right) + \ln I_L - \ln\left(I_H - I_L - I_O\right) - \ln \beta_L(I_L) = 0 \tag{22a}$$

$$-\ln\left(\frac{\mathcal{B}}{A_M N_M}\right) - \ln\left(I_H - I_L - I_O\right) + \frac{1}{\alpha}\ln \beta_O(I_O) + \frac{1-\alpha}{\alpha}\ln \Omega(\cdot) = \frac{1}{\alpha}\ln\omega \tag{22b}$$

$$-\ln\left(\frac{N_M A_M}{N_H A_H}\right) + \ln\left(I_H - I_L - I_O\right) - \ln \beta_H(I_H) - \ln(1 - I_H) = 0. \tag{22c}$$

Now we can compute the impact of easier offshoring on the task margins. We summarize the main results in the following proposition.

Proposition 2 (Easier Offshoring of Medium-Skilled Tasks & Changes in Task Margins)**.** *Qualitatively, easier offshoring* ($\mathrm{d}\omega > 0$)*, due to advances in offshoring-biased technology* ($\mathrm{d}A_O > 0$)*, or lower offshoring cost* ($\mathrm{d}\tau < 0$)*, or a decline in foreign wage costs* ($\mathrm{d}w_O < 0$)*, induces an expansion of the offshorable task range and a contraction of low- and high-skill-intensive tasks ranges*

$$\frac{\mathrm{d}I_L}{\mathrm{d}\omega} < 0, \quad \frac{\mathrm{d}I_O}{\mathrm{d}\omega} > 0, \quad \frac{\mathrm{d}I_H}{\mathrm{d}\omega} > 0, \qquad \textit{and} \qquad \left|\frac{\mathrm{d}I_O}{\mathrm{d}\omega}\right| > \left|\frac{\mathrm{d}I_H}{\mathrm{d}\omega}\right| + \left|\frac{\mathrm{d}I_L}{\mathrm{d}\omega}\right|.$$

The asymmetry in the domestic reallocation of medium-skilled workers is determined by the comparative advantage schedules

$$\left|\frac{\mathrm{d}\ln I_L}{\mathrm{d}\omega}\right| \lesseqgtr \left|\frac{\mathrm{d}\ln I_H}{\mathrm{d}\omega}\right|, \;\Rightarrow \left|\frac{1+\varepsilon_H(1-_H)}{1-I_H}\right| \lesseqgtr \left|\frac{1+\varepsilon_L I_L}{I_L}\right|.$$

Proof. A full analytical proof is provided in Appendix E.1.

The intuition can be explained in the following way. Easer offshoring increases the cost advantage for domestic firms to reallocate domestic job tasks abroad. This effect displaces the marginal medium-skilled workers performing job tasks in the neighborhood of I_O. The Walrasian nature of the labor market implies a downward wage adjustment of medium skilled workers. This in turn indicates that the no-arbitrage conditions in Lemma 1 are off equilibrium, which are reassured by a reallocation of displaced medium-skilled workers to low-skill- (i.e. lower I_L) and high-skill- (i.e. higher I_H) intensive job tasks.

Thus, Proposition 2 highlights what Costinot and Vogel (2010) call a *task upgrading* at the high-skill-extensive margin, i.e. more medium-skilled workers produce former high-skilled tasks, and a *task downgrading* at the low-skill-extensive margin, i.e. more medium-skilled workers produce former low-skilled tasks.[15] Again, the magnitude of the skill down- and upgrading is determined by substitutability of medium-skilled workers at the equilibrium task margins I_L and I_H. Thus a relative low substitutability at the high-skill-intensive job tasks (higher values of ε_H) implies that medium-skilled workers are disproportionately allocated into low-skill-intensive job tasks. The empirical literature has highlighted a gradual increase in skill downgrading in many advanced countries. Particularly, medium-skilled workers are more likely to be

[15] Notice, however, that (easier) offshoring in our framework differs from Costinot and Vogel (2010, section VI.B.). Their results affirm a pervasive rise in wages of more skilled workers, i.e. an increase in inequality, induced by an implicit increase in the size of the relatively skill scarce foreign economy. In contrast, we follow up on the recent empirical findings on the offshoring-induced polarization effect, i.e. a decline in wages of medium-skilled workers relative to low- and high-skilled, and highlight different channels that may lead to easier offshoring.

downgraded into jobs (occupations) that require lower educational attainment (Brynin and Longhi, 2009). In the following section we deviate from the perfect competitive labor market assumption and investigate the potential crowding-out effect of low-skilled workers induced by offshoring.

5.2 Offshoring and Distributional Effects

As discussed in the introduction, in many advanced countries the recent evidence highlights a polarizing wage trend, i.e. a relative decline of medium-skilled wages compared to low- and high-skilled wages. This is in sharp contrast to wage developments in the past, where the burden of globalization regarding wage and employment cuts was mainly borne by low-skilled workers. Particularly, the evolution of earnings inequality followed a monotonic increase between the skill groups (for a neat survey of the literature, see Acemoglu and Autor, 2011). In this section, we address the distributional effect of different stages of global competition and show that offshoring generates income effects similar to factor-biased technology (cf. Acemoglu, 2002). We summarize the main results in the following proposition.

Proposition 3 (Offshoring and Income Distribution)**.** *If offshoring activities are limited to low-skilled-intensive job tasks (indicating globalization trends in the past), then lower offshoring friction induces a distributional impact similar to skill-biased technology change, i.e.* $\frac{\mathrm{d}\left(w_L/w_M\right)}{\mathrm{d}\omega} < 0$ *and* $\frac{\mathrm{d}\left(w_M/w_H\right)}{\mathrm{d}\omega} < 0$*. If offshoring is permitted to medium-skilled job tasks (indicating recent globalization trends), then easier offshoring leads to a wage polarization effect, i.e.* $\frac{\mathrm{d}\left(w_L/w_M\right)}{\mathrm{d}\omega} > 0$ *and* $\frac{\mathrm{d}\left(w_M/w_H\right)}{\mathrm{d}\omega} < 0$*. If offshoring activities are limited to high-skill-intensive job tasks (indicating potential future globalization trends), then a marginal decline in offshoring friction lowers the wage gap between skill groups similar to unskilled-biased technology changes, i.e* $\frac{\mathrm{d}\left(w_L/w_M\right)}{\mathrm{d}\omega} > 0$ *and* $\frac{\mathrm{d}\left(w_M/w_H\right)}{\mathrm{d}\omega} > 0$*.*

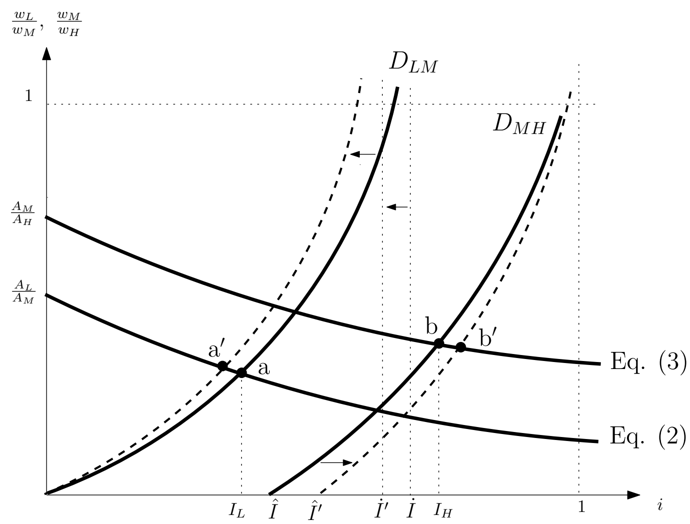
Below we provide a graphical assessment of the distributional impact of each of the offshoring scenarios.

Polarization Effect

Given the results of the comparative statics for changes in the equilibrium task margins, we proceed now with the assessment of the distributional effect of offshoring of medium-skilled job tasks. In doing so, we need to recall the no-arbitrage conditions: Eq. (2): $\frac{w_L}{w_M} = \frac{A_L}{A_M}\beta_L(I_L)$, and Eq. (3): $\frac{w_M}{w_H} = \frac{A_M}{A_H}\beta_H(I_H)$; together with the labor demand functions for medium-skilled workers relative to low-skilled workers Eq. (19a): $D_{LM} \equiv \frac{w_L}{w_M} = \frac{N_M}{N_L}\frac{I_L}{\dot{I}-I_L}$, and relative to high-skilled workers, Eq. (19c): $D_{MH} \equiv \frac{w_M}{w_H} = \frac{N_H}{N_M}\frac{I_H-\hat{I}}{1-I_H}$, where $\dot{I} \equiv I_H - I_O$ and $\hat{I} \equiv I_L + I_O$. Thus, for given I_H and I_O, D_{LM} is increasing in I_L, while higher (lower) values of I_O (I_L) induce a leftward shift of D_{LM}. The relative labor demand curve between medium- and high-skilled workers D_{MH} is increasing in I_H for given values I_L and I_O, while higher values of both I_O and I_H lead to a rightward shift of D_{MH}.

Moreover, it is readily seen that, for all exogenous shocks, but labor-biased technology (A_j), changes in the relative medium-skilled wage compared to low- and high-skilled wages is a movement along the task productivity schedule, $\beta_L(\cdot)$ and $\beta_H(\cdot)$. To fix ideas, we illustrate the four conditions in Figure 2. Notice that the marginal decline in offshoring friction is captured by a decline in $\dot{I}$ and an increase in $\hat{I}$, implying a shift in the relative labor demand curves. From Proposition 2, the task margin I_L will decline, while I_H increases. This will lead to a slight backward shift in the relative labor demand curves. Eventually, the economy will

Figure 2: Offshoring medium-skilled job tasks and wage polarization



reach the points a′ and b′, denoting the new equilibrium task margins. Notice that the internal reallocation of workers mitigates the wage polarization effect. The magnitude of this countervailing effect will have important implications for changes in the level of wages, which we elaborate next.

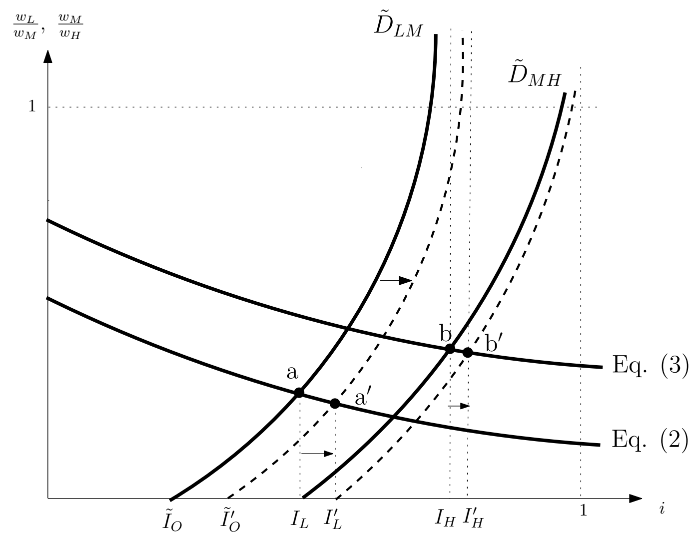
Skill-Biased Effect

We now turn to the case where offshoring is permitted to low-skill-intensive job tasks. A formal discussion of this scenario is provided in Appendix B. Notice again that any changes in relative wages between the skill groups due to exogenous changes in offshoring friction is a movement along the relative task productivity schedules, $\beta_L(I_L)$ and $\beta_H(I_H)$. The only difference compared to the previous case is that the relative labor demand conditions have changed. Now a (sub)range of low-skill-intensive job tasks is offshored, denoted by the equilibrium task margin $\tilde{I}_O$. Thus any changes in offshoring friction will directly shift the relative demand curve between low- and medium-skilled workers (denoted by $\tilde{D}_{LM}$) to the left, while the relative labor demand between medium- and high-skilled workers (denoted by $\tilde{D}_{MH}$) shifts rightward only for changes in the task margin I_L. We derive the relevant conditions in Appendix B and Figure 3 provides a graphical illustration of them.

The intuitive mechanism behind the distributional effect of offshorability of low-skilled job tasks can be explained in the following way. Compared to medium-skilled workers, a decline in the offshoring friction $(\mathrm{d}\omega > 0)$ increases the comparative advantages of offshore workers relative to low-skilled workers. Thus more low-skilled job tasks will be reallocated abroad that were previously performed by low-skilled workers. This implies higher values of the task margin $\tilde{I}_O$. Due to the Walrasian nature of the labor markets, the currently offshoring-induced unemployed low-skilled workers must be reemployed, leading to higher competition for available jobs and consequently to a decline of the low-skilled wage rate. This, in turn, raises the comparative advantages of low-skilled workers relative to medium-skilled workers.

Consequently, a proportion of medium-skill-intensive job tasks will be now allocated to low-skilled workers that were previously performed by medium-skilled workers (i.e. an increase in I_L). This effect is captured by a downward shift of the relative demand curve between low- and medium-skilled workers ($\tilde{D}_{LM}$). Follow-

Figure 3: Offshoring low-skilled job tasks and skill-biased wage effect



ing similar logic, the relative demand curve between medium- and high-skilled workers will shift to the right. These adjustments are depicted in Figure 3, where the economy converges eventually to the new equilibrium points a′ and b′. As is readily seen, the skill premium, i.e. the relative wage between low- and medium-skilled as well as between medium- and high-skilled workers, increases monotonically. This outcome confirms the wage trends in the past due to international competition that were mainly borne by domestic unskilled workers and is in sharp contrast to recent polarizing wage trends in many advanced countries. Thus recent trends highlight the significant shift in the international competition for the domestic workforce.

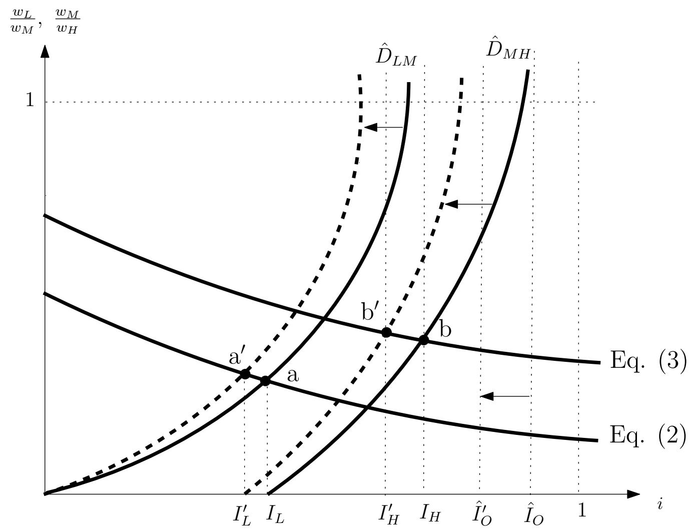
Unskilled-biased Effect

As discussed in the introduction, many occupations that require a high level of skill for cognitive and complex tasks (think of computer programming, statistical analysis) may be at peril in future due to potential international competition (**?**Bhagwati et al., 2004). We provide a intuitive discussion of the distributional impact of this possible future scenario and illustrate graphically the adjustment mechanism.

In this scenario offshoring occurs at the higher end of skill-task distribution, such that a proportion of domestic high-skill-intensive job tasks is only allocated abroad, denoted by $\hat{I}_O$. As in the previous case, the two no-arbitrage conditions (2) and (3) derived in Lemma 1 still hold. Thus the adjustment due to marginal changes in the offshoring friction occurs along the relative labor demand functions, which is depicted now by $\hat{D}_{LM}$ and $\hat{D}_{MH}$. Intuitively, the mechanism works similarly to the previous cases. A lower offshoring friction leads to an expansion of offshore job tasks. The now displaced high-skilled workers are induced to compete for a lower range of job tasks available in the labor market, leading to a decline in their wages.

From the no-arbitrage condition (3) it follows that I_H declines. This is illustrated in Figure 4 by a rightward shift in the relative labor demand curve between high- and medium-skilled workers $\hat{D}_{MH}$. Consequently the competition between medium-skilled workers increases, which pushes their wages down. From no-arbitrage condition (2) it can be verified that I_L declines. This is depicted by a leftward shift of the relative demand curve $\hat{D}_{MH}$ in Figure 4. The labor market converges eventually to the new equilibrium points a′ and

Figure 4: Offshoring high-skilled job-tasks and unskill-biased wage effects



b′, which indicates an increase of relative wages of workers with an inferior skill level. In the next section, we turn back to the polarization case and derive the condition under which the real wage of all domestic workers increases due to the cost-efficiency effect.

5.3 Offshoring-induced Cost-efficiency Effect

As argued earlier, offshoring may generate a cost-efficiency effect similar to advances of technology. The rationale for this is that domestic firms engaged in offshoring benefit from easier offshoring at the *extensive margin*: more tasks can be moved abroad, as well as at the *intensive margin*: tasks that have already been offshored now become cheaper to import – a feature that is omitted in Acemoglu and Autor (2011). Therefore domestic firms experience a reduction in their average production cost, which in turn may lead to beneficial outcomes for the domestic workers.

As will become evident below, the offshoring-induced cost effects depend now on the interaction between the external reallocation and the internal, domestic reallocation of tasks. These two forces will affect the cost of composite labor P_E, which in turn might benefit all workers, even those who have been displaced directly. This new channel is in contrast to the task-based approaches discussed by Grossman and Rossi-Hansberg (2008), where every type of domestic worker is performing a distinct range of tasks and offshoring is characterized by one extensive margin, splitting task produced at home and abroad along this range. It also differs from the approach by Acemoglu and Autor (2011), where the cost index of composite labor (P_E) is held constant.

To derive the effects of easier offshoring on the domestic real wage, we combine the domestic labor demand functions, (14a), (14b) and (14c), with the labor-market-clearing condition (18) to obtain the inverse labor demand functions

$$
\begin{aligned}
w_L &= \frac{I_L}{N_L} P_E E && (23)\\
w_M &= \frac{I_H - I_L - I_O}{N_M} P_E E && (24)\\
w_H &= \frac{1 - I_H}{N_H} P_E E. && (25)
\end{aligned}
$$

Interestingly, the inverse labor demand conditions, (23), (24) and (25), imply that the cost share of each skill group is now denoted by the endogenous equilibrium range of tasks performed by the respective skill group. Moreover, the endogenous cost shares indicate the additional source of substitutability between skill groups across the tasks. Thus this new property can be seen as a generalization of the standard Cobb–Douglas function.[16]

Next, utilizing the optimal demand condition for E, Eq. (11), yields

$$
\begin{aligned}
w_L &= \frac{I_L}{N_L} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B}\\
w_M &= \frac{I_H - I_L - I_O}{N_M} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B}\\
w_H &= \frac{1 - I_H}{N_H} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B}.
\end{aligned}
$$

Recall now the definition of P_E from Eq. (16) and, together with the no-arbitrage conditions (2) and (3) defined in Lemma 1, we obtain, after some rearranging

$$
\begin{aligned}
w_L &= \left(\tfrac{I_L}{N_L}\right)^{\alpha} \left(\beta_L(I_L)^{-1}\Omega(\cdot)\right)^{-(1-\alpha)} A_L^{1-\alpha}\mathcal{B}^{\alpha} && (26)\\
w_M &= \left(\tfrac{I_H - I_L - I_O}{N_M}\right)^{\alpha} \left(\Omega(\cdot)\right)^{-(1-\alpha)} A_M^{1-\alpha}\mathcal{B}^{\alpha} && (27)\\
w_H &= \left(\tfrac{1-I_H}{N_H}\right)^{\alpha} \left(\beta_H(I_H)\Omega(\cdot)\right)^{-(1-\alpha)} A_H^{1-\alpha}\mathcal{B}^{\alpha}. && (28)
\end{aligned}
$$

These equations denote the *generalized* optimal demand for the domestic workforce that accounts for all endogenous equilibrium task margins. It is immediately evident that, given the properties of the task productivity schedules, $\beta_L(\cdot)$ and $\beta_H(\cdot)$, a decline in $\Omega(\cdot)$ will induce a positive demand effect for labor in the economy, thus increasing the wage level of all skills. We denote this effect as the cost-efficiency effect. Computing changes in $\Omega(\cdot)$, we obtain (see Appendix A for the formal derivation)

$$
\frac{\mathrm{d}\ln\Omega(\cdot)}{\mathrm{d}\omega} = \underbrace{\left((1 - I_H)\varepsilon_H \frac{\mathrm{d}I_H}{\mathrm{d}\omega} - \varepsilon_L I_L \frac{\mathrm{d}I_L}{\mathrm{d}\omega}\right)}_{\mathcal{R}>0} - \underbrace{\left(I_O \mu \frac{\mathrm{d}I_O}{\mathrm{d}\omega}\right)}_{\mathcal{E}>0}. \qquad (29)
$$

Changes due to *internal* reallocation, i.e. relocating medium-skilled workers towards low- and high-skill-intensive jobs is denoted by $\mathcal{R}$ and changes due to the *external* reallocation, i.e. moving domestic medium-skill-intensive tasks abroad, is denoted by $\mathcal{E}$. Notice that by the results of the comparative statics in Proposition 2, both the internal $\mathcal{R}$ and the external $\mathcal{E}$ allocation effects are positive.

[16]However, notice the difference compared to Acemoglu and Autor (2011), where the task margins denote the expenditure share of each type of tasks in terms of the value of total output.

Therefore, for the marginal production cost of composite labor (Ω) to decrease in the course of easier offshoring, the necessary condition requires $\mathcal{E} > \mathcal{R}$. As shown in Appendix A, for the following sufficient low substitutability between medium-skilled and offshore workers, the external reallocation effect will be dominating, i.e.

$$\mu > \frac{1-(I_H - I_L)}{I_O(I_H - I_L - I_O)}. \tag{30}$$

However, this condition is not sufficient to ensure an increase in the real wage. The sufficient condition can be derived by straightforwardly differentiating the demand functions, (26), (27) and (28), which yields

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\omega} = \left(\frac{\alpha}{I_L} - (1-\alpha)\varepsilon_L\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} - (1-\alpha)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega} \tag{31a}$$

$$\frac{\mathrm{d}\ln w_M}{\mathrm{d}\omega} = -\frac{\alpha\mathrm{d}I_L}{I_H - I_L - I_O} - \frac{\alpha\mathrm{d}I_O}{I_H - I_L - I_O} + \frac{\alpha\mathrm{d}I_H}{I_H - I_L - I_O} - (1-\alpha)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega} \tag{31b}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\omega} = \left((1-\alpha)\varepsilon_H - \frac{\alpha}{1-I_H}\right)\frac{\mathrm{d}I_H}{\mathrm{d}\omega} - (1-\alpha)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega}. \tag{31c}$$

Now using Eq.(29) to substitute for $\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega}$ in (31b) and rearranging the terms, we obtain

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\omega} = \left(\frac{\alpha}{I_L} - (1-\alpha)\varepsilon_L\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} - (1-\alpha)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega} \tag{31a$'$}$$

$$\begin{aligned}\frac{\mathrm{d}\ln w_M}{\mathrm{d}\omega} &= \left((1-\alpha)I_L\varepsilon_L - \frac{\alpha}{I_H - I_L - I_O}\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} + \left((1-\alpha)I_O\mu - \frac{\alpha}{I_H - I_L - I_O}\right)\frac{\mathrm{d}I_O}{\mathrm{d}\omega} \\ &\quad + \left(\frac{\alpha}{I_H - I_L - I_O} - (1-\alpha)(1-I_H)\varepsilon_H\right)\frac{\mathrm{d}I_H}{\mathrm{d}\omega}\end{aligned} \tag{31b$'$}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\omega} = \left((1-\alpha)\varepsilon_H - \frac{\alpha}{1-I_H}\right)\frac{\mathrm{d}I_H}{\mathrm{d}\omega} - (1-\alpha)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega} \tag{31c$'$}$$

Notice that defining the sign of Eq. (31b′) determines also the relationship between internal (i.e. $\mathcal{E}$) and external (i.e. $\mathcal{R}$) task reallocation. The converse is obviously not true. Therefore, in (31a′) and (31c′) we need to elaborate the sign of the first term on the right hand side. We summarize the results in the following proposition.

Proposition 4 (Offshoring-induced Cost-efficiency Effect)**.** *Given the impact of easier offshoring of medium-skilled tasks on the task reallocation derived in Proposition 2, an offshoring-induced cost-efficiency improvement raises the real wage of all domestic worker for the following jointly sufficient conditions:*

1. $\frac{\alpha}{1-\alpha}\frac{1}{I_L(I_H - I_L - I_O)} > \varepsilon_L > \frac{\alpha}{(1-\alpha)I_L}$
2. $\frac{\alpha}{1-\alpha}\frac{1}{(1-I_H)(I_H - I_L - I_O)} > \varepsilon_H > \frac{\alpha}{1-\alpha}\frac{1}{(1-I_H)}$
3. $\mu > \frac{\alpha}{1-\alpha}\frac{1}{I_O(I_H - I_L - I_O)}$.

Proof. The lower boundary in part 3, as well as the upper limits in parts 1 and 2, follow straightforwardly from (31b′). The lower limits in parts 1 and 2 are, respectively, derived from (31a′) and (31c′). ∎

The intuition behind the jointly sufficient conditions in Proposition 4 is the following. Higher values of ε_L (ε_H) imply a higher comparative advantage of low-skilled (high-skilled) workers in the neighborhood of I_L (I_H) relative to medium-skilled workers. Now recall Eq. (2′) and Eq. (3′) to see the implications. It is readily evident that the size of the parameters ε_L and ε_H importantly determines the magnitude of the change of the task margins I_L and I_H for a one percentage change in relative wages, respectively. This effect is, as put forward in Acemoglu and Autor (2011), the additional source of substitutability between skill groups across tasks, next to the elasticity of substitution of unity arising from the Cobb–Douglas functional form.

The important implication of this additional substitution effect of skills across tasks can be inferred from the following special case. Take the limit $\varepsilon_L \to 0$ (indicating perfect substitutability between low- and medium-skilled workers at I_L), then from Eq. (31a′) it can be easily inferred that the first term reduces to $\frac{\alpha}{I_L}\frac{\mathrm{d}I_L}{\mathrm{d}\omega}$, which by Proposition 2 is unambiguously negative. This is the offshoring-induced labor supply effect as the displaced workers have to be reabsorbed by the labor market. Hence, Proposition 4 highlights that the cost-efficiency effect due to easier offshoring will raise the real wage of all skill groups as the overall labor demand increases more than offshoring-induced increase in labor supply if the comparative advantages of low-skilled and high-skilled workers are sufficiently preserved.

6 Equilibrium Unemployment

So far we have assumed full employment and analyzed the pure distributional effect of offshoring. However, another concern raised in the public debate on offshoring is the displacement effect of workers, leading to unemployment. In this section, we generalize the framework by allowing for equilibrium unemployment. In doing so, we assume that only low-skilled workers face the risk of unemployment. Intuitively and in line with our discussion in the introduction, easier offshoring may indirectly displace low-skilled workers from the labor market due to increasing competition with the offshoring-induced unemployed medium-skilled workers. In the literature, this potential displacement effect is referred to as the crowding-out effect; see Muysken et al. (2015) for a discussion.

We assume two alternative types of labor market friction, without altering the structure of the model. An intuitive source of friction is a minimum wage regime, which is set above the market equilibrium wage rate. Consequently, a proportion of low-skilled workers ends up unemployed. Alternatively, frictions can arise when we allow for endogenous supply of low-skill labor services. In this case the low-skilled wage is set as a mark-up over the unemployment benefits, where the mark-up depends negatively on unemployment rate. While the former is the mirror image of the full-employment case, characterized by a perfect elastic labor supply curve, the latter allows for an elastic labor supply curve and thus includes the standard approach of labor supply.

6.1 Minimum Wage Regime

Let the institutional minimum wage be $\bar{W}$. We assume that the minimum wage is set sufficiently low such that it is still attractive for domestic firms to employ low-skilled workers, but is sufficiently high such that a proportion of low-skilled workers ends up unemployed. Let u_L denote the low-skilled unemployment rate. Formally, we impose the following assumption on the minimum wage scheme.

Assumption 3. *(Minimum wage setting)*

$$w_L < \bar{W} < \frac{w_M A_L}{A_M}\beta_L(0),$$

where w_L and w_M are the equilibrium values resulting from the model analyzed in the previous section.

Given the level of minimum wage, the representative firm will then reallocate the job tasks between low- and medium-skilled workers such that the no-arbitrage condition (2) holds again, though at a lower equilibrium threshold. Moreover, from the general equilibrium perspective, our analysis implies that all other task margins will readjust too. The intuition is the following. A higher minimum wage scheme increases the relative comparative advantage of medium-skilled workers compared to low-skilled workers. Thus, from condition (2), the task margin I_L must decline. This in turn implies that the range of tasks allocated to medium-skilled workers will increase, and from relative medium-skill labor demand conditions (19a)–(19c), medium skill wages will increase too. Consequently, the range of tasks performed by high-skilled $(1 - I_H)$ and offshore workers I_O must increase to satisfy again the no-arbitrage conditions (3) and (7).

In addition, compared to the full-employment case, now only a fraction of low-skilled workers can be hired, i.e. $n_L = (1 - u_L)N_L$, and the resource constraint becomes $l = \frac{n_L}{I_L}$. This implies that the adjustment channel in the low-skill labor market is through employment instead of wages. Thus the relative demand condition for low-skilled workers defined in (19a) has to account for the endogenous adjustment in low-skilled employment. In order to investigate the impact of offshoring on the low-skilled unemployment rate, it is necessary to examine first the impact of offshoring on task allocation under the minimum wage scheme. We proceed with the derivation of the new equilibrium conditions, while, for the sake of illustration, we use the same expression for the equilibrium task margins as in the frictionless labor market scenario.

Recall the first-order condition (14a), where now the marginal productivity of low-skilled workers equals the minimum wage scheme, and utilize the low-skilled labor constraint to obtain

$$\frac{n_L}{I_L} = \frac{P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B}}{\bar{W}}, \tag{32}$$

which is equivalent to the full-employment case, except that now the level of employment, n_L, is endogenous while the level of wage is fixed. Next, notice that the no-arbitrage condition (2) is defined now as

$$\frac{\bar{W}}{w_M} = \frac{A_L}{A_M} \beta_L(I_L). \tag{33}$$

Using this observation together with Eq. (16), we obtain

$$P_E = \frac{\bar{W}}{A_L} \beta_L(I_L)^{-1} \Omega(\cdot). \tag{34}$$

Substituting Eq. (34) for P_E in Eq. (32), we obtain the generalized low-skilled labor demand equation under a minimum wage scheme

$$\frac{n_L}{I_L} = \frac{\left(\frac{\bar{W}}{A_L} \beta_L(I_L)^{-1} \Omega(\cdot)\right)^{-\frac{1-\alpha}{\alpha}} \mathcal{B}}{\bar{W}}. \tag{35}$$

Utilizing this expression in the relative demand equation (19a), together with the new no-arbitrage condition (33), we obtain the modified implicit condition that accounts for the minimum wage scheme and the endogenous low-skilled employment

$$\frac{\mathcal{B}}{A_M N_M}\left(\frac{A_L}{\bar{W}}\right)^{1/\alpha} = \frac{\beta_L(I_L)^{-1/\alpha}\Omega(\cdot)^{\frac{1-\alpha}{\alpha}}}{I_H - I_L - I_O}. \tag{36}$$

This is equivalent to Eq. (21a) in the frictionless labor market scenario. Utilizing Eq. (36) together with Eqs. (21b) and (21c), we obtain the modified implicit 3×3 system of equations for the endogenous task margins.

Taking logs and rearranging slightly yields

$$\begin{aligned}
-\frac{1}{\alpha}\ln\left(\beta_L(I_L)\right) + \frac{1-\alpha}{\alpha}\ln\left(\Omega(I_H, I_L, I_O)\right) - \ln\left(I_H - I_L - I_O\right) - \ln\left(\frac{\mathcal{B}}{A_M N_M}\right) - \frac{1}{\alpha}\ln\left(\frac{A_L}{\bar{W}}\right) &= 0 \\
\frac{1}{\alpha}\ln\left(\beta_O(I_O)\right) + \frac{1-\alpha}{\alpha}\ln\left(\Omega(I_H, I_L, I_O)\right) - \ln\left(I_H - I_L - I_O\right) - \ln\left(\frac{\mathcal{B}}{A_M N_M}\right) - \frac{1}{\alpha}\ln\left(\omega\right) &= 0 \qquad (37) \\
\ln\left(I_H - I_L - I_O\right) - \ln\left(1 - I_H\right) - \ln\left(\beta_H(I_H)\right) - \ln\left(\frac{A_M N_M}{A_H N_H}\right) &= 0.
\end{aligned}$$

By straightforward differentiation of the system (37), one can compute the impact of an increase in the minimum wage scheme as well as easier offshoring on the equilibrium task margins. We summarize the main results in the following proposition and refer readers to the Appendix E.2 and E.3 for a formal proof.

Proposition 5 (Minimum Wage, Offshoring Medium-skilled Tasks, and Task Margins)**.** *If offshoring medium-skilled tasks is permitted and the low-skill labor market is characterized by a minimum wage scheme, then a rise in the minimum wage scheme will lead to a contraction of low-skill-intensive jobs tasks, i.e.* $\frac{\mathrm{d}I_L}{\mathrm{d}\bar{W}} < 0$*, and an expansion of high-skill-intensive and offshorable job tasks, i.e.* $\frac{\mathrm{d}I_H}{\mathrm{d}\bar{W}} < 0$ *and* $\frac{\mathrm{d}I_O}{\mathrm{d}\bar{W}} > 0$*, respectively. Easier offshoring generates similar skill-task reallocation as in Proposition 2.*

Given these results, we can now assess the impact of easier offshoring in the low-skilled unemployment rate. In doing so, recall the low-skilled labor demand condition (35). Rearranging and taking logs, we obtain

$$\ln n_L = \ln(I_L) + \frac{1-\alpha}{\alpha}\ln(\beta_L(I_L)) - \frac{1-\alpha}{\alpha}\ln(\Omega(\cdot)) - \frac{1}{\alpha}\ln\bar{W} + \frac{1-\alpha}{\alpha}\ln A_L + \ln\mathcal{B}.$$

Now total differentiating w.r.t. offshoring friction ($\mathrm{d}\omega > 0$) yields

$$\frac{\mathrm{d}\ln n_L}{\mathrm{d}\omega} = \left(\frac{1}{I_L} - \frac{(1-\alpha)}{\alpha}\varepsilon_L\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} - \frac{(1-\alpha)}{\alpha}\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\omega}.$$

The following proposition summarizes the main results regarding the impact of offshoring on low-skilled unemployment rate.

Proposition 6 (Minimum Wage, Offshoring Medium-Skilled Tasks, and Low-Skilled Unemployment)**.** *If a fraction of low-skilled workers is unemployed due to a minimum wage scheme, then easier offshoring of medium-skilled tasks will lead to a decline in the low-skilled unemployment rate if and only if Proposition 4 holds.*

The intuition is similar to the one discussed in Proposition 4. The difference is that now, with a minimum wage scheme, the low-skill labor market adjustment occurs via employment.

6.2 Endogenous Labor Supply

A more general approach to addressing labor market frictions is to allow workers to supply endogenously labor services, implying an elastic labor supply curve. This feature has important implications for labor market outcomes. In doing so, we follow the standard approach in the literature and assume that the low-skilled wage is a mark-up on unemployment benefits that depends negatively on the unemployment rate. This mark-up can be explained in many ways, such as the standard individual leisure–work choice, wage bargaining (Layard et al., 2005), search and matching theory à la Pissarides (2000) and efficiency wages à la Shapiro and Stiglitz (1984). Imposing such a negative relationship between the mark-up and unemployment induces an elastic labor supply curve. This way, we provide a more general analysis of labor market effects of offshoring compared to the minimum wage case.[17]

Let the endogenous low-skilled wage curve be defined by

$$w_L = f(u_L) b_L, \tag{38}$$

where $f(u_L)$ denotes the mark-up over unemployment benefits, b_L, and has the following properties: $f(u_L) > 1$ and $\frac{\partial f(u_L)}{\partial u_L} < 0$. Moreover, we define the elasticity of the wage curve in absolute value w.r.t. u_L as $\tilde{\delta} \equiv -\frac{\mathrm{d} \ln f(u_L)}{\mathrm{d} \ln u_L} > 0$.

In this case, both the low-skilled wage and employment will adjust to exogenous shocks. Thus, compared to full employment and minimum wage cases, the low-skilled labor demand functions (26) and (35) become

$$w_L = \left(\frac{I_L}{(1-u_L) N_L} \right)^{\alpha} \left(\frac{\beta_L(I_L)}{\Omega(\cdot)} \right)^{1-\alpha} (A_L \mathcal{B})^{\alpha}, \tag{39}$$

where we utilized $n_L = (1-u_L)N_L$. This implies that the relative demand between low- and medium-skilled workers in Eq. (21a) has to account for the endogenous changes in low-skilled employment. Formally, we write this condition by

$$\frac{N_L A_L}{N_M A_M} = \frac{1}{1-u_L} \frac{I_L}{(I_H - I_L - I_O)\beta_L(I_L)}. \tag{40}$$

The model is closed by the adjusted market-clearing condition in the low-skilled labor market, i.e. from Eqs. (39) and (38)

$$f(u_L) b_L = \left(\frac{I_L}{(1-u_L) N_L} \right)^{\alpha} \left(\frac{\beta_L(I_L)}{\Omega(\cdot)} \right)^{1-\alpha} A_L^{1-\alpha} \mathcal{B}^{\alpha}. \tag{41}$$

Thus the new system of equations consists of Eqs. (21b), (21c), (40) and (41). Taking logs and rearranging

[17] It is worth mentioning the important implications of applying different equilibrium unemployment paradigms regarding the adjustment mechanism of the labor market to exogenous shocks. However, our objective is not to explain the efficiency of various adjustment mechanisms, and thus we deliberately leave this to future research. For an application of search-matching and efficiency wage theories to the original task-based approach of Grossman and Rossi-Hansberg (2008), see Kohler and Wrona (2011).

slightly, we obtain

$$
\begin{aligned}
\ln\left(\frac{A_L N_L}{A_M N_M}\right) &= -\ln(1-u_L) + \ln I_L - \ln(I_H - I_L - I_O) - \ln\beta_L(I_L) \\
\ln\left(\frac{\mathcal{B}}{A_M N_M}\right) + \frac{1}{\alpha}\ln\omega &= \frac{1}{\alpha}\ln\beta_O(I_O) + \frac{1-\alpha}{\alpha}\ln\Omega(\cdot) - \ln(I_H - I_L - I_O) \\
\ln\left(\frac{A_M N_M}{A_H N_H}\right) &= -\ln(1-I_H) + \ln(I_H - I_L - I_O) - \ln\beta_H(I_H) \\
\alpha\ln\left(\frac{\mathcal{B}}{N_L}\right) - \ln b_L + (1-\alpha)\ln A_L &= \alpha\ln(1-u_L) + \ln f(u_L) + (1-\alpha)\left(\ln\Omega(\cdot) - \ln\beta_L(I_L)\right) - \alpha\ln I_L.
\end{aligned} \tag{42}
$$

This is the generalized 4×4 implicit system of equations that accounts for endogenous supply of labor services. Notice also that the left-hand side consists of all exogenous variables, while the right-hand side accounts for the four endogenous variables I_L, I_O, I_H and u_L.

Now the marginal impact of offshoring on task margins and the low-skilled unemployment rate can be computed by straightforward differentiation of (42) w.r.t. ω and the endogenous variables. We summarize the main results in the following proposition.

Proposition 7 (Offshoring Medium-skilled Tasks, Labor Supply, and Low-skilled Unemployment)**.** *If the low-skilled labor market is characterized by an endogenous wage curve, where a fraction u_L of low-skilled workers are unemployed, then offshoring of medium-skill job tasks unambiguously reduces the low-skilled unemployment rate for sufficiently low substitutability between medium- and high-skilled workers at task margin I_H and between medium- and low-skilled workers at task margin I_L, i.e.*

$$
\varepsilon_H > \frac{\alpha}{(1-\alpha)I_H(1-I_H)}, \qquad \varepsilon_L > \frac{1}{(1-\alpha)I_L(I_H-I_L)}.
$$

Moreover, by the sufficient conditions in Proposition 4, real wages of all skill groups rise.

Proof. See Appendix E.4.

Thus Proposition 7 highlights again the importance of comparative advantages in performing job tasks, which determine the magnitude of spillover effects induced by offshoring. Recall the interpretation of the parameters ε_L and ε_H, capturing the substitutability of skill groups at the respective equilibrium task margin. The spillover of medium-skilled workers induced by offshoring is dominated by an overall rise in total employment if both high-skilled and low-skilled workers have sufficiently high comparative advantages in performing tasks in the neighborhood of I_H and I_L, respectively. Moreover, notice that the lower boundaries in Proposition 7 dominate those in Proposition 4, but they are not binding since all derived boundaries are not necessary conditions.[18] The intuition behind the real wage effect is equivalent to the one provided in Proposition 4.

In addition, it is important to notice the difference from the minimum wage case, where now higher comparative advantages of low- and high-skilled workers are required due to endogenous labor supply. Intuitively, the low-skilled labor market will adjust to the spillover effect as follows. The bumping down of

[18] As discussed in the supplementary mathematical Appendix E.4, it can be verified that for sufficiently low substitutability between medium-skilled and offshore workers (i.e. high values of μ), the low-skilled unemployment rate might still decline, even if Proposition 7 is violated.

medium-skilled workers (lower I_L) will displace some of low-skilled workers, who were previously performing those tasks in the neighborhood of I_L. The increase in the unemployment rate will in turn lead to a downward wage adjustment due to a lower mark-up (captured by a flatter wage curve), mitigating the relative rise in comparative advantage of medium-skilled workers.

7 Conclusion

In this paper we have analyzed the general equilibrium effects of easier offshoring on task allocation within the domestic economy as well as in the foreign economy. We augment the current literature analyzing the implications of increasing international competition vis-à-vis the domestic labor force with respect to the following features. First, by introducing endogenous offshoring, we have augmented the framework of Acemoglu and Autor (2011), where offshoring is exogenously given. Second, we allow for a heterogeneous labor market, characterized by low-, medium- and high-skilled workers, where the allocation of each skill group to job tasks is based on a Ricardian type of comparative advantages along a unit interval. In doing so, we augment the seminal task-based approach of Grossman and Rossi-Hansberg (2008), where each group of labor is producing a distinct, completely independent range of tasks. In this way, we are able to address two contradictory hypotheses regarding the distributional impact of offshoring for domestic workers: a wage polarization effect and a cost-efficiency effect. Finally, we allow for equilibrium unemployment of low-skilled workers. In doing so, we are able to capture important externalities in the labor market, such as the bumping down of medium-skilled workers inducing a crowding-out effect of low-skilled workers.

The general equilibrium analysis provides several new insights, on which the existing theoretical literature has been silent. First, we show that any scenario of offshoring domestic job tasks can be described by a U-shaped relative productivity schedule between domestic and offshore workers. This allows us to address different stages of globalization, where the burden in terms of wage and employment cuts was borne by different skill groups, e.g. the trends in the past affected mainly low-skilled workers, while recent trends show increasing competition for medium-skilled workers, and potential future developments towards high-skilled workers. Moreover, our analysis reveals that offshoring of low-skill-intensive job tasks generates distributional effects similar to *skill*-biased technology changes – consistent with observations in the past, see Acemoglu (2002), offshoring medium-skill-intensive domestic job tasks induces a wage polarization effect (reflecting recent trends), while offshoring of high-skill-intensive domestic job tasks generates an income distribution effect similar to *unskilled*-biased technology changes (Goldin and Katz, 2009).

Second, we derive clear conditions that characterize the offshoring-induced cost-efficiency effect. We show that the cost-efficiency effect induced by offshoring domestic job tasks, what we refer to as the external reallocation, is countervailed by an internal reallocation of tasks to domestic workers. More precisely, the internal reallocation refers to reallocation of offshoring-induced displaced medium-skilled workers to low-skill- and high-skill-intensive job tasks. The balance between the two forces depends importantly on the substitutability between medium- and low-skilled workers and between high- and medium-skilled workers at the respective equilibrium task margins. More precisely, for sufficient low substitutability, the internal reallocation effect is dominated by the external one. In this case, all domestic skill groups benefit in terms of higher real wages. The importance of this internal reallocation has been put forward in the recent empirical literature (cf. Baumgarten et al., 2013; Hummels et al., 2014).

Finally, we elaborate the implications of the bumping-down effect of medium-skilled workers induced by easier offshoring for the low-skilled unemployment rate. Our analysis reveals that if the substitutability between medium- and low-skilled workers at the equilibrium task margin is sufficiently low (i.e. low-skilled workers have sufficiently higher comparative advantages in performing the domestic job tasks), then the crowding-out effect is offset by the offshoring-induced cost-efficiency effect, boosting the low-skilled labor market.

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Appendix

A Cost-efficiency effect

Recall Eq. (17) and take logs to obtain

$$\ln\Omega(\cdot)=\left[\left(\int_0^{I_L}\ln\left(\frac{\beta_L(i)}{\beta_L(I_L)}\right)\mathrm{d}i+\int_{I_H}^1\ln\left(\frac{\beta_H(i)}{\beta_H(I_H)}\right)\mathrm{d}i+\int_{i\in I_O}\ln\left(\frac{\zeta(i)}{\beta_O(I_O)}\right)\mathrm{d}i-\int_0^1\ln(a_M(i))\mathrm{d}i\right)\right].$$

Next, recall the definition of the semi-elasticities of the comparative advantage schedules

$$\varepsilon_L=-\frac{\partial\ln\beta_L(I_L)}{\partial I_L}>0,\quad \varepsilon_H=-\frac{\partial\ln\beta_H(I_H)}{\partial I_H}>0,\quad \frac{\partial\ln\beta_O(I_O)}{\partial I_O}=\mu>0.$$

Total differentiation w.r.t. the endogenous margins yields

$$\begin{aligned}\mathrm{d}\ln\Omega(\cdot) \;=\;& \ln\beta_L(I_L)\mathrm{d}I_L+I_L\frac{\beta_L'(I_L)}{\beta_L(I_L)}\mathrm{d}I_L-\ln\beta_L(I_L)\mathrm{d}I_L-\ln\beta_H(I_H)\mathrm{d}I_H-(1-I_H)\frac{\beta_H'(I_H)}{\beta_H(I_H)}\mathrm{d}I_H+\ln\beta_H(I_H)\mathrm{d}I_H\\ &+\left(\ln\zeta(I_2)\mathrm{d}I_2-\ln\zeta(I_1)\mathrm{d}I_1-\ln\beta_O(I_O)\mathrm{d}I_O\right)-I_O\frac{\beta_O'(I_O)}{\beta_O(I_O)}\mathrm{d}I_O.\end{aligned}$$

Then, taking into account that the term within the brackets is, by the positive monotone transformation, discussed in Appendix C.3, null, utilizing the definitions of semi-elasticity, and manipulating further, the total differentiation reduces to

$$\mathrm{d}\ln\Omega(\cdot) \;=\; -I_L\varepsilon_L\mathrm{d}I_L+(1-I_H)\varepsilon_H\mathrm{d}I_H-I_O\mu\mathrm{d}I_O. \tag{A.1}$$

Moreover, using the comparative static results derived in Appendix E.1, we can compute the overall sign of $\mathrm{d}\ln\Omega(\cdot)$. Utilizing Eqs. (E.2) and (E.4), the internal reallocation effect, $\mathcal{R}=(1-I_H)\varepsilon_H\frac{\mathrm{d}I_H}{\mathrm{d}\omega}-I_L\varepsilon_L\frac{\mathrm{d}I_L}{\mathrm{d}\omega}$, can be written as

$$\begin{aligned}\mathcal{R} &= \frac{\left[(1-I_H)^2\varepsilon_H(1+\varepsilon_LI_L)+I_L^2\varepsilon_L(1+(1-I_H)\varepsilon_H)\right]}{\alpha\omega I_L(1-I_H)(I_H-I_L-I_O)\Delta}\\ &= \frac{(1-I_H)^2\varepsilon_H+I_L^2\varepsilon_L+(1-I_H)I_L\varepsilon_H\varepsilon_L(1-(I_H-I_L))}{\alpha\omega I_L(1-I_H)(I_H-I_L-I_O)\Delta}.\end{aligned} \tag{A.2}$$

Next, utilizing Eq. (E.2), the external reallocation effect, $\mathcal{E}=I_O\mu\frac{\mathrm{d}I_O}{\mathrm{d}\omega}$, can be written as

$$\mathcal{E} \;=\; \frac{I_O\mu\left[1-I_O+\varepsilon_H(1-I_H)[I_H-I_O+\varepsilon_LI_L(I_H-I_L-I_O)]+\varepsilon_LI_L(1-I_L-I_O)\right]}{\alpha\omega I_L(1-I_H)(I_H-I_L-I_O)\Delta}. \tag{A.3}$$

Now from Eqs. (A.2) and (A.3), we can derive the sufficient condition that determines the sign of $\frac{\mathrm{d}\ln\Omega(\cdot)}{\mathrm{d}\omega}=$

$\mathcal{R} - \mathcal{E} \lesseqgtr 0.$

$$(1-I_H)^2\varepsilon_H + I_L^2\varepsilon_L + (1-I_H)I_L\varepsilon_H\varepsilon_L(1-(I_H-I_L)) \lesseqgtr I_O\mu\Big[1 - I_O + \varepsilon_H(1-I_H)(I_H-I_O) + \varepsilon_H(1-I_H)\varepsilon_L I_L(I_H-I_L-I_O) + \varepsilon_L I_L(1-I_L-I_O)\Big].$$

It can be verified that for a sufficiently low substitutability (i.e. high values of μ) between medium-skilled and offshore workers the marginal cost of composite labor unambiguously declines, i.e. $\frac{\mathrm{d}\ln\Omega(\cdot)}{\mathrm{d}\omega} < 0$. Formally, the sufficient condition follows from the magnitude between the third term on the right hand side and the third term on the left hand side, which yields

$$\mu > \frac{1-(I_H-I_L)}{I_O(I_H-I_L-I_O)}. \tag{A.4}$$

Notice that if the sufficient condition (A.4) holds, the first two terms on the left hand side will also be dominated by the other terms on the right hand side.

B Extension: Alternative offshoring scenarios of domestic job tasks

Offshoring low-skill-intensive domestic job tasks

This section provides an analytical discussion of different stages of globalization trends. Particularly, we discuss the distributional effect when offshoring is limited to other skill segments of domestic tasks, while keeping the structure of the model unchanged. We commence with the special case as in Grossman and Rossi-Hansberg (2008), where offshoring activities are limited to low-skill-intensive job tasks. We refer to this case as globalization trends in the past, where the burden was mainly borne by low-skilled workers in many advanced countries. We then assess the distributional effects of easier offshoring.

First, notice that domestic skill-task allocations defined by Lemma 1 still hold. Then, similar to the case of offshoring medium-skilled tasks, we impose that the task assignment between low-skilled and offshore workers will lead to offshoring of a fraction of all low-skilled job tasks in the interval $(0, I_L)$. This implies that domestic firms find it cheaper to install low-skilled workers at the lower and upper ends of all low-skill-intensive job tasks, while offshoring is cheaper somewhere in the middle. Formally, the assignment problem is defined by

$$c_O(i) \lesseqgtr c_L(i)$$

or equivalently

$$\frac{\tau w_O}{A_O}\frac{1}{a_O(i)} \lesseqgtr \frac{w_L}{A_L}\frac{1}{a_L(i)}, \quad \forall\, i \in I_L \setminus \tilde{I}_O. \tag{B.1}$$

Multiplying both sides of (B.1) by $a_M(i)$ yields the familiar structure of task productivity schedules, i.e. $\beta_L(i) = \frac{a_L(i)}{a_M(i)}$ and $\tilde{\zeta}(i) = \frac{a_M(i)}{a_O(i)}$, with the functional properties defined by Assumptions 1 and 2 in the main text. Notice the difference between the task productivity schedule $\tilde{\zeta}(i)$ (imposed here) and $\zeta(i)$ (used previously). While both have the U-shaped functional form, $\tilde{\zeta}(i)$ indicates a different location over the unit interval. To fix ideas, consider, for example, the following simple quadratic function

$$\zeta(i) = \exp[g_M(i - g_O)^2],$$

where g_M and g_O are parameters and measure the task productivity of medium-skilled and offshore workers respectively. In this example, it can be easily verified that changes in the task productivity of offshore workers (g_O) will induce a horizontal shift of $\zeta(i)$, changes in g_M affect the slope, while lower offshoring friction (i.e. higher values of $\omega \equiv \frac{A_O}{\tau w_O}$) leads to a vertical shift of $\zeta(i)/\omega$. Notice however that if the comparative advantage of offshore workers is sufficiently low (i.e. $g_O = 0$), there are no domestic low-skilled workers employed near the origin, and if they have sufficiently high task productivity (i.e. $g_O = I_L$), there are no low-skilled workers employed between the offshoring interval and the task margin I_L. Thus, as derived below, necessary conditions similar to those derived in Corollary 2 are required for which $\tilde{\zeta}(i) \in (0, I_L)$.

The necessary conditions for offshoring activities to be permitted to a sub-range of low-skilled job tasks are

$$\frac{A_L}{w_L}\beta_L(\dot{I})\tilde{\zeta}(\dot{I}) < \omega < \frac{A_L}{w_L}\beta_L(I_L)\tilde{\zeta}(I_L), \quad \text{and} \quad 0 < \tilde{I}|_{\frac{\partial \ln \tilde{\zeta}(\cdot)}{\partial i}=0} < I_L, \tag{B.2}$$

implying that the offshoring friction is not too large (i.e. low values of $\omega = \frac{A_O}{\tau w_O}$), defined at the tangent point $\dot{I}$ between the productivity schedules $\left|\frac{\partial \beta_L(\dot{I})}{\partial \dot{I}}\right| = \left|\frac{\partial \tilde{\zeta}(\dot{I})}{\partial \dot{I}}\right|$, such that offshoring is permitted, nor too small, avoiding that low-skilled workers lose their comparative advantage in competing with medium-skilled workers, defined by the equilibrium task margin I_L.[19]

Then, given Assumptions 1 and 2, and conditions in (B.2), it follows from (B.1) that there must exist two cut-off margins, similar to Lemma 2, defining the range of offshored low-skilled job tasks, i.e. $\tilde{I}_O = \tilde{I}_2 - \tilde{I}_1$.

However, as discussed in the main text, by means of simple positive monotone transformation (see Lemma 3), we obtain

$$\frac{\tilde{\beta}_O(\tilde{I}_O)}{\omega} = \frac{w_L}{A_L}, \tag{B.3}$$

where $\tilde{\beta}_O(I_O)$ is the positive monotone transformation of comparative advantage schedules $\tilde{\zeta}(\cdot)\beta_L(\cdot)$ at the new task margins, $\tilde{I}_1$ and $\tilde{I}_2$, in terms of the offshoring range $\tilde{I}_O$.

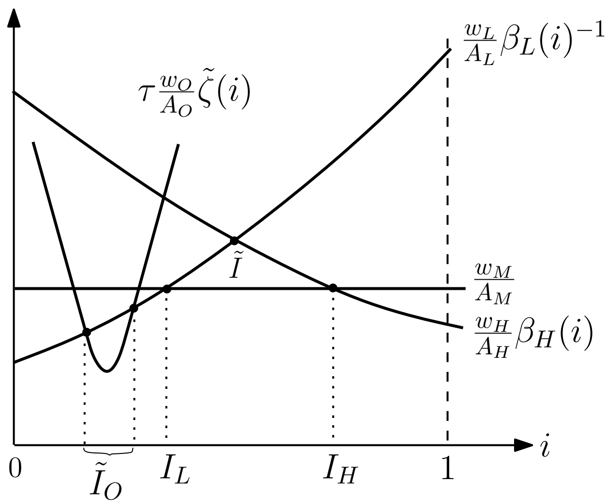
Thus the allocation of tasks is as follows: low-skilled workers perform all tasks $i \in I_L \setminus \tilde{I}_O$ and offshore workers perform all tasks $i \in \tilde{I}_O$. The task allocations between low- and medium-skilled workers and between high- and medium-skilled workers still hold as defined in Lemma 1, where medium-skilled workers perform all tasks $i \in (I_L, I_H)$, while high-skilled workers perform all tasks $i \in [I_H, 1]$. Figure 5 provides a graphical illustration.

General equilibrium solution

Equipped with these conditions, we derive the general equilibrium closed solution as follows. From the cost-minimized first-order conditions, we obtain three equations denoting the relative medium-skilled labor

[19] It is important to notice that at the tangent point $\dot{I}$, the domestic firm is indifferent between low-skilled and offshore workers. Thus, for a positive range of offshoring low-skilled job tasks, $\dot{I}_O > 0$, the offshoring friction must be sufficiently low.

Figure 5: Offshorability of low-skill-intensive job-tasks and skill-task allocation



demand

$$\frac{w_L}{w_M} = \frac{N_M}{N_L}\frac{I_L - \tilde{I}_O}{I_H - I_L} \tag{B.4}$$

$$\frac{w_O}{w_M} = \frac{N_M}{n_O}\frac{\tilde{I}_O}{I_H - I_L} \tag{B.5}$$

$$\frac{w_M}{w_H} = \frac{N_M}{N_H}\frac{I_H - I_L}{1 - I_H}. \tag{B.6}$$

Recall the no-arbitrage conditions (2) and (3)

$$\begin{aligned} \frac{w_L}{w_M} &= \frac{A_L}{A_M}\beta_L(I_L) \\ \frac{w_M}{w_H} &= \frac{A_M}{A_H}\beta_H(I_H). \end{aligned} \tag{B.7}$$

To examine changes in the relative demand between medium-skilled and offshore workers, the no-arbitrage condition (B.3) has to be adjusted for medium-skill unit costs. Thus, dividing both sides of Eq. (B.3) by w_M and using the no-arbitrage condition at I_L, Eq. (2), we obtain

$$\frac{w_O}{w_M} = \frac{A_O}{\tau}\frac{\beta_L(I_L)}{A_M}\frac{1}{\tilde{\beta}_O(\tilde{I}_O)}. \tag{B.8}$$

Moreover, notice the additional endogenous variable in Eq. (B.5): n_O, i.e. the employment level of offshore workers. To account for the endogenous adjustment of offshore employment level, combine the first-order condition for offshore labor per task with the resource constraint for offshore workers to obtain

$$\frac{n_O}{\tilde{I}_O} = \frac{P_E E}{w_O} = \frac{P_E^{-\frac{1-\alpha}{\alpha}}\mathcal{B}}{w_O} = \frac{\mathcal{B}}{w_O}\left(\beta_L(I_L)^{-1}\frac{\tau w_O}{A_O}\tilde{\beta}_O(\tilde{I}_O)\tilde{\Omega}(\cdot)\right)^{-\frac{1-\alpha}{\alpha}}, \tag{B.9}$$

where the second equality accounts for the adjustment in total employment $E = P_E^{-1/\alpha}\mathcal{B}$. In the third equality we utilized the modified no-arbitrage condition (B.8) together with the general equilibrium solution of the price index $P_E = \frac{w_M}{A_M}\tilde{\Omega}(\cdot)$, where

$$\tilde{\Omega}(\cdot) = \exp\left[\int_{i\in\tilde{I}_O} \ln\left(\frac{\beta_L(i)\tilde{\zeta}(i)}{\tilde{\beta}_O(\tilde{I}_O)}\right)\mathrm{d}i + \int_0^{I_L} \ln\left(\frac{\beta_L(I_L)}{\beta_L(i)}\right)\mathrm{d}i + \int_{I_H}^1 \ln\left(\frac{\beta_H(i)}{\beta_H(I_H)}\right)\mathrm{d}i - \int_0^1 \ln a_M(i)\mathrm{d}i\right].$$

Moreover, the convenient structure of the generalized common part of the marginal cost of factor labor $\tilde{\Omega}(\cdot)$ is preserved. To see this, define the semi-elasticities at the offshoring margin $\tilde{\mu} \equiv \frac{\partial \ln \tilde{\beta}_O(\tilde{I}_O)}{\partial \tilde{I}_O} > 0$. Taking the total differentiation w.r.t. the endogenous task margins yields

$$\begin{aligned}
\mathrm{d}\ln\tilde{\Omega}(\cdot) \quad = \quad & \Big(\ln\tilde{\zeta}(\tilde{I}_O) + \beta_L(\tilde{I}_O) - \ln\tilde{\beta}_O(\tilde{I}_O)\Big)\mathrm{d}\tilde{I}_O - \tilde{I}_O\tilde{\mu}\mathrm{d}\tilde{I}_O \\
& + \Big(\ln\beta_L(I_L) - \ln\beta_L(I_L)\Big)\mathrm{d}I_L - I_L\varepsilon_L\mathrm{d}I_L \\
& + \Big(\ln\beta_H(I_H) - \ln\beta_H(I_H)\Big)\mathrm{d}I_H + (1-I_H)\varepsilon_H\mathrm{d}I_H.
\end{aligned}$$

By the positive monotone transformation the bracket in the first line is zero. The brackets in the second and third lines are also zero, yielding

$$\mathrm{d}\ln\tilde{\Omega}(\cdot) = (1-I_H)\varepsilon_H\mathrm{d}I_H - I_L\varepsilon_L\mathrm{d}I_L - \tilde{I}_O\tilde{\mu}\mathrm{d}\tilde{I}_O. \tag{B.10}$$

Thus, similar to the discussion in the main text, the impact on the generalized marginal cost of labor can be decomposed into three terms. As shown in Eq. (B.10), the direct source of the productivity effect due to offshoring is captured by $\tilde{I}_O\tilde{\mu}\mathrm{d}\tilde{I}_O$. The first two terms, $(1-I_H)\varepsilon_H\mathrm{d}I_H$ and $I_L\varepsilon_L\mathrm{d}I_L$, allow for endogenous reallocation of domestic workers. This is what we refer to as the internal reallocation effect. Again, notice the contrast to the original task-based approach by Grossman and Rossi-Hansberg (2008), where only the former channel is captured. Thus Eq. (B.10) can be seen as the generalization of Grossman and Rossi-Hansberg (2008) regarding skill heterogeneity.

Next, utilize the previously derived Eqs. (B.22), (B.8) and (B.9) in the relative demand equations (B.4)–(B.6). Then, after manipulating, rearranging and taking logs, we obtain the general equilibrium closed solution for the three endogenous task margins $\tilde{I}_O$, I_L and I_H:

$$\begin{aligned}
\ln\left(\frac{A_L N_L}{A_M N_M}\right) \quad &= \quad -\ln\beta_L(I_L) + \ln\Big(I_L - \tilde{I}_O\Big) - \ln\left(I_H - I_L\right) \\
\ln\left(\frac{\mathcal{B}}{A_M N_M}\right) + \frac{1}{\alpha}\ln\omega \quad &= \quad -\frac{1}{\alpha}\Big(\ln\beta_L(I_L) - \ln\beta_L(\tilde{I}_O) - \ln\tilde{\beta}_O(\tilde{I}_O)\Big) - \ln\left(I_H - I_L\right) + \frac{1-\alpha}{\alpha}\ln\tilde{\Omega}(\cdot) \\
\frac{A_M N_M}{A_H N_H} \quad &= \quad -\ln\beta_H(I_H) + \ln\left(I_H - I_L\right) - \ln\left(1 - I_H\right).
\end{aligned} \tag{B.11}$$

This 3×3 system of equations (B.11) can be utilized to compute the implicit solutions for the endogenous task margins due to any exogenous changes that are captured on the right hand sides.

Comparative statics: Easier offshoring and skill-task reallocation

Taking the total differentiation of the system (B.11) yields

$$\begin{pmatrix} \left(\varepsilon_L + \frac{1}{I_L - \tilde{I}_O} + \frac{1}{I_H - I_L}\right) & -\frac{1}{I_L - \tilde{I}_O} & -\frac{1}{I_H - I_L} \\ \left([1-(1-\alpha)I_L]\frac{\varepsilon_L}{\alpha} + \frac{1}{I_H - I_L}\right) & [1-(1-\alpha)\tilde{I}_O]\frac{\tilde{\mu}}{\alpha} & \left(\frac{1-\alpha}{\alpha}(1-I_H)\varepsilon_H - \frac{1}{I_H - I_L}\right) \\ \frac{1}{I_H - I_L} & 0 & \left(\varepsilon_H + \frac{1}{I_H - I_L} + \frac{1}{1 - I_H}\right) \end{pmatrix} \begin{pmatrix} \mathrm{d}I_L \\ \mathrm{d}\tilde{I}_O \\ \mathrm{d}I_H \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\alpha\omega} \\ 0 \end{pmatrix} \mathrm{d}\omega. \tag{B.12}$$

Computing the determinant of the 3×3 matrix yields

$$\begin{aligned} \Delta \;=\;& \frac{1}{\alpha(1-I_H)(I_L-\tilde{I}_O)(I_H-I_L)^3} \\ & \left[\left(1 - I_L + (1-I_H)(I_H-I_L)\varepsilon_H\right) \times \right. \\ & \left(\tilde{\mu}(I_H-I_L)\left((I_L-\tilde{I}_O)(I_H-I_L)\varepsilon_L + (I_H-\tilde{I}_O)\right)\left(1-(1-\alpha)\tilde{I}_O\right)\right. \\ & \left.+(I_L-\tilde{I}_O)\big((I_H-I_L)\varepsilon_L\,(1-(1-\alpha)I_L)\big)\right) + \left(1+(1-I_H)\varepsilon_H\right)(I_L-\tilde{I}_O)(I_H-I_L)\alpha \\ & \left.+(1-I_H)(I_L-\tilde{I}_O)\left((1-\alpha)(I_H-I_L)\left((1-I_H)\varepsilon_H - \tilde{\mu}\tilde{I}_O\right) + \tilde{\mu}(I_H-I_L)\right)\right] > 0. \end{aligned}$$

It is evident that all terms in the numerator are positive, implying that Δ is positive too. Given this result, the implicit solutions for the task margins due to easier offshoring are

$$\begin{aligned} \frac{\mathrm{d}I_L}{\mathrm{d}\omega} &= \frac{1}{\Delta}\frac{\varepsilon_H + \frac{1}{1-I_H} + \frac{1}{I_H-I_L}}{\alpha(I_H-I_L)\omega} > 0 \\ \frac{\mathrm{d}\tilde{I}_O}{\mathrm{d}\omega} &= \frac{1}{\Delta}\frac{1}{\alpha(1-I_H)(I_L-\tilde{I}_O)(I_H-I_L)^2\omega} \\ & \quad \left[\left((1-I_H)(I_H-I_L)\varepsilon_H + (1-I_H) + (I_H-I_L)\right)\left((I_L-\tilde{I}_O)(I_H-I_L)\varepsilon_L + (I_L-\tilde{I}_O) + (I_H-I_L)\right)\right. \\ & \quad \left.+(1-I_H)(I_L-\tilde{I}_O)\right] > 0 \\ \frac{\mathrm{d}I_H}{\mathrm{d}\omega} &= \frac{1}{\Delta}\frac{1}{\alpha(I_H-I_L)^2\omega} > 0. \end{aligned}$$

The following proposition summarizes the main results.

Proposition 8 (Offshoring low-skilled job-tasks and skill-task reallocation)**.** *If offshoring activities are permitted only to low-skill-intensive job tasks, then there exists an equilibrium threshold $\tilde{I}_O$ at which domestic firms allocate offshore workers to all tasks $i \in \tilde{I}_O$, where $0 < \tilde{I}_O < I_L < I_H < 1$. Moreover, easier offshoring ($\mathrm{d}\omega > 0$) induces an expansion of offshorable job-tasks as well as a job-task upgrading by low- and medium-skilled workers, i.e.*

$$\frac{\mathrm{d}\tilde{I}_O}{\mathrm{d}\omega} > \frac{\mathrm{d}I_L}{\mathrm{d}\omega} > \frac{\mathrm{d}I_H}{\mathrm{d}\omega} > 0. \tag{B.13}$$

Given these comparative statics, we can now investigate how offshoring affects both real and relative wages of domestic workers. We relegate the discussion on relative wages to the main text and focus here on real wage effects.

Offshoring low-skilled job tasks and productivity effect

To investigate the impact of offshoring low-skilled job tasks on real wages, recall the first-order conditions defining the optimal labor demand, and utilize the resource constraint conditions and the optimal demand condition for total employment, to obtain

$$\begin{aligned}
w_L &= \frac{I_L - \tilde{I}_O}{N_L} P_E E = \frac{I_L - \tilde{I}_O}{N_L} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B} \\
w_M &= \frac{I_H - I_L}{N_M} P_E E = \frac{I_H - I_L}{N_M} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B} \\
w_H &= \frac{1 - I_H}{N_H} P_E E = \frac{1 - I_H}{N_H} P_E^{-\frac{1-\alpha}{\alpha}} \mathcal{B}.
\end{aligned}$$

Next, combine the no-arbitrage condition (B.22) with the cost index of composite labor $P_E = \frac{w_M}{A_M}\tilde{\Omega}(\cdot)$ to substitute for $\frac{w_M}{A_M}$ in the inverse labor demand function of low- and high-skilled workers, respectively. Then, after further manipulation, we obtain

$$\begin{aligned}
w_L &= \left(\frac{I_L - \tilde{I}_O}{N_L}\right)^{\alpha} \beta_L (I_L)^{1-\alpha} \tilde{\Omega}(\cdot)^{-(1-\alpha)} A_L^{1-\alpha} \mathcal{B}^{\alpha} \\
w_M &= \left(\frac{I_H - I_L}{N_M}\right)^{\alpha} \tilde{\Omega}(\cdot)^{-(1-\alpha)} A_M^{1-\alpha} \mathcal{B}^{\alpha} \\
w_H &= \left(\frac{1 - I_H}{N_H}\right)^{\alpha} \beta_H (I_H)^{-(1-\alpha)} \tilde{\Omega}(\cdot)^{-(1-\alpha)} A_H^{1-\alpha} \mathcal{B}^{\alpha}.
\end{aligned}$$

Taking logs and totally differentiating w.r.t. offshoring friction, the wage effect can be decomposed into the following terms:

$$\begin{aligned}
\frac{\mathrm{d}\ln w_L}{\mathrm{d}\omega} &= \left(\frac{\alpha}{I_L - \tilde{I}_O} - (1-\alpha)\varepsilon_L\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} + (1-\alpha)\left(I_L\varepsilon_L\frac{\mathrm{d}I_L}{\mathrm{d}\omega} - (1-I_H)\varepsilon_H\frac{\mathrm{d}I_H}{\mathrm{d}\omega}\right) \\
&\quad + \left((1-\alpha)\tilde{I}_O\tilde{\mu} - \frac{\alpha}{I_L - \tilde{I}_O}\right)\frac{\mathrm{d}\tilde{I}_O}{\mathrm{d}\omega}
\end{aligned} \tag{B.14}$$

$$\frac{\mathrm{d}\ln w_M}{\mathrm{d}\omega} = \left(\frac{\alpha}{I_H - I_L} - (1-\alpha)(1-I_H)\varepsilon_H\right)\frac{\mathrm{d}I_H}{\mathrm{d}\omega} + \left((1-\alpha)I_L\varepsilon_L - \frac{\alpha}{I_H - I_L}\right)\frac{\mathrm{d}I_L}{\mathrm{d}\omega} + (1-\alpha)\tilde{I}_O\tilde{\mu}\frac{\mathrm{d}\tilde{I}_O}{\mathrm{d}\omega} \tag{B.15}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\omega} = \left((1-\alpha)I_H\varepsilon_H - \frac{\alpha}{1 - I_H}\right)\frac{\mathrm{d}I_H}{\mathrm{d}\omega} + (1-\alpha)\left(I_L\varepsilon_L\frac{\mathrm{d}I_L}{\mathrm{d}\omega} + \tilde{I}_O\tilde{\mu}\frac{\mathrm{d}\tilde{I}_O}{\mathrm{d}\omega}\right). \tag{B.16}$$

The last term, $(1-\alpha)\tilde{I}_O\tilde{\mu}$, in all three Eqs. (B.14)–(B.16) captures the aforementioned productivity effect. Notice that in Eq. (B.14) this productivity effect is interacting with the direct displacement effect of low-skilled workers, $\frac{\alpha}{I_L - \tilde{I}_O}$. However, comparing our results with that of Grossman and Rossi-Hansberg (2008), the important difference becomes evident. Next to the direct offshoring-induced productivity effect, there are two additional forces: the first and second terms in Eqs. (B.14)–(B.16) capture the internal reallocation effect. The extent of this internal reallocation effect depends on the substitutability between low- and medium-skilled workers as well as between medium- and high-skilled workers, which are captured by the relative task productivity parameters (semi-elasticities) ε_L and ε_H, respectively. The following proposition summarizes the sufficient conditions.

Proposition 9 (Offshoring low-skilled job tasks and productivity effect)**.** *Given the impact of easier offshoring on the task reallocation derived in Proposition 8, an offshoring-induced cost-efficiency effect raises unambigu-*

ously the real wage of all skill groups for the sufficient conditions

1. $\varepsilon_L I_L > \varepsilon_H(1 - I_H)$,
2. $\frac{\alpha}{1-\alpha}\frac{1}{I_L-\tilde{I}_O} > \varepsilon_L > \frac{\alpha}{1-\alpha}\frac{1}{I_L(I_H-I_L)}$, *for* $\tilde{I}_O > I_L[1-(I_H-I_L)]$
3. $\frac{\alpha}{1-\alpha}\frac{1}{(1-I_H)(I_H-I_L)} > \varepsilon_H > \frac{\alpha}{1-\alpha}\frac{1}{I_H(1-I_H)}$
4. $\tilde{\mu} > \frac{\alpha}{1-\alpha}\frac{1}{\tilde{I}_O(I_L-\tilde{I}_O)}$.

Proof. The sufficient condition in Part 1. follows from the second term in Eq. (B.14) and the comparative static results $\frac{\mathrm{d}I_L}{\mathrm{d}\omega}$ and $\frac{\mathrm{d}I_H}{\mathrm{d}\omega}$ in Proposition 8. It states that for sufficient low substitutability between low- and medium-skilled workers compared to substitutability between medium- and high-skilled workers in the neighborhood of I_L and I_H, respectively, offshoring-induced internal reallocation of workers is limited. If this sufficient condition holds, then the second term in Eq. (B.14) is unambiguously positive. In Part 2., the lower boundary of ε_L is computed from the second term in Eq. (B.15), while the upper boundary follows from the first term in Eq. (B.14). Thus these conditions determine the sufficient range of values of substitutability between low- and medium-skilled workers. Notice however the qualifying necessary condition in this case that requires a sufficiently large range of offshoring. If this necessary condition does not hold, than the lower boundary in Part 2. becomes binding, implying that the first term in Eq. (B.14) will have a negative sign. However, it is worth noting that, even in this case, the real wage of low-skilled workers might increase if the two other positive terms in Eq. (B.14) are sufficiently strong. In Part 3., the lower boundary is computed from the first term in Eq. (B.16), while the upper boundary follows from the first term in Eq. (B.15). Finally, in Part 4., the sufficient condition following from the last term in Eq. (B.14) requires a sufficiently low substitutability between medium-skilled and offshore workers. ■

For these jointly sufficient conditions, the offshoring-induced labor supply effect on wages, characterized by the internal reallocation of domestic workers, is dominated by the overall rise in total domestic employment induced by the cost-efficiency effect due to lower offshoring frictions. This feature extends the models of Grossman and Rossi-Hansberg (2008) and Acemoglu and Autor (2011).

Offshoring high-skill-intensive domestic job tasks

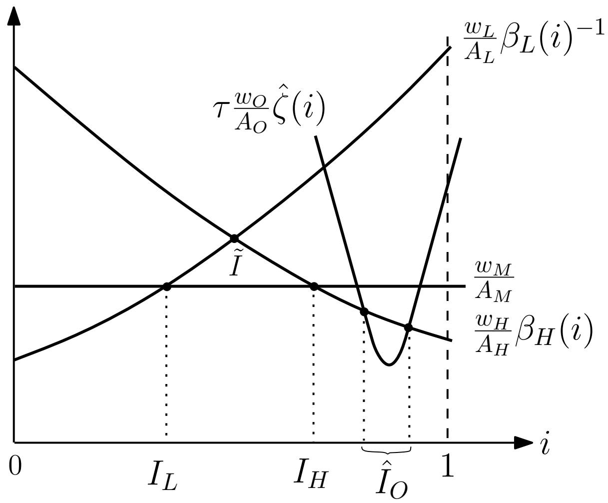
As put forward in **?**, many high-skill-intensive occupations might be affected by offshoring. Our framework can also be easily extended to account for this possible future development of globalization. To get an idea, we sketch the main properties followed by a graphical illustration.

In a similar vein, one can derive conditions characterizing the task allocation between offshore and high-skilled workers. Considering the example of the quadratic function from above, the convenient property of the U-shaped task productivity schedule can be seen. In this case, the rise in international competition on domestic high-skill-intensive job-tasks can be described by an improvement in task productivity of offshore workers. Formally, we state

$$\hat{\zeta}(i) = \exp[g_M(i - \hat{g}_O)^2],$$

where now $\hat{g}_O > g_O > \tilde{g}_O > 0$. Thus the U-shaped curve moves rightwards along the unit interval. Figure 6 provides a graphical illustration.

Figure 6: Offshoring high-skilled job tasks and skill-task allocation



Similar to the case of offshoring low-skilled job tasks, the task assignment between offshore and high-skilled workers will lead to unbundling of a fraction of high-skilled job tasks. This implies again that high-skilled workers are relatively cheaper in producing job tasks in the neighborhoods of I_H and 1, while offshore workers are cheaper somewhere in the middle. Formally, the assignment problem can be written as

$$c_O(i) \lesseqqgtr c_H(i),$$

or equivalently

$$\frac{\tau w_O}{A_O}\frac{1}{a_O(i)} \lesseqqgtr \frac{w_H}{A_H}\frac{1}{a_H(i)}, \quad \forall\, i \in (I_H, 1). \tag{B.17}$$

Now, by a simple transformation, i.e. multiplying both sides in Eq. (B.17) by $a_M(i)$, we obtain the task productivity schedules, $\beta_H(i) = \frac{a_M(i)}{a_H(i)}$ and $\hat{\zeta}(i) = \frac{a_M(i)}{a_O(i)}$, with the same properties defined by Assumptions 1 and 2. Again the structure of the model is preserved. Now the necessary conditions for offshoring to be permitted to a subrange of high-skilled job tasks are

$$\frac{A_H}{w_H}\beta_H(\ddot{I})^{-1}\hat{\zeta}(\ddot{I}) < \omega < \frac{A_H}{w_H}\beta_H(I_H)^{-1}\hat{\zeta}(I_H), \quad \text{and} \quad I_H < \hat{I}|_{\frac{\partial \ln \hat{\zeta}(\cdot)}{\partial i}=0} < 1, \tag{B.18}$$

implying that the offshoring friction is not too large (i.e. low values of $\omega = \frac{A_O}{\tau w_O}$), defined at the tangent point $\dot{I}$ between the productivity schedules $\left|\frac{\partial \beta_H(\ddot{I})}{\partial \ddot{I}}\right| = \left|\frac{\partial \tilde{\zeta}(\ddot{I})}{\partial \ddot{I}}\right|$, such that offshoring is permitted, nor too small, so that high-skilled workers do not lose their comparative advantage in competing with medium-skilled workers, defined by the task margin I_H. Then, given the assumptions and properties in (B.18), it follows from (B.17) that there must exist two cut-off margins defining the range of offshored high-skill-intensive job tasks ($\hat{I}_O$). By means of simple positive monotone transformation, this no-arbitrage condition is given by

$$\frac{\hat{\beta}_O(\hat{I}_O)}{\omega} = \frac{w_H}{A_H}, \tag{B.19}$$

where $\hat{\beta}_O(\cdot)$ denotes the positive monotone transformation of relative task productivity schedules $\frac{\hat{\zeta}(\cdot)}{\beta_H(\cdot)}$ at the new task margins, $\hat{I}_1$ and $\hat{I}_2$, in terms of the offshoring range $\hat{I}_O$.

Notice again that the no-arbitrage conditions defining the domestic allocation of tasks to skills are preserved by Lemma 1. Thus the allocation of tasks across the different types of labor is as follows: low-skilled workers perform all tasks $i \in I_L$, medium-skilled workers perform all tasks $i \in (I_H - I_L)$, high-skilled workers perform all tasks $i \in (1 - I_H - \hat{I}_O)$, and offshore workers perform all tasks $i \in \hat{I}_O$; see Figure 5 for these equilibrium allocations.

Offshoring and domestic income distribution effect

For the computation of the relative wage effects of offshoring, discussed in the main text, we need to adjust the relative labor demand between high- and medium-skilled workers for the range of offshoring $\hat{I}_O$. This follows from the new resource constraint for high-skilled workers and from the cost-minimized first-order conditions, from which the relative medium-skilled labor demand functions can be derived

$$\frac{w_L}{w_M} = \frac{N_M}{N_L}\frac{I_L}{I_H - I_L} \tag{B.20}$$

$$\frac{w_M}{w_H} = \frac{N_M}{N_H}\frac{I_H - I_L}{1 - I_H - \hat{I}_O}. \tag{B.21}$$

Recall the no-arbitrage conditions (2) and (3)

$$\begin{aligned} \frac{w_L}{w_M} &= \frac{A_L}{A_M}\beta_L(I_L) \\ \frac{w_M}{w_H} &= \frac{A_M}{A_H}\beta_H(I_H). \end{aligned} \tag{B.22}$$

This set of Eqs. (B.20), (B.21) and (B.22) can be utilized to assess the impact of a marginal decline of offshoring friction on the relative domestic wages. For an intuitive and graphical discussion we refer the reader to discussion in section 5.2 in the main text.

Supplementary Mathematical Appendix

C Proofs of Lemmata

C.1 Proof of Lemma 1

Proof. We proceed as follows. A task is allocated to a low-skilled worker rather than to a medium-skilled worker as long as

$$\begin{aligned} c_L(i) &\leq c_M(i) \\ \Leftrightarrow \frac{w_L}{A_L a_L(i)} &\leq \frac{w_M}{A_M a_M(i)} \\ \Leftrightarrow \frac{w_L}{A_L}\frac{a_M(i)}{a_L(i)} &\leq \frac{w_M}{A_M}, \end{aligned}$$

which, by Assumption 1, leads to Eq. (2) in Lemma 1 with $\beta_L(I_L) = \frac{a_M(I_L)}{a_L(I_L)}$. Similarly, tasks are allocated to medium-skilled workers as long as they are more productive relative to high-skilled workers. That is,

$$\begin{aligned} c_M(i) &\leq c_H(i) \\ \Leftrightarrow \frac{w_M}{A_M a_M(i)} &\leq \frac{w_H}{A_H a_H(i)} \\ \Leftrightarrow \frac{w_M}{A_M} &\leq \frac{a_M(i)}{a_H(i)}\frac{w_H}{A_H}, \end{aligned}$$

which, by Assumption 1, leads to Eq. (3) in Lemma 1 with $\beta_H(I_H) = \frac{a_M(I_H)}{a_H(I_H)}$. Finally, with a similar argument firms allocate tasks between low and high skills according to

$$\begin{aligned} c_L(i) &\leq c_H(i) \\ \Leftrightarrow \frac{w_L}{A_L a_L(i)} &\leq \frac{w_H}{A_H a_H(i)} \\ \Leftrightarrow \frac{w_L}{A_L}\frac{a_M(i)}{a_L(i)} &\leq \frac{a_M(i)}{a_H(i)}\frac{w_H}{A_H}, \end{aligned}$$

where, in the third inequality, we multiplied both sides by $a_M(i)$. By Assumption 1, we obtain Eq. (4) in Lemma 1 with $\beta_L(\tilde{I}) = \frac{a_M(\tilde{I})}{a_L(\tilde{I})}$ and $\beta_H(\tilde{I}) = \frac{a_M(\tilde{I})}{a_H(\tilde{I})}$. ∎

C.2 Proof of Lemma 2

Similar to the discussion in Lemma 1, the allocation decision in Lemma 2 is based on cost efficiency. However, notice that by Assumption 2 there is a non-linear relationship between the unit costs of medium-skilled and offshore workers. Formally, the allocation problem can be written as

$$\begin{aligned} c_M(i) &\lesseqgtr c_O(i) \\ \Leftrightarrow \frac{w_M}{A_M a_M(i)} &\lesseqgtr \frac{\tau w_O}{A_O a_O(i)} \\ \Leftrightarrow \frac{w_M}{A_M} &\lesseqgtr \frac{\zeta(i)}{\omega}, \end{aligned}$$

where $\frac{\partial\zeta(i)}{\partial i} < 0,\ \forall\, i < \check{I}\big|_{\frac{\partial\zeta(i)}{\partial i}=0}$ and $\frac{\partial\zeta(i)}{\partial i} > 0,\ \forall\, i > \check{I}\big|_{\frac{\partial\zeta(i)}{\partial i}=0}$. Thus, by the jointly necessary conditions in Corollary 2 there must exist two cut-off points at which $c_M(\cdot) - c_O(\cdot) = 0$. These are defined by (5) and (6), where

$$\begin{aligned} \frac{w_M}{A_M} &< \frac{\zeta(i)}{\omega}, \quad \forall\, i < I_1 \text{ and } i > I_2 \\ \frac{w_M}{A_M} &> \frac{\zeta(i)}{\omega}, \quad \forall\, i \in (I_1, I_2). \end{aligned}$$

The boundaries on ω in Corollary 2 defined as follows. The lower boundary follows from

$$\begin{aligned} c_O(\check{I}) &< c_M(\check{I}) \\ \Leftrightarrow \frac{A_M}{w_M}\zeta(\check{I}) &< \omega, \end{aligned}$$

with $\check{I}$ denoting the minimum point of $\zeta(i)$, i.e. $\frac{\partial\zeta(i)}{\partial i} = 0$. In a similar vein, one can derive the upper boundaries, respectively, at the low- and high-skill-extensive margins, i.e. $c_O(I_L) > c_M(I_L)$ and $c_O(I_H) > c_M(I_H)$. Note again that the lower boundaries ensure that offshored job-tasks are a subset of the overall medium skill-intensive range of tasks, i.e. $(I_1, I_2) \in (I_L, I_H)$. ■

C.3 Proof of Lemma 3

To keep the analytical analysis tractable, it is useful to look at changes in the offshoring interval, which reflect implicitly changes in the extensive offshoring margins, I_1 and I_2. In fact, all we need to show is how an endogenous change in the length of offshoring interval affects the domestic job-task margins, I_L and I_H. Thus we need to find a condition that satisfies the no-arbitrage condition between medium-skilled and offshore workers for the length of the offshoring interval (I_O), accounting implicitly for the two endogenous offshoring cut-off points I_1 and I_2.

First, define $\tilde{w}_M \equiv \frac{w_M \omega}{A_M}$ after recalling the two no-arbitrage conditions (5) and (6)

$$\begin{aligned} \tilde{w}_M &= \zeta(I_1), \\ \tilde{w}_M &= \zeta(I_2), \end{aligned}$$

and the definition of the length of offshoring interval

$$I_O = I_2 - I_1.$$

Next, recall the semi-elasticities at the two extensive offshoring margins, i.e. $\varepsilon_1 = -\frac{\partial \ln \zeta(I_1)}{\partial I_1} > 0$ and $\varepsilon_2 = \frac{\partial \ln \zeta(I_2)}{\partial I_2} > 0$. Taking the total differentiation, we obtain

$$\begin{aligned} \mathrm{d}\ln \tilde{w}_M &= -\varepsilon_1 \mathrm{d}I_1, \\ \mathrm{d}\ln \tilde{w}_M &= \varepsilon_2 \mathrm{d}I_2, \\ \mathrm{d}I_O &= \mathrm{d}I_2 - \mathrm{d}I_1. \end{aligned}$$

Utilizing the first two equations in the last one yields

$$\mathrm{d}I_O \quad = \quad \mathrm{d}\ln\tilde{w}_M\left(\frac{1}{\varepsilon_2}+\frac{1}{\varepsilon_1}\right).$$

It is convenient to define $\mu = \frac{\varepsilon_2\varepsilon_1}{\varepsilon_2+\varepsilon_1} > 0$, which is increasing in both arguments. Then, after further manipulation, we obtain

$$\mathrm{d}\ln\tilde{w}_M = \mu\mathrm{d}I_O.$$

This is a simple first-order linear homogeneous ordinary differential equation. Thus, by integration

$$\int \mathrm{d}\ln\tilde{w}_M\mathrm{d}i = \int \mu\mathrm{d}I_O\mathrm{d}i,$$

we obtain a unique solution

$$\frac{w_M}{A_M} = \frac{\beta_O(I_O)}{\omega},$$

where $\beta_O(I_O) = \exp[\mu I_O]$. ■

D Firm's optimization problem

Optimal labor demand

Utilizing (8) in (10), then the Lagrangian to the cost-minimizing problem (12) is read as follows:

$$\begin{aligned}\mathcal{L} \quad = \quad & \left[w_L\int_0^{I_L} l_L(i)\mathrm{d}i + w_M\int_{i\in\mathcal{S_M}} l_M(i)\mathrm{d}i + w_O\int_{i\in I_O} l_O(i)\mathrm{d}i + w_H\int_{I_H}^1 l_H(i)\mathrm{d}i\right] \\ & +\lambda\left[E-\exp\left[\int_0^{I_L}\ln(A_La_L(i)l_L(i))\mathrm{d}i + \int_{i\in\mathcal{S_M}}\ln(A_Ma_M(i)l_M(i))\mathrm{d}i\right.\right. \\ & \left.\left.+\int_{i\in I_O}\ln(A_Oa_O(i)l_O(i)/\tau)\mathrm{d}i + \int_{I_H}^1\ln(A_Ha_H(i)l_H(i))\mathrm{d}i\right]\right],\end{aligned}$$

where λ is the Lagrangian multiplier. The first-order conditions w.r.t. $l_k(i)$, $k = \{L, M, H, O\}$, and E are, respectively, given:

$$\frac{\partial\mathcal{L}}{\partial l_L(i)} \quad = \quad w_L - \lambda\frac{E}{l_L(i)} = 0, \;\Rightarrow\; l_L(i) = l_L(i') = l_L,\; \forall\, i\in[0, I_L] \tag{D.23}$$

$$\frac{\partial\mathcal{L}}{\partial l_M(i)} \quad = \quad w_M - \lambda\frac{E}{l_M(i)} = 0, \;\Rightarrow\; l_M(i) = l_M(i') = l_M,\; \forall\, i\in\mathcal{S_M} \tag{D.24}$$

$$\frac{\partial\mathcal{L}}{\partial l_H(i)} \quad = \quad w_H - \lambda\frac{E}{l_H(i)} = 0, \;\Rightarrow\; l_H(i) = l_H(i') = l_H,\; \forall\, i\in[I_H, 1] \tag{D.25}$$

$$\frac{\partial\mathcal{L}}{\partial l_O(i)} \quad = \quad w_O - \lambda\frac{E}{l_O(i)} = 0, \;\Rightarrow\; l_O(i) = l_O(i') = l_O,\; \forall\, i\in I_O. \tag{D.26}$$

The first-order conditions (D.23)–(D.26) indicate that the marginal productivity of workers across the respective range of tasks is similar and thus the required number of workers per task does not vary across the respective range of tasks. Utilizing these conditions in the constraint (10) and rearranging, we obtain

$$\begin{aligned}\lambda \quad = \quad & \exp\left[\int_0^{I_L} \ln\left(\frac{w_L}{A_L a_L(i)}\right) \mathrm{d}i + \int_{i\in\mathcal{S_M}} \ln\left(\frac{w_M}{A_M a_M(i)}\right) \mathrm{d}i\right. \\ & \left. + \int_{i\in I_O} \ln\left(\frac{\tau w_O}{A_O a_O(i)}\right) \mathrm{d}i + \int_{I_H}^{1} \ln\left(\frac{w_H}{A_H a_H(i)}\right) \mathrm{d}i\right].\end{aligned} \tag{D.27}$$

By the envelope theorem, the marginal (average) cost of the labor composite is denoted by the shadow price, i.e. $\frac{\partial \mathcal{L}}{\partial E} = \lambda$. Thus, under perfect competition, the marginal cost equals the cost index of composite labor mentioned in the text, i.e. $P_E = \lambda$.

E General equilibrium solution for changes in task margins

E.1 Comparative statics for changes in offshoring

Utilizing these observations and taking the total differentiation of the 3×3 system of equations in (22) w.r.t. ω, yields

$$\begin{pmatrix} \left(\frac{1}{I_L} + \varepsilon_L + \frac{1}{I_H - I_L - I_O}\right) & \frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} \\ \left(\frac{1}{I_H - I_L - I_O} - \frac{1-\alpha}{\alpha} I_L \varepsilon_L\right) & \left([1-(1-\alpha)I_O]\frac{\mu}{\alpha} + \frac{1}{I_H - I_L - I_O}\right) & \left(\frac{1-\alpha}{\alpha}(1-I_H)\varepsilon_H - \frac{1}{I_H - I_L - I_O}\right) \\ -\frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} & \left(\frac{1}{1-I_H} + \varepsilon_H + \frac{1}{I_H - I_L - I_O}\right) \end{pmatrix} \begin{pmatrix} \mathrm{d}I_L \\ \mathrm{d}I_O \\ \mathrm{d}I_H \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\alpha\omega} \\ 0 \end{pmatrix} \mathrm{d}\omega. \tag{E.1}$$

Computing the determinant of the 3×3 matrix, we obtain

$$\begin{aligned} \Delta_{3\times 3} = {} & \frac{1}{\alpha(1-I_H)I_L(I_H - I_L - I_O)} \\ & \Big[\alpha + I_L\varepsilon_L(\alpha + \mu(1-I_L-I_O)(1-(1-\alpha)I_O) + (1-\alpha)I_L) + \mu(1-(1-\alpha)I_O)(1-I_O) \\ & + (1-I_H)\varepsilon_H\Big(1-(1-\alpha)I_H + I_L\varepsilon_L\Big[1-(1-\alpha)(I_H-I_L) + \mu M(1-(1-\alpha)I_O)\Big] + \mu(I_H-I_O)(1-(1-\alpha)I_O)\Big)\Big] > 0. \end{aligned}$$

Given the positive sign of the determinant Δ and applying Cramer's Rule, the solution to the 3×3 system (E.1) yields the following effects on the task margins:

$$\frac{\mathrm{d}I_L}{\mathrm{d}\omega} = -\frac{1+\varepsilon_H(1-I_H)}{\alpha\omega(1-I_H)(I_H-I_L-I_O)}\frac{1}{\Delta} < 0 \tag{E.2}$$

$$\frac{\mathrm{d}I_O}{\mathrm{d}\omega} = \frac{1-I_O+\varepsilon_H(1-I_H)[I_H-I_O+\varepsilon_L I_L(I_H-I_L-I_O)]+\varepsilon_L(1-I_L-I_O)I_L}{\alpha\omega(1-I_H)I_L(I_H-I_L-I_O)\Delta} > 0 \tag{E.3}$$

$$\frac{\mathrm{d}I_H}{\mathrm{d}\omega} = \frac{1+\varepsilon_L I_L}{\alpha\omega I_L(I_H-I_L-I_O)}\frac{1}{\Delta} > 0. \tag{E.4}$$

Moreover, it can be verified that

$$\begin{aligned} \left|\frac{\mathrm{d}I_O}{\mathrm{d}\omega}\right| &> \left|\frac{\mathrm{d}I_H}{\mathrm{d}\omega}\right| + \left|\frac{\mathrm{d}I_L}{\mathrm{d}\omega}\right| \\ \Leftrightarrow 1-I_O+\varepsilon_H(1-I_H)[I_H-I_O+\varepsilon_L I_L(I_H-I_L-I_O)]+\varepsilon_L(1-I_L-I_O)I_L &> (1-I_H)(1+\varepsilon_L I_L) + I_L(1+\varepsilon_H(1-I_H)) \\ \Rightarrow (1+\varepsilon_H(1-I_H))(1+\varepsilon_L I_L) &> 0. \end{aligned} \tag{E.5}$$

E.2 Comparative statics for changes in minimum wage

Take the total differentiation of the 3×3 system of equations (37) w.r.t. to the minimum wage $\bar{W}$, and rearrange to obtain

$$\begin{pmatrix} \left(\frac{1}{I_H - I_L - I_O} + \frac{(1-(1-\alpha)I_L)\varepsilon_L}{\alpha}\right) & \left(\frac{1}{I_H - I_L - I_O} - \frac{(1-\alpha)\mu I_O}{\alpha}\right) & \left(\frac{(1-\alpha)(1-I_H)\varepsilon_H}{\alpha} - \frac{1}{I_H - I_L - I_O}\right) \\ \left(\frac{1}{I_H - I_L - I_O} - \frac{(1-\alpha)I_L\varepsilon_L}{\alpha}\right) & \left(\frac{\mu(1-(1-\alpha)I_O)}{\alpha} + \frac{1}{I_H - I_L - I_O}\right) & \left(\frac{(1-\alpha)(1-I_H)\varepsilon_H}{\alpha} - \frac{1}{I_H - I_L - I_O}\right) \\ -\frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} & \left(\varepsilon_H + \frac{1}{1-I_H} + \frac{1}{I_H - I_L - I_O}\right) \end{pmatrix} \begin{pmatrix} \mathrm{d}I_L \\ \mathrm{d}I_O \\ \mathrm{d}I_H \end{pmatrix} = \begin{pmatrix} -\frac{1}{\alpha\bar{W}} \\ 0 \\ 0 \end{pmatrix} \mathrm{d}\bar{W}. \tag{E.6}$$

Computing the determinant of the 3×3 matrix, we obtain

$$\begin{aligned} \tilde{\Delta}_{3\times 3} &= \frac{1}{\alpha^2 (1-I_H)(I_H - I_L - I_O)} \\ &\quad [\alpha\mu + \varepsilon_H (1-I_H)(\varepsilon_L [I_H(-1+\alpha+\mu[1-(1-\alpha)(I_L+I_O)]) - \mu(I_L+I_O)[1-(1-\alpha)(I_L+I_O)] + 1] + \mu[1-(1-\alpha)I_H]) \\ &\quad + \varepsilon_L(\alpha + \mu(1-(I_L+I_O)[1+(1-\alpha)(1-(I_L+I_O))])] > 0. \end{aligned} \tag{E.7}$$

Given the positive sign of $\tilde{\Delta}$ and applying Cramer's Rule, the solution to the 3×3 system (E.6) yields

$$\begin{aligned} \frac{\mathrm{d}I_L}{\mathrm{d}\bar{W}} &= -\frac{1}{\tilde{\Delta}} \frac{1}{\alpha^2 \bar{W}(1-I_H)(I_H - I_L - I_O)} \\ &\quad (\alpha - \mu I_L (1 - \varepsilon_H(1-I_H)))(1-(1-\alpha)I_O) - \varepsilon_H(1-I_H)[-I_H(\alpha+\mu+(\alpha-1)\mu I_O - 1) + \mu I_O(1-(1-\alpha)I_O) - 1] \\ &\quad + \mu(1-I_O)[1-(1-\alpha)I_O]) < 0 \\ \frac{\mathrm{d}I_O}{\mathrm{d}\bar{W}} &= \frac{1}{\tilde{\Delta}} \frac{1}{\alpha^2 \bar{W}(1-I_H)(I_H - I_L - I_O)} \\ &\quad [\alpha + \varepsilon_H(1-I_H)[1+(1-\alpha)(I_H + \varepsilon_L I_L(I_H - I_L - I_O))] - (1-\alpha)\varepsilon_L I_L(1-(I_L+I_O))] > 0 \\ \\ \frac{\mathrm{d}I_H}{\mathrm{d}\bar{W}} &= -\frac{1}{\tilde{\Delta}} \frac{\mu(1-(1-\alpha)I_O) + (1-\alpha)\varepsilon_L I_L}{\alpha^2 \bar{W}(I_H - I_L - I_O)} < 0. \end{aligned}$$

E.3 Comparative statics for changes in offshoring under minimum wage scheme

To compute the impact of easier offshoring on the equilibrium task margins, take the total differentiation of the 3×3 system (37) w.r.t. the offshoring parameter ω, and rearrange to obtain

$$\begin{pmatrix} \left(\frac{1}{I_H - I_L - I_O} + \frac{(1-(1-\alpha)I_L)\varepsilon_L}{\alpha}\right) & \left(\frac{1}{I_H - I_L - I_O} - \frac{(1-\alpha)\mu I_O}{\alpha}\right) & \left(\frac{(1-\alpha)(1-I_H)\varepsilon_H}{\alpha} - \frac{1}{I_H - I_L - I_O}\right) \\ \left(\frac{1}{I_H - I_L - I_O} - \frac{(1-\alpha)I_L\varepsilon_L}{\alpha}\right) & \left(\frac{\mu(1-(1-\alpha)I_O)}{\alpha} + \frac{1}{I_H - I_L - I_O}\right) & \left(\frac{(1-\alpha)(1-I_H)\varepsilon_H}{\alpha} - \frac{1}{I_H - I_L - I_O}\right) \\ -\frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} & \left(\varepsilon_H + \frac{1}{1-I_H} + \frac{1}{I_H - I_L - I_O}\right) \end{pmatrix} \begin{pmatrix} \mathrm{d}I_L \\ \mathrm{d}I_O \\ \mathrm{d}I_H \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\alpha\omega} \\ 0 \end{pmatrix} \mathrm{d}\omega. \tag{E.8}$$

Given the positive sign of the determinant matrix on the left hand side, derived in Eq. (E.7), and applying Cramer's Rule, the solution to the 3×3 system (E.8), we obtain the following solutions of the comparative statics:

$$\begin{aligned} \frac{\mathrm{d}I_L}{\mathrm{d}\omega} &= -\frac{1}{\tilde{\Delta}} \frac{1}{\alpha^2\omega\,(1-I_H)\,(I_H - I_L - I_O)} \\ &\quad \alpha - (1-\alpha)\mu I_O\left(\varepsilon_H\,(1-I_H)\,(I_H - I_L) + 1 - I_L\right) + (1-\alpha)\mu I_O^2\left(1 + (1-I_H)\,\varepsilon_H\right) + \varepsilon_H\,(1-I_H)\,(1-(1-\alpha)I_H) \\ \frac{\mathrm{d}I_O}{\mathrm{d}\omega} &= -\frac{1}{\tilde{\Delta}} \frac{1}{\alpha^2\omega\,(I_H - 1)\,(I_H - I_L - I_O)} \\ &\quad \left[\alpha + \varepsilon_H\,(1-I_H)\left(\varepsilon_L\,(1-(1-\alpha)I_L)\,(I_H - I_L - I_O) + 1 - (1-\alpha)I_H\right) + \varepsilon_L\,(1 - I_L - I_O)\,(1-(1-\alpha)I_L)\right] > 0 \end{aligned}$$

$$\frac{\mathrm{d}I_H}{\mathrm{d}\omega} = \frac{1}{\tilde{\Delta}} \frac{\varepsilon_L\,(1-(1-\alpha)I_L) + (1-\alpha)\mu I_O}{\alpha^2\omega\,(I_H - I_L - I_O)} > 0.$$

E.4 Comparative statics for changes in offshoring under endogenous low-skilled labor supply

Recall the 4×4 system of equations (42). Taking total differentiation w.r.t. ω, we obtain

$$\begin{pmatrix} \varepsilon_L + \frac{1}{I_L} + \frac{1}{I_H - I_L - I_O} & \frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} & \frac{1}{1-u_L} \\ \frac{1}{I_H - I_L - I_O} - \frac{(1-\alpha) I_L \varepsilon_L}{\alpha} & \frac{\mu(1-(1-\alpha)I_O)}{\alpha} + \frac{1}{I_H - I_L - I_O} & \frac{(1-\alpha)(1-I_H)\varepsilon_H}{\alpha} - \frac{1}{I_H - I_L - I_O} & 0 \\ -\frac{1}{I_H - I_L - I_O} & -\frac{1}{I_H - I_L - I_O} & \varepsilon_H + \frac{1}{1-I_H} + \frac{1}{I_H - I_L - I_O} & 0 \\ (1-\alpha)(1-I_L)\varepsilon_L - \frac{\alpha}{I_L} & -(1-\alpha) I_O \mu & (1-\alpha)(1-I_H)\varepsilon_H & -\left(\frac{\alpha}{1-u_L} + \frac{\tilde{\delta}}{u_L}\right) \end{pmatrix} \times \begin{pmatrix} \mathrm{d}I_L \\ \mathrm{d}I_O \\ \mathrm{d}I_H \\ \mathrm{d}u_L \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\alpha\omega} \\ 0 \\ 0 \end{pmatrix} \mathrm{d}\omega. \tag{E.9}$$

Computing the determinant of the matrix yields

$$\begin{aligned} \dot{\tilde{\Delta}}_{4\times4} \quad = \quad & -\frac{1}{\alpha(1-I_H)I_L(I_H - I_L - I_O)(1-u_L)u_L} \\ & \Big(\tilde{\delta}\Big[\alpha + I_L\varepsilon_L[\alpha + \mu(1-I_L-I_O)(1-(1-\alpha)P) + (1-\alpha)I_L] + \mu(1-(1-\alpha)I_O)(1-I_O) \\ & +(1-I_H)\varepsilon_H\big(I_L\varepsilon_L[1-(1-\alpha)(I_H-I_L) + \mu(I_H-I_L-I_O)(1-(1-\alpha)I_O)] + \mu(I_H-I_O)(1-(1-\alpha)I_O) - (1-\alpha)I_H\big)\Big] \\ & +I_L u_L\Big[\alpha\mu + \varepsilon_L[\mu(1-I_L-I_O)(1-(1-\alpha)(I_L+I_O)) + \alpha] \\ & +(1-I_H)\varepsilon_H\left(\mu(1-(1-\alpha)I_H) + \varepsilon_L[1-(1-\alpha)I_H + \mu(I_H-I_L-I_O)(1-(1-\alpha)(I_L+I_O))]\right)\Big]\Big) < 0. \end{aligned}$$

All four terms in the numerator are positive and thus $\dot{\tilde{\Delta}} < 0$. Given the negative sign of $\dot{\tilde{\Delta}}$ and applying Cramer's Rule, the solution to the 4×4 system (E.9) yields the following unambiguous effects on

- the low-skilled job-task margin

$$\begin{aligned} \frac{\mathrm{d}I_L}{\mathrm{d}\omega} \quad = \quad & \frac{1}{\dot{\tilde{\Delta}}} \frac{1}{\alpha\omega(1-I_H)(1-u_L)u_L(I_H-I_L-I_O)} \\ & \Big[\tilde{\delta} + u_L\Big(\alpha + (\alpha-1)\mu I_O(\varepsilon_H(1-I_H)(I_H-I_L) + 1 - I_L) + (1-\alpha)\mu I_O^2(1+\varepsilon_H(1-I_H)) + \varepsilon_H(1-I_H)(1-(1-\alpha)I_H)\Big) \\ & +\varepsilon_H\tilde{\delta}(1-I_H)\Big] < 0 \end{aligned} \tag{E.10}$$

- the offshoring job-task margin

$$
\begin{aligned}
\frac{\mathrm{d}I_O}{\mathrm{d}\omega} = & -\frac{1}{\dot{\tilde{\Delta}}}\frac{1}{\alpha\omega\left(1-I_H\right)I_L\left(1-u_L\right)u_L\left(I_H-I_L-I_O\right)} \\
& \Big[\alpha I_L u_L + \tilde{\delta}\left[1-I_O\left(1+\varepsilon_H\left(1-I_H\right)\right)\right] + \varepsilon_H\left(1-I_H\right)\Big(I_L u_L\left(1-(1-\alpha)I_H\right)+\tilde{\delta}I_H\Big) \\
& +\varepsilon_L I_L\left[\varepsilon_H\left(1-I_H\right)\left(I_H-I_L-I_O\right)+1-I_L-I_O\right]\Big(\tilde{\delta}+\left(1-(1-\alpha)I_L\right)u_L\Big)\Big] > 0.
\end{aligned} \tag{E.11}
$$

- the high-skilled job-task margin

$$
\frac{\mathrm{d}I_H}{\mathrm{d}\omega} = -\frac{1}{\dot{\tilde{\Delta}}}\frac{I_L\left[\mu u_L I_O(1-\alpha)+\varepsilon_L\left(\tilde{\delta}+u_L\left(1-(1-\alpha)I_L\right)\right)\right]+\tilde{\delta}}{\alpha\omega I_L\left(1-u_L\right)u_L\left(I_H-I_L-I_O\right)} > 0 \tag{E.12}
$$

- the low-skilled unemployment rate

$$
\begin{aligned}
\frac{\mathrm{d}u_L}{\mathrm{d}\omega} = & -\frac{1}{\dot{\tilde{\Delta}}}\frac{1}{\alpha(1-I_H)I_L(I_H-I_L-I_O)\omega} \\
& \Big[\alpha+(1-I_H)\varepsilon_H\Big(1-(1-\alpha)[I_H+\mu I_O(I_H-I_O)]\Big) \\
& -(1-\alpha)I_L\varepsilon_L\Big((1-I_H)\varepsilon_H[I_H-I_L+\mu(I_H-I_L-I_O)I_O]+\mu I_O(1-I_L-I_O)+1-I_L\Big)-(1-\alpha)\mu I_O(1-I_O)\Big] \lesseqgtr 0
\end{aligned} \tag{E.13}
$$

To determine the sign of $\frac{\mathrm{d}u_L}{\mathrm{d}\omega}$, notice first the following limiting case, when offshoring is strongly limited, such that $I_O \to 0$. Then, the term in the square brackets reduces to

$$
\alpha+(1-I_H)\varepsilon_H-\Big((1-I_H)\varepsilon_H(1-\alpha)I_H+(1-\alpha)I_L\varepsilon_L(1-I_H)\varepsilon_H(I_H-I_L)+(1-\alpha)I_L\varepsilon_L(1-I_L)\Big), \tag{E.14}
$$

where only the first two terms are positive. Thus, as offshoring becomes easier, the terms in the brackets become larger. However, we can define sufficient conditions so that (E.13) is always negative, irrespective of the level of offshoring. The first condition follows from the interaction between α and the first term within the brackets in (E.14):

$$
\varepsilon_H > \frac{\alpha}{(1-\alpha)I_H(1-I_H)}.
$$

The second condition follows from the interaction between $(1-I_H)\varepsilon_H$ and the second term in (E.14)

$$
\varepsilon_L > \frac{1}{(1-\alpha)I_L(I_H-I_L)}.
$$

The intuition behind this condition is provided in the main text; see discussion of Proposition 7.

F Proof of Proposition 1: Uniqueness of Task Margins

To verify the uniqueness of the equilibrium task margins, we discuss the equilibrium properties of the domestic task margins from system (21). The proof of the claim regarding uniqueness of offshoring margins needs further elaboration. We commence with the equilibrium characteristics of the margins I_L and I_H.

Uniqueness of domestic task margins

From (21a), we can rearrange so that

$$I_H = (I_2 - I_1) + (1 + \mathcal{A}_{\mathcal{L}}\beta_L(I_L))\, I_L \equiv I_O + \mathcal{F}_L(I_L). \tag{21a$'$}$$

Analogously, (21c) can be rearranged so that

$$I_L = -(I_2 - I_1) + I_H - \mathcal{A}_{\mathcal{H}}\beta_H(I_H)(1 - I_H) \equiv -I_O + \mathcal{F}_H(I_H), \tag{21c$'$}$$

where $\mathcal{A}_{\mathcal{L}} \equiv \frac{A_M N_M}{A_L N_L}$ and $\mathcal{A}_{\mathcal{H}} \equiv \frac{A_M N_M}{A_H N_H}$ summarize the exogenous labor-augmenting technology and labor endowment variables. Therefore Eqs. (21a′) and (21c′) highlight the general equilibrium relation between I_L and I_H for any given value of $I_O \in (I_L, I_H)$. Hence any changes in the offshoring extensive margins will shift (21a′) and (21c′), while changes in I_L and I_H, captured by $\mathcal{F}_k(\cdot),\ k = \{L, H\}$, affect the slope. To verify the single-crossing between Eqs. (21a′) and (21c′), we need to examine the properties of the slope of the two curves.

Consider first the properties of Eq. (21a′). It can be shown that I_H is monotonically increasing in I_L. Recalling the property $\beta_L'(I_L) > 0$, then the first derivative implies

$$\frac{\partial \mathcal{F}_L}{\partial I_L} = 1 + \mathcal{A}_{\mathcal{L}}[\beta_L(I_L) + \beta_L'(I_L)I_L] > 0,$$

indicating that the slope of (21a′) is larger than unity.

Moreover, it indicates that the increase at the low-skill-intensive task margin is accompanied by a reduction of high-skill-intensive tasks, i.e. lower $(1 - I_H)$. Intuitively, this implies that the tasks previously performed by medium-skilled workers at I_L are now produced by low-skilled workers. This induces an excess of the medium-skilled labor supply due to the Walrasian nature of the labor markets. Thus medium-skilled wages will decline, so that they become more competitive at the high-skill-extensive task margin I_H. This is then accompanied by increase in I_H.

The monotonic relation between I_L and I_H depends on the properties of the second derivative, which in turn depends on the functional properties of the productivity schedule $\beta_L(\cdot)$. To examine these properties, we proceed as follows. Let $\beta_L(i)$ be a homogeneous function of degree $1 \geq \lambda_L > 0$, indicating a concave function. Then, it is generally valid $|\beta_L^{'}(i)| > |\beta_L^{''}(i)|$. If, on the other hand, $\beta_L(i)$ is homogeneous of degree $\lambda_L > 1$ (indicating a convex function), then $0 < \beta_L^{'}(i) < \beta_L^{''}(i)$. Thus, for any functional property of $\beta_L(\cdot)$, the second derivative yields $\frac{\partial^2 \mathcal{F}_L}{\partial I_L^2} = \mathcal{A}_{\mathcal{L}}(2\beta_L^{'}(I_L) + \beta_L^{''}(I_L)I_L) > 0$. This indicates that $\mathcal{F}_L$ is monotonically increasing in I_L. Furthermore, computing the limits of $\mathcal{F}_L$ over the range defined by Corollaries 1 and 2, yields

$$\lim_{I_L \to 0} \mathcal{F}_L = 0 \quad \Rightarrow \quad \lim_{I_L \to 0} I_H = I_2 - I_1 = I_O$$
$$\lim_{I_L \to I_1} \mathcal{F}_L = I_1 + \mathcal{A}_{\mathcal{L}} \beta_L(I_1) I_1 \quad \Rightarrow \quad \lim_{I_L \to I_1} I_H = I_2 + \mathcal{A}_{\mathcal{L}} \beta_L(I_1) I_1. \tag{F.15}$$

Next consider Eq. (21c′). Similarly, it can be verified that I_L is monotonically increasing in I_H, again for any (fixed) values of $I_O \in (I_L, I_H)$. Recall the property $\beta'_H(I_H) < 0$, then formally

$$\frac{\partial \mathcal{F}_H}{\partial I_H} = 1 + \mathcal{A}_{\mathcal{H}}[\beta_H(I_H) - \beta'_H(I_H)(1 - I_H)] > 0,$$

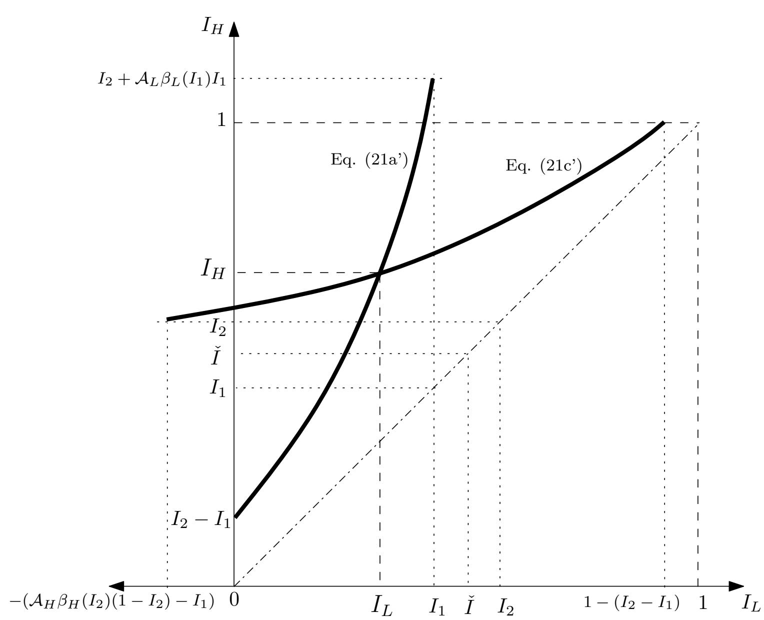
implying a slope of (21c′) larger than unity. In a similar vein, if $\beta_H(i)$ is homogeneous of degree $-1 < \lambda_H < 0$ (i.e. a concave functional form), then $\beta''_H(i) < \beta'_H(i) < 0$. If $\beta_H(i)$ is a homogeneous function of degree $\lambda_H < -1$ (indicating a convex functional form), then $\beta'_H(i) < 0 < \beta''_H(i)$. Thus, computing the second derivative yields $\frac{\partial^2 \mathcal{F}_H}{\partial I_H^2} = \mathcal{A}_{\mathcal{H}}(2\beta'_H(I_H) - \beta''_H(I_H)(1 - I_H)) < 0$. Similarly, computing the limits of $\mathcal{F}_H$ over the range defined by Corollaries 1 and 2, we get

$$\lim_{I_H \to I_2} \mathcal{F}_H = I_2 - \mathcal{A}_{\mathcal{H}} \beta_H(I_2)(1 - I_2) \quad \Rightarrow \quad \lim_{I_H \to I_2} I_L = -(\mathcal{A}_{\mathcal{H}} \beta_H(I_2)(1 - I_2) - I_1)$$
$$\lim_{I_H \to 1} \mathcal{F}_H = 1 \quad \Rightarrow \quad \lim_{I_H \to 1} I_L = 1 - (I_2 - I_1). \tag{F.16}$$

Utilizing the properties of $\mathcal{F}_L$ and $\mathcal{F}_H$ derived in (F.15) and (F.16), we can depict Eqs. (21a′) and (21c′) in the (I_L, I_H)-space. The graphical illustration in Figure 7 reveals the single crossing between the two curves for all values within the bounded intervals.

Obviously, if the offshoring range becomes very large such that $I_O \notin (I_L, I_H)$, then the intersection of the two curves, given by Eqs. (21a′) and (21c′), will be off the unit interval. More precisely, an increase

Figure 7: Unique equilibrium of low- and high-skilled task margins



in $I_O = (I_2 - I_1)$ induces a parallel left-ward shift in the two curves implying unambiguously an increase in I_H and a decline in I_L.[20] The fact that there is a parallel shift can be seen by computing the limits of (21a′) and (21c′) over the unit interval. Formally this is given by $\lim_{I_L \to 1} I_H = 1 + (I_2 - I_1) + \mathcal{A}_{\mathcal{L}}\beta_L(1)$ and $\lim_{I_H \to 0} I_H = -((I_2 - I_1) + \mathcal{A}_{\mathcal{H}}\beta_H(0)$. Thus it is readily evident that changes in the set of offshorable tasks lead to a shift in both curves, whereas changes in factor-biased technology or endowments captured by the terms $\mathcal{A}_{\mathcal{L}}$ and $\mathcal{A}_{\mathcal{H}}$ will also affect the slope of both curves. We summarize by the following Lemma the first sufficient condition.

Lemma 4. *For any given values of $I_O \in (I_L, I_H)$, Eqs. (21a′) and (21c′) determine the unique values of I_L and I_H in the (I_L, I_H)-space.*

Unique solution of offshoring task margins

In order to assess the uniqueness of the offshoring task margins, we need to account explicitly for them. In doing so, recall the offshoring no-arbitrage conditions (5) and (6). Combining these conditions with the equilibrium relative demand conditions, we obtain

$$\frac{\mathcal{B}}{N_M A_M}\omega^{\frac{1}{\alpha}} = \frac{\zeta_1(I_1)^{\frac{1}{\alpha}}\Omega(\cdot)^{\frac{1-\alpha}{\alpha}}}{I_H - I_L - (I_2 - I_1)} \tag{F.17}$$

$$\frac{\mathcal{B}}{N_M A_M}\omega^{\frac{1}{\alpha}} = \frac{\zeta_2(I_2)^{\frac{1}{\alpha}}\Omega(\cdot)^{\frac{1-\alpha}{\alpha}}}{I_H - I_L - (I_2 - I_1)}. \tag{F.18}$$

The equilibrium properties of the offshoring margins can be assessed as follows. For all $I_L \in (0, I_2)$ and $I_H \in (I_2, 1)$, Eqs. (F.17) and (F.18) determine the equilibrium values of I_1 and I_2. However, notice that, due to the non-linearity, we derive the implicit solution by means of the Implicit Function Theorem (IFT).

Consider first Eq. (F.17). Rearranging slightly yields

$$\mathcal{A}_{\mathcal{O}}(I_H - I_L - (I_2 - I_1)) = \zeta(I_1)^{\frac{1}{\alpha}}\left(\Omega(\cdot)\right)^{\frac{1-\alpha}{\alpha}}, \tag{F.17′}$$

with $\mathcal{A}_{\mathcal{O}} \equiv \frac{\mathcal{B}}{N_M A_M}\omega^{\frac{1}{\alpha}}$.

Take logs in (F.17′) and define

$$\mathcal{G}(I_1, I_2) \equiv \ln \mathcal{A}_{\mathcal{O}} + \ln(I_H - I_L - (I_2 - I_1)) - \frac{1}{\alpha}\ln\zeta(I_1) - \frac{1-\alpha}{\alpha}\ln\Omega(I_1, I_2, \cdot) = 0. \tag{F.17′′}$$

Then, taking for the moment I_L and I_H as given, by IFT we obtain

$$\frac{\mathrm{d}I_2}{\mathrm{d}I_1} = -\frac{\mathcal{G}_{I_1}(I_1, I_2)}{\mathcal{G}_{I_2}(I_1, I_2)}, \tag{F.19}$$

where $\mathcal{G}_{I_1}(I_1, I_2)$ and $\mathcal{G}_{I_2}(I_1, I_2)$ denote the partial derivatives w.r.t. the extensive margins and are defined by

$$\frac{\partial\mathcal{G}(I_1, I_2)}{\partial I_1} = \frac{1}{I_H - I_L - (I_2 - I_1)} - \frac{1}{\alpha}\frac{\zeta_{I_1}(I_1)}{\zeta(I_1)} - \frac{1-\alpha}{\alpha}\frac{\Omega_{I_1}(\cdot)}{\Omega(\cdot)}$$

$$\frac{\partial\mathcal{G}(I_1, I_2)}{\partial I_2} = -\frac{1}{I_H - I_L - (I_2 - I_1)} - \frac{1-\alpha}{\alpha}\frac{\Omega_{I_2}(\cdot)}{\Omega(\cdot)},$$

[20] Since the two curves have different slopes in absolute values, any parallel shift induces an unambiguous change in both margins.

where[21]

$$\frac{\Omega_{I_1}(\cdot)}{\Omega(\cdot)} = -(\check{I} - I_1)\frac{\zeta_{I_1}(I_1)}{\zeta(I_1)}, \quad \frac{\Omega_{I_2}(\cdot)}{\Omega(\cdot)} = -(I_2 - \check{I})\frac{\zeta_{I_2}(I_2)}{\zeta(I_2)},$$

with $\zeta_j(j)$ denoting the partial derivative w.r.t. to $j = \{I_1, I_2\}$. Note also that the IFT requires $\mathcal{G}_{I_2}(I_1, I_2) \neq 0$. Recalling the definition of the elasticities of the task productivities at the extensive offshore margins $\tilde{\varepsilon}_{\zeta_1} = -\frac{\zeta_{I_1}(I_1)}{\zeta(I_1)} I_1 > 0$ and $\varepsilon_{\zeta_2} = \frac{\zeta_{I_2}(I_2)}{\zeta(I_2)} I_2 > 0$, then utilizing the solutions of the partial derivatives in (F.19), we get

$$\frac{\mathrm{d}I_2}{\mathrm{d}I_1} = -\left(\frac{\frac{I_1}{I_H - I_L - (I_2 - I_1)} + \left(1 - (1-\alpha)(\check{I} - I_1)\right)\frac{\tilde{\varepsilon}_{\zeta_1}}{\alpha}}{-\frac{I_2}{I_H - I_L - (I_2 - I_1)} + (1-\alpha)(I_2 - \check{I})\frac{\varepsilon_{\zeta_2}}{\alpha}}\frac{I_2}{I_1}\right).$$

Rearranging slightly yields

$$\frac{\hat{I}_2}{\hat{I}_1} = \left(\frac{\alpha I_1 + \left(1 - (1-\alpha)(\check{I} - I_1)\right)(I_H - I_L - (I_2 - I_1))\tilde{\varepsilon}_{\zeta_1}}{\alpha I_2 - (1-\alpha)(I_2 - \check{I})(I_H - I_L - (I_2 - I_1))\varepsilon_{\zeta_2}}\right) \equiv q(I_1, I_2, \cdot), \tag{F.19$'$}$$

where $\hat{x} \equiv \frac{\mathrm{d}x}{x}$ denotes the rate of change. Thus, the right hand side of (F.19′), $q(I_1, I_2)$, denotes the elasticity.

We now turn to the implicit behavior of Eq. (F.18). Rearrange this equation to obtain

$$\mathcal{A}_{\mathcal{O}}(I_H - I_L - (I_2 - I_1)) = \zeta(I_2)^{\frac{1}{\alpha}} \left(\Omega(\cdot)\right)^{\frac{1-\alpha}{\alpha}}. \tag{F.18$'$}$$

Now, following the same steps considered for the derivation of (F.19′), we get a second implicit relation between I_2 and I_1. Formally, it is given by

$$\frac{\mathrm{d}I_2}{\mathrm{d}I_1} = -\left(\frac{-\frac{I_1}{(I_H - I_L - (I_2 - I_1))} + (1-\alpha)(\check{I} - I_1)\frac{\tilde{\varepsilon}_{\zeta_1}}{\alpha}}{\frac{I_2}{(I_H - I_L - (I_2 - I_1))} + \left(1 - (1-\alpha)(I_2 - \check{I})\right)\frac{\varepsilon_{\zeta_2}}{\alpha}}\frac{I_2}{I_1}\right). \tag{F.20}$$

Rearranging slightly, yields

$$\frac{\hat{I}_2}{\hat{I}_1} = \left(\frac{\alpha I_1 - (1-\alpha)(\check{I} - I_1)(I_H - I_L - (I_2 - I_1))\tilde{\varepsilon}_{\zeta_1}}{\alpha I_2 + \left(1 - (1-\alpha)(I_2 - \check{I})\right)(I_H - I_L - (I_2 - I_1))\varepsilon_{\zeta_2}}\right) \equiv z(I_1, I_2, \cdot), \tag{F.20$'$}$$

where the right-hand side, $z(I_1, I_2)$, denotes the elasticity between I_2 and I_1.

In order to assess the behavior of the two implicit relations derived in (F.19′) and (F.20′), we need to elaborate on the sign of the elasticities. It is evident that the sign of the two elasticities $q(\cdot)$ and $z(\cdot)$ depends, respectively, on the sign of the denominator and the numerator. That is, from (F.19′)

$$q(\cdot) = \begin{cases} > 0 & \text{if } \frac{\alpha}{1-\alpha}\frac{I_2}{(I_2 - \check{I})(I_H - I_L - (I_2 - I_1))} > \varepsilon_{\zeta_2} \\ < 0 & \text{if } \frac{\alpha}{1-\alpha}\frac{I_2}{(I_2 - \check{I})(I_H - I_L - (I_2 - I_1))} < \varepsilon_{\zeta_2}. \end{cases} \tag{F.21}$$

[21]The derivative of $\Omega(\cdot)$ w.r.t. any task margin can be computed considering the following general case: if $f(x, \cdot) = \exp[g(x) + \ldots]$, then $\partial f(x)/\partial x = g'(x) f(x)$. Here, $g(I_1) = \int_{I_1}^{\check{I}} \ln\left(\frac{\zeta(i)}{\zeta(I_1)}\right) \mathrm{d}i$ and $g'(I_1) = -\ln\zeta(I_1) - (\check{I} - I_1)\frac{\zeta_{I_1}(I_1)}{\zeta(I_1)} + \ln\zeta(I_1) = -(\check{I} - I_1)\frac{\zeta_{I_1}(I_1)}{\zeta(I_1)}$.

Similarly, from (F.20′)

$$z(\cdot) = \begin{cases} > 0 & \text{if } \frac{\alpha}{1-\alpha}\frac{I_1}{(\check{I}-I_1)(I_H-I_L-(I_2-I_1))} > \tilde{\varepsilon}_{\zeta_1} \\ < 0 & \text{if } \frac{\alpha}{1-\alpha}\frac{I_1}{(\check{I}-I_1)(I_H-I_L-(I_2-I_1))} < \tilde{\varepsilon}_{\zeta_1}. \end{cases} \tag{F.22}$$

Thus the magnitude (in absolute values) of the elasticities of the task productivity schedules at the respective extensive margins determines the implicit relation between I_2 and I_1. Put differently, it is obvious that, if $q(\cdot)$ and $z(\cdot)$ have opposite signs, there must be a single crossing in the (I_1, I_2)-space, for all $I_L \in (0, I_1)$, $I_H \in (I_2, 1)$. Thus, when both have equal signs it is important to verify that one of the elasticities is larger (in absolute values). In doing so, define $\alpha_1 \equiv \alpha I_1$, $\alpha_2 \equiv \alpha I_2$, $\breve{\alpha} \equiv (1-\alpha)(I_H - I_L - (I_2 - I_1))$, $\mathcal{S}_{\mathcal{M}} \equiv (I_H - I_L - (I_2 - I_1))$, then it can be shown that

$$\begin{gathered} |q(\cdot)| > |z(\cdot)| \\ \left|\left(\frac{\alpha_1 + \left(\mathcal{S}_{\mathcal{M}} - (\check{I} - I_1)\breve{\alpha}\right)\tilde{\varepsilon}_{\zeta_1}}{\alpha_2 - (I_2 - \check{I})\breve{\alpha}\varepsilon_{\zeta_2}}\right)\right| > \left|\left(\frac{\alpha_1 - (\check{I} - I_1)\breve{\alpha}\tilde{\varepsilon}_{\zeta_1}}{\alpha_2 + \left(\mathcal{S}_{\mathcal{M}} - \breve{\alpha}(I_2 - \check{I})\right)\varepsilon_{\zeta_2}}\right)\right| \\ [\alpha_2 + \left(\mathcal{S}_{\mathcal{M}} - \breve{\alpha}(I_2 - \check{I})\right)\varepsilon_{\zeta_2}][\alpha_1 + \left(\mathcal{S}_{\mathcal{M}} - (\check{I} - I_1)\breve{\alpha}\right)\tilde{\varepsilon}_{\zeta_1}] > (\alpha_1 - (\check{I} - I_1)\breve{\alpha}\tilde{\varepsilon}_{\zeta_1})(\alpha_2 - (I_2 - \check{I})\breve{\alpha}\varepsilon_{\zeta_2}) \\ \mathcal{S}_{\mathcal{M}}(\alpha_1\varepsilon_{\zeta_2} + \alpha_2\tilde{\varepsilon}_{\zeta_1}) > 0. \end{gathered}$$

The next lemma summarizes the second sufficient condition.[22]

Lemma 5. *For any values of $I_L \in (0, I_1)$ and $I_H \in (I_2, 1)$, the sufficient conditions by the Existence and Uniqueness Theorem (EUT) state that for all values in the intervals $I_1 \in (I_L, \check{I})$ and $I_2 \in (\check{I}, I_H)$ (i) a solution exists if $q(\cdot)$ and $z(\cdot)$ are continuous, and (ii) the solution is unique if both $\frac{\partial q(\cdot)}{\partial I_2}$ and $\frac{\partial z(\cdot)}{\partial I_2}$ are also continuous. Then, there exist unique values of I_1 and I_2 in the (I_1, I_2)-space.*

Thus Lemmas 4 and 5 establish the sufficient conditions for the uniqueness of the equilibrium values of the four endogenous margins. ■

[22] For a general discussion of the Existence and Uniqueness Theorem see Gandolfo (2010, Ch. 23.1.1).