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A Tale of Two Countries: Sovereign Default, Exchange Rate, and Trade

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Abstract

This paper explores the impacts of sovereign defaults on trade and income through a real exchange rate channel, in a DSGE model of two risk-averse open economies, with production. In the model, once the borrower country defaults due to an adverse productivity shock, foreign firms reduce their imports of intermediate goods from the defaulting country, whose income consequently declines. This causes the defaulting country to adjust its consumption portfolio of domestic goods and imports according to its home bias preference, triggers a collapse in its real exchange rate, and leads to a further endogenous plummet in national income. This paper makes three main contributions. First, along business cycles, the model generates countercyclical trade balances, procyclical trade flows, and countercyclical bond spreads with a data-consistent average. Second, following a sovereign default, the model endogenously delivers sharp real exchange rate deterioration, output drops, trade balance improvements, and bilateral trade flow declines. This paper thus also studies a real exchange rate channel, through which default risks and occurrences, income, and trade interact with each other. Lastly, this model predicts lasting welfare gains for the creditor country through the real exchange rate channel, but relatively short-lived welfare losses for the borrower country and the world during and after a sovereign default.

Keywords: sovereign default, real exchange rate, trade, DSGE.

JEL code: E44 - F34 - F31 - F41

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1 Introduction

Sovereign default events are associated with three empirical regularities: (a) deep recessions, (b) international goods trade declines with reversed balance, and (c) falling real exchange rates. Recent evidence shows that, across countries, default episodes have on average been accompanied by a GDP drop of 5 percent below trend, a bilateral trade value decline of 8 percent, a net export increase of about 10 percentage points of GDP, and real depreciation of around 30-50 percent.[1]

The existing quantitative sovereign debt literature, such as Aguiar and Gopinath (2006), Arellano (2008), and Mendoza and Yue (2012), based on Eaton and Gersovitz (1981), has made significant contributions in accounting for countercyclical net exports and other key empirical patterns of the default episodes of developing countries. However, those small open economy models do not address defaults' interactions with 1) bilateral trade flows, 2) real exchange rates, and 3) creditors' welfare. Using a two-country model with endogenous defaultable bonds and goods trade, this paper addresses these three aspects, which are largely left out of the current literature.

First, this paper's theoretical model is able to study default-triggered changes of both trade balances and bilateral trade, with separate import and export flows. As emphasized by the empirical literature, it is not only that trade surpluses improve during default episodes, but also that bilateral trade declines (Rose, 2005). This paper, using a two-country model, helps us understand how a country's consumer preferences regarding domestic products and imports affect its propensity to default (Rose and Spiegel, 2004; Rose, 2005). Past empirical research suggests that less outward-oriented sovereigns are more willing to default. Therefore, if a sovereign government internalizes its citizens' desire for imported goods, we can begin to consider how a country's reduced desire for foreign goods can spur defaults, or how we can motivate the country to service its debt on time.

Moreover, trade balances in previous default models have been calculated as a residual of output from consumption, which means that it depends mainly on capital flows. More importantly, in those models, once default-triggered credit market exclusion happens, the trade balances become zero, which is inconsistent with the data. This paper's trade balances, however, result not only from capital flows, but also from consumers' home bias preferences and their consumption smoothing incentive in both countries.

[1] See Rose (2005), Reinhart and Rogoff (2011), and Mendoza and Yue (2012).

This feature also enables the trade balances to be nonzero *endogenously* during default episodes, even though both countries are in financial autarky.[2]

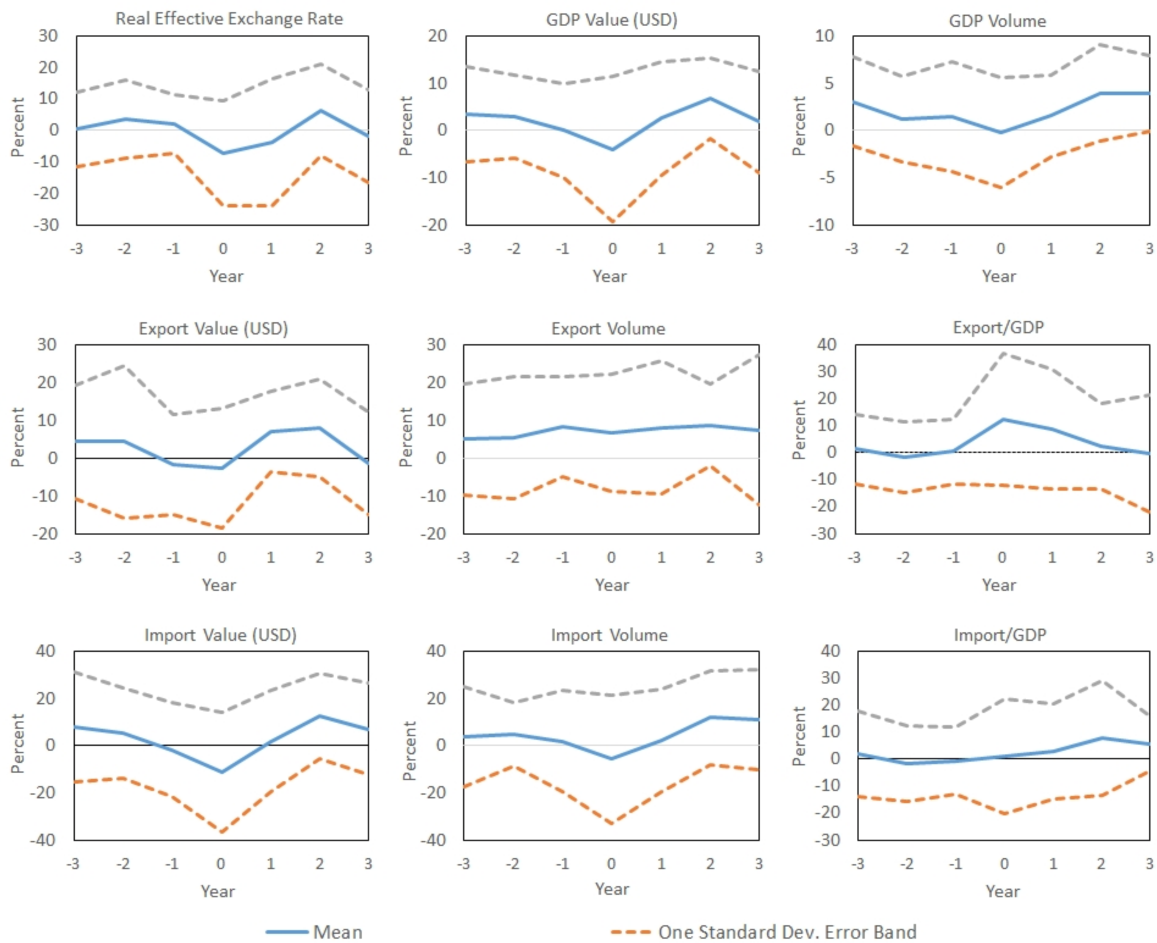
Second, without the exchange rate it is difficult in a model to distinguish the sources of changes to output, import, and export values after a default: by how much the changes are due to depreciation (price effect), and by how much the changes can be attributed to activity volume changes (quantity effect). For example, the first row of Figure 1 shows the average annual growth rates of real effective exchange rate, and GDP value (in USD) and volume around 39 default events over 1977-2009. Upon default (year 0), on average GDP volume growth rate declined a lot less than GDP value, if at all. The majority of the countries' losses of output growth came from real depreciation relative to US dollars.

In addition, the second and third rows of Figure 1 present the annual growth dynamics of trade value (in USD), trade volume, and trade-to-GDP ratio around default events. In general, during the year of sovereign default, the growth rates of both export value and import value turn negative, and the import value declines more significantly than the export value does. Interestingly, most of the decline in export value does not come from changes in export volume but from the real depreciation, since on average export volume growth remains positive through default events. On the contrary, about half of the decline in import value can be attributed to declines in import volumes, and the other half to the real depreciation. It is clear that exchange rate deterioration plays an important role in affecting how the import/export value change. In terms of shares of GDP, the growth of export-to-GDP ratio also becomes much more positive than that of import-to-GDP ratio. All of these facts point to the stylized fact of trade balance improvement during a default event. In sum, it is useful to distinguish the price and quantity effects during default events through an exchange rate channel, in order to study how defaults affect bilateral trade and output losses.

Furthermore, the inclusion of the real exchange rate (i.e., terms of trade) in my model also results in an endogenous default penalty, which, in turn, affects a country's ex ante incentive to default. Therefore, the default penalty of this model does not have to rely on an exogenous output loss as in most previous default models. In this respect, this paper is similar to Mendoza and Yue (2012), where it endogenizes the output loss

[2]Among default models, Mendoza and Yue (2012) is an exception, in that its trade balances are also nonzero during defaults. But they are results of *exogenous* capital flows independent of default or borrowing decisions.

Figure 1: Growth around Sovereign Default Episodes (in percent)



Note: The statistics are based on 39 sovereign default episodes over 1977-2009. Due to data limitation, the sample period and/or the number of default episodes vary slightly for some variables. Raw data sources are detailed in the Appendix.

by a production efficiency loss due to default-triggered exclusion from credit markets that finance parts of the defaulting country's imported intermediate inputs. However, its price index for imported inputs declines during default episodes, since parts of the imports are no longer used, which implies better terms of trade. This leaves it unclear how consistent their paper's real exchange rate movements are with the data. In this paper, the real exchange rate is instead explicitly modeled as terms of trade by two countries' trade flows according to consumer preferences. The endogenous output and the default (risk) affect each other through the real exchange rate channel prior to and during a default event.

Third, with small open economy default models the previous literature is well established in studying defaulting countries' macroeconomic dynamics and welfare changes. However, studies of creditor countries after default events, the other half of this lending-borrowing relationship, are largely absent. Important questions to ask are: how is the welfare of creditor countries affected by default events? Do they register loss or gain from other countries' defaults, and through which mechanisms? This paper's two country model is able to look into the side of creditor countries.

Filling these aforementioned gaps, this paper studies the connection between default, international trade, and exchange rates in a two-country dynamic stochastic general equilibrium model. It features endogenous default and *endogenous* output and international trade losses with *endogenous* real exchange rate deterioration. In the model, two risk-averse open economies trade one-period discount bonds, produce two unique final goods, respectively, and consume both through trade. One of the countries faces stochastic productivity and has an option to default on the bonds, whereas the other country has constant productivity and never defaults. The country with no default risk also purchases intermediate goods that are made by foreign workers in the other country with default risk, to produce its own final goods. In practice, this can be considered as intermediate inputs from abroad through vertical FDI and offshoring. Since I use real economies, the nominal exchange rate is fixed at 1 and the real exchange rate is defined to be terms of trade.

Default can be triggered by negative productivity shocks and can arise in equilibrium. Bond price is determined jointly by the supply from borrower country and the demand from creditor country. The creditor takes into account sovereign default probability in its bond demand function. The bond is denominated by the final goods of the country with no default risk. As the country with default risk borrows more and more, its default risk and equilibrium bond interest rate rise. Higher cost of debt reduces the country's available funds to smooth consumption, and thus due to consumption home bias, the country will consume fewer imports. This puts downward pressure on the real exchange rate, preventing it from appreciating and from helping the country pay back debts.

Once the borrower defaults due to an adverse productivity shock, both countries face financial autarky and only with a certain probability can they resume bond trading again. Moreover, the default affects foreign firms' activities (including FDI, offshoring, intermediate goods purchases, etc.) related to the defaulting country more than it affects domestic firms.[3] In particular, the default triggers an efficiency loss of foreign-firm-related intermediate goods production in the defaulting country (e.g., due to crisis-elevated miscommunication between foreign firms and intermediate goods sector), which results in a decline of intermediate goods exports from the defaulting country to the creditor country.

This is consistent with the data in Table 1. It shows the average annual growth rates of exports for the sample period of 1989-2013, as well as for 15 default episodes during that period (see Table 5 in the Appendix for detailed data availability). On average, intermediate goods exports shrink during defaults, contradictory to its pattern if including normal time. The growth rates of intermediate goods exports are smaller than those of final goods exports during defaults, especially in terms of volume growth. The impact of defaults on intermediate goods export growth distinguishes itself from that on other export growth through a more severe decline.

This change in intermediate goods exports upon default triggers a reallocation of capital and labor inputs within both countries, as well as another income loss additional to that from the initial adverse productivity shock in the defaulting country.[4] Consequently, again because of consumption home bias preference, the defaulting country's imports decline even more, trade balances reverse, and the real exchange rate drops sharply. The real depreciation further takes a toll on the defaulting country's income.

[3]Multiple papers have provided evidence for asymmetric crisis impacts on domestic and foreign agents due to crisis-elevated information asymmetry and risk aversion, for example, Brennan and Cao (1997), Tille and van Wincoop (2008), Milesi-Ferretti and Tille (2010), and Broner, Didier, Erce, and Schmukler (2013). Moreover, Aizenman and Marion (2004) also documents that greater supply uncertainty reduces the expected income from vertical FDI. Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent.

[4]See details in the Model section.

Table 1: Average Growth Rate of Exports During a Default (in percent)

Variable	1989-2013	During Default
Intermediate Goods Exports		
Value	7.51	−1.11
Volume	4.92	0.82
Share of GDP	3.60	−1.92
Final Goods Exports		
Value	7.65	−0.11
Volume*	5.96	7.81
Share of GDP	3.74	−1.02

Note: I collected annual growth data of final goods and intermediate goods export value (in USD), volume (as value in local currency), and as a ratio to GDP for 15 sovereign default episodes over the period of 1989-2013. *Here, due to data limitation, the volume is of total exports.

This builds into the model an endogenous real exchange rate mechanism by which a sovereign default amplifies the effects of adverse productivity shocks on output and trade.

In a quantitative exercise, I apply the model to study the Mexican debt crises in the 1980s and the country's business cycles for the period of 1981Q1-2012Q4.[5] I show that this two-country setup with endogenous real exchange rate changes allows default models to explain the aforementioned stylized facts of sovereign defaults. In particular, this paper generates three important stylized empirical features of emerging markets' business cycles, including sovereign default episodes. First, this paper delivers countercyclical trade balances and procyclical trade flows over business cycles. Second, the model supports high bond spreads that are also countercyclical. Third, this model accounts for sharp real depreciation, GDP drops, trade balance improvements, and bilateral trade flow declines during and right after a default.

Then I further examine the role of the real depreciation in explaining the default-triggered changes to the borrower country's trade value and volume, GDP value and volume, and the creditor country's welfare. The model generates trade and GDP losses that are partially due to real depreciation (price effect) and partially due to volume

[5] I choose Mexico for this two-country model because Mexico has a relatively large open economy among the countries that recently defaulted, as well as relatively large vertically integrated sectors involved with foreign production, including its maquiladora sector (Zlate, 2012).

changes (quantity effect), as in the data. I also find that the creditor country's welfare experiences lasting gains from a default event, mainly because of their improved terms of trade. Moreover, the model can predict the time series of Mexican output and bond spreads in the sample period, as well as its crises around 1986 and 1995. Lastly, in sensitivity analysis, this paper finds that the changes in default penalty (through exchange rate and intermediate goods exports) and consumers' taste in imported final goods versus domestic goods have little impact on the borrower country's default frequency, but do have a great impact on its average debt-to-GDP ratio and on the post-default welfare of the borrower, the creditor, and the world.

In explaining the defaulting country's trade balance reversal and deteriorating real exchange rate and terms of trade, this model is related to papers in the international business cycle literature, such as Backus, Kehoe, and Kydland (1992, 1994), Stockman and Tesar (1995), Heathcote and Perri (2002), Kehoe and Perri (2002), Iacoviello and Minetti (2006), Bodenstein (2008), and Raffo (2008). These papers have addressed many international business cycle features: countercyclical trade balances, cross-country correlations for consumption and output, terms of trade and real exchange rate fluctuations, etc. Some have assumed a complete market for financial assets, while others have incorporated enforcement or borrowing constraints with the result that actual defaults are ruled out at equilibrium. Using a non-state-contingent bond, my model endogenously generates default in equilibrium. Hence, this paper is closely associated with previous small-open economy default models. As explained earlier, however, those models do not focus on default-related trade and real exchange rate changes.

Another strand of literature also focuses on the connection between international trade and defaults, but most of it is empirical studies. For instance, Rose (2005) documents that a default can reduce real bilateral trade value (in USD) by 8 percent for an extended period after the event. However, it remains unclear why trade declines. The four hypotheses of trade sanctions, trade credit collapse, asset seizures, and reputation are commonly mentioned, but their empirical evidence remains ambiguous (Tomz and Wright, 2013). This paper uses a theoretical model to examine the trade impact of sovereign defaults through real exchange rate deterioration.

A few recent sovereign default papers (Cuadra and Sapriza, 2006; and Bleaney, 2008) have examined how *exogenous* exchange rate shocks can impact defaults in small open economy models. The inclusion of *endogenous* exchange rates distinguishes this paper from them. Na, Schomitt-Grohe, Uribe, and Yue (2014) also includes endogenous exchange rate but in nominal terms so that they can focus on optimal default and exchange rate policy. Like this paper, their model achieves concurrent default and depreciation. However, their nominal depreciation is driven by wage rigidity and government's intention to reduce unemployment, whereas this paper's real depreciation is associated with changes to capital and trade flows.

The remainder of this paper is organized as follows. Section 2 describes the model environment and agents' problems, defines a recursive equilibrium, and characterizes its properties. Section 3 provides the model calibration and the results of the quantitative analysis. Section 4 offers concluding remarks. Data sources are in the Appendix.

2 Model

2.1 Environment

In this section, I study sovereign default, international goods trade, and the real exchange rate in a dynamic model of two risk-averse open economies: country 1 and country 2. Each country produces a unique type of final goods. In particular, country 1's firms allocate their capital to produce final goods 1 by pairing the capital either with domestic labor, or with imported intermediate goods produced by country 2's labor (e.g., through vertical FDI and offshoring). Country 2 produces intermediate goods using local workers. Therefore, the labor in country 2 either produces final goods 2 together with domestic capital, or produces intermediate goods for country 1. Households in each country enjoy both final goods, which are imperfect substitutes with constant elasticity. Governments in both countries are benevolent and maximize their households' lifetime utility.

On the bond market, a non-state-contingent one-period bond denominated in final goods 1 is traded between country 1 and country 2, in one direction or the other. I assume that only country 2 has propensity to default with different productivity state realizations, while country 1 is always at steady state, and its bond is always safe. Since it is the case in which country 2 issues risky bonds that is of interest, I will only study the scenario in which country 2 is borrowing. The bond contracts reflect country 2's default probabilities that are endogenous to its debt holding and its fundamental. Hence, the equilibrium interest rate is linked to country 2's default risk. Default can happen along the equilibrium because the asset structure is incomplete, as it does not cover all the states that country 2 encounters. The risk-averse creditors in country 1 are willing to offer debt contracts that in some states may result in default by charging a higher interest rate on these loans. The equilibrium interest rate is also associated with the creditor's risk aversion.

Once country 2 defaults, the event affects foreign firms' activities related to the defaulting country more than it affects domestic firms.[6] In particular, the default triggers an efficiency loss of foreign-firm-related intermediate goods production in the defaulting country (e.g., due to crisis-elevated miscommunication between foreign firms and intermediate goods sector), which results in a decline of intermediate goods exports from the defaulting country to the creditor country. This story is consistent with the data. For instance, the average annual growth rate of intermediate goods export volume for the period of 1989-2013 for 15 countries (see data availability in Table 5 in the Appendix), is 4.92 percent. However, during default episodes their average annual growth rate is only 0.82 percent, suffering from a much larger decline in growth than final goods exports, as shown in Table 1. This decline in intermediate goods exports harms the production of those final goods 1 that involve such imported inputs from country 2. Therefore, country 1's firms reallocate capital away from combining with the imported intermediate goods, but more towards its domestic labor to produce final goods 1 without the imported intermediate goods.

Since there is no unemployment in this model, some workers in country 2 will then shift from the intermediate goods sector to domestic production of final goods 2, enabling the country to export more of its own final goods. The reduced labor income from the intermediate goods sector serves as part of country 2's default costs. Meanwhile, when country 2 defaults on its sovereign bonds, both countries are temporarily excluded from the international financial market for certain periods of time. There is no other direct penalty, such as exogenous output loss or trade sanctions.[7]

I use the following notations throughout the rest of this paper. C_i stands for

[6]As mentioned in introduction, Brennan and Cao (1997), Tille and van Wincoop (2008), Milesi-Ferretti and Tille (2010), and Broner, Didier, Erce, and Schmukler (2013) have provided evidence for asymmetric crisis impacts on domestic and foreign agents due to crisis-elevated information asymmetry and risk aversion. Moreover, Aizenman and Marion (2004) also documents that greater supply uncertainty reduces the expected income from vertical FDI. Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent.

[7]Trade sanctions after default lack of empirical support in the literature (Tomz and Wright, 2013).

country i's total consumption index. c_{ij} stands for country i's consumption of final goods j. e_i stands for country i's productivity, where e_1 is constant and e_2 follows Markov chain. ε stands for the efficiency of foreign-firm-related intermediate goods production in country 2. p_j stands for final goods j's price, and I normalize $p_1 = 1$. Therefore, p_2 is terms of trade for country 2. I assume that the nominal exchange rate between the two countries is 1, and thus p_2 is also the real exchange rate. When p_2 declines, it means the deterioration of the real terms of trade and the real exchange rate for country 2. b_i stands for country i's assets. $\bar{k_1}$ stands for the total capital that country 1's firms have, a constant in this model. It is divided into that used with domestic labor, k_1, and that used with imported intermediate inputs, k_1^*. $\bar{n_2}$ stands for total labor in country 2, also a constant in this model. It is divided into those who produce final goods 2, n_2, and those who produce intermediate inputs for country 1, n_2^*. Country 1 also has constant domestic labor $\bar{n}_1$ that is used to produce final goods 1, and country 2 has constant domestic capital stock $\bar{k}_2$ that is used to produce final goods 2.

2.2 Country 1

In the model, there are three types of agents in country 1: representative firms, households, and a government.

2.2.1 Firms

Firms hold country 1's capital stock, $\bar{k_1}$, and allocate it between that used with domestic labor, k_1, and that used with imported intermediate inputs, k_1^*, to produce final goods 1. The imported intermediate inputs are produced in country 2, using only local labor. Country 1's firms choose how many intermediate goods they need, and thus also how many foreign workers to hire in the intermediate goods sector in country 2, n_2^*. They also pay wage w_2^* to the intermediate goods sector workers denominated in country 2's local final goods 2. Here, I combine the decisions of imported intermediate inputs and final goods 1 production to reflect the practice of vertical FDI and offshoring.[8] The firms' goal is to maximize profits in every period taking w_2^* and p_2 as given. The

[8]This formulation is similar to those used in global sourcing literature where firms have a choice to internalize its input supply, e.g., Antras and Helpman (2004).

one-period profits of country 1's firms are:

$$\Pi_1 = \max_{n_2^*, k_1^*} \left\{ e_1(\varepsilon e_2 n_2^*)^{\alpha_1} k_1^{*1-\alpha_1} - p_2 w_2^* n_2^* + e_1 \bar{n}_1^{\alpha_1} k_1^{1-\alpha_1} \right\} \tag{1}$$

where $k_1 = \bar{k}_1 - k_1^*$. The first two terms are the total profit the firms gain from using intermediate inputs produced by country 2's labor, after deducting wage costs. Notice that the production of final goods 1 using intermediate goods from country 2 uses the same technology as it uses to produce the same final goods 1 with domestic labor. The intermediate goods production is a linear function of country 2's workers in this sector, n_2^*, and is associated with country 2's productivity e_2. Importantly, ε symbolizes the efficiency of foreign-firm-related intermediate goods production in country 2. During normal times, $\varepsilon = 1$, meaning there is no efficiency loss compared with country 2's domestic goods sector. During default periods, $\varepsilon = \min(\epsilon \frac{\bar{e_2}}{e_2}, 1)$, where $0 < \epsilon < 1$ and $\bar{e_2}$ is country 2's average productivity. This ε reflects asymmetric crisis impacts on country 2's domestic and foreign-related activities, e.g., due to crisis-elevated information asymmetry (Brennan and Cao, 1997; Aizenman and Marion, 2004; Tille and van Wincoop, 2008; Milesi-Ferretti and Tille, 2010; and Broner, Didier, Erce, and Schmukler, 2013).

Firms' capital allocation and foreign labor demand satisfy these optimality conditions:

$$(k_1^*) : e_1(\varepsilon e_2 n_2^*)^{\alpha_1}(1-\alpha_1) k_1^{*-\alpha_1} = e_1 \bar{n}_1^{\alpha_1}(1-\alpha_1)(\bar{k_1} - k_1^*)^{-\alpha_1} \tag{2}$$

$$(n_2^*) : e_1 \alpha_1 \varepsilon e_2 (\varepsilon e_2 n_2^*)^{\alpha_1 - 1} k_1^{*1-\alpha_1} = p_2 w_2^* \tag{3}$$

2.2.2 Households and Government

Households in country 1 work at and own the firms. They use the proceeds for consumption. They choose a consumption bundle to maximize a standard time-separable utility function $E[\sum_{t=0}^{\infty} \beta_1^t U(C_{1t})]$, where $0 < \beta_1 < 1$ is the discount factor. C_{1t} denotes country 1's consumption bundle of final goods 1, c_{11}, and final goods 2, c_{12}. More specifically, $C_{1t} = c_{11t}^{\theta_1} c_{12t}^{1-\theta_1}$ with two final goods being imperfect substitutes for each other. I also assume that consumers purchase a relatively large share of domestic goods, i.e., home bias, $\theta_1 > 0.5$. $U(.)$ is the one-period utility function which is continuous, strictly increasing, strictly concave, and which satisfies the Inada conditions. For the purpose of quantitative analysis, I define the utility function in the form of

$U(C_{1t}) = \frac{C_{1t}^{1-\gamma}}{1-\gamma}$ with risk aversion parameter $\gamma > 0$.

Households also choose how many of the one-period non-state-contingent bonds issued by country 2 to purchase, given the bond pricing function $q(b_1', s)$, with s being the aggregate state of the two economies. Alternatively, the households can borrow from country 2 through its government transfers and will never default; the government's objective is to maximize households' expected lifetime utility. The former case is the one that is of interest and the focus of this model, because only country 2 has default risks. So, I assign country 1 as the creditor. Since country 1 does not actively choose whether it will default or not, its welfare depends on country 2's default decisions. When country 2 does not default, country 1's optimization problem can be written recursively as:

$$V_{1c}(s, b_1) = \max_{b_1', c_{11}, c_{12}} \left\{ U(c_{11}, c_{12}) + \beta_1 [\int_{s' \notin \mathcal{D}(b_2')} V_{1c}(s', b_1') dF(s'|s) + \int_{s' \in \mathcal{D}(b_2')} V_{1d}(s') dF(s'|s)] \right\} \tag{4}$$

where $\mathcal{D}$ is the default set for country 2 that I will explain in the next section. The problem is subject to:

$$\Pi_1 + b_1 = c_{11} + p_2 c_{12} + q b_1'. \tag{5}$$

where $q(s', b_1') = \beta_1 \frac{\int_{s' \notin \mathcal{D}(s', b_2')} \partial V_{1c}' / \partial b_1' dF(s'|s)}{\lambda_1}$, and λ_1 is the multiplier of the budget constraint.

When country 2 defaults, both countries will have to undergo financial autarky for a certain period of time.[9] Country 1's constrained maximization problem becomes:

$$V_{1d}(s) = \max_{c_{11}, c_{12}} \{U(c_{11}, c_{12}) + \beta_1 E[re V_{1x}(s', 0) + (1 - re) V_{1d}(s')]\} \tag{6}$$

where $V_{1x} = [V_{1d}(s, b_1)$ or $V_{1c}(s)|$country 2 defaults or not$]$, and re is the probability of both countries' return to the bond market. The problem is subject to

$$\Pi_1 = c_{11} + p_2 c_{12}. \tag{7}$$

[9] Some may argue that it is not realistic to also exclude the creditor from the international financial market. But since country 1 has no productivity shock, its loss from the financial market exclusion is minimal.

Given the above setup, I calculate country 1's GDP as the gross production of final goods 1 minus the cost of imported intermediate goods. Notice that for country 1, its GDP value and volume are the same in the model, because its goods price $p_1 = 1$. Hence, country 1's GDP is defined to be equal to Π_1.

2.3 Country 2

There are three types of agents in country 2: representative firms, households, and a government.

2.3.1 Firms

Since country 2's firms passively hire all the remaining workers that are not hired by the intermediate goods sector, they maximize their profits according to the following first order condition:

$$e_2 \alpha_2 n_2^{\alpha_2 - 1} \bar{k}_2^{1-\alpha_2} = w_2 \tag{8}$$

where w_2 is domestic sector wage. It is worth noting that this model does not have unemployment. However, it would be interesting to study sovereign default related unemployment in this model in the future.

2.3.2 Households

Households in country 2 derive income from two sources: their wage income from producing intermediate goods for country 1, and all the revenue of domestic firms that are owned by the households. Country 2's households choose a consumption bundle to maximize a standard time-separable utility function $E[\sum_{t=0}^{\infty} \beta_2^t U(C_{2t})]$, where $0 < \beta_2 < 1$ is the discount factor, and C_{2t} denotes the consumption bundle of final goods 1, c_{21}, and final goods 2, c_{22}. Similar to the utility function for country 1's consumers, $U(C_{2t}) = \frac{(c_{21t}^{\theta_2} c_{22t}^{1-\theta_2})^{1-\gamma}}{1-\gamma}$ with $\theta_2 < 0.5$ for home bias and $\gamma > 0$ for risk aversion.

Households take as given the wage income from the intermediate goods sector, $p_2 w_2^* n_2^*$, the revenue from domestic firms, $p_2 e_2 n_2^{\alpha_2} \bar{k}_2^{1-\alpha_2}$, and government transfers, T. As in Mendoza and Yue (2012), households do not borrow directly from abroad, but the government borrows, pays transfers to the households, and makes default decisions internalizing its citizens' utility. Alternatively, households may invest directly in the international financial market, but this case is not of interest here since country 1's bond is safe. The households' optimization problem is:

$$\max_{c_{21t}, c_{22t}} E[\sum_{t=0}^{\infty} \beta_2^t U(c_{21t}, c_{22t})] \tag{9}$$

subject to

$$p_{2t} e_{2t} n_{2t}^{\alpha_2} \bar{k}_2^{1-\alpha_2} + p_{2t} w_{2t}^* n_{2t}^* + T_t = c_{21t} + p_{2t} c_{22t}. \tag{10}$$

where $n_{2t} = \bar{n_2} - n_{2t}^*$.

2.3.3 Government

The sovereign government issues one-period non-state-contingent discount bonds, so the asset market is incomplete. The government cannot commit to repaying its debt. It compares the value of repaying debt V_{2c} and that of default V_{2d}, and chooses the option that gives a bigger value, that is:

$$V_{2x}(s, b_2) = \max \{V_{2c}(s, b_2), V_{2d}(s)\} \tag{11}$$

The nondefault value is given by the choice of (b_2', c_{21}, c_{22}) that maximizes the following problem, where the government internalizes its citizens' preferences for domestic final goods 2 and imported final goods 1:

$$V_{2c}(s, b_2) = \max_{b_2', c_{21}, c_{22}} \{U(c_{21}, c_{22}) + \beta_2 E V_{2x}(s', b_2')\} \tag{12}$$

subject to

$$p_2 e_2 n_2^{\alpha_2} \bar{k}_2^{1-\alpha_2} + p_2 w_2^* n_2^* + b_2 = c_{21} + p_2 c_{22} + q b_2'. \tag{13}$$

where $n_2 = \bar{n_2} - n_2^*$ and $q(s', b_2') = \beta_2 \frac{\int_{s' \notin \mathcal{D}(s', b_2')} \partial V_{2c}' / \partial b_2' dF(s'|s)}{\lambda_2}$, and λ_2 is the multiplier of country 2's budget constraint.

The definitions of the default set $\mathcal{D}$ and the probability of default are standard from Eaton-Gersovitz type models (also see Arellano, 2008). Default set $\mathcal{D}$ at each current debt level is a collection of exogenous states when country 2's government will strategically choose to default to maximize its own value:

$$\mathcal{D}(s, b_2) = \{s \in S : V_{2c}(s, b_2) < V_{2d}(s)\} \tag{14}$$

Because no one is certain about the aggregate state tomorrow, the default probability π_2 is the sum of all the probabilities of tomorrow's states where country 2 will choose to default, given the current debt level. This default probability exists even if country 2 does not consider the default risks when issuing bonds:

$$\pi_2(s, b_2') = \int_{s' \in \mathcal{D}(s', b_2')} f(s, s')ds' \tag{15}$$

In the event of a default due to an adverse productivity shock to country 2, both countries are temporarily excluded from the international bond market, and country 2 can not smooth its consumption by borrowing any more. Furthermore, the production of intermediate goods in country 2 suffers from an efficiency loss due to a larger crisis impact on foreign-firm-related activities than that on domestic activities (Brennan and Cao, 1997; Aizenman and Marion, 2004; Tille and van Wincoop, 2008; Milesi-Ferretti and Tille, 2010; and Broner, Didier, Erce, and Schmukler, 2013). Therefore, country 1's firms shift capital towards pure domestic production; country 2's workers get lower income from the intermediate goods sector as part of default costs. Consequently, country 2's workers flow into the domestic sector, which encourages the production of final goods 2. Nevertheless, due to the initial adverse productivity shock and income loss from the intermediate goods sector, country 2's overall income declines and its consumers adjust their consumption portfolio according to home bias preferences. This causes the country to import fewer and export more of final goods. Meanwhile, country 2's terms of trade and real exchange rate deteriorate, further lowering its income and magnifying the initial adverse productivity shock. Country 2's default value is as follows:

$$V_{2d}(s) = \max_{c_{21}, c_{22}} \{U(c_{21}, c_{22}) + \beta_2 E[reV_{2x}(s', 0) + (1 - re)V_{2d}(s')]\} \tag{16}$$

subject to

$$p_2 e_2 n_2^{\alpha_2} \bar{k}_2^{1-\alpha_2} + p_2 w_2^* n_2^* = c_{21} + p_2 c_{22} \tag{17}$$

where $n_2 = \bar{n_2} - n_2^*$.

Given the above setup, I calculate country 2's GDP as the gross production of final goods 2 plus the intermediate goods exports. Hence, country 2's GDP value is defined as $p_2 e_2 n_2^{\alpha_2} \bar{k}_2^{1-\alpha_2} + p_2 w_2^* n_2^*$, and its GDP volume is defined as $e_2 n_2^{\alpha_2} \bar{k}_2^{1-\alpha_2} + w_2^* n_2^*$.

2.4 Equilibrium

Finally, in equilibrium all goods and financial markets clear for both countries in default and nondefault regimes, and there is no wage difference between different sectors in country 2:

$$e_1(s)\bar{n}_1^{\alpha_1}(\bar{k_1} - k_1^*)^{1-\alpha_1} + e_1(\varepsilon e_2 n_2^*)^{\alpha_1} k_1^{*1-\alpha_1} = c_{11} + c_{21} \tag{18}$$

$$e_2(s)(\bar{n_2} - n_2^*)^{\alpha_2} \bar{k}_2^{1-\alpha_2} = c_{12} + c_{22} \tag{19}$$

$$b_1{}'(s, b_1) + b_2{}'(s, b_2) = 0 \tag{20}$$

$$w_2^* = w_2. \tag{21}$$

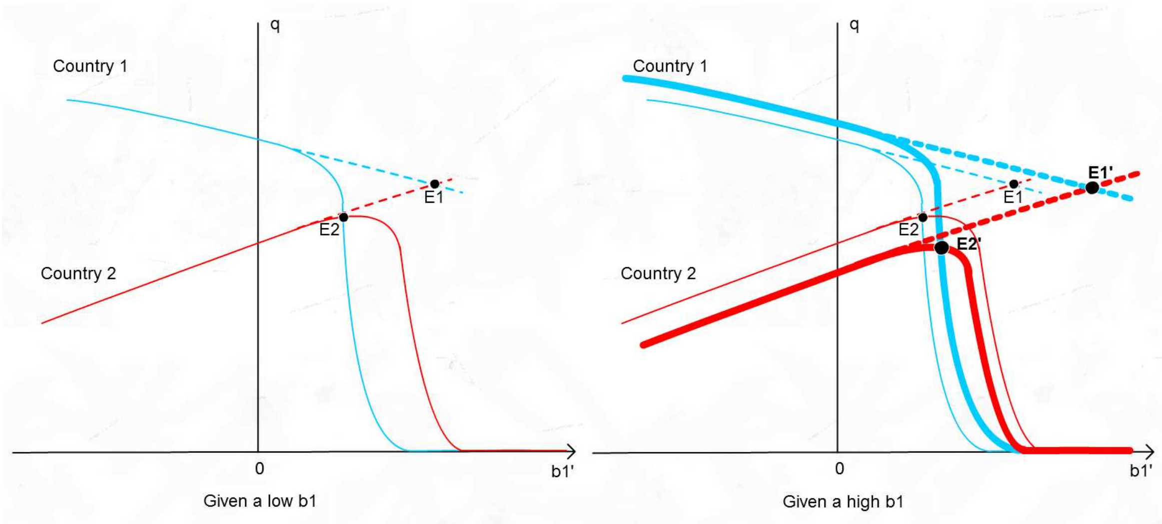
Definition 1 *A recursive competitive equilibrium is defined as a set of functions for (a) country 1's capital allocation and hiring decisions; (b) both countries' households' consumption policy c and saving policy b'; (c) welfare value V at default and repayment; and (d) the law of motion for the aggregate states, such that: (i) the borrowing and lending policies satisfy the problem's first order conditions; (ii) the two countries' value functions satisfy Bellman Equations; (iii) p_2 and q clear the goods and bond markets; (iv) w_2^* and w_2 stabilize labor flows between the two sectors in country 2; and (v) the law of motion is consistent with the stochastic processes of e_2.*

Now I illustrate how bond prices are determined in this model. Let's backpedal to a simpler case where there is no default risk. Figure 2 plots bond price q against country 1's asset level *tomorrow* b_1' (country 2's borrowing *tomorrow*) for a given productivity state s and *current* asset level of country 1 b_1 (country 2 *current* borrowing). As shown in the left panel, in the case of no default risk, the bond demand curve and bond supply curve (dash lines) are close to linear and intersect at point $E1$.[10] Point $E1$ pins down the market equilibrium bond price and the next period's quantity.

If the *current* bond holding is at a higher level, as in the right panel of Figure 2, then country 1's bond demand curve will shift up to the thicker dash line because of a

[10] Even in this risk-free bond case, the bond supply and demand curves are not exactly linear, because of the agents' risk aversion.

Figure 2: Bond Price (given current b_1)



Note: Here I assume productivity state s is constant. The x-axis in the above plots is b_1'. As b_1' is positive (right hand side) and becomes larger, country 2 borrows more and more.

lower current marginal utility of domestic consumption (i.e., λ_1), and country 2's bond supply curve will shift down because of a higher current marginal utility of imported consumption (i.e., λ_2). This results in a new intersect point $E1'$, which provides a larger equilibrium bond quantity and a slightly lower price, depending on the two countries' risk aversion.

Now let's consider the default risks of bonds issued by country 2. In this case, bond supply and demand curves take into account the borrower and the creditor's perspectives on default probability, respectively, as in the following equations:

$$\begin{aligned} q(b, b', s) &= \beta_1 \int_{s' \notin \mathcal{D}(s', b_2'^0)} \frac{\frac{\partial U}{\partial C_1'} \frac{\partial C_1'}{\partial c_{11}'}}{\frac{\partial U}{\partial C_1} \frac{\partial C_1}{\partial c_{11}}} dF(s'|s) \\ &= \beta_2 \int_{s' \notin \mathcal{D}(s', b_2'^0)} \frac{\frac{\partial U}{\partial C_2'} \frac{\partial C_2'}{\partial c_{21}'}}{\frac{\partial U}{\partial C_2} \frac{\partial C_2}{\partial c_{21}}} dF(s'|s) \end{aligned} \tag{22}$$

The first equation represents the bond demand curve of country 1, and the second equation is the bond supply curve of country 2. As default risk increases, both the demand and supply curves imply lower bond prices, since both countries take into account the default risk. That is to say, in the left panel of Figure 2, given current b_1, both curves bend downward as tomorrow's b'_1 (i.e., country 2 tomorrow's borrowing) becomes larger, reflecting a higher default risk (solid lines).[11] This results in a different equilibrium point from the no-default-risk case, at $E2$: both the equilibrium bond quantity and the price are lower than those at $E1$.

Again, if the *current* bond holding is at a higher level, as in the right panel of Figure 2, then country 1's bond demand curve will shift upward and country 2's bond supply curve will shift downward. The new intersect point $E2'$, again, provides a larger equilibrium bond quantity and a lower price. However, due to the default risk, the increase in the quantity is much smaller and the decrease in bond price is much larger than the no-default-risk case.

Computationally, the inclusion of default risks in both bond supply and demand functions poses a difficult challenge in solving this problem numerically. This is because if both curves bend downward at similar levels of b'_1 and when both of them reach where $q = 0$, they create multiple equilibria. Therefore, in practice I solve the model with country 1's expectation about country 2's default probability, but assume that country 2 itself is not concerned about its own default risk. Hence, country 1's bond demand curve is the solid bending curve, while country 2's bond supply curve is the dash line in Figure 2.

Theorem 1 *Given a productivity shock to e_2 and a pair of bond assets $b_2^0 < b_2^1 \leq 0$, if default is optional for b_2^1, then default is also optimal for b_2^0 and the probability of default at equilibrium satisfies $\pi_2(s, b_2^0) \geq \pi_2(s, b_2^1)$.*

Proof 1 *Given a productivity shock to e_2, the value of default $V_{2d}(s)$ is independent of b_2. The value function of repaying debt $V_{2c}(s, b_2)$ is increasing in b_2. Therefore, if $V_{2c}(s, b_2^1) \leq V_{2d}(s)$, then it must be the case that $V_{2c}(s, b_2^0) \leq V_{2d}(s)$ since $V_{2c}(s, b_2^0) \leq V_{2c}(s, b_2^1)$. Hence, for any $s \in \mathcal{D}(s, b_2^1)$, we must also have $s \in \mathcal{D}(s, b_2^0)$, that is, $\mathcal{D}(s, b_2^1) \subseteq \mathcal{D}(s, b_2^0)$. Therefore, by definition, we have $\pi_2(s, b_2^0) \geq \pi_2(s, b_2^1)$.*

[11] At which levels of b'_1 the supply and demand curves will bend down, however, may differ depending on which country is more observant and/or more sensitive about the default risk. In Figure 2, I draw that the bond demand curve starts to bend downward at a lower bond quantity level (b'_1) than the bond supply curve does. This implies that creditor country 1 is more aware and/or more sensitive about default risks than the borrower country 2 is.

3 Quantitative Results

3.1 Baseline Calibration

In this section, I study the quantitative implications of the model by conducting numerical simulations at the quarterly frequency and using a baseline calibration based largely on data for Mexico and Canada. Table 2 shows the calibrated parameter values.

Table 2: Parametrization

Calibrated Parameter	**Value**	**Target Statistics**
Risk aversion	$\gamma = 2$	Standard RBC value
Financial market re-entry probability	$re = 0.083$	Dias and Richmond (2007)
Country 1 home goods consumption share	$\theta_1 = 0.60$	Canadian (CA) national accounts
Country 2 home goods consumption share	$1 - \theta_2 = 0.70$	Mexican (MX) national accounts
Country 1 domestic production labor share	$\alpha_1 = 0.63$	From OECD data for CA
Country 2 domestic production labor share	$\alpha_2 = 0.45$	From OECD data for MX
Intermediate goods share in final goods 1 production	$\alpha_1 = 0.63$	Same as country 1's domestic labor share
Country 1 labor endowment	$\bar{n}_1 = 2.5$	Average CA-to-MX employment ratio
Country 1 capital endowment	$\bar{k}_1 = 27.56$	0.65, average FDI-to-GDP ratio for MX
Country 2 labor endowment	$\bar{n}_2 = 1$	Normalized to 1
Country 2 capital endowment	$\bar{k}_2 = 1$	Normalized to 1
Country 1 productivity	$e_1 = 0.82$	2, average CA-to-MX GDP ratio
Country 2 productivity steady state	$E(e_2) = 1$	Normalized to 1
Country 2 autocorrelation of TFP	$\rho = 0.4162$	From production function
Country 2 std. dev. of TFP shocks	$\sigma = 0.0377$	From production function
Country 1 discount factor	$\beta_1 = 0.99$	1%, US government bond interest rate
Parameter by Simulation	**Value**	**Target Statistics**
Country 2 discount factor	$\beta_2 = 0.9655$	1%, quarterly default frequency for MX
Intermediate goods sector discount upon default	$\epsilon = 0.85$	-0.09, average intm. goods export income deviation from trend upon default for MX

The risk aversion parameter γ is set to 2, which is the standard value in quantitative business cycle and sovereign default studies. The probability of reentering the international financial market after a default is 0.083, which implies that the country stays in exclusion for an average of three years after default. This is the estimate obtained by Dias and Richmond (2007) for the median duration of exclusion periods. It is also consistent with the finding by Gelos, Sahay, and Sandleris (2011) and is applied by Mendoza and Yue (2012).

The share of domestic products in final consumption for the two countries are obtained from Canadian and Mexican national accounts and set to be 0.6 and 0.7, respectively. They correspond to one minus the average ratio of goods and services imports to final consumption expenditure calculated using annual data for the period of $1981 - 2012$ from the World Bank (WDI).

The labor share in final production is set at 0.63 for Canada and 0.45 for Mexico, which are the average labor income shares using annual data for the period of 1981-2009 (1981-2008 for Canada) from OECD Statistics. The share of the imported intermediate goods in final goods production in Canada is the same as the labor share in final production in Canada, since those intermediate goods are used to produce the same Canadian final goods in the model, and the intermediate goods production is a function of labor only.

I calibrate the sizes of the two countries assembling some regularities of Canada and Mexico. The capital and labor endowments of country 2 are normalized to 1. Therefore, country 1's labor size is $\bar{n}_1 = 2.5$ to match the average CA-to-Mexico employment ratio for 1981Q1-2012Q4. Country 1's capital endowment is chosen such that at steady state country 1's capital used with intermediate goods is 65 percent of country 2's GDP, which is approximated by the average of FDI-to-GDP ratio for Mexico during 1981Q1-2012Q4 assuming the majority of the FDI to Mexico is vertical. However, it is important to note that this approximated target is by no means a complete calibration for the actual amount of foreign capital used, along with intermediate goods from Mexico to produce foreign final goods.

At this stage, the only productivity shock in the model is to country 2's productivity e_2. The steady state of e_2 is normalized to 1. Therefore, the constant productivity of country 1, e_1, is calibrated to be 0.8247, so that at steady state the CA-to-Mexico GDP ratio is 2, the same as the average CA-to-Mexico GDP ratio for $1981 - 2011$ according to annual data from IMF. I model the productivity of country 2 as an AR(1) process:

$$\log e_{2,t} = \rho \log e_{2,t-1} + \epsilon_t$$

with ϵ being iid and following $N(0, \sigma^2)$. I estimate the process using the model's production functions, the HP-filtered data of GDP, (average) capital stock, and employment at both domestic sector and FDI sector for 1981Q1-2012Q4 in Mexico. Using Tauchen and Hussey (1991), I further construct a Markov approximation to this process with 5 states of productivity realization for e_2.

The targets for setting β_1, β_2, and ϵ are, respectively, the risk free interest rate from US or Canadian treasury bills, quarterly frequency of Mexico defaults, and the loss in intermediate goods exports upon default. Both US and Canadian treasury bills bear interest rates that are lower than 1 percent, hence, we have $\beta_1 = 0.99$. Mexico's quarterly default frequency is about 1%, since it had eight default episodes between 1828 and 2012 according to Reinhart (2010).[12] Moreover, I include more default incidents in order to study the dynamics around those events in later sections. I distinguish three separate default occurrences drawn from Paris Club data for this paper's sample period 1981Q1-2012Q4, even though Reinhart (2010) counts them as one. They are 1982Q3, 1986Q1, and 1989Q1. Upon these three most recent onsets of sovereign default, Mexico's intermediate goods export value, on average, was about 9 percent below trend. Given these two targets, the simulated procedure yields $\beta_2 = 0.9655$ and $\epsilon = 0.85$.

I solve the model with a discretized state space of 5 realizations for country 2's productivity and 107 points for asset holdings. The model is considered as solved when the convergence distance diminishes to $1.0000e-06$. In the following sections, I first examine the properties of the calibrated model, then study the simulated results both along business cycles and around default events.

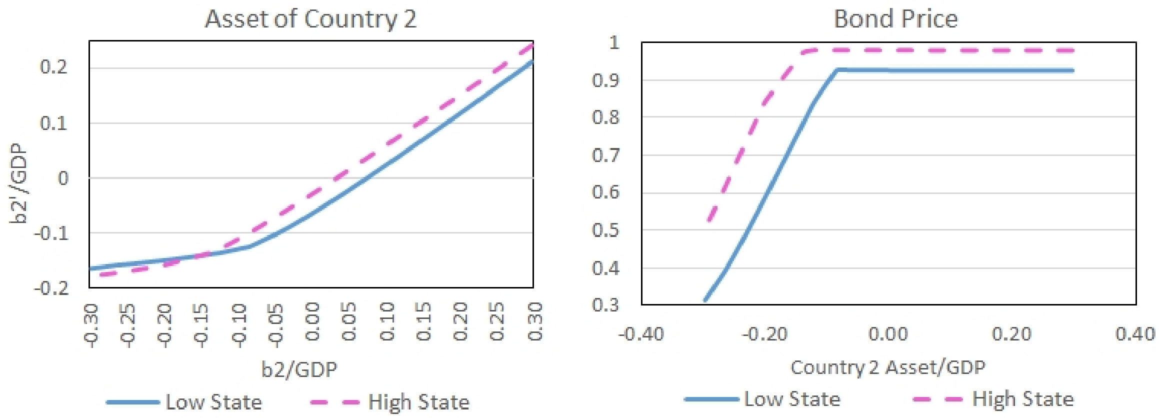
3.2 Policy Functions

The properties of bond quantity and its price in this model are in line with existing papers. Figure 3 left plot graphs the next period assets for country 2 against its current assets, at a high productivity state and a low productivity state in the current period. As country 2 borrows more and more (to the left of the bottom axis), its marginal borrowing capacity diminishes. Moreover, when the country is at a low state, its bond function starts to flatten out at a lower current debt amount than if it were at a high state.

Figure 3 right plot graphs the bond price functions. It shows that the bond price

[12] The average default frequency is 4 percent annually, or 1 percent quarterly.

Figure 3: Policy Functions



decreases with debt level (i.e., the interest rate rises). Across productivity states, the bond price is significantly higher for a high state, which implies countercyclical interest rates.

3.3 Cyclical Movements in the Baseline Model

This section starts the assessment of the quantitative performance of the model by comparing moments from the data with moments from the model's dynamics. To compute the latter, I feed the TFP process into the model and conduct 1000 simulations, each with 600 periods. Then I truncate the first 100 observations and use the rest to compute the statistics of this model's results.

Table 3 compares the moments produced by the baseline model with those from Mexico data and Mendoza and Yue (MY, 2012). Notice that Mendoza and Yue (2012) calibrate their model partially to Argentine data and partially to Mexican data. All the data used in this model are quarterly from 1981Q1 to 2012Q4. The data sources are provided in the Appendix. First, this model produces a debt-to-GDP ratio of more than 11 percent on average while at the same time matching the 1 percent default frequency observed in the data. Because the time discount factor of country 2 is lower than that of country 1 in calibration, the difference in consumption patience can support a certain level of debt. However, this model's debt-to-GDP ratio is lower than that in Mendoza and Yue (2012). Their paper adopts a lower default frequency and a lower time discount factor ($\beta = 0.88$) that allow their model to produce a higher

Table 3: Statistical Moments of Country 2's Business Cycles

Variable	**Data**	**Baseline**	**MY (2012)**
Average debt/GDP ratio (in percent)	74.94	11.73	22.88
Average bond spreads (in percent)	4.35	4.51	0.74
Bond spreads std. dev. (in percent)	4.71	3.83	1.23
Real exchange rate std. dev.	0.17	0.07	n.a.
Domestic product consumption std. dev./GDP std. dev.	1.23	0.73	n.a.
Total consumption std. dev./GDP std. dev.	1.12	0.98	1.05
Trade balances excl. intm goods exp/GDP std. dev. in percent	2.01	1.71	n.a.
Final goods export std. dev./GDP std. dev.	0.82	0.12	n.a.
Intm. goods export std. dev./GDP std. dev.	1.02	0.42	n.a.
Total import std. dev./GDP std. dev.	1.13	0.31	n.a.
Correlation with GDP			
Bond spreads	−0.39	−0.67	−0.17
Real exchange rate	0.53	0.37	n.a.
Trade balances excl. intm goods exp/GDP	−0.65	−0.61	−0.54
Total exports	0.21	0.84	n.a.
Intermediate goods exports	0.18	0.80	n.a.
Total import	0.75	0.82	n.a.
GDP volume	0.65	0.80	n.a.
Default occurrence	−0.14	−0.25	−0.09
Default duration	−0.39	−0.60	n.a.
Correlation with bond spreads			
Real exchange rate	−0.76	−0.86	n.a.
Trade balances excl. intm goods exp/GDP	0.30	0.37	0.15
Total exports	−0.02	−0.59	n.a.
Intermediate goods exports	−0.08	−0.67	n.a.
Total import	−0.28	−0.86	n.a.
GDP volume	−0.19	−0.16	n.a.
Default occurrence	0.18	0.29	n.a.
Default duration	0.56	0.94	n.a.

Note: Except bond spreads, the real exchange rate, default occurrence and duration, and the averages, all other data in the table are HP-filtered. Trade balances are calculated as ratios to GDP.
All data are in real terms and at quarterly frequency.

debt-to-GDP ratio.

Moreover, unlike Mendoza and Yue (2012) and many other sovereign default models, this paper incorporates creditors' risk aversion. As default risks mount, creditors' risk aversion can potentially cause bond prices to decrease faster and suppress country 2's debt level more severely than in a model with risk neutral creditors. This also implies that country 2 needs to pay a higher interest rate, which in turn further increases its default probability. Hence, creditors' risk aversion limits the model's ability to generate data-matching debt-to-GDP ratios, which is another reason that this model generates a lower debt-to-GDP ratio than Mendoza and Yue (2012). However, risk aversion does help my model support a higher average bond spread, which is more consistent with the data than existing literature. I will elaborate on this point below.

The sustainable debt level in the model also depends on the costs of default. As the default costs get larger as a percentage of GDP, country 2 becomes less likely to default, and thus can sustain a higher debt level. The costs of default to the borrower in the model include exclusion from the financial market, and endogenous income losses from the intermediate goods sector and the real depreciation. The effect of income loss from intermediate goods exports on debt level is restricted by the magnitude of the income loss, consumers' preferences with respect to home goods and imports, and their impacts on the real exchange rate. Additionally, the income loss from intermediate goods exports increases with country 2's productivity state. Hence, country 2 has a larger incentive to default at a lower state.

The mean and variance of bond spreads are close to the data. Notice that the bond price reflects not only the expected return due to the probability of default, but also compensation to the risk-averse creditors for bearing consumption risk. Therefore, unlike previous papers with risk-neutral investors, the risk aversion of investors in country 1 in this model break the close link between the probability of default and bond pricing. This enables the result to be consistent with the data, where the average bond spread is several times higher than historical default frequency.

The volatility of real exchange rates, domestic consumption, and total consumption are smaller in the model than in the data. Real exchange rate fluctuation upon default is greatly influenced by the home bias preference of consumption in the model (see Sensitivity Analysis section). In the model, when a large share of a country's utility depends on its consumption of foreign goods, it is optimal for the country not to reduce imports too much when income is low and to have a stable real exchange rate. However, in reality many other trade and monetary policies can affect the fluctuations of real exchange rates along business cycles, and these are not taken into account by this model. Domestic consumption is much smoother in the model than in the data because of borrowing, home bias preference, and labor movement from the intermediate goods sector to the domestic sector in default crises. These three factors support domestic goods production and their consumption in spite of adverse productivity shocks. Total consumption is less smoothed than the domestic counterpart in the model on account of the variations in imports and real exchange rate deterioration during default.

In terms of the volatility of trade balances, imports, and exports, the model results are also lower than those of the data. Other sovereign default models have generated similar results for trade balances, for example, Aguiar and Gopinath (2007) produces trade balance standard deviation to be 0.95, Arellano (2008) 1.5, and Yue (2010) 2.81. But the difference here is that the trade balances in this paper are not residuals of output from consumption. Instead, they are the combined result of capital flows and consumers' home bias and risk aversion. Moreover, this model is distinguished from previous default models in separating exports and imports. In both the data and the model, final goods exports are less volatile than imports and intermediate goods exports.

Next, Table 3 shows that this model does a good job of delivering the correlation between GDP and bond spreads, as well as their correlations with other variables. The model yields a negative correlation between bond spreads and GDP, consistent with the data, because the bond bears higher default risk in bad states. As in Mendoza and Yue (2012), this model produces countercyclical default risk in a setting where both output and default risks are endogenous and affect each other, unlike in the models of sovereign default alone or of business cycles alone.

However, this model distinguishes itself from Mendoza and Yue (2012) in that the endogenous output and default risks affect each other mainly through the deterioration of the real exchange rate. In my model, the real exchange rate has a positive relation with GDP and a strong negative relation with bond spreads, which is consistent with the data. As country 2 borrows more and more, the bond interest rate rises, resulting in a tighter budget constraint with more and more funds flowing out of country 2 into country 1. Therefore, due to the home bias preference, country 2 consumers adjust their consumption portfolio to reduce imports, and the real exchange rate is under pressure to fall. This prevents real appreciation from helping the country to pay back debt that is denominated in country 1's final goods, and also from easing the budget constraint.

Once country 2 defaults due to an adverse productivity shock, it is penalized by exclusion from the financial market and another endogenous income loss from the intermediate goods sector that is additional to the income loss from the initial adverse productivity shock. As country 2's budget constraint further tightens, its consumption portfolio is once again adjusted away from imports and its final goods exports rise. Consequently, the trade balances become more positive than before, while the real exchange rate drops sharply.

This mechanism, in which increased borrowing and default risk raise the real interest rate and exports, and decrease income and imports, explains why this model generates a negative relation between trade balances and GDP, while producing a positive relation between trade balances and bond spreads. This is consistent with the Mexican data. It is also in line with the stylized fact of other developing countries' business cycles and previous models of sovereign default. For instance, Mendoza and Yue (2012) uses annual Argentine data for 1980-2005, and show that its trade balances' correlations with GDP and bond spreads are -0.87 and 0.82, respectively. Their model generates the trade balances' correlations with GDP and bond spreads to be -0.54 and 0.15, respectively. Moreover, consistently with the data, this model also delivers procyclical bilateral trade flows and a negative relation between bilateral trade flows and bond spreads that have not been captured by previous papers.

Furthermore, the model predicts the correlations between country 1's use of intermediate goods from country 2 and both GDP and bond spreads, qualitatively in line with the correlations observed in Mexican data. More broadly, the business cycle correlations between output and intermediate goods export value vary across countries but are usually positive. For instance, using annual growth data (1988-2013), I compute the correlation for 17 countries, for which I have intermediate goods exports data. On average, their correlation between output growth and intermediate goods exports growth is 40 percent.[13]

Additionally, with the real exchange rate in this model, I can disentangle the default-related losses in GDP value and GDP volume. As I show in the next section, during default periods, about one third of GDP value loss is due to GDP volume declines, while the other two thirds are attributed to real depreciation. In Table 3, even though GDP value is positively correlated to GDP volume, it is not a perfect correlation–only 65 percent in the data and 80 percent in the model. The real exchange rate does play a role in GDP value variations in both the model and the data. Also, consistently with the data, the model generates declining production activities when bond spreads increase along business cycles.

Lastly, I report in Table 3 the correlations between default and output, and between default and bond spreads. In particular, the onset of a default event is positively

[13] The correlation for Argentina is 34.8%, Brazil 66.4%, Croatia 13.1%, Ecuador 5.2%, Greece 17.1%, Iceland 41.8%, Indonesia 24.4%, Moldova 26.2%, Peru 52.6%, Russia 66.1%, South Africa 56.4%, Spain 36.3%, Thailand 55.5%, Turkey 7.2%, Ukraine 81.8%, Uruguay 58.5%, and Venezuela 43.3%.

correlated with output and negatively correlated with bond spreads in both the data and the model. As for the duration of a default episode, it is more closely related to bond spreads than to output in the model, which is also the case in the data.

3.4 Dynamics around Default Events

Next I study the model's macroeconomic, trade, and welfare dynamics *around default events* by comparing the simulated results with the time series data for Mexico. This paper identifies three sovereign default occurrences by Mexico since 1981, in 1982Q3, 1986Q1, and 1989Q1. Those dates are inferred from Paris Club data, which shows that Mexico was treated on 1983 June 22nd, 1986 September 17th, and 1989 May 30th by foreign creditors for its sovereign debts. Each episode window covers 6 quarters before and after a default onset. Date 0 is the quarter of default occurrences. I plot the mean of these three default episodes for each of the variables from the data, as well as the mean from the model simulation surrounded by a one-standard-deviation band.[14]

3.4.1 Macroeconomic Dynamics

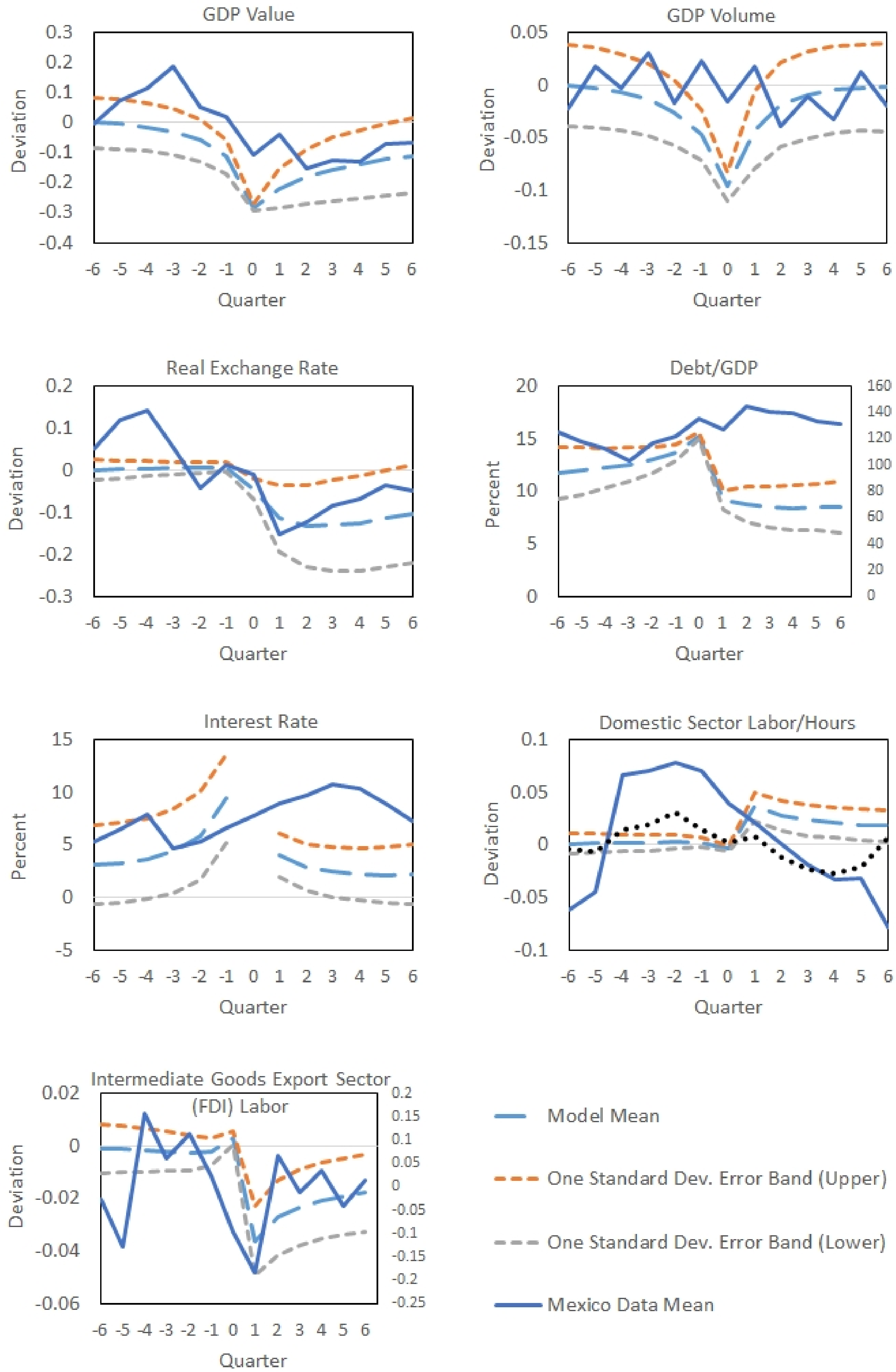
Figure 4 shows the model's macroeconomic dynamic results, i.e., deviations from steady state (or trend in the data). The model's V-shaped GDP value dynamics are qualitatively consistent with the data for Mexico, although the model generates a deeper drop. In the model, the decline of output upon default comes from two sources: GDP volume decrease and real exchange rate deterioration. On average, the lasting exchange rate deterioration after a default in the model matches well with the data, about 15 percent. However, Mexico's GDP volume (seasonally adjusted) does not seem affected much by the default events, whereas in the model GDP volume decreases by almost 10 percent on average, less than the decline of the real exchange rate. Overall, the GDP loss upon default is mostly driven by the lower real exchange rate in the model, and even more so in Mexico data.[15]

On the sovereign debt market, this model predicts that the debt-to-GDP ratio is relatively stable over the four quarters prior to a default, then surges as it approaches

[14]This section uses the same simulation results as in the previous section.

[15]It is worth noting the fact that little GDP volume declines around the default episodes for Mexico is not uncommon for other countries. The changes to GDP volume growth upon a default are diverse across countries. For instance, Paraguay grew by 4.32 percent in GDP volume during its 2003 default, while Indonesia's GDP volume declined by 13.13 percent during 2008 default.

Figure 4: Macroeconomic Dynamics around Default Events



Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1. Except for the interest rate and debt-to-GDP ratio, all other data are HP-filtered. GDP value, GDP volume, domestic sector labor, hours worked, and intermediate goods export sector (FDI) labor are logged before being detrended. For the subplots with a different scale on the right axis, the scale is for the data of the variable.

and reaches the default, and drops afterwards. To make the model's debt ratio comparable to the data, I follow Mendoza and Yue (2012) and adjust the mean of the post-default debt ratios from the model to be the average of the pre-default debt ratio and the debt ratio chosen once the country reenters the international financial market. However, the post-default debt ratio in the model still declines faster than in the data. Also, the model's debt ratio is significantly lower than the data's. This is an issue that is common to the previous strategic sovereign default models. It can be partially attributed to creditors' risk aversion, as analyzed in the previous section.

Moreover, the model does well in creating the qualitative features of the increase in the real interest rate around default events. I do not show the model interest rate at the default quarter because it does not exist due to exclusion from the financial market. The model is able to support high interest rates and bond spreads prior to a default, due to creditors' risk aversion. Here the interest rates not only incorporate the default risk, but also compensates the creditor country for its risk aversion.

In addition, the model matches the intermediate goods exports (through vertical FDI and offshoring) sector employment especially well, in both its decline and recovery, consistent with the finding of Bergin, Feenstra, and Hanson (2009, 2011).[16] But the model does not match the domestic sector employment very well. This is mainly because the model does not include unemployment. More specifically, country 2's domestic production sector has to take whatever amount of labor remains after the intermediate goods sector's employment. If this limitation were eliminated, the model could potentially have another labor market channel affecting the costs of default for country 2, as in Mendoza and Yue (2012). However, if we look at total hours worked (the dotted line), it did not decline as much as employment upon default and matches better with the model result.[17] Overall, I simplify the labor market in this paper to highlight the real exchange rate channel and international trade results. Nevertheless, adding more labor market dynamics to this model would be of interest for future research.

[16]Bergin, Feenstra, and Hanson (2009, 2011), using Mexico's maquiladora sectors, find that the country's offshoring industries experience employment fluctuations that are much more volatile than those in the U.S.

[17]Notice that due to data limitation, here total hours worked data is for manufacturing sector and is not necessarily for domestic production only.

3.4.2 Trade Dynamics

I now analyze the model's performance in variables related to international trade in Figure 5 and Figure 6. Before going into detail in Figure 5, one thing to notice from the data is that there are significant fluctuations in the measures of exports around $t = -4$. This is due to the rise of Mexico's real exchange rate from mid-1980 to early 1982, as shown in Figure 4's real exchange rate data. It is not a consequence of default and is exogenous to the components of this model. Disregarding this irregularity in the data, the model captures the qualitative features of different measures of exports in the data, as shown in Figure 5.

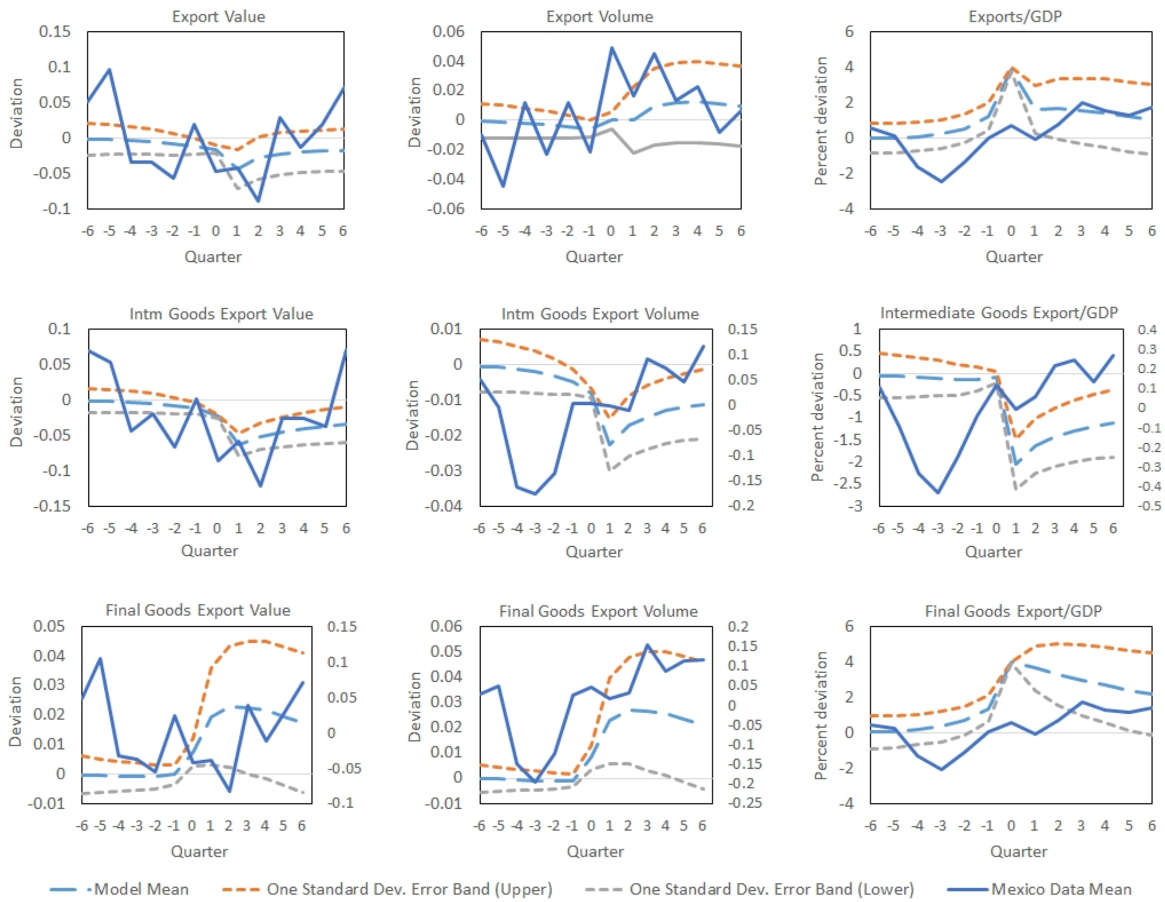
In particular, total export value declines slightly, even though export volume and export share of GDP rise on average. The main cause of this difference is real exchange rate deterioration upon default. I then separate the total exports data into intermediate goods and final goods, as in the model. Since there is no intermediate goods and final goods export volume data available for Mexico, I use intermediate goods and final goods exports in peso (logged) to approximate the volumes. For final goods exports, the model matches the post-default rising value, volume, and share of GDP, although it fails to capture the initial declines of their value that are due to real exchange rate declines upon default.

For the intermediate goods exports, their value, volume, and percentage of GDP all decline in the model as in the data, even though the volume and the percentage of GDP declines are slight and they recovered faster in the data than in the model after the defaults. It is worth noting that declines of intermediate goods exports are not uncommon for other developing countries' default episodes. As shown in Table 1, on average during default periods, its average annual growth rate is -1.11 percent. A similar picture is true for the growth rates of intermediate goods export volume (0.82 percent) and its share of GDP (-1.92 percent).

Figure 6 plots the dynamics for imports and trade balances (including intermediate goods exports). The model does well in matching the qualitative patterns of imports upon default. It succeeds especially in being in line with the data of import-to-GDP ratio, given that most of the data path is within the error band of the model result. Adding import value and export value together, bilateral trade value is more than 8 percent below trend upon default, which is consistent with the finding of Rose (2005).

Lastly and importantly, the rise in the trade balances when a default occurs is a

Figure 5: Export Dynamics around Default Events



Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1. All data are real and HP-filtered. Export value (in USD), export volume, and final goods exports are logged before being detrended. For the subplots with a different scale on the right axis, the scale is for the data of the variable.

Figure 6: Trade Dynamics around Default Events

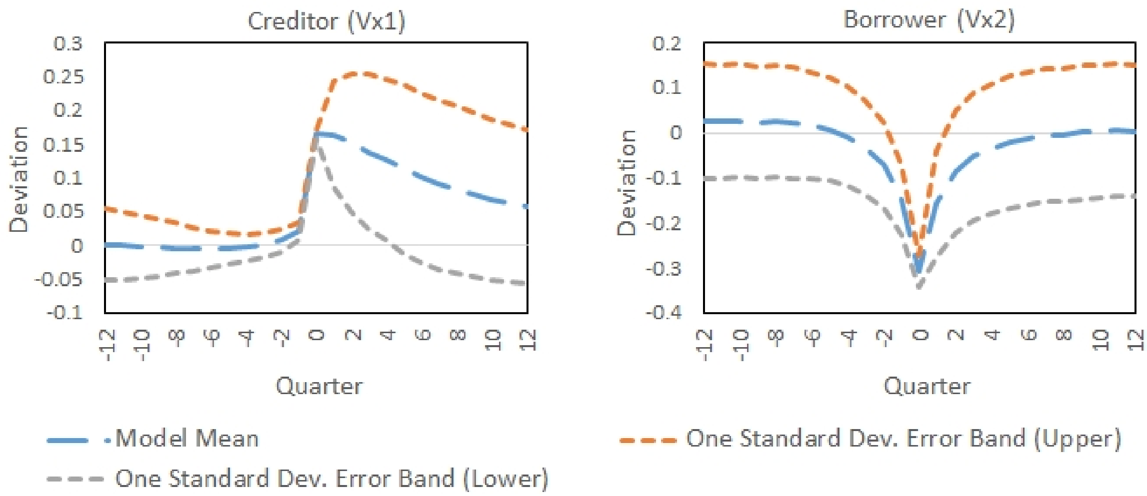
Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1. All data are HP-filtered. Import value and volume are logged before being detrended. For the subplots with a different scale on the right axis, the scale is for the data of the variable.

result of a larger increase in the export-to-GDP ratio than the increase in the import-to-GDP ratio, in both the data and the model. In previous default models, during the default period the defaulting country usually registers a zero trade balance, because it is excluded from the international financial market for at least one period, and thus there is no capital flow to finance the trade imbalance. In this respect, Mendoza and Yue (2012) is an exception. It introduces default-triggered exogenous capital flows that are independent of the borrowing and the default decisions, to support the surge of trade balances upon default in their model. This paper differs in that no exogenous element is needed. Here, the trade balances are naturally generated by the trading of two final goods for consumption and intermediate goods for production.

3.4.3 Welfare Dynamics

Since this paper uses a two-country model, I am able to study the welfare of both creditor and borrower (Figure 7). For the creditor country, on average a default triggers a 15 percent surge in welfare; this gain remains positive and withers slowly back to zero during the next 12 quarters and beyond. It is not surprising to see a welfare gain as

Figure 7: Welfare around Default Events



Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1.

a default approaches, because the creditor country harvests higher interest rates. But the slow decay of this extra gain after a default is less obvious. This happens mainly because after a default, the borrower's real exchange rate remains low. Therefore, the creditor country is still able to enjoy inexpensive imports. However, we should interpret this result of welfare gain for the creditor country and the magnitude of the gain with caution. This paper aims to point out that it is worth considering the real exchange channel, through which a creditor country can experience gains from another country's sovereign default. Meanwhile, we should also acknowledge that in practice the impact of a sovereign default to a creditor country's welfare depends on many other factors. For instance, it hinges on the substitutability between imports from the defaulting country and other goods for consumption and production in the creditor country.[18]

For the borrower country, a default delivers, on average, a 30 percent drop in welfare. However, this loss can be recovered in fewer than 12 quarters, as the country restores its productivity and reenters the international financial market. The welfare pattern before and after a default is more symmetric than it is for the creditor country. Additionally, in the model the world welfare worsens upon default, since the creditor's gain is smaller than the borrower's loss. But the world is able to recover and register

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a net gain within 12 quarters.

3.5 Results for the Sample Period

The model can also replicate the time series of Mexico output and bond spreads for the sample period of 1981Q1-2012Q4. I feed the corresponding productivity shocks into the model and compare the results with the data. Figure 8 plots the HP-filtered output value and volume, along with the simulated results. The model matches the data well in terms of the two output measures. The grey areas in the figures show the model-predicted default occurrences in 1986Q1 and 1995Q3. Even though 1995Q3 is not officially documented as a sovereign default, Mexico would have defaulted following its 1994 crisis without the aid it received from foreign countries (mainly the US).

Figure 9 plots the bond spreads. The model matches the bond spreads data well for the period of 1986Q1-2001Q4, and less so for the periods before and after. In particular, the model underestimates the interest rate spreads for 1982Q1-1985Q4. The main reason for this is that the productivity shocks fed into the model show a big spike for the output boom in Mexico right before 1985.[19] Overall, the model results indicate that Mexico faces countercyclical bond spreads, and it defaults on sovereign debts when the output is low and the interest rate is high.

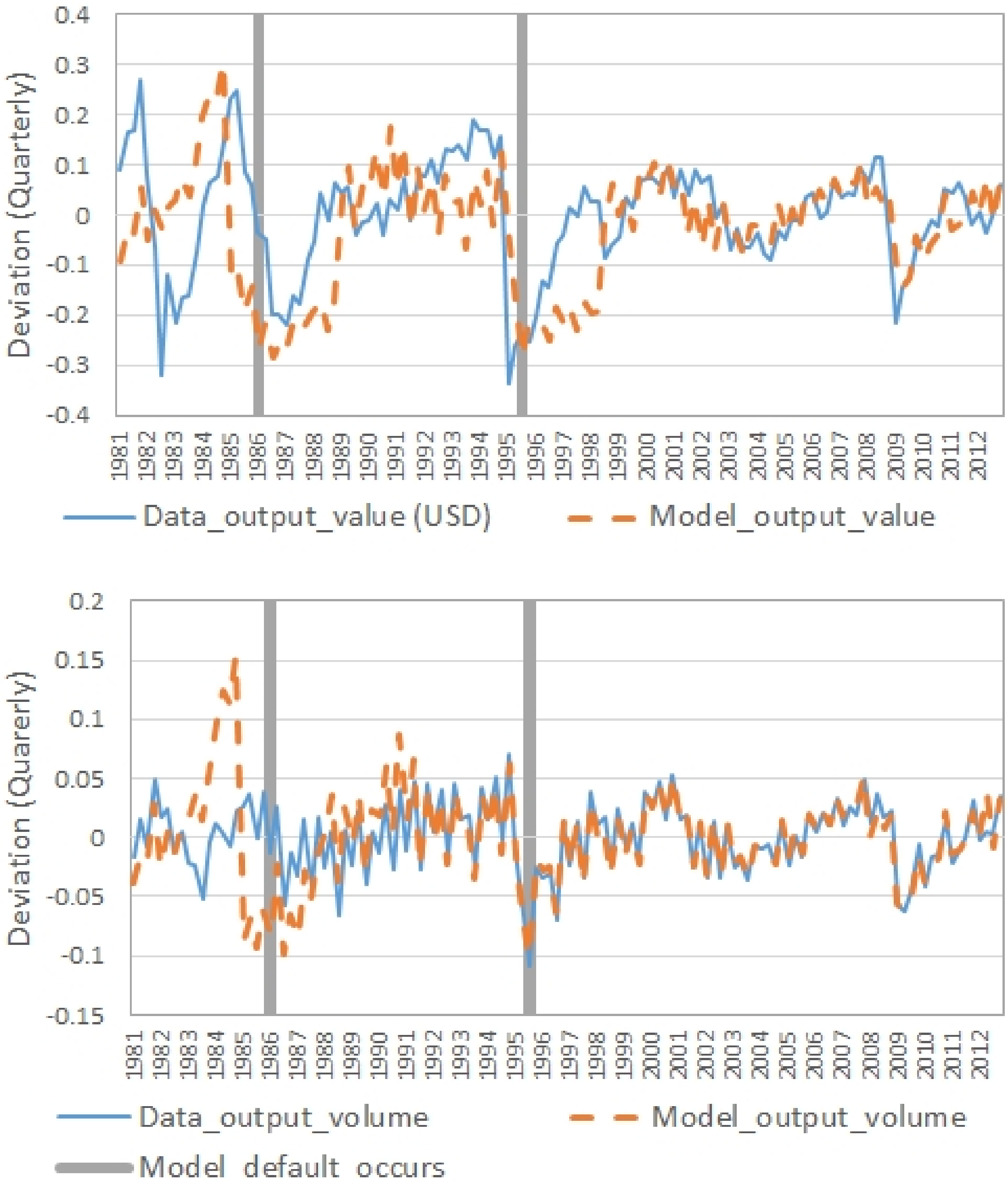
3.6 Sensitivity Analysis

In this section I conduct a sensitivity analysis to evaluate the robustness of the model's quantitative results regarding the real exchange rate, bond spreads, and trade flows. The model results are robust to changes in data filter, country 2's shock persistence (ρ_2), patience (β_2), consumption preference to imports (θ_2), and post-default efficiency loss in intermediate goods exports (ϵ). The results are summarized in Table 4.[20] This table shows the main business cycle statistical moments as in Table 3 for each alternative scenario, plus default frequency and several around-default values for variables of interest.

[19]The spike appears using HP filter or BP filter.

[20]I also generated around-default dynamics plots comparable to Figure 4, 5, and 6. The quantitative differences are small and qualitative patterns are the same, so I do not put them in the paper due to space limitation. But several around-default values for variables of interest are reported in Table 4.

Figure 8: Mexico Output in the Data and in the Model (1981Q1-2012Q4)



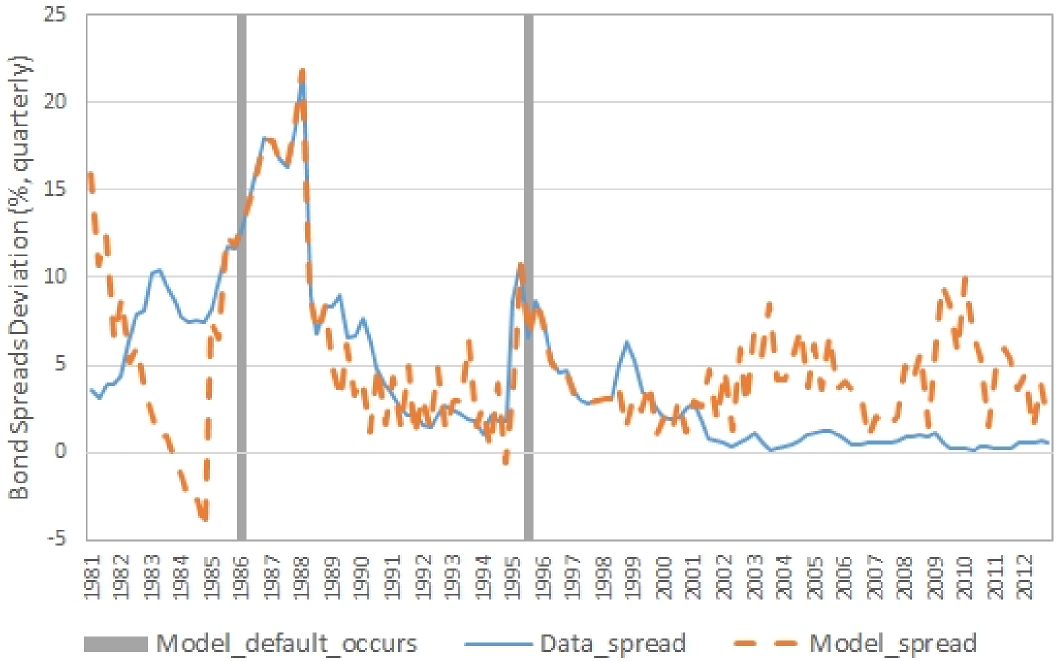
Note: All Mexican data are HP-filtered.

Table 4: Sensitivity Analysis

Statistics	**HP**		**BP**		**Shock Persistence**		**Patience**		**Imports Preference**		**Intm. Goods**	
	Data	Baseline	Data	Model	$\rho = 0.2$	$\rho = 0.6$	$\beta_2 = 0.9255$	$\beta_2 = 0.9755$	$\theta_2 = 0.2$	$\theta_2 = 0.4$	$\epsilon = 0.82$	$\epsilon = 0.88$
Default frequency	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.007	0.01	0.01	0.01	0.01
Ave. debt/GDP ratio (%)	74.94	11.73	74.94	15.46	12.29	10.90	9.85	11.73	10.35	13.31	14.35	8.94
Ave. spreads (%)	4.35	4.51	4.35	3.46	4.67	4.42	8.13	3.72	4.54	4.57	4.48	4.58
Spreads std. dev. (%)	4.71	3.83	4.71	0.90	4.56	3.23	5.16	3.32	3.76	3.87	3.85	3.84
REXR std. dev.	0.17	0.07	0.17	0.05	0.06	0.07	0.08	0.05	0.14	0.04	0.07	0.05
Dom. prod. cons. std/GDP std.	1.23	0.73	1.15	0.73	0.71	0.75	0.73	0.71	0.83	0.60	0.72	0.75
Total cons. std./GDP std.	1.12	0.98	1.10	1.01	0.96	0.99	1.03	0.92	0.98	0.98	0.99	0.96
Trade balance std. (%)	2.01	1.71	1.82	1.47	1.61	1.70	2.19	1.37	1.18	1.96	1.99	1.34
Final goods exp. std./GDP std.	0.82	0.12	0.73	0.10	0.12	0.10	0.13	0.11	0.07	0.15	0.12	0.11
Intm. Exp. std./GDP std.	1.02	0.42	0.99	0.40	0.44	0.38	0.47	0.39	0.29	0.53	0.44	0.38
Total imp. std./GDP std.	1.13	0.31	1.13	0.32	0.31	0.29	0.35	0.27	0.19	0.43	0.33	0.27
Correlation with GDP												
Spreads	−0.39	−0.67	−0.23	−0.76	−0.66	−0.64	−0.78	−0.58	−0.70	−0.62	−0.70	−0.59
REXR	0.53	0.37	0.44	0.59	0.34	0.30	0.58	0.12	0.54	0.10	0.47	0.18
Trade balances	−0.65	−0.61	−0.74	−0.79	−0.57	−0.61	−0.76	−0.42	−0.53	−0.67	−0.68	−0.50
Total exports	0.21	0.84	0.27	0.88	0.84	0.84	0.81	0.87	0.76	0.91	0.84	0.84
Intm. exp.	0.18	0.80	0.22	0.86	0.81	0.79	0.79	0.82	0.77	0.83	0.82	0.77
Total import	0.75	0.82	0.87	0.92	0.80	0.84	0.89	0.75	0.78	0.88	0.85	0.79
GDP volume	0.65	0.80	0.77	0.84	0.79	0.84	0.73	0.85	0.76	0.84	0.77	0.85
Default occ.	−0.14	−0.25	−0.08	−0.24	−0.25	−0.24	−0.27	−0.22	−0.24	−0.24	−0.26	−0.23
Default dur.	−0.39	−0.60	−0.38	−0.71	−0.59	−0.58	−0.70	−0.51	−0.64	−0.56	−0.64	−0.52
Correlation with bond spreads												
REXR	−0.76	−0.86	−0.76	−0.92	−0.86	−0.83	−0.89	−0.77	−0.90	−0.76	−0.90	−0.79
Trade balances	0.30	0.37	0.16	0.68	0.22	0.52	0.65	0.09	−0.07	0.63	0.41	0.33
Total exports	−0.02	−0.59	−0.04	−0.39	−0.61	−0.60	−0.36	−0.68	−0.39	−0.75	−0.54	−0.65
Intm. exp.	−0.08	−0.67	−0.05	−0.51	−0.68	−0.68	−0.51	−0.73	−0.44	−0.82	−0.63	−0.72
Total import	−0.28	−0.86	−0.17	−0.91	−0.79	−0.92	−0.90	−0.79	−0.74	−0.92	−0.85	−0.88
GDP volume	−0.19	−0.16	−0.13	−0.35	−0.13	−0.20	−0.21	−0.14	−0.15	−0.16	−0.16	−0.15
Default occ.	0.18	0.29	0.18	0.29	0.30	0.28	0.26	0.32	0.29	0.29	0.29	0.29
Default dur.	0.56	0.94	0.56	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
Upon default												
Ave. REXR deviation in a year	-0.09	−0.11	-0.10	−0.10	−0.12	−0.08	−0.11	−0.10	−0.26	−0.05	−0.14	−0.07
Ave. creditor's welfare deviation	n.a.	0.16	n.a.	0.13	0.17	0.15	0.11	0.18	0.20	0.12	0.20	0.12
Ave. defaulter's welfare deviation	n.a.	−0.31	n.a.	−0.70	−0.24	−0.45	−0.25	−0.34	−0.27	−0.34	−0.33	−0.30
Ave. world welfare deviation	n.a.	−0.15	n.a.	−0.56	−0.07	−0.30	−0.13	−0.16	−0.07	−0.22	−0.13	−0.18

Note: Except for bond spreads, the real exchange rate, default occurrence and duration, and the averages, all other data in the table are HP-filtered. Trade balances here exclude intermediate goods exports, and are measured as a ratio to GDP. All data are in real terms and at quarterly frequency. For BP filter model results, $\beta_2 = 0.9797$, shock process $\rho = 0.8876$ and $\sigma = 0.0143$.

Figure 9: Mexico Bond Spreads in the Data and in the Model (1981Q1-2012Q4)



3.6.1 BP Filter

In the first scenario, I re-calibrate the entire model with BP-filtered data. BP filter may extract the desired frequency from the quarterly data more precisely than HP filter. In fact, in this case BP filter generates a much higher persistence of productivity shock for Mexico (0.8876) than the baseline using HP filter (0.4162), and a lower volatility (0.0143) than the baseline (0.0377). Again, to calibrate the model's default frequency so that it is consistent with the data (0.01), I adjust β_2 to 0.9797 (baseline is 0.9655).

The results are close to the baseline and qualitatively robust to different filters. Yet three differences stand out. First, the average debt-to-GDP ratio and the volatility of total consumption are higher than those of the baseline, becoming more consistent with the data. Second, the variation of bond spreads declines. Third, the defaulter registers a larger welfare loss upon default than baseline. However, switching the filter makes it difficult to disentangle the causes for those changes, because multiple parameters have been changed at the same time (i.e., the entire model is re-calibrated). The following scenarios have only one parameter's value changed at a time.

3.6.2 Shock Persistence

The next two columns of Table 4 report the results for shocks with lower persistence ($\rho = 0.2$) and higher persistence ($\rho = 0.6$) than the baseline ($\rho = 0.4162$). Comparing the three scenarios, I find that most variables have small changes that are monotonic with the change of shock persistence. Some more significant changes occur to the average debt-to-GDP ratio, the correlations of GDP value and bond spreads with trade balances and GDP volume, and the defaulter's and the world's welfare losses.

First, lower shock persistence allows a country to get back on track faster after an adverse productivity shock and thus to maintain a higher average debt level. Second, lower shock persistence also generates lower and short-lived output and income declines on average. So although home bias preference will shift consumers towards domestic goods during downturns, imports do not have to decline as much as in baseline. Therefore, both imports and trade balances become less correlated with GDP and bond spreads. Third, these conditions also lead to a smaller welfare loss for the defaulting country, as well as the world. These changes are reversed in the case of higher shock persistence.

3.6.3 Patience

The discount factor $\beta_2 = 0.9655$ in the baseline is much higher than the value commonly used in the sovereign default literature (e.g., the discount factor in Aguiar and Gopinath 2006, Arellano 2008, Yue 2010, and Mendoza and Yue 2012 are all below 0.953). However, it is lower than the typical value in real business cycle literature. Here I conduct some analysis to see how the results vary with the value of the discount factor. A lower discount factor of 0.9255 and a higher one of 0.9755 are used to compare the results with the baseline's 0.9655. All the variables change monotonically with the discount factor.

Intuitively, less care about the future brings more frequent defaults, a lower average debt-to-GDP ratio, and a higher average bond spread. Meanwhile, less patience also generates more volatility in total consumption and trade, moving the model results closer to the data. The resulting real exchange rates and trade balances also fluctuate more in correlation with GDP and bond spreads. However, the borrower's post-default welfare loss is significantly smaller than that in the baseline. The main reason is that the borrower is not as concerned as in the baseline about their losses in the near future due to default.

3.6.4 Consumer Preference to Imports

The next two columns in Table 4 report the model results with a lower consumer preference towards imports ($\theta_2 = 0.2$) and a higher imports preference ($\theta_2 = 0.4$) than that of the baseline ($\theta_2 = 0.3$). Hypothetically, with a higher preference for imports, the borrower would become more concerned about its imported consumption once it defaults, leading it to default less frequently. However, its impact on default reduction is little according to the model result. Even though a higher imports preference changes little about the frequency of defaults, it does allow the borrower to sustain a higher average debt-to-GDP ratio.

Moreover, with a higher consumption preference for imports, it is optimal for the country to maintain lower real exchange volatility, and thus the real exchange rate's correlations with GDP and bond spreads both decline. Upon default, the real exchange rate's drop (-0.05) is about half of that in the baseline (-0.11). In terms of welfare, being more open to imports does expose the borrower to more welfare losses during a sovereign default crisis.

Worldwide, welfare also declines upon default, but the decline is much larger in the case of a higher imports preference of the borrower, because of a larger decline in the defaulting country's welfare and a smaller gain by the creditor country. Here, creditor country's smaller gain is due to limited real exchange rate change upon default. This result does not necessarily imply that increasing trade openness causes more damage to the world during a default crisis. For instance, if the model allows the creditor country to raise its appetite for the borrower's imports at the same time, the creditor would be able to harvest more gains from a sovereign default event through increased imports from the defaulter. World welfare would not decrease as much upon default in a bilaterally-integrated world as it would in a unilaterally-integrated world.

One result that is qualitatively different from the baseline is that when the imports are desired less ($\theta_2 = 0.2$, stronger home bias), the business cycle correlation between trade balances and bond spreads turns slightly negative. Two effects play a role in this change. First, even though trade balances still rise upon default, these increases are weakened because of weaker increases of final goods exports as a GDP ratio given the stronger home bias. Second, when the borrower country has not defaulted yet, its bond spreads rise, and its budget constraint is tightened, the country will decrease some of its imports and even more of its exports as ratios to GDP to satisfy the strong preference for domestic goods. Despite the slight negative correlation between trade balances and bond spreads, it is worth noting that the correlations of GDP with trade balances and bond spreads are still negative and consistent with data.

3.6.5 Post-default Efficiency Loss in Intermediate Goods Exports

Finally, I report results with a lower value of $\epsilon = 0.82$ (i.e., a greater efficiency loss of intermediate goods exports upon default) and a higher $\epsilon = 0.88$ than that in the baseline $\epsilon = 0.85$. It is important to experiment with different values of ϵ because it governs the magnitude of post-default losses in the defaulter's intermediate goods exports, and thus also the post-default tightness of its budget constraint, and its changes in real exchange rates and trade flows. Even though the results change slightly under different values of ϵ, the signs of all the statistics remain consistent with the data across the two scenarios.

With a lower value of $\epsilon = 0.82$, the borrower suffers from a larger loss upon default, which induces two main effects. First, it helps the borrower to maintain a higher level of average debt-to-GDP ratio.[21] Second, together with home bias preference, the larger loss and tighter budget make the borrower's trade flows and real exchange rates more responsive to a default crisis. Hence, the model delivers a larger real depreciation of 14 percent upon default. Meanwhile, the volatility of the real exchange rates and trade balances are slightly higher, and their correlations with GDP and bond spreads are also higher.

As for welfare, it is clear that under a lower value of $\epsilon = 0.82$, the defaulter registers a larger loss, while the creditor gains more through favorable exchange rate movements and trade than in the baseline. In terms of world welfare, this model predicts that, other conditions being equal, a higher default penalty achieved by reducing the use of intermediate goods from the defaulting country can help reduce the loss of world welfare. This is because of a larger welfare transfer from the borrower to the creditor through more severe real depreciation.

Summing up, this sensitivity analysis shows that although the model's statistical moments vary somewhat as I change key parameters, the main quantitative and qual-

[21] Default frequency also declines but by very little, beyond the digit shown in the table.

itative findings are robust to these changes. The model produces a sharp decline in the real exchange rate upon default, a high average bond spread, a negative correlation between trade balances and GDP, and other data-consistent correlations between various trade flows and GDP or bond spreads.

4 Conclusion

This paper proposes a two-country open-economy model of sovereign default, including both production and risk-averse agents. Its quantitative predictions are in general consistent with observed empirical regularities around emerging markets' sovereign defaults and along their business cycles. The model contributes to the literature with its endogenous trade balance reversal upon default, as well as its inclusion of imports and exports, the real exchange rate, and creditor welfare analysis.

The model features a real exchange rate amplification channel that links default with trade and income. As a country borrows more and more, its default risk increases, the interest rate rises, and the budget constraint tightens. Then, due to home bias consumption preference, the country adjusts its consumption portfolio by reducing imports. Now, with reduced imports, the real exchange rate is subject to downward pressure, and does not increase enough, if at all, to help the borrower pay back debt and ease its budget constraint.

Once the borrower does default due to an adverse productivity shock, an imposed financial autarky for a random period of time and a decline of intermediate goods exports from the defaulter to the creditor trigger another income loss additional to that by the initial adverse productivity shock in the defaulting country. The further tightened budget constraint induces another even bigger decline in imports, and thus a sharp deterioration of the real exchange rate. This real depreciation then contributes to a third income loss of the defaulter and to trade balance reversal. The real exchange rate channel produces a novel feedback loop among the borrower's default, income, and trade.

The model results are consistent with three important stylized facts about emerging markets' business cycles and sovereign defaults. First, the model delivers countercyclical trade balances and procyclical trade flows over business cycles. Second, it produces countercyclical bond spreads with a data-consistent average. Third, this model accounts for sharp real depreciation, trade balance improvements, and bilateral trade declines upon default. Moreover, following a default the model does not need an exogenous output loss but endogenously generates GDP losses, partially from real depreciation, partially from production activity decline, as in the data.

This model also predicts lasting welfare gains through the real exchange rate channel for the creditor country, but relatively short-lived welfare losses for the borrower country and the world during and after a sovereign default. Furthermore, this paper offers interesting perspectives on how default penalty and consumers' taste of foreign imports may interact with default frequency. Surprisingly, default frequency is hardly affected by either. The most important factor impacting default frequency lies in the borrower's degree of patience. However, a higher openness to foreign imports for consumption or a higher default penalty through intermediate goods exports does allow the borrower country to sustain a higher average debt-to-GDP ratio. But in the meantime, the borrower country will also suffer from a larger welfare loss once it defaults.

Default penalty and consumers' taste in foreign imports also impact the post-default welfare of the creditor. All else being equal, if the borrower country increases its unilateral openness to foreign import consumption, its post-default loss will become larger, and the creditor country will enjoy a smaller gain due to limited favorable real exchange rate changes. Therefore, in this case the world registers a larger loss. Alternatively, all else being equal, a higher default penalty through intermediate goods exports allows the creditor to gain more from the other country's default through much more favorable real exchange rates. Hence, the world registers a smaller loss.

It is worth noting that the story behind this model has the borrower country exporting intermediate goods. This allows sovereign defaults to interact with vertical integration. However, exporting intermediate goods is not necessarily the only story that can be told by this model. The model setup here is sufficiently versatile to be compatible with other stories that are also consistent with empirical observations. For instance, instead of borrower country 2 exporting intermediate goods, it could receive FDI k_1^* from country 1 to produce final goods 1 and exports them back to country 1. When country 2 defaults, the FDI may decline, triggering changes in trade and the real exchange rate.[22] Or, as in Mendoza and Yue (2012), borrower country 2 could instead import intermediate (capital) goods from country 1; and a default causes a decline in

[22]Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent.

such imports. Those alternative stories can be supported by the same model setup used here. The only adjustments needed are accounting for GDP, trade balances, and trade flows. I have calculated the model results for the business cycle moments of these two alternative stories, and have found that the key results remain the same.[23]

This line of research into the connections between default, income, trade, and exchange rate is far from complete. For instance, it would be interesting to study the case when *both* countries suffer from productivity shocks. Valid questions to ask include: how are the shocks transmitted across countries in a default model, and how are the risks shared? In particular, this model, with risk-averse investors, has the potential to explain why risk sharing worsens for emerging markets after financial integration (Bai and Zhang, 2012). Moreover, introducing a better labor market, exchange rate regimes (see Na, Schomitt-Grohe, Uribe, and Yue, 2014), and debt restructuring would all be promising subjects for future research.

Appendix: Data Sources

GDP value (in USD) and volume (in index), trade value (in USD) and volume (in index), trade as a share of GDP, real exchange rate, and consumption are from the IMF and the World Bank. Real Effective Exchange Rates for Mexico come from FRED maintained by the Federal Reverse Bank of St. Louis. Mexico's domestic capital stock is calculated by the author from combined data from FRED and the IMF.

I use Mexico's treasury bill rates from the IMF as its sovereign bond interest rates, and calculate the bond spreads with the U.S. government bond interest rates (FRED). Mexico's external public debt is from the combined sources of Mexico's Secretary of Finance and Public Credit and the IMF.

The frequency of defaults over the long term is calculated with information from Reinhart (2010). Sample default episodes are based on Reinhart (2010), the treatment dates from the Paris Club, and Mendoza and Yue (2012). Slight date adjustments according to GDP fluctuations have been made with regard to the Paris Club dates to reflect delayed treatments after defaults. The results are not sensitive to the default date specifications.

[23]These results are not reported in this paper due to space limitation, but they are available upon request.

Table 5: Default Events and Data Availability

Country	Event	Available Data
Argentina	1982	GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
	2002	GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Brazil	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Chile	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Croatia	1992	REXR, GDP vol., Exp. val., Imp. val.
Dom. Rep.	1993	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Ecuador	1999	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Egypt	1984	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Greece	2009	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Iceland	2008	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Indonesia	1998	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Mexico	1982	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
	1986	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
	1989	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Moldova	2002	REXR, GDP val., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Morocco	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
	1986	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Nigeria	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
	1986	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Pakistan	1998	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Paraguay	1986	REXR, GDP val., GDP vol., Exp. vol., Imp. vol.
	2003	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Peru	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Philipines	1983	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Romania	1981	GDP val., GDP vol.
	1986	GDP val., GDP vol.
Russia	1998	REXR, GDP val., GDP vol., Exp. val., Exp/GDP, Imp. val., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
S. Africa	1985	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
	1993	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Thailand	1998	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Turkey	1978	GDP val., GDP vol., Exp/GDP, Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
	1982	REXR, GDP val., GDP vol., Exp. vol., Exp/GDP, Imp. vol., Imp/GDP
	2000	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Ukraine	1998	REXR, GDP val., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Uruguay	1990	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
	2003	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP
Venezuela	1995	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
	1998	REXR, GDP val., GDP vol., Exp. val., Exp. vol., Exp/GDP, Imp. val., Imp. vol., Imp/GDP, Intm. exp. val., Intm. exp. vol., Intm. exp./GDP
Vietnam	1993	GDP val., GDP vol., Exp. val., Imp. val.

The annual data for the intermediate goods exports of multiple countries (see Table 5) come from the World Bank (WITS). The quarterly data for Mexico's intermediate goods exports is from Mexico's National Institute of Statistics and Geography (INEGI). It is also cross-checked with Mexico's annual intermediate goods exports data from the World Bank.

Mexico's total hours worked (quarterly) comes from FRED, and its total employment comes from the combined sources of the World Bank (WDI), the IMF, and Mexico's INEGI. Total FDI stock in Mexico is calculated using data from the OECD, U.S. Bureau of Economic Analysis (BEA), and the IMF. Furthermore, according to the UN's 2013 report on Foreign Direct Investment in Latin America and Caribbean, about 4.4 jobs are created in Mexico for every 1 million USD invested from abroad during 2003-2013. Using this number, I compute Mexico's FDI related employment according to its FDI stock data. Hence, Mexico's domestic sector employment is its total employment minus its FDI employment. In calibration, the data for labor share in production is from the OECD. Canadian employment is from Statistics Canada.

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