The Social Benefit of War

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October 2007

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November 22, 2007

Abstract

Recent findings in economic theory show that cooperation (settlement) between two identical players with conflicting interests in a valuable and contestable resource always Pareto dominates violent dispute (war), given that cooperation is presented using a symmetric bargaining norm. Necessary conditions for settlement to arise are the destructibility of war, and the costless and exogenous enforcement of any agreement made by the two players. We show that endogenous enforcement of the agreements alters the incentives of the players to bargain. This causes a shift in the Pareto frontier so that - under certain conditions - war Pareto dominates settlement.

Keywords: contests, property rights, endogenous enforcement, bargaining
JEL classification: C72, C78, D30, D70, D72, K42

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I am grateful to Martin Kolmar, Marco Runkel and Jan Schumacher for helpful comments. The usual disclaimer applies.

Even peace may be purchased at too high a price.

— Benjamin Franklin

1 Introduction

Since the seminal contributions of Nash (1950) and Schelling (1956) bargaining has become a major tool in economics to explain the phenomena of cooperation between agents with conflicting interests. Examples include labor and trade negotiations,[1] as well as peace negotiations.[2] Recently, economic theory began to illuminate the reason for the violent and non-violent resolution of territorial disputes using different bargaining norms in models of contests, i.e. in models in which agents make irreversible investment in arms in order to influence the probability of winning (Garfinkel and Skaperdas (2000), Anbarci et al. (2002), Cai (2003), Skaperdas (2006), McBride and Skaperdas (2006) and Garfinkel and Skaperdas (2007)), most of them using the classical guns-versus-butter tradeoff of nations.[3]

Garfinkel and Skaperdas (2000) and Skaperdas (2006) have shown that, in a one-period game, cooperation between two identical players with conflicting interests in a valuable and contestable resource always Pareto dominates violent dispute (*war*), given that cooperation is presented using a symmetric bargaining norm. The reason for this finding is the fact that violent measures involve destruction of the above resource (henceforth referred to as *resource under consideration*). The fact that the investment in arms made by the agents in order to compel cooperation is equal to (Skaperdas (2006)) or less than (Garfinkel and Skaperdas (2000)) the investment in arms made in case of *war*, creates a Pareto inferior outcome in case of *war*. Central to this finding is the assumption that, even without third party enforcement of contracts, an agreement between the two agents about the allocation of the resource under consideration affords them perfectly secure property rights, which means that the settlement remains uncontested. Given that in case of settlement arms are produced prior to the settlement in order to influence the negotiating position and that arms are therefore not exhausted in case of settlement, this seems to be an awkward assumption.

Contrary to Garfinkel and Skaperdas (2000) and Skaperdas (2006) we assume that the agreed-upon property rights are not necessarily self-enforcing. In fact, we believe that any agreement leading to a peaceful settlement affords initial claims to property which have to be enforced by the agents using the arms initially produced as a bargaining chip. Therefore, in the spirit of Grossman (2001), we distinguish between the conflict about a common pool resource (*war*) and the conflict about a resource to which agents have initial claims to property, obtained by mutual agreement.

Any violent dispute over the resource under consideration, without former mutual

[1] See, for example, Crawford (1981), McDonald and Solow (1981), Rubinstein and Wolinsky (1985), and Acemoglu and Pischke (1999).

[2] Early examples are Morrow (1986), Powell (1987), Morrow (1987), and Downs and Rocke (1990).

[3] See Samuelson (1958), page 21.

agreement on how to divide the resource, is called *war* (about a common pool resource). In the case of *war*, the resource under consideration is divided according to the technology of conflict, which maps the investment in arms of either side into a share function.[4] Additionally, we assume that, unlike any non-violent partition of the resource, *war* is destructive, by which we mean that *war* destroys a part of the resource under consideration. It is to emphasize that arms are exhausted in case of *war*. Thus, similar to Grossman (2001), any post-*war* allocation of the common pool resource remains uncontested, i.e. *war* creates effective property rights.
If *both* agents decide to bargain over the common pool resource and find an agreement, then each one obtains initial claims to a part of the resource under consideration through the agreement. In other words, any agreement on the resource allocation turns a former common pool resource into a commodity to which agents have initial claims to property. Unlike Garfinkel and Skaperdas (2000) and Skaperdas (2006), we are emphasizing that these are *initial* claims, since any agreement met through a bargaining procedure is basically not more than a mutual deselecting of the payoffs in case of a disagreement (here: payoffs in case of *war*).[5] However, as will become apparent, the bargaining procedure involves positive investment in arms in order to influence the disagreement point. Therefore, in contrast to *war*, arms are not *exhausted* in the bargaining procedure since they are produced in order to compel cooperation (an agreement). This is the reason why settlement, unlike *war*, does not necessarily create effective property rights. Effective property rights only emerge if the arms, produced in order to compel cooperation, are not exhausted in order to contest the initial claims afforded by the agreement.[6]
To display the potential violent use of arms after settlement, we make use of the Grossman modification of the TCSF which discriminates between the offensive use of arms in order to contest the initial claims of others (*appropriation*) on the one hand, and the use of arms for defensive fortification (*defense*) on the other.[7] Consequently, any agreement on the division of the resource under consideration which remains unchallenged (*uncontested settlement*) is only possible in the following case: Either side's investment in arms that associates the bargaining procedure is exclusively used to defend the initial claims to property.[8] In other words: In this case, both agents are able to deter appropriation, given the investment in arms already made.[9] This fact leads to our first question: *Is uncontested settlement possible in the shadow of arms that are associated with the bargaining procedure?* The answer is

[4] As in Garfinkel and Skaperdas (2000) and Skaperdas (2006) we use a Tullock contest success function (TCSF) to display the technology of conflict in case of war.

[5] In Garfinkel and Skaperdas (2000) these initial claims are perfectly secure. That implies that any agreement about the division of the resource under consideration creates simultaneously perfectly secure property rights, i.e. property rights that remain uncontested.

[6] This is what Neary (1996) calls *redistribution by force*.

[7] See Grossman and Kim (1995).

[8] As will become clear, given the preferences of agents and the Grossman modification of the TCSF, arms are allways exhausted ex-post, i.e. after the settlement.

[9] There is still another possibility: If initial claims to property are converted into effective property rights by an outside enforcement agency. But this raises the question why this outside enforcement agency does not ex-ante, i.e. prior to the bargaining procedure, enforces an allocation of the resource under consideration, which comprises zero inefficient use scarce resources.

yes, and we will work out the exact circumstances under which self-enforcing property rights emerge in the case of settlement.
The second question is straightforward: Since initial claims are contestable, this will directly influence the investment in arms in the first stage. Any unit of the uncontestable resource, spent by an agent in order to influence the disagreement point has a further function: It is also a means of defending the part of the resource under consideration obtained through the bargaining procedure (*domestic share*), and of appropriating the remaining part of the resource (*foreign share*). Thus, unlike in Skaperdas (2006), the investment in arms in both cases (*war* and *settlement*) are different with the latter case exhibiting a higher investment in arms.[10] This will - at least partly - offset the disadvantages of *war*, namely the destruction of the resource under consideration. Consequently, the second question arises: *Does settlement always Pareto dominate war?* The answer is no. We will show that if *war* is not too destructive, settlement always produces a lower payoff in equilibrium for both agents than *war*, i.e. we will demonstrate the social benefit of *war*. This finding emerges, in contrast to Garfinkel and Skaperdas (2000) and Skaperdas (2006), although we will exclusively consider a one-period game.
We will proceed in the following manner: First, we will present the basic structure of the game, than we explicitly describe the bargaining procedure and the game in case of *war*. Second, we will compare the equilibrium outcomes under *war* and *settlement*. Third, we will conclude.

2 The modell

We assume two risk-neutral agents, indexed $i = 1, 2$, that have conflicting interests in a common-pool resource with size $T > 0$. Both agents are in possession of an uncontestable resource (R) from which they derive positive and constant marginal utility. This resource can be converted into arms (e_i for agent i) on a one-to-one basis. These arms are means to determine the share of the common pool resource each agent obtains in case of *war* (T_i^{W}). In the other case (*settlement*), these arms determine the negotiation position in the bargaining procedure. Moreover, these arms, for agent i, are means to defend the initial claims obtained through the agreement (the domestic share, T_i^{S}) or to appropriate the other part of the resource (the foreign share, T_j^{S} with $T \equiv T_1^{\mathrm{S}} + T_2^{\mathrm{S}}$).[11] In either case we assume that R is large enough so that corner solutions can be ruled out.
We will start with the bargaining procedure which we will compose as a two-stage-game. In the first stage both agents, given the bargaining norm, simultaneously and independently choose their investment in arms (e_i^{S}). Moreover, since we like to capture the role of defense as a deterrent to appropriation, we assume that in the first stage each agent simultaneously and independently chooses the level of defense (d_i) given the investment in arms (e_i^{S}), with $d_i \leq e_i^{\mathrm{S}}$. In the second stage, each

[10] Hence, analogous to Garfinkel (1990), the production of arms may additionally arise from a deterrence motive in case of *settlement*.
[11] The index j, when it appears, refers to agent other than i. To avoid needless repetition of "$i = 1, 2$", i alone will be understood to refer to each agent.

agent decides simultaneously and independently about his investment in appropriation (a_i), with $a_i \leq e_i^{\mathrm{S}} - d_i$. The part of domestic share each agent is able to defend is as follows:

$$\phi_i^i(d_i, a_j, \theta) = \begin{cases} 1 & \text{for } a_j = 0, \\ \frac{d_i}{d_i + \theta\, a_j} & \text{else.} \end{cases} \tag{1}$$

$\phi_i^i(d_i, a_j, \theta)$ is a function of the efforts of agents, raised in order to defend (d_i) and to appropriate (a_j) the initial claims of agent i. The exogenous parameter θ, with $\theta \in [0, 1]$, measures the effectiveness of units of R invested in arms and allocated for appropriation of initial claims, relative to units of R invested in arms and allocated into defending initial claims.[12] For θ equal to zero challenging initial claims is impossible. In this case, $\phi_i^i = 1$, irrespective of the investment in appropriation, which corresponds to the structure of the model used by Skaperdas (2006). For $\theta \in]0, 1[$, appropriation is no longer precluded but is still inferior to defense in terms of effectiveness. $\theta = 1$ represents a challenge technology that does not discriminate between protecting and seizing: No advantage emanates from the initial claims to a consumption good. In order for ϕ_i^i to be well defined, we assume that ϕ_i^i is 1 if agent j allocates no resources to appropriation.
To get a simple function of the appropriated fraction of the foreign initial claim, we define:

$$\phi_i^j(d_j, a_i, \theta) \;\; = \;\; 1 - \phi_j^j(d_j, a_i, \theta) = \begin{cases} 0, & \text{for } a_i = 0, \\ \frac{\theta a_i}{d_j + \theta\, a_i}, & \text{else.} \end{cases} \tag{2}$$

That is, ϕ_i^j represents the fraction of the foreign share (T_j^{S}) that agent i can successfully appropriate.
In line with the literature on emerging property rights (see e.g. Grossman (2001) and Kolmar (2003)), the formal design allows us to distinguish between two different levels of property rights:

Definition 1 (*The level of security of property rights*)

1. *A settlement is said to be* **uncontested** *if the property rights of both agents are perfectly secure, i.e. if $\phi_1^1 = \phi_2^2 = 1$ in equilibrium.*

2. *A settlement is said to be* **contested** *if the property rights of at least one agent is insecure, i.e. if $\phi_1^1 \times \phi_2^2 < 1$ in equilibrium.*

We now turn to the second stage in the case of settlement: The decision about the allocation of arms to appropriation.

3 Settlement - second stage

Given any allocation of the resource under consideration (T_1^{S} and T_2^{S}), any investment in arms (e_1^{S} and e_2^{S}) and any allocation of arms to defensive actions (d_1 and

[12]It may represent formal property rights or it may reflect some technological gap between appropriating and defending initial claims. See Grossman (2001).

d_2) in the first stage, the utility function of both agents are

$$u_i(\boldsymbol{a}, \chi, \boldsymbol{d}, e_i^{\mathrm{S}}, R_i, \theta, \boldsymbol{T}) = \phi_i^i(d_i, a_j, \theta)\, T_i^{\mathrm{S}} + \phi_i^j(d_j, a_i, \theta)\, \chi\, T_j^{\mathrm{S}} + R_i - e_i^{\mathrm{S}}, \tag{3}$$

with $\boldsymbol{a} = (a_1 \;\; a_2)$, $\boldsymbol{d} = (d_1 \;\; d_2)$, $\boldsymbol{T} = \left(T_1^{\mathrm{S}} \;\; T_2^{\mathrm{S}}\right)$, $i \neq j$ and $\chi \in [0, 1]$. Analogous to Grossman and Kim (1995), we allow for the possibility that appropriation destroys part of the resource, so that the appropriator gains less than the defender looses. Thus, if the level of destruction parameter (χ) equals zero, appropriation is totally destructive.

Given equation 3, the utility maximization problem of each agent becomes

$$\begin{aligned} &\max_{a_i} \;\; u_i(\cdot) \\ &\text{s.t.} \quad e_i^{\mathrm{S}} \geq a_i + d_i, \end{aligned} \tag{4}$$

which delivers the following Lagrangian function

$$\mathcal{L}_i(\boldsymbol{a}, \chi, \boldsymbol{d}, e_i^{\mathrm{S}}, R_i, \theta, \boldsymbol{T}) = u_i(\cdot) + \lambda_i(e_i^{\mathrm{S}} - a_i - d_i), \tag{5}$$

where λ_i just represents the Lagrangian multiplier which, in optimum, measures the shadow price of a marginal increase in the investment in arms (e_i^{S}). The shadow price has to be distinguished from the marginal cost of production of arms. The shadow price solely represents the marginal willingness to pay for an incremental increase in arms in order to defend the domestic share and to appropriate the foreign share of the resource under consideration.

Partial derivation of equation (5) with respect to a_i leads to the first order condition (FOC) in the first stage

$$\frac{\theta\, d_j}{(\theta a_i + d_j)^2}\, \chi\, T_j^{\mathrm{S}} \;\; = \;\; \lambda_i, \tag{6}$$

which shows that, in optimum, the marginal benefit of appropriation (left hand side (LHS) of equation (6)) equals the shadow price of arms (right hand side (RHS) of equation (6)). Taking into account that $a_i \geq 0$, equation (6) leads to the following reaction function:

$$a_i(\chi, d_j, \lambda_i, \theta, T_j) = \begin{cases} \bar{a}_i & \text{for } d_j < \bar{d}_j, \\ 0 & \text{else}, \end{cases} \tag{7}$$

with

$$\bar{a}_i = \sqrt{\frac{d_j\, T_j\, \chi}{\theta\, \lambda_i}} - \frac{d_j}{\theta}, \tag{8}$$

and

$$\bar{d}_j = \frac{T_j\, \theta\, \chi}{\lambda_i}. \tag{9}$$

Without loss of generality, figure 1 represents the reaction function of agent 1 ($a_1(\cdot)$), contingent on the allocation of arms to defensive actions by agent 2 (d_2) for various allocations of the resource under consideration (T_2^{S}). As long as d_2 is sufficiently low ($d_2 < \bar{d}_2$) the level of appropriation of agent 1 contingent on d_2 is represented by a hump-shaped function. For $d_2 \geq \bar{d}_2$, the level of appropriation is zero, which

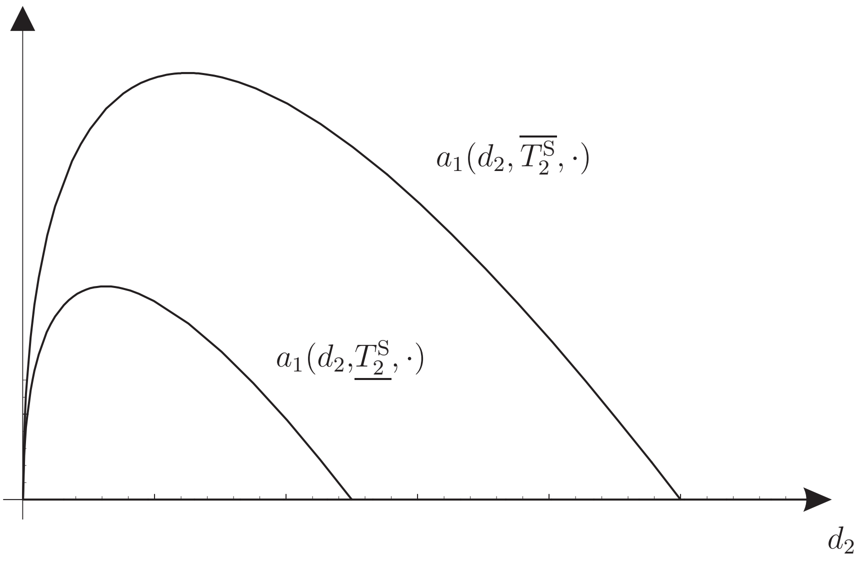

Figure 1: $a_1(\cdot)$ contingent on d_2 for various allocations of the resource (T_2^{S})

shows that for the given allocation of arms to defense, appropriation is endogenously deterred. It is worth mentioning that the higher the level of the foreign share (T_2^{S}), the higher the investment in appropriation from agent 1: $a_1(d_2, \overline{T_2^{\mathrm{S}}}, \cdot) > a_1(d_2, \underline{T_2^{\mathrm{S}}}, \cdot)$ if $\overline{T_2^{\mathrm{S}}} > \underline{T_2^{\mathrm{S}}}$.
Given the reaction of both agents, we can now turn to display the first stage problem of both agents.

4 Bargain - first stage

Before turning to the first stage decision of both agents regarding the investment in arms and the allocation of arms to defensive actions, we will present the bargaining norm. The supposed bargaining norm explicitly defines the share each agent obtains as a result of the agreement as a function of their investment in arms:

$$T_i^{\mathrm{S}} \equiv \psi_i(\boldsymbol{e}^{\mathrm{S}}, \gamma)\, T, \tag{10}$$

with

$$\psi_i(\boldsymbol{e}^{\mathrm{S}}, \gamma) = \frac{e_i^{\mathrm{S}}}{e_i^{\mathrm{S}} + e_j^{\mathrm{S}}}\,\gamma + \frac{1}{2}\,(1-\gamma), \tag{11}$$

$\boldsymbol{e}^{\mathrm{S}} = \begin{pmatrix} e_1^{\mathrm{S}} & e_2^{\mathrm{S}} \end{pmatrix}$ and $\gamma \in [0,1]$. $1-\gamma$ represents the part of the common pool resource (T) which will be destroyed in case of *war*. Thus, the bargaining norm splits the surplus which will arise in case of settlement ($(1-\gamma)\,T$).[13] Given that $\gamma = 0$, i.e. in case *war* destroys the whole resource, the resource is divided in half, irrespectively

[13] All axiomatic bargaining solutions (such as Nash (1950) and Kalai and Smorodinsky (1975)) in a symmetric environment yield the same outcome. This occurs since T and R are complete substitutes and preferences are identical. Therefore, the resulting payoff frontier is linear and has a gradient of -1. All calculations can be found in an appendix, which will be sent to the reader upon request.

of the investment in arms. For $\gamma \in]0,1[$ part of the resource is destroyed in case of *war* and for $\gamma = 1$ *war* has no impact on the size of T.
Implementing the bargaining norm and the level of appropriation contingent on the parameters of the first stage in equation (3) yields the following indirect utility function:

$$\begin{aligned} v_i(\chi, \boldsymbol{d}, \boldsymbol{e}^{\mathrm{S}}, \gamma, \theta, R, T) = &\Big[\phi_i^i\Big(d_i, \widetilde{a_j}(\cdot), \theta\Big)\, \psi_i(\cdot) + \phi_i^j\Big(d_j, \widetilde{a_i}(\cdot), \theta\Big)\, \psi_j(\cdot)\Big]\, T \\ &+ R - e_i^{\mathrm{S}}, \end{aligned} \tag{12}$$

with $\widetilde{a_i}(\cdot) = \bar{a}_i(\chi, d_j, \theta, \psi_i(\boldsymbol{e}^{\mathrm{S}}, \gamma), T)$. The utility maximization problem of the agents thus becomes

$$\begin{aligned} &\max_{d_i, e_i^{\mathrm{S}}} \quad v_i(\cdot) \\ \text{s.t.} \quad &a_j(\cdot) \geq 0,\ d_i \geq 0,\ e_i^{\mathrm{S}} \geq 0, e_i^{\mathrm{S}} \geq a_i(d_j) + d_i, \end{aligned} \tag{13}$$

which gives us the following Lagrangian function:

$$\mathcal{K}_i(\chi,\, \boldsymbol{d},\, \boldsymbol{e}^{\mathrm{S}},\, \gamma,\, \mu_i,\, R,\, \theta,\, T) = v_i(\cdot) + \mu_i\, a_j(\cdot) + \lambda_i \Big(e_i^{\mathrm{S}} - d_i - a_i(\cdot)\Big), \tag{14}$$

where μ_i just represents the non-negativity constraint on the level of appropriation in equilibrium. We are able to show that the restrictions on e_i^{S} and d_i are never binding and that $\lambda_i > 0$ in equilibrium, i.e. the corresponding constraint ($e_i^{\mathrm{S}} \geq a_i(d_j) + d_i$) is exactly satisfied.[14] Therefore, concerning the equilibrium allocation of e_i^{S} and d_i, we only need to discriminate between an interior and a corner solution with respect to the investment in appropriation in stage 2. In the interior solution the level of appropriation is positive ($a_j(\cdot) > 0$), in the corner solution it is zero ($a_j(\cdot) = 0$).
Equation (15a) and (15b) display the FOC in the first stage with respect to the investment in arms (e_i^{S}) and its allocation to defense (d_i) if the restriction on a_j ($a_j \geq 0$) is non-binding, i.e. $\mu_i = 0$:

$$\left[\frac{\partial\, \phi_i^i(\cdot)}{\partial\, d_i} + \frac{\partial\, \phi_i^i(\cdot)}{\partial\, a_j(\cdot)} \frac{\mathrm{d}\, a_j(\cdot)}{\mathrm{d}\, d_i}\right] \psi_i(\cdot)\, T = \lambda_i, \tag{15a}$$

$$\left[\phi_i^i(\cdot)\, \frac{\partial\, \psi_i(\cdot)}{\partial\, e_i^{\mathrm{S}}} + \phi_i^j(\cdot)\, \frac{\partial\, \psi_j(\cdot)}{\partial\, e_i^{\mathrm{S}}} + \frac{\partial\, \phi_i^i(\cdot)}{\partial\, a_j(\cdot)} \frac{\partial\, a_j(\cdot)}{\partial\, \psi_i(\cdot)} \frac{\mathrm{d}\, \psi_i(\cdot)}{\mathrm{d}\, e_i^{\mathrm{S}}}\, \psi_i(\cdot)\right] T = 1 - \lambda_i. \tag{15b}$$

Equation (15a) displays the FOC with respect to the allocation of arms to defensive actions. The first term in brackets on the LHS of equation (15a) represents the direct effect of d_i on ϕ_i^i, which is unambiguously positive. The second term in brackets represents the effect of a change in the level of appropriation on ϕ_i^i triggered by a marginal increase in defense (indirect effect of d_i), which can be either positive or negative (cf. figure 1). The term on the RHS displays the shadow price of arms. In equilibrium it represents, as already mentioned, the marginal willingness to pay for an incremental increase in arms in order to defend the domestic share or to appropriate the foreign share of the resource.

[14] See mathematical appendix.

Equation (15b) displays the FOC with respect to the investment in arms. The first two terms in brackets on the LHS represent the direct effect of an increase in e_i^{S} on the defended domestic share (first term) and the appropriated foreign share (second term), where the former one is unambiguously positive and the latter is unambiguously non-positive.[15] The third term in brackets on the LHS of equation (15b) shows the indirect effect of an incremental increase in e_i^{S} on the defended domestic share. This effect is unambiguously negative since an increase in the domestic share ($T_i^{\mathrm{S}} \equiv \psi_i(\cdot)\, T$) also increases the investment in appropriation of agent j (cf. figure 1). The RHS of equation (15b) represents the opportunity cost of arms, which consists of two parts. The first part simply represents the marginal cost of production of arms, the second part is the shadow price of arms. As long as $\lambda_i > 0$, the opportunity costs of arms are below one. This shows the additional benefit which arises from the fact that each unit of the resource R invested in arms is also a means of both defending the domestic share and of appropriating the foreign share.
It is easy to verify that these FOCs are symmetric. Thus, $e_1 = e_2 = e$ and $d_1 = d_2 = d$ is a Nash-equilibrium and the symmetric Nash-equilibrium level of arms and defense are

$$e^{\mathrm{S}^*}(\chi,\, \gamma,\, \theta, T) = \begin{cases} \frac{\left(\gamma+(2-\gamma)\theta\,\chi\right)T}{4} & \text{for } \theta \in \left[0, \tilde{\theta}\right], \\ \frac{\left(-2+\theta(1+2\,\chi)(2+\gamma)-4\,\gamma\,\theta^2\,\chi^2\right)T}{16\,\theta^2\,\chi} & \text{for } \theta \in \left]\tilde{\theta}, 1\right], \end{cases} \tag{16}$$

$$d^*(\chi,\, \gamma,\, \theta, T) = \begin{cases} \frac{\left(\gamma+(2-\gamma)\theta\,\chi\right)T}{4}, & \text{for } \theta \in \left[0, \tilde{\theta}\right], \\ \frac{\left(-2+\theta(1+2\,\chi)(2+\gamma)-4\,\gamma\,\theta^2\,\chi^2\right)T}{16\,\theta\,\chi\left(\theta(1+2\,\chi)-1\right)} & \text{for } \theta \in \left]\tilde{\theta}, 1\right], \end{cases} \tag{17}$$

with

$$\tilde{\theta} = \frac{1}{2\,\chi}.$$

$\tilde{\theta}$ represents the case-separating level of the exogenous effectiveness parameter.[16] If appropriation is sufficiently relatively ineffective ($\theta \in [0, \tilde{\theta}]$), the Nash-equilibrium level of defense equals the investment in defense just sufficient to deter appropriation. In this case, the investment in defense equals the effort in the bargaining game ($e^{\mathrm{S}^*}(\cdot) = d^*(\cdot)$, cf. equations (16) and (17), upper case) showing that the whole investment in arms is indeed exclusively used for defensive means. Thus, the bargaining norm produces an allocation of the common-pool resource T which remains uncontested in this case.
If $\theta \in]\tilde{\theta}, 1]$, the investment in arms (e^{S^*}) is higher than the investment in defense

[15]Since $e_i^{\mathrm{S}} > 0$, $\frac{\partial\,\psi_i(\cdot)}{\partial\, e_i^{\mathrm{S}}} > 0$ and $\frac{\partial\,\psi_j(\cdot)}{\partial\, e_i^{\mathrm{S}}} < 0$. Moreover, since $\mu_i = 0$ (i.e. $a_j(\cdot) \geq 0$) and $d_i > 0 \Rightarrow \phi_i^i \in]0,1]$ and $\phi_i^j \in [0,1]$.

[16]For convenience, we are not explicitly display the equilibrium value of the shadow price ($\lambda^*(\chi,\gamma,\theta,T)$). As it turns out, $\lambda^*(\cdot)$ is positive, which proofs that indeed the corresponding constraint is exactly satisfied, i.e. arms are totally exhausted in case of settlement. See appendix for calculation.

(d^*). This demonstrates that in this case the level of defense in equilibrium is insufficient for deterring appropriation (cf. equations (15a) and (15b), lower case), since the shadow price of an investment in arms is positive, i.e. arms are exhausted in case of settlement. Hence, the bargaining norm produces an allocation of T which becomes contested.
We now turn to determine the level of appropriation and the level of claims to property in equilibrium.

5 The full game

Given the investment in arms (e^*) and their allocation to defensive means (d^*), we are now able to determine the level of appropriation and the level of security of property rights in equilibrium:

$$a^*(\chi,\,\gamma,\,\theta,T)=\begin{cases}0, & \text{for } \theta\in\left[0,\tilde{\theta}\right],\\ \frac{T(2\,\theta\,\chi-1)\left(-2+\theta(1+2\,\chi)(2+\gamma)-4\,\gamma\,\theta^2\,\chi^2\right)}{16\,\theta^2\,\chi\left(\theta(2\,\chi+1)-1\right)} & \text{for } \theta\in\left]\tilde{\theta},1\right],\end{cases} \tag{18}$$

and

$$\phi_i^{i*}(\chi,\,\gamma,\,\theta,T)=\begin{cases}1 & \text{for } \theta\in\left[0,\tilde{\theta}\right],\\ \frac{1}{2\,\theta\,\chi}, & \text{for } \theta\in\left]\tilde{\theta},1\right].\end{cases} \tag{19}$$

As already mentioned, in case θ is low enough, appropriation will be endogenously deterred, due to the investment in defense ($a^*(\cdot)=0$, cf. equation (18), upper case). In this case property rights are perfectly secure ($\phi_i^i(\cdot)=1$, cf. equation (19), upper case). Thus, $a^*(\cdot)$ and $\phi_i^{i*}(\cdot)$ remain unchanged by a change in θ, as long as $\theta\le\tilde{\theta}$ (see figure 2 and 3). In the case of $\theta>\tilde{\theta}$, property rights become insecure due to the positive investment in appropriation (cf. equations (18) and (19), lower case).
Figure 2 shows the investment in arms and the allocation of arms for appropriation and defense. As long as θ is sufficiently low ($\theta<\tilde{\theta}$), appropriation is deterred and $e^*(\cdot)=d^*(\cdot)$ becomes a linear increasing function of θ. At the case-separating level of the exogenous relative effectiveness parameter ($\tilde{\theta}$) appropriation emerges and stays positive for $\theta\in]\tilde{\theta},1]$. The investment in arms increases furthermore in the interior solution until $\theta=\breve{\theta}$ if $(\chi,\gamma)\in\xi$, with

$$\breve{\theta}=\frac{1}{(2+\gamma)(1+2\,\chi)} \tag{20}$$

and

$$\xi=\left\{(\chi,\gamma)\middle|\frac{1}{2}<\chi\le 1, 0\le\gamma\le\frac{4\,\chi-2}{2\,\chi-1}\right\}. \tag{21}$$

In this case the negative impact on the investment in arms due to the decrease in defense is overcompensated by the positive impact due to the increase in appropriation, as long as $\theta<\breve{\theta}$. Nevertheless, a^* stays below d^* as long as $\theta,\chi\ll 1$. Taking

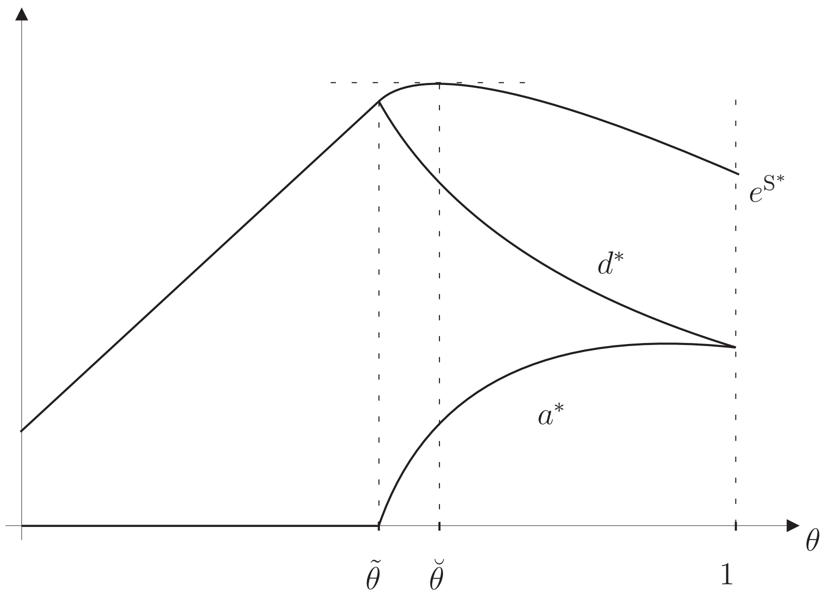

Figure 2: e^*, d^* and a^* contingent on θ for $\chi = 1$

a look at the level of security of claims to property (figure 3) will demonstrate that, as long as $\theta \in [0, \tilde{\theta}]$, perfectly secure property rights emerge in equilibrium. Given that $\theta > \tilde{\theta}$, appropriation emerges and with it insecure property rights ($\phi_i^{i*} < 1$). Thus, the level of security of property rights is a monotonically decreasing function of θ for $\theta > \tilde{\theta}$, with $\phi_{i\min}^{i*} = \phi_i^{i*}\big|_{\chi,\theta\to 1} = \frac{1}{2}$.
The following proposition recapitulates our findings:

Proposition 1 (*Contested and uncontested settlement in equilibrium*)

1. *If the level of the exogenous relative effectiveness parameter is sufficiently low ($\theta \in [0, \tilde{\theta}]$), the investment in arms equals the level of defense in equilibrium ($e^{S^*} = d^*$). In this case, appropriation is endogenously deterred ($a^* = 0$), perfectly secure property rights emerge in equilibrium ($\phi_i^{i*} = 1$) and with them* **uncontested settlement**.

2. *If $\theta \in]\tilde{\theta}, 1]$ the investment in arms exceeds the level of defense ($e^{S^*} > d^*$) and appropriation emerges in equilibrium ($a^* > 0$). In this case, property rights become insecure ($\phi_i^{i*} < 1$), i.e.* **contested settlement** *emerges.*

We now turn to answering the second question: Does *settlement* always Pareto dominate *war*? Therefore we have to determine the investment in arms and the payoff in equilibrium under *war*.

6 War

Equation (22) represents the payoff of both agents in case of *war*, given a TCSF:

$$u_i^{\mathrm{W}}(\boldsymbol{e}^{\mathrm{W}}, R, T) = \frac{e_i^{\mathrm{W}}}{e_i^{\mathrm{W}} + e_j^{\mathrm{W}}}\gamma T + R - e_i^{\mathrm{W}}, \tag{22}$$

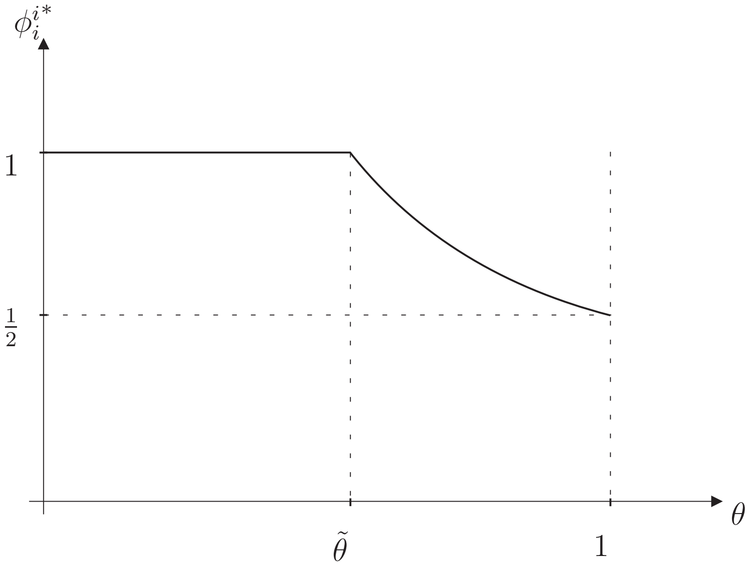

Figure 3: The level of security of property rights (ϕ_i^{i*}) contingent on θ for $\chi = 1$

with $\boldsymbol{e}^{\mathrm{W}} = \left(e_1^{\mathrm{W}} \;\; e_2^{\mathrm{W}}\right)$. In case of *war* both parties compete over the part of the common pool resource which is not destroyed by *war* ($\gamma\, T$), where e_1^{W} and e_2^{W} represent the investment in arms in order to seize the common pool resource. Anticipating *war*, both agents try to maximize their payoff, given by (22), over e_i^{W}:

$$\max_{e_i^{\mathrm{W}}} \quad u_i^{\mathrm{W}}(\cdot) \qquad s.t. \quad e_i^{\mathrm{W}} \geq 0,$$

where it is easy to verify that in the symmetric Nash-equilibrium

$$e^{\mathrm{W}^*} = \frac{\gamma\, T}{4}. \tag{23}$$

Given the investment in arms in case of *war* (e^{W^*}) and the investment in arms and its allocation to offensive and defensive actions in case of settlement (e^{S^*}, a^* and d^*) we are now able to compare the payoffs under contested and uncontested settlement with the payoff under *war* in equilibrium.

7 War versus Bargaining

The indirect utility function under settlement in equilibrium becomes

$$v^{\mathrm{S}^*}(\chi,\gamma,\theta,R,T) = \phi_i^{i*}(\chi,T)\,\frac{T}{2} + \left(1-\phi_i^{i*}(\chi,T)\right)\frac{T}{2} + R - e^{\mathrm{S}^*}(\chi,\gamma,\theta,T)$$

$$\Leftrightarrow v^{\mathrm{S}^*}(\cdot) = \begin{cases} \frac{T}{2} + R - \frac{\gamma+(2-\gamma)\theta\,\chi}{4}\,T, & \text{for } \theta \in \left[0,\tilde{\theta}\right], \\ \frac{T\left(1-\chi+2\,\chi^2\,\theta\right)}{4\,\chi\,\theta} + R - \frac{T\left(-2+\theta(1+2\,\chi)(2+\gamma)-4\,\gamma\,\theta^2\,\chi^2\right)}{16\,\theta^2\,\chi} & \text{for } \theta \in \left]\tilde{\theta},1\right]. \end{cases} \tag{24}$$

Since the efforts of both agents are identical, $T_1^{\mathrm{S}} = T_2^{\mathrm{S}} = \psi_i(\boldsymbol{e}^{\mathrm{S}^*},\gamma)\,T = \frac{T}{2}$, i.e. both agents obtain initial claims to a half of the common pool resource in case of

settlement. If θ is sufficiently low, the domestic share remains unchallenged ($\phi_i^{i*} = 1$) since appropriation is deterred (equation (24), upper case). If $\theta > \tilde{\theta}$ appropriation emerges and thus $\phi_i^{i*} < 1$ (equation (24), lower case). In either case, the investment in arms is positive ($e^{S*}(\cdot) > 0$).
Under *war* a part of the common pool resource ($(1-\gamma)T$) is destroyed. Since the investment in arms is identical for both agents and arms are exhausted in *war*, each agent creates effective property rights to a half of the remaining common pool resource, i.e. the part which is not destroyed. The payoff under *war* thus becomes in equilibrium:

$$\begin{aligned} v^{W^*}(\gamma, R, T) &= \frac{\gamma T}{2} + R - e^{W^*}(\gamma, T), \\ \Leftrightarrow v^{W^*}(\gamma, R, T) &= R + \frac{\gamma T}{4}. \end{aligned} \tag{25}$$

Given these results we are now able to compare the payoffs in equilibrium under *settlement* and under *war*. To keep the analysis simple, we assume that $\chi = 1$, i.e. we assume that appropriation is non-destructive. This is *primae facie* an ideal condition to detect *settlement* rather than *war* in equilibrium since, even if *settlement* becomes contested, any ex post forced redistribution does not involve demolishment of the resource under consideration. However, comparing v^{W^*} with v^{S^*} shows, that even under these circumstances, *war* delivers a higher payoff in equilibrium than the payoff under contested and, respectively, uncontested *settlement* if *war* is not too destructive (if γ is not to high):

$$v^{W^*} \left\{ \begin{array}{c} > \\ \leq \end{array} \right\} v^{S^*} \text{ iff } \gamma, \theta \in \left\{ \begin{array}{c} \mathcal{A} \\ \mathcal{B} \end{array} \right\}, \tag{26}$$

with

$$\begin{aligned} \mathcal{A} &= \left\{ (\gamma, \theta) \,\middle|\, 0 \leq \theta \leq \bar{\theta},\, \widetilde{\gamma} < \gamma \leq 1 \right\}, \\ \mathcal{B} &= \left\{ (\gamma, \theta) \,\middle|\, 0 \leq \theta \leq \bar{\theta},\, 0 \leq \gamma \leq \widetilde{\gamma} \,\middle|\, \bar{\theta} < \theta \leq 1, 0 \leq \gamma \leq 1 \right\}, \end{aligned}$$

for

$$\widetilde{\gamma} = \begin{cases} \frac{\theta - 1}{\frac{\theta}{2} - 1}, & \text{for } \theta \in \left[0, \tilde{\theta}\right], \\ \frac{2}{3\theta} + \frac{8\theta}{3} - 2, & \text{for } \theta > \tilde{\theta} \end{cases} \qquad \text{and} \qquad \bar{\theta} = \frac{9 + \sqrt{17}}{16}. \tag{27}$$

This is graphically represented in figure 4, where the u-shaped curve represents the continuum of γ/θ-combinations which delivers equal payoff under *war* and *settlement*. To the north of this curve we find the γ/θ-combinations which deliver a higher payoff under *war* than under settlement ($\mathcal{A}$).
If $\theta \leq \tilde{\theta}$, the corner solution applies in the bargaining game (to the west of the dotted line). Here, the investment in arms in case of settlement (e^{S^*}) is strictly monotonically rising in θ, with $e^{S^*}\big|_{\theta \to 0} \to e^{W^*}$ (cf. equation (16), upper case and equation (23)). Thus, as already mentioned, for $\theta = 0$ the model presented here replicates the findings of Skaperdas (2006), where appropriation is exogenously foreclosed. Hence, as long as $\theta \in]0, \tilde{\theta}]$, the (positive) difference between the investment in arms in case

of *settlement* and in case of *war* ($\Delta e^* = e^{S^*} - e^{W^*}$) is solely owed to the investment in defense necessary to deter appropriation. The more relative effective appropriation becomes compared to defense (the higher the level of θ), the higher the value of Δe^*. Furthermore, if, in addition, γ is sufficiently high, i.e. the costs of *war* in terms of destroyed units of the resource under consideration are sufficiently low, the effect of the additional investment in arms (Δe^*) on the payoff in equilibrium overcompensates the negative effect due to the destruction of the resource under consideration.
Therefore, we were able to demonstrate the circumstances under which *war* Pareto dominates *settlement*, or in other words: The social benefit of *war*.
Moreover, the model presented here comprises the structure of various models used extensively in contest theory.

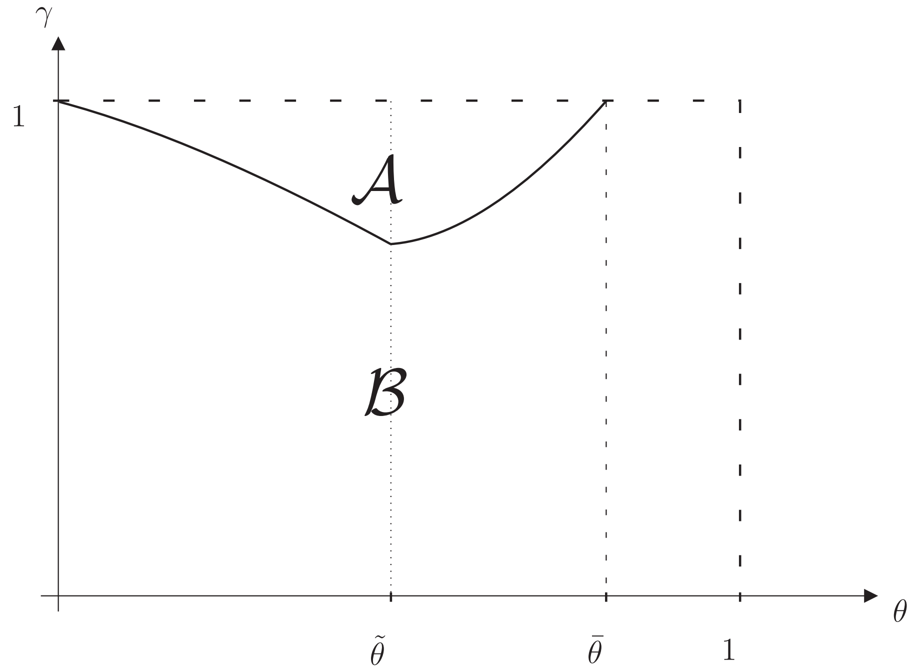

Figure 4: The dominance of *war* over settlement and v.v. for $\chi = 1$

1. If $(\gamma, \theta) \in \mathcal{C}$ the model presented here replicates the findings of Skaperdas (2006), with

$$\mathcal{C} = \{(\gamma, \theta)\, | 0 \leq \gamma < 1, \theta = 0\}\ ,$$

   and $\mathcal{C} \subset \mathcal{B}$. In this case *settlement* always Pareto dominates *war*, since in both cases (*settlement* and *war*) there is an equal investment in arms, but in case of *war*, part of the resource is destroyed:

$$e^{S^*}\Big|_{(\gamma,\theta)\in\mathcal{C}} = e^{W^*} = \frac{\gamma T}{4}, \tag{28}$$

$$v^{S^*}\Big|_{(\gamma,\theta)\in\mathcal{C}} = \frac{T(2-\gamma)}{4} + R > \frac{\gamma T}{4} + R = v^{W^*}. \tag{29}$$

2. If $(\gamma, \theta) \in \mathcal{D}$, with
$$\mathcal{D} = \{(\gamma, \theta) \,|\gamma = 0, 0 \leq \theta \leq 1\} \,, \tag{30}$$
and $\mathcal{D} \subset \mathcal{B}$, the model presented here replicates the findings of Grossman (2001), in his case of initial claims to property, and Grossman and Kim (1995), where a resource to which two agents have initial claims to property is contested in a two stage game. In this case *war* is totally destructive, thus the resource under consideration is divided equally between both agents, irrespective of their investment in arms.

3. If $\gamma = 1$, *war* is not destructive, thus *war* always Pareto dominates *settlement*. The modell presented here replicates the findings of Grossman (2001) in case of a common pool resource, if $(\gamma, \theta) \in \mathcal{E}$, with
$$\mathcal{E} = \left\{(\gamma, \theta) \,\middle|\gamma = 1, 0 \leq \theta \leq \bar{\theta}\right\} \,, \tag{31}$$
with $\mathcal{E} \subset \mathcal{A}$.

The following proposition recapitulates our findings.

Proposition 2 (*Settlement vs. war*)

1. *If* $(\gamma, \theta) \in \mathcal{A}$*,* **war Pareto dominates settlement** $(v^{\mathrm{W}^*} > v^{\mathrm{S}^*})$*. In this case the level of destructibility (γ) is insufficient to endogenously foreclose war.*

2. *If* $(\gamma, \theta) \in \mathcal{B}$*,* **settlement Pareto dominates war** $(v^{\mathrm{S}^*} > v^{\mathrm{W}^*})$*. In this case the level of destructibility is sufficient to foreclose war.*

3. *If* $(\gamma, \theta) \in \mathcal{C}$*, the model presented here replicates the findings of Garfinkel and Skaperdas (2000) and Skaperdas (2006).*

4. *If* $(\gamma, \theta) \in \mathcal{D}$*, the model presented here replicates the findings of Grossman (2001), in his case of initial claims to property, and Grossman and Kim (1995).*

5. *If* $(\gamma, \theta) \in \mathcal{E}$*, the model presented here replicates the findings of Grossman (2001) in his case of a common pool resource.*

8 Conclusion

Disputes between groups of people, such as states, confronts the parties with a guns-versus-butter tradeoff, i.e. a tradeoff between the productive and unproductive use of scarce resources. Given these opportunity costs, we illuminated the terms and conditions necessary to endogenously preclude the violent settlement of disputes (*war*).
To begin with we have shown the exact circumstances under which self-enforcing property rights (*uncontested settlement*) arise using a bargaining solution. We found out that the investment in arms associated with the bargaining procedure is exclusively used in order to defend either side's initial claims to property - obtained

through the agreement - if appropriation is sufficiently relatively ineffective. Accordingly, we were able to show that the supposed bargaining procedure is able to convert initial claims to property into effective property rights. However, in contrast to Garfinkel and Skaperdas (2000) and Skaperdas (2006), uncontested settlement appears not by assumption but in equilibrium: Both agents, given their investment in arms, independently decide to back-off from appropriation. Moreover, we were able to determine the exact costs of the endogenous deterrence of appropriation, i.e. we have found an endogenously determined measure for the transaction costs underlying an uncontested bargaining solution.

Further, we have shown that, in contrast to Garfinkel and Skaperdas (2000) and Skaperdas (2006), even in a one-period consideration, *war* may Pareto dominate *settlement*. The reason for this is that the investment in arms is always higher in case of *settlement* than in case of *war*, if appropriation is not technologically impossible. This effect emerges since any unit of the uncontestable resource, spent in order to influence the threadpoint has a further function: It is also a means of defending the part of the resource under consideration obtained through the bargaining procedure, and of appropriating the other part of the resource. This partly offsets the disadvantages of *war*, namely the destruction of the resource under consideration. Consequently, if *war* is not too destructive, *settlement* always produces a lower payoff for both agents in equilibrium than *war*. Hence, we were able to demonstrate the social benefit of *war*.

Thirdly, the model presented here is able to replicate the findings of (*i*) Skaperdas (2006) if appropriation is technologically impossible and *war* is destructive. In this case, *war* and *settlement* exhibit equal investment in arms but *war* delivers a lower value of the resource. According to this, even in a one period consideration *settlement* Pareto dominates *war*. (*ii*) If *war* is absolutely destructive then, according to the bargaining norm, the resource under consideration is divided in half, irrespective of the investment in arms by both agents. If appropriation is not exogenously precluded, then the investment in arms made by the two agents, solely represents the investment made in order to defend and to appropriate initial claims. On this account, the model presented here replicates the findings of Grossman (2001) in his case of initial claims to property and Grossman and Kim (1995). (*iii*) If *war* evades no destruction of the resource under consideration, then the model presented here replicates the findings of Grossman (2001) in his case of a common pool resource.

Mathematical Appendix

A Utility Maximization in the First Stage

The utility maximization problem of the agent in stage one becomes

$$\begin{aligned} \max_{d_i,e_i} \quad & v_i(\chi, \boldsymbol{d}, \boldsymbol{e}, \gamma, \theta) \\ \text{s.t.} \quad (i) \quad & e_i \geq 0, \\ (ii) \quad & d_i \geq 0, \\ (iii) \quad & e_i \geq a_i(d_j) + d_i, \\ (iv) \quad & a_j(\cdot) \geq 0, \end{aligned} \tag{32}$$

with $i \neq j$. The Lagrangian thus becomes

$$\mathcal{K}_i(\cdot) = v_i(\chi, \boldsymbol{d}, \boldsymbol{e}, \gamma, \theta) + \mu_i\, a_j(\cdot) + \lambda_i(e_i - d_i - a_i(\cdot)). \tag{33}$$

Partial derivation leads to the following Kuhn-Tucker-conditions

$$\frac{\partial \mathcal{K}_i}{\partial d_i} \leq 0, \qquad d_i \geq 0 \qquad \text{and} \qquad \frac{\partial \mathcal{K}_i}{\partial d_i}\, d_i = 0, \tag{34a}$$

$$\frac{\partial \mathcal{K}_i}{\partial e_i} \leq 0, \qquad e_i \geq 0 \qquad \text{and} \qquad \frac{\partial \mathcal{K}_i}{\partial e_i}\, e_i = 0, \tag{34b}$$

$$\frac{\partial \mathcal{K}_i}{\partial \mu_i} \geq 0, \qquad \mu_i \geq 0 \qquad \text{and} \qquad \frac{\partial \mathcal{K}_i}{\partial \mu_i}\, \mu_i = 0, \tag{34c}$$

$$\frac{\partial \mathcal{K}_i}{\partial \lambda_i} \geq 0, \qquad \lambda_i \geq 0 \qquad \text{and} \qquad \frac{\partial \mathcal{K}_i}{\partial \lambda_i}\, \lambda_i = 0, \tag{34d}$$

where each third term represents the complementary slackness condition. Since these conditions are symmetric, we know that $e_1 = e_2 = e$ and $d_1 = d_2 = d$ is a Nash-equilibrium.[17]
Implementing κ_i and ν_i as the shadow price for violating the non-negativity constraints on e_i and d_i respectively delivers the modified Lagrangian

$$\widetilde{\mathcal{K}}_i(\cdot) = v_i(\chi, \boldsymbol{d}, \boldsymbol{e}, \gamma, \theta) + \mu_i\, a_j(\cdot) + \lambda_i(e_i - d_i - a_i(\cdot)) + \kappa_i\, e_i + \nu_i\, d_i, \tag{35}$$

[17]Therefore also $\lambda_1 = \lambda_2 = \lambda$ and $\mu_1 = \mu_2 = \mu$ in equilibrium.

where partial derivation leads to the following FOCs:

$$\left.\frac{\partial \widetilde{\mathcal{K}}_i}{\partial e_i}\right|_{e_1=e_2,\, d_1=d_2,\, \lambda_1=\lambda_2,\, \mu_1=\mu_2, \kappa_1=\kappa_2,\, \nu_1=\nu_2} = 0, \tag{36a}$$

$$\Leftrightarrow \sqrt{\frac{2\,d\,T}{\theta\,\lambda\,\chi}}\,\frac{\gamma(\lambda(1+2\,\chi)+\mu\,\chi)}{8\,e} - \frac{T\,\gamma\,\chi}{4\,e} + \lambda - 1 + \kappa = 0, \tag{36a$'$}$$

$$\left.\frac{\partial \widetilde{\mathcal{K}}_i}{\partial d_i}\right|_{e_1=e_2,\, d_1=d_2,\, \lambda_1=\lambda_2,\, \mu_1=\mu_2,\, \kappa_1=\kappa_2,\, \nu_1=\nu_2} = 0, \tag{36b}$$

$$\Leftrightarrow \sqrt{\frac{2\,T}{d\,\theta\,\lambda\,\chi}}\,\frac{\lambda+\mu\,\chi}{4} - \frac{\mu}{\theta} - \lambda + \nu = 0, \tag{36b$'$}$$

$$\left.\frac{\partial \widetilde{\mathcal{K}}_i}{\partial \mu_i}\right|_{e_1=e_2,\, d_1=d_2,\, \lambda_1=\lambda_2,\, \mu_1=\mu_2} = 0, \tag{36c}$$

$$\Leftrightarrow \sqrt{\frac{d\,T\,\chi}{2\,\theta\,\lambda}} - \frac{d}{\theta} = 0, \tag{36c$'$}$$

$$\left.\frac{\partial \widetilde{\mathcal{K}}_i}{\partial \lambda_i}\right|_{e_1=e_2,\, d_1=d_2,\, \lambda_1=\lambda_2,\, \mu_1=\mu_2} = 0, \tag{36d}$$

$$\Leftrightarrow e - d - \sqrt{\frac{d\,T\,\chi}{2\,\theta\,\lambda}} + \frac{d}{\theta} = 0, \tag{36d$'$}$$

For either $d = 0$ or $e = 0$, this system of equations is not solvable since the LHS of either of equation (36a$'$) or (36b$'$) would go to infinity. Hence, the restrictions on e_i and d_i are never binding in the symmetric equilibrium and consequently, the shadow prices are zero in equilibrium:

$$\kappa^* = 0 \qquad \wedge \qquad \nu^* = 0. \tag{37}$$

Moreover, $\lambda \neq 0$ in equilibrium, since otherwise the LHS of equations (36a$'$-36d$'$) would go to infinity. Thus, arms are always exhausted, i.e. the sum of efforts raised in order to defend or appropriate equals the investment in arms.

Given these calculations, we know that there are only two different cases that need to be examined in equilibrium: In the first case the restriction on a_i ($a_i \geq 0$) is non-binding, in the second case it binds.

Case	a_i	μ_i
1.	≥ 0	$= 0$
2.	$= 0$	≥ 0

Given, that $\mu_1 = \mu_2 \geq 0$, the solution to the systems of equations (36a$'$ - 36d$'$) becomes:

$$e^{S^*}(\chi, \gamma, \theta, T) = \frac{(\gamma + (2-\gamma)\theta\chi)\,T}{4}, \tag{38a}$$

$$d^*(\chi,\gamma,\theta,T) = \frac{(\gamma + (2-\gamma)\theta\chi)\,T}{4}, \tag{38b}$$

$$\lambda^*(\chi,\gamma,\theta,T) = \frac{2\,\theta\,\chi}{\gamma + (2-\gamma)\,\theta\,\chi}, \tag{38c}$$

$$\mu^*(\chi,\gamma,\theta,T) = \frac{2\,\theta(1-2\,\theta\,\chi)}{\gamma + (2-\gamma)\,\theta\,\chi}. \tag{38d}$$

Given, that $\mu_1 = \mu_2 = 0$, the solution to the systems of equations (36a′ - 36d′) becomes:

$$e^{S^*}(\chi,\,\gamma,\,\theta,T) = \frac{\big(-2 + \theta(1+2\,\chi)(2+\gamma) - 4\,\gamma\,\theta^2\,\chi^2\big)T}{16\,\theta^2\,\chi}, \tag{39a}$$

$$d^*(\chi,\,\gamma,\,\theta,T) = \frac{\big(-2 + \theta(1+2\,\chi)(2+\gamma) - 4\,\gamma\,\theta^2\,\chi^2\big)T}{16\,\theta\,\chi\big(\theta(1+2\,\chi)-1\big)}, \tag{39b}$$

$$\lambda^*(\chi,\,\gamma,\,\theta,T) = \frac{2(\theta(2\,\chi+1)-1)}{-2 + \theta(1+2\,\chi)(2+\gamma) - 4\,\gamma\,\theta^2\,\chi^2}, \tag{39c}$$

where it is easy to verify that the corner solution ($\mu_1^* = \mu_2^* \geq 0$) applies as long as $\theta \leq \frac{1}{2\,\chi}$.

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