Exchange Rate Determination and Out of Sample Forecasting: Cointegration Analysis

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2015
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Abstract:
Forecasting the nominal exchange rate has been one of the most difficult exercises in economics. This study employs the Frankel (1979) monetary model of exchange rate to examine the long run behavior of Pakistan rupee per unit of US dollar over the period 1982:Q1 to 2012:Q2. Johansen and Juselious (1988,1992) likelihood ratio test indicates one long-run cointegrating vector among the fundamentals. Cointegrating vector is uniquely identified as Dornbusch (1976) monetary model by imposing plausible economic restrictions. Finally, the short-run dynamic error correction model is estimated on the bases of identified cointegrated vector. Out of sample forecasting analysis of parsimonious short run dynamic error correction model is able to beat the naïve random walk model on the basis of root mean square error, Theil’s U coefficient and Diebold and Mariano (1995) test statistics.

\textit{JEL classification:} F31; F37; F47
\textit{Keywords:} Exchange rate determination; Unit root; Cointegration; Error correction model; Forecasting; Random walk model

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1. **Introduction:**

Exchange rate modeling and forecasting is important for policy making. It has been received momentous consideration in the international finance since the inception of flexible exchange rate system. Pakistan has experienced a multiple exchange rate regimes after its independence to achieve the objective of export competitiveness, improve balance of payments and rapid economic growth.

Since independence in 1947, Pakistan has adopted a fixed exchange rate system and pegged its currency - Pakistan rupee (PKR hereafter) - with pound sterling up to 1971. With the collapse of the Pound Sterling in June 1972, the Sterling area has been demolished and US dollar became dominant currency across the globe. Pakistan, therefore, decided to delink PKR from pound sterling and tied its currency to the US dollar.

In 1980-81, Pakistan had lost its export in non-dollar area, when PKR appreciated against French franc and pound sterling. It raised the import bills and deficit in balance of trade increased. Under these circumstances, in Jan 1982, the decision was made to delink PKR from US $ and exchange rate system was moved away from fixed exchange rate system to flexible exchange rate system. However, Pakistan’s exchange rate system was not completely flexible; it is a hybrid of fixed and flexible exchange rate system, in which market forces determine the level of currency price and central bank does intervention in the forex market via open market operations and foreign exchange operations to prevent the exchange rate volatility.

Pakistan became the nuclear power on 28th May, 1998. Nuclear test hit the economy and created financial crisis because very soon after the test foreign aid sanctions were imposed and this drastically decreased the remittances. In order to alleviate the financial and exchange
rate crisis, State Bank of Pakistan has frozen the foreign currency accounts and introduced two-tier exchange rate system, in July 1998.

In May 1999, State Bank of Pakistan replaced the two tier exchange rate system by market based unified exchange rate system and put the PKR on dirty float. Under dirty float exchange rate policy, PKR was pegged to US dollar and allowed to float within the narrow band of 52.10 - 52.30 rupees per US dollar. In this period, exchange rate was relatively highly volatile. Dirty float exchange rate band was abolished by the State Bank of Pakistan in July 2000 and flexible exchange rate system was finally achieved. This exchange rate system remains unaltered until the present time. Theoretically this system is helpful to maintain the stability of exchange rate as it allows the self correction in response to market pressure. In practice, exchange rate is highly volatile under floating exchange rate and can have a major impact on local economies. Especially when a country is in an economic spiral the flexible exchange rate system put downward pressure on the local currency and led to decline in the purchasing power of the citizens. In this period exchange rate of Pakistan has experienced wide fluctuations as compared to earlier exchange rate regimes.

The 9/11 event had affected the Pakistan economy. The rupee started appreciating against the dollar and kept on appreciating because of abundant inflows of capital in form of remittances. During 2001 to 2003, nominal exchange rate against dollar appreciated by 6 percent.

Recent GFC which originated in US had affected the Pakistan economy like other economies of the world. The real effects of GFC such as slowdown in global demand and fall in commodity prices have been felt by Pakistan’s economy through trade imbalances, and significant reduction in remittances and capital inflows. Pakistan rupee exchange rate against US dollar has lost its value by 21 percent during 2008 and this has caused the exchange rate to reach the level more than 86 Rupees per US dollar in June 2010. The fluctuations in
exchange rate will further adversely affect the value of trade, amount of debt liability, economic growth and development. Therefore, a greater understanding of exchange rate fluctuations is necessary to understand the determinant of PKR- US dollar exchange rate and provide better economic stability.

In literature of international finance a numerous models have been found to determine exchange rate with fundamentals. For example, the tradition Keynesian approach (Mundell, 1962 and Fleming ,1962) considers the market for flow of funds in exchange rate determination. In this approach foreign exchange is determined by the demand and supply of flow of foreign money, which is required by the domestic and foreign residents to purchase the goods from another country (Mussa, 1976). This approach totally ignored the market for stock of assets that is monetary factors in exchange rate determination. During 1970s, asset approach came up to fill this gap and assimilate the international financial markets stock equilibrium in exchange rate determination (Taylor, 1995). This approach is further classified into monetary and portfolio balance models. These models are capable of addressing the high volatility of exchange rate under flexible exchange rate system. The aim of this study is to investigate and forecast the exchange rate determination mechanism of PKR-US dollar by employing the fundamentals of monetary models.

Following the introduction, rest of the study is organized as follows: Section 2 presents the literature review. Section 3 describes the model specification of exchange rate model. Section 4 deals with the econometric methodology. Data and construction of variables is subject of section 5. Section 5 describes the empirical results and section 6 reports the out of sample forecast. Section 7 concludes the study.
2. **Literature Review:**

Before going to discuss the economic modeling of exchange rate monetary model, this section provides the snapshot of literature review relating to the empirical estimation and forecasting of exchange rate. First part of this section broadly discusses the studies relating to empirical estimation of the exchange rate monetary models by using various techniques of estimation. Latter this section considers the literature on exchange rate forecasting.

The advancement in econometric techniques, from simple to rigorous techniques of estimation, enables to review the implications of economic model from different dimensions. Pearce (1983) have tested the empirical validity of the monetary model using simple OLS method. The estimated parameters turned out to be statistically insignificant and residuals of the regression model had the problem of serial correlation. The empirical results suggest that the model cannot be considered an adequate guide for economic policy. Frankel (1984) has employed the iterative Cochrane-Orcutt technique in order to correct for high serial correlation and improve the results of sticky price monetary model and portfolio balance model. However, the results of monetary and portfolio balance models do not support the theoretical expectations and indicate no improvement. The author then combined the monetary model with portfolio model. The estimates of synthesis equation only improve the results on the variables from portfolio model such as domestic bonds and foreign bonds. The author argued that one cannot claim that the synthesis works better than the sum of the parts, because the coefficients on the variables from the monetary model (relative money supply, interest rate, output and inflation rates) are almost insignificant.

With the development of time series literature particularly relating to cointegration and unit root testing enables the researches to re-estimate the economic theories and solve not only the spurious results of OLS technique but also explore the long run cointegrating vectors among the variables (MacDonald, 1995). Engle and Granger (1987) two step procedure is applicable
to probe the cointegration relationship between two variables only. The application of cointegration approach on economic models consisting of more than two variables is done through the aid of Johansen and Juselius (1988, 1990, 1992) maximum likelihood method of cointegration. Numerous studies have considered it for the empirical validation of exchange rate monetary model. For example, Diamandis and Kouretas (1996) examined the Frenkel (1976) flexible price monetary approach for Greek currency and concluded that study of monetary model by technique of cointegration provides a suitable structure for investigating long-run movements of the Greek drachma.

Wong (2004) argued that flexible price exchange rate is performed well on a floating exchange rate period and examined the long run behavior of British Sterling against per unit of US dollar. After establishing the multiple cointegrating vectors, the author proceeds to test whether the homogeneity and symmetry restriction held to support the exchange rate model in relative form or not. But have failed to accept the homogeneity and symmetry restrictions on money supply, output and interest rate coefficients. Similarly, Abas and Yusof (2009) have also rejected the proportionality between nominal exchange rate and money supplies, symmetry among the domestic and foreign price levels, interest rates and money supplies in case of Malaysian Ringgit and Japanese Yen against US $. Generally concluded, if these conditions are not satisfied then it is preferable to estimate exchange rate model in absolute form rather than in relative form.

Dibooglu and Enders (1995) documented that multiple cointegrating vectors contain valuable information and should be carefully interpreted. This paper provides convincing interpretations of multiple cointegration vectors by considering the Dornbusch(1973, 1974) monetary model in case of dollar/ Franc exchange rate. Johansen procedure revealed two independent cointegration vectors between nominal exchange rate and the fundamentals (relative price levels, relative money supplies, relative income levels, relative interest rates
and relative productivity levels). The first long run relationship is statistically identified as money market equilibrium relationship and second cointegration relationship is identified as modified PPP relationship. However, this paper did not find any evidence to support the symmetry of domestic and foreign variables. Finally, the use of long run structural relationships in the dynamic error correction model provides consistent result with the Keynesian model, that is, expansionary monetary policy is related to domestic inflation, exchange rate depreciation and permanent increase in domestic output level. Diamandis et al. (1998) have restricted the cointegrating vectors of flexible price monetary model, to identify forward looking version of monetary model and uncovered interest parity (UIP) by considering deutschemark-dollar, dollar-pound and yen-dollar exchange rate. likelihood ratio (LR )test is unable to accept the forward looking version of monetary model for all three bilateral exchange rates. Gebreselasie et al. (2005) have identified three long run cointegrating vectors on Dornbush (1976) sticky price model of exchange rate overshooting by considering South African Rand against US dollar. The first cointegrating vector is uniquely defined as original Dornbush (1980) model by imposing unity restriction on exchange rate and relative money supply, and zero restriction on the relative inflation rate coefficients. Second cointegrating vector is normalized on relative money supply and zero restriction on nominal exchange rate coefficients, to identify a money demand equation.

According to Dornbusch (1976) Sticky-Price monetary model, an increase in money supply causes to overshoot the current exchange rate over its long run value. This overshooting effect will vanish when commodity prices adjust in long run to clear the goods market and this will result to reduce the real money supply and thus increase interest rate and would appreciate the domestic currency in the long run. Oskooee and Kara (2000) investigated that rapid increase in the relative money supply in Turkey has a tendency to overshoot the lira against US dollar not only in short run but also in the long run.
In literature some studies have extended the monetary models by incorporating the different variables along the conventional variables. For example, Kouretas and Zarangas (1998) examined the long run movement of parallel (black) market exchange rate of Greek Drachma against US dollar. They employed the extended version of flexible price monetary model, provided by Blejer (1978a). Accordingly, there are two types of arbitrage, financial and goods market arbitrage, which enter the market when black market rate diverges from official rate and PPP. after constructing the long run cointegrating vector through Johansen and Juselius (1993) likelihood ratio test and Stock and Watson (1993) dynamic OLS method, they suggested that monetary model is valid to examine the long run behavior of the parallel drachma-US dollar exchange rate and Greek authorities should consider the black market for US dollar in Greece for effective trade restrictions and foreign exchange.

Moosa (2000) has tested the flexible price monetary model of exchange rate under the German hyperinflation. This paper modifies the conventional monetary model by incorporating expected rate of change in the exchange rate as an additional variable through money demand function. This additional variable is added to measure currency substitution, which is a common practice under hyperinflation.

Friedman (1988) suggested that there is a positive relationship between the level of stock market and money demand. Morley (2009) investigated the relationship between exchange rate and stock price by incorporating the incorporated the Friedman’s (1988) demand function in conventional flexible price monetary model of Frenkel (1976). Hwang (2001) examined the role of stock prices in Dornbusch-Frankel sticky price monetary model. They suggest that the exchange rate equity type monetary model produces more appropriate long run coefficients than the conventional monetary model.
Craigwell et al. (2011) analyzed the long run behavior of Jamaica exchange against US dollar rate by combining the microstructures variables such as sales of foreign US dollars by the central bank for intervention purposes, volume of sales of US dollars in the market, volume of US dollars purchased in the market and bid-ask spread on US dollars in the market, with the macroeconomic fundamental models of exchange rate. They suggested that micro market variables are important factors in examining the Jamaican exchange rate movements.

There is a lack of studies that analyze the validity of exchange rate monetary models in Pakistan. Kemal and Haider (2004) have estimated monetary model for Pakistan. They did not get the satisfactory significant estimates of the monetary model by using Vector autoregressive (VAR) procedure. Their monetary model is limited to reflect the impact of money differential, interest rate differential on exchange rate and did not take into account the effect of output differential on exchange rate. The reason behind the rejection may be the consideration of very short time period from 2000 to 2004. Obstfeld and Taylor (1997), Mark and Choi (1996) and Rogoff (1996) rejected the PPP phenomena under flexible exchange rate in short run, which is assumed under monetary models.

Saeed et al. (2012) examined the long run and short run behavior of Pakistan exchange rate against the US dollar within the framework of monetary model. The monetary model takes into account relative money supply, relative foreign exchange reserve, total debt of Pakistan relative to United States and political instability as explanatory variables. The ARDL bound test confirms the long run relationship among the nominal exchange rate and the explanatory variables. All variables play a significant role to determine the behavior of Pakistan rupee against dollar. They suggest that along fiscal and monetary policy, political stability is required to maintain the exchange rate of Pakistan.
In another study, Malik (2011) predicts the exchange rate behavior of Pakistan relative to the US dollar using a wider set of models: Autoregressive model, Autoregressive moving average model, time series decomposition model, purchasing power parity model, Dornbusch Frankel sticky price monetary model, and combined forecast model. Results suggest that ARCH model beats all other models for exchange rate forecasting. In the short term, forecasting from Dornbusch and Frankel sticky price monetary model with share price performed better than conventional monetary model without share prices.

Literature on exchange rate forecasting is further divided into two categories, one who believes the predicting ability of the structural models of exchange rate and other who supports the random walk model of exchange rate. In forecasting analysis Johnston and Sun (1997), MacDonald (1997), Hwang (2001), Cheung et al. (2002), Anaraki (2007), Khalid (2007) and Abbas et al. (2011) among others observed that the exchange rate equation based on fundamental determinants outperforms a random walk model in out-of-sample forecast tests. Korap (2008) found that fundamental monetary model performed well for forecasting inside a 2.5 years as compare to the random walk with and without drift models.

On the other hand the promoters of random walk model argued that exchange rate is a random walk phenomenon. It efficiently analyzes the exchange rate fluctuations and provides better future forecast such as Rashid (2006) and Malik (2011). According to these studies there is no need to worry about the macroeconomic variables of exchange rate determination. Meese and Meese and Rogoff (1983) and Najand and Bond (2000) suggested that the poor performance of structural models is characterized by instable parameters. The stability of parameters is usually disturbed by the existence of outlier in the series. Therefore, it is necessary to control the outliers in order to get better forecast (Balke and Famby, 1994 and Dijk, et al.1999).
From the above review, specifically for Pakistan no study has been analyzed the validity of real interest rate differential model of exchange rate. Moreover, the existing studies have not tested the homogeneity and symmetry restriction on monetary model and no one address how to get the restricted cointegrating vector. This study will not only try to fill these gaps but also verify the forecasting performance of the fundamental based monetary model against random walk models.

3. Model Specification
Monetary approaches were initiated by Frenkel and Johnson (1976), Mussa (1976), Bilson (1978a) and Dornbush (1976a). They have developed different models to capture the behavior of exchange rate through monetary variables. These are flexible price (FEM), sticky price and Frankel real interest rate differential monetary models. The similarity among these models is that supply and demand for money are the main determinants of exchange rate and assumes a stable money demand function. Monetary models also assumes perfect substitutability among domestic and foreign bonds and utilizes the uncovered interest rate parity condition as it holds continuously due to perfect capital mobility. Besides these similar characteristics, there are some significant differences among these models, such as, flexible price exchange rate model assumes PPP holds continuously, and commodity prices, wages and exchange rate are perfectly flexible both in short run and long run. In Sticky price monetary model (Dornbusch, 1976), introduced dynamic aspects of exchange rate via short run exchange rate volatility in terms of exchange rate overshooting above long run equilibrium exchange rate. The starting point of this model is the different speeds of adjustment in goods and asset markets, that is in short run exchange rate and prices in asset markets adjust quickly relative to prices in goods market and wages in labor market, in response to various shocks such as change in money supply. In the short run, prices in goods and wages in labor market are determined by the sticky prices. Hence, PPP does not hold
continuously as assumed in the FEM, it retains only in the long run. Long run exchange rate is determined by PPP condition symbolically

\[ s = p - p^* , \]

where \( s, p, p^* \) represents the log of long run exchange rate, log of domestic and foreign long run price level. Finally, it assumes regressive exchange rate expectations, which is given by

\[ \Delta s^e = \theta (s - \bar{s}) \quad \text{where} \ 0 < \theta < 1 \]  

Equation (4.7) implies that expected change in current exchange rate (∆s^e) is proportional to the difference between long run exchange rate (\( s^e \)) and current exchange rate (s). In other words exchange rate is expected to converge towards its long run equilibrium level at rate \( \theta \).

Frankel (1979) presented a real interest rate differential model (RIEM). It is a more general monetary model of exchange rate determination. This model has modified the Doronbush’s sticky price exchange rate model by incorporating the expected rate of inflation in the exchange rate expectation equation.i.e.

\[ \Delta s^e = \theta (s - \bar{s}) + (\pi^e - \pi^{e^*}) \]  

Where \( \pi^e - \pi^{e^*} \) is expected long run differential between domestic and foreign inflation.

Equation (2) states that in short run, current exchange rate converges to its long run equilibrium value \( \bar{s} \) at speed of \( \theta \). Utilizing UIP condition \( \Delta s^e = i - i^* \) with equation (2):

\[ i - i^* = \theta (\bar{s} - s) + (\pi^e - \pi^{e^*}) \]

Rearranging we have
Equation (3) explains that the deviation of spot exchange rate from its long run equilibrium exchange rate is proportional to the real interest rate differential. The higher expected real interest rate on domestic assets relative to foreign assets, will attract the investors to invest in domestic asset and cause the capital inflow in domestic country. This will appreciate the domestic currency, until the real interest rates are equalized toward the long run steady state.

Frankel (1979) further argues that in long run equilibrium, $s = \ddot{s}$, which implies

$$i - i' = \pi' - \pi'^*$$

and terms in square brackets of equation (3) can be written as

$$[(i - i') - (i - i')]$$.

Now consider the conventional money demand functions of domestic country ($p = m - b_y y + b_i i$) and foreign country ($p^* = m^* - b_y^* y^* + b_i^* i^*$), for simplicity take $b_y = b_y^* = b$ and $b_i = b_i^* = c$, substitute in the long run PPP condition to obtain an expression for the long run exchange rate as

$$s = p - p^* = m - m^* - b (y - y^*) + c (i - i'^*)$$

Use $i - i' = \pi' - \pi'^*$

$$s = m - m^* - b (y - y^*) + c (\pi' - \pi'^*)$$

Equation (4) is the long run steady state equilibrium value of exchange rate. To obtain the reduce form expression of short run exchange rate determination under general monetary model, substitute for $s$ from equation (3) in (4)

$$s + 1 / \theta [(i - \pi') - (i^* - \pi'^*)] = m - m^* - b (y - y^*) + c (\pi' - \pi'^*)$$

Solve for $s$
\[ s = m - m^* - b(y - y^*) - \frac{1}{\theta}(i - i^*) + \frac{1}{\theta + c}(\pi^e - \pi^{e*}) \]

Let \( a = \frac{1}{\theta} \) and \( d = \frac{1}{\theta + c} \)

\[ s = m - m^* - b(y - y^*) - a(i - i^*) + d(\pi^e - \pi^{e*}) \] (5)

Assuming that the current equilibrium money supplies and income levels are given by their current actual levels, we obtain a complete equation that represents the real interest rate model given below.

\[ s = m - m^* - b(y - y^*) - a(i - i^*) + d(\pi^e - \pi^{e*}) \] (6)

Equation (6) is the general exchange rate equation of monetary model. It explains that short run exchange rate is determined by the relative money supply, income level and expected inflation rate.

In RIEM model prices in goods and labor market tend to adjust slowly. Therefore, if there is unanticipated increase in the money supply, it leads to a decrease in the domestic real interest rate relative to foreign interest rate immediately (because money markets are continuously in equilibrium), while the domestic prices are sticky in short run and initially remain unchanged due to monetary shock but is expected to rise in long run. The result according to equation (6) is that in the short run exchange rate overshoots its long run equilibrium value. This will depreciates the domestic currency more than the increase in money supply. The reason for short run exchange rate overshooting is that foreign investors will want compensation, when domestic interest rate falls, in form of appreciation of domestic currency to maintain UIP. This appreciation is possible only when short run exchange rate overshoots the long run
depreciation and then expected to appreciate. So there are expectations of a future real appreciation of the currency to compensate for the lower real rate of return.

In general form the exchange rate equation under RIEM from equation (6) is

$$ s = f (\pi^e, \delta^e, m^*, m^e, y^*, y^e, i^*, i^e) $$

(7)

FEM can be obtained from RIEM model if prices in all markets are fully flexible and adjust instantaneously due to change in economic policy. FEM is depending on domestic and foreign money supply, output and interest rate.

As far as the empirical validity of the monetary model of exchange rate is concern, it can only be tested in the long run given the short run deviation of the exchange rate from its PPP. More precisely, the RIEM incorporates short run influences and hence it is only the FEM that can be tested in a long run setting (Diamandis and Koutretas, 1996). Therefore, we will use test of cointegration which provide evidence for the existence of a long run relationship between the exchange rate and money supply, real income and interest rate in the domestic and foreign economies. Moreover, in the presence of non stationary variables, the Dornbusch’s SEM can be consistent with the presence of long run equilibrium, even though a temporary overshooting of exchange rate is implied.

4. **Estimation methodology**

According to Granger Representation Theorem (Engle and Granger, 1987), if there is long-run stable relationship among the non-stationary variables then the dynamic exchange rate model can be represented by Error Correction Model (ECM).

Following Johansen (1988) and Johansen and Juselius (1992) the k=9 dimensional dynamic error correction exchange rate function is represented by the vector autoregressive (VAR) model. The VAR representation of exchange rate function is as
\[
\begin{align*}
\varepsilon_t & = A_1 \varepsilon_{t-1} + A_2 \varepsilon_{t-2} + \ldots \ldots + A_i \varepsilon_{t-i-1} + \mu + \phi D_t + \varepsilon_t, \quad t = 1, 2, \ldots, T \tag{8}
\end{align*}
\]

Where \( \varepsilon_t = [s, m, m^*, y, y^*, i, i^*, \sigma, \sigma^*] \) is \((k \times 1)\) dimensional vector,

\( \varepsilon_t \sim N(0, \Sigma) \) is \((k \times 1)\) vector of Gaussian random error terms and \( \Sigma \) is \((k \times k)\) variance covariance matrix of error terms and \( D_t \) is consist of centered seasonal dummies and intervention dummies.

Given that \( \varepsilon_t \) is vector of non-stationary and \( \Delta z_t = (1 - B) \) is vector of stationary variables, therefore, we can infer the following dynamic ECM of exchange rate.

\[
\Delta z_t = \sum_{i=1}^{l-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-i} + \phi D_t + \mu + \varepsilon_t \tag{9}
\]

Where \( l = 1, 2, \ldots \), \( t - 1 \) is the lag length, \( \Gamma_i = -(I - A_1 - \ldots - A_i) \) is short- run dynamic coefficients, \( \Pi = -(I - A_1 - \ldots - A_i) \) is \((k \times k)\) matrix containing long-run information and \( \mu \) is consisting of deterministic component.

The investigation of \( \Pi \) matrix is important to analyze the long run relationship among the exchange rate and its determinants. The number of cointegrating vectors \( r \) are determined by rank of \( \Pi \) matrix. Johansen (1988) maximum likelihood methods; trace and maximum eigenvalue test statistics are commonly used to measure the rank of \( \Pi \) matrix. If \( 0 < \text{rank} \ (\Pi) < k - 1 \) then it is further decompose into two matrices i.e. \( \Pi = \alpha \beta^t \cdot \alpha \) is \((k \times r)\) matrix contains error correction or adjustment coefficients and \( \beta^t \) is the \((r \times k)\) matrix of long run cointegrating vectors.

Three step procedure is applied to obtain the stable dynamic exchange rate function. The first step addresses the stationarity and non stationarity of individual series by applying unit root test. Second step estimate the long run exchange rate function by applying the maximum
likelihood method. Final step attain a parsimonious short run dynamic exchange rate function through the error correction mechanism.

After obtaining the short run dynamic exchange rate function, this study will move to compare the forecasting performance of exchange rate monetary model to random walk models. The recursive regression methodology is adopted to generate multi-step-ahead forecast. The forecasting performance of each forecast horizon will be evaluated by using standard root mean squared error (RMSE) and Theil’s U statistics. Finally, Diebold and Mariano (DM) (1995) test statistics will evaluate the statistical significance of each forecasting horizon.

5. **Data and Construction of Variables**

This study has considered the quarterly data over the period of flexible exchange rate spanning from 1982:Q1 to 2012:Q2. All variables are measured in the currency units of each country. The data are obtained from International Financial Statistics (IFS) and State Bank of Pakistan (SBP) Monthly Statistical Bulletin (various issues).

The nominal exchange rate is measured in terms of units of Pakistan rupee (PKR) per unit of US dollar. Domestic and foreign money supply is measured by broad money M2. Real Gross domestic product (GDP) is commonly used as a measure of real output level. Quarter wise nominal GDP of US is accessible from IFS. The real GDP at constant base of 2000 is found by deflating nominal GDP on GDP deflator (2000=100). In case of Pakistan only annual real GDP is available. Quarterisation of annual real GDP from 1982:Q1 to 2003:Q4 is done by using the percentage share of each quarterly to annual GDP at market price (1980-81), as estimated by Kemal and Arby (2004). However, quarterisation of 2004:Q1 to 2012: Q2 is made by utilizing the average share of each quarter to annual GDP in 2000s (2000 to 2003) i.e. 22.07, 27.15, 25.21 and 25.57 percent in first, second, third and fourth quarter respectively. Call money rate for Pakistan and federal fund rate for US is used as a measure
of interest rate. Unobservable expected inflation rate for domestic country \( (\pi') \) and for foreign country \( (\pi') \) in percentage is calculated by taking the logarithmic change of the CPI (2000=100), in respective countries, over the preceding four quarters, i.e. \( \pi_s = (\ln CPI_{q_s} - \ln CPI_{q_{s-1}}) \times 100 \) where \( s=1,2,3,4 \).

During the analysis period exchange rate of Pakistan is also influenced by the critical events such as 1998 Pakistan’s nuclear, 9/11 event in year 2001, US war against terror in Afghanistan after 9/11 and recent global financial crisis of 2007. Dummy variables \( D_{98} \) (1 for 1998:Q2 and 0 otherwise), \( D_{911} \) (1 for \( t = 2001:Q3 \) and 0 otherwise), \( D_{afgwar} \) (0 for \( t < 2001:Q4 \) and 1 otherwise) and \( D_{fc} \) (1 for \( 2007:Q1 \leq t \leq 2009:Q2 \) and 0 otherwise) are used to capture the influence of intervention events on the exchange rate.

6. Results and Discussion

Cointegration analysis is based on the assumption that variables are integrated of the same order. Hylleberg, Engle, Granger and Yoo (HEGY) (1990) unit root test has been used to test for nonseasonal zero frequency, biannual and annual frequency seasonal unit roots on quarterly data. The results of the HEGY test both at level and at first difference of variables are presented in Table 1. It is clear that on the level of series the null hypothesis of a nonseasonal unit root cannot be rejected whereas the null hypothesis of seasonal unit root at both biannual and annual frequency are rejected at 5% critical values for all of the variables. Therefore, \((1-B)\) is an appropriate filter to make the series stationary. After applying the HEGY (1990) test on the filtered series we found no evidence of seasonal and nonseasonal unit roots at 5% level of significance. Therefore, all variables in our cointegration analysis are integrated of order one and we may suspect multiple long run cointegrating vectors.
Table 1: HEGY Test for Non-Seasonal and Seasonal Unit Roots

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>Drift</th>
<th>Trend</th>
<th>Seasonal Dummies</th>
<th>Regressors</th>
<th>Null &amp; Alternative Hypothesis</th>
<th>Test Statistic</th>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$i'$</td>
<td>$\pi_5 = \pi_5 &lt; 0$</td>
<td>$F_{s_5,s_5}$</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\pi'$</td>
<td>$\pi_6 = \pi_6 &lt; 0$</td>
<td>$F_{s_6,s_6}$</td>
<td>2.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\pi''$</td>
<td>$\pi_7 = \pi_7 &lt; 0$</td>
<td>$F_{s_7,s_7}$</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$m$</td>
<td>$\pi_8 = \pi_8 &lt; 0$</td>
<td>$F_{s_8,s_8}$</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$m'$</td>
<td>$\pi_9 = \pi_9 &lt; 0$</td>
<td>$F_{s_9,s_9}$</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Multivariate Cointegration Analysis

To investigate the existence of long run cointegration relationship among the fundamental variables of RIEM, it is necessary to set the appropriate lag length of the model and deciding the appropriate model regarding the deterministic components in the multivariate system.
The optimal lag length of VAR model is selected by multivariate LM test statistics which ensures that lag length is sufficient to remove serial correlation in the residuals of VAR model. 5 quarters is chosen as optimal lag length of unrestricted VAR model. Three central seasonal dummies and four intervention dummies $D_{98}$, $D_{911}$, $D_{afgwar}$, $D_{fc}$ are also included.

Most of the variables of monetary models have linear trend and grow over time. Therefore, intercept term enters unrestricted in the cointegrating analysis (Johansen, 1995; Harris and Sollis, 2003 and Qayyum, 2005). The results of Johansen’s likelihood ratio test after adjusting by factor $(T-kl)/T$ to correct the small sample bias (Reimers, 1992), is reported in Table 2.

**Table 2: Cointegration Test Results of RIEM**

<table>
<thead>
<tr>
<th></th>
<th>Trace statistic</th>
<th>Maximum Eigenvalue statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$H_A$</td>
<td>Chi- Square</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>204.80*</td>
</tr>
<tr>
<td>$r ≤ 1$</td>
<td>$r &gt; 1$</td>
<td>154.09</td>
</tr>
<tr>
<td>$r ≤ 2$</td>
<td>$r &gt; 2$</td>
<td>107.78</td>
</tr>
<tr>
<td>$r ≤ 3$</td>
<td>$r &gt; 3$</td>
<td>64.38</td>
</tr>
<tr>
<td>$r ≤ 4$</td>
<td>$r &gt; 4$</td>
<td>36.39</td>
</tr>
<tr>
<td>$r ≤ 5$</td>
<td>$r &gt; 5$</td>
<td>20.40</td>
</tr>
<tr>
<td>$r ≤ 6$</td>
<td>$r &gt; 6$</td>
<td>9.27</td>
</tr>
<tr>
<td>$r ≤ 7$</td>
<td>$r &gt; 7$</td>
<td>2.83</td>
</tr>
<tr>
<td>$r ≤ 8$</td>
<td>$r &gt; 8$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note: ‘*’ indicates the rejection of null hypothesis at the 5 percent level of significance.

Using trace statistic, the null hypothesis of $r = 0$ (against alternative that $r > 0$) can be rejected at 5% level of significance. On the basis of trace statistics it is feasible to conclude that there is one cointegrating vector, as the null hypothesis that $r ≤ 1$ is not rejected. Maximum eigenvalue test suggests that null hypothesis of zero cointegrating vectors ($r = 0$) cannot be rejected at 5% level.

However, according to maximum eigenvalue test, RIEM model contains no long run cointegrating vector. We continue our analysis on the basis of trace test, as it considers all $k-r$ values of smallest eigenvalues (Kasa, 1992 and Serletis and King 1997). Now we proceed
to test the commonly imposed monetary restrictions on one cointegrating vector. The most important restrictions, discussed in literature, is to test whether proportionality exists between the nominal exchange rate and relative money supplies, opposite and equal coefficient restriction on relative income, interest rates and inflation rates (Diamandis et al., 1998). Table 3, summarizes the results of these hypotheses;

Table 3: Proportionality Restrictions on Monetary Model

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Hypothesis</th>
<th>$\chi^2$ (df)</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Symmetry</td>
<td>$H_1$: $\beta_1 = -\beta_2 = 1$</td>
<td>4.33(1)${}^a$</td>
<td>0.04</td>
</tr>
<tr>
<td>Output Symmetry</td>
<td>$H_2$: $\beta_3 = -\beta_4$</td>
<td>4.70(1)${}^a$</td>
<td>0.03</td>
</tr>
<tr>
<td>Interest rate Symmetry</td>
<td>$H_3$: $\beta_4 = -\beta_5$</td>
<td>1.17(1)${}^{aaa}$</td>
<td>0.28</td>
</tr>
<tr>
<td>Inflation Symmetry</td>
<td>$H_4$: $\beta_7 = -\beta_8$</td>
<td>0.74(1)${}^{aaa}$</td>
<td>0.38</td>
</tr>
<tr>
<td>$H_5$: $H_1 \cap H_2$</td>
<td>8.59(3)${}^a$</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$H_6$: $H_1 \cap H_3$</td>
<td>7.50(3)${}^{aaa}$</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>$H_7$: $H_1 \cap H_4$</td>
<td>5.35(3)${}^{aaa}$</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$H_8$: $H_2 \cap H_3$</td>
<td>17.24(3)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$H_9$: $H_2 \cap H_4$</td>
<td>14.78(3)</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>$H_{10}$: $H_3 \cap H_4$</td>
<td>22.04(3)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$H_{11}$: $H_1 \cap H_3 \cap H_5$</td>
<td>22.29(5)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$H_{12}$: $H_1 \cap H_5 \cap H_4$</td>
<td>17.08(5)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$H_{13}$: $H_3 \cap H_5 \cap H_4$</td>
<td>30.04(5)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Joint Symmetry</td>
<td>$H_{14}$: $H_1 \cap H_3 \cap H_5 \cap H_4$</td>
<td>57.68(7)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: ${}^a$, ${}^{aa}$, and ${}^{aaa}$ indicates the significance at 1%,5% and 10%.

Based on the $\chi^2$ values, it is found that individual proportionality restriction of money supplies ($H_1$), output ($H_2$), interest rate ($H_3$) and inflation rate ($H_4$) are not rejected. Similarly, the joint hypothesis of proportionality between money supplies and output ($H_5$), money supplies and interest rate ($H_6$) and money supplies and inflation ($H_7$) are also accepted. These results provide direct support to long run monetary model in case of rupee dollar exchange rate and indirect verification of long run PPP, as PPP develops a linkage between money demand and exchange rate monetary equation.
The long-run exchange rate monetary function is estimated by normalizing the cointegrating vector on log of exchange rate is as follows

\[ s_t = 76.45 + 2.66 \cdot m_r - 0.79 \cdot m_r^* - 3.54 \cdot y_r - 3.03 \cdot y_r^* - 0.10 \cdot i_r + 0.02 \cdot i_r^* + 0.01 \cdot \pi_r - 0.07 \cdot \pi_r^* \]  

(10)

The estimated cointegrating vector indicates that long run exchange rate function is determined by relative money supplies, output level, interest rate and rate of inflation. Estimated parameters are according to expectation of model.

The results in equation (10) can be explained as follows

- A positive significant coefficient of domestic money supplies on nominal exchange rate reveals that one percent increase in domestic money result in a 2.66 percent depreciation of domestic currency. A corresponding increase in foreign money supplies causes the nominal exchange rate to appreciate by 0.79 percent. These coefficients are according to theory. Therefore, increase in domestic money supply relative to foreign money supply leads to depreciate the currency in the long-run. The elasticity obtained for domestic money supply is greater than unity (2.66). This result is according to overshooting hypothesis and consistent with Oskooee and Kara (2000).

- Negative coefficient of domestic real output on exchange rate suggests that an increase in domestic output level has a favorable effect on nominal exchange rate of Pakistan. A one percent increase in domestic output level leads to appreciate the domestic currency against US dollar significantly by 3.54 percent. However, the corresponding foreign real output level also results into the appreciation of domestic currency, which is opposite of what is recommended under RIEM.

- A one percent increase in domestic interest rate is significantly associated with appreciation of the nominal exchange rate by 0.10 percent. Whereas, a one percent
point increase in foreign interest rate causes the domestic currency to depreciate by 0.02. Thus, an increase in domestic interest rate relative to foreign interest rate appreciates the domestic currency.

- The estimated parameter of domestic inflation rate indicates that one percent increase in domestic inflation rate results into depreciation of the Pakistan rupee against US dollar by 0.01 percent. Foreign inflation rate leads to appreciate the domestic currency by 0.07 percent.

The most striking result of monetary model is the high coefficients of domestic variables (except inflation rate), implies the dominant role of domestic variables in affecting exchange rate.

The critical point is whether it is possible to impose FEM relationship on equation (10) and still find reasonable values for the other behavioral coefficients. The issue is whether it is statistically possible to restrict the $\beta^*$ matrix as

$$
\beta^* = \begin{bmatrix}
1 & * & * & * & * & * & 0 & 0 \\
\end{bmatrix}
$$

However, the zero restrictions on the coefficients of domestic and foreign inflation rate cannot be rejected. The LR test statistics for these restrictions is $\chi^2_{df=2} = 0.594$. The resulting Flexible exchange rate equilibrium relationship is:

$$
s = \{119 \quad .06 + 3.63 \quad m, - 0.85 \quad m', - 4.21 \quad y, - 5.89 \quad y', - 0.16 \quad i, + 0.12 \quad i'\} (11)
$$

The estimated cointegrating vector indicates that long run exchange rate function is determined by relative money supplies, output level and interest rate. Estimated parameters are according to expectation of flexible exchange rate model except the signs of domestic and foreign interest rate.
The Short-Run function For Monetary Exchange Rate: Dynamic Error Correction Model

The short-run dynamic error correction model (ECM) is estimated by using the residual deduced from long run cointegration function (11), known as error correction term. Parsimonious dynamic model is achieved by considering Hendry’s (1992) general-to-specific methodology. General model is started by having drift term, three central seasonal dummies, intervention dummies \((D_{98}, D_{911}, D_{afgwar}, D_{fc})\), lag of error correction term and lag length of five for each first difference variables (exchange rate, money supplies, output levels, interest rates and inflation rates). The specific model is achieved by dropping the insignificant regressors. The parsimonious ECM model with t-ratios in parentheses is reported as follows:

\[
\Delta x_t = \begin{align*}
0.11 & -0.16 \Delta s_{t-1} - 0.57 \Delta m_{t-2} - 0.33 \Delta m_{t-2} - 0.20 \Delta m_{t-4} - 0.91 \Delta m_{t-1} - 0.76 \Delta m_{t-4} \\
(6.51) & (-1.70) (-4.45) (-2.78) (-1.90) (-2.56) (-2.82) \\\n-0.39 & \Delta y_{t-2} - 0.29 \Delta y_{t-3} - 0.69 \Delta y_{t-4} - 0.03 \Delta y_{t-1} - 0.06 D_{yl} - 0.03 D_{hy} + 0.02 D_{y} \\
(-2.61) & (-1.90) (-4.60) (-4.14) (-2.36) (-3.67) (2.01) \\
-0.03 & EC_{t-1} \\
(-3.34) &
\end{align*}
\]

\(Adj \ R^2 = 0.33 \quad F_{(14, 90)} = 3.61 \quad \text{prob (0.000)}\)

Finally, certain diagnostic tests are performed on the residuals of dynamic error correction model (equation 12), to determine its appropriateness. It passes the diagnostic test of no autocorrelation \((X^2_{(1)} = 0.40 \text{ and } X^2_{(4)} = 2.17)\) and no ARCH \((X^2_{(1)} = 0.31 \text{ and } X^2_{(4)} = 1.14)\) at 5 percent level of significance. Further the residuals are normally distributed \((X^2_{(2)} = 5.35)\).

The estimated ECM indicates that in the short run exchange rate is responsive to change in money supplies, domestic real output level and foreign interest rate. The estimated parameters of almost all variables are found to be negative and significant in short run dynamic error correction model.
The presence of lag of dependent variable makes the short run dynamic ECM as an autoregressive model. Its estimated parameter indicates that a one percent depreciation in preceding third quarter (approximately nine month back) change in exchange rate results into the appreciation of change in current exchange rate by 0.16 percent.

The short run effect of change in foreign money supply and domestic output level on nominal exchange rate is similar to their effects on exchange rate in long run. Increase in these variables tends to exert upward pressure on the domestic currency both in short run and long run. Interestingly exchange rate is appreciated by change in domestic money supply in short run, which is opposite to its long run effect. The policy recommendation that comes out from the result of monetary model is that measures of monetary policy to strengthen the rupee by rising the money supply will appreciate the currency in short run but ultimately would lead to depreciation of the currency in long run.

Equation 12 does not contain change in inflation rate. It indicates that there is no short run association between change in inflation rate and exchange rate as reported by Choudri and Khan (2002).

Inclusion of intervention dummy variables in the dynamic model suggests that exchange rate is vulnerable to external shocks. The coefficients for $D_{911}$ and $D_{afgwar}$ are found to be negative and significant, which represents the appreciation of rupee after September 9, 2001. This appreciation is due to high inflows of remittances and foreign reserves during this period (Kemal and Qadir, 2005). $D_{fc}$ captures the effect of recent financial crisis on nominal exchange rate. The positive coefficient of $D_{fc}$ suggests that 2007 financial crisis exerts downward pressure on Pakistani rupee.

Finally, the coefficient of error correction term shows the speed of adjustment which comes out to be -0.03 and is significant at 5 percent level of significance. The feedback coefficient
suggests that short run deviation of nominal exchange rate from its long run equilibrium path is being corrected by 3 percent in each quarter. The time required to remove the 50 percent of disequilibrium from its exchange rate equilibrium path is 23 quarters (five years and three quarters). The slow speed of adjustment toward equilibrium path indicates the great instability and large shocks during the flexible exchange rate period which is captured by the error correction model of RIEM.

The forecasting power of the model depends on the stability of parameters. CUSUM and CUSUMSQ test are applied for this purpose. The plots of CUSUM and CUSUMSQ are provided in Figure 1 and Figure 2. The plots show that CUSUM and CUSUMSQ remain within the 5% critical bound. Suggesting that there is no significant structural instability and residual variance is stable during the analysis period.

![Figure 1: Cumulative Sum of Recursive Residuals](image-url)
7. Out of Sample Forecasts

The recursive regression methodology is adopted to generate multi-step-ahead forecast from monetary and random walk exchange rate models. The basic procedure for the construction of subsample is similar to that of Mark (1995) and Cushman (2006) among others. The procedure starts by dividing the data set, containing \( t=1, 2, \ldots, T \) number of observations into thirty seven subsamples \( t_1, t_2, \ldots, t_{37} \). The first subsample contains \( T-37 \) (smallest subsample) number of observations. Denote it by \( t_1 \) (ends at period 2003:1). The next subsample \( t_2 \) is extended by one observation; it contains \( T-36 \) number of observations (ends at period 2001:2), and so on the largest and last sample ends with \( T-1 \) number of observations, denote it by \( t_{37} \) with ending period 2012:1.

The parsimonious error correction model equation is then estimated for each subsample. This recursive procedure updates the estimated parameters in each subsample due to the inclusion of new data point. Each subsample estimated error correction model will be used to construct
a one quarter ahead forecast to sixteen quarter ahead forecast. This will result in 37 one quarter ahead forecast, 36 two quarter ahead forecast and so on 22 sixteen quarter ahead forecast. Forecasted values are also obtained from random walk models for each subsample.

Table 4, gives the result for RMSE of different models at 1, 4, 8, 12 and 16 forecasting horizons. It can be noted that RMSE of fundamental based monetary model is smaller than the RMSE of benchmark random walk models, with and without drift, at all out of sample forecast horizons. Therefore, it is easy to conclude that monetary model yields better forecast for exchange rate than theory less random walk models.

Theil’s U statistics computes the ratio of the RMSE of the monetary model to the RMSE of random walk models. If this ratio is less than one then structural model on average provide better forecast than benchmark. Theil’s U coefficient at each forecasting horizon is reported in Table 5. This coefficient again supports the dominance of structural model over the random walk models at every horizon.

RMSE and Theil’s U factor do not provide any idea of the significance of the difference in the forecasting performance. Therefore, final conclusion will draw on DM test statistics. Table 8 lists the DM statistics and its associated probability values at various horizons, to significantly test whether the mean square error of one forecast is better than another.

First part of Table 6, takes random walk model without drift as benchmark model in loss differential function. The DM test statistics confirm that the predictive accuracy of monetary model is significantly more accurate than the random walk model at long forecast horizon i.e. \( k = 12 \), and 16. The success of structural models at long horizons is consistent with Mark (1995) and Chinn and Meese (1992). Second part of Table 6 is comparing the difference in the forecasting performance of the structural models to the benchmark random walk with drift model. DM test statistics clearly states that parsimonious cointegrated monetary model easily
beat the random walk model with drift at every horizon except the first. This finding confirms the remarks of Faust et al. (2003) that is easy to beat the random walk model with drift than the random walk model without drift.

Table 4: Out-of-Sample Forecast Evaluation: RMSE

<table>
<thead>
<tr>
<th>RMSE</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW Model</td>
<td>0.030</td>
<td>0.089</td>
<td>0.152</td>
<td>0.177</td>
<td>0.199</td>
</tr>
<tr>
<td>RW with Drift Model</td>
<td>0.048</td>
<td>0.103</td>
<td>0.162</td>
<td>0.201</td>
<td>0.247</td>
</tr>
<tr>
<td>RIEM Model</td>
<td>0.032</td>
<td>0.025</td>
<td>0.030</td>
<td>0.030</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 5: Out-of-Sample Forecast Evaluation: Theil’s U

<table>
<thead>
<tr>
<th>Model</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark: RW Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIEM</td>
<td>1.050</td>
<td>0.282</td>
<td>0.199</td>
<td>0.171</td>
<td>0.163</td>
</tr>
<tr>
<td>Benchmark: RW with drift Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIEM</td>
<td>0.671</td>
<td>0.245</td>
<td>0.187</td>
<td>0.151</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Table 6: Out-of-Sample Forecast Evaluation: DM Test Statistic

<table>
<thead>
<tr>
<th>Model</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark loss Function: RW Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIEM</td>
<td>-0.169&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.457&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.847&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2.177</td>
<td>2.259</td>
</tr>
<tr>
<td></td>
<td>(0.867)</td>
<td>(0.155)</td>
<td>(0.075)</td>
<td>(0.039)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Benchmark loss Function: RW with Drift Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIEM</td>
<td>0.902&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.027</td>
<td>4.040</td>
<td>3.490</td>
<td>2.836</td>
</tr>
<tr>
<td></td>
<td>(0.373)</td>
<td>(0.005)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.010)</td>
</tr>
</tbody>
</table>

*Note: ‘<sup>a</sup>’ represents the acceptance of null hypothesis of equal forecast. A probability value of DM statistics is in brackets.
7. **Conclusion**

In this paper, an empirical exchange rate monetary model based on Dornbusch, (1976) and Frankel (1979) is specified for the Pakistan–US exchange rate over the flexible exchange rate period. Cointegration test found one cointegrating vector. In monetary model the estimated signs of all coefficients in the long run cointegrating equation of monetary model are significant and according to expectations. The large magnitude of coefficients of domestic variables implies the significant role of domestic variables in affecting exchange rate. In addition, the commonly imposed proportionality restrictions on the coefficients of monetary model are supported by likelihood ratio test and confirm the validity of the model. Finally, error correction model revealed a slow speed of adjustment towards its equilibrium path that is 3 percent and 23 quarters are required to remove 50 percent of deviation. The interesting finding of the error correction model first is the opposite sign of money supply on exchange rate. Therefore, the expansion of money supply appreciates the currency in short run and depreciates the currency in long run. Second, it overshoots itself in reaction to rapid increase in domestic money supply relative to foreign money supply not only in the short-run but also in the long-run.

Out of sample forecasting performance of monetary model was supported by RMSE and Theil’s U and DM test statistics over naïve random walk models. This finding is attributable to the parsimonious error correction model, which includes lags of dependent variable and fundamental variables to exchange rate determination, error correction term and intervention dummies.

**References**


