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Abstract

Borrowing from our experience in agent-based computational economic research from 'bottom-up', this paper considers economic system as multi-level dynamical system that micro-level agents' interaction leads to structural transition in meso-level, which results in macro-level market dynamics with endogenous fluctuation or even market crashes. By the concept of transition matrix, we develop technique to quantify meso-level structural change induced by micro-level interaction. Then we apply this quantification to propose the method of dynamic projection that delivers out-of-sample forecast of macro-level economic variable from micro-level big data. We testify this method with a data set of financial statements for 4599 firms listed in Tokyo Stock Exchange for the year of 1980 to 2012. The Diebold-Mariano test indicates that the dynamic projection has significantly higher accuracy for one-period-ahead out-of-sample forecast than the benchmark of ARIMA models.

Keywords: economic forecasting, dynamic projection, multi-level dynamical system, transition matrix

JEL Classification: C53, C63, E27.

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1 Introduction

As human beings, we admit diversification among individuals in our species. We are not isolated in our society. Instead, we connect and interact with each other in one way or another along time horizon. Thus, it is natural to understand our economy from the perspective of dynamical system of heterogeneous interacting agents. This perspective implies complexity that agents interaction in micro-level leads to transition or structural change in meso-level, which in turn leads to macro-level dynamics with endogenous market fluctuation or even market crashes (crises). In this regard, economic system can be regarded as multi-level dynamical system that micro-level agents' interaction, through meso-level structural change, results in macro-level complex dynamics. The crucial point to understand the linkage between microfoundation of agents' interaction and macro-level complex dynamics is on the structural change in meso-level induced by agents' interaction.

When macroscopic market equilibrium is highlighted in economic model, the micro-macro linkage goes to the microfoundation with boundedly rational economic agents. It leaves no room for explicitly modeling meso-level structural change. However, the recent financial crisis and the subsequent Great Recession indicate the necessity of understanding our economy not only in market equilibrium but also in market fluctuation or even under crises. This necessity evokes demand in academia and in the practice of policy-making to develop an economic framework with the capability of explaining market fluctuation and crises from bottom-up, which is an open issue in current economic research. As a response, efforts have been devoted to attacking this problem from different angles in economic research. On one hand, the branch of agent-based economics considers that agents' interaction in micro-level endogenously leads to macro-level market fluctuation through micro-macro linkage. The linkage can be represented by network of economic agents, with market crashes (crises) represented by bankruptcy cascade in agents' network, e.g. see Battiston, Delli Gatti, Gallegati, Greenwald, and Stiglitz (2012) and Delli Gatti, Gallegati, Greenwald, Russo, and Stiglitz (2012). Another representation of the micro-macro linkage in agent-based economics is through market mechanism such that agents' interaction triggers large imbalance in demand-supply scheme which induces economic crises, e.g. with a decentralized matching mechanism proposed in Riccetti, Russo, and Gallegati (2014). Similar idea has been applied in another branch of complex economics. This branch mainly considers that macro-level economic phenomena are characterized with emergent properties endogenously generated by micro-level interaction among agents' behavior, e.g. see Kirman (2011). On the other hand, the branch of Dynamic stochastic general equilibrium (DSGE) modeling, instead of considering the micro-macro linkage endogenously, attacks the same problem by exogenously adding Markov-switching mechanism to represent the system-level regime change or switching in policy rules, e.g. see Sims, Waggoner, and Zha (2008). No matter endogenously or exogenously, what lies in centrality in these different branches is the linkage between microfoundation and macro-level dynamics. In this regard, modeling meso-level structural change expects to play an important role in this open topic.

Modeling meso-level structural change requires appropriate measurement and quantification. From the mathematical foundation of multi-level dynamical system shown in Pfante, Bertschinger, Olbrich, Ay, and Jost (2013), if the finite-dimensional discrete-time dynamical system has lower-level dynamics that follows a Markov process, under deterministic coarse-graining on the lower-level state of the system, one can construct a representation of Markov process for upper-level dynamics. Reflecting this finding in economic system indicates the meso-level dynamics can be represented by Markov process, with transition matrix as the quantification of the meso-level transition. By admitting this viewpoint, we develop in this paper a technique to quantify meso-level structural change by transition matrix. This technique of quantification requires the input of large volume of micro-level economic data. The dawning era of big data satisfies this requisite condition. It inspires us to apply this quantification to establish the method of dynamic projection that aims at computing out-of-sample forecast of macro-level economic variable from micro-level big data.

In this work, we show under the following structure the development on this method of dynamic projection. Section 2 studies existing economic literature and reviews related works that have direct connection to our topic. Section 3 describes the technique of dynamic projection. It starts with the quantification of meso-level structural change by the concept of transition matrix in section 3.1. Section 3.2 explains how to develop the technique of dynamic projection by applying the quantification of meso-level structural change. Then we testify the performance of dynamic projection on one-period-ahead out-of-sample forecast with the dataset of firms listed in Tokyo Stock Exchange depicted in section 3.3. Section 3.4 and 3.5 illustrate the result of forecast on firms' aggregate equity and aggregate profit respectively. Section 4 concludes.

2 Related Literature

Our work is related with existing branches of economic research. The first linkage is on the agent-based economic study with Markov chain, which considers the structural change from microscopic level. Several attempts have been made in this branch to model with the concept of transition matrix. For example, Gintis (2013) proposes an agent-based general equilibrium model of decentralized markets with

g types of money. It considers, in the micro-level, the agent-based model as a finite-dimension Markov chain and the state of the system as a g-dimensional vector with its element representing the number of economic agents who accept certain type of money. The limitation of this work is that the number of states is comparatively large, which requires brute force on computing numerically high-dimensional transition matrix for micro-level state transition.

Another related work has been found in the strand of complexity economics and econophysics, e.g. see Foley (1994), Aoki (1998, 2004), Lux (2008), Aoki and Yoshikawa (2011), Landini, Gallegati, and Stiglitz (2014a), Landini, Gallegati, Stiglitz, Li, and Di Guilmi (2014b), among others. This type of work considers the state of the system from meso-level, and derives the dynamics of the system by depicting the transition rate with technique such as FokkerPlanck equation or master equation, which refers to an implicit application of the concept of transition matrix to depict the structural change on meso-level. The limitation of this work is that, if the system has more than two-dimensional states, the derivation of an analytic solution for the dynamical system becomes difficult, if not impossible.

The third related work is on Markov-switching models, which regards systemic change as switching among different macro-states of the system under Markov process with associated transition matrix, e.g. see the early work in Hamilton (1989) for a study of US business cycle by considering the states of economic expansion and recession. This type of models has been widely applied in the filed of econometrics, e.g. see Diebold, Lee, and Weinbach (1994), Engel and Hamilton (1990), Garcia and Perron (1996), Goodwin (1993), and Kim and Nelson (1998). Markov-switching models have recent comeback in studying regime changes in monetary and fiscal policy under DSGE framework, see e.g. Davig and Leeper (2005), Sims and Zha (2006), Davig and Leeper (2008), Liu, Waggoner, and Zha (2011), Chen and Macdonald (2012), Bianchi (2012), Foerster (2013), among others. This line of research focuses on macro-states of the system, with the quantification of the system change on macro-level by transition matrix. It has limitation that the parametrization of transition matrix is by and large from either educated guess or from ad-hoc procedure on estimation with the usage of macro-level time series data. Notice that the transition matrix is for a quantification of the system change in macro-level, and macroscopic change has its root from microfoundation. Using macro-level data for parametrization of the transition matrix, while ignoring the micro-macro linkage, pales the predictive power of this type of Markov-switching models.

To escape from these limitations aforementioned, we first admit the micro-macro linkage and use micro-level economic data and as the data input. Then we work out the construction of meso-level states to avoid the difficulty in high-dimensional transition matrix. Instead of insisting on analytic or closed-form solution of the system dynamics, we take the pragmatic viewpoint to develop the numerical technique on computing transition matrix.

3 Dynamic Projection

We show the quantification and computation of transition matrix by assuming an economy with $i \in \{1, \ldots, I\}$ firms for dynamics of $t \in \{1, \ldots, T\}$ periods. At each period t, firm i has the equity level of $A[i, t]$. Each firm conducts its activities to generate profit, and updates its equity level $A[i, t+1]$ for the next period $t+1$. For information disclosure, each firm distributes at the end of each period t its financial statements, including the balance sheet and the income statement.

Firms financial statements disclose information of two types of economic variables: stock variable and flow variable. *Stock variable* measures quantities at a time point, e.g. firms equity in the balance sheet. *Flow variable* measures quantities at a time interval, e.g. firms profit in the income statement. Stock variable measures the state of the firm, while flow variable are interrelated with the state of the firm, and thus with stock variable. We choose from the firms stock variables as the basis to quantify transition matrix.

3.1 Quantification of Transition Matrix

We consider in this paper the equity level for each firm as the micro-level state of the system at the end of each period t. According to the firms equity $A[i, t]$, we construct in the following a coarse-graining on the micro-level state of the economic system by partitioning firms into different bins such that one firm only belongs to one bin.

Suppose a partition with N bins on the firms equity level. We rank all firms by their equity level in non-decreasing order. Then we compute a series of thresholds of equity level $\theta_n^A[t]$, $n = 1, \ldots, N-1$, and construct bins as follows:

$$\begin{cases} s_1[t] = & \{\text{firm } i : A[i,t] \in (-\infty, \theta_1^A[t]]\}, \\ & \text{where firms are with their equity level in the first } \frac{1}{N} \text{ quantile;} \\ s_n[t] = & \{\text{firm } i : A[i,t] \in (\theta_{n-1}^A[t], \theta_n^A[t]]\},\ n = 2, \ldots, N-1, \\ & \text{where firms are with their equity level in the } n\text{-th } \frac{1}{N} \text{ quantile;} \\ s_N[t] = & \{\text{firm } i : A[i,t] \in (\theta_N^A[t], +\infty)\}, \\ & \text{where firms are with their equity level in the } N\text{-th } \frac{1}{N} \text{ quantile.} \end{cases} \tag{1}$$

The series of thresholds $\theta_n^A[t]$, $n = 1, \ldots, N-1$, defines a measurement on the structure of equity level in the population of firms. It is obvious that this series of thresholds and the associated partition have no element of randomness, and thus what we have constructed is a deterministic coarse-graining.

Denote the number of firms in each bin as $\sharp s_n[t]$, $n = 1, 2, \ldots, N$. Then the frequency number for each bin is:

$$fr[s_n, t] = \frac{\sharp s_n[t]}{\sum_{j=1,\ldots,N} \sharp s_j[t]}, \quad n = 1, 2, \ldots, N. \tag{2}$$

We have $fr[s_n, t] \geq 0$ for $n = 1, 2, \ldots, N$, and $\sum_{n=1,\ldots,N} fr[s_n, t] = 1$. The vector $\mathbf{fr_s[t]} = (fr[s_1, t], \ldots, fr[s_N, t])$ represents in the meso-level the percentage of the firm being in these bins, and is thus called the *distribution vector* (or simply distribution) of the firms equity structure.

After firms update their equity level to $A[i, t+1]$ at the end of period $t+1$, the construction of bins is as follows:

$$\begin{cases} s_1[t+1] = \{\text{firm } i : A[i, t+1] \in (-\infty, \theta_1^A[t])\}; \\ s_n[t+1] = \{\text{firm } i : A[i, t+1] \in (\theta_{n-1}^A[t], \theta_n^A[t])\}, \ n = 2, \ldots, N-1; \\ s_N[t+1] = \{\text{firm } i : A[i, t+1] \in (\theta_N^A[t], +\infty)\}. \end{cases}$$

Notice that we use thresholds $\theta_n^A[t]$, $n = 1, \ldots, N-1$, derived at period t to construct the partition at period $t+1$, with the purpose of using the same criterion to measure the equity structure in the population of firms at period t and $t+1$.

The distribution vector $\mathbf{fr_s[t+1]} = (fr[s_1, t+1], \ldots, fr[s_N, t+1])$ is then known. The shift of firm i in bin $s_n[t]$ to $s_{n'}[t+1]$ for $n, n' \in \{1, 2, \ldots, N\}$ aggregates in the meso-level the transition from $fr_s[t]$ to $fr_s[t+1]$. Denote the number of firms in $s_n[t]$ that shift to $s_{n'}[t+1]$ as $\sharp\{s_n[t] \cap s_{n'}[t+1]\}$. The transition rate between bin $s_n[t]$ and bin $s_{n'}[t+1]$ is:

$$m[n, n', t] = \frac{\sharp\{s_n[t] \cap s_{n'}[t+1]\}}{\sharp s_n[t]}. \tag{3}$$

Define the transition matrix as:

$$\mathbf{M[t]} = \begin{bmatrix} m[1,1,t] & \ldots & m[1,n',t] & \ldots & m[1,N,t] \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ m[n,1,t] & \ldots & m[n,n',t] & \ldots & m[n,N,t] \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ m[N,1,t] & \ldots & m[N,n',t] & \ldots & m[N,N,t] \end{bmatrix}, \tag{4}$$

which fully describes the transition among all bins $s_n[t]$ and $s_{n'}[t+1]$ for $n, n' \in \{1, 2, \ldots, N\}$. The dynamics from $\mathbf{fr_s[t]}$ to $\mathbf{fr_s[t+1]}$ is modeled as:

$$\mathbf{fr_s[t+1]} = \mathbf{fr_s[t]} \cdot \mathbf{M[t]}. \tag{5}$$

3.2 Principle of Dynamic Projection

We aim at developing a technique of dynamic projection that projects the future state of macro-level economic variable by using related micro-level stock and flow variable with the consideration of the structural change in meso-level. The principle of dynamic projection can be stated in a loose form by the following formula:

$$\text{Macro-level economic variable} = \text{aggregation in micro-level economic variable} + \text{aggregation in meso-level structural change.} \tag{6}$$

Assume we stand at the end of period t, and would like to project the future state of some target macro-level economic variable for the next period $t+1$, "as-if" the economy, *ceteris paribus*, from period t to period $t+1$.[1] By Equation (6), the crucial point to compute the dynamic projection is on the quantification for the meso-level structural change, which refers to the usage of the transition matrix. There are two types of target: macro-level stock variable and macro-level flow variable. For macro-level stock variable, the transition matrix is based on the associated micro-level stock variable. We recognize the relation that flow variable is attached with certain stock variable. For macro-level flow variable, we consider the transition matrix based on the attached micro-level stock variable. In this sense, we call it *stock-flow dynamic projection*. We demonstrate how to derive the dynamic projection by example with a data set of Japanese firms listed in Tokyo Stock Exchange.

3.3 Data

We use the data set for 4599 Japanese firms listed in Tokyo Stock Exchange, for the year of 1980 to 2012. This data set contains for all firms 33 years of annual balance sheet and the profit and loss (PL) statement, an equivalence to the income statement. We are concerned with two macro-level economic variables, the macro-level stock variable as the firms aggregate equity and the macro-level flow variable as the firms aggregate profit.

This data set contains a small portion of the firms accounted for Japan GDP. The aggregate equity and aggregate profit contained in this data set are somehow related to the dynamic pattern of Japan GDP, see Figure 1 for the dynamics of aggregate equity and GDP and Figure 2 for the dynamics of aggregate profit and GDP.[2]

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[1]One can consider the dynamic projection for the future period $t+t'$, which is equivalent to the dynamic projection for the next period $t+1$ by t' times.

[2]The data source for Japan GDP is obtained from the World Bank. The link to the data is: http://data.worldbank.org/indicator/NY.GDP.MKTP.CN.

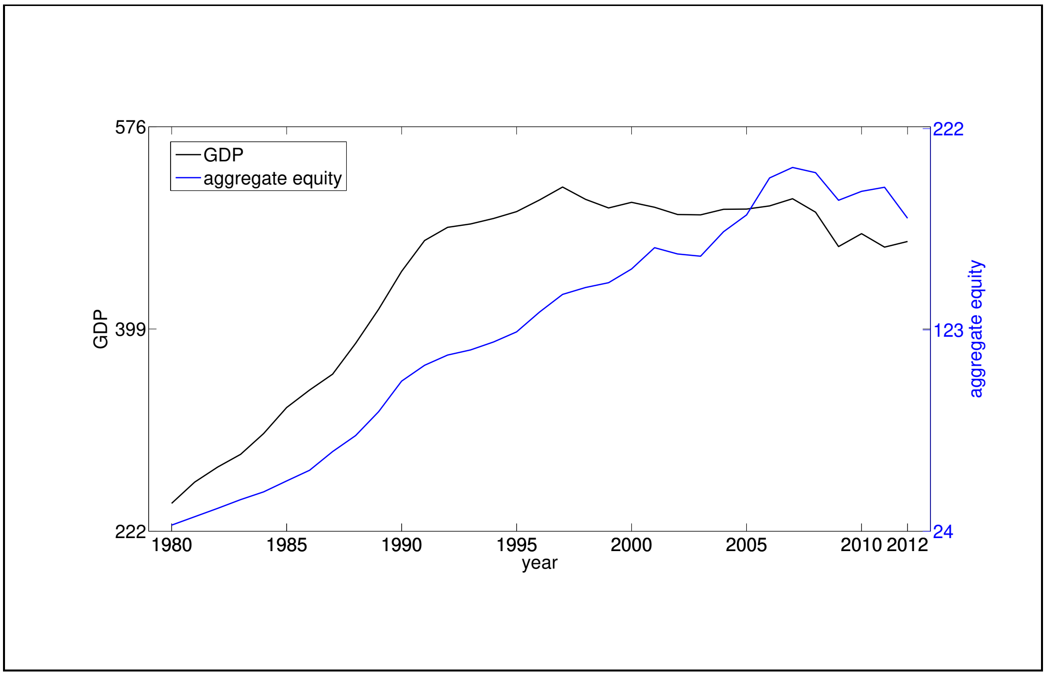

Figure 1: Aggregate equity and Japan GDP (in Trillion Yen).

3.4 Dynamic Projection of Aggregate Equity

By Equation (6), the dynamic projection of aggregate equity requires the quantification of the aggregation in micro-level firms equity and the transition matrix based on firms equity level. Specifically, at the end of year t, we have by computation the following data:

1. The number of bins $N = 10$ by setup;[3]
2. The distribution of the firms equity structure at year t: $\mathbf{fr_s[t]} = (fr[s_1, t], \ldots, fr[s_N, t])$;
3. The transition matrix computed by Equation (4) at the end of year t: $\mathbf{M[t-1]}$;
4. The average equity level for each bin $s_n[t]$, $n = 1, \ldots, N$ at year t, denoted in vector form: $\overline{\mathbf{A_S}}[\mathbf{t}] = (\overline{A_1}[t], \ldots, \overline{\mathbf{A_n}}[t], \ldots, \overline{A_N}[t])$.

Suppose "as-if" the economy, *ceteris paribus*, from period t to period $t+1$. By Equation 5, the aggregation of meso-level structural change at next year $t+1$ is estimated as: $\widehat{\mathbf{fr_s}}[\mathbf{t+1|t}] = \mathbf{fr_s[t]} \cdot \mathbf{M[t-1]}$. The aggregation in micro-level firms equity at next year $t+1$ is estimated by $\overline{\mathbf{A_S}}[\mathbf{t}]$, the average equity level for each bin

[3]In section 3.6, we have a come back to discuss the choice on the number of bins N.

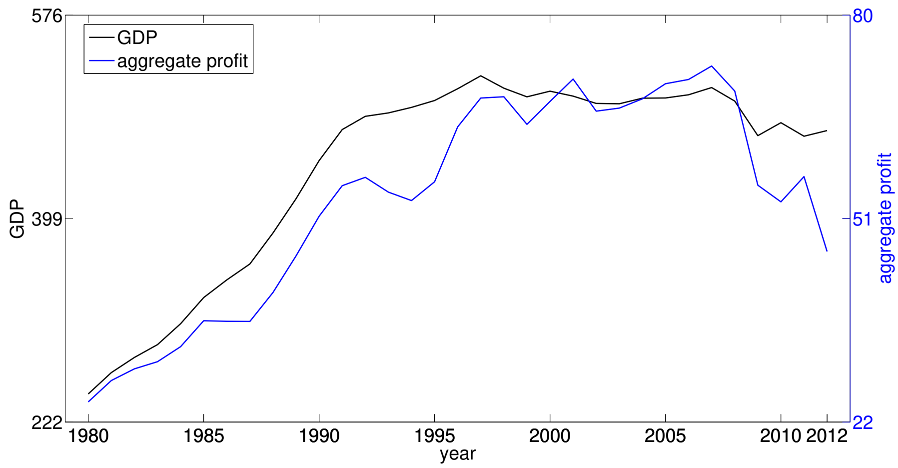

Figure 2: Aggregate profit and Japan GDP (in Trillion Yen).

at year t. With the notation $<\ ,\ >$ for scalar product, the one-period-ahead dynamic projection of aggregate equity at year t for year $t+1$ is formulated as:

$$\widehat{DP}(A)_{t+1|t} =< \widehat{\mathbf{fr_s}}[\mathbf{t+1|t}], \overline{\mathbf{A_S}}[\mathbf{t}] >=< \mathbf{fr_s}[\mathbf{t}] \cdot \mathbf{M}[\mathbf{t-1}], \overline{\mathbf{A_S}}[\mathbf{t}] > . \tag{7}$$

$\widehat{DP}(A)_{t+1|t}$ can be regarded as one-period-ahead out-of-sample forecast of the aggregate equity computed at year t for year $t+1$. To testify its forecasting performance, we split the data set into two halves, with the first half as the training set that contains 17 annual data for the year of 1980 to 1996 and the second half as the target set that contains 16 annual data for the year of 1997 to 2012. We compare the one-period-ahead out-of-sample forecast by dynamic projection with the benchmark of the classical ARIMA model on the aggregate equity at the year of 1997 to 2012.

We use the aggregate equity for the year 1980 to 1996 as the initial information set for ARIMA model. At the end of each year $t = 1996, \ldots, 2011$, use the up-to-date information set on aggregate equity. For example, at the end of year $t = 2000$, the up-to-date information set has the aggregate equity for the year 1980 to 2000. We apply two information criteria, BIC and AICc, to choose the optimal ARIMA model to conduct the one-period-ahead out-of-sample forecast on aggregate equity, denoted as:

$$\widehat{ARIMA}(A)_{t+1|t} = \{\widehat{ARIMA}(A)_{1997|1996}, \ldots, \widehat{ARIMA}(A)_{2012|2011}\}. \quad (8)$$

Then compare with the dynamic projection $\widehat{DP}(A)_{t+1|t} = \{\widehat{DP}(A)_{1997|1996}, \ldots,$ $\widehat{DP}(A)_{2012|2011}\}$, by applying Diebold-Mariano test with linear loss function (power =1) and quadratic loss function (power = 2). We construct the null hypothesis: ARIMA method has at least the same forecast accuracy as the dynamic projection on aggregate equity. The alternative hypothesis is thus: ARIMA is less accurate than dynamic projection on aggregate equity. Table 1 shows the p-value for the comparison with optimal ARIMA on BIC and on AICc information criterion with linear loss function and quadratic loss function.

Table 1: Diebold-Mariano test result on forecast of aggregate equity.

p-value	Vs. optimal ARIMA with BIC	Vs. optimal ARIMA with AICc
Power = 1	0.036	0.028
Power = 2	0.042	0.029

The p-values shown in Table 1 indicate that we reject the null hypothesis with 95% confidence, and accept the alternative hypothesis that dynamic projection for aggregate equity has higher forecasting accuracy than optimal ARIMA with BIC and AICc information criterion. As a visualization, Figure 3 shows in the upper plot the dynamics of the actual time series of aggregate equity for the year 1997 to 2012 (blue line), the dynamic projection (red line), and the optimal ARIMA with BIC information criterion (green line). It shows in the lower plot the forecast error for the dynamic projection (red line) and that for the optimal ARIMA (green line).

3.5 Dynamic Projection of Aggregate Profit

We consider the firms profit is related to the stock variable of firms equity level. By Equation (6), the dynamic projection of aggregate profit hence requires the quantification of the aggregation in micro-level firms profit and the transition matrix based on firms equity structure. Specifically, at the end of year t, we have by computation the following data:

1. The number of bins $N = 10$ by setup;[4]
2. The distribution of the firms equity structure at year t: $\mathbf{fr_s[t]} = (fr[s_1, t], \ldots,$ $fr[s_N, t])$;

[4]In section 3.6, we have a come back to discuss the choice on the number of bins N.

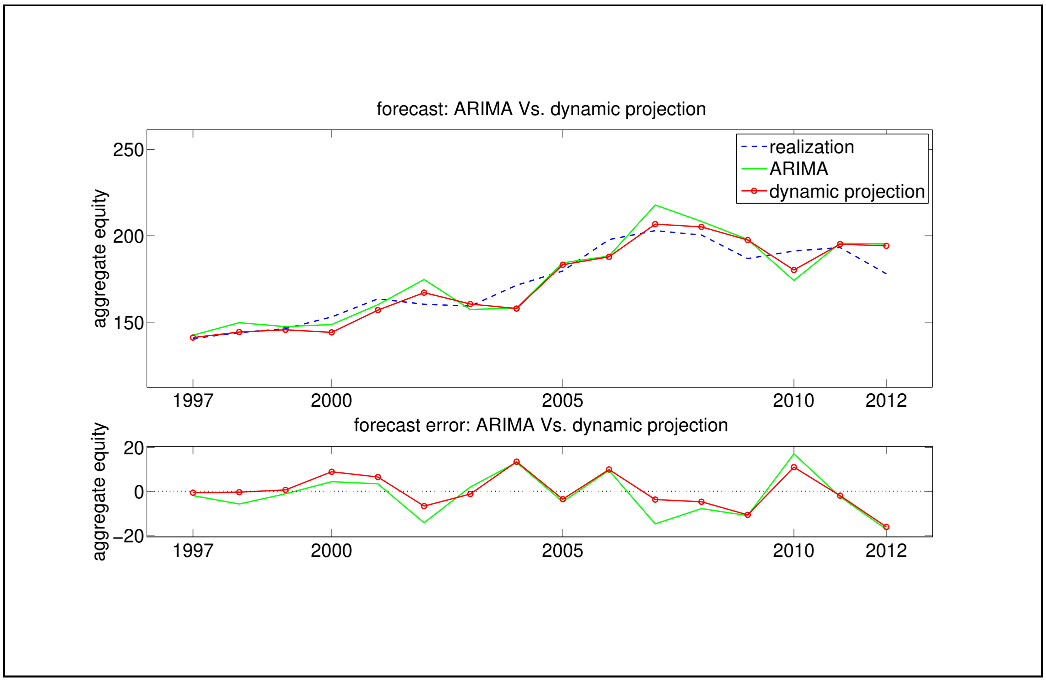

Figure 3: Forecast for aggregate equity: ARIMA Vs. dynamic projection (in Trillion Yen).

3. The transition matrix computed by Equation (4) at the end of year t: $\mathbf{M[t-1]}$;
4. The average profit for each bin $s_n[t]$, $n = 1, \ldots, N$ at year t, denoted in vector form: $\overline{\mathbf{\Pi_S}}\mathbf{[t]} = (\overline{\Pi_1}[t], \ldots, \overline{\Pi_N}[t])$.

Suppose "as-if" the economy, *ceteris paribus*, from period t to period $t+1$. By Equation 5, the aggregation of meso-level structural change at next year $t+1$ is estimated as: $\widehat{\mathbf{fr_s}}\mathbf{[t+1|t]} = \mathbf{fr_s[t]} \cdot \mathbf{M[t-1]}$. The aggregation in micro-level firms profit at next year $t+1$ is estimated by $\overline{\mathbf{\Pi_S}}\mathbf{[t]}$, the average firm's profit for each bin at year t. The one-period-ahead dynamic projection of aggregate profit at year t for year $t+1$ is formulated as:

$$\widehat{DP}(\Pi)_{t+1|t} =< \widehat{\mathbf{fr_s}}\mathbf{[t+1|t]}, \overline{\mathbf{\Pi_S}}\mathbf{[t]} >=< \mathbf{fr_s[t]} \cdot \mathbf{M[t-1]}, \overline{\mathbf{\Pi_S}}\mathbf{[t]} > . \tag{9}$$

$\widehat{DP}(\Pi)_{t+1|t}$ can be regarded as one-period-ahead out-of-sample forecast of the aggregate profit computed at year t for year $t+1$. Analogously, we testify its forecasting performance, by comparing with the benchmark of the classical ARIMA model on the aggregate profit at the year of 1997 to 2012.

We use the aggregate equity for the year 1980 to 1996 as the initial information set for ARIMA model. At the end of each year $t = 1996, \ldots, 2011$, use the up-to-date information set on aggregate profit. We also apply two information criteria, BIC and AICc, to choose the optimal ARIMA model to conduct the one-period-ahead out-of-sample forecast on aggregate profit, denoted as:

$$\widehat{ARIMA}(\Pi)_{t+1|t} = \{\widehat{ARIMA}(\Pi)_{1997|1996}, \ldots, \widehat{ARIMA}(\Pi)_{2012|2011}\}. \tag{10}$$

Then we compare with the dynamic projection $\widehat{DP}(\Pi)_{t+1|t} = \{\widehat{DP}(\Pi)_{1997|1996}$, ..., $\widehat{DP}(\Pi)_{2012|2011}\}$, by applying Diebold-Mariano test with linear loss function (power =1) and quadratic loss function (power = 2). We construct the null hypothesis: ARIMA method has at least the same forecast accuracy as the dynamic projection on aggregate profit. The alternative hypothesis is thus: ARIMA is less accurate than dynamic projection on aggregate profit. Table 2 shows the p-value for the comparison with optimal ARIMA on BIC and on AICc information criterion with linear loss function and quadratic loss function.

Table 2: Diebold-Mariano test result on forecast of aggregate profit.

p-value	Vs. optimal ARIMA with BIC	Vs. optimal ARIMA with AICc
Power = 1	0.014	0.022
Power = 2	0.027	0.033

The p-values shown in Table 2 indicate that we reject the null hypothesis with 95% confidence, and accept the alternative hypothesis that dynamic projection for aggregate profit has higher forecasting accuracy than optimal ARIMA with BIC and AICc information criterion. As a visualization, Figure 4 shows in the upper plot the dynamics of the actual time series of aggregate profit for the year 1997 to 2012 (blue line), the dynamic projection (red line), and the optimal ARIMA with BIC information criterion (green line). It shows in the lower plot the forecast error for the dynamic projection (red line) and that for the optimal ARIMA (green line).

3.6 Sensitivity Analysis on Number of Bins

The choice of the number of bins may influence the performance of dynamic projections. To analyze, we employ different values for the number of bins $N = 2, \ldots, 13$, compute the one-period-ahead out-of-sample dynamic projection for aggregate equity and aggregate profit for each N, then use Diebold-Mariano test to compare with the performance of optimal ARIMA models under BIC and AICc information criterion. The alternative hypothesis is: ARIMA is less accurate than dynamic projection.

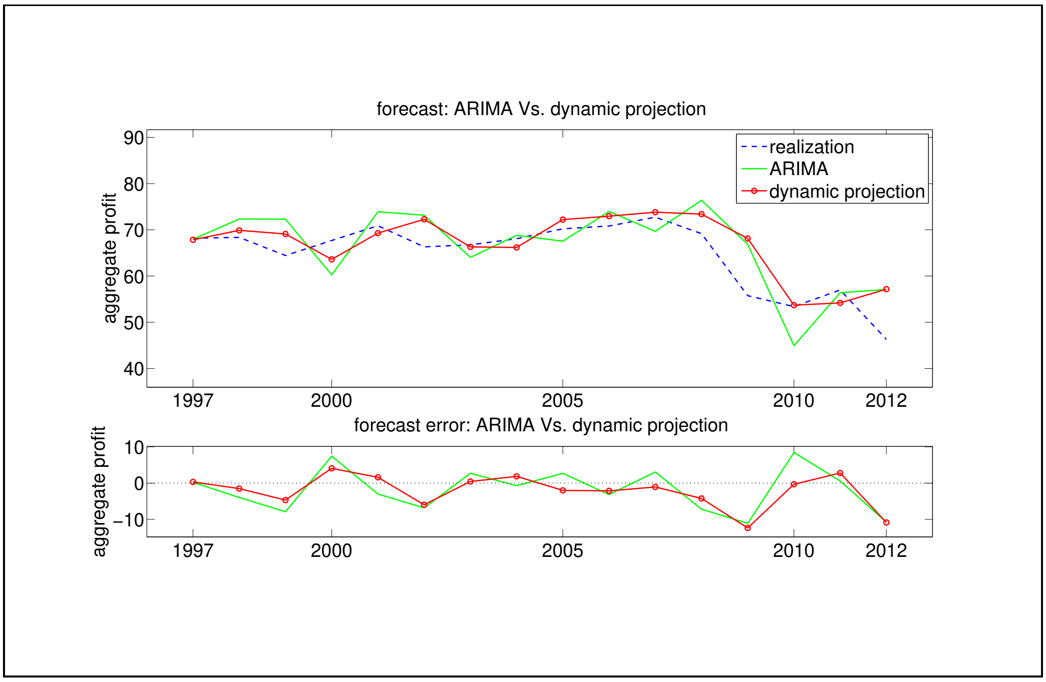

Figure 4: Forecast for aggregate profit: ARIMA Vs. dynamic projection (in Trillion Yen).

Table 3 and Table 4 show p-values of Diebold-Mariano test with linear loss function (power =1) and quadratic loss function (power = 2), for $N = 2, \ldots, 7$ and $N = 8, \ldots, 13$ respectively.

Table 3: Diebold-Mariano test results with number of bins $N = 2, \ldots, 7$

	number of bins	2	3	4	5	6	7
aggregate equity	BIC, power = 1	0.049	0.045	0.024	0.034	0.037	0.056
	BIC, power = 2	0.087	0.069	0.036	0.059	0.041	0.092
	AICc, power = 1	0.040	0.038	0.018	0.027	0.029	0.040
	AICc, power = 2	0.065	0.051	0.025	0.042	0.029	0.065
aggregate profit	BIC, power = 1	0.007	0.014	0.007	0.015	0.017	0.021
	BIC, power = 2	0.016	0.015	0.006	0.018	0.021	0.033
	AICc, power = 1	0.008	0.020	0.012	0.022	0.027	0.033
	AICc, power = 2	0.019	0.019	0.008	0.022	0.027	0.040

These two tables indicate that, for number of bins $N = 2, \ldots, 12$, we can generally reject the null hypothesis with 90% confidence, and accept the alternative hypothesis that dynamic projection has higher forecasting accuracy than optimal

Table 4: Diebold-Mariano test results with number of bins $N = 8, \ldots, 13$

	number of bins	8	9	10	11	12	13
aggregate equity	BIC, power = 1	0.023	0.074	0.036	0.049	0.038	0.185
	BIC, power = 2	0.039	0.109	0.042	0.062	0.048	0.175
	AICc, power = 1	0.016	0.055	0.028	0.037	0.025	0.117
	AICc, power = 2	0.026	0.078	0.029	0.044	0.032	0.109
aggregate profit	BIC, power = 1	0.010	0.033	0.014	0.042	0.041	0.061
	BIC, power = 2	0.010	0.056	0.027	0.091	0.039	0.060
	AICc, power = 1	0.016	0.045	0.022	0.061	0.061	0.085
	AICc, power = 2	0.014	0.065	0.033	0.104	0.047	0.070

ARIMA with BIC and AICc information criterion. The only two exceptions are on the scenario of $N = 9$ for aggregate equity, against optimal ARIMA models under BIC criterion, where Diebold-Mariano test under quadratic loss function (power = 2) gives the p-value = 0.109; and on the scenario of $N = 11$ for aggregate profit, against optimal ARIMA models under AICc criterion, where Diebold-Mariano test under quadratic loss function (power = 2) gives the p-value = 0.104.

We may argue a potential explanation on the poor performance for $N = 9$ and $N = 11$ from the perspective of *round-off error*. For instance, for $N = 9$, we need to construct the bins $s_n[t]$ for $n = 1, \ldots, 9$ by Equation (1), where $s_n[t]$ equally contains $\frac{1}{9}$ population of firms. In numerical computation, $\frac{1}{9}$ can only be approximated by floating-point number with certain digits of rounding, i.e. $\frac{1}{9} \approx 0.1111$ under 4-digits of chopping or $\frac{1}{9} \approx 0.11$ under 2-digits of chopping. In any case of chopping, we face the round-off error for $N = 9$, which might not have insignificant impact on the computation. Similar situation happens for $N = 11$.

Another observation from these two tables is that, for number of bins $N = 13$, the highest p-value is $0.185 > 0.10$ for aggregate equity, while the highest p-value is $0.085 > 0.05$ for aggregate profit. It shows no strong support with statistical significance that dynamic projection has higher forecasting accuracy than optimal ARIMA with BIC and AICc information criterion. This indicates that increasing the number of bins does not necessarily lead to better performance of dynamic projection. We conduct another tests with higher number of bins $N = 14, \ldots, 17$, which gives us the highest p-value larger than 0.10 in general, with some scenarios even larger than 0.30.

These findings suggest us to select $N \in \{2, \ldots, 12\}$. Moreover, we prefer less impact from round-off errors on computation, which suggests choosing N that less likely generates round-off errors, i.e. $\frac{1}{N}$ can be exactly represented by floating-point number under finite digits. In this regard, we suggest a rule of thumb of choosing $N = 2, 4, 5, 8, 10$ for dynamic projection.

4 Concluding Remark

Employing the viewpoint that economic system can be regarded as multi-level dynamical system, we have developed in this paper a quantification on the meso-level structural change by applying the concept of transition matrix with appropriate partition on the micro-level state of the economic system. We have utilized this method of quantification to develop a technique of dynamic projection that can be used to compute out-of-sample forecasting of macro-level economic variable from micro-level Big Data. We have shown this technique of dynamic projection gives us potential to project the future state of macro-level economic variable of aggregate equity and aggregate profit from the data set of firms listed in Tokyo Stock Exchange for the year of 1980 to 2012. In the coming era of Big Data, this technique can be regarded as the building block for a Central Guidance System (CGS) to monitor and to guide the direction of the economy in a real-time manner.

In this paper, we have developed the technique of dynamic projection without the consideration of network topology among economic agents in the system. Since network structure can be represented by adjacency matrix that shares similarity to transition matrix, it is reasonable to believe the technique of dynamic projection can be applied in this scenario, with the inclusion of adjacency matrix with transition matrix, which is left as one of the future tasks in our research agenda.

By our research, it seems that our construction of transition matrix works as an appropriate measurement and quantification on the meso-level structural change. It suggests, behind this finding, there might exist mathematical foundation for supporting and guiding us on developing technique on quantifying meso-level structural change. Another future task is concerned with exploring the mathematical foundation in this sense.

In our work, we have shown the number of bins N that defines the dimension of the transition matrix plays an important role in the technique of dynamic projection. By statistical test, we have conducted sensitivity analysis of this key parameter on the performance of dynamic projection and have demonstrated the rule of thumb on how to choose N. We expect a more comprehensive analysis from mathematical perspective on this topic in our future study, in relation with information theory and statistical mechanics.

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