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Knowing me, imagining you: Projection and overbidding in auctions

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Abstract

People overestimate the probability that others share their values or preferences. I introduce type projection equilibrium (TPE) to capture such projection in Bayesian games. TPE allows each player to believe his opponents share his type with intermediate probability ρ. After establishing existence, I address my main question: How does projection affect behavior in games? I analyze auctions and distribution games. In auctions, projection implies an increased sense of competition, which induces overbidding in all (first-price) auctions. In addition, it biases the perceived value distribution, which induces cursed bidding in common value auctions. Thus, projection induces a hitherto neglected bias in bidding. It is novel in that it explains behavior across conditions and it is independently founded in psychology. I test projection equilibrium in multiple ways on existing data and find that it substantially improves on alternative concepts, both in isolation and in addition to them. The findings are cross-validated by testing projection of social preferences in distribution games.

JEL–Codes: C72, C91, D44
Keywords: auctions, overbidding, winner's curse, projection, risk aversion, cursed equilibrium, level-*k*, social preferences

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1 Introduction

The false consensus bias is the tendency to assume that one's own opinions, preferences and values are typical and shared by others. Following its original observation of Ross et al. (1977), this form of "projection" has been confirmed in a variety of experiments, as Mullen et al. (1985) show in a meta-analysis. Projection may persist even if subjects are provided with factually contradicting information (Krueger and Clement, 1994). Thus, projection is of intuitive relevance in all choices under incomplete information—not just in the non-strategic environments on which the psychological literature traditionally focuses, but also in strategic interactions. Current concepts studying projection in "games" focus on one-sided incomplete information. In their seminal paper, Loewenstein et al. (2003) study projection of utility onto future selves, finding that it explains anomalies in purchases of durable goods. In a different context, Madarász (2012) studies projection of information from an informed player to an uninformed one, which explains the hindsight bias in agency problems.

The purpose of the present paper is to study projection of generic "types" in games with two-sided incomplete information. Types may capture social preferences or object valuations. I set up a simple model of projection of types in Bayesian games where players may overestimate the probability that their opponents share their type—ranging from the special cases of zero projection (the original Bayesian case) to full projection (disregarding all prior information).[1] The degree of projection is denoted by $\rho \in [0,1)$. In equilibrium, players know their opponents' strategies but compute their expected payoffs with a projection bias. For each ρ, a corresponding type projection equilibrium is shown to exist. In the analysis, I estimate the degree of projection ρ to be around 0.5 in simple distribution games and auctions, and find that allowing for projection better describes and predicts behavior in these games. Aside from explaining behavior, the relevance of projection also has policy implications, as the projection bias is reduced when subjects are educated explicitly (Engelmann and Strobel, 2012).

My main application is the analysis of auctions with players who project their values. Object values are personal traits, and thus intuitively projected in general, and their projection has been observed explicitly in bargaining (Bottom and Paese, 1999; Galinsky

[1] The case of full projection is regularly considered in analyses of social preferences. The present paper provides a conceptual framework allowing the analyst to capture the more intricate case of imperfect projection. This is critical for several reasons. First, full projection is rarely observed in psychology. Second, I will estimate imperfect degrees of projection in auctions (where full projection does not fit), and third, imperfect projection also explains behavior under social preferences, as shown below.

and Mussweiler, 2001) and consumption decisions (Frederick, 2012; Kurt and Inman, 2013). The basic intuition for projection in auctions is simple. When bidding on say a house, projecting bidders neglect competitors whose values are vastly inferior, against whom they surely win, and competitors whose values are vastly superior, against whom they surely lose. Rather, they focus on competitors with similar values, trying to ensure winning against them. This focus on close competitors is a form of the projection bias—implicitly, the bidders overestimate the probability that competitors have values similar to their own. It increases the sense of competition, which induces overbidding in first-price auctions and explains the loser regret observed by Filiz-Ozbay and Ozbay (2007). In addition, the biased perception of the value distribution obscures the inference of the object value in common value auctions, which "curses" one's estimates and yields the Winner's Curse.

The increased sense of competition also relates to risk aversion in private value auctions, but the projection bias is special in that it induces a similar sense of competition in common value auctions, on top of the cursed value perception. Thus, the projection bias provides a unified explanation of overbidding in both information conditions. This is interesting, as differences between information conditions may be rather intransparent to both bidders and analysts. For example, the differences are small between affiliated private values and common values. If one believes that the behavioral source of overbidding does not change between these fairly similar regimes, then the unified explanation by the independently observed projection bias is promising. From this point of view, the research question may be refined as follows: Knowing that people project their values, which aspects of behavior does projection explain in auctions, and which aspects are to be explained differently, by say risk aversion or cursedness (Eyster and Rabin, 2005)? The short answer is that projection equilibria seem to fit behavior fairly well and alternative models do not seem to add significantly to its explanatory content.

I test projection equilibria in many ways, by evaluating the testable predictions derived in the theoretical analysis and by a structural analysis evaluating its quantitative fit to individual choices. I allow for a variety of behavioral assumptions including non-equilibrium beliefs and subject heterogeneity, and test the adequacy of projection to describe, predict, and infer from behavior. The main findings are that projection fits about similarly well as risk aversion in private values auctions, about as well as cursed equilibrium in common value auctions, but overall it describes and predicts behavior better than these concepts (also in combination). That is, a specific degree of projection, estimated to be around $\rho \approx 0.5$, captures behavior consistently across conditions. In itself it explains around 65% of observed variance. Additionally accounting for risk aversion does not improve the descriptive or predictive adequacy significantly, but accounting for subject heterogeneity improves the model adequacy by around 10 percentage points.

I complement the findings by examining imperfect projection ($\rho = 0.5$) of social preferences. Relatedly, projection of guilt aversion was observed by Bellemare et al. (2011), and projection of inequity aversion was observed by Blanco et al. (2014). In addition, I show that projection allows to predict within-subject changes of behavior, using the data of Blanco et al. (2011). In their experiment, each subject played six games, the individual degrees of envy and guilt were calibrated using two of these games and used to predict behavior in the remaining four games (assuming either equilibrium or rational expectations). Blanco et al. (2011) found that these predictions did not correlate with the actual choices. Blanco et al. (2014) show that in simple sequential-move games, projection of both strategies and preferences can be observed. Using type projection equilibrium,[2] I show that predictions using intermediate values of ρ do correlate with behavior and thus help explain the apparent behavioral changes.

I conclude that the projection bias matters in games with incomplete information, which extends existing evidence from psychology. At least in the games analyzed here, behavior is better described and predicted by considering projection. Since understanding behavior in distribution games and auctions is of independent interest, as large stocks of literature have been devoted to analyses of behavior in these games, the result that the projection bias organizes much of the variance still remaining in these games is a positive finding. In turn, as the projection bias may be reduced by explicit information of subjects, the finding that projection helps explaining observed behavior leads to potential policy implications that are discussed in the conclusion.

Section 2 defines type projection equilibrium and relates it to existing literature. Section 3 derives theoretical predictions for auctions. Section 4 describes the data. Section 5 examines the testable implications of projection equilibrium using these data and Section 6 analyzes the adequacy to capture individual behavior. Section 7 concludes. There is comprehensive supplementary material with further analyses.

[2]Note that Blanco et al. (2011) consider either simultaneous-move games or games with strictly sequential moves where each player moves once. Projection equilibrium can be applied straightforwardly.

2 Type projection equilibrium

2.1 Definition

Type projection equilibrium extends Bayesian Nash equilibrium by incorporating the projection bias in a stylized manner. To simplify notation, I focus on games where all players are ex-ante symmetric in the sense that their type spaces are identical, but generalizations are possible. My formulation does not require "types" to represent preference parameters, but the idea of projection equilibrium may appear particularly natural if the reader wishes to keep this interpretation in mind.

Let $\Gamma = \langle N, (A_i)_{i\in N}, T_0, (T_i)_{i\in N}, p, (u_i)_{i\in N}\rangle$ denote a Bayesian game, including the set of players N, and sets of actions A_i and types T_i for each player $i \in N$. The set of Nature's types is T_0, the set of type profiles is $T = T_0 \times T_1 \times \cdots \times T_n$, and the probability distribution over T is denoted by p. Types may be correlated. The marginal distribution on T_i is denoted by p_i, and action profiles are denoted by $a \in A = A_1 \times \cdots \times A_n$.

Definition 1. A Bayesian game Γ is called **finite** if N, A, T are finite. It is called **type-symmetric** if $T_i = T_j$ and $p_i = p_j$ for all $i, j \in N$.

Given action profile $a \in A$ and type profile $t \in T$, i's utility is $u_i(a,t)$. The strategy $\sigma_i(\cdot|t_i) \in \Delta A_i$ of i maps i's actions to probabilities contingent on type t_i. As usual, $A_{-i} = \times_{j\in N\setminus\{i\}} A_j$ and $T_{-i} = \times_{j\in N\cup\{0\}\setminus\{i\}} T_j$, but slightly abusing notation, define $\sigma_{-i}(a_{-i}|t_{-i}) = \prod_{j\neq i} \sigma_j(a_j|t_j)$ as the *product* of the probabilities. The expected utility of type $t_i \in T_i$ from action a_i in response to σ_{-i} may thus be written as

$$\pi_i\big(a_i|t_i,\sigma_{-i}\big) = \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} p(t_{-i}|t_i)\; u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big]\, \sigma_{-i}(a_{-i}|t_{-i}).$$

A strategy profile $\sigma = (\sigma_1,\ldots,\sigma_n)$ is a Bayesian Nash equilibrium (BNE) if all types $t_i \in T_i$ of all players $i \in N$ put positive weight only on optimal actions,

$$\sigma_i(\cdot|t_i) \in \arg\max_{\sigma'_i\in\Delta A_i} \sum_{a_i\in A_i} \sigma'_i(a_i)\; \pi_i\big(a_i|t_i,\sigma_{-i}\big).$$

Type projection equilibrium extends BNE by allowing players to overestimate the probability that their opponents are of the same type as they are. This biases their beliefs about types. Formally, player $i \in N$ assigns weight $1-\rho$, $\rho \in [0,1)$, to their prior information that the types adhere to the prior p and weight ρ to their projection that all players' types are equal to i's type. The parameter ρ is called *degree of projection*. Due to the projection bias, players believe that their opponents' strategy profile is summarized as

$$\tilde{\sigma}_{-i}(a_{-i}|t_{-i},t_i) = (1-\rho)\prod_{j\neq i}\sigma_j(a_j|t_j) + \rho\prod_{j\neq i}\sigma_j(a_j|t_i) \tag{1}$$

rather than $\sigma_{-i}(a_{-i}|t_{-i})$. A detailed discussion in relation to existing concepts and psychological evidence follows shortly, but briefly, let me discuss two assumptions inherent in this formulation. On the one hand, players project their types onto *all* their opponents simultaneously. Alternatively, one might assume that players project their type independently onto their opponents. The resulting predictions appear to be qualitatively rather similar. Besides its analytical tractability, I assume correlated projection, as it corresponds with the observations of Camerer et al. (2004) and Costa-Gomes et al. (2009) that experimental subjects tend to believe their opponents make correlated choices. On the other hand, players project their exact type. In Bayesian games with ordered type sets, an intuitive alternative might be that players project by putting extra weight on the belief that the opponents are of similar types, i.e. of types that are proximate to theirs under the assumed ordering. I choose the simpler model of exact projection, as it equally applies to environments with both ordered and unordered type sets (an example of the latter will be the projection of social preferences), and as it avoids measures of the degree of similarity between types.

The type projection equilibrium is defined as the BNE in response to $\tilde{\sigma}_{-i}$. That is, as in every equilibrium concept, players anticipate their opponents' strategies σ_{-i}, but due to their projection bias, they use $\tilde{\sigma}_{-i}$ when computing expected payoffs.

Definition 2. For any $\rho \in [0,1)$, a strategy profile $\sigma = (\sigma_1,\ldots,\sigma_n)$ is a ρ-type projection equilibrium (ρ-TPE) if for all players $i \in N$ and all types $t_i \in T_i$,

$$\sigma_i(\cdot|t_i) \in \arg\max_{\sigma'_i\in\Delta A_i}\sum_{a_i\in A_i}\sigma'_i(a_i)\,\pi_i\big(a_i|t_i,\tilde{\sigma}_{-i}\big).$$

Existence can be established as any ρ-TPE of Γ is a Bayesian Nash equilibrium of an augmented game $\tilde{\Gamma}$ where the projected events are possible draws by Nature. This relates to the argument establishing existence of cursed equilibrium in Eyster and Rabin (2005).

Proposition 1. *If Γ is a finite and type-symmetric Bayesian game and $\rho \in [0,1)$, a ρ-type projection equilibrium of Γ exists.*

Proof. See appendix. □

2.2 Relation to the theoretical literature

Type projection incorporates a projection bias similar to the one defined by Loewenstein et al. (2003). They consider a decision maker predicting his own utility in future states of the world. Given consumption c and current state s', the decision maker predicts the utility will be $(1-\alpha)\,u(c,s)+\alpha u(c,s')$, $\alpha \in [0,1]$ in alternative state s. In many games of interest, types and utilities are related and then a type-projecting player predicting utilities of her opponents would predict very similarly to the utility-projecting players defined by Loewenstein et al. (2003). Frequently, the relationship between types and utilities is even linear, and in such cases a type-projecting player would predict the same utilities as a utility-projecting one. The difference materializes in Bayesian games. Since players understand that they respond to a mixture of types, each of which plays a distinct strategy, a type-projecting player anticipates *mixed strategies* of each opponent's type—with probability $1-\rho$ the Bayesian type plays and with probability ρ the projected type plays. In turn, utility projection as in Loewenstein et al. (2003) implies that players believe their opponents' types have "average" utilities and play pure strategies each.[3]

Type projection also relates to information projection (Madarász, 2012, 2013). An information projecting player believes that his opponents know all he knows, in addition to their existing knowledge. In an auction, for example, information projection implies that the opponents know my values, in addition to knowing their own values. Type projection assumes instead that the opponents share i's type. Arguably, information projection appears to be less appealing in the context of say auctions, while it appears appealing in cases of one-sided incomplete information, and it provides an intriguing explanation of the hindsight bias.

Another concept related to type projection is cursed equilibrium (Eyster and Rabin, 2005). Similarly to TPE, cursed equilibrium is a solution concept for Bayesian games introducing a subjective type distribution differing from the objective one. The difference is that a type projecting player projects his own type onto his opponents, while

[3] Also note the difference to pure strategy projection: A type projecting player projects what he would play in his opponents' shoes, assuming all are of his type but they keep their individual incentives. In contrast, both utility projection and strategy projection implicitly assume the opponents neglect their original incentives and adopt my utilities or use my strategies.

a cursed player projects a random type. Both models distort the correlation between opponents' actual types and their strategies, which distorts the information that can be inferred from their actions. Thus, both concepts predict distorted inference in common value auctions, implying a Winner's Curse. Type projection affects behavior beyond information inference, however, e.g. by inducing loser regret in first-price auctions. Aside from behavioral implications, type projection seems to more closely implement the projection bias as observed in the psychological literature, that people project their own traits or opinions. Such evidence usually draws from interactions with ex-ante symmetric type sets. In turn, cursed equilibrium appears more appropriate to capture beliefs if type sets are asymmetric, as result of which projection of the own type appears less intuitive. Market interactions with one-sided incomplete information as analyzed in Eyster and Rabin (2005) appear to be a prototypical example of a Bayesian game that is more intuitively captured by cursed equilibrium than by type projection equilibrium.

A concept inverting the idea of cursed equilibrium is the level-k model as applied to auctions by Crawford and Iriberri (2007). Contrary to cursed equilibrium, where strategies are correct but projected types are random, level-k assumes types are correct but strategies are random. The predictions are rather similar. Players at level 1 assume their opponents randomize uniformly, which again weakens the amount of information contained in the opponents' bids and thus induces cursed bidding in standard common value auctions. Since level 1 is contained as a special case in many models of asymmetric beliefs about strategies, e.g. in cognitive hierarchy (Camerer et al., 2004), noisy introspection (Goeree and Holt, 2004), and asymmetric logit equilibrium (Weizsäcker, 2003), the results of Crawford and Iriberri suggest that belief asymmetry as such may well affect behavior in Bayesian games. The purpose of this paper is to investigate the comparative relevance of these concepts with respect to explaining behavior in Bayesian games with ex-ante symmetric type sets. In these games, I hypothesize type projection to be of relevance.

Question 1. *To which degree is behavior in type-symmetric Bayesian games explained by type projection, cursedness, belief asymmetry, and risk aversion?*

A distinctive feature of level-k models is their assumption of discrete subject types. Thus, as a test for the level-k model, and to be able to adequately model subject heterogeneity, I will also address the following, more specific question.

Question 2. *In auctions, is subject heterogeneity discrete?*

2.3 Empirical evidence on false consensus and projection

False consensus and projection bias are used largely synonymously to describe the tendency to assume that the own opinions, preferences and values are shared by others. Ross et al. (1977) present evidence on projection with respect to both, choices in everyday decision making and individual characteristics (such as personal problems, expectations, and preferences). They show that subjects' beliefs about others' choices correlate with their own choices, and their beliefs about the others' characteristics correlate with their own. An early meta-analysis illustrating the robustness of projection is provided by Mullen et al. (1985), showing that its magnitude is largely invariant with respect to the generality of the reference population. However, projection is strongest in relation to people similar to oneself (Clement and Krueger, 2002). Projection appears to have evolved, as it is helpful in predicting characteristics of other subjects. Hoch (1987) shows that the majority of subjects would actually improve their predictive accuracy if they weighted their own positions even stronger, as they seem to face difficulties in using available information about the targets. Krueger and Clement (1994) show that the projection bias persists even if subjects are provided with factually contradicting information, which is called the truly false consensus effect. Engelmann and Strobel (2000, 2012) qualify this result, showing that the extent of truly false consensus depends on the kind and clarity of information provided to the subjects.

There is ample evidence of projection both within individuals and between individuals. On the one hand, individuals tend to project their current preferences on their future selves. For example, Gilbert et al. (1998) show that subjects overestimate the duration of affective reactions to negative events, Read and Van Leeuwen (1998) show that individuals project their current state of appetite when ordering meals in advance, and analyzing catalog orders, Conlin et al. (2007) show that individuals underestimate how tastes change over time.[4] In their seminal theoretical study of utility projection, Loewenstein et al. (2003) show that it also explains phenomena such as overspending early in life and misguided purchases of durable goods.

On the other hand, projection is similarly strong between individuals, even in anonymous interactions such as games played in laboratory experiments. In such environments, subjects appear to project their own choices and preferences. That is, subjects tend to consider their preferences and choices to be more common than they actually

[4]Further, Simonsohn (2010) shows that preference projection affects college enrollment decisions, via the cloud cover observed on visiting day, Grable et al. (2004), Grable et al. (2006), and Kliger and Levy (2008) show that projection in reaction to stock market price changes explain investment decisions.

are.[5] Relatedly, Blanco et al. (2011) analyzed the within-subject stability of inequity aversion (Fehr and Schmidt, 1999, FS) using experimental data with six observations per subject from different games. They estimate individual FS preferences based on two of the six decisions per subject and use these estimates to predict the subject's remaining four decisions (assuming rational expectations about preferences or choices of other subjects). These predictions did not significantly correlate with the actual choices. In one of the games, however, Blanco et al. observed that projection potentially affects behavior. Namely, in a sequential Prisoner's Dilemma, subjects' choices as first mover can be predicted based on their own choices as second mover. In order to verify its robustness, Blanco et al. (2014) test the predictiveness of own second-mover choices for own first-mover choices in a larger variety of sequential social dilemmas. Their new results indicate evidence of both, strategy projection and preference projection. In order to verify the relevance of projection with respect to their first paper (Blanco et al., 2011), I re-analyzed their data allowing for either strategy projection or preference projection. The results, provided as supplementary material, indicate that both forms of projection indeed allow to successfully predict individual behavior across games, confirming projection also in these cases.

Preference projection as observed in the literature can be directly captured by type projection equilibrium in Bayesian games. Laboratory games are played under anonymity and preferences are private information. In a Bayesian game, one's type may characterize one's preferences, and preference projection is therefore a particular instance of type projection in games where (social) preferences are behaviorally relevant. Regarding type projection in the case of auctions, the underlying assumption is that subjects project their object valuation, exaggerating the probability that other subjects value the object similarly. There also exists explicit evidence of precisely this form of projection. Frederick (2012) and Kurt and Inman (2013) show that estimates of others' willingness to pay are highly correlated with the respondents' own willingness.[6] This evidence implies the hypothesis that projection also affects bidding in auctions. To my knowledge, there is but one paper linking projection and bidding, namely Engelmann and Strobel (2012), who mention the idea in their discussion. In addition, Güth and Ivanova-Stenzel (2003) experimentally analyze auctions and observe that behavior is

[5]For example, Messé and Sivacek (1979) observe strategy projection in the one-shot Prisoner's Dilemma and Offerman et al. (1996) observe it in public goods games. Iedema and Poppe (1995) and e.g. Aksoy and Weesie (2012) show that subjects project their social value orientation, and Bellemare et al. (2011) find preference projection with respect to guilt aversion.

[6]In addition, they show that the estimates are biased upward, which however is less commonly reported. Similarly, Bottom and Paese (1999) and Galinsky and Mussweiler (2001) show that subjects tend to use their own reservation price when they estimate others' reservation prices in negotiations.

largely invariant with respect to the subjects' knowledge of the distributions of values. In their experiment, subjects do not seem to use information on the distribution of values effectively, which also indicates the existence of a projection bias.

3 Type projection in auctions

3.1 Projection bias and incentives in bidding

Initially, I focus on auctions with either affiliated private values (APV) or common values (CV). The case of independent private values (IPV) is qualitatively similar to APV in many ways, but the notation of mixed strategies needs to be modified, obfuscating a joint discussion. The notation is standard. A player gets a signal denoted by x. The expectation of the object value conditional on the signal x is denoted by $v(x)$, its expectation conditional on both the own signal x and the highest opponent signal y is denoted by $v(x,y)$. Note that the notation does not yet entail a restriction of the information condition toward either APV or CV. The density of the highest opponent signal y conditional on my signal x is $f_Y(y|x)$. A pure strategy $b_\star$ is a continuous, monotonic function mapping signals x to bids $b \in \mathbb{R}$. The expected payoff of bidding $b \in \mathbb{R}$, conditional on my signal x and in response to opponents bidding function $b_\star$, is

$$\Pi(b|b_\star,x) = E\left[(V_i - b)\, I_{b_\star(Y)<b)}|X_i = x\right] = \int_{\underline{x}}^{b_\star^{-1}(b)} \big(v(x,y) - b\big) f_Y(y|x)\, dy.$$

The symmetric BNE solves $b = b_\star(x)$. In contrast, assume the player in question projects his type x onto his opponents, as defined above, with degree ρ. Now, the expected payoff of b conditional on signal x depends on the relation of b and $b_\star(x)$. Define one's share of the prize as $s = 0$ if $b < b_\star(x)$, $s = 1/n$ if $b = b_\star(x)$, and $s = 1$ if $b > b_\star(x)$. The expected payoff is a weighted sum of objective and projected payoff,

$$\Pi_\rho(b|b_\star,x) = (1-\rho)\int_{\underline{x}}^{b_\star^{-1}(b)} \big(v(x,y)-b\big) f_Y(y|x)\, dy + s\cdot\rho \int_{\underline{x}}^{\overline{x}} \big(v(x,y)-b\big) f_Y(y|x)\, dy,$$

where the second summand captures the case of projection. Note that in the case of projection, the payoff does not depend on the opponents' signal or their strategies beyond their implications with respect to s. The implications of projection can be dissected into two components, loser regret and cursed value perception, as follows.

Loser regret Let $w(b|x,b_\star)$ denote the probability of winning without projection, and $w_\rho(b|x,b_\star)$ denote the respective probability with ρ-projection. Again using the "projection share" $s=0$ if $b<b_\star(x)$, $s=1/n$ if $b=b_\star(x)$, and $s=1$ if $b>b_\star(x)$,

$$w(b|x,b_\star) = \int_{\underline{x}}^{b_\star^{-1}(b)} f_Y(y|x)\,dy,$$

$$w_\rho(b|x,b_\star) = (1-\rho)\int_{\underline{x}}^{b_\star^{-1}(b)} f_Y(y|x)\,dy + s\cdot\rho\int_{\underline{x}}^{\overline{x}} f_Y(y|x)\,dy.$$

If the projecting player bids less than opponents with the same signal, $b<b_\star(x)$, he underestimates the probability of winning, as $w_\rho(b)=(1-\rho)\,w(b)+\rho\cdot 0$ is then less than the objective probability $w(b)$. In turn, if he outbids opponents with the same signal, he overestimates the probability of winning, as $w_\rho(b)>w(b)$ results then. In the latter case, the projecting player ensures that he wins against opponents with the same signal, the probability of which he exaggerates. This induces an incentive to outbid opponents with the same signal, under all information conditions. These incentives resemble *loser regret* (Filiz-Ozbay and Ozbay, 2007), i.e. to feel regret if a higher bid would have won the auction profitably. Projecting players act as if they felt "conditional loser regret", i.e. regret if a higher bid would have won the auction against opponents with the same valuation. The technical differences appear minor, as loser regret materializes only if the opponents' values are similar to mine. Thus, I will say that projection induces loser regret as observed by Filiz-Ozbay and Ozbay (2007).[7]

Cursed value perception Let $\tilde{v}(x|b,b_\star)$ denote the object value conditional on winning without projection, and let $\tilde{v}_\rho(x|b,b_\star)$ denote the respective value with ρ-projection.

$$\tilde{v}(x|b,b_\star) = \frac{\int_{\underline{x}}^{b_\star^{-1}(b)} v(x,y)\,f_Y(y|x)\,dy}{\int_{\underline{x}}^{b_\star^{-1}(b)} f_Y(y|x)\,dy}$$

$$\tilde{v}_\rho(x|b,b_\star) = \frac{(1-\rho)\int_{\underline{x}}^{b_\star^{-1}(b)} v(x,y)\,f_Y(y|x)\,dy + s\cdot\rho\int_{\underline{x}}^{\overline{x}} v(x,y)\,f_Y(y|x)\,dy}{(1-\rho)\int_{\underline{x}}^{b_\star^{-1}(b)} f_Y(y|x)\,dy + s\cdot\rho\int_{\underline{x}}^{\overline{x}} f_Y(y|x)\,dy}$$

If one outbids opponents with the same signal, i.e. if $b>b_\star(x)$, the expected object value under projection is a weighted average of conditional and unconditional value. In

[7]Note that the projected probability of winning is discontinuous in b if the opponents play a pure strategy. It jumps at $b=b_\star(x)$ where one "overtakes" opponents with the same signal. The discontinuity will disappear once we allow for mixed strategies, but the incentive to slightly outbid opponents with similar values is robust to allowing for mixed strategies.

this case, the projected expectation is equal to the expectation under cursedness (Eyster and Rabin, 2005). Alternatively, if $b < b_\star(x)$, the projected expectation equates with the actual expectation. That is, the projected expectation is biased only if one outbids opponents with the same value. In standard common value auctions, the bias is an upward bias, i.e. the object value is overestimated. Thus, the projected expectation exhibits an upward jump at $b = b_\star(x)$, but once we allow for mixed strategies, the transition will be smooth again. Besides inducing cursed object valuations, this increment of the expectation adds to the loser regret discussed above. Thus, the incentives of projecting players to outbid opponents with the same signal are particularly strong in common value auctions. On a qualitative basis, type projection therefore predicts that if we hold the degree of projection constant, overbidding occurs in both information conditions, but the normalized degree of overbidding (suitably defined) is larger in common value auctions than in private value auctions.

Question 3. *Is the degree of overbidding larger in common value auctions than in private value auctions?*

Such a comparative prediction is not implied by existing concepts such as cursedness, level-k, or risk aversion. In the canonical models of common value auctions and private value auctions, cursedness and level-k predict overbidding only for common values, while risk aversion predicts overbidding only for private values. Thus, type projection is unique in that it makes comparative predictions across information conditions, but the testability critically depends on whether we find a suitable measure of the degree of overbidding. A few such measures are proposed and tested below.

3.2 Mixed bidding strategies

Consider an auction with private values, a player with signal x, and assume his opponents bid according to the pure strategy $b_\star$. Outbidding the opponents by some $b = b_\star(x) + \varepsilon$ increases the probability of winning but decreases the expected payoff conditional on winning. Without projection, there exists an Bayesian Nash equilibrium bid $b_\star(x)$ where these effects are balanced.

With projection, outbidding the opponents by $b = b_\star(x) + \varepsilon$ yields a marginal decrease of the conditional payoffs but a discrete upwards jump of the winning probability—reflecting the insurance of winning against opponents with the same signal. Thus, whenever the conditional payoff after bid increment is positive, $\tilde{v}_\rho(x|b, b_\star) - b > 0$, the projecting player prefers outbidding the opponents to matching their bids. In turn, a symmetric, pure strategy profile can be an equilibrium only if it induces zero expected payoffs. Then, however, even projecting players can realize positive profits by deviating to bids $b < b_\star(x)$. They would lose against players with similar valuations (probability $\rho < 1$), but they win profitably against some players with lower valuations. Thus, they make positive profits with positive probability (if $\rho < 1$), and in turn, pure symmetric equilibria do not exist. As a result, type projection equilibria of auctions with near-continuous bids must be mixed. This raises the following question.

Question 4. *Do individual bidding strategies exhibit strategic randomization?*

This question is addressed below. Next, let us examine mixed bidding strategies and mixed equilibria. To this end, let us focus on two-player auctions. This simplifies notation substantially, as there is a curse of dimensionality in analyses of mixed strategy equilibria.[8] I further restrict the environment as follows.

Assumption 1. Consider a two player auction with the signals x and y of the players. The distribution of $d = x - y$ is independent of x, has density f_D and support $[\underline{d}, \overline{d}]$.

The key assumption here is independence of x, which is invoked by many empirical analyses. In canonical auctions with affiliated or common values, such as those implemented by Kagel and Levin (1986) and Kagel et al. (1987), independence is satisfied almost exactly unless one's signal x is rather near the bounds of the signal space. The assumption of independence is thus valid in the interior of the signal space and allows us to focus on the properties of strategic bidding abstract of distortions induced by the signal space bounds. Note that independence is not satisfied in the case of independent private values, which prevents a joint analysis assuming independence.

The assumption of independence between $d = y - x$ and x allows us to express the strategic problem of the bidders independently of the signal x. Given signal x, the own bid is normalized toward $r = b - x$ and the opponent's bid is normalized toward $r_\star = b_\star(y) - y$. Normalized bids express the "degree of bid shading", i.e. the amount by which the players undercut their signals x and y, respectively. Theoretically, these normalizations are valid in standard auctions, as the normalized BNE bids are independent

[8] In contrast to pure strategies, where it is sufficient to focus on the opponent with the highest signal, with mixed strategies, even players with lower signals may place the winning bid. Derivations of equilibria for more than two players must account for this, which appears technically straightforward but tedious, without offering any obvious additional insights in relation to the case of two players.

of the signals.[9] Empirically, their validity has not yet been analyzed.

Question 5. *Are normalized bids $r = b - x$ in APV and CV auctions independent of x?*

Given independence, define the normalized expected object value as $\tilde{v}(d) := v(x, x + d) - x$, with $d = y - x$ as above. The normalized expected payoff without projection is

$$\tilde{\Pi}(r \mid r_\star) = \int_{\underline{d}}^{r - r_\star} \big(\tilde{v}(d) - r\big) f_D(d)\, dd.$$

Thus, with Assumption 1, a two-player auction is fully characterized by the duple $\langle \tilde{v}, f_D \rangle$. In equilibrium, the normalized bid r is negative and the expected normalized payoffs are positive. More generally, by normalization of all terms in relation to x, we reduce the dimensionality of the strategy space, which helps both analytically and econometrically. In particular, we can express mixed strategies conveniently. Let $R \subset \mathbb{R}$ denote the set of normalized strategies r. A mixed strategy $\sigma \in \Delta R$ is the density of a distribution on R. The expected (normalized) payoff of bidding r in response to the mixed strategy σ is therefore

$$\tilde{\Pi}(r \mid \sigma) = \int_R \sigma(r_\star)\, \tilde{\Pi}(r | r_\star)\, dr_\star = \int_R \sigma(r_\star) \int_{\overline{d}}^{r - r_\star} \big(\tilde{v}(d) - r\big) f_D(d)\, dd\, dr_\star.$$

3.3 Type projection equilibria in auctions

Under full projection, players assume that their opponent's signal equates with theirs and that they do not learn anything new when winning the auction. Hence, the object value conditional on winning equates with the unconditional object value, $\tilde{V} = \int_{\underline{d}}^{\overline{d}} \tilde{v}(d) f_D(d)\, dd$, and the probability of winning in response to σ equates with σ's cumulative density $F_\sigma(r) = \int_{r_\star < r} \sigma(r_\star) dr_\star$. Again it is independent of the opponent's actual signal. Under ρ-projection, with $0 < \rho < 1$, both conditional value and probability of winning are weighted averages of full-projection and zero-projection. Under ρ-projection, the expected payoff is

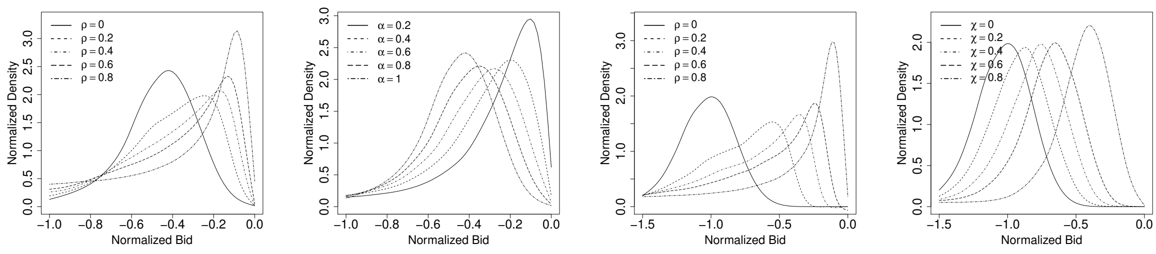
$$\tilde{\Pi}_\rho(r|\sigma) = \int_R \sigma(r_\star)\, \tilde{\Pi}_\rho(r|r_\star)\, dr_\star = (1 - \rho)\, \tilde{\Pi}(r|\sigma) + \rho\, (\tilde{V} - r) F_\sigma(r)$$

To characterize the respective (mixed) equilibria, let us first consider their support. The support of a strategy σ is denoted by $S_\sigma = \{r \in R | \sigma(r) > 0\}$ with bounds $\underline{r} = \inf S_\sigma$

[9] Details follow, but for example, in the CV auction of Kagel and Levin (1986), the BNE bids are $b = x - w$, where w is exogenous (w is the "bandwidth" of the distribution). Hence the normalized BNE bids $r = -w$ are indeed independent of x.

Figure 1: Projection predicts skewed overbidding in both APV and CV auctions. Risk aversion and cursedness predict symmetric overbidding in APV and CV, respectively

(a) APV: Projection ρ (b) APV: Risk aversion α (c) CV: Projection ρ (d) CV: Curse χ



and $\bar{r} = \sup S_\sigma$. Taking the derivative of the payoff with respect r (in response to σ), we obtain (in the interior of the support)

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r\in(\underline{r},\bar{r})} = \rho\,(\tilde{V}-r)\,\sigma(r) - \rho F_\sigma(r) + (1-\rho)\,\tilde{\Pi}'(r|\sigma).$$

Now, along the support of the mixed equilibrium, $\tilde{\Pi}'_\rho(r|\sigma) = 0$ is satisfied. At the lower bound $r = \underline{r}$, this implies $\tilde{\Pi}'(\underline{r}|\sigma) = 0$ and $\sigma(\underline{r}) = 0$. For interior $r \in (\underline{r}, \bar{r})$, we obtain

$$\sigma(r) = \frac{F_\sigma(r)}{\tilde{V}-r} - \frac{1-\rho}{\rho}\cdot\frac{\tilde{\Pi}'(r|\sigma)}{\tilde{V}-r},$$

which implies that $\sigma(r)$ is increasing in r, since $F_\sigma(r)$ is increasing and $\tilde{\Pi}'(r|\sigma)$ is decreasing. The upper bound of the support follows from $\tilde{\Pi}'_\rho(\bar{r}|\sigma)\big|_{dr<0} = 0$, which yields $\sigma(\bar{r}) = -1/(v-\bar{r}) + \frac{1-\rho}{\rho}\tilde{\Pi}'(\bar{r}|\sigma)/(v-\bar{r})$. At the upper bound, the density σ drops to zero, and overall the equilibrium strategy is thus left-skewed (i.e. the mean is left to the median). The following proposition establishes skewness and characterizes the bounds for a variety of conditions including standard APV and CV auctions.

Definition 3. An auction $\langle \tilde{v}, f_D\rangle$ exhibits *strategic complementarity* if $d\tilde{\Pi}(r|r_\star)/dr$ is increasing in $r_\star$.

Proposition 2. *Consider a first-price auction $\langle \tilde{v}, f_D\rangle$ that exhibits strategic complementarity and non-decreasing $\tilde{v}$. For any $\rho \in (0,1)$, any symmetric ρ-TPE is mixed, its support satisfies $r^{BNE} < \underline{r} < \bar{r} < \tilde{V}$, and its density is monotonically increasing.*

The upper bound $\bar{r}$ of the support equates with r^{BNE} in case $\rho = 0$ and it converges to $\tilde{V}$ in case $\rho = 1$. Figure 1 plots the predictions of type projection in APV and CV auctions, alongside those of risk aversion in APV auctions and cursedness in CV auctions.[10] The predictions are plotted for logit equilibria as analyzed in the econometric analysis below, which illustrates that the predicted bounds and shape of the equilibrium strategies are robust to (small) logit errors. Both risk aversion and cursedness predict symmetric distributions, while type projection predicts left-skewed strategies.

Question 6. *Are strategies left-skewed?*

The bounds of the support may alternatively accommodate for risk aversion. For example, consider constant relative risk aversion, $u(\pi) = \pi^{\alpha}/\alpha$, $\alpha \neq 0$, and let r^{α} denote the respective BNE. Risk aversion shifts the lower bound of the support of the corresponding ρ-TPE from r^{BNE} to r^{α}, i.e. under risk aversion the ρ-TPE has support $[r^{\alpha}, r^{\max}]$ where $r^{\max} < \tilde{V}$ can be characterized similarly to above.

Finally, let us briefly look at second-price auctions. Due to the second-price payment rule, the loser-regret component of projection vanishes, but the cursed value perception continues to affect behavior. The projected expectation exhibits a jump discontinuity at $b = b_{\star}(x)$, where one overtakes the opponent, if the object has a common value. In such cases, players again perceive to benefit from overtaking opponents, i.e. their conditional value is not uniquely defined but depends on $b \gtreqless b_{\star}(x)$. This rules out optimality of bidding one's value and the existence of pure equilibria. Otherwise, e.g. in private value auctions, projection equilibria are pure and players bid their values.[11]

4 The data

I re-analyze a total of eight experiments to cover a variety of auction formats and information conditions. Type projection yields distinct behavioral predictions, and thus is testable, across these conditions, while we might expect the projection bias to be of varying relevance if we look at a sufficiently large set of auction formats. Thus, the following analysis constitutes a rather challenging but informative test of projection in auctions. The data sets used in the analysis are chosen with two further objectives in mind. On the one hand, all main information conditions should be included, i.e. independent private values, affiliated private values, and common values. I intend to cover these conditions in standard auction formats (e.g. continuous bids and signals) and in non-standard auction formats (e.g. a non-standard common value, as explained

[10]Risk aversion does not affect equilibrium predictions in common value auctions and cursedness does not affect predictions in private value auctions. Hence, the corresponding plots are skipped.

[11]A more comprehensive analysis can be found in the supplementary material.

Table 1: Data sources

Format	Source	Values	Signals	Inexperienced #Subj	Inexperienced #Obs	Experienced #Subj	Experienced #Obs
Standard auctions							
First price,	Kagel and Levin (2002)	$v = X_0$	$X_i \mid X_0 \sim U[s \pm \varepsilon]$	51	255		
common value	Kagel and Levin (1986)	$v = X_0$	$X_i \mid X_0 \sim U[s \pm \varepsilon]$			49	237
Second price, common value	Garvin and Kagel (1994)	$v = X_0$	$X_i \mid X_0 \sim U[s \pm \varepsilon]$	28	140		
First price, affiliated private	Kagel et al. (1987)	$v = X_i$	$X_i \mid X_0 \sim U[x_0 \pm \varepsilon]$	42	210	42	210
First price,	Dyer et al. (1989)	$v = X_i$	$X_i \sim U[0,30]$	18	180	18	180
Independ. private	Kagel and Levin (1993)	$v = X_i$	$X_i \sim U[0,28.3]$	10	50	10	100
Non-standard auctions							
First price, Independ. private	Goeree et al. (2002b)	$v = X_i$	X_i discrete	80	400	80	400
Second price, Common value	Avery and Kagel (1997)	$v = X_1 + X_2$	$X_i \sim U[1,4]$	23	115	23	115

Note: The discrete signals in Goeree et al. (2002b) are uniform draws from either $\{0,2,4,6,8,11\}$ or $\{0,3,5,7,9,12\}$. The data for inexperienced subjects are mostly from Crawford and Iriberri (2007). In most rounds of Dyer et al. (1989) and Kagel and Levin (1993), the subjects played two auction markets simultaneously. Focusing on the first and last five rounds they played, we mostly have ten observations per subject. Due to bankruptcies in CV auctions, there are not always five observations per subject.

shortly, and discrete bids and signals), of which the latter will be used for strict out-of-sample analysis. On the other hand, I intend to cover exactly those data sets that had been used in previous analyses to test the adequacy of behavioral theories. Indeed, the eight data sets considered here form the union of the data sets used by Goeree et al. (2002b), Bajari and Hortacsu (2005), Eyster and Rabin (2005), and Crawford and Iriberri (2007), to test quantal response equilibrium, risk aversion, cursed equilibrium, and level-*k*, respectively. Thus, if data selection has any influence on the results, then it would be in favor of existing theories, and my estimates will indicate a lower bound of the relevance of type projection. In addition, the repetitive use of these data sets in the literature indicates the existence of a consensus on the adequacy of the underlying experimental designs and on their feasibility to test behavioral theories.

An overview of the data sets used in the analysis is provided in Table 1. The four auction experiments implementing conditions that I call "standard" are the top entries in this table. The data on first price, common value auctions are from the experiment of Kagel and Levin (1986) and Kagel and Levin (2002, Chapter 4). The common value is $v = X_0$ where individual signals are distributed as $X_i \mid X_0 \sim U[x_0 \pm w]$. The BNE strategy is $b(x_i) \approx x_i - w$.[12] The data on second price, common value auctions

[12]The exact BNE strategy is $b(x_i) = x_i - w + Y$ with $Y = \frac{2w}{N+1} \times \exp\{-N(x_i - \underline{x} - w)/2w\}$, but $Y \approx 0$

are from Kagel and Levin (1986) and Garvin and Kagel (1994). Signals and value are distributed as in the first-price case, but now the BNE strategy is $b(x_i) = x_i - w + \frac{2w}{N}$. The data on first price auctions with affiliated private values are from Kagel et al. (1987). The private value $v = X_i$ is distributed as $X_i|X_0 \sim U[x_0 \pm w]$ with BNE strategy $b(x_i) \approx x_i - \frac{2w}{N}$. Finally, the first price auctions with independent private values are from Dyer et al. (1989) and Kagel and Levin (1993), where the private value $v = X_i$ is distributed as $X_i \sim U[0,30]$ and $X_i \sim U[0,28.3]$, respectively. The BNE strategy is $b(x_i) = x_i\,(n-1)/n$ using n to denote the number of players.

The two auction experiments implementing conditions that I call "non-standard" are the bottom two entries in Table 1. Goeree et al. (2002b, GHP02) implement a first price auction with independent private values where bids and signals are small integers. As discussed above, type projection predicts overbidding in IPV auctions primarily due to the low costs of outbidding opponents. In the experiment of GHP02, the smallest bid increment is 1 and thus comparably large. Avery and Kagel (1997) implement a second-price auction with non-standard common value. There are two players who draw independent signals $X_i \sim U[1,4]$ and the common value is $v = X_1 + X_2$ (in their "symmetric case", on which I focus).[13] This auction is non-standard for its violation of Assumption 1, i.e. the distribution of the difference between the signals is not independent of the own signal, and for the experimental observation that subjects do not uniformly overbid. Rather, subjects with high values underbid. For these reasons, I consider these two experiments to be suitable out-of-sample tests of projection.

As indicated in Table 1, I distinguish behavior of experienced subjects and inexperienced subjects. This follows a tendency in the existing literature, and in particular Crawford and Iriberri (2007), who suggest that non-equilibrium concepts such as level-k are most suitable to capture the "initial" behavior of inexperienced subjects, while equilibrium concepts with say risk aversion or cursedness are most suitable to capture the "converged" behavior of experienced subjects. The comparative analysis of both experienced subjects and inexperienced subjects will allow me to detect paradigm shifts in behavior as a function of experience. I closely follow Crawford and Iriberri (2007) by calling a subject "inexperienced" during the first five auctions the subject played, and by inversion, I call a subject "experienced" during the last five auctions the subject played (usually out of approximately 20 auctions in a session). The latter accounts for the fact that in common value auctions, in particular, behavior has not

if the signal x_i is not very close to the bounds of the signal space.

[13] Turocy (2008) discusses the auction in detail. Such common values arise if one might find himself in a position to sell the object later (thus, the opponent's value matters) or e.g. due to prestige effects.

converged after five auctions, which precludes me from using all observations from the sixth auction on in the analysis of experienced subjects. In turn, I show in the supplementary material that behavior is independent of time during the first five auctions and during the last five auctions, respectively, indicating that these partitions of the data set meet the necessary conditions of the ensuing analysis.

Are normalized bids independent of x?

Concluding the data description, let me address Question 5, as it affects the analysis of bids. In Bayesian Nash equilibrium, normalized bids are approximately independent of the signal x if one's signal is not close to the bounds of the signal space. This condition applies to most observations.[14] If independence is empirically valid, Assumption 1 can be invoked and mixed strategies in APV and CV auctions simplify to the normalized strategies defined above. In order to facilitate comparisons of treatments, let me further normalize bids with respect to the signal width w in APV and CV auctions. That is, unless stated otherwise, I refer to $r = (b - x)/w$ as the normalized bid.[15]

I test the independence by regressing bids b on signals x, with subject-level random effects and bootstrapping p-values to account for the panel structure of the data (and the possible non-normality of errors, namely skewed errors). The results are provided in Table 2 in the column "Estimate $B(X)$".[16] The normalized bid $r = b - x$ is statistically independent of x if the regression coefficient of x does not differ from 1. It does not differ significantly in eleven of the twelve treatments distinguished for APV and CV auctions (at $\alpha = .05$). In this multiple testing problem, this is well within the limits of chance assuming a family-wise error rate of .05.

Result 1. *In APV and CV auctions, normalized bids $b(x) - x$ are independent of x.*

Table 2 additionally presents the results of corresponding tests for IPV auctions. The BNE prediction is $b = a + r \cdot x$ where a is not statistically different from zero and $r = (n-1)/n$ under risk neutrality. This prediction is tested in an analyses similar to the previous one and confirmed in the sense that a is statistically insignificant in all

[14] Alternatively, independence obtains if subjects simply disregard the bounds entirely.

[15] This second normalization with respect to w does not obstruct the validity of the independence test, as I test independence treatment-wise, and w is constant in each treatment (or nearly constant, reflecting the treatment distinctions of Kagel and Levin, 1986). For example, with $r = -0.4$, subjects bid $0.4 \cdot w$ less than their signal, $r = 0$ indicates bidding one's signal, $r = -2/N$ is the BNE strategy in APV auctions, and $r = -1$ is the BNE strategy in CV auctions.

[16] The technical details on the bootstrapping procedure are provided in the note to the Table 2.

Table 2: Summary statistics of bidding in the first-price auctions

	Inexperienced subjects					Experienced subjects				
		Degree of	Standard Deviation				Degree of	Standard Deviation		
Condition	Bidding function	Overbidding	within Ss	between Ss	Skewness	Bidding function	Overbidding	within Ss	between Ss	Skewness
Independent private values, First price (DKL89, KL93)										
$N=3$	$b=-0.031+0.803^{\star\star}\cdot x$ (0.234) (0.018)	$0.104^{\star\star}$ (0.017)	0.161 (0.031)	0.025	$-3.17^{\star\star}$	$b=0.028+0.822^{\star\star}\cdot x$ (0.149) (0.01)	$0.143^{\star\star}$ (0.014)	0.126 (0.018)	0.054	$-3.26^{\star\star}$
$N=6$	$b=-0.039+0.849^{\star\star}\cdot x$ (0.196) (0.013)	-0.021 (0.02)	0.162 (0.034)	0.053	$-3.35^{\star\star}$	$b=0.037+0.875^{\star\star}\cdot x$ (0.182) (0.009)	$0.034^{\star\star}$ (0.012)	0.108 (0.018)	0.044	$-4.49^{\star\star}$
$N=5$	$b=0.195+0.886^{\star\star}\cdot x$ (0.241) (0.01)	$0.08^{\star\star}$ (0.021)	0.145 (0.053)	0.028	$-4.34^{\star\star}$	$b=-0.873+0.896^{\star\star}\cdot x$ (0.496) (0.017)	-0.021 (0.042)	0.264 (0.036)	0.129	$-1.65^{\star}$
Affiliated private values, First price (KHL87)										
$N=6, w=6$	$b=0.986^{\star}\cdot x-0.284^{\star\star}\cdot w$ (0.006) (0.079)	$-0.127^{\star\star}$ (0.038)	0.366 (0.085)	0.148	$-3.83^{\star\star}$					
$N=6, w=12$	$b=0.992\cdot x-0.172^{\star\star}\cdot w$ (0.006) (0.048)	$0.104^{\star\star}$ (0.026)	0.052 (0.006)	0.15	0.1	$b=1\cdot x-0.247^{\star\star}\cdot w$ (0.005) (0.037)	$0.088^{\star\star}$ (0.021)	0.168 (0.044)	0.129	$-1.52^{\star}$
$N=6, w=24$						$b=1\cdot x-0.168^{\star\star}\cdot w$ (0.006) (0.02)	$0.164^{\star\star}$ (0.022)	0.09 (0.015)	0.135	-0.24
Common value auctions, First price (KL86)										
$N\leq 4, w=6$	$b=0.996\cdot x-0.22\cdot w$ (0.003) (0.139)	$0.657^{\star\star}$ (0.066)	0.341 (0.033)	0.28	$0.57^{\star\star}$					
$N\leq 4, w\leq 18$	$b=1.014\cdot x-0.676^{\star\star}\cdot w$ (0.013) (0.172)	$0.551^{\star\star}$ (0.099)	0.43 (0.163)	0.095	$0.54^{\star}$	$b=1.002\cdot x-0.905^{\star\star}\cdot w$ (0.016) (0.576)	$0.106^{\star}$ (0.038)	0.276 (0.087)	0.011	-1.77
$N\leq 4, w\geq 24$						$b=1\cdot x-0.63^{\star\star}\cdot w$ (0.021) (0.091)	$0.373^{\star\star}$ (0.05)	0.31 (0.058)	0.178	$0.7^{\star}$
$N=7, w=6$	$b=0.999\cdot x-0.322^{\star\star}\cdot w$ (0.002) (0.088)	$0.629^{\star\star}$ (0.067)	0.333 (0.036)	0.313	$0.54^{\star}$					
$N\geq 5, w=12$						$b=0.99\cdot x-0.575^{\star\star}\cdot w$ (0.007) (0.084)	$0.338^{\star\star}$ (0.051)	0.151 (0.025)	0.225	$1.02^{\star}$
$N\geq 5, w=18$						$b=1\cdot x-0.654^{\star\star}\cdot w$ (0.008) (0.082)	$0.348^{\star\star}$ (0.045)	0.296 (0.025)	0.206	$1.09^{\star\star}$
$N\geq 5, w\geq 24$						$b=0.999\cdot x-0.714^{\star\star}\cdot w$ (0.012) (0.085)	$0.279^{\star\star}$ (0.046)	0.231 (0.025)	0.201	$1.33^{\star}$

Notation: b is the bid, x is the signal, w is the interval width in APV and CV auctions, N is the number of players
Normalized bids: The normalized bids are $r=(b-x)/w$ in APV and CV auction and $r=b/x$ in IPV auctions
Degree of overbidding is the difference between the mean normalized bid and the normalized equilibrium bid, it is estimated controlling for subject-level random effects ("between-subject standard deviation"). The within- and between-subject standard deviations refer to the distribution of normalized bids
Skewness: Skewness of the normalized bids after controlling for subject-level random effects (i.e. skewness of the errors in the regressions of normalized bids on intercept, controlling for random effects).
Experience: Subjects are "inexperienced" in their first five auctions and "experienced" in their last five auctions.
Asterisks indicate the bootstrapped p-values of the null hypotheses that the respective parameters are either 1 (in case of the coefficients of x in APV and CV auctions, which are predicted to be 1) or 0 (in all other cases). "$\star\star$" indicates p-values less than .005, and "$\star$" indicates p-values between .005 and .05. The lower threshold .005 roughly implements the Bonferroni correction for multiple testing across treatments (note that there are more than 10 treatments, so the correction is implemented in a rather conservative way).
Bootstrapped p-value All p-values are bootstrapped, resampling the data set 10.000 times at the subject level (reflecting the panel structure of the data). To define the p-value of the null hypothesis that some statistic s is zero, let s_b denote its value in sample b and let s_0 denote its original value. The p-value of the two-sided test is $\frac{1}{2R}\#\{b:|s_b-\bar{s}|>|s_0|\}+\frac{1}{2R}\#\{b:|s_b-\bar{s}|\geq|s_0|\}$, where $\bar{s}$ is the mean of (s_b) and R the number of samples. Other p-values are defined similarly.

cases. This rules out the alternative hypothesis that subjects' behavior is captured by the simple heuristic $b = x - a$ or by a blend of these approaches as in $b = r \cdot x - a$. Thus, it is feasible to normalize bids also in IPV auctions, namely towards $r = b/x$, and to define corresponding mixed strategies as mixtures over normalized bids $r = b/x$. Again, this normalization reduces the strategic dimensionality and enables econometric analyses of equilibria in mixed strategies (including QRE and ρ-TPE).

Result 2. *In IPV auctions, normalized bids $b(x)/x$ are independent of x.*

These results show that bids in standard auctions can be normalized such that they are statistically independent of x, namely toward $r = (b-x)/w$ in APV and CV auctions and toward $r = b/x$ in IPV auctions. Figure 2 provides histograms of the distributions of these normalized bids across treatments. Due to the independence of x, histograms of normalized bids contain all the information that is also available in scatter plots of bids on signals (as frequently found in the literature), but by reducing the dimensionality of the distribution, the information is provided in a more condensed way. Essentially, these histograms are left to be explained by behavioral theories. The normalization also allows us to look at the data from novel perspectives, e.g. skewness and within-subject variances are meaningful moments of the data after normalization of bids. Thus, we can make and test novel statements about facets of behavior that had to be disregarded so far, as discussed next.

5 Evaluating the testable predictions of projection

Now, I address the remaining questions raised in the theoretical analysis. Those questions relate particular moments of the data to predictions of type projection. Later, I will address the question to which degree the various behavioral theories allow to capture the actual distributions of bids.

Is subject heterogeneity of discrete nature? The shape of subject heterogeneity is interesting for two reasons. The possible existence of discrete components in the population is predicted by two of the asymmetric-belief models, level-k and CHM. Accordingly, the population consists of subjects spread across various discrete levels of reasoning, and different levels of reasoning induce different (normalized) strategies. Thus, if subjects' strategies can be organized into different clusters, it would be indicative of strategic reasoning according to level-k or CHM. Further, the potential necessity

Figure 2: First-price auctions, affiliated private values (KHL87). Inexperienced subjects (a–b) vs. experienced subjects (c–d). Plots are histograms of $r = (Bid - Signal)/w$

(a) Inexp: $N = 6, w = 6$ (b) Inexp: $N = 6, w = 12$ (c) Exp: $N = 6, w = 12$ (d) Exp: $N = 6, w = 24$

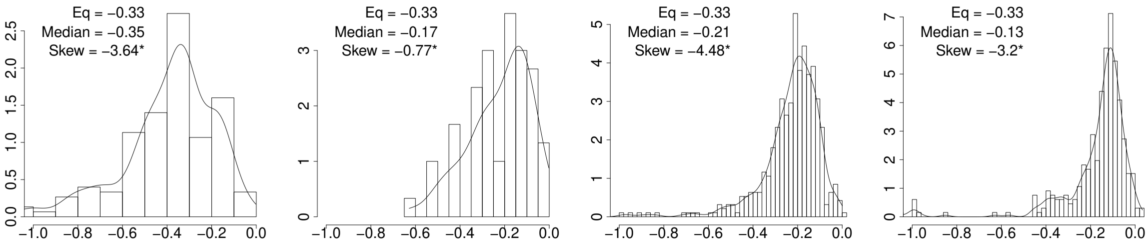
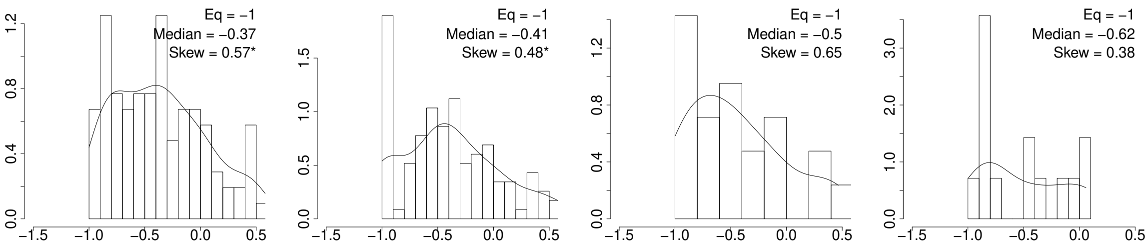


Figure 3: First-price auctions with common values (KL86), inexperienced subjects (a–d) vs. experienced subjects(e–h). Plots are histograms of $r = (Bid - Signal)/w$

(a) $N = 4, w = 6$ (b) $N = 7, w = 6$ (c) $N = 4, w = 12$ (d) $N = 7, w = 12$



(e) $N = 3-4, w = 12, 18$ (f) $N = 5-7, w = 12, 18$ (g) $N = 3-4, w = 24, 30$ (h) $N = 5-7, w = 24, 30$

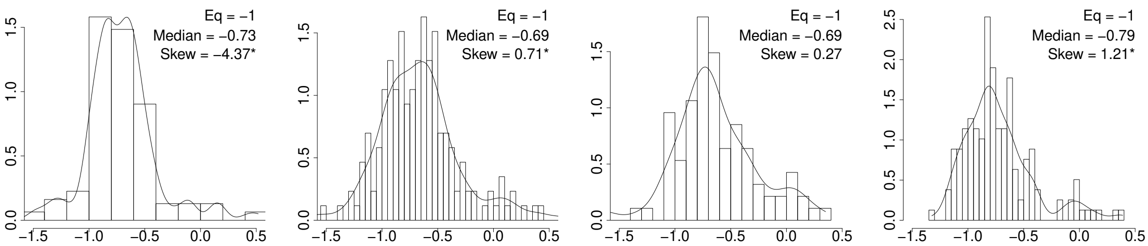
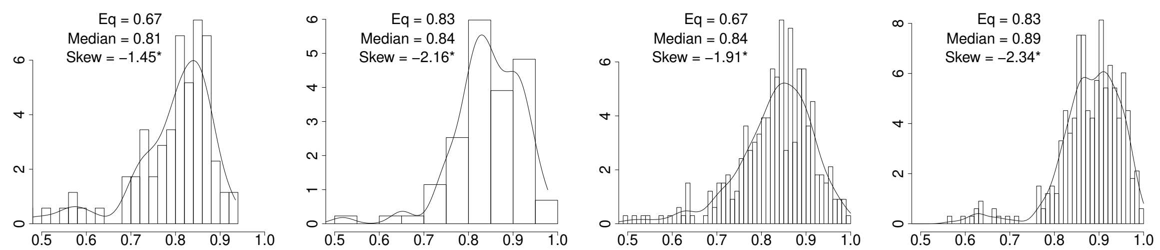


Figure 4: First-price auctions with independent private values (DKL89). Inexperienced subjects (a–c) and experienced subjects (d–f). Histograms of $Bid/Signal$

(a) Inexp: DKL89, $N = 3$ (b) Inexp: DKL89, $N = 6$ (c) Exp: DKL89, $N = 3$ (d) Exp: DKL89, $N = 6$



to differentiate discrete components on top of the random effects affects the modeling strategy below. Continuity of subject heterogeneity simplifies the analysis, as random effects, or random coefficients, would then suffice to capture subject heterogeneity.

The histograms of normalized strategies in Figures 2, 3, and 4, and the respective kernel density estimates, suggest that the distributions are uni-modal and therefore consist of single (continuous) components in all cases. In order to verify this impression, I estimate finite mixture models with up to three components. Each component is a characterized by a mean normalized strategy, by a between-subject variance regarding the subjects making up the component, and a within-subject variance to capture individual randomization.[17] The details are relegated to the supplementary material, but the impression given by the histograms is confirmed and summarized as follows.

Result 3. *Across information conditions and experience levels, secondary components are either insignificant (16 of 18 treatments) or contain less than 10 percent of the subjects (2 of 18 treatments, both with experienced subjects in the APV auction).*

I conclude that random effects suffice to capture subject heterogeneity.

Are strategies left-skewed? Type projection equilibrium predicts distributions of normalized bids to be left-skewed (the mean left of the median). Thus, the estimated skewness should be negative in the data sets. Figures 2, 3, and 4 present histograms of the normalized bids in APV, IPV, and CV auctions, respectively. First, looking at the overall distribution, the results appear rather clear-cut.[18] The distributions of bids are significantly left-skewed in private value auctions, i.e. in both IPV and APV auctions, while skewness tends to be inverted in CV auctions. In all five cases where exact treatment-wise comparisons between inexperienced and experienced subjects are possible, the estimated skewness further shifts toward left-skewed distributions as subjects gain experienced. Second, due to subject heterogeneity, the overall skewness may not equate with the average individual skewness. The individual skewness is the skewness of the errors when regressing the normalized bids on the intercept controlling for

[17]In order to focus on whether discrete levels of reasoning need to be distinguished, the within-subject variances are held fixed constant across components. The structural model below will allow for heterogeneity in within-subject variances. Further, the models are estimated using the EM algorithm using 25 different starting values in each case, and the number of model components is estimated by maximizing the integrated classification likelihood (ICL), following Biernacki et al. (2000). Maximizing the ICL estimates the correct number of components of finite mixtures more consistently than say Bayes Information Criterion (BIC). See McLachlan and Peel (2000) for further information.

[18]All of the histograms additionally present information on the skewness of the distributions of normalized bids. An asterisk is printed next to the skewness estimate if it deviates significantly from zero. As above, I evaluate the significance of the skewness by bootstrapping, resampling at the subject level.

subject-level random effects. These estimates, reported in Table 2, are very similar to the overall skewness estimates. Therefore, I arrive at the following conclusion.

Result 4. *Distributions of bids are left-skewed in private value auctions and right-skewed in common-value auctions.*

Risk aversion and cursed equilibrium predict neither left-skewed nor right-skewed distributions. Thus, the observed left-skewness in private value auctions is compatible with type projection, while the observed right-skewness in common value auctions is not compatible with any theory but "closer" to existing theories. This observation will be reconsidered below.

Does individual variance decline with experience? In first-price auctions, type projection equilibria are generally mixed. In contrast, risk aversion and cursedness predict pure equilibria. Assuming that subjects gaining experience converge to pure equilibria, the within-subject variance is predicted to be less for experienced subjects than it is for inexperienced ones. Estimates of the within-subject standard deviations are obtained in regression analyses corresponding with those for individual skewness, and presented in Table 3 in the columns entitled "Standard Deviation within Ss". Their interaction with experience is estimated in regression models with different within-subject variances for the two levels of experience. Table 4 presents the results in the columns on the "Within-Subject Variance". I test the hypothesis in multiple ways, either holding the conditions such as number of players N or signal bandwidth w constant, or pooling the data and then controlling for N or w.[19] Summarizing, the overall results strongly indicate that the within-subject variance does not decline as subjects gain experience. This holds both in treatment-wise comparisons when they are possible, noting that treatment parameters in some experiments are changed as subjects gain experience (see Table 2), and after pooling treatments. Between the 13 tests in Table 3, there is one significant result for either direction, but none of them is significant at the .005 level, as would be required by the Bonferroni correction. These observations are summarized as follows.

Result 5. *The within-subject variance does not differ between experienced and inexperienced subjects. There is no indication that subjects converge to a pure strategy as they gain experience, and even experienced subjects seem to randomize consistently. This confirms the prediction of type projection equilibrium.*

[19] The supplementary material contains related evidence on the treatment effects with respect to the degree of overbidding and the within-subject variance (holding experience constant). Here, I focus on the tests of the hypotheses developed above.

Table 3: Statistical tests of the degree of overbidding and within-subject variance (with respect to the degree of overbidding) as a function of experience

	Degree of Overbidding			Within-Subject Variance			Between-Subj
Data	Inexperienced		Experienced	Inexperienced		Experienced	Variance
Independent private values auctions (DKL89, KL93)							
$N = 3$	0.104	$\approx$	0.148	0.16	$\approx$	0.123	0.033
$N = 5$	0.08	$>$	-0.144	0.142	$<$	0.36	0.077
$N = 6$	-0.021	$<$	0.036	0.164	$\approx$	0.11	0.039
all N	0.05	$\approx$	0.04	0.156	$\approx$	0.178	0.119
all, contr. for N	0.05	$\approx$	0.041	0.155	$\approx$	0.179	0.103
Affiliated private values auctions (KHL87)							
$w = 12$	0.104	$\approx$	0.062	0.051	$\approx$	0.172	0.17
All data	-0.058	$\ll$	0.142	0.331	$>$	0.15	0.134
All, contr. for w	0.058	$\approx$	0.04	0.192	$\approx$	0.209	0.117
Common value auctions (KL86)							
$N \leq 4$, $w \in \{12, 18\}$	0.538	$>$	0.228	0.415	$\approx$	0.343	0.208
$N \leq 4$, all w	0.63	$\gg$	0.316	0.37	$\approx$	0.363	0.233
$N \geq 5$, all w	0.613	$\gg$	0.389	0.344	$\approx$	0.309	0.254
all N, $w \in \{12, 18\}$	0.517	$\approx$	0.404	0.397	$\approx$	0.332	0.263
all N, all w	0.621	$\gg$	0.359	0.357	$\approx$	0.332	0.242
all N, all w, contr. for w	0.573	$\approx$	0.411	0.349	$\approx$	0.337	0.247

Description: The table reports the results of one set of statistical tests per row. Given the subset of data specified in column 1, two null hypotheses are simultaneously tested: (i) H_0 : the degree of overbidding does not differ between inexperienced and experienced subjects, and (ii) H_0 : the residual (i.e. within-subject) variances do not differ between them. These nulls are tested in regression models with the degree of overbidding as independent variable and the level of experience as independent variable (without intercept). $\gg, \ll$ indicate rejection of H_0 at the .005 level and $>, <$ indicate rejection at .05, where the p-values are bootstrapped as described above. Considering the Bonferroni correction for the multiple testing problem inherent in this analysis, results should be significant roughly at the .005 level. Terms such as the degree of overbidding are used as defined above (e.g. Table 2).

Table 4: Statistical tests of differences in the degree of overbidding and within-subject variance between auctions with affiliated private values and common values

	Degree of Overbidding			Within-Subject Variance			Between-Subj
Data	APV		CV	APV		CV	Variance
Inexperienced, $w = 6$	-0.128	$\ll$	0.641	0.359	$\approx$	0.341	0.246
Inexperienced, $w = 12$	0.104	$\ll$	0.523	0.052	$\ll$	0.4	0.147
Inexperienced, all w	-0.058	$\ll$	0.621	0.326	$\approx$	0.361	0.21
Experienced, $w \leq 18$	0.062	$\ll$	0.403	0.159	$<$	0.336	0.248
Experienced, $w \geq 24$	0.179	$<$	0.329	0.113	$\ll$	0.32	0.13
Experienced, all w	0.142	$\ll$	0.357	0.15	$\ll$	0.343	0.162

Description: The sole difference to Table 3 is that the comparison is between APV and CV auctions, instead of inexperienced and experienced subjects.

Is the degree of overbidding larger in CV than in PV auctions? In auctions with common values, projection induces both loser regret and cursed value perception, and in this sense, projecting players have stronger incentives to overbid (in relation to BNE) in common value auctions than in private value auctions. This hypothesis can be tested by comparing bidding in KL86's CV auctions and with bidding in KHL87's APV auctions. These experiments implement common values and (affiliated) private values in otherwise equivalent conditions: signal bandwidths w are similar, numbers of players N are similar, and even experimental instructions and logistics are similar. This hypothesis is evaluated using an econometric approach similar to the one used to compare experienced and inexperienced bidders, i.e. by regressing the degree of overbidding on the information condition (APV or CV), controlling for subject-level random effects and bootstrapping p-values. The degree of overbidding is the difference between normalized bid and BNE bid. Table 4 presents the results: Across all treatment conditions and experience levels, the degree of overbidding is highly significantly higher in CV auctions than in APV auctions.[20]

Result 6. *The degree of overbidding is higher in CV auctions than in APV auctions.*

6 Quantifying the empirical content of projection

In this section, I investigate to which degree projection explains the distribution of bids. First, the *descriptive adequacy* measures the adequacy after fitting the degree of projection (ρ) to the data. I look at both, the fit with respect to behavior in particular information conditions, and the fit with respect to the data pooled across conditions. We may expect that projection fits well in any given condition, due to the qualitative properties discussed above, but the fit of projection with respect to the pooled data is less obvious. Projection fits to pooled data only if a specific degree of projection applies to all information conditions. This cannot be guaranteed or denied on a qualitative case-by-case basis.

Second, the *predictive adequacy* measures the adequacy after fitting the degree of pro-

[20]The supplementary material contains further robustness checks by testing a variety of alternative measures of the degree of overbidding derived from differences of realized and hypothetical payoffs. Without overbidding, those differences would be zero, while they are non-zero if subjects overbid. These tests have substantially less power, as both realized and hypothetical payoffs are zero for most subjects in any given auction. Their results still indicate highly significant more overbidding in CV auctions if subjects are inexperienced, though differences are not quite significant for experienced ones (presumably due to the lack of power, as all differences point into the predicted direction).

jection to one information condition and using the estimate to predict the remaining data. This allows me to determine the robustness of the fit.[21] Clearly, a high descriptive adequacy does not guarantee predictive adequacy if the model overfits. In particular, even if a specific degree of projection allows us to explain behavior after pooling across information conditions, we cannot rule out that an entirely different degree of projection would have provided a better fit in a particular condition while ruining predictions in alternative conditions. Ideally, projection is both, descriptively and predictively adequate, but this requires robustness of the degree of projection.

Third, *inferential adequacy* measures the adequacy of projection (out of sample, again) to infer the bidders' valuations from their bids. Besides measuring the goodness of fit in an inverted way, this measure may be of explicit interest for practitioners.[22]

In addition, it may be interesting to know to which degree projection works across conditions. To this end, the empirical content of projection is quantified by the Cox-Snell pseudo-R^2 (Nagelkerke, 1991) in addition the above measures, and I relate the goodness-of-fit of projection to that of existing explanations of auction behavior (risk aversion, cursedness, and belief asymmetry). I find that projection describes behavior more adequately than existing concepts in all dimensions, and even combined they do not fit better than projection. Projection seems to explain all we currently can explain.

The econometric methodology is standard. I consider a structural model of bidding that allows for individual errors, belief asymmetry, and individual heterogeneity. The errors follow from logistic utility perturbations. The basic model underlying the analysis is the quantal response equilibrium (McKelvey and Palfrey, 1995). In the context of type projection in Bayesian games, it can be defined as follows.

Definition 4. For any $\lambda \geq 0$ and $\rho \in [0,1)$, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is a (λ, ρ)-type projection logit equilibrium ((λ, ρ)-TPLE) if for all players $i \in N$, all types $t_i \in T_i$, and all actions $a_i \in A_i$,

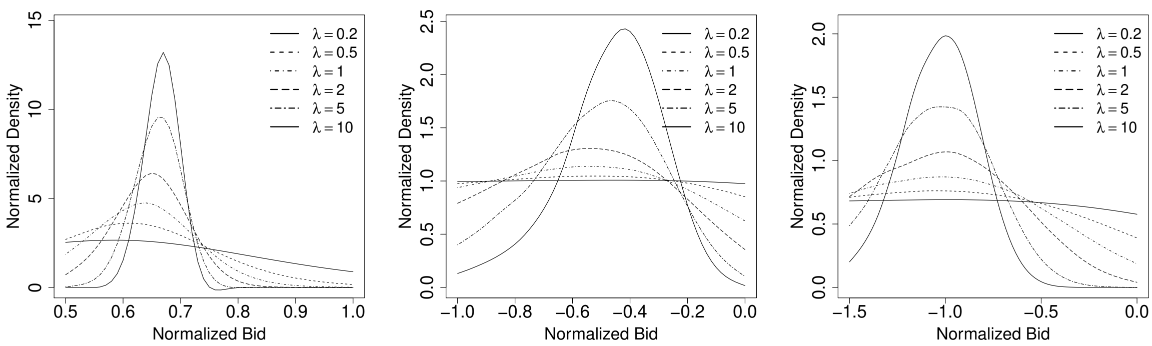
$$\sigma_i(a_i|t_i) = \frac{\exp\{\lambda\, \pi_i\big(a_i|t_i, \tilde{\sigma}_{-i}\big)\}}{\sum_{a_i' \in A_i} \exp\{\lambda\, \pi_i\big(a_i'|t_i, \tilde{\sigma}_{-i}\big)\}}$$

[21]The tendency to distinguish descriptive and predictive adequacy is a rather recent development in analyses of decision-theoretic models (Wilcox, 2008; Hey et al., 2010), learning models (Erev and Roth, 1998; Camerer and Ho, 1999; Tang, 2003; Ho et al., 2008), and simple games (Blanco et al., 2011; Shapiro et al., 2014). I am not aware of existing analyses in Bayesian games in general or auctions in particular. The approach adopted here is known as cross-validation (Browne, 2000) with nonrandom holdout samples (Keane and Wolpin, 2007).

[22]This approach towards model validation follows Bajari and Hortacsu (2005), who also discuss further applications.

Figure 5: Logit equilibria with varying precisions λ predict almost invariant modes and provide no systematic explanation of overbidding

(a) Independent private values (b) Affiliated private values (c) Common values



with $\tilde{\sigma}_{-i}(a_{-i}|t_{-i}) = \rho \prod_{j\neq i} \sigma_j(a_j|t_i) + (1-\rho)\prod_{j\neq i}\sigma_j(a_j|t_j)$.

For simplicity, I abbreviate (λ,ρ)-TPLE as QRE. First, it constitutes the standard model in behavioral game theory,[23] and as such, its choice does not exploit a degree of freedom. Secondly, QRE captures errors and by being structural, it allows me to estimate parameters such as the degree of projection. Thirdly, QRE is a fairly neutral base model in auctions. Changes in the precision λ do not affect the degree of overbidding, as Figure 5 shows. Finally, by pooling various data sets and by restricting QRE to a single degree of freedom (λ), I avoid the potential critique that QRE with sufficient freedom in the correlation structure allows me to fit any single data set.[24]

Non-equilibrium models may be plausible bases in alternative classes of games, but they are not invariant with respect to the degree of overbidding. Non-equilibrium models partially explain overbidding and thus constitute alternative explanations rather than base models in their own rights. Amongst all alternatives, the main candidate that stands out shall be dubbed asymmetric quantal response equilibrium (AQRE). AQRE allows that players believe their opponents play a QRE with precision $\lambda \geq 0$ and in response to this belief, they play a strategy with precision $\kappa \geq 0$.[25] AQRE contains the most important models discussed in the literature as special cases. Besides QRE,

[23]QRE with logistic errors explains behavior in games as diverse as the centipede game (Fey et al., 1996), the traveler's dilemma (Capra et al., 1999), public goods games (Goeree et al., 2002a), monotone contribution games (Choi et al., 2008), and beauty contests (Breitmoser, 2012).

[24]Haile et al. (2006) show that QRE with fully flexible correlation of perturbations allows to fit any single set of choice probabilities. By using multiple choices per subject, pooling multiple treatments, and assuming independence of irrelevant alternatives, this flexibility is ruled out here.

[25]AQRE differs from the asymmetric logit equilibrium defined by Weizsäcker (2003) insofar as opponents do not know that I use some $\kappa \neq \lambda$. They simply play the QRE with precision λ.

which is contained for $\lambda = \kappa$, it contains level-1 behavior for $\lambda = 0$ and logit responses to BNE for $\lambda \to \infty$. The level-1 case has been discussed by Crawford and Iriberri (2007) and noisy responses to BNE are the standard assumption in structural analyses of auctions, see e.g. Bajari and Hortacsu (2005). Thus, by using AQRE as control, these standard models are implicitly contained as special cases and thus also included in the analysis.[26] The detailed results are in the supplement. I find that these models of belief asymmetry do not explain auction behavior robustly better than QRE.

6.1 Descriptive adequacy in standard auctions

Throughout the analysis, I allow for heterogeneity of subjects. Consistent with the lack of time trends found above, each subject behaves according to a constant set of parameters, but the parameters are randomly distributed across subjects. The precision parameters λ and κ are bounded at zero and have independent gamma distributions, whereas the degrees of risk aversion, projection and cursedness are bounded at both 0 and 1 and have independent beta distributions. Thus, each subject is described by a parameter vector $\mathbf{p} \in \mathbf{P}$ with joint density $f()$. Using $o_s = (o_{s,t})$ to describe the observations of subject $s \in S$ at time $t \in T$, and $\sigma(o_{s,t}|f)$ as the probability of observation $o_{s,t}$ under density f, the individual likelihood given the observations o_s of subject s is

$$l_s(f|o_s) = \int_{\mathbf{P}} \prod_{t \in T} \sigma(o_{s,t}|\mathbf{p}) \cdot f(\mathbf{p}) d\mathbf{p}.$$

The predictions $\sigma(o_{s,t}|f)$ implicitly depend also on the underlying belief model, e.g. QRE or AQRE. The integral is evaluated by simulation, using quasi random numbers, see Train (2003) and e.g. the supplement to Bellemare et al. (2008). Aggregating across subjects, the log-likelihood of the respective belief model with parameter density f is

$$ll(f) = \sum_{s \in S} \log l_s(f|o_s).$$

Parameters are estimated by maximizing the log-likelihood,[27] and the estimates are tested by extensive cross-analysis to ensure that global maxima are found. All parameter estimates and likelihood-ratio tests are provided as supplementary material.

[26] To be conservative, I additionally estimate level-k (Stahl and Wilson, 1995; Nagel, 1995), cognitive hierarchy (Camerer et al., 2004)1 and noisy introspection models (Goeree and Holt, 2004).

[27] I sequentially applying two maximization algorithms. Initially, I use the robust, gradient-free NEWUOA algorithm (Powell, 2006) and I verify convergence using a Newton-Raphson algorithm.

Table 5: The models' adequacy to capture behavior in "standard auctions" – As for all measures: Less is better

	Plain models (assuming homogeneous parameters)									Mixture models (assuming heterogeneous parameters)										
	RiskAv		P&RA		C&RA		Proj		Curse	Mix Log		Mix RA		Mix C&RA		Mix P&RA		Mix Proj		Mix Curse
Descriptive Adequacy (Bayes Information Criterion)																				
Inexperienced subjects																				
CV1	1349	$\gg$	1322	$\approx$	1318	$\approx$	1320	$\approx$	1316	1347	$\ll$	1351	$\gg$	1312	$<$	1322	$\approx$	1321	$>$	1308
CV2	746	$\approx$	735	$\approx$	737	$\approx$	734	$\approx$	736	744	$\ll$	747	$>$	726	$\approx$	731	$\approx$	729	$\approx$	726
APV	883	$\approx$	883	$\approx$	886	$\approx$	881	$\approx$	885	964	$\gg$	864	$\approx$	864	$\approx$	863	$\approx$	871	$\approx$	878
IPV	918	$\gg$	871	$\ll$	919	$\gg$	870	$\ll$	1089	1076	$\gg$	911	$\approx$	912	$\gg$	859	$\approx$	853	$\ll$	1072
Pooled	3998	$\gg$	3898	$\ll$	3956	$\gg$	3893	$\ll$	4152	4149	$\gg$	3924	$\gg$	3796	$\approx$	3780	$\approx$	3771	$\ll$	3962
Experienced subjects																				
CV1	1478	$>$	1447	$\approx$	1442	$\approx$	1464	$\approx$	1440	1468	$\approx$	1465	$\gg$	1405	$\ll$	1428	$>$	1422	$\gg$	1403
APV	965	$\approx$	965	$\approx$	967	$<$	991	$\ll$	1101	1293	$\gg$	957	$<$	961	$\approx$	960	$\approx$	961	$\ll$	1111
IPV	1136	$\gg$	1047	$\ll$	1139	$\gg$	1042	$\ll$	1349	1327	$\gg$	1106	$\approx$	1106	$\gg$	1027	$\approx$	1034	$\ll$	1327
Pooled	3718	$\gg$	3496	$\ll$	3611	$\approx$	3570	$\ll$	4029	4086	$\gg$	3603	$\gg$	3418	$>$	3377	$\approx$	3406	$\ll$	3833
Predictive Adequacy (Absolute Value of Log-Likelihood)																				
Inexp Ss	4499	$\gg$	4088	$\ll$	4560	$\gg$	4075	$\ll$	5943	4288	$\ll$	4604	$\gg$	4123	$\ll$	4299	$\gg$	4123	$\approx$	4082
Exp Ss	4043	$\gg$	3870	$\ll$	3978	$\approx$	4029	$\ll$	4268	4116	$\gg$	3873	$\gg$	3686	$\approx$	3633	$\approx$	3599	$\ll$	4399
Inferential Adequacy (Mean Absolute Deviation)																				
Inexp Ss	2481	$\gg$	2109	$\ll$	2224	$\gg$	2108	$\ll$	2334	2414	$\approx$	2442	$\gg$	2358	$>$	2319	$\gg$	2121	$\approx$	2065
Exp Ss	4248	$\gg$	3042	$\ll$	3414	$\ll$	4611	$\approx$	4453	4655	$\gg$	3962	$\gg$	3259	$\approx$	3184	$\approx$	3508	$<$	3728

Note: The relation signs describe the results of Vuong tests of the likelihood ratios (nested or non-nested as appropriate, with bootstrapped p-values, using individual subjects as independent observations). In all cases, the less the score the better. $\ll$ indicates that the left model outperforms the right one at $p < .005$, $<$ indicates $.005 < p < .05$, $\gg$ and $>$ indicate corresponding inference in favor of the right model, and $\approx$ indicates insignificance of the difference.

Notation: "RA" stands for constant relative risk aversion, "P" for projection, and "C" for cursed equilibrium. "&" indicates models merging these approaches.

Table 6: Pseudo-R^2 of the various models (Higher is better)

	Inexperienced Subjects													Experienced Subjects												
	Curse		RA		Proj		Mix Logit		Mix RA		Mix Proj		Mix Curse	Curse		RA		Proj		Mix Logit		Mix RA		Mix Proj		Mix Curse
Descriptive Adequacy (In-Sample BIC)																										
Standard Pooled	0.33	$\ll$	0.48	$\approx$	0.48	$>$	0.48	$\ll$	0.61	$\ll$	0.69	$\gg$	0.61	0.46	$\approx$	0.46	$\ll$	0.74	$\gg$	0.36	$\ll$	0.79	$\ll$	0.84	$\gg$	0.61
Non-Stand CV	0.22	$>$	0.1	$\ll$	0.32	$\gg$	0.1	$\approx$	0.1	$\ll$	0.35	$\approx$	0.27	0.24	$\gg$	0.1	$<$	0.23	$\approx$	0.19	$\approx$	0.22	$\approx$	0.32	$\approx$	0.32
Discrete IPV	0.16	$\ll$	0.62	$\approx$	0.65	$\gg$	0.17	$\ll$	0.73	$\approx$	0.72	$\gg$	0.15	0.14	$\ll$	0.58	$\approx$	0.64	$\gg$	0.17	$\ll$	0.72	$\approx$	0.73	$\gg$	0.18
Predictive Adequacy (Out-Of-Sample $\|LL\|$)																										
Standard Pooled	0.12	$\ll$	0.54	$\ll$	0.77	$\gg$	0.66	$\gg$	0.36	$\ll$	0.75	$<$	0.79	0.57	$\approx$	0.53	$\ll$	0.76	$\gg$	0.54	$\ll$	0.77	$\ll$	0.86	$\gg$	0.41
Non-Stand CV	0.45	$\gg$	0.21	$\ll$	0.4	$>$	0.32	$\ll$	0.35	$<$	0.42	$\ll$	0.53	0.45	$\gg$	0.21	$\ll$	0.4	$>$	0.32	$\ll$	0.35	$<$	0.42	$\ll$	0.53
Discrete IPV	0.37	$\approx$	0.4	$\ll$	0.59	$\gg$	0.38	$\ll$	0.59	$\ll$	0.71	$\gg$	0.44	0.37	$\approx$	0.4	$\ll$	0.59	$\gg$	0.38	$\ll$	0.59	$\ll$	0.71	$\gg$	0.44

Note: The testing procedure is equivalent (i.e. p-values are bootstrap using subjects' scores as independent observations), the difference being that pseudo-R^2 are used.

The main results of the analysis are summarized in Tables 5 and 6 and in Figures 6–8. I focus on the fit using QRE with heterogeneity of subjects, but as shown in the supplementary material, the results are very similar for all non-equilibrium models and with homogeneous parameters. For convenience, Tables 5 and 6 also provide the main results on the adequacy measures for homogenous parameters.

As for inexperienced subjects, Table 5 provides the results for first- and second-price common value auctions, and for two kinds of private values auctions (affiliated and independent ones), as described in the overview of the data. Regarding these information conditions in isolation, the main hypotheses are confirmed. In CV auctions, where both cursedness and projection predict overbidding, these concepts fit about similarly well. In PV auctions, both risk aversion and projection predict overbidding, and these concepts fit about similarly well. In common value auctions, the left-skewness predicted by projection is not observed, and cursedness fits slightly better than projection. In turn, left-skewness predicted by projection is observed in private value auctions, and there, projections fits slightly better than risk aversion. In aggregate, projection is the only concept that is compatible with overbidding across information conditions, and it fits the pooled set of observations highly significantly better than the other concepts in isolation. The picture is very similar for experienced subjects.

Intuitively, a model combining both cursedness and risk aversion attains a better fit overall than these concepts in isolation, and indeed, this combined model has a descriptive adequacy that is statistically not different from that of projection. In turn, projection explains everything that these concepts explain in combination. Also, enriching projection by risk aversion does not significantly improve its descriptive adequacy, but the resulting adequacy is significantly higher than that of the model combining cursedness and risk aversion (if subjects are experienced). In order to quantify the amount of variance explained by the models, Table 6 provides the respective pseudo-R^2 for the pooled samples.[28]

Result 7 (Descriptive adequacy)**.** *The descriptive adequacy of projection is about as high as the best of risk aversion and cursedness in each information condition. Overall, projection fits significantly better than these concepts in isolation, it fits about as well as the approaches combined, and it explains around 65% of the observed variance.*

[28] The pseudo-R^2 measures how much a given model explains in relation to a baseline model. Let $l_{1,s}$ denote the likelihood of the model in question with respect to subject s, and let $l_{0,s}$ denote the baseline likelihood with respect to the subject in question (= the minimum of all likelihoods that the range of models estimated here yield). The Cox-Snell pseudo-R^2 of subject s is $r_s^2 = 1 - (l_{0,s}/l_{1,s})^{2/N_s}$ where N_s is the number of observations. The mean of the individual pseudo-R^2 over the entire sample is used in comparisons. Testing the differences between two pseudo-R^2 follows the procedure for Vuong tests.

Figures 6–8 plot the predicted densities of the models over the histograms. As discussed above, risk aversion predicts the wrong mode in the common value auctions (for all α, it predicts that BNE bids, $r = -1$ in these cases, have the highest probability), and cursedness predicts the wrong mode in private value auctions (in particular in IPV auctions). Projection has no such weakness and fits better than these concepts overall. The data on CV auctions are particularly interesting, as it shows that subjects are rather unlikely to bid below the BNE bid of $r = -1$. That is, subjects hardly bid less than an interval width w below their signal. As a result, the observations look like they are censored at $r = -1$, in particular for inexperienced subjects. Bidding less than $r = -1$ implies that all signals were certainly higher than the own bid, and for subjects, bid shading to such an extent seems to be difficult—although they should do so, based on expected payoffs. Bidding below $r = -1$ is associated with non-negative expected payoffs for all beliefs, whereas outbidding the own signal (i.e. bidding $r > 0$) is associated with non-positive payoffs for all beliefs (assuming symmetric strategies). As a result of this apparent self-censoring, observed bids do not exhibit the predicted tail to the left in the data on CV auctions, but as all concepts fail to predict such self-censoring, the resulting descriptive adequacies do not differ substantially.

6.2 Predictive adequacy in standard auctions

Having estimated the model parameters for all data sets, I look at their adequacy to actually predict behavior. The observed adequacy of projection to ex-post explain behavior does not imply that it is predictive. Predictive adequacy requires the degree of projection to be robust across data sets not just in the ex-post sense, but also in an ex-ante sense: a chosen degree of projection needs to explain behavior before fitting ρ to the data. I test the even stronger assertion that ρ needs to explain behavior after fitting it to the wrong data. Thus, measures of descriptive and predictive adequacy may be seen as necessary and sufficient measures of the general model adequacy.

All parameters are estimated on "training data" and used to evaluate the model's log-likelihood on the entire data set (for the respective level of experience). For each model, this yields a total of seven basic measures, derived from each of the four training data sets for inexperienced subjects and each of the three training data sets for experienced ones. These basic measures still contain the goodness-of-fit with respect to the training data, alongside the remaining data, and thus they are partially in-sample. In order to evaluate the predictive adequacy fully out-of-sample, I next remove the in-

Figure 6: The predictions of **mixed QRE with risk aversion** in relation to histograms of the data

(a) CV, Inexperienced (b) APV, Inexperienced (c) IPV, Inexperienced (d) CV, Experienced (e) APV, Experienced (f) IPV, Experienced

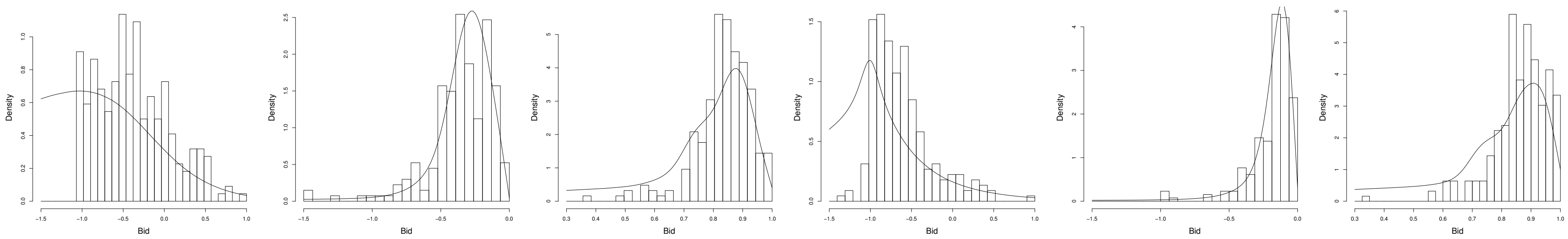


Figure 7: The predictions of **mixed QRE with projection** in relation to histograms of the data

(a) CV, Inexperienced (b) APV, Inexperienced (c) IPV, Inexperienced (d) CV, Experienced (e) APV, Experienced (f) IPV, Experienced

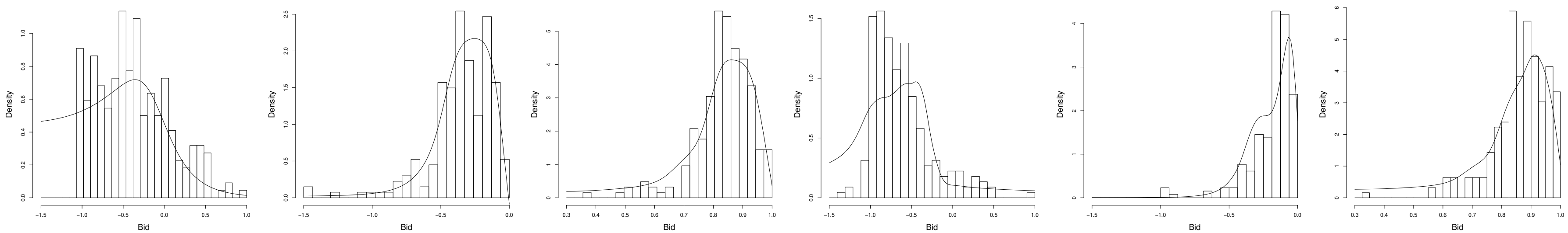
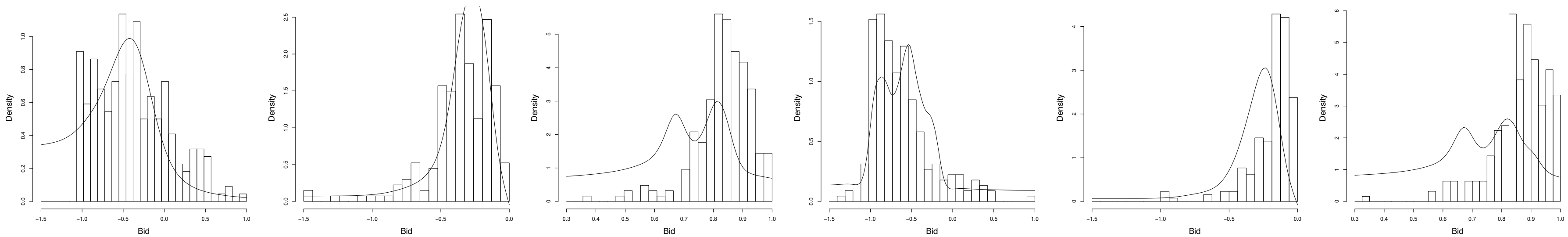


Figure 8: The predictions of **mixed QRE with cursedness** in relation to histograms of the data

(a) CV, Inexperienced (b) APV, Inexperienced (c) IPV, Inexperienced (d) CV, Experienced (e) APV, Experienced (f) IPV, Experienced



Note: In all cases, the histogram is the distribution of normalized bids in the respective information condition, and the line plotted above it is the density of the prediction of the respective concept aggregated across treatments in the respective information condition.

sample segments of the data sets, aggregate the remaining out-of-sample LLs over all training data sets given the respective level of experience, and divide by the number of times that each data set was used in the out-of-sample evaluation stage.[29] This measure is called "pooled out-of-sample LL", and I report the absolute values. It is purely out-of-sample and its magnitude is comparable to that of the in-sample BIC.

By the qualitative properties, I hypothesize that projection yields higher predictive adequacy than the other concepts in isolation, as projection predicts overbidding across information conditions. Further, if subjects are inexperienced, risk aversion is predicted to predict worse than for cursedness (then, cursedness has descriptive adequacy in three out of four conditions, see above),[30] but if subjects are experienced, the circumstances seem to favor risk aversion (which fits in two out of the three conditions then). The respective section of Table 5 shows that these relations are significant in the data. That is, risk aversion predicts well for experienced subjects but less so for inexperienced ones, and cursedness predicts well for inexperienced subjects but not for experienced ones. Surprisingly, in the cases where these concepts do not predict well, they actually predict poorly in an absolute sense—their predictive adequacies fall below that of mixed logit equilibrium, and thus their structural explanations actually have negative validity for the respective levels of experience. In turn, the predictive adequacy of projection is significantly higher than each of these if subjects are experienced, it is about as high as that of cursedness if subjects are inexperienced, it improves on mixed logit in all cases, it is as high as that of the model combining cursedness and risk aversion if subjects are inexperienced, and projection predicts best if subjects are experienced.

Result 8 (Predictive adequacy)**.** *For both inexperienced and experienced subjects, projection equilibrium belongs to the most predictive models in all cases, it predicts better than each model in at least one case, and it always improves on mixed logit. Extending projection by risk aversion does not improve its predictive adequacy.*

[29] Each of the four data sets for inexperienced subjects is used thrice in the out-of-sample evaluation, and each of the three data sets for experienced subjects is used twice. This measure is leaned on cross-validation (Browne, 2000) with nonrandom holdout samples (Keane and Wolpin, 2007).

[30] In addition to the common value auctions, cursedness fits as well as projection in the affiliated private value auctions of inexperienced subjects, where behavior is rather noisy and exhibits little overbidding.

6.3 Inferential adequacy in standard auctions

The inferential adequacy is similar to the predictive adequacy in that it is purely out-of-sample, but instead of determining log-likelihoods, we infer subject values in a procedure following Bajari and Hortacsu (2005). Given an observation and a set of parameters (estimated using training data), the theoretical bidding function for the respective out-of-sample treatment is determined and the expectation of the signal conditional on the observed bid is computed. This conditional expectation is called *inferred signal*. The inferential adequacy is the mean absolute deviation (MAD) to the actual signal. The supplementary material additionally also lists the results for the mean squared deviation (MSD), which are very similar. The inferential adequacy is complementary to the predictive adequacy due to its focus on the *expectation* of the underlying signal. That is, the first moment of its distribution needs to be predicted, rather than the full distribution. In turn, the likelihood allows one to substitute getting the moment right for getting the distribution right. Testing a concept's adequacy in predicting both first moment and full distribution therefore is challenging, but by its qualitative properties, I hypothesize projection to be adequate also in this dimension.

The results are presented in the third panel of Table 5. They are qualitatively similar to those for the predictive adequacy, which shows that projection adequately predicts both the first moment and the distribution, as predicted. The main differences to the previous observations can be observed for risk aversion and cursedness. Risk aversion fits rather poorly when focusing on the first moment, while cursedness fits rather well. In particular, risk aversion fits worse than projection now for both levels of experience, and also worse than cursedness, which in turn significantly improves on mixed logit for both levels (though not on projection).

Result 9 (Inferential adequacy)**.** *For both levels of experience, projection is one of the models exhibiting the highest inferential adequacy, and it improves on all alternative models for at least one of the levels of experience.*

6.4 Adequacy in non-standard auctions

I finally test the fit to behavior in the "non-standard" auctions with private values (Goeree et al., 2002b, GHP02) and common values (Avery and Kagel, 1997, AK97).[31] The

[31] The first-price IPV auctions of Goeree et al. (2002b) have two players. The values are drawn from $\{0,2,4,6,8,11\}$ or $\{0,3,5,7,9,12\}$. The former case is called "Low Signals" treatment, the latter is

Table 7: The models' adequacy to capture behavior in "non-standard auctions" (less is better)

	Mix Log		Mix RA		Mix C&RA		Mix P&RA		Mix Proj		Mix Curse
Descriptive Adequacy (Bayes Information Criterion)											
Inexperienced subjects											
AK97	553	$\ll$	556	$>$	546	$>$	537	$\gg$	534	$<$	542
GHP02	694	$\gg$	423	$\ll$	431	$\gg$	425	$<$	439	$\ll$	700
Experienced subjects											
AK97	524	$\approx$	525	$\approx$	519	$\approx$	519	$\gg$	515	$\approx$	517
GHP02	631	$\gg$	359	$\ll$	362	$>$	358	$>$	349	$\ll$	627
Predictive Adequacy (Non-standard $\rightarrow$ Standard; absolute value of log-likelihood)											
Inexperienced subjects											
AK97	4269	$>$	4256	$\gg$	4126	$>$	4027	$\ll$	4040	$\approx$	4080
GHP02	4252	$\gg$	3981	$\gg$	3834	$\ll$	3973	$\gg$	3842	$\ll$	4157
Experienced subjects											
AK97	4214	$\gg$	3836	$\gg$	3602	$\approx$	3574	$\ll$	3647	$\ll$	3888
GHP02	4261	$\gg$	3750	$\gg$	3595	$\gg$	3502	$\approx$	3474	$\ll$	3984

analysis of the discrete GHP02 auction is interesting, as projection partially predicts overbidding because of the assumption that outbidding opponents with similar values is possible at low costs in standard auctions. In the simple model of projection considered here, any increment of $\varepsilon > 0$ suffices if bids are continuous. If bids are discrete, opponents bidding 2 monetary units, such as those with signals 4 or 5 in GHP02, need to be outbid by 50% to break the tie. If the signal is just 4, breaking the tie reduces the expected profits from 2 units (bidding 2 with signal 4) to 1 unit (bidding 3). Since this is just a two-player auction, the player is indifferent in this case. In equilibrium, projecting players still overbid with positive probability, even with ρ-projection and $\rho < 1$. In general, though, overbidding is predicted to be more limited than in standard auctions, and more pronounced for players with high values. Qualitatively, this matches the observations of GHP02.

The common value auction of AK97 is non-standard in the sense that the common value is simply the sum of (two) independent signals. The BNE prediction is to bid twice the own signal, which is the expectation of the common value in case one is just able to win the auction (i.e. if both signals are equal). A fully projecting player always assumes the opponent's signal is equal to the own signal. The expected object value in this case is equal to the own signal (x_i) plus the average opponent's actual signal (2.5), yielding the conditional expectation $x_i + 2.5$. This equates with the conditional expectation of a fully cursed player. More generally, a χ-cursed player has expectation $\chi(x_i + 2.5) + (1 - \chi)(2x_i)$, which he also bids in AK97's second price auction

called "High Signals" treatment. The unique BNE equilibrium bids are $\{0, 1, 2, 3, 4, 5\}$ in either case. Avery and Kagel (1997) implemented second-price common-value auctions of two players. The players draw independent signals $X_i \sim U[1, 4]$ and the common value common value is $v = X_1 + X_2$ (in their "symmetric case", on which I focus). The BNE strategy is $b(x_i) = 2x_i$.

(Crawford and Iriberri, 2007, provide further details). A ρ-projecting player has the corresponding expectation $x^* = \rho(x_i + 2.5) + (1 - \rho)(2x_i)$ conditional on winning all ties, but bidding this value is winning only with probability of $\rho/2$ in the projection case (if the opponent bids x^*, too). Conditional on this, the projected expectation is $(\rho/2)(x_i + 2.5) + (1 - \rho/2)(2x_i)$, i.e. higher than x^* if $x_i > 2.5$. This generates an incentive to outbid the opponent—but outbidding x^* throws him back to winning all ties and thus to the conditional expectation x^*. In equilibrium, the projecting player uses a mixed strategy with support between the cursed equilibrium $(x_i + 2.5)$ and the BNE $(2x_i)$. Thus, a projecting player underbids the BNE if $x_i > 2.5$, and similarly, he outbids the BNE if $x_i < 2.5$. Overall, this pattern matches AK97's observations.

Table 7 presents the quantitative adequacies to explain these observations. I focus on descriptive and predictive adequacy, of which the latter concerns the adequacy of the estimates taken from these non-standard auctions in predicting the pooled standard auctions. This appears to be the main direction of interest and the predictive adequacy in the other direction (from standard toward non-standard) is largely insignificant due to the smaller sizes of the non-standard data sets. Information about the fit in this second direction and the inferential adequacy is provided as supplementary material.

Result 10 (Non-standard auctions)**.** *Projection fits about as well as risk aversion in the non-standard private value auction, about as well as cursed equilibrium in the non-standard common-value auction, and in all cases, about as well as these concepts combined. It predicts better than each of these models in at least one of the cases.*

7 Conclusion

The purpose of the paper is to model and test projection of types in Bayesian games. It is well-established in psychological research that people project their preferences and beliefs onto others in cases of incomplete information, and thus, we may plausibly expect that projection affects behavior also in games. First, I define type projection equilibrium, leaving the choice of the degree of projection to the analyst, and show that a projection equilibrium exists for all degrees $\rho \in [0, 1)$. Second, its predictions were tested mainly on a class of Bayesian games, auctions, where projection was perhaps least expected to affect behavior—despite the large amounts of studies dedicated to either, auctions in economics and projection in psychology. To my knowledge, the only paper mentioning a potential link between bidding and projection is Engelmann and Strobel (2012).

I studied the properties of projection equilibrium in auctions, tested the testable implications, and analyzed the overall fit to the data. Projection equilibria tend to be mixed and left-skewed, and in each information condition, projection fits about as well as the best existing explanation—risk aversion and cursed equilibrium, for private and common values, respectively. These existing explanations of overbidding are orthogonal, however, i.e. depending on information condition a different explanation is being called upon. Projection, in turn, offers a unified explanation based on a psychologically well-founded concept, and as an advantage, it offers an explanation that is overall more descriptive and robust. Thus, we may conclude that projection of values affects behavior in auctions and that analyses of bidding seem to benefit from considering projection as a potential explanation. This corroborates independent findings that negotiating subjects project their willingness-to-pay and their reservation prices onto their opponents, and was further validated by showing that projection of preferences helps explain the within-subject variance of behavior in simple distribution games.

This suggests that type projection is relevant in all games with incomplete information—assuming the player types are symmetric ex ante. This symmetry assumption was made partially for simplicity, but it also reflects the range of interactions where psychologists studied projection and false consensus effects. In interactions of distinctively asymmetric players, e.g. buyers and sellers, or informed and uninformed players, the alternative concepts of cursed equilibrium (Eyster and Rabin, 2005) and information projection (Madarász, 2012) appear to be more intuitive starting points.

From a more general perspective, three points may be worth noting. First, projection affects behavior in games, but subjects tend not to fully project their types. While this may not be surprising to psychologists, who studied projection in decision-theoretic frameworks, the comprehensive analysis of projection in an important class of games is novel, and the empirical content of projection is surprisingly high. Further, experimental work in economics tends to attribute most of the deviations from Nash equilibrium to either preferences, such as risk aversion or inequity aversion, or belief asymmetry, such as level-k. Needless to say, each of these intuitions impacts behavior in general, but projection should not be neglected as a confound simply because the literature focused on other issues so far. Second, practitioners of auction theory may consider projection at least alongside risk aversion as an explanation of overbidding. This has both a downside and an upside. On the downside, projection equilibria are mixed and their computation may require information that analysts do not immediately have, e.g.

the upper bound of values in private value auctions. Less information is required if one is willing to assume Bayesian Nash equilibrium and thus to neglect projection (for a review, see Bajari and Hortacsu, 2005). This assumption is highly debatable, though, as Crawford and Iriberri (2007) challenge the equilibrium assumption and my results challenge the neglect of projection. Further on the upside, projection equilibria fit more robustly than BNE with risk aversion across information conditions, which suggests that they are less prone to misspecification of the information conditions and the value parameters. It is unclear which of these effects dominates in which conditions, but it is probably not always the first one.

Finally, Engelmann and Strobel (2012) have shown that subjects are less likely to project if they are provided with the objective information in the best possible way. This suggests that the fallacy to projection may be subject to policy intervention, even if the best way of providing information is not obvious in all cases. Further, to the degree that overbidding is due to risk aversion, information does not help efficiency. To the degree that overbidding is due to projection, educating subjects increases the efficiency in at least two ways: Subjects stop randomizing in equilibrium, which ensures that the bidder with the highest value wins, and in cases where not just the winners pay their bids (e.g. contests), a reduction of overbidding increases efficiency. Thus, the above findings also have novel policy implications.

A Relegated proofs

Proof of Proposition 1

Consider the Bayesian game $\tilde{\Gamma} = \langle N, (A_i)_{i \in N}, T_0, (T_i)_{i \in N}, p, \rho, (u_i)_{i \in N} \rangle$ with the following move order: (1) With probability $1-\rho$, Nature sends all players individual signals (with joint distribution p), and with probability ρ, Nature sends all of the players the same signal (drawn from any of the marginal distributions p_i, which are equal by type-symmetry). (2) The players observe their signals and simultaneously choose actions (a_i). The expected payoff of $i \in N$ in $\tilde{\Gamma}$, given action a_i, type t_i, and opponents' strategies σ_{-i}, is

$$\tilde{\pi}_i\big(a_i|t_i,\sigma_{-i}\big) = (1-\rho) \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} p(t_{-i}|t_i)\; u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big] \prod_{j\neq i} \sigma_j(a_j|t_j)$$
$$+\rho \sum_{a_{-i}\in A_{-i}} u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big] \prod_{j\neq i} \sigma_j(a_j|t_i).$$

As t_i has the same distribution in either case and $\sum_{t_{-i}} p(t_{-i}|t_i) = 1$, we obtain

$$\tilde{\pi}_i\big(a_i|t_i,\sigma_{-i}\big) = (1-\rho) \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} p(t_{-i}|t_i)\; u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big] \prod_{j\neq i} \sigma_j(a_j|t_j)$$
$$+\rho \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} p(t_{-i}|t_i)\; u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big] \prod_{j\neq i} \sigma_j(a_j|t_i)$$

and thus

$$\tilde{\pi}_i\big(a_i|t_i,\sigma_{-i}\big) = \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} p(t_{-i}|t_i)\; u_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big]\, \tilde{\sigma}_{-i}(a_{-i}|t_{-i},t_i)$$

using $\tilde{\sigma}_{-i}$ as defined in Eq. (1). Hence, the expected utility of $i \in N$ in $\tilde{\Gamma}$ equals i's expected utility under ρ-projection in Γ, and any Bayesian Nash equilibrium of $\tilde{\Gamma}$ is a ρ-TPE of Γ. The finiteness of Γ implies finiteness of $\tilde{\Gamma}$ and thus existence of a Bayesian Nash equilibrium of $\tilde{\Gamma}$, which in turn implies existence of a ρ-TPE of Γ. □

A.1 Proof of Proposition 2

Fix any symmetric ρ-TPE. It has already been established that σ must be mixed and have increasing density on its support. It remains to characterize its bounds. First, consider the lower bound $\underline{r}$ of the support. The directional derivative with respect to $dr < 0$

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r=\underline{r},dr<0} = (1-\rho)\,\tilde{\Pi}'(r|\sigma)$$

must be non-negative. Otherwise, one benefits through deviating by putting probability mass on bids $r < \underline{r}$. Thus, $\tilde{\Pi}'(\underline{r}|\sigma) \geq 0$. Second,

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r=\underline{r},dr>0} = \rho\,(\tilde{V}-r)\,\sigma(r) + (1-\rho)\,\tilde{\Pi}'(r|\sigma)$$

must be zero, since σ is mixed. Further, $\tilde{V} - r > 0$; otherwise the assumption that $\tilde{v}$ is increasing would imply $\tilde{\Pi}_\rho(\underline{r}|\sigma) < 0$, contradicting the assumption that σ is an equilibrium. Hence, $\tilde{\Pi}'(\underline{r}|\sigma) = 0$ and $\sigma(\underline{r}) = 0$.

Second, I show that this implies $\underline{r} \geq r^{BNE}$. Let $BR : \Delta R \to \mathcal{P}(R)$ denote the best-response correspondence (allowing for mixed strategies as arguments) of the auction without projection. For simplicity, let $BR(r')$ also denote the best response to the pure strategy r'. Define $r := BR(\underline{r})$. Strategic complementarity implies $d\tilde{\Pi}(r|r')/dr > 0$ for all $r' > \underline{r}$. Hence, $d\tilde{\Pi}(r|\sigma)/dr > 0$ and $\inf BR(\sigma) > BR(\underline{r})$. Since we know $\underline{r} = \inf BR(\sigma)$, we obtain $\underline{r} > BR(\underline{r})$, and by strategic complementarity $\underline{r} > r^{BNE}$.

Third, I characterize the upper bound $\bar{r}$. To begin with, $\underline{r} \geq r^{BNE}$ implies $\bar{r} \geq r^{BNE}$, and the fact that it is a best response to σ implies that it yields non-negative expected payoffs. That is

$$\tilde{\Pi}_\rho(\bar{r}|\sigma) = (1-\rho)\,\tilde{\Pi}(\bar{r}|\sigma) + \rho\,(\tilde{V} - \bar{r})F_\sigma(r) \geq 0.$$

Since $\tilde{v}$ is non-decreasing, $\tilde{V} - \bar{r} < 0$ implies $\tilde{\Pi}(\bar{r}|\sigma) < 0$. Thus, $\tilde{\Pi}_\rho(\bar{r}|\sigma) \geq 0$ implies $\tilde{V} - \bar{r} \geq 0$ for all $\rho \geq 0$. □

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