Investor Sentiment and Irrational Speculative Bubble Model

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Abstract

This paper dynamically extends the noise trading model (DSSW model) via describing the limited rational investors' sentiment more specifically, and using the bipolar sigmoid activation function in the neural network system to depict noise traders' overreaction to the past changes of fundamental value. And then we construct an irrational speculative bubble model according to some relevant theoretical hypothesis, which can measure the scale of stock market bubbles precisely. Moreover, we also explore the plausible rang of speculative bubbles on the basis of the irrational bubble model. Finally, we can conclude from the results of corresponding simulations that the existence of irrational bubbles in the market is strongly linked to noise traders' misperceptions and their inherent sentiments during the investment, as well as their overreaction to the historical impacts of fundamental value. Particularly, we find that, under the condition of given simulation parameters, the larger the proportion of noise traders exists in the market, the higher the degree of irrational speculative bubbles is included in the risky assets, and the more violent the fluctuations of stock market bubbles are.

Keywords: Noise traders; Investor sentiment; Irrational speculative bubbles; Behavioral finance;

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1. Introduction

Black (1986) first introduced the concept of noise into bubble theory, he thought that noise trading makes the market price become noise, so it can't fully reflect the content of the information contained, which greatly reduces the effectiveness of the market. In stock trading, noise traders constantly accumulate the "noise" to the stock price, which leads it to deviate from its intrinsic value and imply a definite bubble component. This is how the stock bubbles come into being. Shiller (1984, 1990) and Summers (1986) established a Fashion model, which gives an explanation of speculative bubble formation. Shiller (1984) thought that stock prices are highly susceptible to pure fashion trends and the social dynamics, and the close attention between investors may cause stock bubbles. Summers (1986) considered that the deviation between asset price and market fundamental value is produced from the change of investor sentiment or fashion. Shiller (2000) analyzed the stock market speculative bubbles from the angle of investor's psychology, he recognized that investors' psychological reliance, herd mentality and the feedback loop mechanism will result in the stock market bubbles. De Long、Shleifer、Summers and Waldmann (1990) (hereafter DSSW) creatively established the noise trading model (DSSW model), they explained the continuous deviation of stock market price relative to the fundamental value from the perspective of microscopic behavior, and argued that there are two main reasons for the formation and continuation of bubbles, one is the unpredictable of noise traders' beliefs which brings a risk to asset price, the other is arbitrageurs' risk aversion and short-horizon which limit their abilities to correct the mispricing. Binswanger (1999) dynamically extended the DSSW model in the case of the fundamental value is invariant and changed respectively. Since considering the change of fundamental value in the stock market, it makes the DSSW model more realistic to describe the evolutionary process of speculative bubbles. Based on the extended DSSW model, Wang and Yang (2005) introduced the noise traders' overreaction to the historical impacts of fundamental value into their model to describe the formation of irrational bubbles. They found that the irrational bubbles are related to such factors as noise traders' misperceptions and overreactions. And given certain parameters, the proportion of noise traders affects the scale of irrational bubbles.

In particular, DSSW (1990) argued that higher investor sentiment generates more noise trading. This is because investor sentiment indicates noise traders' misperceptions of the expected price of the risky asset and, as demonstrated by DSSW, noise traders' demand for securities is proportional to their sentiments. Other researchers such as Trueman (1988), Shleifer and Summers (1990), Lakonishok et al. (1992), Campbell and Kyle (1993), Shefrin and Statman (1994), Palomino (1996), Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999) provided the theoretical framework to describe the role of investor sentiment in determining stock prices. These studies believed that noise traders, who do not make investment based on the fundamental value of company, are capable of affecting stock prices by way of unpredictable changes in their responds. On the basis of DSSW model and relevant theories, several empirical studies examined the influence of investor sentiment on stock returns (De Bondt, 1993; Clarke and Statman, 1998; Fisher and Statman, 2000; Lee et al., 2002; Brown and Cliff, 2004, 2005; Verma et al., 2008). These studies provided evidence for the existence of strong co-movements between individual and institutional investor sentiment and stock market returns. Only a few papers have investigated the relationship between investor sentiment and stock volatility (Brown, 1999; Lee, Jiang and Indro, 2002; Wang, Keswani and Taylor, 2006).

Throughout the above literatures, these scholars have studied irrational speculative bubbles from various angles, and obtained some preliminary achievements. On the one hand, they derived and verified the objective existence of speculative bubbles in asset prices, and further discussed the causes of the formation of irrational speculative bubbles. On the other hand, they preliminarily built a series of irrational speculative bubble models, and explained the evolution of irrational speculative bubbles from different perspectives. However, there also exist some deficiencies in the past studies of irrational speculative bubbles, which mainly reflect in the following aspects: first of all, due to the emergence and development of the stock market bubbles are not only affected by macroeconomic factors, also restricted by the micro investors' psychologies. Therefore, studying the speculative bubbles from the aspect of investor psychology is of great significance. In recent decades, the rise of behavioral finance provides a bridge and a link to the combination of psychology and finance. Throughout the current research achievements of irrational speculative bubbles, the combination is inadequate. Because they can be thought of as putting forward the phenomenon of behavior with the combination of the psychology, but didn't put forward it in nature. Second, the existing irrational bubble models were focused on the market investors' different opinions on the future trend of stock market, and they only recognized the impact of investor sentiment on the stock market bubble (or price), but didn't specifically measure the investor sentiment. While in the factors which will affect the formation of irrational speculative bubbles, investor sentiment is often occupies a pivotal position. So constructing the irrational bubble model combined with the characteristic of investor sentiment can help to better study the emergence and development of irrational speculative bubbles. Third, the bursting of stock market bubbles will bring very bad influence on all aspects of economy, social life, and the whole society. Thus researching the recognition and plausible range of speculative bubbles, and the method of pre-controlling the bubbles are of great significance. But the current researches are lack of them and mainly focus on examining the existence of speculative bubbles.

Given all this, we expand the extended DSSW model from two main aspects in this paper. First, in terms of investor sentiment, this paper argues that it can affect not only the noise traders' bullish or bearish on stock price, but also their levels of risk aversion. Barberis, Huang and Santos (2001) (hereafter BHS) proposed and calibrated a model in which investors have linear loss aversion preferences and derived gain-loss utility only over fluctuations in financial wealth. BHS (2001) pointed out that, if an investor cumulates his gains and losses, value inflection would seem to imply that he is more likely to take risk after a series of good outcomes, and less likely after a series of bad outcomes. Follow the BHS model, we posit an monotone decreasing function which is regard to investor sentiment to measure the degree of noise traders' risk aversion. When noise traders are bullish (bearish), they think the possibility of future stock market gains (losses) is big, so their degree of risk aversion is low (high). Another striking feature of our model is that we explicitly measure noise traders' overreaction to the past changes of stock fundamental value by means of using the bipolar sigmoid activation function in the neural network system to replace the corresponding normal random variable in the extended DSSW model proposed by Binswanger(1999). This function measures the degree of the investors' overreaction to historical information of fundamental value from two aspects: gains and losses, further explains the impact of investors' overreaction on the formation of speculative bubbles. And then we construct an irrational speculative bubble model on the basis of the noise traders' sentiment function and their overreaction function. Via these two extensions, the irrational speculative bubble model shows the effect of noise traders' misperceptions and intrinsic sentiments, and their overreaction to the historical shocks of the fundamental value on the formation and persistence of stock market bubbles. Since the model is a blend of factors such as investors' sentiment and their instinctive reaction, it can describe the evolution of the irrational speculative bubbles more realistically and measure the scale of irrational bubbles more precisely. After that, based on the principle of the consistency of the level of stock market bubbles and economic development, we give the plausible scope of irrational speculative bubbles on the basis of our irrational bubble model, which provides a reference for judging the rationality of the development of stock market bubbles.

Finally, the research results indicate that the irrational bubbles are strongly linked to the noise traders' misperceptions and sentiments during investment, as well as their overreaction to the historical changes of fundamental value. Moreover, in order to explore the influence of the changing of the proportion of noise traders on speculative bubbles, we simulate the irrational bubble model with different proportion of noise traders. We find that, given the fixed parameters of simulation, with the increasing proportion of noise traders in the market, the scale of irrational speculative bubbles in risky asset is larger, and the stock market bubbles will fluctuate more dramatically. However, the crescent proportion of noise traders will not necessarily result in the gradually increment of irrational bubbles.

The remainder of this paper is structured as follows. In the next section, we construct the extend DSSW model. And the irrational bubble model follows in Section 3. Section 4 presents the plausible range of the stock market bubbles. Section 5 shows the multi-period extension of irrational bubble model and simulation. Section 6 summarizes our conclusions.

2. The extend DSSW model

DSSW (1990) separated the investors in the market into sophisticated investors and noise traders, and put forward the noise trading model to analyze the level of asset pricing. They assumed that all investors are risk aversion and used a stripped down overlapping generations model with two-period lived agents to describe the issues. And for simplicity, there is no first period consumption, no labor supply decision, and no bequest in their model. The only decision agents make is to choose a portfolio when young.

As the DSSW model, we assume that there are two types of traders in the market: informed traders (denoted by the superscript "i") and noise traders (denoted by the superscript "n"). Among them, the informed traders who have rational expectations about the future price of risky asset are homogeneous, and they also treat the changes of fundamental value rationally. While noise traders are heterogeneous, who tend to predict asset prices according to some pseudo-signals from technical analysts, stock brokers, or economic consultants and irrationally believe that these signals carry information. Similarly, there is assumed to be a proportion μ of noise traders inhabiting the market, leaving the remainder of the market to be populated by a proportion $1-\mu$ of informed traders.

The market contains two assets that pay identical dividends. One of the assets, the safe asset (s), pays a fixed riskless rate r. Asset (s) is in perfectly elastic supply: a unit of it can be created out of and a unit of it turned back into a unit of the consumption good in any period. Taking consumption each period as numeraire, the price of the safe asset is always fixed at one. The other asset, the unsafe asset (u), pays the same fixed real dividend r as asset (s). But (u) is not in elastic supply: it is in fixed and unchangeable quantity, normalized at one unit. The price of (u) in period t is denoted by p_t. We usually interpret (s) as a riskless short-term bond and (u) as aggregate equities. Both types of traders meet in a simple two-period model in which they choose their portfolios when young to maximize perceived expected utility given their own beliefs about the ex-ante mean of the distribution of the price of (u) as of time $t+1$. When old, they must transform their holdings of (s) into consumer goods, sell their holdings of (u) for price p_{t+1} to next generation and consume away all the wealth. Note that this type of "overlapping generation" framework means that although individual lives are described in a very stylized way, the continuity of financial markets can still be captured.

The representative sophisticated investor young in period t accurately perceives the distribution of returns from holding the risky asset, and so maximizes expected utility given that distribution. While, the representative noise trader young in period t misperceives the expected price of the risky asset (u) by a time-varying amount ρ_t. ρ_t is an i.i.d. normal random variable, whose mean is denoted by $\rho*$ and variance is denoted by σ_ρ^2, i.e. $\rho_t \sim N\left(\rho*, \sigma_\rho^2\right)$. where $\rho*$ is the average amount of "bullishness/bearishness" of the noise traders around which individual values of ρ_t vary each time period, and reflects the noise traders' optimistic or pessimistic mood. And σ_ρ^2 is the variance of noise traders' misperceptions of the expected return per unit of the risky asset. Note that all variance in the price of the unsafe asset (u) is induced by variations in the misperceptions of price held by noise traders, ρ_t.

Each informed trader's utility is a constant absolute risk aversion function of wealth when old: $U\left(\omega^i\right)=-e^{-(2\gamma)\omega^i}$. Where γ is the coefficient of absolute risk aversion, ω^i is informed traders' wealth. When the returns of risky assets is normal distribution, the expected utility function is equivalent to $E\left[U\left(\omega^i\right)\right]=\bar{\omega}^i-\gamma\sigma^2_{\omega^i}$. Where $\bar{\omega}^i$ is informed traders' final expected wealth, $\sigma^2_{\omega^i}$ is the one-period ahead variance of expected wealth.

However, in consideration of noise traders' optimistic or pessimistic mood may influence their degree of risk aversion to invest risky assets, we assume that their risk aversion function is $\gamma g(s)$, and then the noise traders' expected utility function is $U\left(\omega^n\right)=-e^{-\left[2\gamma g(s)\right]\omega^n}$. Where $g(s)$ is the influence function of noise traders' sentiment, $g(s)=\frac{1}{\mathrm{n}}\sum_{i=1}^{n}g\left(s_i\right)$, n is the amount of noise traders in the market, s_i is the sentiment of noise trader $\left(\mathrm{n}_i\right)$ during investment. When $s_i=0$, denotes that the noise traders is not with their own emotions in investment. When $s_i>0$, denotes that they hold optimism about investment. On the contrary, when $s_i<0$, denotes that they hold pessimism. BHS (2001) argued that investor is more likely to take risk after a series of gains, and less likely after a series of losses. So when noise traders are bullish (or optimistic), they are more willing to invest. Then according to BHS model, we let $g\left(s_i\right)>0$, and posit $g\left(s_i\right)$ is a monotone decreasing function. Which means the higher the investor sentiment is, the closer the value of $g\left(s_i\right)$ to zero. Particularly, when noise trader without any emotion, i.e. $s_i=0$, then $g(0)=1$, which means the level of noise trader's risk aversion equals to that of informed trader during investment.

Different with the DSSW model, Wang and Yang (2005) emphasized that the fundamental value of risky asset (u) will change, then the corresponding dividend it pays is $r+\varepsilon_t$. Where ε_t is an i.i.d. normal random variable, $\varepsilon_t\sim N\left(0,\sigma^2_{\varepsilon}\right)$. Due to

ε_t is only recognized by informed traders as of time t, so it only directly affect the expected utility of informed traders but not the noise traders'. They also assumed that there is a change of fundamental value as of time $t-1$, which is denoted by ε_{t-1}. Noise traders realized this historical impact, and overreacted to this information, which is measured by the overreaction coefficient θ. In addition, the noise traders will still misperceive the expected price of the risky asset, which is also denoted by ρ_t, but it won't include the noise trader's misperception of the shock of fundamental value.

It is worth noting that, different with the setting of overreaction coefficient by Wang and Yang (2005), this article follow the command from investors' psychological reaction -- the reaction of brain neurons, and use the bipolar sigmoid activation function in the neural network system to depict noise traders' average overreaction to the change of fundamental value. And the expression of sigmoid function is:

$$y = h(u) = \frac{1-e^{-\lambda u}}{1+e^{-\lambda u}} = 1 - \frac{2}{1+e^{\lambda u}}$$

Where parameter λ is the gain of sigmoid function, whose value determines the slope of unsaturated section of the function. And the greater the value of λ is, the steeper the curve is.

As for the dividend payment of risky asset, we also assume the dividend it pays is $r+\varepsilon_t$. Where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. But we creatively consider noise traders' average overreaction to the changes of fundamental value during the past m periods, then the overreaction function can be expressed as:

$$h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right) = k \cdot \left(1 - \frac{2}{1+\exp\left(\lambda \cdot \frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)}\right)$$

Where $k \geq 1$, $\lambda > 0$.

Synthesize above assumptions, the objective of the informed traders is to choose the amount of risky assets, ϕ_t^i, to maximize their expected utility function:

$$E\left[U\left(\omega^i\right)\right]=\bar{\omega}^i-\gamma\sigma_{\omega^i}^2=c_0^i+\phi_t^i\left[r+\varepsilon_t+{}_tp_{t+1}-\left(1+r\right)p_t\right]-\gamma\left(\phi_t^i\right)^2\left({}_t\sigma_{p_{t+1}}^2\right) \quad (1)$$

Where c_0^i is a function of informed traders' first-period labor income, ${}_tp_{t+1}$ denotes the informed traders' rational expectation to p_{t+1} as of time t, ${}_t\sigma_{p_{t+1}}^2$ denotes the one-period variance of p_{t+1}, which is defined as ${}_t\sigma_{p_{t+1}}^2=E_t\left\{\left[p_{t+1}-E_t\left(p_{t+1}\right)\right]^2\right\}$.

Similarly, on the premise that the distribution of wealth is normal distribution, the noise traders' expected utility function can be defined as:

$$E\left[U\left(\omega^n\right)\right]=-\int e^{-\left[2\gamma g(s)\right]\omega^n}f\left(\omega^n\right)d\omega^n=-e^{-2\gamma g(s)\left(\bar{\omega}^n-\gamma g(s)\sigma_{\omega^n}^2\right)} \quad (2)$$

Where $f\left(\omega^n\right)=\left(\sqrt{2\pi}\sigma_{\omega^n}\right)^{-1}e^{-\left(\omega^n-\bar{\omega}^n\right)/\left(2\sigma_{\omega^n}^2\right)}$ is the probability density function of wealth, which is normally distributed, i.e. $\omega^n\sim N\left(\bar{\omega}^n,\sigma_{\omega^n}^2\right)$. Due to the expected utility function is the increasing function of $\left(\bar{\omega}^n-\gamma g\left(s\right)\sigma_{\omega^n}^2\right)$, so equation (2) can be converted into $E\left[U\left(\omega^n\right)\right]=\bar{\omega}^n-\gamma g\left(s\right)\sigma_{\omega^n}^2$. And the objective of noise traders is to choose the amount of risky assets, ϕ_t^n, to maximize their expected utility function:

$$E\left[U\left(\omega^n\right)\right]=\bar{\omega}^n-\gamma g\left(s\right)\sigma_{\omega^n}^2=c_0^n+\phi_t^n\left[r+h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}+{}_tp_{t+1}+\rho_t-\left(1+r\right)p_t\right]$$
$$-\gamma g\left(s\right)\left(\phi_t^n\right)^2\left({}_t\sigma_{p_{t+1}}^2\right) \quad (3)$$

Where c_0^n is the function of noise traders' first-period labor income, $h\left(\cdot\right)$ is the noise traders' overreaction to the historical changes of fundamental value, ρ_t is noise traders' misperceptions of the expected price of the risky asset, and $g\left(\cdot\right)$ is the influence function of noise traders' sentiment.

To obtain the optimum allocation of the portfolio to the unsafe asset for informed traders and noise traders respectively, we need to maximize the equation (1) and (3) with respect to the proportion in the total market portfolio of the unsafe asset as of time t held by informed traders and noise traders respectively, i.e. ϕ_t^i and ϕ_t^n. Then we have:

$$\phi_t^i = \frac{r + {}_t p_{t+1} - (1+r) p_t}{2\gamma\left({}_t\sigma^2_{p_{t+1}}\right)} + \frac{\varepsilon_t}{2\gamma\left({}_t\sigma^2_{p_{t+1}}\right)} \tag{4}$$

$$\phi_t^{\mathrm{n}} = \frac{r + {}_t p_{t+1} - (1+r) p_t}{2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)} + \frac{h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j} + \rho_t}{2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)} \tag{5}$$

Recalling the restriction that we weighted sum of total asset demands must equal one, i.e. $(1-\mu)\phi_t^i + \phi\lambda_t^n = 1$. Then combining equation (4) and (5) can yield the expression for the equilibrium price of risky asset (u):

$$p_t = \frac{r + {}_t p_{t+1}}{1+r} + \frac{(1-\mu) g(s)\varepsilon_t + \mu\left[h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j} + \rho_t\right] - 2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)}{\left[(1-\mu) g(s) + \mu\right](1+r)} \tag{6}$$

Besides period t's misperception by noise traders (ρ_t), the technological (r) and behavioral (γ) parameters of the model, and the moments of one-period ahead distribution of p_{t+1}, Equation (6) also expresses the risky asset's price in period t as a function of noise traders' average sentiment $g(s)$, and their overreaction to the changes of fundamental value during the past m periods. According to the calculation method of DSSW (1990), We also consider only steady-state equilibria by imposing the requirement that the unconditional distribution of p_{t+1} be identical to the distribution of p_t. The endogenous one-period ahead distribution of the price of asset (u) can then be eliminated from (6) by solving recursively.

Specifically, we denote $\Lambda = (1-\mu) g(s) + \mu$, $h(\overline{\varepsilon}_t) = h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}$, then equation (6) can be represented as:

$$p_t = \frac{1}{\Lambda(1+r)}\left[\Lambda r + \Lambda\left({}_t p_{t+1}\right) + \mu h(\overline{\varepsilon}_t)\cdot\overline{\varepsilon}_t + \mu\rho_t - 2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)\right]$$

$$\Rightarrow p_t=\frac{1}{\Lambda(1+r)}\left\{\begin{array}{l}\Lambda r+\dfrac{\Lambda}{\Lambda(1+r)}\left[\Lambda r+\Lambda\left({}_{t+1}p_{t+2}\right)+\mu h(\bar{\varepsilon}_{t+1})\cdot\bar{\varepsilon}_{t+1}+\mu\rho_{t+1}-2\gamma g(s)\left({}_{t+1}\sigma^2_{p_{t+2}}\right)\right]\\+\mu h(\bar{\varepsilon}_t)\cdot\bar{\varepsilon}_t+\mu\rho_t-2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)\end{array}\right\}$$

$$\Rightarrow p_t=\frac{1}{\Lambda(1+r)}\left\{\begin{array}{l}\Lambda r+\dfrac{\Lambda}{\Lambda(1+r)}\left\{\begin{array}{l}\Lambda r+\dfrac{\Lambda}{\Lambda(1+r)}\left[\Lambda r+\Lambda\left({}_{t+2}p_{t+3}\right)+\mu h(\bar{\varepsilon}_{t+2})\cdot\bar{\varepsilon}_{t+2}+\mu\rho_{t+2}-2\gamma g(s)\left({}_{t+2}\sigma^2_{p_{t+3}}\right)\right]\\+\mu h(\bar{\varepsilon}_{t+1})\cdot\bar{\varepsilon}_{t+1}+\mu\rho_{t+1}-2\gamma g(s)\left({}_{t+1}\sigma^2_{p_{t+2}}\right)\end{array}\right\}\\+\mu h(\bar{\varepsilon}_t)\cdot\bar{\varepsilon}_t+\mu\rho_t-2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)\end{array}\right\}$$

$$\cdots\cdots$$

In this way, p_t becomes a weighted average of the following parts after discount: a. the risk-free rate (r) of assets; b. the expected price of unsafe asset in the next period, such as ${}_tp_{t+1}$, ${}_{t+1}p_{t+2}$, ${}_{t+2}p_{t+3}$, etc; c. the current forecast of future volatility of asset price, such as ${}_t\sigma^2_{p_{t+1}}$, ${}_{t+1}\sigma^2_{p_{t+2}}$, ${}_{t+2}\sigma^2_{p_{t+3}}$, etc; d. noise traders' misperception of the current and future price of risky asset, such as $\mu\rho_t$, $\mu\rho_{t+1}$, $\mu\rho_{t+2}$, etc.

Through forward iterating the expression of p_t infinitely, we can get the equilibrium price of risky asset in period t *:

$$p_t=1+\frac{(1-\mu)g(s)\varepsilon_t}{\left[(1-\mu)g(s)+\mu\right](1+r)}+\frac{\mu h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}}{\left[(1-\mu)g(s)+\mu\right](1+r)}+\frac{\mu(\rho_t-\rho^*)}{\left[(1-\mu)g(s)+\mu\right](1+r)}$$
$$+\frac{\mu\rho^*}{\left[(1-\mu)g(s)+\mu\right]r}-\frac{2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)}{\left[(1-\mu)g(s)+\mu\right]r}\qquad(7)$$

Due to the one-step ahead variance of p_t is an unchanging function of the constant variance of the change of fundamental value (ε_t) and a generation of noise traders' misperception (ρ_t), and the product of σ^2_ε and the square of overreaction function $\left(h^2(\cdot)\right)$:

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* The expression of informed traders' expectation to p_{t+1} at time t is ${}_tp_{t+1}=1+\dfrac{\mu\rho^*-2\gamma g(s)\left({}_t\sigma^2_{p_{t+1}}\right)}{\left[(1-\mu)g(s)+\mu\right]r}$.

$$
{}_t\sigma^2_{p_{t+1}} = \sigma^2_{p_{t+1}} = \frac{(1-\mu)^2\sigma^2_\varepsilon + \mu^2\left[\sigma^2_\rho + h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\sigma^2_\varepsilon\right]}{(1+r)^2} \tag{8}
$$

Finally, substituting the equation (8) into (7) can derive the final form of the pricing rule for risky asset (u) as of time t:

$$
p_t = 1 + \frac{(1-\mu)g(s)\varepsilon_t}{\left[(1-\mu)g(s)+\mu\right](1+r)} + \frac{\mu h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}}{\left[(1-\mu)g(s)+\mu\right](1+r)} + \frac{\mu(\rho_t-\rho^*)}{\left[(1-\mu)g(s)+\mu\right](1+r)}
$$
$$
+\frac{\mu\rho^*}{\left[(1-\mu)g(s)+\mu\right]r} - \frac{2\gamma g(s)\left[(1-\mu)^2\sigma^2_\varepsilon + \mu^2\sigma^2_\rho + \mu^2h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\sigma^2_\varepsilon\right]}{\left[(1-\mu)g(s)+\mu\right](1+r)^2 r} \tag{9}
$$

The last five terms that appear in equation (9) show the impact of noise traders on the price of asset (u). As the distribution of ρ_t and ε_t converge to a point mass at zero, the equilibrium pricing function (9) converges to its fundamental value of one. The second and third term in (9) capture the fluctuations in the price of the risky asset (u) due to the changes of fundamental value, the noise traders' overreaction and sentiment respectively. The forth term in (9) captures the influence of the variation of noise traders' misperceptions of the expected price of risky asset. When a generation of noise traders is more "bullish" ("bearish") than the average generation, which means their sentiments are higher (lower), then the value of $g(s)$ is smaller (bigger), so they will bid up (down) the price of (u). The fifth term in (9) measures the deviations of p_t from its fundamental value due to the fact that the average misperception by noise traders is not zero. Similarly, if noise traders are "bullish" ("bearish") on average, this "price pressure" effect will make the price of the risky asset higher (lower) than it would otherwise be. The last term in (9) expresses the "price suppression" effect caused by risks, which include the risks from the changes of fundamental value and the noise traders' misperception of the future price of risky asset.

3. The irrational speculative bubble model

When informed traders and noise traders coexist in the market, the equilibrium price will be decided jointly by these two types of traders, so we can use both the proportion of risky assets they held and the corresponding prices they expected to denote it:

$$p_t = (1-\mu)\phi_t^i p_t^i + \mu\phi_t^n p_t^n = (1-\mu)\phi_t^i p_t^i + \mu\phi_t^n\left(p_t^i + b_t\right) = p_t^i + \mu\phi_t^n b_t \tag{10}$$

Where p_t^i is the reasonable price of risky asset when there are only informed traders in the market; p_t^n is the price of risky asset which contains the irrational speculative bubbles, and it's determined only by noise traders; b_t represents the irrational speculative bubbles which is generated when the noise traders are irrational; $\mu\phi_t^n b_t$ denotes the scale of irrational bubbles when these two types of traders coexist in the market, which is the difference between market equilibrium price and the reasonable price of risky asset. Fortunately, we can calculate the scale of irrational speculative bubbles contained in the market equilibrium price only by confirming the reasonable price of risky assets.

When there are only informed investors exist in the market, their utility function is still $E\left[U\left(\omega^i\right)\right]$, and the risk they meet only comes from the change of fundamental value. So we can calculate the price of risky asset decided by informed traders. Due to the amount of risky assets is fixed and standardized on one, so the supply of risky assets is equal to demand when the market is balanced, i.e. $\phi_t^i = 1$. Similar to above calculation method, we can derive the equilibrium price of risky assets: $p_t = \frac{1}{1+r}\left[r + \varepsilon_t + {}_t p_{t+1} - 2\gamma\left({}_t\sigma_{p_{t+1}}^2\right)\right]$. In view of the risk at the moment is only caused by the change of fundamental value, i.e. ${}_t\sigma_{p_{t+1}}^2 = \sigma_{p_{t+1}}^2 = \frac{\sigma_\varepsilon^2}{(1+r)^2}$, we can calculate the price of risky asset which is decided by informed traders through the method of recursive computation. Then we have:

$$p_t^i = 1 + \frac{\varepsilon_t}{1+r} - \frac{2\gamma\left({}_t\sigma^2_{p_{t+1}}\right)}{r} = 1 + \frac{\varepsilon_t}{1+r} - \frac{2\gamma\sigma_\varepsilon^2}{r(1+r)^2} \tag{11}$$

Finally, substituting the equation (9) and (11) into (10) can conclude the scale of irrational speculative bubbles at time t when the market is balanced:

$$\begin{aligned} B_t &= \mu\phi_t^n b_t = \mathrm{p}_t - p_t^i \\ &= \frac{\mu(\rho_t - \rho^*)}{\left[(1-\mu)g(s)+\mu\right](1+r)} + \frac{\mu\rho^*}{\left[(1-\mu)g(s)+\mu\right]r} - \frac{\mu\varepsilon_t}{\left[(1-\mu)g(s)+\mu\right](1+r)} \\ &+ \frac{\mu h\left(\frac{1}{m}\sum_{j=1}^m \varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^m \varepsilon_{t-j}}{\left[(1-\mu)g(s)+\mu\right](1+r)} - \frac{2\gamma\mu^2 g(s)\sigma_\rho^2 + 2\gamma\left[\mu^2 g(s)h^2\left(\frac{1}{m}\sum_{j=1}^m \varepsilon_{t-j}\right) - \mu(1-\mu)g(s) - \mu\right]\sigma_\varepsilon^2}{\left[(1-\mu)g(s)+\mu\right](1+r)^2 r} \end{aligned} \tag{12}$$

The five terms situated on the right side of equation (12) show the influence of noise traders on the scale of irrational speculative bubbles. Among them, the first term captures the impact of the variation of noise traders' misperceptions and their sentiment on the volatility of irrational speculative bubbles. For example, when a generation of noise traders are more "bullish" ("bearish") than the average generation, which means their sentiment are higher (lower), then the value of $g(s)$ is smaller (bigger). Finally, the interplay of these two factors will accelerate the expansion (contraction) of irrational speculative bubbles. Similarly, the second term shows the movements of irrational speculative bubbles caused by noise traders' average misperceptions and sentiments. Specifically, when $\rho* > 0$ ($\rho* < 0$), which means noise traders are "bullish" ("bearish") , then this optimistic (pessimistic) sentiment will lead to the expansion (contraction) of bubbles with the gradual increase (decrease) of $\rho*$ and the gradual decrease (increase) of $g(s)$. The third term mainly reflects the negative effect of the shock of fundamental value at time t on the bubbles, which noise traders didn't realized. When the shock of dividend is positive, informed traders will take active trading strategies, while noise traders won't take the corresponding trading strategies because their under-reaction to current information, which will cause the equilibrium price of risky asset can't fully reflect the positive shock brought by the fundamental value. At the same time, with the increase of the reasonable price of risky asset, the scale of irrational bubbles will relatively reduce. All this effect is because of the noise traders' under-reaction on the current shock of fundamental value. The forth term captures the fluctuations in the irrational speculative bubbles due to the changes of fundamental value during past m periods which noise traders have recognized, and the noise traders' overreaction to these changes and their sentiments. When the mean value of the changes of fundamental value during the past m periods is greater than zero, noise traders will take active trading strategies, which leads to the generation of irrational bubbles followed by the rising of equilibrium price. Note that the overreaction function $h(\cdot)$ is related to the speed of the expansion or contraction of the irrational bubbles. When the value of $h(\cdot)$ is greater than one, the noise traders' reaction to the shocks of fundamental value will increase multiply, which will result in the accelerating expansion or contraction of irrational bubbles. In particular, if noise traders find that the average changes of fundamental value during the past m periods is positive, they will mistakenly think that buying risky assets will profit and take active trading strategies, which leads to the rapid swelling of irrational bubbles with the exaggerated value of overreaction function. On the contrary, if the mean value of these shocks is negative, noise traders mistakenly believe that they are losing money and decide to hold the risky assets. As a result, following by the smaller value of overreaction function, the scale of irrational bubbles decrease quickly. In the end, the last item in equation (12) expresses the "bubble suppression" effect caused by risks, which include the risks from the changes of fundamental value and the noise traders' overreaction to these changes during past m periods, as well as their sentiments.

4. The plausible range of stock market bubbles

In many functions of the stock market, financing is one of the most basic functions among them. It is well known that support from the financing function of stock market to the economic development of a country is particularly important. Thus, we can't neglect the impact of the emergence of stock market bubbles on the investment and financing of whole market, as well as the development of the real economy. And so is the change of the scale of stock market bubbles. As inflation is a double-edged sword, a moderate amount of stock market bubbles are able to activate the investment and financing of the whole market, while large stock market bubbles will cause an devastating consequence on the stock market and real economy. Therefore, no matter how the scale of stock market bubbles is too large or too small, it will be not conducive to the healthy development of stock market and the real economy. In view of this, we consult the model proposed by Feng and Sun (2005) to explore the plausible range of stock market bubbles by the relationship between the stock market bubbles and the real economy.

From the perspective of speculation, we suppose that the objective of investors' participation in the stock trading is to gain from the spread of stock prices. On the one hand, because of the profit instinct of capital, the yield of the stock market must be higher than bonds and bank deposits or other investment channels which can attract capital, or it will not perform the market function of funding. On the other hand, the yield of the stock market can't exceed the ROE of industrial investments, the resources of the real economy will otherwise come back to the stock market for speculation, which will be bad for the healthy development of the real economy. At the mean time, since the trading time is so short that the investors will face bigger risks, thus we should take the risk premium of the stock market into account.

To sum up, the quantitative relationship between the investors' yields in stock market at time t, the risk-free rate and the average ROE of industrial investments can be express as follows:

$$r \le E(R_t) - \overline{erp} \le \bar{R} \tag{13}$$

Where r denotes the risk-free interest rate, $E(R_t)$ is the expected rate of return which investors obtain in period t by investing the stock market, $\overline{erp}$ indicates the average risk premium in the stock market, and $\bar{R}$ denotes the ROAE (Return On Average Equity) of all the list companies.

According to the assumption of above irrational speculative bubble model which is related to investor sentiment, we can obtain the expected yield of all investors in period t:

$$E(R_t) = r + (1-\mu)\varepsilon_t + \mu h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j} \tag{14}$$

Combining equation (12) and (14), we have:

$$E(R_t) = \left[(1-\mu)g(s)+\mu\right](1+r)B_t + \varepsilon_t - \mu\rho_t + \Delta \tag{15}$$

Where $\Delta = r - \frac{\mu\rho^*}{r} + \frac{2\gamma\mu^2 g(s)\sigma_\rho^2 + 2\gamma\left[\mu^2 g(s)h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right) - \mu(1-\mu)g(s) - \mu\right]\sigma_\varepsilon^2}{\left[(1-\mu)g(s)+\mu\right](1+r)^2 r}$.

At the end, according to equation (13) and (15), the plausible range of stock market bubbles can be derived as follows:

$$B_{\min} \le B_t \le B_{\max} \tag{16}$$

Where $B_{\min} = \frac{r + \overline{erp} - \varepsilon_t + \mu\rho_t - \Delta}{\left[(1-\mu)g(s)+\mu\right](1+r)}$, $B_{\max} = \frac{\bar{R} + \overline{erp} - \varepsilon_t + \mu\rho_t - \Delta}{\left[(1-\mu)g(s)+\mu\right](1+r)}$.

From the expression of the plausible range of stock market bubbles, we can find that the range is also relevant to the noise traders' misperceptions of the expected price of the risky asset and overreactions to the historical shocks of fundamental value, as well as their intrinsic sentiments during investment.

5. The multi-period extension and simulation of the irrational bubble model

This section will simulate the impact of noise traders on the tendency of stock prices and irrational bubbles according to equation (9) and (12) respectively, and accomplish a numerical simulation of the plausible range of stock market bubbles on the basis of equation (16).

5.1 The simulation on the tendency of stock prices

Since the extended DSSW model is a two-period model, in order to study the dynamic movements of the irrational bubbles more than two periods, we must expand it to more periods. We assume that $\rho *$ obeys a random walk process, which is defined as $\rho_t^* = \rho_{t-1}^* + \varphi_t$. Through iteration we can get $\rho_t^* = \rho_0^* + \sum_{i=0}^{t} \varphi_i$. Where $\varphi_t \sim N\left(0, \sigma_\varphi^2\right)$. In addition, there exist erratic swings on noise traders' misperception in period t, which is defined as ζ_t. It's also an i.i.d. normal random variable and its specific distribution is $\zeta_t \sim N\left(0, \sigma_\zeta^2\right)$, so we have $\rho_t = \rho_t^* + \zeta_t = \rho_{t-1}^* + \varphi_t + \zeta_t$. Where φ_t denotes the long-term fluctuation of noise traders' misperception, and has a permanent impact on the price of risky asset; while ζ_t measures the temporary shocks on noise traders' misperception, and only affects the price of risky asset in a short time. Now, we posit these two random variables are independent, then we have $\sigma_\rho^2 = \sigma_\varphi^2 + \sigma_\zeta^2$, so the final form of the pricing rule for risky asset (u) (i.e. equation (9)) can be converted into:

$$p_t = 1 + \frac{(1-\mu) g(s) \varepsilon_t}{\left[(1-\mu) g(s) + \mu\right](1+r)} + \frac{\mu h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right) \cdot \frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}}{\left[(1-\mu) g(s) + \mu\right](1+r)} + \frac{\mu\left(\rho_t - \rho_t^*\right)}{\left[(1-\mu) g(s) + \mu\right](1+r)} + \frac{\mu \rho_t^*}{\left[(1-\mu) g(s) + \mu\right] r} - \frac{2\gamma g(s)\left[(1-\mu)^2 \sigma_\varepsilon^2 + \mu^2(\sigma_\varphi^2 + \sigma_\zeta^2) + \mu^2 h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\sigma_\varepsilon^2\right]}{\left[(1-\mu) g(s) + \mu\right](1+r)^2 r} \tag{17}$$

Where $g(s) = \frac{1}{\mathrm{n}}\sum_{i=1}^{n} g(s_i)$, $h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right) = k \cdot \left(1 - \frac{2}{1 + \exp\left(\lambda \cdot \frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)}\right)$.

Moreover, for the purpose of simulating the noise traders' sentiments concretely, this paper assumes that the specific form of the influence function of noise traders' sentiment is a negative exponential function, i.e. $g(s_i)=e^{-\alpha s_i}$, which satisfies the constraint conditions above. Where $\alpha>0$ is called the influence coefficient of noise traders' sentiment, which expresses the effect degree of noise traders' sentiment on their levels of risk aversion. According to the research result of Baker and Wurger (2006), we let the value of noise trader' sentiment s_i follow the uniform distribution which is from minus two to three in the simulation, i.e. $s_i \sim U(-2,3)$.

In order to reflect the impact of the increasing proportion of noise traders on the tendency of stock price p_t, under the condition of given parameters and with the assumption that the proportion of noise traders is 5%、10% and 50% separately, this paper simulates the corresponding movements of stock price respectively. Detailed simulative results are shown in figure 1.

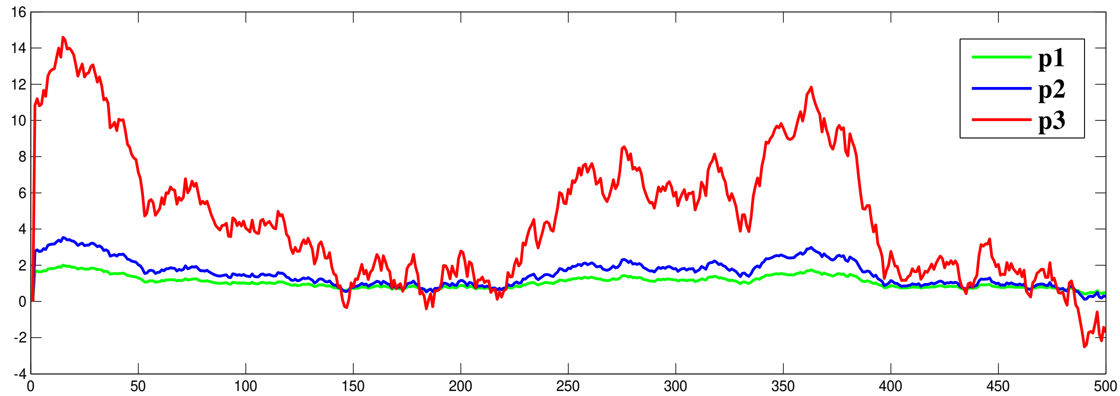

Figure 1 The simulation on the tendency of stock prices within 500 periods

Fig.1. Random number generator is used to determine the values of normal random variable ε_t、φ_t and ζ_t within 500 periods respectively, whose mean is all zero, while its variance is $\sigma_\varepsilon^2=1,\sigma_\varphi^2=0.05$ and $\sigma_\zeta^2=1$ respectively; and we posit the initial price is one; there are a total of one million investors in the market, i.e. $N=1{,}000{,}000$; the risk-free rate $r=0.05$, the coefficient of investor's risk aversion is equal to one, i.e. $\gamma=1$, the periods in which the noise traders will overreact to the changes of fundamental value is equal to five, i.e. $m=5$; the corresponding parameters in noise traders' overreaction function are set to $k=2$, $\lambda=1$; noise

traders' average misperception of ${}_{t}p_{t+1}$ at the beginning is also equal to one, i.e. $\rho_1^* = 1$ (illustrates that noise traders are "long" at the beginning). In addition, we also assume that the influence coefficient of noise traders' sentiment is equal to one, i.e. $\alpha = 1$, and use the random number generator again to generate the values of the $(-2\ ,\ 3)$ uniformly distributed random variable s_i; n denotes the amount of noise traders in the market (When the proportion of noise traders in the market is 0.05, 0.1 and 0.5, the corresponding number of noise traders is $n_1 = 0.05N$, $n_2 = 0.1N$ and $n_3 = 0.5N$ respectively). Finally, we simulate the corresponding sequences $\{p_t\}$ with different proportion of noise traders, which is denotes by p_i, where $i = 1, 2, 3$ represents the proportion of noise traders is 0.05、0.1 and 0.5 respectively.

Figure 1 shows that with an increasing proportion of noise traders in the market compared to the informed traders, the overall trend of stock price will gradually deviate from its fundamental value. And the volatility of stock price is becoming more and more dramatic, which increases the risk of the stock market crash. Therefore, combining with the pricing function of risky asset, we can infer that the noise traders' proportion, their overreaction to historical changes of fundamental value and their sentiments in the process of investment can lead the stock price deviate from its fundamental value, and cause the instability of the stock market.

5.2 The simulation on the tendency of irrational speculative bubbles

Similarly, by multi-period extension, the expression (12) of irrational speculative bubbles can be converted into:

$$B_t = \frac{\mu\left(\rho_t - \rho_t^*\right)}{\left[(1-\mu)g(s)+\mu\right](1+r)} + \frac{\mu\rho_t^*}{\left[(1-\mu)g(s)+\mu\right]r} - \frac{\mu\varepsilon_t}{\left[(1-\mu)g(s)+\mu\right](1+r)} \qquad (18)$$

$$+ \frac{\mu h\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)\cdot\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}}{\left[(1-\mu)g(s)+\mu\right](1+r)} - \frac{2\gamma\mu^2 g(s)(\sigma_\varphi^2+\sigma_\zeta^2)+2\gamma\left[\mu^2 g(s)h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right)-\mu(1-\mu)g(s)-\mu\right]\sigma_\varepsilon^2}{\left[(1-\mu)g(s)+\mu\right](1+r)^2 r}$$

With the purpose of exploring different proportion of noise traders how to affect the tendency of speculative bubbles B_t, under the condition of given parameters and with the assumption that the proportion of noise traders is 5%、10% and 50% separately, we simulate the corresponding movements of irrational speculative bubbles respectively. Detailed simulative results are shown in figure 2.

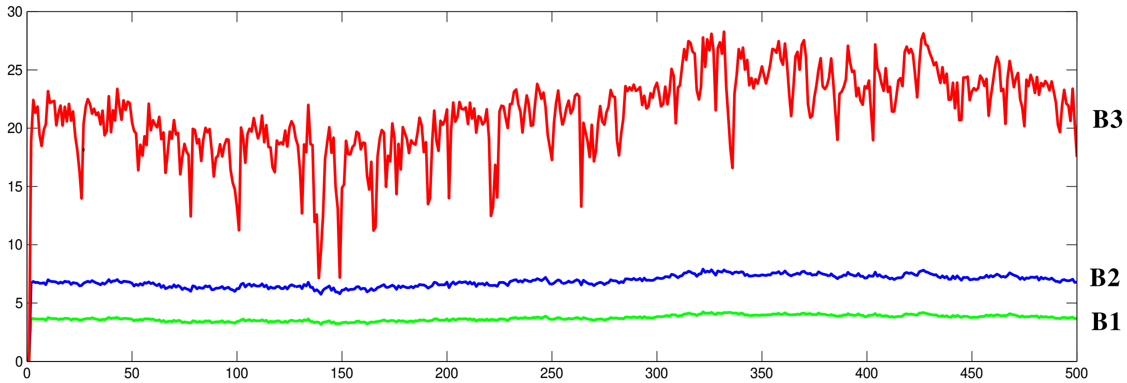

Figure 2 The simulation on the tendency of irrational bubbles within 500 periods

Fig.2. Random number generator is used to determine the values of normal random variable ε_t、φ_t and ζ_t within 500 periods respectively, whose mean is all zero, while its variance is $\sigma_\varepsilon^2 = 1, \sigma_\varphi^2 = 0.05$ and $\sigma_\zeta^2 = 1$ respectively; and we posit the initial price is one; there are a total of one million investors in the market, i.e. $N = 1{,}000{,}000$; the risk-free rate $r = 0.05$, the coefficient of investor's risk aversion is equal to one, i.e. $\gamma = 1$, the periods in which the noise traders will overreact to the changes of fundamental value is equal to five, i.e. $m = 5$; the corresponding parameters in noise traders' overreaction function are set to $k = 2$, $\lambda = 1$; noise traders' average misperception of ${}_t p_{t+1}$ at the beginning is also equal to one, i.e. $\rho_1^* = 1$

(illustrates that noise traders are "long" at the beginning). In addition, we also assume that the influence coefficient of noise traders' sentiment is equal to one, i.e. $\alpha = 1$, and use the random number generator again to generate the values of the $(-2\ ,\ 3)$ uniformly distributed random variable s_i; n denotes the amount of noise traders in the market (When the proportion of noise traders in the market is 0.05, 0.1 and 0.5, the corresponding number of noise traders is $n_1 = 0.05N$, $n_2 = 0.1N$ and $n_3 = 0.5N$ respectively). Finally, we simulate the corresponding sequences $\{B_t\}$ with different proportion of noise traders, which is denotes by B_i, where $i = 1,\ 2,\ 3$ represents the proportion of noise traders is 0.05、0.1 and 0.5 respectively.

From figure 2 we can find that in the condition of giving related parameters, the larger the proportion of noise traders is, the vaster the scale of irrational speculative bubbles exists, and the larger the amplitude of fluctuation is. And with the enhancement of the volatility of irrational bubbles, the possibility of their bursting is gradually increased. Therefore, according to the setting of the irrational bubble model, we can deduce that the reasons for the generation of irrational speculative bubbles in the stock market are including the noise traders' misperception of the future price and their inherent sentiments in the process of investment, as well as their overreaction to the historical changes of fundamental value.

5.3 The simulation on the plausible range of stock market bubbles

Finally, building on the assumptions and theories we summarized above, we can get the plausible range of stock market bubbles as follows:

$$B_{\min} \le B_t \le B_{\max}$$

Where $B_{\min} = \dfrac{r + \overline{erp} - \varepsilon_t + \mu\rho_t - \Delta}{\left[(1-\mu)\,g(s) + \mu\right](1+r)}$, $B_{\max} = \dfrac{\overline{R} + \overline{erp} - \varepsilon_t + \mu\rho_t - \Delta}{\left[(1-\mu)\,g(s) + \mu\right](1+r)}$,

$$\Delta = r - \frac{\mu\rho_t^*}{r} + \frac{2\gamma\mu^2 g(s)(\sigma_\varphi^2+\sigma_\zeta^2) + 2\gamma\left[\mu^2 g(s) h^2\left(\frac{1}{m}\sum_{j=1}^{m}\varepsilon_{t-j}\right) - \mu(1-\mu)g(s) - \mu\right]\sigma_\varepsilon^2}{\left[(1-\mu)g(s)+\mu\right](1+r)^2 r}$$

In this section we only simulate the trend of the stock market bubbles and their plausible range when the proportion of noise traders is 0.1, and posit the average risk premium in the stock market is 0.08, i.e. $\overline{erp}=0.08$, the ROAE (Return On Average Equity) of all the list companies is 0.16, i.e. $\bar{R}=0.16$. Detailed simulative results are shown in figure 3.

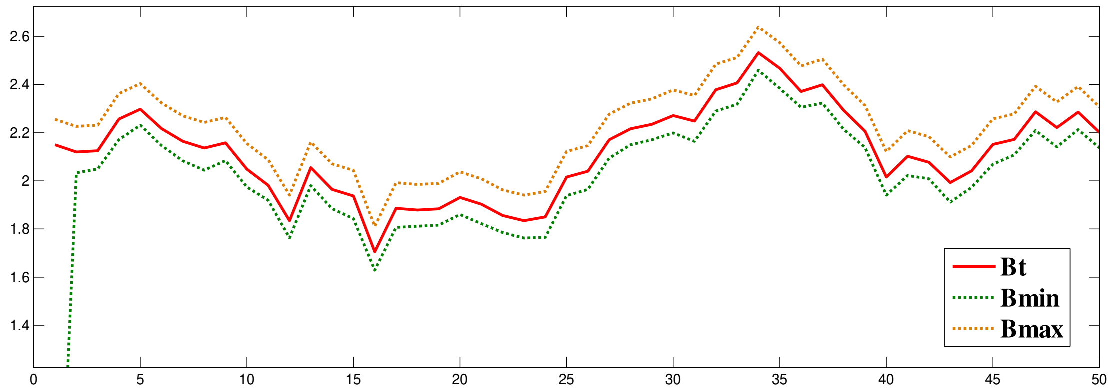

Figure 3 The simulation on the plausible range of bubbles within 50 periods

Fig.3. Random number generator is used to determine the values of normal random variable ε_t 、 φ_t and ζ_t within 50 periods respectively, whose mean is all zero, while its variance is $\sigma_\varepsilon^2=1, \sigma_\varphi^2=0.05$ and $\sigma_\zeta^2=1$; and we posit the initial price is one; there are a total of one million investors in the market, i.e. $N=1{,}000{,}000$; the risk-free rate $r=0.05$, the coefficient of investor's risk aversion is equal to one, i.e. $\gamma=1$, the periods in which the noise traders will overreact to the changes of fundamental value is equal to five, i.e. $m=5$; the corresponding parameters in noise traders' overreaction function are set to $k=2$, $\lambda=1$; noise traders' average misperception of ${}_t p_{t+1}$ at the beginning is also equal to one, i.e. $\rho_1^*=1$ (illustrates that noise traders are "long" at the beginning). In addition, we also assume that the influence coefficient of noise traders' sentiment is equal to one, i.e. $\alpha=1$, and use the random number generator again to

generate the values of the $(-2\,,3)$ uniformly distributed random variable s_i, where n denotes the amount of noise traders in the market (When the proportion of noise traders in the market is 0.1, the corresponding number of noise traders is $n = 0.1N$).

5.4 The influence of changing proportion of noise traders on irrational bubbles

In order to concretely research the impact of changing proportion of noise traders on irrational bubbles, we will assume that the change of the proportion of noise traders obeys the following rule: $\mu_t = \left[\cos\left(-\pi + \frac{M\pi t}{500}\right) + 1.2\right] \div 3$, which is proposed by Yang (2008). Due to the proportion is posited to obey the cosine function, so the cycle of changing proportion of noise traders is $\frac{2\pi}{M\pi t/500}$, and the interval for the change of the proportion is $\left[\frac{0.2}{3}, \frac{2.2}{3}\right]$. This assumption conforms to the shift relationship between informed traders and noise traders. Then we simulate the trend of irrational bubbles according to equation (18) in the conditions of setting $M = 2$ and $M = 4$ respectively, and the other parameters are in accordance with above. The simulative results are shown in figure 4 and figure 5.

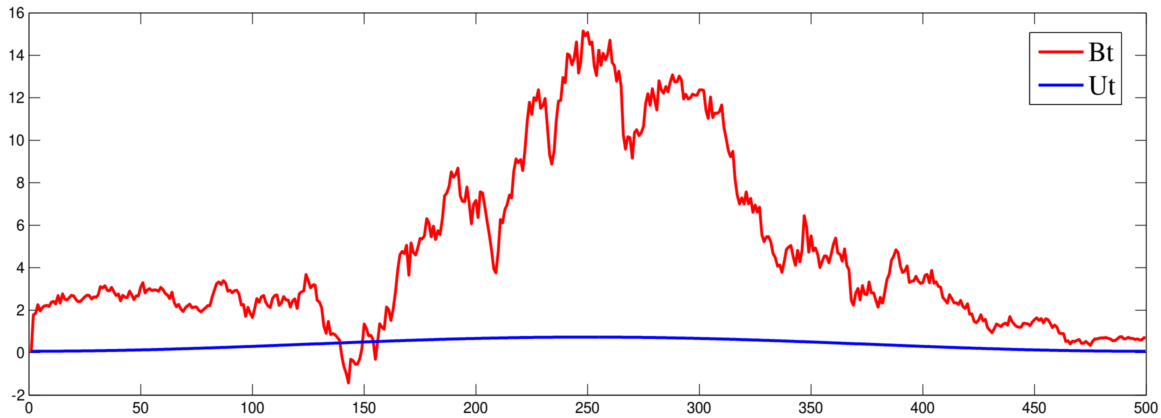

Figure 4 The simulation on the trend of irrational bubbles within 500 periods ($M = 2$)

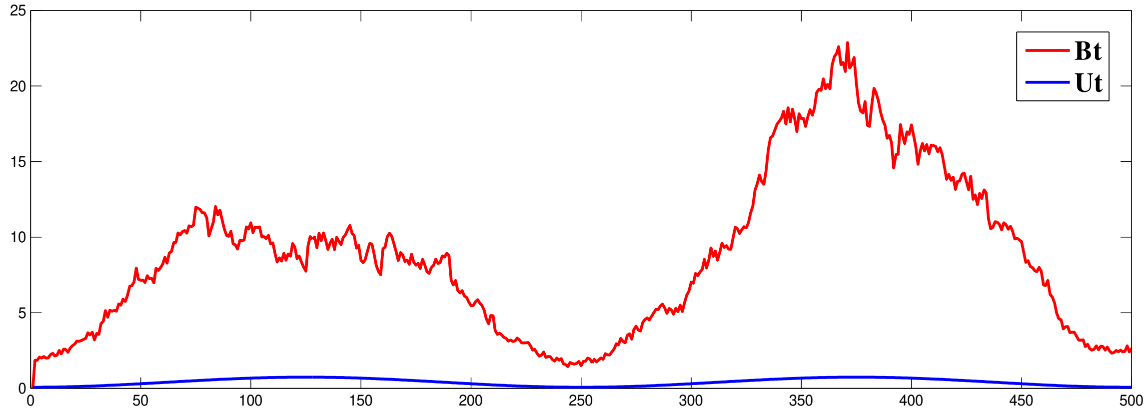

Figure 5 The simulation on the trend of irrational bubbles within 500 periods ($M = 4$)

Fig.4. & Fig.5. Random number generator is used to determine the values of normal random variable ε_t 、 φ_t and ζ_t within 500 periods respectively, whose mean is all zero, while its variance is $\sigma_\varepsilon^2 = 1, \sigma_\varphi^2 = 0.05$ and $\sigma_\zeta^2 = 1$; and we posit the initial price is one; there are a total of one million investors in the market, i.e. $N = 1{,}000{,}000$; the risk-free rate $r = 0.05$, the coefficient of investor's risk aversion is equal to one, i.e. $\gamma = 1$, the periods in which the noise traders will overreact to the changes of fundamental value is equal to five, i.e. $m = 5$; the corresponding parameters in noise traders' overreaction function are set to $k = 2$, $\lambda = 1$; noise traders' average misperception of ${}_t p_{t+1}$ at the beginning is also equal to one, i.e. $\rho_1^* = 1$ (illustrates that noise traders are "long" at the beginning). In addition, we also assume that the influence coefficient of noise traders' sentiment is equal to one, i.e. $\alpha = 1$, and use the random number generator again to generate the values of the $(-2\,,3)$ uniformly distributed random variable s_{ti}, n_t denotes the amount of noise traders in the market (When the proportion of noise traders in the market is μ_t, the corresponding number of noise traders is $n_t = \mu_t N$). Finally, we simulate the corresponding sequences $\{B_t\}$ with different μ_t.

With the combination of figures 4 and 5, we can find that when the proportion of noise traders is small, the scale of irrational bubbles is also small, while the bubbles will inflate with the increasing of the percentage of noise traders. But the bigger the proportion of noise traders is, the larger scale of the irrational bubbles becomes is not always true. This is because noise traders' psychologies are changing during the inflation of irrational bubbles, only when there are exist adverse changes in the noise traders' misperceptions and other related random variables, the irrational bubbles will lessen or even burst. In addition, figures 4 and 5 also show that when the proportion of noise traders is very small, the irrational bubbles are hardly formative and inflated, and only increasing the proportion of noise traders can the irrational bubbles will expand. In the end, through the simulations we can conclude that the irrational behavior of noise traders is indeed an important reason for the generation and expansion of the irrational speculative bubbles.

6. Conclusion

According to the different assumptions for investors, stock market bubbles can be classified into rational bubbles and irrational bubbles. On the basis of investors' rational expectation hypothesis, the rational bubble model can study the existence of rational bubbles, but it can't analyze the specific factors of causing price to deviate from its fundamental value. While behavioral finance avoids the hypotheses of investors are rational and market is completely effective, it depict the composition of asset price bubble from the perspective of the investors are irrational, which is more realistic and can effectively explain the reason of the generation mechanism of stock market bubbles more than other theories. In view of this, this article constructs the irrational speculative bubble model which is based on the noise trading model (DSSW model). Through relevant extension, the irrational speculative bubble model analyzes the role of noise traders' misperceptions and typical sentiments, and their overreaction to the historical changes of the fundamental value in the generation of stock market bubbles. It also identified the scale of irrational bubbles and contributed a lot to the sureness of the plausible range of irrational bubbles. The research result shows that the irrational bubbles is closely related to the noise traders' misperceptions of the expected return of the risky asset and their intrinsic sentiments in the process of investment, as well as their overreaction to the historical changes of fundamental value. In particular, by setting the parameters of simulation to fixed values, we find that under certain conditions, the more the noise traders are in the market, the more the irrational speculative bubbles contained in risky asset, and the greater the fluctuation of the stock market bubbles is. But the crescent proportion of noise traders will not necessarily result in the gradual inflation of irrational bubbles. At last, we suggest that the market should constantly foster the investors' rational investment philosophy and improve the investors' diathesis to restrain the growth and expansion of irrational bubbles and stabilize the stock market.

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