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Collaborative Research and Rate of Interests

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Abstract

This paper makes an attempt to link collaborative research in industry with Government initiative and market rate of interests. Two firms involved in Cournot competition in the market are deciding whether to conduct research to device a technique for cost reduction. Amount of cost reduction after the research and the initial amount of capital possessed by each firm are private information to each of the firms. In particular both of them are having capacity constraint. Our objective here is to figure out the impacts of the lending and borrowing rates of interest on collaborative research. In the process we study the effectiveness of different policies to encourage collaborative R&D.

Keywords: Collaborative research, Government policy, Subsidy, Interest rate

JEL Classifications: E43, H71, L50, O31, O38

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1 Introduction

In oligopolistic markets, firms conduct Research and Development (R&D) in order to achieve or maintain their competitive edge over their contenders. But conducting R&D, irrespective of whether it is directed towards product or process innovation, mostly involves huge capital investment¹. Also, as is quite well known, the outcomes of such R&D activities are quite uncertain, and there is always the possibility that the new innovation will be soon imitated by the competing firms in the industry. As suggested by Hall and Lerner (2009)[8], because of this reason, there is likely to be under-investment in R&D.

When firms conduct research in collaboration with each other, both the cost of conducting the R&D activity and the risk involved are shared to a certain extent. In fact, collaborative research might enhance the probability of success. The problem of free riding is also reduced in case of collaborative research². d'Aspremont and Jacquemin (1988)[6] show that technological advances are higher under cooperative R&D as compared to competitive R&D. Marjit (1991)[11] establishes that firms cooperate in R&D when the probability of success is either very high or very low. Combs (1992)[4], however, points out that, Research Joint Ventures (RJV) occur only for very high probabilities of success. Bayona, Garcia-marco and Huerta (2001)[1] suggest that, the complexity of technology and the fact that innovation is costly and uncertain, motivate firms to conduct cooperative R&D. According to Miotti and Schawald (2003)[12], co-operative R&D efforts are higher for firms from sectors with relatively high R&D intensity and also for firms that draw on scientific resources to innovate compared to firms further away from the technological frontier. As pointed out by Mukherjee and Ray (2009)[13], uncertainty in patent approvals also might induce cooperative R&D. Kabiraj and Chattopadhyay (2014)[9] show that Marjit's (1991)[11] findings hold under incomplete information framework as well, moreover the range of probability values of success, for which cooperative R&D occurs, increases.

However, there are certain disincentives for collaborative R&D. When firms collaborate only in the R&D stage, the research outcome is equally shared. If the same firms compete in the aftermarket, then sharing the R&D outcome equally does not provide any competitive edge to any individual firm over its contenders, who are also part of the collaborative R&D team. It happens since each of the firms participating in collaborative R&D enjoys the same level of benefits e.g. same level of cost reduction resulting from the R&D. This scenario is more prevalent for low rates of spillover of the concerned R&D activity and thus private firms prefer cooperative R&D when such spillover rates are high, as noted by Choi(1992)[3].

The benefits of R&D activities do not remain confined in just yielding higher profits to firms who undertake such activities. These activities very often benefit the consumers, e.g. in the form of lower prices for the same products, resulting from the lowering of costs of production or better quality products. R&D activities also have crucial implications for the overall development prospects of any economy. For instance, R&D activities leading to generation of new technologies might push out the technological frontier of an economy. Kamien, Muller and Zang (1992)[10] suggest that equilibrium level technological improvements are low and equilibrium prices are high under competitive R&D than under cooperative R&D. Also, R&D resulting in innovation of less polluting technologies reduces pollution. Thus R&D activities potentially possess huge positive externalities.

R&D often remains unfunded by private capital e.g. venture capital since private capital does not have any incentive to internalise positive externalities. This situation is more prevalent in cases where public equity markets are underdeveloped³. Therefore R&D activities in industries are often consciously promoted by policy makers (Governments in most cases). Brocas (2004)[2] mentions that in context of R&D rivalry, socially optimal

¹The amount of investment in any R&D activity might vary depending on maturity time for the concerned activity

²Kabiraj and Chattopadhyay (2014)[9]

³Hall and Lerner (2009)[8]

outcome might not materialise, and therefore, the policymakers have developed various tools to encourage and promote collaborative R&D to mitigate the inefficiencies generated in the process. As noted by Ghosh and Ghosh (2014)[7] in US the National Cooperative Research Act of 1984 was designed to promote cooperative R&D ventures among firms. The Governments can adopt different types of policies for this, e.g. allowing for tax credits for research collaboration, provide research funds at cheaper rates of interest or directly share the costs by providing subsidies etc. Thus for both the situations, mentioned above, the problem of under-investment in R&D can be mitigated to a certain extent, even though not completely eliminated⁴.

From the above discussion it emerges that both collaborative research as well as incentives from Governments can enhance the scale of R&D activities in industries. Sakakibara and Cho (2002)[14] show that due to the fact that cooperative R&D had been actively encouraged by the Japanese Government, R&D output in Japan was significantly higher as compared to Korea, where the Government didn't take such initiatives. Thus empirical evidences also suggest that Government support for cooperative R&D indeed results in higher R&D output. The Government, however, can offer supports for cooperative R&D in various different ways. For example, in Japan, in case of the VLSI (Very Large Scale Integrated circuit) project, the Government used to finance 22 per cent of the R&D budget. Czarnitzki, Ebersberger and Fier (2007)[5] mention that the German Federal Government offers direct subsidies for collaborative research, and the number of such funding has increased substantially over the years⁵. Schacht(2010)[15] notes that the US Government offers funds for collaborative research at the industry level. An alternative possibility is offering funds in the form of concessional loans i.e. loans at cheaper rates as compared to prevailing market rates. This paper makes an attempt to explore the possibility where Government provides incentives to conduct research collaboratively in the form of cheaper loans.

The remaining part of the paper is organised as follows: Section 2 describes the model, where subsections 2.1 elaborates the case of non-collaborative research, comparing the profit levels under different possibilities, 2.2 talks about the case of collaborative research and 2.3 discusses the Government policy, in terms of offering cheap capital and providing subsidy and finally Section 3 concludes the paper.

2 The Model

We begin this section by describing the model's framework. We assume that there is a market, which we term the aftermarket, where two firms (Firm 1 and Firm 2) are involved in a Cournot type competition. The market demand function is given by $P = a - bQ$. The parameters a and b are common knowledge to the firms. The cost function of each firm i is cq_i , where q_i is the quantity produced by the firm i and the parameter c is common knowledge. We assume $a, b, c > 0$ with $a > 2c$ ⁶.

Each firm has K_i amount of capital that it can invest in any R&D activity. K_i is private information to firm i . It is common knowledge to the firms that all K_i -s are independently and identically distributed with distribution function $G(\cdot)$ with corresponding density function $g(\cdot)$ in the domain $[0, \bar{K}]$ and has full support. Before entering the market each of the firms has the option to conduct a cost reducing R&D activity either singlehandedly or collaboratively. If a firm conducts the research singlehandedly then following the success of the research, the new cost function of firm i is $(c - \epsilon_i)q_i$, where ϵ_i is private information to firm i and

⁴We need to note here that if firms collaborate both at the level of R&D as well as production in the aftermarket, the market becomes less competitive leading to loss of consumer surplus. Thus if any policy-maker's objective is to ensure higher level of social welfare, then incentives for R&D collaboration needs to be complemented with strong anti-trust laws.

⁵100 collaborative projects received funding from the German Federal Government in 1980, and this number reached 2100 in 1990 and more than 7500 in 2001 (Czarnitzki, Ebersberger and Fier (2007))[5].

⁶This means that the demand is "sufficiently" high.

independently and identically distributed with distribution function $F(\cdot)$ with corresponding density function $f(\cdot)$ in the domain $[0, \bar{\epsilon}]$ and has full support. On the other hand if they conduct the research collaboratively then after the research the new cost function of each firm is $(c - \epsilon_{max})q_i$ where $\epsilon_{max} = \max\{\epsilon_1, \epsilon_2\}$. Cost of the research is \mathcal{K} . We assume that $\mathcal{K} > \bar{K}$ i.e. both the firms are capacity constrained. However \mathcal{K} is common knowledge to both the firms.

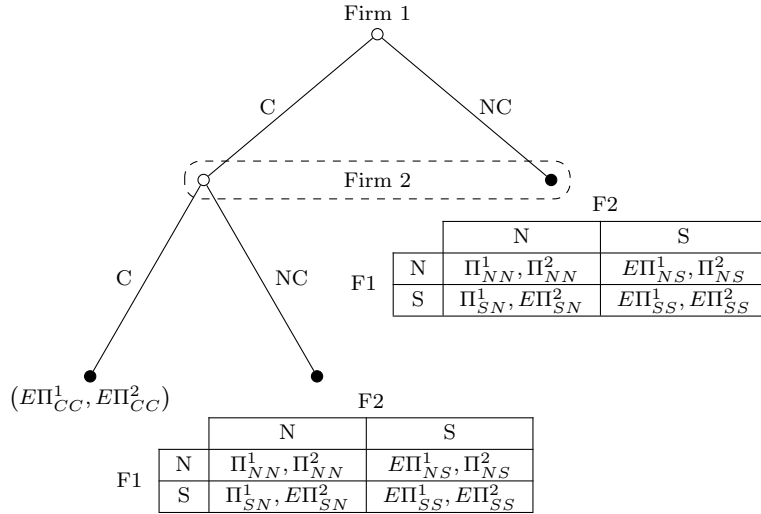
There is a bank from which both the firms can borrow. The borrowing rate of interest for the firms is \hat{t} and the interest rate which a firm would earn by depositing its capital in the bank is \hat{r} with $\hat{t} > \hat{r}$. Henceforward, we will call this latter rate, the lending rate. Let us denote $t = 1 + \hat{t}$ and $r = 1 + \hat{r}$. So if a firm has capital K_i then she will obtain $\hat{r}K_i$ extra profit if she does not conduct the research. If she conducts the research singlehandedly, then she should borrow from the bank $\mathcal{K} - K_i$ amount of capital for which she has to pay $t(\mathcal{K} - K_i)$ amount to the bank. If they perform collaborative R&D then the total capital they have is $K = K_1 + K_2$. If $K = \mathcal{K}$ then they don't have to borrow. If $K < \mathcal{K}$ then again they have to borrow from the bank $\mathcal{K} - K$ amount for which they have to pay $t(\mathcal{K} - K)$ to the bank. We assume that in this case they share this extra cost equally. Finally, if $K > \mathcal{K}$ then after performing the R&D they get $\hat{r}(K - \mathcal{K})$ extra profit. We assume that this profit will also be shared equally.

The sequence of the game is as follows:

Stage 1 Both the Firms are simultaneously deciding whether to do collaborative research. If both of them decides to do collaborative R&D then they does collaborative research, otherwise a Firm either do research singlehandedly or does not do research at all.

Stage 2 Firms decide how much to produce in the aftermarket.

The extensive form of the game is given below:



Here, we assume that every firm knows whether its rival wants to conduct R&D or not. The action of actually conducting R&D is denoted by S and that of not conducting R&D is denoted by N . When none of the firms is conducting R&D then the profit in the aftermarket is the usual Cournot profit for each firm. This is denoted by Π_{NN}^i where $i = 1, 2$. If just one firm i conducts R&D while its rival does not, then its profit in the aftermarket is denoted by Π_{SN}^i for $i = 1, 2$. In just the opposite case, where firm i does not conduct R&D, while its rival does, firm i has an expected profit in the aftermarket, since it does not know the actual cost and therefore the

actual output of its rival. This expected profit is denoted by $E\Pi_{NS}^i$ where $i = 1, 2$. Finally, when both the firms choose to undertake R&D activity, each firm i has expected profit level $E\Pi_{SS}^i$ since each firm i , where $i = 1, 2$, is uncertain about its rival's actual cost and hence actual output.

Interestingly, since this is a dynamic game of incomplete as well as imperfect information, none of the firms knows ex-ante any profit levels of the contending firm. Also, except for the case where none of them is performing any research, they don't even know their own profit levels, since own profit level for each firm is dependent on the type of its rival firm.

2.1 Non-collaborative Research

In this subsection we analyse the case of non-collaborative R&D. First we consider the different profit levels and then compare them.

2.1.1 Characterization of different profit levels

In this subsection we will derive four different profit levels of a firm. Since the firms are symmetric a priori, therefore, without any loss of generality, we use Firm 1 as the representative firm. The same profit levels for Firm 2 can be derived analogously. Let us denote

- $K_m = \int_0^{\bar{K}} kdG(k)$, the mean of the random variable K .
- $\epsilon_m = \int_0^{\bar{\epsilon}} \epsilon dF(\epsilon)$, the mean of the random variable ϵ .
- $\epsilon_{max}^i = \int_0^{\bar{\epsilon}} \max\{\epsilon_i, \epsilon_j\} dF(\epsilon_j)$ where $i \neq j$.

Note that all the profit levels are calculated when the firms are at the stage of deciding whether to go for a collaborative research i.e. all the profits are ex-ante profits.

First we calculate the profit level when no firm undertakes any R&D activity. Cost of production for each firm remains at its original level. In this case, each firm deposits its capital in the bank and thus earns interest. Thus the total earning of a firm is its profit in the aftermarket and the interest earning from its deposit as established in the following Lemma 2.1.

Lemma 2.1. *If no firm conducts R&D, then the profit of Firm 1 is given by*

$$\Pi_{NN}^1 = \frac{(a-c)^2}{9b} + \hat{r}K_1$$

Proof. If both the firms are not doing research then the profit they get in the market is the standard Cournot profit plus the interest on their capital. □

Next we calculate the profit level for the other extreme case, where both the firms conduct R&D. Here both experience some reduction in cost, each firm has to invest its own capital for the R&D and also has to borrow from the bank in order to meet the cost of R&D. So the net earning of every firm is its profit in the aftermarket less the sum of its capital and the loan payment as shown in the following Lemma 2.2.

Lemma 2.2. *If each firm conducts R&D, then the profit of firm 1 is given by*

$$E\Pi_{SS}^1 = \frac{\left(a - c + \left(\frac{3\epsilon_1 - \epsilon_m}{2}\right)^2\right)^2}{9b} - K_1 - (\mathcal{K} - K_1)t$$

Proof. Note that, for Firm 1,

$$E\Pi_1 = \int_0^{\bar{\epsilon}} [(a - c + \epsilon_1) q_1 - bq_1^2 - bq_1 q_2(\epsilon_2)] dF(\epsilon_2) - K_1 - (\mathcal{K} - K_1) t$$

Similarly, for Firm 2,

$$E\Pi_2 = \int_0^{\bar{\epsilon}} [(a - c + \epsilon_2) q_2 - bq_2^2 - bq_2 q_1(\epsilon_1)] dF(\epsilon_1) - K_1 - (\mathcal{K} - K_2) t$$

So the reaction functions are

$$q_1 = \frac{a - c + \epsilon_1 - b \int_0^{\bar{\epsilon}} q_2(\epsilon_2) dF(\epsilon_2)}{2b}$$

$$q_2 = \frac{a - c + \epsilon_2 - b \int_0^{\bar{\epsilon}} q_1(\epsilon_1) dF(\epsilon_1)}{2b}$$

Solving these two reaction functions we get

$$q_i^* = \frac{(a - c) + \frac{3\epsilon_i - \epsilon_m}{2}}{3b}$$

And putting q_1^* and q_2^* in to $E\Pi_1$ we get the Lemma. □

We next proceed to calculate the profit level of Firm 1, first, when it does not conduct R&D while Firm 2 does and second, when just the reverse happens. In the first case, firm 1 does not experience any cost reduction, while Firm 2 does, but the actual amount of cost reduction is known to firm 2 alone, so that Firm 1 has to act totally on the basis of expectations about firm 2's cost and output. In the reverse case, Firm 2 experiences no reduction in its cost level and Firm 2 has to act on the basis of expectation about Firm 1's cost and output, for the same reason. The following Lemma 2.3 shows the profit levels of firm 1 in both these cases.

Lemma 2.3. *The profit level of Firm 1 when Firm 1 does not undertake R&D activities but Firm 2 does, and that when just the opposite holds true are given respectively by*

$$E\Pi_{NS}^1 = \frac{(a - c - \epsilon_m)^2}{9b} + \hat{r}K_1$$

and

$$\Pi_{SN}^1 = \frac{\left((a - c) + \frac{(3\epsilon_1 + \epsilon_m)}{2}\right)^2}{9b} - K_1 - (\mathcal{K} - K_1) t$$

Proof. Consider the situation when Firm 1 conducts the research and Firm 2 does not. Then,

$$\Pi_1 = (a - c + \epsilon_1) - bq_1^2 - bq_1 q_2 - K_1 - (\mathcal{K} - K_1) t$$

And

$$E\Pi_2 = \int_0^{\bar{\epsilon}} [(a - c) - bq_2^2 - bq_2 q_1(\epsilon_1)] dF(\epsilon_1) + \hat{r}K_2$$

So, the reaction functions are,

$$q_1 = \frac{a - c + \epsilon_1 - bq_2}{2b}$$

$$q_2 = \frac{a - c - b \int_0^{\bar{\epsilon}} q_1(\epsilon_1) dF(\epsilon_1)}{2b}$$

Solving these two reaction functions, we get,

$$q_1^* = \frac{(a - c) + \frac{3\epsilon_1 + \epsilon_m}{2}}{3b}$$

$$q_2^* = \frac{a - c - \epsilon_m}{3b}$$

Rest of the proof is trivial. \square

2.1.2 Comparison of different profit levels

Here we compare the different profits levels derived in the previous subsection. To start with, we consider the profit levels for Firm 1, when it does not conduct R&D in two situations, first, when Firm 2 also does not conduct R&D as opposed to when Firm 2 conducts R&D. We resort to the following Lemma 2.4 for this comparison.

Lemma 2.4. *The profit level for Firm 1 when no firm conducts R&D is higher than the expected profit when Firm 1 does not conduct R&D but Firm 2 does, i.e.*

$$\Pi_{NN}^1 > E\Pi_{NS}^1$$

Proof. Trivial. \square

The intuition behind Lemma 2.4 is that when Firm 2 is conducting R&D, there is an expected reduction in Firm 2's cost. So Firm 2 experiences a competitive edge over Firm 1. So the profit of Firm 1 is less when Firm 2 is conducting R&D. We next compare the profit levels of Firm 1 when it undertakes R&D under two situations, first when Firm 2 also conducts R&D and second, when Firm 2 does not, in the following Lemma 2.5.

Lemma 2.5. *The profit level for Firm 1 when Firm 1 conducts R&D but Firm 2 does not, is higher than the expected profit when both the firms conduct R&D activities, i.e.*

$$\Pi_{SN}^1 > E\Pi_{SS}^1$$

Proof. Trivial. \square

The intuition behind the result in Lemma 2.5 is similar to that behind Lemma 2.4. Our next comparison involves the profit levels of Firm 1 when it performs R&D and Firm 2 does not, and when it does not perform R&D and Firm 2 does. The following Lemma 2.6 makes this comparison and we find that the ranking of the above mentioned profit levels varies depending on the parametric values.

Lemma 2.6. *For the ranking of the two profit levels, Π_{SN}^1 and Π_{NS}^1 , the following condition holds:*

$$\Pi_{SN}^1 \geq E\Pi_{NS}^1 \Leftrightarrow \epsilon_1 \geq \frac{2\sqrt{(a - c - \epsilon_m)^2 + Z_1} - (2(a - c) + \epsilon_m)}{3}$$

where $Z_1 = 9b[t(\mathcal{K} - K_1) + rK_1]$.

Proof. Note that

$$\begin{aligned}
& \frac{\left((a-c) + \left(\frac{3\epsilon_1 + \epsilon_m}{2}\right)\right)^2}{9b} - K_1 - (\mathcal{K} - K_1)t \geq \frac{(a-c - \epsilon_m)^2}{9b} + \hat{r}K_1 \\
\Leftrightarrow & \left((a-c) + \left(\frac{3\epsilon_1 + \epsilon_m}{2}\right)\right)^2 \geq (a-c - \epsilon_m)^2 + Z_1 \\
\Leftrightarrow & \frac{3\epsilon_1 + \epsilon_m}{2} \geq \sqrt{(a-c - \epsilon_m)^2 + Z_1} - (a-c) \\
\Leftrightarrow & \epsilon_1 \geq \frac{2\sqrt{(a-c - \epsilon_m)^2 + Z_1} - (2(a-c) + \epsilon_m)}{3}
\end{aligned}$$

□

Next we compare the profit levels of Firm 1 when it performs R&D and Firm 2 does not, and when no firm performs R&D. The following Lemma 2.7 makes this comparison. The result suggests that no a priori ranking is possible as the ranking of the above mentioned profit levels again varies depending on the parametric values.

Lemma 2.7. *For the ranking of the two profit levels, Π_{SN}^1 and Π_{NN}^1 , the following condition holds:*

$$\Pi_{SN}^1 \geq \Pi_{NN}^1 \Leftrightarrow \epsilon_1 \geq A_1$$

where $A_1 = \frac{2\sqrt{(a-c)^2 + Z_1} - [2(a-c) + \epsilon_m]}{3}$.

Proof. Note that

$$\begin{aligned}
& \frac{\left((a-c) + \frac{3\epsilon_1 + \epsilon_m}{2}\right)^2}{9b} - K_1 - (\mathcal{K} - K_1)t \geq \frac{(a-c)^2}{9b} + \hat{r}K_1 \\
\Leftrightarrow & \left((a-c) + \frac{3\epsilon_1 + \epsilon_m}{2}\right)^2 \geq (a-c)^2 + Z_1 \\
\Leftrightarrow & \frac{3\epsilon_1 + \epsilon_m}{2} \geq \sqrt{(a-c)^2 + Z_1} - (a-c) \\
\Leftrightarrow & \epsilon_1 \geq \frac{2\left[\sqrt{(a-c)^2 + Z_1} - (a-c)\right] - \epsilon_m}{3}
\end{aligned}$$

□

This result can be intuitively explained in terms of the fact that if Firm 1 conducts R&D then it will enjoy lower cost of production. However in this case it has to forgo the interest earning. Firm 1 will, therefore, conduct R&D only if the increment in her profit by conducting R&D is higher than the forgone interest earning. Therefore, Firm 1 will conduct R&D only when her cost reduction is more than the threshold value as given in the above Lemma 2.7.

Our next comparison in the following Lemma 2.8 considers the profit levels of Firm 1 when both the firms undertake R&D as opposed to when only Firm 2 undertakes R&D. The ranking of these profit levels also varies depending on parametric values.

Lemma 2.8. *For the ranking of the two profit levels, Π_{SS}^1 and Π_{NS}^1 , the following condition holds:*

$$E\Pi_{SS}^1 \geq E\Pi_{NS}^1 \Leftrightarrow \epsilon_1 \geq B_1$$

where $B_1 = \frac{2\sqrt{(a-c-\epsilon_m)^2 + Z_1} - [2(a-c) - \epsilon_m]}{3}$.

Proof. Note that

$$\begin{aligned}
& \frac{(a-c + \frac{3\epsilon_1 - \epsilon_m}{2})^2}{9b} - K_1 - (\mathcal{K} - K_1)t \geq \frac{(a-c - \epsilon_m)^2}{9b} + \hat{r}K_1 \\
\Leftrightarrow & \left(a-c + \frac{3\epsilon_1 - \epsilon_m}{2}\right)^2 \geq (a-c - \epsilon_m)^2 + Z_1 \\
\Leftrightarrow & \frac{3\epsilon_1 - \epsilon_m}{2} \geq \sqrt{(a-c - \epsilon_m)^2 + Z_1} - (a-c) \\
\Leftrightarrow & \epsilon_1 \geq \frac{2\sqrt{(a-c - \epsilon_m)^2 + Z_1} - [2(a-c) - \epsilon_m]}{3}
\end{aligned}$$

□

The intuition of the above Lemma 2.8 is similar to that of Lemma 2.7. For deriving the next set of results, it is important to establish the following Lemma 2.9 which makes a comparison between A_1 and B_1 .

Lemma 2.9. *Comparing A_1 and B_1 we obtain the following ranking:*

$$B_1 > A_1$$

Proof. Note that,

$$\begin{aligned}
& \frac{2\sqrt{(a-c - \epsilon_m)^2 + Z_1} - [2(a-c) - \epsilon_m]}{3} > \frac{2\sqrt{(a-c)^2 + Z_1} - [2(a-c) + \epsilon_m]}{3} \\
\Leftrightarrow & 2\sqrt{(a-c - \epsilon_m)^2 + Z_1} - [2(a-c) - \epsilon_m] > 2\sqrt{(a-c)^2 + Z_1} - [2(a-c) + \epsilon_m] \\
\Leftrightarrow & 2\sqrt{(a-c - \epsilon_m)^2 + Z_1} + 2\epsilon_m > 2\sqrt{(a-c)^2 + Z_1} \\
\Leftrightarrow & \sqrt{(a-c - \epsilon_m)^2 + Z_1} + \epsilon_m > \sqrt{(a-c)^2 + Z_1} \\
\Leftrightarrow & (a-c - \epsilon_m)^2 + Z_1 + \epsilon_m^2 + 2\epsilon_m\sqrt{(a-c - \epsilon_m)^2 + Z_1} > (a-c)^2 + Z_1 \\
\Leftrightarrow & 2\epsilon_m\sqrt{(a-c - \epsilon_m)^2 + Z_1} + 2\epsilon_m^2 - 2(a-c)\epsilon_m > 0 \\
\Leftrightarrow & \sqrt{(a-c - \epsilon_m)^2 + Z_1} - (a-c - \epsilon_m) > 0
\end{aligned}$$

□

Next we consider the four profit levels $E\Pi_{SS}^1$, $E\Pi_{NS}^1$, Π_{SN}^1 and Π_{NN}^1 and obtain the relationship derived in the following Lemma 2.10.

Lemma 2.10. *The following condition holds for profit levels $E\Pi_{SS}^1$, $E\Pi_{NS}^1$, Π_{SN}^1 and Π_{NN}^1 :*

$$E\Pi_{SS}^1 \geq E\Pi_{NS}^1 \Rightarrow \Pi_{SN}^1 \geq \Pi_{NN}^1$$

Proof. This follows from Lemma 2.9, Lemma 2.7 and Lemma 2.8. □

Lemma 2.10 implies that when the rival firm is performing R&D, if a firm is also interested in performing R&D then it will conduct R&D irrespective of whether the other firm is undertaking R&D or not. Note that it also implies that $\Pi_{SN}^1 \leq \Pi_{NN}^1 \Rightarrow E\Pi_{SS}^1 \leq E\Pi_{NS}^1$, which happens when $\epsilon_1 \leq A_1$.

Comparing the profit levels Π_{SN}^1 , Π_{NN}^1 , Π_{SN}^1 and $E\Pi_{NS}^1$ in the following Lemma 2.11, we derive the implication.

Lemma 2.11. *Comparison of profit levels Π_{SN}^1 , Π_{NN}^1 , Π_{SN}^1 and $E\Pi_{NS}^1$ suggests that*

$$\Pi_{SN}^1 \geq \Pi_{NN}^1 \Rightarrow \Pi_{SN}^1 \geq E\Pi_{NS}^1$$

Proof. Trivial. □

After establishing the required relationships in the various Lemmas above, we now state the following Proposition 2.12 which establishes the sufficient conditions for Firm 1's decision about whether to undertake R&D or not.

Proposition 2.12. • *If $\epsilon_1 > B_1$ then Firm 1 will always do research*

- *If $\epsilon_1 < A_1$ then Firm 1 will never do research*
- *If $B_1 \geq \epsilon_1 \geq A_1$ then at equilibrium Firm 1s will perform R&D sif she expects that the other firm will not conduct R&D. Otherwise it will not conduct R&D.*

Proof. The proofs of the first two results we obtain by combining all the Lemmas stated above.

For the proof of the last result, first note that in this case $\Pi_{SN}^1 \geq \Pi_{NN}^1 > E\Pi_{NS}^1 \geq E\Pi_{SS}^1$ holds. Second, here Firm 1 needs to consider the strategy of Firm 2. Suppose Firm 1 expects that Firm 2 will conduct R&D, then $E\Pi_{NS}^1 \geq E\Pi_{SS}^1$ dictates her not to undertake R&D. On the other hand, suppose Firm 1 expects that Firm 2 will not conduct R&D, then $\Pi_{SN}^1 \geq \Pi_{NN}^1$ dictates her to conduct R&D. □

2.2 Collaborative Research

In this subsection we consider the case where the firms collaborate in the R&D activity. We can express the expected profit of Firm 1 as in the following Lemma 2.13.

Lemma 2.13. *The expected profit of Firm 1 when both the firms conduct research collaboratively is given by:*

$$E\Pi_{CC}^1 = \frac{(a - c + \epsilon_{max1})^2}{9b} + \frac{T_1}{9b}$$

where $T_1 = 9b \left[\frac{r}{2} \int_{\mathcal{K}-K_1}^{\bar{\mathcal{K}}} (1 - G(K_2)) dK_2 - K_1 - \frac{t}{2} \int_0^{\mathcal{K}-K_1} G(K_2) dK_2 \right]$ and note that $T_1 < 0$.

Proof. Note

$$\Pi_{CC}^1 = \frac{(a - c + \epsilon_{max}^1)^2}{9b} + \begin{cases} -K_1 - \frac{t}{2} (\mathcal{K} - K_1 - K_2) & , \text{If } K_1 + K_2 < \mathcal{K}; \\ -K_1 & , \text{If } K_1 + K_2 = \mathcal{K}; \\ \frac{r}{2} (K_1 + K_2 - \mathcal{K}) - K_1 & , \text{Otherwise.} \end{cases}$$

So,

$$\begin{aligned}
E\Pi_{CC}^1 &= \int_0^{\bar{K}} \Pi_{CC}^1 dG(K_2) \\
&= \frac{(a-c + \epsilon_{max}^1)^2}{9b} + G(\mathcal{K} - K_1) \int_0^{\mathcal{K} - K_1} \left[-K_1 - \frac{t}{2}(\mathcal{K} - K_1 - K_2) \right] dG(K_2 | K_1 + K_2 \leq \mathcal{K}) \\
&\quad + (1 - G(\mathcal{K} - K_1)) \int_{\mathcal{K} - K_1}^{\bar{K}} \left[\frac{r}{2}(K_1 + K_2 - \mathcal{K}) - K_1 \right] dG(K_2 | K_1 + K_2 \geq \mathcal{K}) \\
&= \frac{(a-c + \epsilon_{max}^1)^2}{9b} + \int_0^{\mathcal{K} - K_1} \left[-K_1 - \frac{t}{2}(\mathcal{K} - K_1 - K_2) \right] dG(K_2) \\
&\quad + \int_{\mathcal{K} - K_1}^{\bar{K}} \left[\frac{r}{2}(K_1 + K_2 - \mathcal{K}) - K_1 \right] dG(K_2)
\end{aligned}$$

Rest of the proof is trivial. \square

Here we again consider Firm 1 only and the strategies of Firm 2 can be deduced routinely in a similar way. Note that the profit of Firm 1 depends ϵ_{max}^1 because now the reduction in cost due to conducting of research not only depends on her own type but also the type of the other firm. We know that $\epsilon_{max}^i > \epsilon^i$ so in case of collaborative research both the firms will expect to have greater reduction in cost than whatever they can achieve by themselves. The following Lemma 2.14 makes a comparison of expected profits of Firm 1, when both the firms conduct collaborative R&D vis-a-vis when both the firms undertake R&D activity singlehandedly.

Lemma 2.14. *Suppose $\epsilon_1 \geq B_1$, then,*

$$E\Pi_{CC}^1 \geq E\Pi_{SS}^1 \Leftrightarrow \epsilon_1 \leq \frac{2\sqrt{(a-c + \epsilon_{max}^1)^2 + [T_1 + 9b(K_1 + t(\mathcal{K} - K_1))]} - [2(a-c) - \epsilon_m]}{3}$$

Proof. First note that $[T_1 + 9b(K_1 + t(\mathcal{K} - K_1))] > 0$. Rest of the proof is trivial. \square

The intuition is that they will perform collaborative R&D only if both of them think that the additional reduction in cost due to collaboration is significant. The next Lemma 2.15 compares the profit levels when the firms conduct collaborative R&D as against when they do not perform any R&D at all.

Lemma 2.15. *Suppose $\epsilon_1 \leq A_1$, then,*

$$E\Pi_{CC}^1 \geq \Pi_{NN}^1 \Leftrightarrow \epsilon_{max}^1 \geq \left[\sqrt{(a-c)^2 + (9b\hat{r}K_1 - T_1)} - (a-c) \right]$$

Proof. Trivial. Note that $(9b\hat{r}K_1 - T_1) > 0$. \square

The following Lemma 2.16 ranks expected profit for Firm 1 from collaborative R&D against profit of Firm 1 when only this firm undertakes R&D but Firm 2 does not.

Lemma 2.16. *Suppose $B_1 > \epsilon_1 > A_1$, then,*

$$E\Pi_{CC}^1 \geq \Pi_{SN}^1 \Leftrightarrow \epsilon_1 \leq \frac{2\sqrt{(a-c+\epsilon_{max}^1)^2 + [T_1 + 9b(K_1 + t(\mathcal{K} - K_1))]} - [2(a-c) + \epsilon_m]}{3}$$

Proof. The proof is similar to the proof of the Lemma 2.14. □

The following Proposition 2.17 establishes the sufficient condition for expected profit from collaborative R&D being higher than that from non-collaborative R&D by both the firms.

Proposition 2.17. *Suppose $\epsilon_1 \geq B_1$ then,*

$$(\epsilon_{max}^1 - \epsilon_1) \geq \frac{\epsilon_1 - \epsilon_m}{2} \Rightarrow E\Pi_{CC}^1 > E\Pi_{SS}^1$$

Proof. Since $[T_1 + t(\mathcal{K} - K - 1)] > 0$, we have

$$\begin{aligned} (\epsilon_{max}^1 - \epsilon_1) &\geq \frac{\epsilon_1 - \epsilon_m}{2} \\ \Leftrightarrow (a - c + \epsilon_{max}^1) &\geq \left(a - c + \frac{3\epsilon_1 - \epsilon_m}{2} \right) \\ \Rightarrow E\Pi_{CC}^1 &> E\Pi_{SS}^1 \end{aligned}$$

□

Finally, the following Proposition 2.18 states the sufficient condition for expected profit of Firm 1 from collaborative R&D being higher than that when only Firm 1 undertakes R&D but Firm 2 does not.

Proposition 2.18. *Suppose $B_1 > \epsilon_1 > A_1$, then,*

$$(\epsilon_{max}^1 - \epsilon_1) \geq \frac{\epsilon_1 + \epsilon_m}{2} \Rightarrow E\Pi_{CC}^1 > E\Pi_{SN}^1$$

Proof. The proof is similar to that of Proposition 2.17. □

Remark 1. Several remarks can be made. First if both the Firms otherwise interested in conducting research will consider collaborative R&D if Lemma 2.14 holds for both of them. Second if both the Firms otherwise not interested in performing R&D will undertake collaborative R&D if Lemma 2.15 holds for both of them. Finally suppose they only want to conduct research if they expect that the rival will not be going to do research even then rather doing research alone they will prefer collaborative research if Lemma 2.16 holds for both of them. Finally, without any Government intervention a firm with very high efficiency or a firm with very low efficiency will not do collaborative research. This particular result marks a departure from Marjit (1991)[11] and Kabiraj and Chattopadhyay (2014)[9] where it is suggested that firms engage in collaborative R&D when the probability of success is either very high or very low. We, however, must note that, Marjit (1991)[11] deals with complete information framework and two discrete contingencies (e.g. success or failure), whereas Kabiraj and Chattopadhyay (2014)[9] deal with discrete cases of success and failure in R&D in presence of incomplete information. Our framework involves continuous types and incomplete information and also entails two separate types about each of which the agents have private information.

2.3 Government Policies

Government can try to facilitate co-operative R&D when at least one firm is not conducting R&D or each firm is conducting research singlehandedly. In these cases Government can have several possible policies. First, it can provide capital at lower rates of interest as compared to the market rates. Second, it can provide capital and share the profit with the firms. Third, it can give subsidy for collaborative research. Finally, it can use tax reduction as a policy to encourage collaborative research. We will study the first and the third policies.⁷ But first we are assuming here that the sum of capital they have is strictly less than the amount of capital needed to conduct the R&D activity.

2.3.1 Cheap capital

Suppose Government wants to provide capital at lower rates of interest. In this case, as usual, the firms will divide the burden of interest equally. Suppose firms are interested in collaborative research if Government provides them cheap capital. Suppose Government wants to give $K_G > 0$ amount of capital (this is equal to $\mathcal{K} - K_1^R - K_2^R$, where K_i^R is the reported amount of capital by Firm i) at a rate of interest \hat{r} . Define $\tau = 1 + \hat{r}$. The following Proposition 2.19 establishes the required condition to ensure truthful reporting of level of capital possessed by each firm.

Proposition 2.19. *If Government wants that the firms report their level of capital truthfully then the minimum rate of interest that the Government should offer is $\tau = 2r$. Also it must be the case that $t \geq 2r$, otherwise Government can't use cheap capital as a policy instrument.*

Proof. Suppose $t \geq 2r$. Then two cases are possible:

Case 1 ($\tau < 2r$). In this case we will show that both the firms will report that they have no capital for research. Note that each firm will have to return $\frac{\tau}{2}K_G$ amount of money to the Government. So the total capital Firm 1 is investing if she reports truthfully, is $K_1 + \frac{\tau}{2} \int_0^{\mathcal{K}-K_1} (\mathcal{K} - K_1 - K_2) dG(K_2 | K_1 + K_2 < \mathcal{K})$; which is equal to $K_1 + \frac{\tau}{2G(\mathcal{K}-K_1)} \int_0^{\mathcal{K}-K_1} G(K_2) dK_2$.

On the other hand, suppose Firm 1 declares that she has zero capital. Then $K_G = \mathcal{K} - K_2$. Firm 1's investment here is $\frac{\tau}{2} \int_0^{\mathcal{K}-K_1} (\mathcal{K} - K_2) dG(K_2 | K_1 + K_2 < \mathcal{K}) - \hat{r}K_1$; that is $\frac{\tau}{2}K_1 - \hat{r}K_1 + \frac{\tau}{2g(\mathcal{K}-K_1)} \int_0^{\mathcal{K}-K_1} G(K_2) dK_2$. Finally,

$$[\tau < 2r] \Leftrightarrow \left[\left[K_1 + \frac{\tau}{2G(\mathcal{K}-K_1)} \int_0^{\mathcal{K}-K_1} G(K_2) dK_2 \right] > \left[\frac{\tau}{2}K_1 - \hat{r}K_1 + \frac{\tau}{2g(\mathcal{K}-K_1)} \int_0^{\mathcal{K}-K_1} G(K_2) dK_2 \right] \right]$$

That is, when $\tau < 2r$ the investment of Firm 1 is higher when she reports her capital endowment truthfully than when she reports zero. So Firms are better off putting all their money to the bank and asking the Government to provide all the capital required for the research.

Case 2 ($\tau \geq 2r$). First note that the firms are not going to be better off by putting their money in the bank. Second they can't borrow money from the bank because $t \geq \tau$. These two ensure that they will confess their true types to the Government. □

⁷We did not study tax reduction as a policy because we did not assume that the output or the profit of the firms are taxed by the Government. Also the analysis of lump-sum tax reduction is identical to that of the subsidy, which we have studied below.

Remark 2. As established in Lemma 2.19, the required condition for truthful reporting about capital possessed by each firm, is $\hat{\tau} = 1 + 2\hat{r}$, i.e. the interest rate at which the firms can borrow, has to be more than twice the deposit rate. Now, in actual reality, the market rate of interest for borrowing is much less than this level, for example, the State Bank of India offers a deposit rate 8.50 per cent per annum on deposits for one to five years⁸, while the interest rate for loans against gold ornaments or mortgage of property, which is generally availed of by industrial houses along with domestic households at times, is 12.75-13.00 per cent per annum⁹, far less than double the deposit rate. This holds true for other major banks in India as well. Thus the Government has no scope for providing cheaper loans, while also ensuring truthful reporting of capital held by the firms. So, as a policy for encouraging collaborative research, providing cheap capital is not a very attractive option for the Government. In real life also irrespective of the countries Government did not exercise this policy to encourage collaborative research.

2.3.2 Subsidy

Now we will study subsidy as a policy for encouraging collaborative research. Assume Government wants to offer S amount of money as subsidy if the firms agree to perform collaborative research. Denote $\mathcal{K} - S = \mathcal{K}_S$. It may be sensible to assume $\mathcal{K}_S > 0$ i.e. Government does not want to fund the collaborative research fully by itself. Here two cases can occur:

Case 1 ($\mathcal{K}_S > K_1$). Define $T_{S1} = 9b \left[\frac{r}{2} \int_{\mathcal{K}_S - K_1}^{\bar{K}} (1 - G(K_2)) dK_2 - K_1 - \frac{t}{2} \int_0^{\mathcal{K}_S - K_1} G(K_2) dK_2 \right]$. Note that $T_{S1} < 0$ and $T_{S1} > T_1$. Now all the results of Section 2.2 holds by replacing T_1 with T_{S1} .

Case 2 ($\mathcal{K}_S \leq K_1$). Define $\hat{T}_{S1} = 9b \left[\frac{r}{2} (K_1 + K_m - \mathcal{K}_S) \right]$. Obviously, $\hat{T}_{S1} > 0$.

The following Lemma 2.20 gives the expected profit in case of collaboration under subsidy.

Lemma 2.20. *The expected profit of Firm 1 when both the firms conduct research collaboratively is given by:*

$$E\Pi_{CC}^1 = \frac{(a - c + \epsilon_{max}^1)^2}{9b} + \frac{\hat{T}_{S1}}{9b}$$

Proof. Note that in this case, the expected profit of Firm 1 is given by

$$E\Pi_{CC} = \frac{(a - c + \epsilon_{max}^1)^2}{9b} + \frac{1}{9b} \int_0^{\bar{K}} (K_1 + K_2 - \mathcal{K}_S) dG(K_2)$$

□

When amount of cost reduction following R&D singlehandedly is higher than a threshold value but not very high, then the firms have incentive to conduct collaborative R&D as opposed to both conducting R&D singlehandedly as suggested by the following Lemma 2.21.

Lemma 2.21. *Suppose $\epsilon_1 \geq B_1$, then,*

$$E\Pi_{CC}^1 \geq E\Pi_{SS}^1 \Leftrightarrow \epsilon_1 \leq \frac{2\sqrt{(a - c + \epsilon_{max}^1)^2 + \left[\hat{T}_{S1} + 9b(K_1 + t(\mathcal{K} - K_1)) \right]} - [2(a - c) - \epsilon_m]}{3}$$

⁸<https://www.sbi.co.in/portal/web/interest-rates/domestic-term-deposits> as accessed on February 4,2015.

⁹<https://sbi.co.in/portal/web/interest-rates/loans-against-gold-ornaments-mortgage-of-property> as accessed on February 4,2015.

Proof. First note that $\left[\hat{T}_{S1} + 9b(K_1 + t(\mathcal{K} - K_1))\right] > 0$. Rest of the proof is trivial. \square

The following Lemma 2.22 establishes the necessary and sufficient condition under which the firms always prefer to conduct R&D collaboratively than both not conducting any R&D at all.

Lemma 2.22. Suppose $\epsilon_1 \leq A_1$, then,

$$E\Pi_{CC}^1 \geq \Pi_{NN}^1 \Leftrightarrow \epsilon_{max}^1 \geq \left[\sqrt{(a-c)^2 + (9b\hat{r}K_1 - \hat{T}_{S1})} - (a-c) \right]$$

Proof. Trivial. Note that $(9b\hat{r}K_1 - \hat{T}_{S1}) \gtrless 0$. \square

Remark 3. Note that if $(9b\hat{r}K_1 - \hat{T}_{S1}) \leq 0$ and $\epsilon_1 \leq A_1$ then $E\Pi_{CC}^1 > \Pi_{NN}^1$ always holds.

Comparing Lemma 2.14 and Lemma 2.21, we find that, since $T_1 < \hat{T}_{S1}$ (as noted earlier, that $T_1 < 0$ and $\hat{T}_{S1} > 0$), therefore the upper bound of ϵ_1 in the required condition for expected profit from collaborative R&D being higher than single handed R&D, goes up in presence of subsidy. Similarly, comparing Lemma 2.15 and Lemma 2.22, we observe, that, again because $T_1 < \hat{T}_{S1}$, the upper bound of ϵ_{max}^1 goes up. These two together suggest, that for a wider range of values of the cost reduction parameter, collaborative R&D will be chosen over either singlehanded R&D or no R&D at all. So, subsidy provides an effective way of encouraging collaborative research. In fact in most of the countries Governments either provide subsidy or reduce tax to encourage collaborative research. As noted earlier in the paper, the study by Sakakibara and Cho (2002)[14] suggests that Japan could successfully catch up with the US in semiconductor technology through direct support from the Japanese Government in the form of subsidy to the VLSI projects. This is the most celebrated example and believed to be one of many such success stories of Government support to collaborative R&D¹⁰.

3 Conclusion

The benefits of collaborative R&D among firms in industries is widely acknowledged in standard industrial organisation literature and the role of Governments in encouraging collaborative R&D, thus pushing out the technology frontier is also supported by theoretical analysis and empirical findings. The governments can resort to various alternative methods of supporting collaborative R&D. This paper has made an attempt to compare two alternative policy options in this regard e.g. providing subsidies as opposed to offering funds in the form of loans at cheaper rates of interest. As noted earlier, the results in our model suggest that offering cheap capital by Government is not a suitable policy tool, since a truthful reporting on the level of capital possessed by firms requires that the Government needs to charge an interest rate which is more than twice the deposit rate. But market rates of interest on borrowing is much less than this level. Thus the Government is not left with any scope for providing loans at cheaper rates as compared to the market rates. Thus offering cheaper loans, although a suitable policy option, used in encouraging various industrial activities, e.g. used for promotion small scale industries in India, is not at all an option for encouraging collaborative R&D. On the other hand, provision of subsidy for encouraging R&D appears to be a good policy option as suggested by our results. Subsidy, as noted earlier in the paper, is a standard tool for promoting collaborative R&D. All the real life examples as observed in the cases of Japan, Germany, USA etc. bear testimony to the success stories of this policy. Provision of subsidy for any project in developing countries, where there is scarcity of capital is a

¹⁰Sakakibara and Cho (2002)[14]

widely debated issue. In presence of capital scarcity in the private sector, subsidy can definitely help growth of industry. Particularly, promotion of collaborative R&D, given the positive externalities associated with it, has very important social welfare implications. Also, the range of values of the cost reduction parameters of the firms, for which collaborative R&D occurs under subsidy is wider as compared to the occurrence of any R&D activity, collaborative or non-collaborative. This means to say that subsidy encourages relatively “non-efficient” firms to engage in collaborative research, who otherwise without subsidy will prefer not to invest in R&D. This observation is quite explicit from the results in our model. Thus, subsidy is an appropriate tool for encouraging R&D. We, however, have not studied the impact of tax rebate on a proportional basis as opposed to the standard practice of providing subsidy here, which can be taken up as a further research question in this direction.

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