Returns to tail hedging

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13 February 2015

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Abstract

Tail hedging is a portfolio management strategy meant to reduce the risk of large losses. For an investor who holds a stock market index fund, the strategy entails buying out of the money put options on the index. Research suggests the strategy works well in practice and I explore the returns to tail hedging in a simple theoretical model. I calculate descriptive statistics for the returns to tail hedging when the stock price has either a normal or fat tailed distribution. I find that tail hedging is rewarding when stock prices have fat tails.

Keywords: Portfolio management, tail option, fat tail, simulation.

JEL Classification: B50, C63, G11, G32.

Returns to tail hedging

1. Introduction

Diversification can reduce risk in a stock portfolio, but cannot eliminate the risk of large losses due to widespread crisis. Investors can reduce this risk by using a technique called tail hedging, which entails buying out of the money put options. However, tail hedging requires specific expertise to trade in illiquid options and portfolio managers' time may be better spent assessing their investments than preparing for systemic crises. This creates an opportunity for firms like Universa Investments to provide tail hedging services to institutional investors. Spitznagel, Yarckin, Mann (2015), who are affiliated with Universa Investments, describe the performance of a typical tail hedging strategy from 2004-2014. They find that the strategy has higher risk adjusted returns than many other types of portfolios. The evidence suggests that tail hedging works in practice, but does it work in theory?

Spitznagel, Yarckin, Mann (2015) describe a simple tail hedged portfolio as one unit of the SP500 index and several out of the money put options on the index. They roll the options positions as they expire in order to ensure that "the tail-hedged portfolio breaks even for a down 20% move in the S&P 500 over a month" (2015, p.2). It is possible to construct such a portfolio of options in different ways based on the expiry dates, strike prices, and quantity of options used. However, the authors state simply that they buy "one delta which has a strike roughly 30-35% below spot" (2015, p.2), which helps me determine ballpark figures for appropriate quantity of options and strike price for a tail hedged portfolio.

I use a simple model with two time periods to explore tail hedging in theory. At the initial time period, the investor buys the stock and one put option. At the final time period, the option expires and they calculate their returns. I use simulation to estimate the distribution of returns under different assumptions about the data generating process and investor portfolio. I consider an unhedged portfolio in stock, a fully hedged portfolio where the notional value of the put option equals the value of the stock, an over-hedged portfolio where the notional is larger, and a portfolio where the investor only buys put options. For each portfolio, I simulate stock returns according to either a normal or fat tail distribution. However, I always use the Black-Scholes option price to create some mispricing in options that could make tail hedging particularly valuable.

2. Model

I denote stock price at expiry as S_i where the returns are either normal i=Z or fat tailed i=T. The normal model is given in Equation (1), where Z is a standard normal random variable. The average return is $\mu=0$ and standard deviation is $\sigma=0.10$. The initial stock price is $S_0=100$.

$$(1) \qquad S_Z = S_0 \exp(\mu + \sigma Z).$$

The fat tail model for stock prices is given in Equation (2). I assume returns have a T(v) distribution, with degree of freedom v>2. I use v=8 to create a mild degree of fat tails in the returns. As before, the average return is $\mu=0$ and initial price $S_0=100$. However, to ensure that the standard deviation of returns under fat tails is equal to the normal model, I use a different scale parameter σ_T. I use $\sigma_T=\sigma/s(T)$ where s() denotes sample estimate of standard deviation for the T variable.

$$(2) \qquad S_T = S_0 \exp(\mu + \sigma_T T).$$

Throughout the paper, I calculate the put option price P using Black-Scholes model as in Equations (3) and (4). Black-Scholes model is based on a normal probability model for returns, which implies that the options are mispriced when stock returns have fat tails. I assume the option price uses true volatility, $\sigma=0.10$, time to expiry is $\tau=1$, and the interest rate is zero $r=0$.

(3) $d_1=(1/\sigma\sqrt{\tau})(\log(S_0/K)+(r+½\sigma^2)\tau)$, $d_2=d_1-\sigma\sqrt{\tau}$.

(4) $P=Ke^{-r\tau}\Phi(-d_2)-S_0\Phi(-d_1)$.

I calculate the agent's wealth at expiry as in Equation (5). I denote the quantity of put options in the portfolio as q_P and quantity of stock as q_S.

(5) $W(S_i)=q_S S_i+q_P(\max[K-S_i,0])$.

I calculate the distribution for wealth at expiry and returns as in Equation (6). I denote returns as $R(q_S,q_P,K)$ to emphasize the key variables: quantity of options q_p, stock q_S, and strike price K. I consider different combinations of values for these variables to explore different types of portfolios.

(6) $R(q_S,q_P,K)=W(S_i)/(q_S S_0+q_P P)$

To analyze the model, I specify all parameters above and draw a sample of observations on the stock price with sample size $n=10^7$. I use the sample to estimate the distribution of returns for the portfolio, then discuss several statistics that characterize the distribution.

3. Results

The first portfolio that I consider is an unhedged portfolio, with $q_S=1$, $q_P=0$. I report statistics from the distribution of returns under both the normal and fat tail model in Table 1. The table shows that the average and standard deviation of returns are equal for the normal and fat tail models, but the skewness and kurtosis is larger in the fat tail model. These two features occur by my construction of the model.

Table 1: Statistics for returns with unhedged portfolio

	Average	Standard Deviation	Skewness	Kurtosis
Normal	0.50%	10.1%	30.2%	3.17
Fat tails	0.50%	10.1%	54.8%	5.71

Table 2 describes returns with a tail hedge, q_S=1, q_P=1, and K=0.8S_0. The statistics suggest that the tail hedge has a very small effect on the distribution of returns, but the effect is apparent. Notice how the skewness increases and kurtosis decreases for both the normal and fat tail model, this occurs because the tail option removes extreme downward move in portfolio. Notice also how the average returns increase and standard deviation of returns decrease under the fat tail model. In other words, the tail hedge improves the risk adjusted returns under the fat tail model. This encouraging result suggests it may be valuable to consider a larger tail hedge where q_P>1.

Table 2: Statistics for returns with tail hedge

	Average	Standard Deviation	Skewness	Kurtosis
Normal	0.50%	9.99%	35.4%	3.09
Fat tails	0.53%	9.94%	69.8%	5.61

Table 3 describes returns with an extremely large tail hedge, q_S=1, q_P=10, and K=0.8S_0. The results show that a large tail hedge decreases average returns under the normal model, but actually increases them under fat tails. Although the standard deviation of returns increase in this case, the average returns increase even faster, which suggests that a large tail hedge can improve risk adjusted returns even further. However, these gains may be due to the fact that the options are mispriced under the fat tail model.

Table 3: Statistics for returns with large tail hedge

	Average	Standard Deviation	Skewness	Kurtosis
Normal	0.45%	9.99%	62%	4.63
Fat tails	0.82%	11.1%	311%	41.72

Table 4 describes returns when an investor buys put options with no stock, q_S=0 and q_P=1. One section of the results describes returns with a tail option K=0.8S_0 and the other describes an option with strike at-the-money K=S_0. The average returns are generally negative because options function as insurance. However, the results show that buying tail options under the fat tail model provides large positive average returns relative to the cost of option (80.9%). This large return is due to the mispricing of options under fat tails, which is related to the trading activities of Spitznagel and Nassim Taleb at Empirica Capital.

Table 4: Statistics for returns with buying naked put options

	Tail option		At-the-money options	
	Average	Standard Deviation	Average	Standard Deviation
Normal	-13.1%	1024%	-6.0%	134%
Fat tails	80.9%	1990%	-10.0%	135%

Table 5 describes returns with an at-the-money hedge, q_S=1, q_P=1, K=S_0. The average returns are much lower than the unhedged portfolio for both models because at-the-money options are much more expensive than tail options. However, the options also cause the standard deviation of returns to decrease substantially. These results begin to show just how different it can be to hedge a portfolio with at-the-money put options versus tail options.

Table 5: Statistics for returns with at-the-money hedge

	Average	Standard Deviation	Skewness	Kurtosis
Normal	0.26%	6.15%	180%	6.35
Fat tails	0.10%	6.38%	264%	16.17

Although the differences between the average returns in this section are generally very small, keep in mind that 1/100th of one percent is known as a basis points in finance and serves a useful function for measuring returns. Furthermore, it is not clear from my analysis that the differences described here are statistically significant. However, I would suggest that the differences are economically significant because I am working with a model of one time step. In practice, these differences can be compounded over many periods and grow to a large amount.

4. Discussion

I present a simple theoretical model to explore the distribution of returns for portfolios with tail hedging. I compare portfolio returns when prices are generated according to a normal probability model versus one with fat tails and show that tail hedging is associated with an increase in risk adjusted returns under the fat tail model, which may be driven by the fact that the options are underpriced in this model.

It is possible to extend the analysis presented here in several ways. One way is to use another measure of risk to compare portfolios. Spitznagel, Yarckin, Mann (2015) use semi-variance instead of variance because it only measures downward variation in prices and the benefits of tail hedging may be even more apparent using that measure than standard deviation. Another way to extend my analysis is to further develop the concept of time in the model. A theoretical model with more time steps would allow for a more sophisticated tail hedging strategy, such as scaling in or out of options positions over time, and important stylized features of financial time series, such as volatility clustering. I believe there are many opportunities to develop theory to better understand the empirical results described by Spitznagel, Yarckin, Mann and demonstrated in related business activity.

References

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