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TIME SERIES ANALYSIS OF MACROECONOMIC CONDITIONS
IN OPEN ECONOMIES

by

Gover Barja

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Economics

UTAH STATE UNIVERSITY
Logan, Utah

1995

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ABSTRACT

Time Series Analysis of Macroeconomic Conditions
in Open Economies

by

Gover Barja, Doctor of Philosophy
Utah State University, 1995

Major Professor: Dr. Terrence Glover
Department: Economics

Three macroeconomic issues are examined in separate self-contained studies. The first study tests the business cycle theory with application of an enhanced Augmented Dickey-Fuller test on the U.S. time series of real gross national product. Unlike previous studies, the null hypothesis of a unit root is rejected. The second study tests for IS-LM conditions in the U.S. during the post-Bretton Woods era by combining the Johansen's approach to cointegration with bootstrap algorithms. The estimated model produces a dynamic version of the IS-LM that permits short-term evaluations of fiscal and monetary policies. The third study seeks to explain the observed persistence in the Bolivian dollarization process. It is found that dollarization is now an irreversible process, with the Bolivian economy in transition toward equalization with U.S. prices and interest rates.

(125 pages)

To Sonia, my partner in life, with love,
to Rebeca, my daughter, for bringing me
such joy, and to my parents, for their love
and support.

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CONTENTS

	Page
ABSTRACT	iii
DEDICATION	iv
ACKNOWLEDGMENTS	v
LIST OF TABLES	ix
LIST OF FIGURES	xi
CHAPTER	
1. INTRODUCTION	1
2. BOOTSTRAPPING FOR UNIT ROOTS	4
Abstract	4
Introduction	4
Classical Procedures, The Bootstrap, and Robust Regression	8
Bootstrapping the ADF Test	12
Assessing the Power of the ADF Test	24
Concluding Remarks	31
References	32
3. A BOOTSTRAP EVALUATION OF COINTEGRATION: TESTING THE IS-LM	34
Abstract	34
Introduction	34
A Simple Open Economy IS-LM Model	37
Cointegration and Model Estimation	39
Conclusions and Final Comments	56
References	61

4.	SOME DYNAMIC ASPECTS OF THE BOLIVIAN DOLLARIZATION PROCESS	63
	Summary	63
	Introduction	63
	A Currency Substitution Model	67
	Testing for Irreversibility	72
	Cointegration and Dollarization	79
	Concluding Remarks	94
	References	98
5.	FINAL REMARKS	101
	APPENDICES	104
	Appendix A	105
	Appendix B	108
	Appendix C	114
	Appendix D	116
	Appendix E	118
	Appendix F	120
	CURRICULUM VITAE	122

LIST OF TABLES

Table	Page
2-1. Critical Values for Tau-Test	20
2-2. Critical Values for F-Test	20
2-3. OLS Mean Square Error Assessment	23
2-4. RREG Mean Square Error Assessment	23
2-5. Power Assessment of the Tau-Test under OLS	30
2-6. Power Assessment of the Tau-Test under RREG	30
3-1. Lag Order Selection Criteria	45
3-2. Bootstrap Trace Test	49
3-3. Bootstrap Maximum Eigenvalue Test	49
4-1. Application of the ADF test to the Dollarization Series	76
4-2. Test of Structural Change on the Dollarization Series	77
4-3. Order Selection Criteria	83
4-4. Trace Test for Cointegration Rank	84
4-5. Maximum Eigenvalue Test for Cointegration Rank	84
C1. Significance of HC_{1j} on GDP Equation	114
C2. Significance of HC_{2j} on Money Equation	114
C3. Significance of HC_{3j} on Inflation Equation	115
C4. Significance of HC_{4j} on Interest Rate Equation	115
C5. Significance of HC_{5j} on Budget Deficit Equation	115

D1.	Significance of B_{1j} on GDP Equation	116
D2.	Significance of B_{2j} on Money Equation	116
D3.	Significance of B_{3j} on Inflation Equation	117
D4.	Significance of B_{4j} on Interest Rate Equation	117
D5.	Significance of B_{5j} on Budget Deficit Equation	117
E1.	Significance of HC_{1j} on Dollarization Equation	118
E2.	Significance of HC_{2j} on Inflation Equation	118
E3.	Significance of HC_{3j} on the Dollar Deposits Equation	119
E4.	Significance of HC_{4j} on the Boliviano Deposits Equation	119
E5.	Significance of HC_{5j} on the Depreciation Equation	119
F1.	Significance of B_{1j} on the Dollarization Equation	120
F2.	Significance of B_{2j} on the Inflation Equation	120
F3.	Significance of B_{3j} on the Dollar Deposits Equation	121
F4.	Significance of B_{4j} on the Boliviano Deposits Equation	121
F5.	Significance of B_{5j} on the Depreciation Equation	121

LIST OF FIGURES

Figure	Page
2-1. Some statistical properties of differenced log GNP	13
2-2. Comparing weights with detrended and differenced lgnp	15
2-3. Empirical distributions of tau and F under OLS and RREG	19
2-4. Empirical distributions of the parameters beta under OLS and RREG	22
2-5. Some statistical properties of detrended log GNP	25
2-6. Cumulative effect of a unit shock at horizon h	27
3-1. Time series of macroeconomic variables	43
3-2. Empirical distributions of $(H^*C^* - \hat{H}\hat{C})/se(H^*C^*)$	54
3-3. Empirical distributions of $(B_1^* - B_1)/se(B_1^*)$	55
4-1. Some statistical properties of the time series of dollarization	75
4-2. Dollarization series versus fitted regression under the alternative	78
4-3. Time series of macroeconomic variables of interest	80

CHAPTER 1

INTRODUCTION

The novelty of the present dissertation research is in the combination of time domain time series methods with bootstrap techniques. The former is widely used in the econometrics literature for estimation of dynamic models. The latter is used here for hypothesis testing purposes. The combined technique is here used to analyze three macroeconomic issues of current interest where accuracy in inference, rather than the lack of macroeconomic theory, has increasingly become the central concern. Each of the three issues is examined and discussed in a separate self-contained study.

The objective of the first study is to test the business cycle theory, which is a basic building block of current macroeconomic theory. The test is performed using the univariate time series of real gross national product (GNP) from the United States. Questioning the cycle is, in practice, an issue of whether the time series of real GNP should be made stationary by detrending or differencing. The question is empirically answered using an enhanced unit root testing procedure that combines the classical test with robust regression and bootstrapping techniques. These techniques improve the accuracy of the classical test. This study is also used to develop the basic structure of the bootstrap algorithm needed for hypothesis testing in a single equation of the autoregressive kind, and to set the ground work for its application in a multivariate multiple equation framework.

The objective of the second study is to test for IS-LM conditions in the United States for the post-Bretton Woods era. The most general IS-LM framework is used to identify the

relevant macroeconomic variables allowing the time series of these variables to reveal their own story. This is achieved by treating each variable as equally endogenous in the estimation of a multivariate dynamic system. This is strictly a statistical-based test, and the statistical methods that accomplish this are of the autoregression kind. These methods are by their own nature data based and atheoretical. However, they can fit short-term dynamic interactions with the use of vector autoregression models that are long-term equilibrium relationships. Testing for cointegration, and short-term/long-term interactions with error correction models are likewise carried out. Given the sample size and problems of nonstandard distributions associated with these methods, the cointegration test and multivariate parameter evaluation are performed by implementing the bootstrap at a multivariate level.

The objective of the third study is to explain the observed irreversibility in the Bolivian dollarization process. Although current theory implicitly contains an explanation for the possibility of irreversibility, traditional empirical models fail to capture the complex dynamics that have given rise to it. Irreversibility is first verified with use the of a test for structural change applied to the monthly univariate time series of dollarization for the period from 1987 to 1994. Then, a multivariate dynamic system is estimated using identified relevant variables from a general model of currency substitution. Estimation is done in a cointegration-error correction framework as well in order to capture the relevant short-term, long-term, and short-term/long-term interactions among variables. Given the peculiarities of the model, foreign variables are treated as exogenous while domestic variables are treated as equally endogenous during estimation. The latter has the purpose of producing a data-

driven structure of the empirical model. Multivariate parameter evaluation and the cointegration test are also performed by implementing the bootstrap at a multivariate level.

CHAPTER 2

BOOTSTRAPPING FOR UNIT ROOTS

Abstract. The Augmented Dickey-Fuller (ADF) test is used to determine if the time series of U.S. GNP should be detrended or differenced for stationarity. The statistics of interest and power of the test are evaluated using a nonparametric bootstrap procedure. Estimation of the ADF equation is done by ordinary least squares and robust regression. The robust method downweights observations at the beginning and ending of the natural cycles in the data according to their strength, and thus has the effect of decreasing the degree of bias. Unlike the findings in the previous literature, in both cases the null of a difference stationary process is rejected at a 5% alpha level for a one-sided test. This result is also supported by a test power of above 80% when considering a plausible trend stationary alternative.

Introduction

One of the most interesting debates for over a decade now has been about the existence and behavior of the business cycle, which is a basic building block of current macroeconomic theory. It is generally assumed that real GNP tends to grow in cycles around a positive trend representing a long-term growth path. Growth is believed to be determined by real variables, like capital accumulation, population growth, and technological change, and therefore changes in these variables are a source of permanent change in the growth path. It is also believed that cycles are transitory deviations from the long-term growth path caused by monetary variables and, to a lesser extent, by real variables. The empirical consequence of this theory is that a macroeconomic time series could be regressed against a time variable

in order to eliminate the long-term component and generate a stationary series useful for business cycle short-term analysis. In statistical terms this is equivalent to detrending in order to generate a trend stationary (TS) representation of the time series of real GNP. However, in a paper by Nelson and Plosser (1982), the unit root testing procedure developed by Dickey and Fuller (1979; 1981) showed that almost all U.S. macroeconomic variables are random walks, and in particular that real GNP is a random walk with drift. A random walk simply meanders without exhibiting a tendency to return to a trend line, and therefore, real GNP is more correctly transformed into a stationary series by taking a first difference. In statistical terms this transformation generates a difference stationary (DS) representation of the time series of real GNP. The theoretical implication of this finding is that permanent sources of business cycles dominate transitory sources. Therefore, real GNP in the short run is always moving away from a previous position and never coming back to a within-cycle long-term growth path. That is, there is no within-cycle long-term growth path to come back to because the long-term is always changing through short-term adjustments.

Questioning the existence of the cycle involves in practice an assessment of whether the time series of real GNP should be made stationary by detrending or differencing. However, the Dickey-Fuller (DF) procedure, which tests the null of a DS representation, is sensitive to how it is performed. Dickey and Fuller (1979) find that it is sensitive to whether a nonzero mean or a time trend is assumed. Schwert (1987) finds it is sensitive to whether a moving average or autoregressive data-generating process is assumed. Sims (1988) finds it is sensitive to whether the test is performed using classical or Bayesian statistical inference. Simkins (1994) finds it is sensitive to the number of breaks in the trend when

considering a more flexible trend specification. These sensitivities are partly due to the lack of power this test has against an alternative hypothesis of a stationary but large root. Evidence of this sort has been provided by Phillips and Perron (1988), and by DeJong, Nankervis, Savin, and Whiteman (1989).

These studies of power analyze the performance of the DF test in general circumstances. However, given the macroeconomic implications of a unit root in real GNP, it is of importance to evaluate the power of the DF test in this particular case. The work of Rudebush (1993) goes in this direction. He evaluates the ability of the DF test to distinguish between two data-based plausible TS and DS alternative representations of real GNP. More specifically, within a Monte Carlo approach, Rudebush draws random values from a normal distribution and, together with the estimated coefficients of the TS and DS representations, generates an empirical distribution of the statistic of interest for each case. Then, by computing the p-value of the sample statistic for the two alternative distributions, he concludes that the existence of a unit root in real GNP is uncertain due to low test power.

The procedure used by Rudebusch is called parametric bootstrap by Efron and Tibshirani (1986), and its main advantage is that it takes into account the sample size and the nonstandard distribution problem associated with the DF test. However, instead of substituting the actual residuals with random draws from a normal distribution, another method would be to draw random values from the residuals themselves to obtain an approximation to the true distribution of the statistics of interest. This approach is called the nonparametric bootstrap by the same authors. The bootstrap was introduced by Efron (1979) and applications to stationary autoregressive processes were implicitly introduced by

Freedman (1981) and Freedman and Peters (1984). One aspect of bootstrap applications to regression equations, emphasized by Freedman and Peters (1984), is that the fitted regression equation and its estimated parameters are the true model of the phenomena of interest. In the context of unit root testing, however, neither the Dickey-Fuller (DF) equation nor Rudebush's TS and DS representations constitute models of real GNP. The TS and DS representations are simply two alternative transformations of real GNP, and the DF equation is a way to decide which transformation is correct. A model of real GNP could contain many other macroeconomic variables, probably within a dynamic system of equations. However, it is these characteristics of the TS, DS, and DF equations that justify the use of the nonparametric bootstrap based on observed residuals since these residuals retain historic distributional information from the other variables not included in these equations. This is a further advantage of the nonparametric bootstrap, in addition to taking into account the sample size and nonstandard distribution problems.

The purpose of this study is to use the nonparametric bootstrap to evaluate the performance of the Dickey-Fuller (DF) test in a near unit root case. The data used are the quarterly time series of real GNP for the period 1948.3-1989.4. The DF equation is estimated by ordinary least squares and robust regression. This latter procedure is used to improve accuracy in estimation. The evaluation itself is done in three ways; first, by building critical values from the estimated empirical distributions of the statistics of interest and comparing them with the usual DF critical values; second, by determining the contribution of squared bias and variance to total mean square error in the estimated parameters resulting from ordinary least squares and robust regression estimation; third, by

assessing the power of the DF test against a plausible TS alternative.

Classical Procedures, the Bootstrap, and Robust Regression

To fix the idea of a unit root, consider the following model for a time series y_t :

$$y_t = \alpha_0 + \alpha_1 t + u_t \quad (1)$$

$$u_t = \rho u_{t-1} + \epsilon_t, \quad (2)$$

where ϵ_t is a zero mean white noise process. The reduced form of model (1)-(2) yields the following equation:

$$y_t = \gamma + \delta t + \rho y_{t-1} + \epsilon_t, \quad (3)$$

where $\gamma = [\alpha_0(1-\rho) + \alpha_1\rho]$ and $\delta = \alpha_1(1-\rho)$. Equation (3) and y_t are said to have a "unit root" when $\rho = 1$ (which in turn implies $\delta = 0$). In this case, y_t is nonstationary, but the process is said to be difference stationary (DS) because stationarity is induced by first differencing: $\Delta y_t = y_t - y_{t-1} = \gamma + \epsilon_t$. Alternatively, when $\rho < 1$ and $\alpha_1 \neq 0$, y_t is stationary about the linear trend $\alpha_0 + \alpha_1 t$, and hence is said to be trend stationary (TS). The case when $\rho < 1$ and $\alpha_1 = 0$ implies y_t is stationary itself.

Unit root tests based on equation (3) were pioneered by Dickey and Fuller (1979), and the most common procedure is to test the null hypothesis $H_0: \rho = 1$ based on the statistics $\tau = (\hat{\rho} - 1)/\text{se}(\hat{\rho})$, and $F(0,1)$. Here, $\hat{\rho}$ is the ordinary least squares (OLS) estimator of ρ , $\text{se}(\hat{\rho})$ is the standard error of $\hat{\rho}$, and $F(0,1)$ is the usual F-statistic for testing $H_0: \delta = 0, \rho = 1$. Dickey and Fuller have shown that these statistics do not have the traditional t and F distributions but, rather, some nonstandard distributions. However, they tabulated critical values for the asymptotic case based on Monte Carlo simulation.

The above Dickey-Fuller (DF) test requires the error term ϵ_t to be white noise. Said and Dickey (1984) further developed the test to accommodate AR serial correlation in the error term and introduced the "augmented" Dickey-Fuller (ADF) test, which involves estimating $\tau(\text{tau})$ and F using

$$y_t = \gamma + \delta t + \rho y_{t-1} + \sum_{i=1}^g \theta_i \Delta y_{t-i} + \epsilon_t, \quad (4)$$

where Δ is the first difference operator and with the number of lags, g , taking into account the presence of AR serial correlation. Said and Dickey also showed that the "augmentation" of the ADF test leaves the statistics asymptotically distributed according to the Dickey-Fuller tables even when the errors are ARMA processes of unknown order.

Subtracting y_{t-1} from both sides produces the following equation:

$$\Delta y_t = \gamma + \delta t + \beta y_{t-1} + \sum_{i=1}^g \theta_i \Delta y_{t-i} + \epsilon_t \quad (5)$$

where $\beta = \rho - 1$. The null hypothesis for the DF test now changes to the more convenient form of $H_0: \beta = 0$ for the τ -test, and $H_0: \delta = 0, \beta = 0$ for the F-test. When the null hypothesis is true, equation (5) becomes a random walk with drift plus g short-term dynamic components. The present study uses this last equation.

The bootstrap is a computer-intensive method used to establish the accuracy of a parameter estimate. To illustrate the bootstrap in autoregressive time series models, assume the following second order autoregressive model:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \epsilon_t \quad (6)$$

where the sample size for y_t is n , the α_i are unknown parameters and the errors are identified with an unknown distribution having a zero mean. Assume it is of interest to test $H_0: \alpha_i = 0$ and let $t(\alpha_i)$ be any measure of statistical accuracy, say the t-statistic. Following the work of Freedman and Peters (1984), and Hall and Wilson (1991), the bootstrap algorithm for hypothesis testing in a time series context would be the following:

Algorithm 1:

1. Begin by obtaining an estimate $\hat{\alpha}_i$ of the parameters, say by least squares, and compute the original $t(\alpha_i) = (\hat{\alpha}_i - 0)/se(\hat{\alpha}_i)$.
2. Compute the observed residuals $\hat{\epsilon}_t = y_t - \hat{\alpha}_0 - \hat{\alpha}_1 y_{t-1} - \hat{\alpha}_2 y_{t-2}$.
3. Take a random sample of size n drawn with replacement from ϵ_t to generate new residuals ϵ_t^* .
4. Construct new values y_t^* using the residuals ϵ_t^* , and the estimated dependence mechanism from step 1, $y_t^* = \hat{\alpha}_0 + \hat{\alpha}_1 y_{t-1}^* + \hat{\alpha}_2 y_{t-2}^* + \epsilon_t^*$, with $y_1^* = y_1$ and $y_2^* = y_2$.
5. Use the new values y_t^* to estimate equation (6) and generate bootstrap estimates α_i^* .
6. Compute bootstrapped $t(\alpha_i^*) = (\alpha_i^* - \hat{\alpha}_i)/se(\alpha_i^*)$ to approximate the null distribution of $t(\alpha_i) = (\hat{\alpha}_i - 0)/se(\hat{\alpha}_i)$. Go back to step 3 and repeat the process enough times (say 1,000) to obtain the empirical distribution function (df) for the statistic $t(\alpha_i)$.
7. Finally, based on this df evaluate the significance of the original $t(\alpha_i)$ estimated in step 1 by computing its (bootstrapped) p-value.

An alternative to the OLS estimates found in Steps 1 to 5 are these found using robust regression. Robust regression is a method useful in the presence of contamination in the data. To explain how robust regression operates, we begin with the classical regression

problem: $Y = X\beta + \epsilon$, where $Y(n \times 1)$ is a vector of observations, $X(n \times p)$ is a design matrix of full rank p , $\beta(p \times 1)$ is a vector of unknown parameters, and ϵ is a vector of errors which satisfy $E[\epsilon]=0$ and $\text{Cov}[\epsilon]=\sigma^2 I$. The method of least squares obtains an estimate of β that minimizes $\sum \epsilon_i^2$. The possibility of outliers in the observations or the design are handled by estimating weights for those outliers, then using the weights to obtain an estimate of β that minimizes $\sum \eta(\epsilon_i/s)$. This is the classical M-estimation approach, where $\eta(\cdot)$ is a symmetric loss function and s is a scale parameter. The derivative respect to β produces the following system of nonlinear equations:

$$\begin{aligned} \sum_{i=1}^n \psi\left(\frac{y_i - x_i^T \beta}{s}\right) &= 0 \\ \sum_{i=1}^n x_i \psi\left(\frac{y_i - x_i^T \beta}{s}\right) &= 0 \end{aligned}$$

where $\psi(\cdot)$ is the derivative of $\eta(\cdot)$. The computational procedure is the following:

1. Obtain an initial estimate β^0 of β .
2. Find residuals $\epsilon_i^j = y_i - x_i^T \beta^j$ associated with the j th estimate and compute an estimate of scale s^j .
3. Compute weights to be used in the next weighted least squares estimate

$$w_i^j = \frac{\psi(\epsilon_i^j/s^j)}{\epsilon_i^j/s^j}$$

4. Use the weights obtained in 3 to solve the following weighted least squares equations for β^{j+1}

$$\sum_{i=1}^n (y_i - x_i' \beta^{j+1}) w_i = 0$$

$$\sum_{i=1}^n (y_i - x_i' \beta^{j+1}) w_i x_{ij} = 0$$

5. Return to 2, unless the estimates differ from β^j by less than a desired accuracy.

Bootstrapping the ADF Test

Figure 2-1 presents plots describing some statistical properties of the first difference of the log of real GNP (lgnp). Plot (a) is lgnp before differencing. Plot (b) is the first difference of lgnp, called dlnp. Taking logs has a double objective. First, it decreases the magnitude of observed values of GNP. Second, its first difference expresses GNP in its growth rate form. Plots (c) and (d) show that the growth rate of GNP has a distribution close to normal and there are no obvious outliers. Plots (e) and (f) are the autocorrelation and partial autocorrelation functions (ACF and PACF) of the growth rate of GNP. These show first order autocorrelation, which suggests the following ADF equation¹:

$$\Delta \text{lgnp}_t = \gamma + \delta t + \beta \text{lgnp}_{t-1} + \theta \Delta \text{lgnp}_{t-1} + \epsilon_t \quad (7)$$

Equation (7) is the same as equation (5) but with $g = 1$. If $\delta = 0$ and $\beta = 0$ then (7) reduces to the null representation of lgnp suggested by the data, that is, a random walk with drift and first order autocorrelation. Following algorithm 1, the bootstrap procedure starts by estimating (7) (instead of (6)) using OLS and robust regression (RREG). The numbers in parentheses are the estimated coefficients which are subsequently divided by their

¹ Notice that the first difference of lgnp = Δlgnp (used in equations) = growth rate of GNP = dlnp (used in plots).

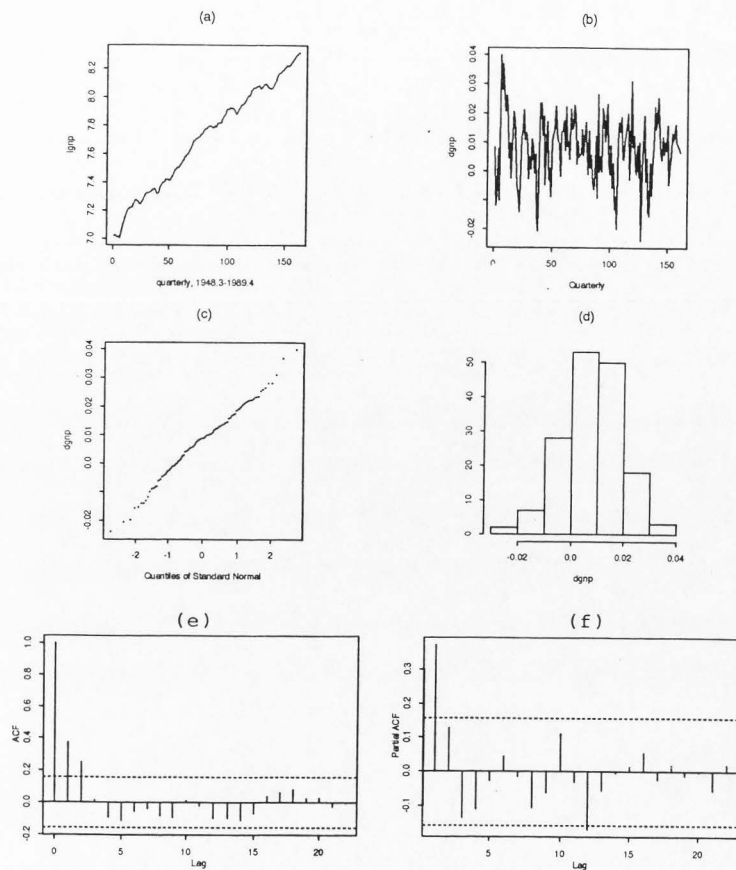


Fig. 2-1.—Some statistical properties of differenced log GNP.

standard errors:

$$\text{OLS: } \Delta \lg np_t = 0.384 + 0.0004 t - 0.0535 \lg np_{t-1} + 0.389 \Delta \lg np_{t-1} + \varepsilon_t \quad (8)$$

(2.810) (2.664) (-2.766) (5.334)

F-test = 4.08; Q = 9.75 (0.46); Q² = 7.40 (0.68); VN = 0.759 (0.77)

$$\text{RREG: } \Delta \lg np_t = 0.387 + 0.0004 t - 0.0538 \lg np_{t-1} + 0.406 \Delta \lg np_{t-1} + \varepsilon_t \quad (9)$$

(3.364) (3.219) (-3.312) (6.676)

F-test = 5.67; Q = 10.86 (0.36); Q² = 9.97 (0.44); VN = 1.00 (0.84)

Although plots (c) and (d) show no obvious outliers, it is worth noting that robust regression improves the accuracy of the estimated parameters. The best way to see this is by comparing the computed weights with the differenced and detrended series of $\lg np$. Plot (a) in Figure 2-2 is the detrended series, (b) is a plot of the weights, and (c) is the differenced series that produces the weights. The weights are values between zero and one, and they indicate the proportion by which the original values are downweighted in order to achieve the most robust fit. What is interesting to note is that the downweighted values correspond to the beginning or ending of the natural cycles in the data. That is, what robust regression achieved is similar to a regression with dummy variables at the beginning or ending of cycles with the difference that each dummy is not weighted equally, but rather according to the initial or ending strength of each cycle.

The estimated coefficients in both regressions do not show much difference, but in all cases the RREG statistics are greater in magnitude. The τ -statistics are $\tau_{\text{OLS}} = -2.766$ and $\tau_{\text{RREG}} = -3.312$, which are generally evaluated using the DF critical values. The F-statistics are $F_{\text{OLS}} = 4.08$ and $F_{\text{RREG}} = 5.67$, which are also evaluated using their DF critical values. The

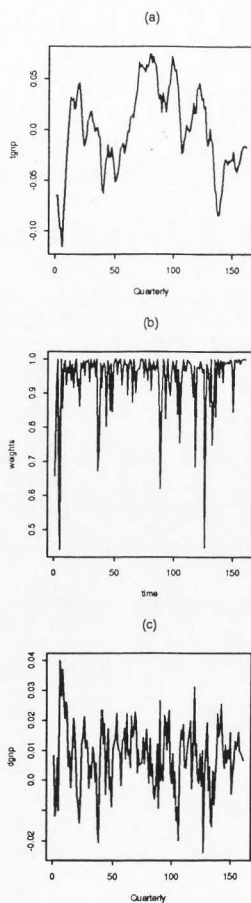


Fig. 2-2.—Comparing weights with detrended and differenced lgnp.

statistics Q , Q^2 , and VN where computed with the objective of verifying if the residuals from both equations are in fact independent before continuing with the bootstrap algorithm. The first is the Box-Ljung Q -statistic designed to test the null of zero autocorrelation at all lags in the residuals. The second, Q^2 , is also the Box-Ljung Q -statistic but this time it is used to test the null of zero autocorrelation at all lags in the squared residuals. This last test is used to identify possible heteroskedasticity. The Q -statistic is given by

$$Q = n(n+2) \sum_{j=1}^p \frac{r_j^2}{(n-j)}$$

where r_j is the j th autocorrelation and n is the number of observations. Under the null, Q has a χ^2 -distribution with degrees of freedom equal to the number of autocorrelations, p . For $p=10$, the 5% critical value for the χ^2 -distribution is 18.3, which is above the computed values for Q and Q^2 in both regressions. Values of $p > 10$ were not considered given that they reduce the power of the Q -test. The numbers in parentheses beside the computed values for this statistics are p -values. In all cases we fail to reject the null of independence. The third statistic, VN , is the Rank von Neumann Ratio designed to test the null of independence of the residuals with a nonparametric approach. The VN statistic is given below, where v is the Neumann ration and r_i is the rank associated with the i th residual. For large n and under the null, v is approximately distributed $N(2, 20/(5n+7))$, and therefore $VN \sim N(0,1)$. The numbers in parentheses beside the computed values for VN are p -values. In both

$$v = \frac{\sum_{i=2}^n (r_i - r_{i-1})^2}{n(n^2-1)/12}$$

$$VN = \frac{v-2}{\sqrt{(20/(5n+7))}}$$

regressions we fail to reject the null of independence of the residuals.

Having parameter estimates and independent residuals, a new bootstrap algorithm was developed to account for the requirements of the unit root testing procedure, but following the main structure of algorithm 1. It is of interest to test two hypotheses. First, $H_0: \beta=0$, for which the statistic tau is computed. Second, $H_0: \delta=0$ and $\beta=0$, for which the F-test is computed. The original values of tau are shown in the above estimated ADF equations (8) and (9). The following algorithm describes the steps followed in bootstrapping residuals from the OLS regression:²

Algorithm 2:

1. Take a random sample of size n drawn with replacement from the standardized residuals $\hat{\epsilon}_t$, of the original regression, to generate new residuals ϵ_t^* .
2. Construct new values $\lg np_t^*$ using the residuals ϵ_t^* , together with the estimated ADF structure and dependence mechanism from the OLS regression (8),

$$\Delta \lg np_t^* = \hat{\gamma} + \hat{\delta}t + \hat{\beta} \lg np_{t-1}^* + \hat{\theta} \Delta \lg np_{t-1}^* + \epsilon_t^*,$$

with $\lg np_1^* = \lg np_1$ and $\lg np_2^* = \lg np_2$.

²All computations for estimation and bootstrap hypothesis testing were programmed in Splus, version 3.2 release 1 for Sun SPARC, SunOS 5.x: 1993. Computer programs are presented in appendix A.

3. Use lgnp^*_i to estimate equation (7) and generate bootstrapped parameter estimates.
4. Also compute bootstrapped $\tau^* = (\beta^* - \hat{\beta})/\text{se}(\beta^*)$ and $F^*(0,0)$ to approximate the distribution of $\tau = (\hat{\beta} - 0)/\text{se}(\hat{\beta})$ and $F(0,0)$.
5. Go to step 1 and replicate 1000 times.
6. Finally, use the 1000 values of τ^* and F^* to build their empirical distribution functions (df's) to evaluate the significance of the original τ and F .

The algorithm used for bootstrapping residuals from the RREG regression follows the same steps as algorithm 2 (see Shorack, 1982) except that at step 1 it samples from the standardized weighed residuals, and at step 3 it estimates a robust regression to produce bootstrapped parameter estimates.

Plots (a) and (c) in Figure 2-3 present the estimated empirical df's for the tau-test under OLS and RREG, respectively. In both cases the distribution of τ appears not much different from a normal distribution. However, the range of values τ can take does suggest important differences compared to the Dickey-Fuller (DF) critical values, as can be seen in Table 2-1. The DF critical values are shifted to the left compared to the bootstrap RREG and bootstrap OLS critical values. By using the estimated τ statistics from equations (8) and (9), a one-sided test of the null hypothesis of $\beta=0$ can be rejected at a 5% alpha level for $\tau_{\text{OLS}} = -2.766$, but cannot be rejected at a 2.5% level. In the RREG case, the conclusion is exactly the same for $\tau_{\text{RREG}} = -3.312$. However, the DF test fails to reject the null at even a 10% alpha level when considering the value of τ_{OLS} , and it rejects the null at a 10% alpha level for τ_{RREG} .

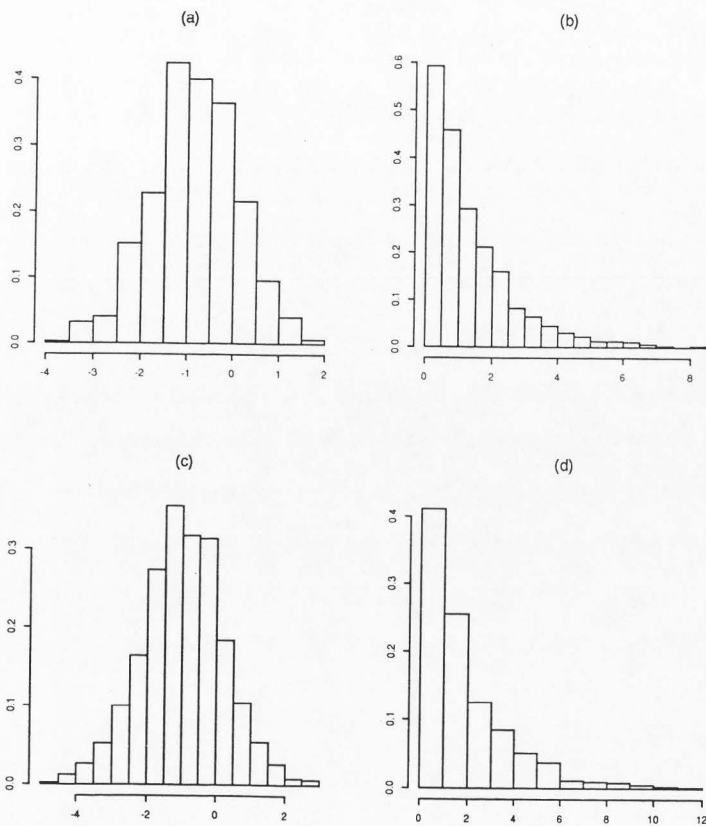


Fig. 2-3.—Empirical distributions of tau and F under OLS and RREG.

TABLE 2-1
CRITICAL VALUES FOR TAU-TEST

	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
Bootstrapped Tau_{OLS}	-3.15	-2.78	-2.41	-2.06	0.32	0.69	0.96	1.22
Bootstrapped Tau_{RREG}	-3.88	-3.36	-2.96	-2.47	0.48	0.94	1.35	1.74
Dickey-Fuller	-3.99	-3.69	-3.43	-3.13	-1.23	-0.92	-0.64	-0.31

TABLE 2-2
CRITICAL VALUES FOR F-TEST

	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
Bootstrapped F_{OLS}	0.01	0.03	0.07	0.15	3.00	3.89	4.90	5.92
Bootstrapped F_{RREG}	0.01	0.03	0.10	0.21	4.29	5.43	6.65	8.63
Dickey-Fuller	0.76	0.94	1.13	1.39	5.39	6.34	7.25	8.43

Plots (b) and (d) in Figures 2-3 present the estimated empirical df 's for the F-test under OLS and RREG. Once again the range of values of these distributions suggest important differences compared to the DF critical values, as can be seen in Table 2-2.

By using the computed F-statistics from equations (8) and (9), the null hypothesis of $\delta=0$ and $\beta=0$ can be rejected at a 5% alpha level for bootstrapped $F_{OLS}=4.08$ and bootstrapped $F_{RREG}=5.67$, but it cannot be rejected for a 2.5% alpha level. This result is consistent with the findings above. When using the DF critical values, the value of F_{OLS} cannot be rejected while F_{RREG} can be rejected at a 10% alpha level. This result is also consistent with the findings above.

Failure to reject the null is the stylized fact that has emerged from applications of the DF test, but as found here, the small sample results in a nonparametric bootstrap context are somewhat different. These results suggest that US real GNP is a trend stationary series (at a 5% alpha level and if we are willing to accept a one sided test) which grows in cycles around a positive time trend. By using the estimated equations (8) and (9), and the definitions given in (1)-(5), the estimated time trends are $7.045 + 0.00747t$ for the OLS case, and $7.062 + 0.00743t$ for the RREG case.

Figure 2-4 shows the distribution of the bootstrapped parameters of interest. Plots (a) and (b) correspond to the distributions of β^* and $se(\beta^*)$, respectively, and under OLS. Plots (c) and (d) are their RREG counterparts. In all of the plots there is some degree of skewness either toward the right or left tail. This degree of skewness is more accentuated in the distribution of the parameters rather than on their standard errors. It is interesting to note, however, that dividing parameters by standard errors produces symmetric distributions as in the distribution for tau. One useful aspect of these distributions is that they can help establish the contributions of squared bias and variance to total mean squared error (MSE). Table 2-3 shows the proportions of squared bias and variance to MSE, plus a measure of relative bias for all OLS parameter estimates. Table 2-4 presents the same for all RREG parameter estimates. The definitions used were variance = $var(b^*)$, bias = $E(b^*) - \hat{b}$, and relative bias = $bias/se(b^*)$ for $b = (\gamma, \delta, \beta, \theta)$.

In comparing the tables, OLS produces smaller MSE for γ and δ , but RREG produces smaller MSE for β and especially for θ . If an estimator is unbiased, then its variance should equal its MSE. This does not happen with the OLS estimates where squared

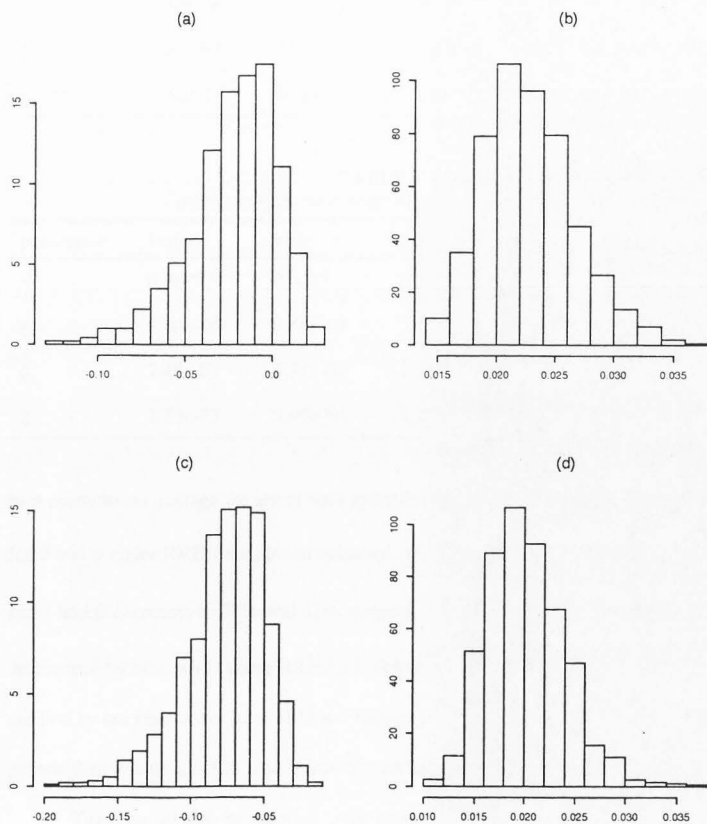


Fig. 2-4.—Empirical distributions of the parameters β under OLS and RREG.

TABLE 2-3
OLS MEAN SQUARE ERROR ASSESSMENT

parameter	var(b*)	bias ²	MSE	bias ² /MSE	bias/se(b*)
γ	0.0317	0.0510	0.0827	0.6166	1.2683
δ	3.88e-08	5.21e-08	9.09e-08	0.5733	1.1592
β	6.33e-04	9.65e-04	1.59e-03	0.6038	1.2347
θ	5.32e-03	0.1518	0.1571	0.9661	5.3389

TABLE 2-4
RREG MEAN SQUARE ERROR ASSESSMENT

parameter	var(b*)	bias ²	MSE	bias ² /MSE	bias/se(b*)
γ	0.0899	0.0417	0.1316	0.3168	0.6810
δ	5.40e-06	9.46e-09	5.41e-06	1.74e-03	0.0418
β	7.48e-04	5.31e-04	1.27e-03	0.4149	0.8421
θ	4.70e-03	5.96e-06	4.70e-03	1.26e-03	0.0356

bias accounts on average for about 60% of MSE (and 96% for θ). However, the estimates for δ and θ under RREG are almost unbiased, and the proportion of squared bias on MSE for γ and β decreases to 31% and 41%, respectively. That is, MSE under OLS is mainly determined by bias, while under RREG it is determined by variance. This result can also be verified by our measure of relative bias = $\text{bias}/\text{se}(b^*)$, which consistently presents numbers greater than one under OLS, and below one under RREG.

These results may be pointing to the main problem of why the unit root uncertainty in real GNP could not be solved. It is not necessarily a problem of bad inference methods, but rather a problem of estimation methods. In our case OLS produces biased parameter

estimates at a given level of variance. RREG reduces bias but at a higher variance level compared to OLS. Neither of them produces unbiased estimates for β , the key parameter.

Assessing the Power of the ADF Test

The nonparametric bootstrap is again used here to assess the power of the ADF test in the particular case of $\lg np_t$. The analysis starts with an estimate of a plausible trend stationary (TS) representation of real GNP as the alternative hypothesis. There could be several alternative hypotheses but, here, it is of interest to consider one that shows opposite persistence behavior compared to the null of a DS representation.

Figure 2-5 presents various plots that describe some statistical properties of the detrended version of the log of real GNP ($\lg np$). Plot (a) is $\lg np$ before detrending. Plot (b) is detrended $\lg np$, called $tg np$. Plots (c) and (d) show that $tg np$ departs somewhat from normality, suggesting a possible mixture distribution, but there are no obvious outliers. Plots (e) and (f) are the ACF and PACF of $tg np$, and these show up to third order autocorrelation, which sets to three the number of lags for a plausible TS model. Following the ACF and PACF plots of the differenced $\lg np$ presented in Figure 2-1, the DS model will continue with one lag only. The following are the TS and DS equations estimated by OLS:

$$TS_{OLS}: \lg np_t = 0.384 + 0.0004 t + 1.336 \lg np_{t-1} - 0.389 \lg np_{t-2} + e_t \quad (10)$$

(2.810) (2.664) (18.155) (-5.334)

$$R^2 = 0.999; Q = 9.75 (0.462); Q^2 = 7.407 (0.686); VN = 0.75 (0.776)$$

$$DS_{OLS}: \Delta \lg np_t = 0.005 + 0.372 \Delta \lg np_{t-1} + w_t \quad (11)$$

(4.992) (5.049)

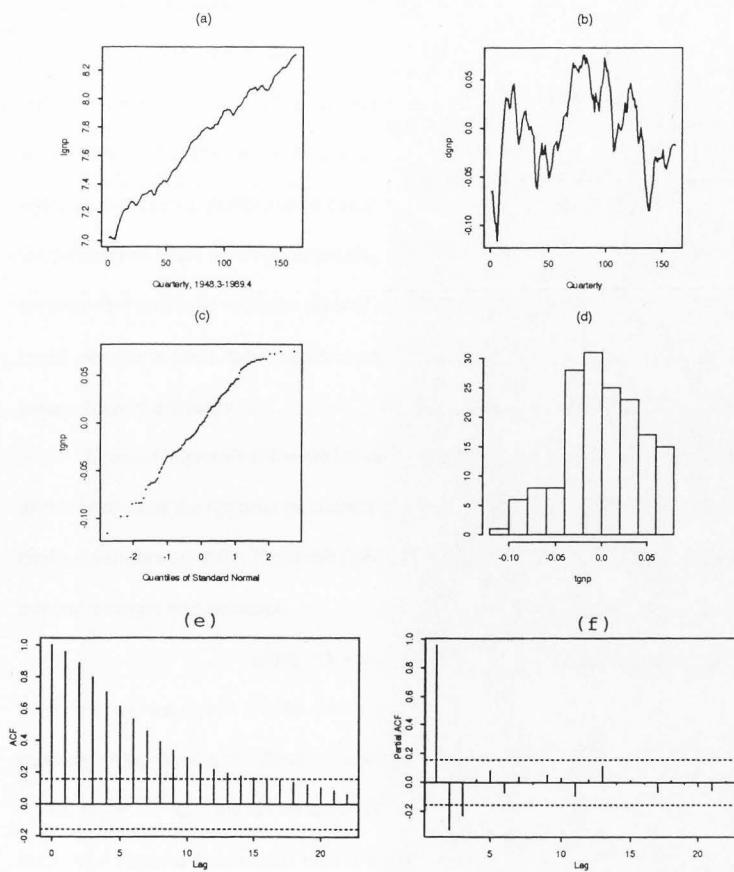


Fig. 2-5.—Some statistical properties of detrended log GNP.

$$R^2 = 0.138; Q = 10.18 (0.424); Q^2 = 4.91 (0.897); VN = 0.11 (0.544)$$

The estimated TS model contains two lags rather than three because the third lag was not significant. The numbers in parentheses under the estimated coefficients are t-statistics and they all are significant. The p-values for the Q-statistics and the von Neumann ratio all suggest the residuals are independent. Although both equations appear to do well, the difference in R^2 is dramatic. The DS equation suggest there must be other variables that account for most of the variability of the GNP growth rate, which was expected. The TS equation, however, suggests that no other macroeconomic variables are needed to explain the behavior of GNP, which is unrealistic. The histogram and normal probability plots of the detrended series showed some signs of a possible mixture distribution. This observation could provide a basis for questioning the estimated t-statistics, which are based on assumptions of normality.

A second important difference between the two equations, and relevant to the purpose of this section, is the opposite persistence behavior of the dynamic response of \lgnp to a random disturbance. After Rudebush (1993), both models can be written in the following moving averages representation:

$$\Delta \lgnp_t = k + \epsilon_t + a_1 \epsilon_{t-1} + a_2 \epsilon_{t-2} + \dots$$

where k is a constant and ϵ_t is the innovation. For the TS equation $\epsilon_t = e_t$, and for the DS equation $\epsilon_t = w_t$. Now, a unit shock in period t affects $\Delta \lgnp_{t+h}$ by a_h and affects \lgnp_{t+h} by $c_h = 1 + a_1 + a_2 + \dots + a_h$. This last measures the cumulative effect of a unit shock on \lgnp at a horizon h . Figure 2-6 shows the plots of these cumulative shocks at different horizons for both equations. The DS equation suggests increasing persistence up to a certain horizon and

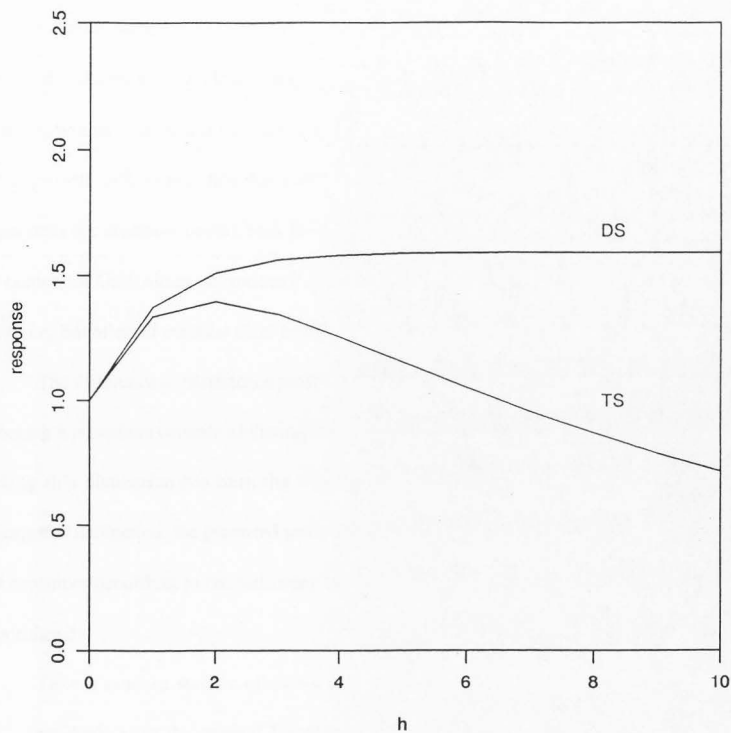


Fig. 2-6.—Cumulative effect of a unit shock at horizon h .

then it stabilizes at a positive number. This implies shocks persist forever and cause \lgnp to continuously move away from its previous position. The TS equation, on the contrary, suggests increasing persistence up to a certain horizon and then decreasing to zero persistence. This implies shocks are eventually absorbed and \lgnp moves back to its initial position after a certain period of time. This second behavior corresponds to the behavior described in the business cycle and macroeconomic theory. A series of random events in the economy will have the effect of many unit shocks, which will push the economy to a period of fast growth. After some time this initial push will lose steam and a period of decline will begin until the economy comes back to its initial long-term growth rate. Similarly, and in the context of GNP alone, a random event will cause GNP to go out of equilibrium in the short run, but after all markets clear again, GNP comes back to its long-term equilibrium.

The difference in persistence properties of the equations demonstrates the importance of having a procedure capable of distinguishing between them. The classical procedure for making this distinction has been the ADF test. To assess the power of the ADF test in making this distinction, the proposed procedure starts from the estimated TS representation and continues according to the following algorithm:

Algorithm 3:

1. Take a random sample of size n drawn with replacement from the standardized residuals \hat{e}_t , of the original TS equation to generate new residuals e^*_t .
2. Construct new values \lgnp^*_t using the residuals e^*_t , together with the estimated TS structure and dependence mechanism from the OLS regression (10),

$$\lgnp_t^* = \hat{\alpha}_0 + \hat{\alpha}_1 t + \hat{\alpha}_2 \lgnp_{t-1}^* + \hat{\alpha}_3 \lgnp_{t-2}^* + e^*_t,$$

with $\lg np^*_1 = \lg np_1$ and $\lg np^*_2 = \lg np_2$.

3. Use the new values $\lg np^*_i$ to estimate the ADF equation (7) and compute $\tau^* = \beta^*/\text{se}(\beta^*)$.
4. Go to step 1 and replicate 1000 times.
5. Finally, use the 1000 values of τ^* to build its empirical probability distribution.

This algorithm is used to obtain the empirical probability distribution of τ_{TS} under OLS or TS_{OLS} . The same steps are followed to obtain the empirical probability distribution of τ_{REG} , with the difference that in step 3 the ADF equation is estimated by robust regression. Table 2-5 presents the type II error rates and power of the tau-test under OLS at different significance levels given that only one alternative hypothesis is being considered. The first column of Table 2-5 displays the τ_{OLS} critical values obtained from the first row of Table 2-1. The second column shows the probabilities of obtaining values more extreme or equal to the critical values when the null is true. These probabilities correspond to the alpha levels in Table 2-1. The third column shows probabilities of obtaining values of tau greater than the critical values when the alternative is true, that is, the probability of a type II error. The last column is $1 - P(\text{type II error})$, and represents the power of the tau test for the specific TS alternative considered. In the previous section it was found that, with the computed value of $\tau_{OLS} = -2.766$, it was possible to reject the null at a 5% alpha level. According to Table 2-5 (last column), the probability of rejecting the null given that the alternative is true is 86.5%, which is very good power. Similarly, in the previous section we could not reject the null at a 2.5% alpha level, but according to the type II error, the probability of accepting the null given that it is false is 28.9%, which seems high. In this

TABLE 2-5
POWER ASSESSMENT OF THE TAU-TEST UNDER OLS

τ_0	$\Pr(\tau \leq \tau_0 \mid \text{DS})$	$\Pr(\tau > \tau_0 \mid \text{TS})$	$\Pr(\tau \leq \tau_0 \mid \text{TS})$
-3.15	0.01	0.515	0.485
-2.78	0.025	0.289	0.711
-2.41	0.05	0.135	0.865
-2.06	0.10	0.049	0.951

TABLE 2-6
POWER ASSESSMENT OF THE TAU-TEST UNDER RREG

τ_0	$\Pr(\tau \leq \tau_0 \mid \text{DS})$	$\Pr(\tau > \tau_0 \mid \text{TS})$	$\Pr(\tau \leq \tau_0 \mid \text{TS})$
-3.88	0.01	0.608	0.392
-3.36	0.025	0.360	0.640
-2.96	0.05	0.177	0.823
-2.47	0.10	0.051	0.949

case, rejection of the null is a better choice.

Table 2-6 presents the type II errors and power of the tau-test under RREG. The first column of Table 2-6 displays the τ_{RREG} critical values obtained from the second row of Table 2-1. The second column shows the probabilities of obtaining values more extreme or equal to the critical values when the null is true. Again, the third column shows the probability of a type II error and the last column is the power of the tau-test for the specific TS alternative considered. In the previous section it was found that, with the computed value of $\tau_{\text{RREG}} = -3.312$, it was possible to reject the null at a 5% alpha level. According to Table 2-6 (last column), the probability of rejecting the null given that the alternative is true is 82.3%, which is good power. Similarly, in the previous section there was failure to reject

the null at a 2.5% alpha level, but according to the type II error, the probability of accepting the null given that it is false is 36%, which also seems to suggest rejection of the null as a better choice.

Concluding Remarks

Applications of the Dickey-Fuller (DF) testing procedure to macroeconomic time series abound in the literature. The stylized fact that has emerged from these applications is that it is difficult to reject the null of the existence of a unit root, and therefore this has justified the use of differencing rather than detrending for achieving stationarity. This was our experience as well when using the DF critical values to determine if the time series of real GNP contains a unit root. However, the small sample results in a nonparametric bootstrap context are quite different. The bootstrapped critical values suggest rejection of the null at a 5% alpha level for a one-sided test.

The DF procedure has also been criticized for its lack of power against an alternative hypothesis of a trend stationary process, especially in a near unit root case. However, when considering the most interesting trend stationary alternative to evaluate the type II error and power of the test in a nonparametric bootstrap context, the finding suggests that rejection of the null at a 5% alpha level is accompanied by a power of above 80%.

Problems in the Dickey-Fuller testing procedure were found when using the bootstrapped empirical distributions in computing the contributions of squared bias and variance to total mean square error. While OLS produces biased parameter estimates at a given variance level, the use of robust regression does reduce bias (bias is zero for some parameters) although at the cost of slightly higher variance compared to OLS. However,

neither of the two estimation methods produces unbiased estimates for the key parameter in the testing procedure. This result illustrates the main problem of the Dickey-Fuller test in that it is not necessarily a problem of bad inference methods, but rather a problem of estimation methods.

The dramatic difference in residual sums of squares when comparing two plausible trend stationary and difference stationary models also suggests that the Dickey-Fuller test may not be the most appropriate method for testing the business cycle. An alternative approach is to perform the test in a multivariate level. It is known from economic theory that income, or real GNP, is endogenous in a macroeconomic system, and therefore, the business cycle is determined by the simultaneous action of several macroeconomic variables. The test should be performed in an error correction framework, which would be a multivariate version of the Augmented Dickey-Fuller equation.

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CHAPTER 3
A BOOTSTRAP EVALUATION OF COINTEGRATION:
TESTING THE IS-LM

Abstract. A simple open economy IS-LM model is used to identify relevant macroeconomic variables and their theoretical interrelationships. The Johansen's maximum likelihood approach to cointegration is then used to estimate an error correction model by treating all variables as equally endogenous. Estimation is done with US post-Bretton Woods data. Only one cointegrating vector is found when using Johansen's asymptotic critical values and also when using critical values derived from a nonparametric asymptotic bootstrap. A second nonparametric bootstrap is implemented to study the distribution and evaluate the significance of estimated short-term and long-term parameters. The bootstrap is found to be a powerful method for hypothesis testing in a multivariate time series framework, small samples, and in the presence of multiple nonstandard distributions.

INTRODUCTION

Of the many theoretical macroeconomic models used in the design and assessment of macroeconomic policy, the IS-LM and the Mundell-Fleming models continue to be the workhorses of applied macroeconomics. The IS-LM model was introduced by Hicks in the late 1930s to summarize the work of Keynes. The open economy version of the IS-LM is the Mundell-Fleming model introduced in the early 1960s. The model identifies the most important variables that explain the dynamics of aggregate demand. It describes how these

variables interact within identified aggregate markets and how these markets interact to achieve macroeconomic equilibrium. Furthermore, based on some assumptions about the direction of causality between variables, it predicts the short run effects of shocks to the economy as all markets adjust to their long-term equilibrium. Despite criticisms, the IS-LM model survives in use past its fiftieth year probably because, as Blanchard and Fischer (1994) conclude, it is appropriate for the study of short run adjustments.

Recent time series methodologies introduced in the econometrics literature permit estimation of the IS-LM model and the study of short-run dynamics. Engle and Granger (1987) introduced the concept of cointegration to represent long-term relationships among variables, and the estimation of short-run adjustment of those variables toward their long-term equilibrium is studied through estimation of error correction models. Johansen and Juselius (1990) developed a maximum likelihood approach to test for cointegration and for estimation of error correction models. More specifically, they developed likelihood ratio statistics to test for the number of cointegrating relationships. One problem with these tests, however, is that they do not have standard distributions. Nevertheless, Johansen and Juselius did compute critical values for the asymptotic case based on Monte Carlo experiments. Unfortunately, most of the available macroeconomic time series tend to be of short length, and studies of specific periods of time tend to have series of even shorter length. A complementary approach for treating the small sample problem and nonstandard distributions problem simultaneously is to derive critical values from a nonparametric bootstrap.

The bootstrap was introduced by Efron (1979) and applications to stationary autoregressive processes was implicitly introduced by Freedman (1981) and Freedman and Peters (1984). A useful reference for applications of the bootstrap is Efron and Tibshirani (1993). A good discussion of bootstrap applications to hypothesis testing is Hall and Wilson (1991), and Tibshirani (1992). One aspect of bootstrap applications to regression-type equations, emphasized by Freedman and Peters (1984), is that the equation and its estimated parameters represent the true model of the phenomena of interest. This condition is not easy to achieve, but selecting the variables from a sound theoretical model should approximate the true model. In any case, the estimated model residuals retain information on other events outside the scope of the theoretical model, which makes bootstrapping residuals even more attractive.

The purpose of this study is to show the usefulness of the bootstrap in hypothesis testing in a multivariate time series framework, and under conditions of nonstandard distributions and small samples. Specifically, the objectives are first, use the IS-LM framework to select the most relevant macroeconomic variables and establish their theoretical interrelationships; second, under the framework of Johansen's approach to cointegration, use a nonparametric bootstrap to test for the number of cointegrating vectors and then estimate an error correction model in which all variables are treated as equally endogenous; third, evaluate the significance of the estimated parameters by bootstrapping from the residuals of the estimated error correction model; and finally, derive some implications of the estimated model.

A SIMPLE OPEN ECONOMY IS-LM MODEL

In the open economy IS-LM, the basic macroeconomic relationships are explained in terms of the simultaneous equilibrium in three markets: the goods market, the money market, and the foreign exchange market. In the goods market, equilibrium is referred to the equality of aggregate supply and aggregate demand for goods. Aggregate demand (AD) is determined by the sum of a consumption function, C, an investment function, I, fiscal policy, F, and a current account function, CA:

$$AD = C(\lambda Y, r^e) + I(r^e) + F + CA(\lambda Y, EP^*/P, r^e) \quad (1)$$

or
$$AD = AD(Y, r^e, F, EP^*/P), \quad (2)$$

$$0 < \partial AD / \partial Y < 1; \quad \partial AD / \partial (r^e) < 0; \quad (3)$$

$$\partial AD / \partial F > 0; \quad \partial AD / \partial (EP^*/P) > 0, \quad (4)$$

where Y is total real income or the value of total output, $\lambda = 1 -$ the current income tax rate, for $0 < \lambda < 1$. Disposable income is λY , E is the nominal exchange rate, P is the price level in the domestic economy, P^* is the price level in the foreign economy, $r^e = r - \pi^e$ is the expected real interest rate with r the nominal interest rate and π^e expected inflation. The government's budget deficit is $F = G - (1 - \lambda)Y$, with G being government spending and $(1 - \lambda)Y$ representing government tax revenues. Given that aggregate supply equals the value of total output of the economy, then in equilibrium we have:

$$Y = AD(Y, EP^*/P, r^e, F) \quad (5)$$

or simply
$$Y = f(EP^*/P, r^e, F). \quad (6)$$

Money demand, L , is assumed to have a positive relationship with income Y and a negative relationship with the nominal interest rate r . In equilibrium, money supply (M/P) equals money demand, and interest rate can be derived as:

$$r = r(M/P, Y). \quad (7)$$

In the foreign exchange market, equilibrium is achieved with the exchange rate that allows for the equality between the nominal interest rate of the domestic and foreign capital markets. This is the nominal interest parity condition:

$$r = r^* + \log(E^*/E). \quad (8)$$

Equations (6), (7), and (8) above describe the macro economy with the traditional view of IS-LM principles applied to an open economy. Additional assumptions imposed suggest that the variables P , P^* , r^* and E^* are known, and that under a flexible exchange rate system F and M are exogenous, while r , Y and E are the endogenous variables.

The system can be reduced to two equations and two endogenous variables by solving (8) for $E = E^*/\exp(r-r^*)$, and substituting into (6) for the real exchange rate given by $EP^*/P = E^*P^*/\exp(r-r^*)P = g(r)$ given E^* , P , P^* , r^* . Then (1) becomes $Y = f(g(r), r^*, F)$ or simply $Y = h(r, r^*, F)$. In this last equation, r is representing the behavior of the real exchange rate, while r^* is related to investment decisions.

The price level is assumed to be fixed in the short-run ("sticky prices"), while in the long-run it can vary. A convenient way to close the model is to assume an expected inflation rate that depends positively on the divergence between real aggregate demand and full employment output $\pi^e = \alpha(h(r, r^*, F) - Y)$. The IS-LM can now be represented in the following equations:

$$Y = h(r, r^e, F) \quad \delta h / \delta(r) > 0 \quad \delta h / \delta(r^e) < 0 \quad \delta h / \delta F > 0 \quad (9)$$

$$r = r(M/P, Y) \quad \delta r / \delta(M/P) < 0 \quad \delta r / \delta Y > 0 \quad (10)$$

$$\pi^e = \sigma(h(r, r^e, F) - Y) \quad \sigma > 0 \quad (11)$$

From these equations it is now easy to see that Y , r , π^e , F , and M/P are the most relevant variables for understanding the macro economy (notice that $r^e = r - \pi^e$). In the short-run it is expected that any variable could change due to temporary shocks to the economy, including fiscal and monetary policies, but then the variable would always adjust toward its long-run equilibrium with the other variables of the system. This long-run equilibrium with the other variables is represented by the solution of the above system as the overall macroeconomic equilibrium, i.e., the simultaneous equilibrium in the goods and money markets considering inflation expectations. The adjustment process itself, however, would involve movement in the other variables as well, following the connections and structure of the same goods market and money market equations. This is why the IS-LM model is useful for understanding short-run adjustment. Estimation of the model requires the simultaneous consideration of long-term equilibrium relationships and short-term dynamics.

COINTEGRATION AND MODEL ESTIMATION

In the econometrics literature, the requirement of long-term equilibrium among variables is referred to as variables being cointegrated. The idea of cointegration can most easily be explained by considering the case of two nonstationary time series x_t and y_t . Assume the two series can be made stationary after first differencing, in which case they are said to be integrated of order 1, or $I(1)$. If this is the case, then it is generally true that a

linear combination, $y_t - \alpha x_t$, will also be nonstationary. However, if there exists a value of α such that $y_t - \alpha x_t$ is stationary, then the series x_t and y_t are said to be cointegrated. In other words, although individually each series tends to wander aimlessly, there is some kind of steady-state relationship when considered together. In the case of $\alpha = 1$, the steady-state relationship is such that x_t and y_t cannot drift too far apart. Similarly, in a multivariate setting, if y_t is a $K \times 1$ vector with all its components $I(1)$ and there exists some linearly independent vectors $\alpha_1 \dots \alpha_k$, $0 < k < K$, such that $z_{it} = \alpha_i' y_t$ ($i=1 \dots k$) are stationary, then the components of the vector are said to be cointegrated and the α_i are called the cointegrating vectors. Based on these definitions, Granger (1986) and Engle and Granger (1987) proved the Granger representation theorem, which states that if a $K \times 1$ vector y_t is cointegrated, with cointegration rank k (number of possible cointegrating relationships), then there exists a vector autoregressive representation

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t, \quad (12)$$

and an error correction representation

$$\Delta y_t = B_1 \Delta y_{t-1} + \dots + B_{p-1} \Delta y_{t-p+1} - \Pi y_{t-1} + u_t, \quad (13)$$

where p is the number of lags, $B_1 = -A_p - A_{p-1} - \dots - A_{i+2} - A_{i+1}$, and $\Pi = HC = I_K - A_1 - \dots - A_p$ with $\text{rank}(\Pi) = \text{rank}(H) = \text{rank}(C) = k$. C ($k \times K$) is the matrix of cointegrating vectors and H ($K \times k$) is the matrix of adjustment or error correction coefficients. The errors u_t are assumed Gaussian white noise. Short-run movements from one period to the next are represented in the left-hand side of (13). These short-run dynamics are explained in part by the short-run dynamics occurring in previous periods, $B_{p-1} \Delta y_{t-p+1}$ ($p=1, 2, \dots$), and in part by adjustments, H , toward the long-term equilibrium relationships, $Cy_t=0$. When the long-term equilibrium

relationships are expressed as $Cy_{t-1} \neq 0$, then they are measuring the degree of out-of-equilibrium experienced in the previous period. Then the expression $-HCy_{t-1}$ measures the amount of correction that should take place in the present period.

Equation (13) is a reparameterized version of equation (12). Alternatively equation (12) can be reparameterized into the following error correction form:

$$\Delta y_t = D_1 \Delta y_{t-1} + \dots + D_{p-1} \Delta y_{t-p+1} - \Pi y_{t-p} + u_t, \quad (14)$$

where $D_1 = -I_K + A_1 + \dots + A_p$. Equation (13) is used here for interpretation purposes while (14) is the form used by Johansen and Juselius for estimation purposes. In either equation the rank of Π , i.e., k , is the number of linearly independent cointegrating relations among the variables in y_t . If Π is full rank, then any linear combination of y_t will be stationary, while if it is a matrix of zeros, then any linear combination of y_t will be a nonstationary unit root process. The most interesting case is when Π is less than full rank, $0 < k < K$. In this situation, k is the number of cointegrating vectors and $K - k$ is the number of unit roots or number of nonstationary linear combinations.

Let $y_t = (g_t, m_t, \pi_t^e, r_t, f_t)$ contain our macroeconomic variables of interest, where $g = \log(Y)$, $m = \log(M/P)$, and $f = \log(F)$. Estimation of model (14) is done using the time series of those variables from the post-Bretton Woods period of 1974.2 to 1993.3. The data definitions are

g_t = quarterly $\log(\text{GDP})$, where GDP is in billions of 1987 dollars.

m_t = quarterly $\log(\text{money supply } M1)$, where M1 is in billions of 1987 dollars.

r_t = quarterly nominal annual interest rate.

f_t = quarterly $\log(\text{budget deficit})$, with budget deficit in billions of 1987 dollars.

π_t^e = quarterly expected inflation (π_t^e). Annualized two period ahead first difference of the price deflator.¹

Figure 3-1 presents plots of these variables. The first column of plots corresponds to the variables in their levels, and the second column of plots presents them after taking a first difference. Plots (a) and (b) belong to g_t , (c) and (d) to m_t , (e) and (f) to π_t^e , (g) and (h) to r_t^e , and, (i) and (j) to f_t . It is important to notice that all variables in their levels present a changing mean over time, and are therefore nonstationary. However, in all cases they become stationary after taking a first difference. This satisfies the requirement of working with variables integrated of order one, or $I(1)$. When observing normal probability plots of the variables in their first difference form, in most cases they tend to depart somewhat from normality due to contamination. The presence of outliers and/or structural change explains such contamination. The following dummy variables were included in order to diminish their influence:

$d1 = 1$ in 1984.2 and 0 elsewhere; extreme observation in first difference of m_t .

$d2 = 1$ in 1979.1 and 0 elsewhere; extreme observation in first difference of f_t .

$d3 = 1$ in 1980.2, 1 in 1981.4, 1 in 1982.3, and 0 elsewhere; extreme observations in first difference of r_t .

¹ This lead was selected based on observed correlations with the nominal interest rate. Also the inflation rate was multiplied by four for annualization.

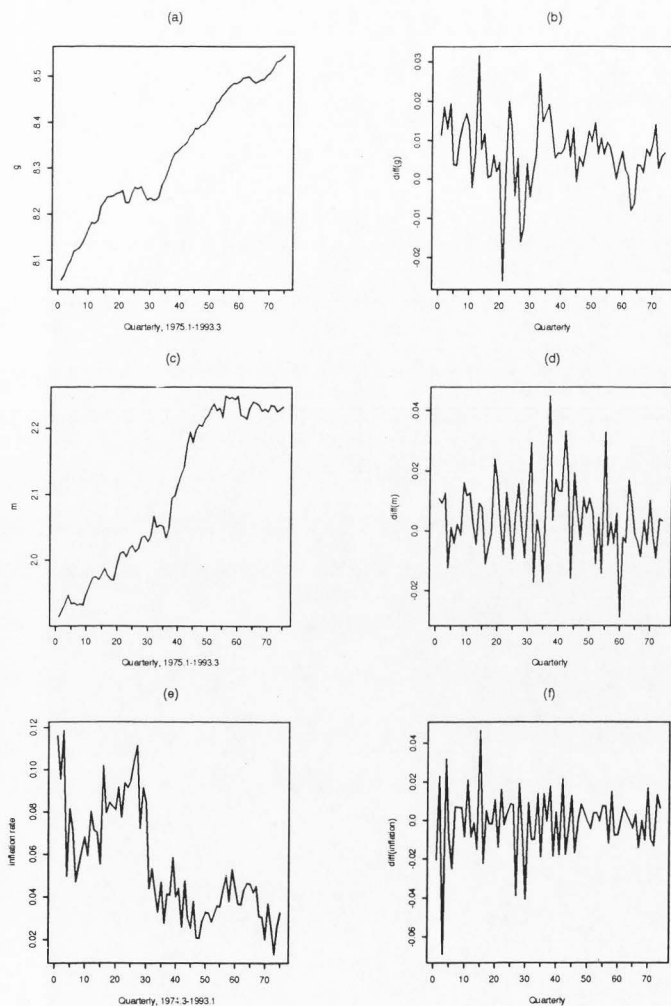


Fig. 3-1.—Time Series of Macroeconomic Variables

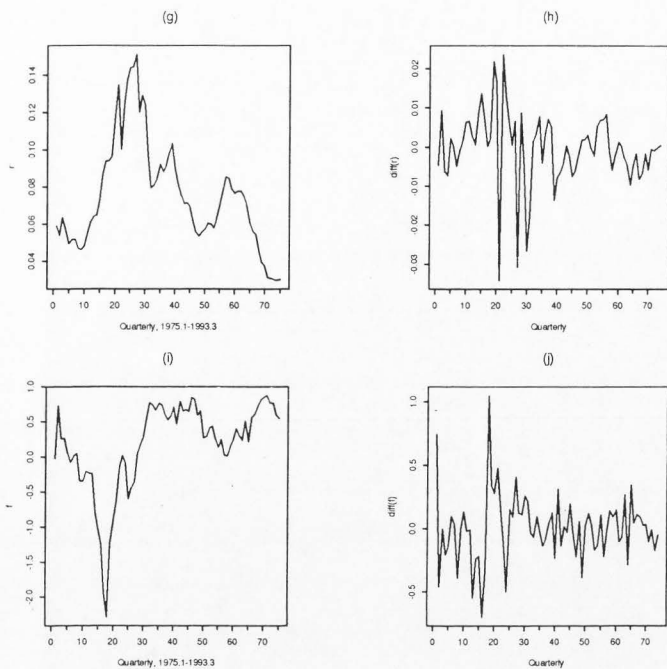


Fig. 3-1.—Continuation

The Johansen and Juselius (1990) maximum likelihood (ML) approach is followed for estimation of the error correction model (14).² Computation of the ML estimators requires previous knowledge of the lag order, p , and the cointegration rank, k . The methods followed for determining the lag order were Akaike's Information Criterion, the Hannan-Quinn Criterion, and the Schwarz Criterion. Each of these procedures uses the above variables in computing vector autoregressive systems (VARs) at alternative lag orders. The optimum lag is then selected as the one that minimizes the criterion expressed in the following formulas:

$$AIC(m) = \ln |\hat{U}(m)| + 2mK^2/T$$

$$HQ(m) = \ln |\hat{U}(m)| + (2 \ln \ln T)/T$$

$$SC(m) = \ln |\hat{U}(m)| + (\ln T \ mK^2)/T$$

where $m = 0, 1, \dots, M$ is the number of alternative lags considered, T is the length of the series, K is the number of variables, and $\hat{U}(m)$ are the residuals from the estimated VARs at different lags, m . Table 3-1 shows the computed values.

Table 3-1. Lag Order Selection Criteria

Lag	1	2	3	4	5	6	7	8	9	10
AIC	-34.5	-34.1	-33.2	-32.3	-31.4	-30.5	-30.4	-29.2	-27.8	-26.3
HQ	-34.2	-33.5	-32.3	-31.1	-29.8	-28.6	-28.1	-26.6	-24.8	-23.0
SC	-33.8	-32.5	-30.8	-29.1	-27.4	-25.7	-24.7	-22.6	-20.3	-17.9

²Parameter estimation and computation of their standard errors were programmed following Lütkepohl's (1993) propositions 11.1-2, pp. 356-358. All computations for estimation and bootstrap hypothesis testing were programmed in Splus, version 3.2 release 1 for Sun SPARC, SunOS 5.x: 1993. The computer code is in Appendix B.

In all three cases the criterion suggested one lag for the optimum lag order. In equation (14), a lag order of one in the VAR portion of the error correction model (ECM) implies $p=2$ in its cointegration portion. Given that (14) is the equation used for ML estimation, then p is set to two.

With knowledge of p we can now test for the cointegration rank, k . The idea is to test the null hypotheses $H_0: k = k_0$ against the alternative $H_1: k_0 < k < K$, or alternatively to test $H_0: k = k_0$ against $H_1: k = k_0 + 1$. Johansen showed that ML estimation of (14) produces the following likelihood ratio (LR) statistic for such type of test, respectively:

$$LR(k_0, k_1) = 2[\ln l(k_1) - \ln l(k_0)] = -T \sum_{i=k_0+1}^{k_1} \ln(1-\lambda_i) \approx \sum_{i=k_0+1}^{k_1} T\lambda_i$$

$$LR(k, k+1) = 2[\ln l(k_1) - \ln l(k_0)] = -T \ln(1-\lambda_1) \approx T\lambda_1$$

where $l(k_0)$ denotes the maximum of the Gaussian likelihood function for cointegration rank k_0 , the λ_i are computed eigenvalues, and T the sample size. The distribution of these LR statistics, however, is not the traditional χ^2 -distribution, but rather some unknown non-standard distribution. Nevertheless, noticing that it depends on the difference $K-k$, Johansen and Juselius tabulated critical values for the asymptotic case. They show that the first LR statistic converges weakly to the trace of the following matrix:

$$\int_0^1 (dU) F' \left(\int_0^1 F F' dt \right)^{-1} \int_0^1 F (dU)'$$

here $U(t)$ is a $(K-k)$ dimensional Brownian motion and $F(t)$ is a $(K-k)$ dimensional stochastic process defined by $U \cdot \bar{U}$. For this reason, the first test is referred to as the trace test. Similarly, they show that the second LR test converges weakly to the maximum eigenvalue of the same matrix above. Thus, the second test is referred to as the maximum eigenvalue test. Johansen and Juselius computed critical values for $T=400$ and 6000 simulations. However, use of those critical values might be questioned for our small sample case because of (i) the nonstandard distribution of the LR statistic and (ii) the use of critical values valid only for the asymptotic case. These concerns justify use of bootstrapping techniques to assess the significance of the trace test and maximum eigenvalue test, especially for finite samples. Implementation of the bootstrap in testing for cointegration rank is achieved with the following algorithm:

Algorithm 1:

1. Estimate model (14) assuming full rank and obtain its residuals e_{it} , ($t=1,2,\dots,T$, $i=1,\dots,K$).
2. Take a random sample of the stacked standardized residuals e_{it} and generate e_{it}^* , ($t=1,2,\dots,T$, $i=1,\dots,K-k$).
3. Compute X_t from $X_{it} = \sum e_{si}^*$, ($t=1,2,\dots,T$, $i=1,\dots,K-k$), with $X_{0i} = 0$.
4. Approximate the matrix $\int (dU)F' (\int FF'dt)^{-1} \int FdU'$ by

$$V = \sum e_t^* (X_{t-1} - \bar{X}_{-1})' [\sum (X_{t-1} - \bar{X}_{-1})(X_{t-1} - \bar{X}_{-1})']^{-1} \sum (X_{t-1} - \bar{X}_{-1}) e_t^{*'}.$$
5. Compute the trace and maximum eigenvalue of V .
6. Go back to step 2 for next simulation.

Given that Algorithm 1 follows the same procedure developed by Johansen and Juselius for the asymptotic case,³ then critical values generated this way can be referred to as "bootstrap asymptotic critical values." The advantage of this algorithm is that it is based on a small sample and takes into account the possibility that the errors are not i.i.d. normally distributed.

Algorithm 1 was implemented for $K=5$, $T=75$, and 2500 simulations. Tables 3-2 and 3-3 use the critical values obtained from the asymptotic bootstrap distributions to evaluate the computed values for the trace test and maximum eigenvalue test. In both cases the null of rank zero is rejected leading to the conclusion that there is only one cointegrating vector. Acceptance of rank one is based on a p-value between 1% and 2.5% for the trace test, and a p-value between 2.5% and 5% for the maximum eigenvalue test. When using Johansen's critical values the conclusion is the same; however, in this second case, rank one is accepted based on p-values of less than 1% for both tests. Application of the bootstrap shows evidence of longer right tails for the distributions of both tests when compared to Johansen's critical values. This is a result of greater variability contained in the residuals when performing the simulations, and also from use of dummy variables in estimation.

Equation (14) was estimated to obtain its reparameterized version using the lag order and cointegration rank,

$$\Delta y_t = B_0 + B_1 \Delta y_{t-1} - \Pi y_{t-1} + u_t, \quad (15)$$

³See Johansen and Juselius (1990), page 207.

Table 3-2. Bootstrap Trace Test

Hypothesis		Test value	Critical Values				mean	var
Ho	H ₁		90%	95%	97.5%	99%		
k = 4	k = 5	0.353	6.795	9.237	12.421	15.465	2.86	10.82
k = 3	k ≥ 4	2.039	17.608	21.646	25.570	33.629	9.72	40.27
k = 2	k ≥ 3	16.399	32.349	36.732	41.785	48.695	19.51	82.59
k = 1	k ≥ 2	39.078	49.865	56.385	63.046	72.049	33.22	151.27
k = 0	k ≥ 1	87.219*	71.779	79.191	86.773	95.164	49.98	259.67

Table 3-3. Bootstrap Maximum Eigenvalue Test

Hypothesis		Test value	Critical Values				mean	var
Ho	H ₁		90%	95%	97.5%	99%		
k = 4	k = 5	0.353	6.794	9.237	12.421	15.465	2.86	10.82
k = 3	k = 4	1.686	15.062	18.943	22.705	29.745	8.21	31.84
k = 2	k = 3	14.360	22.156	26.189	30.865	36.706	13.27	50.25
k = 1	k = 2	22.678	30.265	34.487	39.862	48.815	18.98	72.65
k = 0	k = 1	48.141*	36.879	42.545	49.041	55.802	24.51	96.21

where B_0 is a vector of constants, and the other parameters are as defined in (13). The following are the estimated parameters:⁴

$$B_0' = \begin{vmatrix} 0.1157 & -0.0460 & -0.1952 & -0.1098 & -3.7685 \\ (3.708) & (-0.833) & (-3.397) & (-3.485) & (-3.882) \\ (0.003) & (0.329) & (0.010) & (0.002) & (0.562) \end{vmatrix}$$

⁴Estimated parameters for dummy variables are not reported. However, evaluation of their t-statistics indicated that dummies operated as intended.

$$\hat{B}_1 = \begin{vmatrix} 0.2029 & -0.0810 & -0.1061 & 0.2547 & -1.1e-04 \\ (1.968) & (-1.329) & (-1.968) & (2.538) & (-0.037) \\ (0.064) & (0.209) & (0.056) & (0.022) & (0.975) \\ \\ 0.2500 & 0.0112 & 0.0257 & -0.0775 & 0.0099 \\ (1.371) & (0.104) & (0.269) & (-0.437) & (1.786) \\ (0.192) & (0.926) & (0.798) & (0.676) & (0.094) \\ \\ 0.0726 & 0.1134 & -0.4945 & 0.0833 & 0.0135 \\ (0.382) & (1.010) & (-4.983) & (0.451) & (2.322) \\ (0.730) & (0.338) & (0.000) & (0.643) & (0.034) \\ \\ 0.1411 & 0.0116 & 0.0576 & 0.0420 & 0.0109 \\ (1.357) & (0.189) & (1.060) & (0.415) & (3.434) \\ (0.186) & (0.859) & (0.311) & (0.691) & (0.001) \\ \\ -3.3559 & 0.7286 & 3.5118 & -5.9650 & 0.0377 \\ (-1.047) & (0.384) & (2.099) & (-1.912) & (0.384) \\ (0.374) & (0.756) & (0.114) & (0.092) & (0.748) \end{vmatrix}$$

$$\hat{C} = \begin{vmatrix} -5.8704 & 5.9267 & 26.4075 & -14.5297 & 1.6004 \end{vmatrix}$$

$$\hat{H} = \begin{vmatrix} -0.00305 \\ 0.00138 \\ 0.00542 \\ 0.00306 \\ 0.10609 \end{vmatrix}$$

HC =	0.0179 (3.533) (0.002)	-0.0180 (-3.533) (0.002)	-0.0805 (-3.533) (0.002)	0.0443 (3.533) (0.002)	-0.0048 (-3.533) (0.001)
	-0.0081 (-0.908) (0.411)	0.0082 (0.908) (0.414)	0.0366 (0.908) (0.410)	-0.0201 (-0.908) (0.410)	0.0022 (0.908) (0.409)
	-0.0318 (-3.414) (0.005)	0.0321 (3.414) (0.004)	0.1433 (3.414) (0.002)	-0.0788 (-3.414) (0.002)	0.0086 (3.414) (0.002)
	-0.0179 (-3.516) (0.002)	0.0181 (3.516) (0.003)	0.0809 (3.516) (0.001)	-0.0445 (-3.516) (0.002)	0.0049 (3.516) (0.002)
	-0.6228 (-3.952) (0.487)	0.6287 (3.952) (0.475)	2.8016 (3.952) (0.096)	-1.5415 (-3.952) (0.084)	0.1697 (3.952) (0.179)

All numbers without associated parentheses are estimated parameters. Numbers in parentheses immediately below parameter estimates are t-statistics, and the second number in parentheses is the estimated p-value. It is not possible to obtain standard errors for H and C separately but only jointly as HC. P-values were obtained via a second nonparametric bootstrap designed to evaluate the significance of computed t-statistics. The bootstrap takes random samples from the residuals of the estimated equations in order to consider the effects of the small sample size and departures from normality. Then it generates new sets of data based on the estimated parameters that define the structure of model (12). Given the dynamic connections among equations, the bootstrap is designed to simulate these connections. The new sets of data are then used to reestimate the model and by doing so generate empirical distributions that approximate the true distributions of the parameters' t-statistics. These distributions are useful for two reasons. First, they show the distributional

properties of parameters and their statistics, and second, they can be used to evaluate significance of parameters by evaluating the significance of their computed t-statistics. The following is the algorithm used for these purposes:⁵

Algorithm 2:

1. Take a random sample from each of the standardized residual vectors contained in $\hat{u}_t = y_t - \hat{A}_0 - \hat{A}_1 y_{t-1} - \hat{A}_2 y_{t-2}$, and generate new residuals u_t^* .
2. Use the residuals u_t^* to build new values y_t^* based on the structure of the estimated model and using the first two observed values of y_t as starting points. That is,

$$y_t^* = \hat{A}_0 + \hat{A}_1 y_{t-1}^* + \hat{A}_2 y_{t-2}^* + u_t^*.$$

3. Use the generated values y_t^* to compute model (14) with $k=1$,

$$\Delta y_t^* = D_1 \Delta y_{t-1}^* - \Pi y_{t-2}^* + u_t,$$

and obtain new matrices D^* , H^*C^* and their matrix of standard errors $se(D^*)$ and $se(H^*C^*)$. Based on these also compute B_1^* and its standard error, $se(B_1^*)$.

4. Then compute the following matrices:

$$(B_1^* - \hat{B}_1)/se(B_1^*) \text{ to approximate the distribution of } (\hat{B}_1 - 0)/se(\hat{B}_1),$$

$$(H^*C^* - \hat{H}\hat{C})/se(H^*C^*) \text{ to approximate the distribution of } (\hat{H}\hat{C} - 0)/se(\hat{H}\hat{C})$$

5. Go to step 1 and repeat for next bootstrap sample.
6. Finally, use the absolute values of the vectors generated for each element of the above matrices to build their empirical probability distributions for a two-sided evaluation of the significance of the originally estimated parameters.

⁵ The general principles used for the development of this algorithm follow the work of Freedman and Peters (1984) and Hall and Wilson (1991).

The experiment was performed with 2500 simulations. Figure 3-2 presents the empirical distributions of $(H^*C^* - \hat{H}\hat{C})/\text{se}(H^*C^*)$, where the first row of plots corresponds to the distribution of t-statistics of the first row of the matrix $\hat{H}\hat{C}$, and the remaining rows follow the same order. In general these distributions are symmetric around zero, however, they are nonstandard distributions when compared to the traditional t-distribution. This was verified from the computation of critical values at several alpha levels and comparing them to traditional t-tables (see Appendix C). The bootstrapped distributions consistently show longer tails, furthermore, they also tend to vary from equation to equation revealing the presence of multiple nonstandard distributions. This result implies that use of standard t-tables would overestimate the significance of the parameters, and in some cases by a large degree, for example, the fifth row of matrix $\hat{H}\hat{C}$. The absolute value of $(H^*C^* - \hat{H}\hat{C})/\text{se}(H^*C^*)$ was then computed to perform a two-sided test of the null hypothesis that each parameter is equal to zero, and the resulting p-values are presented in the matrix $\hat{H}\hat{C}$ above.

Figure 3-3 shows the empirical distributions of $(B_1^* - \hat{B}_1)/\text{se}(B_1^*)$. The first row of plots corresponds to the distribution of t-statistics of the first row of the matrix \hat{B}_1 , and the remaining rows follow the same order. These plots are also symmetric around zero and close to a t-distribution. However, comparison of their critical values to traditional t-tables also shows evidence of multiple nonstandard distributions (see Appendix D). The absolute value of $(B_1^* - \hat{B}_1)/\text{se}(B_1^*)$ was also computed to perform a two-sided test of the null hypothesis that each parameter is equal to zero, and their p-values are presented in B_1 above.

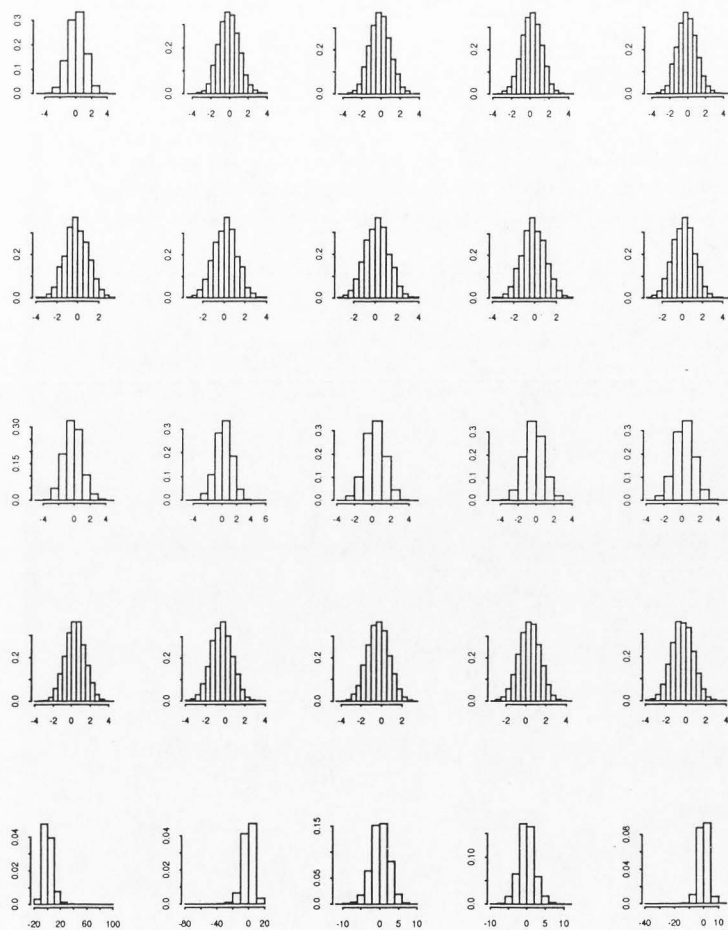


Fig. 3-2.—Empirical Distributions of $(H^*C^* - \hat{H} \hat{C}) / \text{se}(H^*C^*)$.

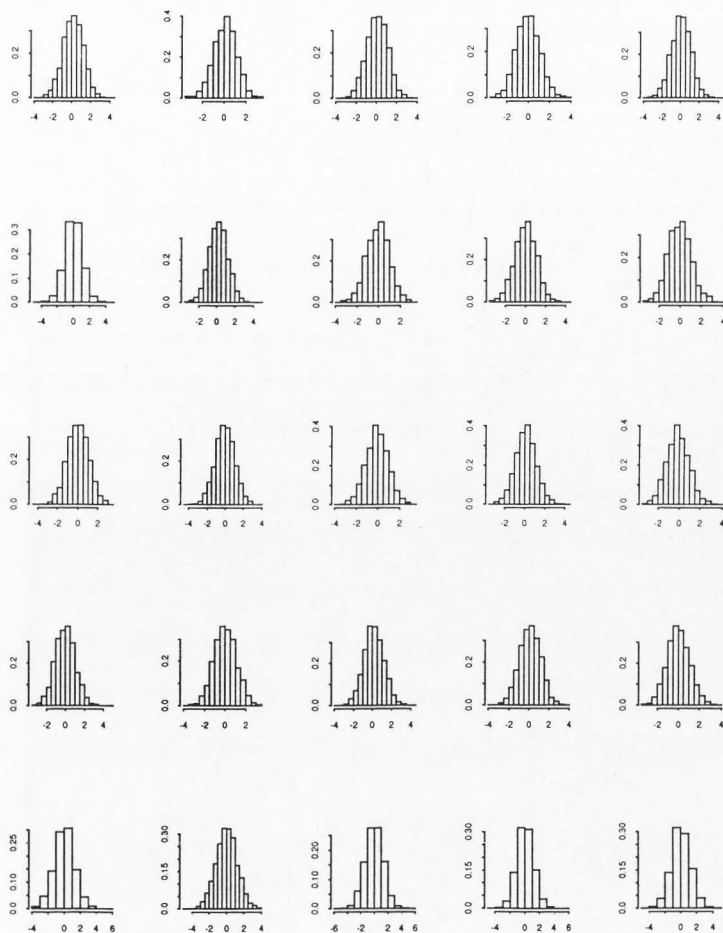


Fig. 3-3.—Empirical Distributions of $(B_1^* - \hat{B}_1)/\text{se}(B_1^*)$.

Given the significance of the parameters in the HC, \hat{B}_0 and \hat{B}_1 matrices as evaluated by their p-values, the resulting estimated model has the following structure:

$$\Delta g_t = 0.11 + 0.25\Delta r_{t-1} - 0.003(-g_t + m_t - 4.5r_t^e + 2.02r_t + 0.27f_t)_{t-1} + u_{1t} \quad (16)$$

$$\Delta m_t = u_{2t} \quad (17)$$

$$\Delta \pi_t^e = -0.19 - 0.5\Delta \pi_{t-1}^e + 0.013\Delta f_{t-1} - 0.005(0.22g_t - 0.22m_t - \pi_t^e + 0.55r_t - 0.06f_t)_{t-1} + u_{3t} \quad (18)$$

$$\Delta r_t = -0.11 + 0.011\Delta f_{t-1} - 0.003(0.5g_t - 0.5m_t + 2.22r_t^e - r_t - 0.13f_t)_{t-1} + u_{4t} \quad (19)$$

$$\Delta f_t = u_{5t} \quad (20)$$

The cointegrating vector contained in \hat{C} affects three of the five equations according to p-values of the $\hat{H}\hat{C}$ matrix. The vector \hat{C} , however, has been divided by the coefficient of the variable whose equation is being affected.

CONCLUSIONS AND FINAL COMMENTS

One overall long-term macroeconomic equilibrium relationship among all variables was found in the following linear combination:

$$-5.87g_t + 5.92m_t + 26.40\pi_t^e - 14.53r_t + 1.60f_t = 0 \quad (21)$$

$$\text{or} \quad -5.87g_t + 5.92m_t - 26.40r_t^e + 11.87r_t + 1.60f_t = 0 \quad (22)$$

where g_t is log(real GDP), m_t is log(real money supply, M1), π_t^e is expected inflation, r_t is nominal interest rate, f_t is government budget deficit, and $r_t^e = r_t - \pi_t^e$ is expected real interest rate. This linear combination represents the solution of the IS-LM system, that is, it is the locus of points in time where the IS crosses the LM, and therefore it contains the goods and money markets equilibrium conditions. For example, solving for g_t produces:

$$g_t = m_t - 4.5r_t^e + 2.02r_t + 0.27f_t \quad (23)$$

which describes a negative relationship between GDP and expected real interest rate, a positive relationship between GDP and an active fiscal policy, f_t , and a positive relationship with the nominal interest rate, which is representing some function of the real exchange rate. All of these relationships are predicted by the IS-LM through the goods market equation.

Solving for r_t produces:

$$r_t = 0.5g_t - 0.5m_t - 2.22r_t^e - 0.13f_t, \quad (24)$$

which describes a positive relationship of the nominal interest rate with income, and a negative relationship with money. Both of these relationships are also predicted by the IS-LM model through its money market equation.

Computing the short-run expected real interest rate as $\Delta r_t^e = \Delta r_t - \Delta \pi_t^e$, that is subtracting equation (18) from equation (19) in the estimated model, produces:

$$\Delta r_t^e = 0.08 + 0.5\Delta \pi_{t-1}^e - 0.003(0.06g_t - 0.06m_t + r_t^e - 0.86(r_t - 0.87\pi_t^e) - 0.01f_t)_{t-1} + u_{6t}, \quad (25)$$

where $u_{6t} = u_{4t} - u_{3t}$. Now, solving its long-term component for r_t^e produces:

$$r_t^e = -0.06g_t + 0.06m_t + 0.86(r_t - 0.87\pi_t^e) + 0.01f_t. \quad (26)$$

This equation shows a positive relationship between long-term real interest rate and a fraction of $r_t - 0.87\pi_t^e$, which approximates the long-term Fisher condition, i.e., the relationship of short-term interest rates to expected inflation. However, given that increases in inflation are not fully reflected in nominal interest rates, then the long-term real interest rate is not fully free of monetary disturbances.

The estimated framework of this chapter is different than the models of the Fisher effect found in the recent literature [cf. Charmichael and Stebling (1983), and Mishkin (1992)]. Here all the variables are considered together in a multivariate and error correction

specification following Johansen, and unlike the two-step two-stage estimation procedures used to estimate short-run dynamics once long-run relationships are known as found in Cumby, Huizinga, and Obstfeld (1983), Ogaki and Park (1991), and Vahid and Engle (1992). Each of these latter studies views the short-run conditional on the long-run relationships. However, Mishkin's direct model of the relationship of the change in inflation and short-term interest rates is incorrect given that he found cointegration of inflation and short-term interest rates. Given cointegration then a pure regression in differences is misspecified without the inclusion of an error correction term resulting in inconsistent and biased coefficients. Furthermore, the power of the tests used in these studies is weak as pointed out by Kremers, Ericsson and Dolado (1992), and which has prompted the estimation of the nonstandard distributions in this paper.

The estimation results here suggest a form of the IS-LM framework is observed in the aggregate U.S. economy for the post-Bretton Woods era. Correlation between expected inflation and short-term nominal interest rates appears to be the result of an imperfect long-run Fisher effect in which inflation and interest trend together in the long run, but there are co-movements among variables of the IS-LM framework as well. This finding has interesting implications in the light of the dominance of policy targeting the federal funds rate for the period of the sample. Looking solely at the level of short-term interest rates is in part misleading. A high interest rate that has persisted for some time is an indication that expected inflation is high and may not be an indication that monetary policy is tight. Short-run changes in the short-term interest rate can reflect the stance of monetary policy as implied by the estimation results.

Three of the variables considered adjust their short-term dynamics toward the overall long-term equilibrium relationship. Those variables are GDP, expected inflation, and the nominal interest rate, which are predicted as the endogenous variables in an open economy IS-LM system. This result has important implications for the specific case of GDP. The history of its short-term dynamic adjustments resulting from its out-of-equilibrium position with respect to the overall long-term equilibrium relationship is what generates the business cycle. Unlike the general literature in which the business cycle is viewed as the out-of-equilibrium position of GDP with respect to a positive linear trend, here the business cycle results as adjustments to the co-movement of the macroeconomic fundamentals.

The variables money supply and budget deficit are found to follow random walk processes, which drive the system. These are identified as monetary and fiscal policy instruments by the IS-LM, and are in fact predicted to drive the system. Both variables are identified as long-term policy instruments given that they are contained in the equilibrium relationship. However, fiscal policy also appears as a short-term instrument affecting the short-term dynamics of the nominal interest rate and expected inflation variables. This last result has important implications when considering the promotion of long term growth. An active fiscal policy, which would continue increasing the budget deficit, has a direct positive impact on GDP. Over time it can lead to an inflationary period given its effect in increasing the speed of inflationary expectations. On the other hand, growth based on monetary policy aimed at real interest rate targets cannot be effective under volatile inflationary expectations. Under these conditions, a less active fiscal policy directed at containing inflationary expectations plus a monetary policy directed at real interest rate targets can be an appropriate

policy mix.

The bootstrap is proven a powerful method for improving the accuracy of hypothesis testing in a multivariate time series framework, in small samples, and in the presence of multiple nonstandard distributions. First, acceptance of the alternative hypothesis of rank one was verified by use of a nonparametric asymptotic bootstrap. Acceptance resulted from a p-value between 2.5% and 1% for the trace test, and a p-value between 2.5% and 5% for the maximum eigenvalue test. These results contrast with the p-value of less than 1% for both tests when using Johansen's asymptotic critical values. From this result we can say that the Johansen's asymptotic critical values can be considered a good guideline for small sample testing, at least as a first approximation. Second, by empirical approximations to the true distributions of computed t-type statistics, the bootstrap was able to distinguish with accuracy the significant from the nonsignificant parameters. Moreover, given the interrelationships characteristic of a multivariate time series system, the bootstrap was able to evaluate all parameters simultaneously not only by taking into account the small sample, or the information remaining in the residuals not captured by the model, but also by the fact that each parameter's t-statistic follows a unique nonstandard distribution. An important aspect, however, is that critical values either from bootstrapped distributions or t-distributions can be misleading in the presence of extreme observations. Once contamination is considered (with the inclusion of dummy variables in this chapter), then the superiority of bootstrapped critical values over those presented in t-tables becomes evident.

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CHAPTER 4

SOME DYNAMIC ASPECTS OF THE BOLIVIAN DOLLARIZATION PROCESS

Summary. Perron's test for structural change is used to show that the Bolivian time series of dollarization is a trend stationary process around a positive trend. Rejection of a random walk with drift hypothesis leads to the conclusion that dollarization in Bolivia is an irreversible process. The Johansen's maximum likelihood approach to cointegration is then used to estimate a partial model of the dynamics of dollarization. Parameter evaluation is performed via a nonparametric bootstrap. The empirical model reveals a dollarization process driven by foreign inflation and the return on dollar deposits, and an economy in transition toward equalization with US prices and interest rates.

INTRODUCTION

One of the most interesting economic phenomena currently taking place throughout the world is that of "dollarization" of entire economies. According to Calvo and Végh (1992), dollarization refers to the replacement of the domestic currency by US dollars in the unit of account and store of value roles of money. Currency substitution is another term used in the literature, and it refers to the situation when, in addition, the domestic currency is also replaced as a medium of exchange. Either dollarization or currency substitution has been observed and documented in Latin America, Eastern Europe, Russia, and the Middle East. As El-Erian (1988, p. 88) has put it, "The essence of this phenomena [sic] reflects

individuals' attempts to protect the value of their wealth and income, and has usually taken place in the context of deteriorating economic and financial conditions...." However, the perception from a policy point of view was best described by Guidotti and Rodriguez (1992: p. 527), "What is being discussed under the heading of dollarization is the survival of national monies in the face of the competitive challenge posed by other 'superior' currencies such as the dollar."¹

The dollarization process experienced in Bolivia has been particularly dramatic. In a period of almost ten years, from 1986 to 1994, the degree of dollarization has gone above 90%, with most of it occurring in a period of five years. As in all cases, the Bolivian dollarization experience has its own peculiarities in terms of the process itself. The perception of deteriorating economic conditions during the late 1970's and early 1980's lead to substitution of the domestic currency by the US dollar as a store of value. Economic deterioration became a full economic crisis when international interest rates increased in the early 1980's together with a cut off in international lending. The result was a fiscal imbalance which led to the 1983-85 hyperinflation period. The bulk of dollarization, however, occurred after 1985 when the economy was stabilized. During this period Bolivia was experiencing low inflation, stable foreign exchange market and disciplined fiscal policy. This contradiction in the Bolivian case is frequently mentioned in the literature. One important reason why it occurred is that all dollar operations were forbidden by decree in November 1982. As a consequence, deposits were shifted abroad, and domestically it

¹For a good discussion and literature review see Giovannini and Turtelboom (1992). For an overview of the Latin American experience see Savastano (1992).

generated an underground economy in which the US dollar became the most precious commodity. It is believed that if dollar operations had not been forbidden during the crisis, then an important degree of dollarization would have occurred at that time. Once dollar operations were reinstated as part of a stabilization program in August of 1985, the degree of dollarization rose to 50% after the first two years of 1986-87. This time, however, while the US treasury bill rate was around 6%, the domestic saving rate for dollar deposits was at the 15%-16% level. This differential indeed attracted domestic and foreign dollar deposits into the Bolivian banking system from those who were willing to accept the risks. This argumentation is important for understanding at least the first two years of dollarization after stabilization. Although high returns on dollar deposits persisted in later years, it was not enough to explain the rise to 90% dollarization reached by 1994.

Clements and Schwartz (1992) tried to explain the Bolivian dollarization based on a model that uses variables that reflect deteriorating economic conditions.² The following was their estimated model (but with different notation) for the period 1986-91:

$$m_t = -0.204 + 0.002 t + 0.372 d_t^e + 0.365 rdif_t + 0.821 m_{t-1} + \delta \Theta + \epsilon_t, \quad (1)$$

$$(4.77) \quad (3.10) \quad (3.08) \quad (2.03) \quad (21.76)$$

where m_t is log of the share of foreign currency deposits in total broad money. The variable d_t^e is expected depreciation, and is proxied by the difference between the Bolivian and U.S. monthly inflation rates. The variable $rdif_t$ measures the differential in returns between domestic and foreign currency deposits in the Bolivian banking system. A stock adjustment

² This model and its structure was originally introduced by El-Erian (1988).

mechanism is captured by a lag of the dependent variable, m_{t-1} , and, together with a trend variable t , they are both said to capture "inertia" factors in the dollarization process. The vector, Θ , was used as monthly dummies. Two basic conclusions were derived from the estimated model. First, there is a positive relationship between the dollarization rate and both expected depreciation and the return differential as suggested by the literature. However, given the size of the estimated parameters, these relationships are found to be weak and hardly the driving factors of the dollarization process. Second, "inertia" factors appeared as the most significant variables in the dollarization process.

It is not clear what the "inertia" factors mean, but they may be related to a self-feeding process. Clements and Schwarz mention the asymmetry it suggests, where dollarization increases rapidly with macroeconomic instability, but is difficult to reverse even after years of macroeconomic stability. A similar idea was advanced by Guidotti and Rodriguez (1992), who observed persistence of the phenomenon in Uruguay, Mexico, Peru, and Bolivia, despite wide fluctuations in inflation differentials and interest rates differentials. Furthermore, they developed a theoretical model which shows that dollarization irreversibility is possible when considering the costs involved in switching the currency denomination of transactions. In their model, transaction costs of dollarization define a band for the inflation differential, and dollarization irreversibility occurs above the upper value of the band.

The concepts of irreversibility, "inertia" factors, persistence, or hysteresis all imply either that the Bolivian dollarization process might be following a long-term natural trend to full currency substitution, or that it is a random walk with drift, or maybe even both considering the theoretical possibility of a band for the inflation differential. On the other

hand, Guidotti and Rodriguez have suggested that traditional models like (1) cannot fully capture the dynamic relationships between dollarization and the interest rate or inflation differential, which may be a more complex one.

The purpose of the present chapters to perform a data driven study of the dynamics of the Bolivian dollarization process. First, a sketch of a currency substitution model is developed to reconcile current theory with the possibility of dollarization irreversibility. The hypothesis of irreversibility is then tested with use of unit root testing procedures. Later, the structure of equation (1) is used only to identify the fundamental variables theoretically related to the dollarization process, and then the Johansen's maximum likelihood approach to cointegration is used to estimate a system of dynamic equations in an error correction framework. Finally, conclusions and policy options are derived from the theoretical and the estimated models.

A CURRENCY SUBSTITUTION MODEL

Currency substitution in developing nations is usually represented by a framework which simultaneously explains the transactions demand for domestic and foreign currency and their role as stores of value among alternative portfolio assets. The basic idea is that risk components that determine the return on money holdings in addition to interest rates on bonds and devaluation expectations (the portfolio variables) influence the holding of domestic currency relative to foreign currency. The model follows from the theory developed by Thomas (1985), and more recently by work at the International Monetary Fund [El-Erian (1988), and Giovannini and Turtelboom (1992)], and the work of Clements and

Schwartz (1992) more specifically for the Bolivian case.

The issue in characterizing currency substitution is to explain the shift in the currency composition of money demand derived from individual behavior in money demand. This is essentially a microeconomic framework embedded in a macroeconomic model but accounting for stylized facts of developing countries. The use of foreign currency for transactions purposes follows by including money in the utility function of a representative individual, or by including money in the production function of a representative firm as a factor of production. Both financial restrictions of households and liquidity constraints of firms due to inefficient capital markets are of equal importance as issues of growth and the effects of monetary policy on that growth in the developing nation context. Viewed in this way, currency substitution is the outcome of rational decisions of both households and firms using money for the transactions services it provides.

Real financial wealth denominated in domestic currency can be written as

$$w/p = M/p + e M^*/p + B/p + e B^*/p, \quad (2)$$

for w = wealth, M = domestic money balances, M^* = foreign money balances, and B and B^* are, respectively, domestic and foreign bond holdings. The deflators, p and p^* , are respectively the domestic and foreign price level. The exchange rate e is p/p^* , and for simplicity here purchasing power parity is assumed. Wealth is allocated between domestic and foreign balances as well as domestic and foreign bonds.

Shares of the two currency holdings that contribute to the total stock of domestic currency comprise (a) liquid forms of money (cash and demand deposits) and (b) longer-term commitments such as savings and time deposits. There are three forms of foreign money:

liquid foreign currency balances, foreign currency deposits in the domestic banking system, and foreign currency in institutions abroad. Foreign currency bonds, B^* , can be held both domestically and abroad.

Real demand for each of the four components of the portfolio of financial assets (M/p , M^*/p , B/p , B^*/p) becomes a function of total real wealth, w/p , the real interest rate on each respective asset (r_M , r_{M^*} , r_B , r_{B^*}), and the real interest rates on the alternative portfolio assets, as expressed in

$$\begin{aligned} M/p &= f^M(w/p, r_M, r_{M^*}, r_B, r_{B^*}, (M/p)_{t-1}) \\ M^*/p &= f^{M^*}(w/p, r_M, r_{M^*}, r_B, r_{B^*}) \\ B/p &= f^B(w/p, r_M, r_{M^*}, r_B, r_{B^*}) \\ B^*/p &= f^{B^*}(w/p, r_M, r_{M^*}, r_B, r_{B^*}), \end{aligned} \quad (3)$$

where f is a functional notation.

Total real wealth of the economy reflects the scale of the economy, which is expected to have a positive influence on asset demand. The transactions theory of money demand would suggest a relatively high wealth elasticity of the demand for particularly liquid domestic money balances. The wealth effect also reflects the positive influence that the level of trade has on foreign exchange balances. Generally, one can expect a positive impact of real wealth on the demand for both currencies. To the extent that economic growth raises confidence in the stability of the domestic currency, the ratio of domestic to foreign money demand might rise with an increase in real income. However, it is likely that the relative demand for foreign exchange rises parallel to the share of foreign trade transactions in total transactions, a direction from the transactions cost approach to foreign money demand. The

resulting empirical scale effect on the currency composition of money demand depends on which of the above mentioned effects dominates in a particular developing nation setting.

An additional, and perhaps more important, scale effect is the capture of the hysteresis, or the Clements-Schwartz "inertia factor," which follows from the above theory as well as being observed particularly in the work of Guidotti and Rodriguez as discussed above. There is an increasing returns to scale property of money holdings.

If there is a potential for efficiency gains through such scale effects, then the present stock of liquid domestic or foreign currency can be expected to be a positive function of the respective currency stock accumulated in previous periods, $(M/p)_{t-1}$, allowing the possibility of a downward trend if there has been an increase of the absolute level of foreign liquid money balances in the particular economy of interest. That is, a hypothesis of irreversible dollarization derives from the theory.

The demand for each asset is expected to be positively related to the real rate of interest of this asset and negatively related to the interest rate on the alternative assets in the portfolio. The real rate of interest is the nominal rate minus the expected inflation rate. The nominal rates provided by liquid domestic and foreign money balances are actually liquidity premiums which indicate the efficiency with which domestic and foreign balances contribute to reduced transaction costs in production and consumption. They are a negative function of the degree of financial development in any particular country and positive functions of the accumulated stocks of liquid money balances that are held in the respective currencies. The degree of financial development determines the efficiency with which individuals can sell and purchase financial assets or invest in the banking system. The wider the availability of

assets that reduce transaction costs and serve as store of value, the lower the efficiency of cash balances in providing these same reductions and services. For an already dollarized economy, the liquidity premium for foreign money balances is likely to fall as the government permits the banking system to introduce longer-term foreign currency deposits or to develop foreign currency denominated financial instruments.

The expected inflation rate in the country of interest depreciates the real return on domestic currency-denominated assets. Foreign currency-denominated financial assets then yield additional returns if the domestic currency can be expected to depreciate and vice versa.

Therefore, the real interest rates are positively related to the own-asset liquidity premiums and the nominal yields on bank deposits. However, they are negatively related to expected inflation and other risk factors such as the risk of inflation variability or political risk.

The El-Erian and Clements-Schwartz characterizations of the currency substitution model incorporate this theory as generally expressed in the relationships (3) and the logic of the theory discussed above in differential and summary form as the logarithmic share of foreign currency deposits in total broad money,

$$m_t = \ln(M^*/M) = f(\pi - \pi^*, r_M - r_M^*, m_{t-1}) \quad (4)$$

for inflation being denoted as π (domestic country) and π^* (foreign country). Vector dummy variables are used by El-Erian and Clements-Schwartz to capture changes in institutional and country environment. A time trend is also used in the model to capture the hysteresis in addition to the stock adjustment mechanism reflected in m_{t-1} . Bonds assets are not included

in these studies because of the low degree of financial development assumption.

TESTING FOR IRREVERSIBILITY

The previous section provided a sketch of the theory explaining currency substitution and provided a basis for the possible observance of irreversibility in the dollarization process. Irreversibility has important implications for policy and policy options in Bolivia.

If dollarization is in fact an irreversible process, then the time series of the dollarization should be stationary with respect to a positive trend. The alternative is a random walk, or difference stationary process, by which the tendency of future dollarization cannot be known. A random walk process simply meanders without exhibiting any tendency to increase or decrease, and the perception of a positive trend in dollarization might simply describe a random walk with drift process. The question of irreversibility of the dollarization process is in practice a question of whether the time series of dollarization is trend stationary or difference stationary. The Dickey-Fuller unit root test is an appropriate procedure for testing this hypothesis. The time series of dollarization is proxied by the share of foreign currency deposits in total broad money during the period of 1987.08 to 1994.09.

To fix the idea of a unit root, consider the following model for a time series y_t :

$$y_t = \alpha_0 + \alpha_1 t + u_t \quad (5)$$

$$u_t = \rho u_{t-1} + \epsilon_t, \quad (6)$$

where ϵ_t is a zero mean white noise process. The reduced form of model (5)-(6) yields the following equation:

$$y_t = \gamma + \delta t + \rho y_{t-1} + \epsilon_t, \quad (7)$$

where $\gamma = [\alpha_0(1-\rho) + \alpha_1\rho]$ and $\delta = \alpha_1(1-\rho)$. Equation (7) and y_t are said to have a "unit root" when $\rho=1$ (which in turn implies $\delta = 0$). In this case y_t is nonstationary, and the process is said to be "difference stationary" (DS) because stationarity is induced by first differencing: $\Delta y_t = y_t - y_{t-1} = \gamma + \epsilon_t$. Alternatively, when $\rho < 1$ and $\alpha_1 \neq 0$, y_t is stationary about a linear trend $\alpha_0 + \alpha_1 t$, and hence is said to be "trend stationary" (TS). The case when $\rho < 1$ and $\alpha_1 = 0$ implies y_t is stationary itself.

Unit root tests based on equation (7) were pioneered by Dickey and Fuller (1979), and the most common procedure is to estimate (7) by ordinary least squares, and test the null hypothesis $H_0: \rho = 1$ based on the statistic $t(\rho) = (\hat{\rho}-1)/\text{se}(\hat{\rho})$. Dickey and Fuller have shown that this statistic does not have the traditional t-distribution but, rather, follows some nonstandard distribution. They tabulated critical values for the asymptotic case based on Monte Carlo simulation.

The above Dickey-Fuller (DF) test requires the error term ϵ_t to be white noise. Said and Dickey (1984) further developed the test to accommodate autoregressive (AR) serial correlation in the error term and introduced the Augmented Dickey-Fuller (ADF) test, which involves estimating the test statistic $t(\beta)$ using

$$\Delta y_t = \gamma + \delta t + \beta y_{t-1} + \sum_{i=1}^g \theta_i \Delta y_{t-i} + \epsilon_t \quad (8)$$

where Δ is the first difference operator, g is the number of lags required to account for serial

correlation, and $\beta = \rho - 1$. Said and Dickey showed that the "augmentation" leaves the statistic $t(\beta)$ distributed according to the Dickey-Fuller tables. However, the null hypothesis changes to $H_0: \beta = 0$.

Notice the similarity between equations (7) and (1). The estimated coefficient for the lag dependent variable in (1) is 0.821 with standard error of 0.0377, then $t(\rho) = -4.748$, which suggests rejection of the null of a unit root at less than a 1% alpha level according to DF critical values. Use of the DF critical values in this case, however, can be questioned by the presence of other regressors in the estimation of Clements-Schwartz model (1) as compared to (7), namely d_t^* , $rdif_t$, and Θ . A formal ADF test would be more appropriate.

Figure 4-1 presents some statistical properties of the time series of dollarization. Plot (a) is the logarithm of the time series of dollarization, m_t . One important feature of this series is the sudden jump that occurred in August of 1989, when a political regime change was taking place. This jump appears to have dramatically increased the mean of the dollarization process. Plot (b) is the first difference of the dollarization rate. Plots (c) and (d) are, respectively, the autocorrelation function and partial autocorrelation function of the first difference of dollarization. The first difference of the dollarization rate will be used as the dependent variable in (8), and given the autocorrelation of order 12 detected by plot (d), then g in (8) is set to 12. Table 4-1 presents estimation of (8) performed for different periods to consider the possible effects of the political regime change.

The first row of Table 4-1 shows the estimated parameters and their statistics when considering the entire length of the data set. The coefficient for the estimate $\beta = -0.09$ is very

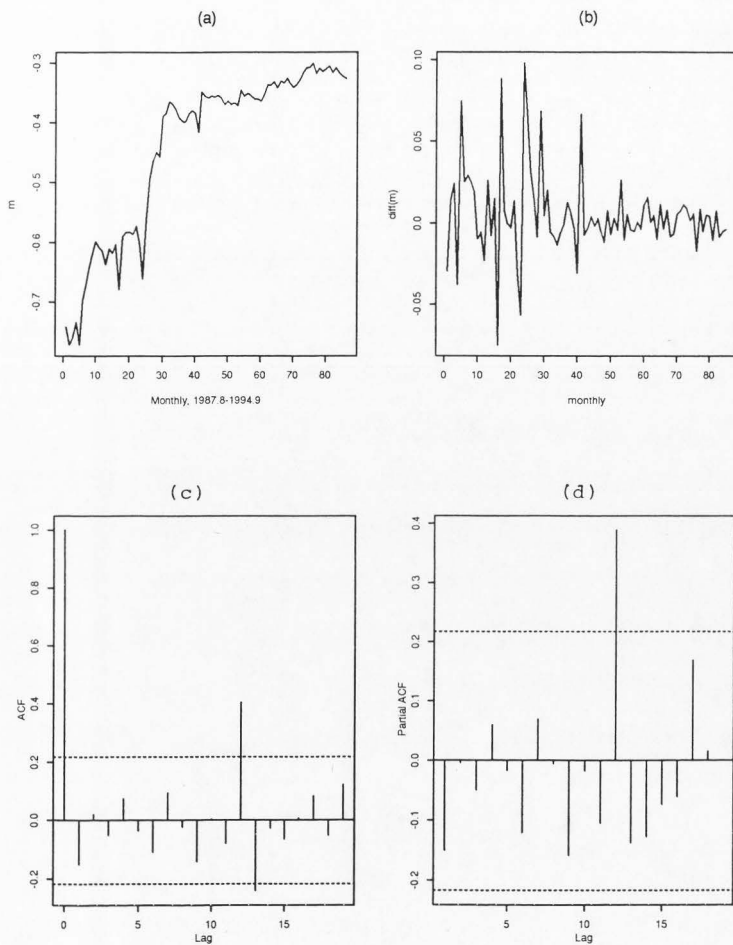


Figure 4-1. Some statistical properties of the time series of dollarization.

Table 4-1. *Application of the ADF Test to the Dollarization Series*

Period	g	γ	$t(\gamma)$	δ	$t(\delta)$	β	$t(\beta)$	DF-2.5%
1987.08 - 1994.09	12	-0.04	-1.08	0.0002	0.61	-0.09	-1.60	-3.73
1989.09 - 1994.09	12	-0.17	-5.90	0.0005	3.64	-0.41	-6.58	-3.80
1987.08 - 1989.08	0	-0.36	-2.56	0.003	1.94	-0.49	-2.65	-3.95

close to zero, and in comparing the statistic, $t(\beta)$, with the DF critical value at 2.5%, the conclusion is that a unit root process exists. When the sample is split in two periods (pre-1989.08 and post-1989.08), the estimated value of β has an important change: -0.41 for the post-1989.08 sample and -0.49 for the pre-1989.08 sample. Moreover, by the magnitude of $t(\beta)$ for the post-1989.08 sample, the null of a unit root can be rejected at even less than the 1% alpha level. That is, dollarization is trend stationary after 1989.08. Given the size of the pre-1989.08 sample, a unit root cannot be rejected. It appears that the political regime change is responsible for the near unit root value of β when considering the entire sample.

Perron (1989) developed a procedure which improves the performance of the ADF test in the presence of shocks which are not a realization of the underlying data-generating mechanism (shocks are exogenous). Perron's "Test for Structural Change," as it is referred to in the literature, has the following structure:

$$\Delta y_t = \gamma + \mu_1 D_p + \mu_2 D_L + \delta t + \beta y_{t-1} + \sum_{i=1}^g \theta_i \Delta y_{t-i} + \epsilon_t \quad (9)$$

where $D_p=1$ in 1989.09 and zero otherwise, and $D_L=1$ for all t beginning in 1989.09 and zero otherwise. D_p and D_L are referred to as the pulse dummy and the level dummy. Under the

Table 4-2. *Test of Structural Change on the Dollarization Series*

	γ	μ_1	μ_2	δ	β	$\lambda=0.3; 1\%$	$g=12$
Coefficients	-0.423	-0.059	0.131	0.001	-0.67		
t-statistics	-6.587	-1.908	6.796	3.836	-6.840	-4.39	

null hypothesis we have a unit root with a one-time change in its mean, so the null becomes $H_0: \beta=0, \delta=0$, and $\mu_2=0$. Under the alternative we have a trend stationary series with a permanent one-time break in the trend, i.e., $H_1: \beta<0$ and $\mu_1=0$. Figure 4-2 shows the plot of the dollarization series against the fitted regression under the alternative, that is, $y_t = \gamma + \mu_2 D_L + \delta t$. Perron also showed that given the extra regressors and the split sample nature of these regressors, the distribution of the statistic of interest, $t(\beta)$, changes depending on the proportion of observations before the break, λ . Application of Perron's procedure to the dollarization series produced the result given in Table 4-2.

Comparing $t(\beta)$ to Perron's critical value suggests rejection of the unit root hypothesis at less than the 1% alpha level. Also μ_2 and δ appear to be significant at usual t-table values. There is, however, an important loss of degrees of freedom given the number of lags used in estimation. This produces parameter estimates strongly influenced by the post-1989.08 dollarization process. These results suggest that the Bolivian dollarization, and specifically the post-1989.08 period, is an irreversible process. By using the estimated coefficients of equation (9) presented in Table 4-2, and the definitions given in (5) to (9), the estimated time trend is $-0.632 + 0.0015 t$. The coefficient for trend is positive but small, which suggests that

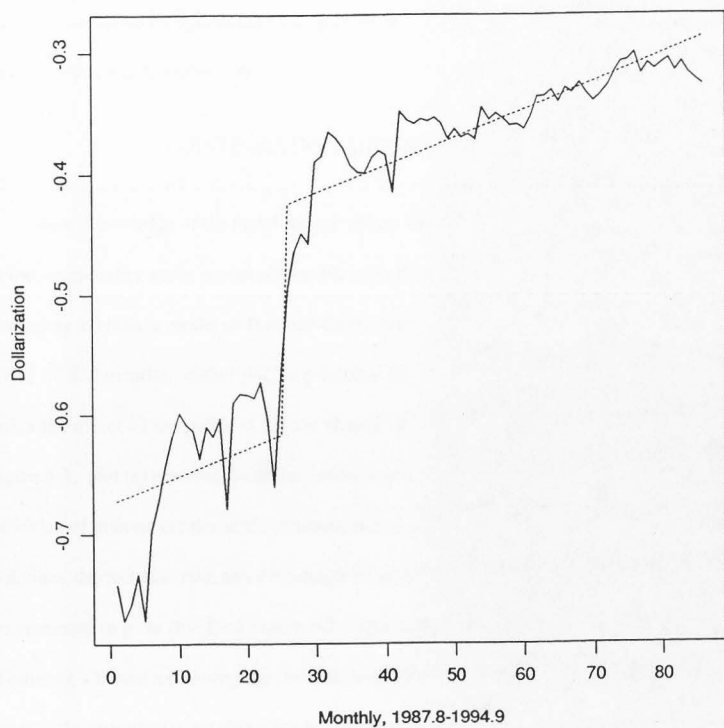


Figure 4-2. *Dollarization series versus fitted regression under the alternative.*

dollarization will continue to full currency substitution at a slow but steady pace. In connection to the theory presented above, the degree of dollarization caused by the political regime change may have introduced a scale effect, after which there were more efficiency gains to be realized by operating in foreign currency in the form of reduced transaction costs in consumption and production.

COINTEGRATION AND DOLLARIZATION

With knowledge of the variables and effects that determine dollarization, as presented in the comparative static model above, the objective now is to improve over model (1) by estimating a dynamic model of Bolivian dollarization process. For this purpose, a new time series of dollarization, called dol_t , is produced by taking out of the original dollarization series the effect of the political regime change, that is, $dol_t = y_t - \hat{\mu}_2 D_L = \hat{\gamma} + \hat{\delta}t + \hat{\epsilon}_t$. In Figure 4-3, plot (a) presents the dollarization series after making this correction. The other variables of interest are domestic inflation, return on dollar deposits, return on deposits in Bolivians, depreciation rate, and the foreign inflation rate. The time series of these variables are presented in plots (b)-(f) of Figure 4-3, respectively. More specifically, it is of interest to estimate a model exhibiting the short-term dynamic connections among these variables, and how these dynamics could be, in part, adjustments to long-term equilibrium relationships among them.

In the econometrics literature, the requirement of equilibrium relationships among variables is referred to as variables being cointegrated. Let y_t be a $K \times 1$ vector of nonstationary variables that can be made stationary after first differencing. If there exists

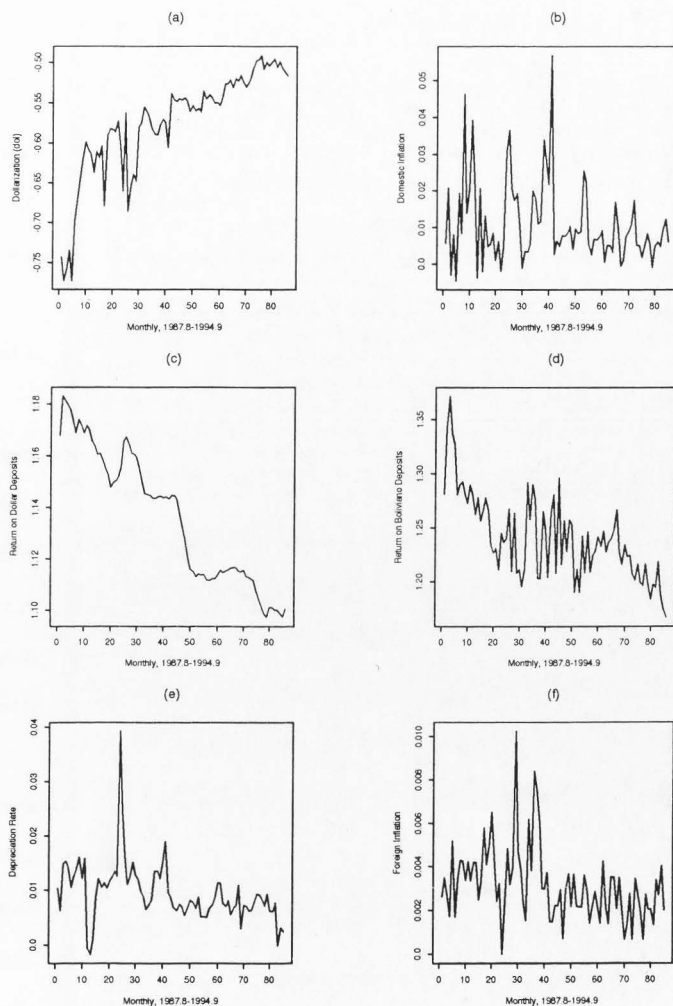


Figure 4-3. Time series of macroeconomic variables of interest.

some linearly independent vectors $\alpha_1, \dots, \alpha_k$, $0 < k < K$, such that $\alpha_i' y_t$ ($i=1 \dots k$) are stationary, then the components of the vector y_t are said to be cointegrated with cointegration rank k (number of possible stationary linear combinations). Based on these definitions, Granger (1986) and Engle and Granger (1987) proved the Granger Representation Theorem, which states that if a $K \times 1$ vector y_t is cointegrated, with cointegration rank k , then there exists a vector autoregressive representation,

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (10)$$

and an error correction representation

$$\Delta y_t = B_1 \Delta y_{t-1} + \dots + B_{p-1} \Delta y_{t-p+1} - \Pi y_{t-1} + u_t \quad (11)$$

where p is the number of lags, $B_i = -A_p - A_{p-1} - \dots - A_{i+2} - A_{i+1}$, and $\Pi = HC = I_K - A_1 - \dots - A_p$ with $\text{rank}(\Pi) = \text{rank}(H) = \text{rank}(C) = k$. C is a $k \times K$ matrix containing the cointegrating vectors, and H is a $K \times k$ matrix of adjustment or error correction coefficients. The errors u_t are assumed to be gaussian white noise. Equation (11) is a reparameterized version of equation (10). Alternatively, equation (11) can be reparameterized into the following error correction form:

$$\Delta y_t = D_1 \Delta y_{t-1} + \dots + D_{p-1} \Delta y_{t-p+1} - \Pi y_{t-p} + u_t \quad (12)$$

where $D_i = -I_K + A_1 + \dots + A_p$. Equation (12) is the form used by Johansen and Juselius. In either equation the rank k of the matrix Π is the empirical number of linearly independent cointegrating relations among the variables in y_t . If Π is of full rank, then any linear combination of y_t will be stationary, while if it is a matrix of zeros, then any linear combination of y_t will be a nonstationary unit root process. The most interesting case is

when Π is less than full rank, $0 < k < K$. In this situation, k is the number of cointegrating vectors and $K-k$ is the number of nonstationary linear combinations.

Let $y_t = (\text{dol}_t, \pi_t, r_t^*, r_t, \text{dep}_t, \pi_t^*)$ contain our variables of interest, where dol_t is the time series of dollarization, π_t is domestic inflation, r_t^* is the return on foreign currency deposits (Dollar deposits), r_t is the return on domestic currency deposits (Boliviano deposits), dep_t is short for the time series of depreciation, and π_t^* is foreign inflation. Estimation is done by treating all variables as equally endogenous (except for foreign inflation, which is treated as exogenous because it cannot be determined within the Bolivian economy, but it is expected to influence it) with the objective of letting the data determine the structure of the empirical model. The data used correspond to the time series of these variables for the period 1987.08 to 1994.09 collected from the Bolivian Central Bank statistical bulletins. In addition to these variables, two dummy variables are included representing the 1989 political event and the December bonus. The latter is an important institutional component of the Bolivian economy.

Computation of the ML estimators requires previous knowledge of the lag order, p , and the cointegration rank, k . Three methods were used to establish an optimum lag order: the Akaike's Information Criterion (AIC), the Hannan-Quinn Criterion (HQ), and the Schwarz Criterion (SC). Each of these procedures uses the above variables in computing VAR systems at alternative lags. Then the optimum lag is selected as the one that minimizes the criterion represented by the following formulas:

$$\text{AIC}(m) = \ln |\hat{U}(m)| + 2mK^2/T$$

$$HQ(m) = \ln |\hat{U}(m)| + (2 \ln \ln T)/T$$

$$SC(m) = \ln |\hat{U}(m)| + (\ln T \cdot mK^2)/T,$$

where $m=0,1,\dots,M$ is the number of alternative lags considered, T is the length of the series, K is the number of variables, and $\hat{U}(m)$ are the residuals from the estimated VARs at different lags, m . Table 4-3 shows the computed values.

Table 4-3. *Order Selection Criteria*

Lag	1	2	3	4	5	6	7	8	9	10
AIC	-44.1	-43.6	-42.9	-42.3	-41.4	-40.6	-39.7	-39.0	-38.0	-36.9
HQ	-43.8	-43.0	-42.0	-41.1	-39.9	-38.8	-37.6	-36.6	-35.2	-33.8
SC	-43.4	-42.2	-40.7	-39.3	-37.7	-36.1	-34.4	-32.9	-31.1	-29.1

In all three cases the suggested optimum lag order is one. In equation (6), a lag order of one in the VAR portion of the ECM implies $p=2$ in the cointegrating portion of the ECM. Given that (6) is the equation used for ML estimation, then p is set to two, which gives the following equation for estimation purposes:

$$\Delta y_t = D_1 \Delta y_{t-1} - \Pi y_{t-2} + u_t. \quad (13)$$

To establish cointegration rank, the trace statistic and maximum eigenvalue statistic were computed and then evaluated using critical values obtained from a nonparametric bootstrap according to Algorithm 1 (see below). This procedure was necessary given the specific treatment of endogenous and exogenous variables in the ECM. Algorithm 1 was developed following the same procedure used by Johansen and Juselius (1990) in producing their tables of critical values.

Algorithm 1:

1. Estimate the ECM model (12) assuming full rank and obtain its residuals e_{it} , ($t=1...T$; $i=1...K-k$).
2. Take a random sample of the stacked residuals e_{it} and generate e_{it}^* .
3. Compute $X_{it} = \sum e_{it}^*$, ($t=1,...,T$; $i=1,...,K-k$), with $X_{0i} = 0$.
4. Compute the matrix $V = \sum e_t^* (X_{t-1} - X_{-1})' [\sum (X_{t-1} - X_{-1})(X_{t-1} - X_{-1})']^{-1} \sum (X_{t-1} - X_{-1}) e_t^{*'}.$
5. Compute the trace and maximum eigenvalue of V , and go back to step 2.
6. Obtain critical values for the trace and maximum eigenvalue test.

Table 4-4 presents the computed statistic and critical values for the trace test. Table 4-5 presents the computed statistic and critical values for the maximum eigenvalue test. In both cases cointegration of rank three is accepted at an alpha level of less than 1%. Three cointegrating vectors imply that there are three long-term equilibrium relationships among the variables, and therefore their relationship is stable in three directions.

With knowledge of p and k , then equation (13) can be estimated to obtain the following

Table 4-4. Trace Test for Cointegration Rank

H_0	H_1	Value	0.90	0.95	0.975	0.99	mean	variance
$k=4$	$k=5$	5.97	6.777	8.221	10.027	12.521	3.00	8.11
$k=3$	$k \geq 4$	17.45	15.686	17.802	21.140	25.985	9.48	21.94
$k=2$	$k \geq 3$	43.22*	28.481	31.436	34.224	37.287	19.90	40.46
$k=1$	$k \geq 2$	74.85	44.508	48.847	52.664	57.611	33.48	72.07
$k=0$	$k \geq 1$	117.10	64.756	69.089	72.851	77.975	50.61	110.57

Table 4-5. *Maximum Eigenvalue Test for Cointegration*

H_0	H_1	Value	0.90	0.95	0.975	0.99	mean	variance
$k=4$	$k \geq 5$	5.97	6.777	8.221	10.027	12.521	3.00	8.11
$k=3$	$k \geq 4$	11.48	12.869	14.886	16.935	21.674	7.74	15.57
$k=2$	$k \geq 3$	25.76*	19.294	21.303	23.671	25.470	12.82	22.55
$k=1$	$k \geq 2$	31.63	24.971	27.516	30.387	32.368	17.82	28.40
$k=0$	$k \geq 1$	42.24	30.992	34.018	35.916	39.754	22.90	38.62

reparameterized version:

$$\Delta y_t = B_0 + B_1 \Delta y_{t-1} - \Pi y_{t-1} + u_t \quad (14)$$

where $B_1 = -A_2$, $\Pi = HC = I_5 - A_1 - A_2$, $A_1 = I_5 + D_1$, and $A_2 = -HC - D_1$. The matrix B_1 reports the short-term coefficients, and the matrix B_0 reports the coefficients of the constant term, the dummy variables³, and the exogenous variable, π^*_t . The cointegrating vectors are contained in the matrix C , and the error correction coefficients are in the matrix H . The matrix Π simply multiplies H and C and therefore it contains both coefficients, that of the cointegrating vectors and that of the error correction. The following were the estimated parameters:

0.428 (0.005)	-0.275 (0.271)	0.561 (0.011)	0.220 (0.044)	-0.058 (0.910)
-9.8e-03 (0.986)	0.479 (0.002)	-0.242 (0.025)	0.129 (0.016)	0.471 (0.090)

³Estimated parameters for the dummy variables are not reported. However, evaluation of their t-statistics indicated that dummies operated as intended.

$$\hat{H}\hat{C} = \begin{vmatrix} 5.6\text{e-}02 & -0.024 & 0.019 & -0.005 & -1.6\text{e-}03 \\ (0.660) & (0.358) & (0.360) & (0.642) & (0.997) \\ -0.091 & 0.814 & -0.682 & 0.421 & -1.339 \\ (0.460) & (0.011) & (0.010) & (0.002) & (0.058) \\ 0.020 & 0.080 & 0.012 & -0.011 & 0.466 \\ (0.158) & (0.039) & (0.651) & (0.470) & (0.000) \end{vmatrix}$$

$$\hat{C} = \begin{vmatrix} 6.20 & -51.64 & 17.33 & 11.53 & -217.85 \\ 27.95 & -66.75 & 71.38 & -13.52 & 65.07 \\ 27.26 & 58.99 & -6.71 & 40.54 & 26.35 \end{vmatrix}$$

$$\hat{H} = \begin{vmatrix} 3.41\text{e-}03 & 7.69\text{e-}02 & 7.03\text{e-}03 \\ -2.53\text{e-}03 & -2.49\text{e-}02 & 3.09\text{e-}03 \\ 6.83\text{e-}05 & 2.55\text{e-}03 & -7.18\text{e-}05 \\ 3.90\text{e-}03 & -9.94\text{e-}02 & 5.95\text{e-}03 \\ -1.88\text{e-}03 & 6.70\text{e-}03 & 4.79\text{e-}04 \end{vmatrix}$$

$$B_1 = \begin{vmatrix} -0.217 & -0.003 & -1.439 & 0.022 & -1.432 \\ (0.075) & (0.988) & (0.201) & (0.800) & (0.030) \\ -0.024 & -0.274 & 1.461 & 0.053 & 0.635 \\ (0.643) & (0.015) & (0.013) & (0.242) & (0.042) \\ -0.002 & 0.001 & 0.345 & 0.001 & 2.7\text{e-}03 \\ (0.847) & (0.981) & (0.008) & (0.922) & (0.995) \\ -0.061 & 0.853 & 0.515 & -0.326 & -0.906 \\ (0.667) & (0.003) & (0.701) & (0.006) & (0.261) \\ 0.002 & 0.049 & 0.227 & -0.014 & -0.050 \\ (0.884) & (0.118) & (0.156) & (0.298) & (0.621) \end{vmatrix}$$

$$B_0 = \begin{vmatrix} 0.670 & -0.990 \\ (0.001) & (0.535) \\ -0.100 & -0.746 \\ (0.232) & (0.350) \\ 0.011 & 0.113 \\ (0.464) & (0.528) \\ -0.198 & -2.151 \\ (0.322) & (0.284) \\ -0.008 & 0.450 \\ (0.682) & (0.042) \end{vmatrix}$$

The numbers in parenthesis are bootstrapped p-values that were obtained with use of the following algorithm:⁴

Algorithm 2:

1. Take a random sample from each of the residual vectors contained in

$$\hat{u}_t = y_t - \hat{A}_0 - \hat{A}_1 y_{t-1} - \hat{A}_2 y_{t-2} \text{ and generate new residuals } u_t^*.$$

2. Use the residuals u_t^* to build new values y_t^* based on the structure of the estimated model and using the first two observed values of y_t as starting points. That is,

$$y_t^* = \hat{A}_0 - \hat{A}_1 y_{t-1}^* - \hat{A}_2 y_{t-2}^* + u_t^*.$$

3. Use the generated values y_t^* to compute model (12) with $p=2$ and $k=3$, and generate new D^* , H^*C^* , and their matrix of standard errors $se(D^*)$ and $se(H^*C^*)$. Based on these also compute new B_1^* and $se(B_1^*)$.

⁴The general principles used for the development of this algorithm follow the work of Freedman and Peters (1984) and Hall and Wilson (1991).

4. Then compute the following matrices:

$(B_1^* - \hat{B}_1)/\text{se}(B_1^*)$ to approximate the distribution of $(\hat{B}_1 - 0)/\text{se}(\hat{B}_1)$,

$(HC^* - \hat{HC})/\text{se}(HC^*)$ to approximate the distribution of $(\hat{HC} - 0)/\text{se}(\hat{HC})$.

5. Go to step 1 and repeat the simulation 1000 times.
6. Finally, use the absolute values of the vectors generated for each element of the above matrices to build their empirical probability distributions for a two-sided evaluation of the significance of the originally estimated parameters.

Use of the nonparametric bootstrap (bootstrapping from residuals) described in algorithm 2, has a multiple purpose in the context of error correction models. First, it considers the small sample problem and possible departures from normality in the residuals. Second, bootstrapping from residuals takes into account the information contained in them relative to other variables not considered in estimation. Third, the bootstrap produces p-values for each individual parameter based on their individual empirical distribution function. Each distribution is obtained simultaneously for all parameters by using the estimated model as the data generating mechanism and by drawing random samples from the residuals. The result is superior inference in hypothesis testing compared to use of traditional t-table values. Appendix E presents critical values derived from these distributions for a two-sided evaluation of the significance of each parameter contained in the matrix HC. Appendix F does the same for each parameter contained in the matrix B_1 .

The final model can be summarized in the following equations:

$$\begin{aligned} \Delta \text{dol}_t = & 0.67 - 1.43 \Delta \text{dep}_{t-1} - 0.0034(-\text{dol}_t - 2.79r_t^* - 1.86r_t)_{t-1} \\ & - 0.0076(-\text{dol}_t - 2.55r_t^* + 0.48r_t)_{t-1} \\ & - 0.0070(-\text{dol}_t + 0.24r_t^* - 1.48r_t)_{t-1} + \varepsilon_{1t} \end{aligned} \quad (14)$$

$$\begin{aligned}\Delta\pi_t = & -0.27\Delta\pi_{t-1} + 1.46\Delta r^*_{t-1} + 0.63\Delta dep_{t-1} - 0.00253(-\pi_t + 0.33r^*_t + 0.22r_t)_{t-1} \\ & - 0.00249(-\pi_t + 1.06r^*_t - 0.20r_t)_{t-1} \\ & - 0.0031(-\pi_t + 0.11r^*_t - 0.68r_t)_{t-1} + \varepsilon_{12}\end{aligned}\quad (15)$$

$$\Delta r^*_t = 0.34\Delta r^*_{t-1} + \varepsilon_{13}\quad (16)$$

$$\begin{aligned}\Delta r_t = & 0.85\Delta\pi_{t-1} - 0.32\Delta r_{t-1} - 0.0039(4.47\pi_t - 1.50r^*_t - r_t + 18.89dep_t)_{t-1} \\ & - 0.0099(-4.93\pi_t + 5.28r^*_t - r_t + 4.81dep_t)_{t-1} \\ & - 0.0059(-1.45\pi_t + 0.16r^*_t - r_t - 0.65dep_t)_{t-1} + \varepsilon_{14}\end{aligned}\quad (17)$$

$$\begin{aligned}\Delta dep_t = & -0.0018(-0.23\pi_t - dep_t - 239.0\pi^*_t)_{t-1} \\ & - 6.7e-04(1.02\pi_t - dep_t - 670.8\pi^*_t)_{t-1} \\ & - 4.7e-04(-2.23\pi_t - dep_t - 938.04\pi^*_t)_{t-1} + \varepsilon_{15}\end{aligned}\quad (18)$$

The cointegrating vectors contained in \hat{C} affect four of the five equations according to p-values of the \hat{H} matrix. Each vector, however, has been divided by the coefficient of the variable whose equation is being affected.

Equations (14)-(18) describe a partial structure of the Bolivian economy directly related to the dollarization process. Before analyzing equation (14), which describes the dynamics of dollarization, it is convenient to understand first the dynamics of the other variables. This makes sense because the variable for dollarization, dol_t , does not appear either in the short-term or long-term components of any of the other equations. That is, there is no feedback effects from dollarization on the behavior of the other variables, but all of these other variables do enter directly or indirectly into equation (14).

The dynamics of depreciation is explained in equation (18). Depreciation results strictly as an adjustment of its out-of-equilibrium position with respect to its long-term relationship with the domestic and foreign inflation rates. This long-term relationship,

however, is of a multiple and contradictory nature. One cointegrating vector describes a positive relationship between depreciation and domestic inflation, and a negative relationship between depreciation and foreign inflation. This is the relationship suggested by theory, but, by the size of its error correction coefficient ($6.7e-04$), this is not the dominating vector. The other two cointegrating vectors (one of them being the dominant) describe a negative relationship between depreciation and domestic inflation, and between depreciation and foreign inflation, implying that decreases in both inflation rates promote further depreciation. However, in both vectors the coefficient for foreign inflation is several times greater than the coefficient for domestic inflation, implying that depreciation is basically determined by foreign inflation. Even though monthly domestic inflation has been low by Bolivian standards, with an average of 1% (although volatile), the US monthly inflation has averaged 0.3%-0.4% and is less volatile. That is, the "low" Bolivian inflation could have become an illusion being more and more evident as dollarization proceeded. Inflation tax from using Boliviano currency is still high compared to the inflation tax that Bolivians would pay to the US government from dollar currency use. Theoretically, as dollarization proceeds, the last two cointegrating vectors will dominate until full equalization with US prices occurs.

Domestic inflation is an important variable in equations (14) and (18). Equation (15) shows that the dynamics of domestic inflation is a result of two components. First, in the short run it has a negative relationship with the speed of inflation experienced in the previous period, a positive relationship with the rate of change in the return on dollar deposits, r^*_d , and a positive relationship with the speed of depreciation (which is determined by foreign inflation). Then, inflation in the short run is determined by the return on dollar deposits and

foreign inflation. Second, it results from an adjustment to its out-of-equilibrium position with respect to three equilibrium relationships with the return on dollar and Boliviano deposits. Two of the vectors, which together dominate the long-run behavior of inflation, present a positive relationship of inflation with the return on Dollar deposits and a negative relationship with the return on Boliviano deposits. In one case, the coefficient for return on dollar deposits is several times larger than the coefficient on Boliviano returns, and in the other case the opposite occurs.

The return on Boliviano deposits is also an important variable in explaining dollarization. Equation (17) shows three equilibrium relationships among the return on Boliviano deposits, r_t , the return on Dollar deposits, r_t^* , the domestic inflation rate, π_t , and depreciation. These relationships simply show that r_t follows closely r_t^* adjusted by inflation and depreciation. The short-term dynamics of r_t has a positive relationship with the speed of inflation experienced in the previous period, a negative relationship with changes in r_t itself in the previous period, and of course it includes the adjustment with respect to the equilibrium relationships.

The time series of return on Dollar deposits, r_t^* , appears to have a life of its own. In equation (16), the short-term dynamics of r_t^* does not include adjustments to any equilibrium relationships or to the short-term movements of the other variables in the previous period. It only has a positive relationship with its own changes occurring in the previous period. However, this variable has an important impact on the inflation rate, first by affecting directly the short-term dynamics of inflation, second, by being part of the cointegrating vectors to which inflation adjusts, and third, indirectly through its effects on

r_t . Although r_t^* does not appear to be affecting depreciation directly, it does influence it indirectly through domestic inflation. However, r_t^* does affect the dollarization process directly by being part of the cointegrating vectors to which dollarization adjusts, and also indirectly through r_t , given that this variable is determined by r_t^* . From these results it can be concluded that r_t^* drives the system from inside the economy. The other variable that drives the system, although from outside the system, is foreign inflation, which determines directly the rate of depreciation, and indirectly inflation and dollarization through depreciation.

As the literature suggests, there is in fact a relationship between the dollarization rate and the return on dollar and Boliviano deposits as shown by the long-term components of (14). However, this relationship is also found to be of a multiple and contradictory nature. One of the cointegrating vectors describes a positive relationship between dollarization and the return on dollar deposits, and a negative relationship between dollarization and the return on Boliviano deposits. This is the relationship suggested by theory, but, given the size of its error correction coefficient (0.0070), this is not the dominant vector, which coincides with the findings of Clements and Schwartz. Notice that the coefficient on Boliviano deposits is about six times greater than the coefficient on dollar deposits, suggesting that Bolivian savers do shift strongly in favor of Boliviano deposits when the opportunity comes. A second cointegrating vector describes a negative relationship between dollarization and the return on Dollar deposits, and a positive relationship between dollarization and the return on Boliviano deposits. This relationship is exactly the opposite to what theory suggests; furthermore, the coefficient for the return on dollar deposits is about 5.5 times greater than

the coefficient for the return on Boliviano deposits, and by the size of its error correction coefficient (0.0076), this is the cointegrating vector that dominates the other two. That is, decreases in the return on dollar deposits actually promote further dollarization. The third cointegrating vector is really a mixture of the other two. It describes a negative relationship between dollarization and the return on dollar deposits, and a negative relationship between dollarization and the return on Boliviano deposits. In this case, decreases in both returns promote further dollarization; however, by the size of its error correction coefficient this vector is the weakest of the three. Besides adjusting to its out-of-equilibrium position with respect to the three cointegrating vectors, the short-term dynamics of dollarization also responds negatively to changes in the speed of depreciation. But, given that this variable is determined by the domestic and foreign inflations, then this is the way these variables enter the dollarization equation.

The question now is, why would people prefer more dollar deposits over Boliviano deposits when both rates are decreasing over time? In Figure 4-1, (c) and (d) show continuously decreasing returns, while (a) shows continuously increasing dollarization. The answer to this question could be intimately linked to a third alternative of dollar deposits abroad. The return on dollar deposits in Bolivia is still high compared to the international rate, and given its past behavior it is probably expected to continue decreasing toward full equalization with the international rate. Therefore, there is still a period in which one can realize high returns before full equalization occurs. This argument could justify the preference of more deposits in Bolivia rather than abroad despite decreasing interest rates. The preference of dollar deposits over Boliviano simply results from a decreasing return on

Boliviano deposits as theory suggests. Moreover, it turns out that, by equation (17), the return on Boliviano deposits is simply equal to the return on dollar deposits adjusted for inflation and depreciation. That is, the return on dollar deposits drives the return on Boliviano deposits.

In addition, the speed of dollarization rate was found to decrease with the speed of change in the inflation differential. Domestic inflation and the return on Boliviano deposits were found directly and indirectly affected by the domestic variable that drives the system, the return on dollar deposits. Domestic inflation together with foreign inflation affects indirectly the short-term dynamics of dollarization, and the return on dollar and Boliviano deposits determines a long-term equilibrium with dollarization. Then, it turns out that it is the return on dollar deposits and the speed of foreign inflation that greatly contributed to the dollarization process.

CONCLUDING REMARKS

As the literature suggests, there is in fact an equilibrium relationship between the dollarization rate and the return on dollar and Boliviano deposits. There is also an equilibrium relationship between depreciation and domestic and foreign inflations. It was also found that in the long run the return on Boliviano deposits simply follows the return on Dollar deposits adjusted for inflation and depreciation, and therefore does not play any important role in the dollarization process. Further, there is an equilibrium relationship between domestic inflation and the returns on dollar and Boliviano deposits. However, these relationships appear to be of a multiple and perhaps contradictory nature. The existence of

multiple equilibria among these variables indicates that, as the dollarization process has proceeded, the Bolivian economy has apparently been in a transition period toward full equalization of prices and interest rates with the US. This conclusion derives from the fact that the return on dollar deposits and foreign inflation are the variables that in practice drive the dollarization process.

Even though the return on dollar deposits has consistently been decreasing, it has remained high compared to the US treasury bill rate. Similarly, even though domestic inflation has been low by Bolivian standards, it has remained high compared to the US inflation rate. These differentials have become more and more evident as dollarization proceeded, making in fact the differentials in interest rates and inflation rates the driving forces for further dollarization. It is now expected that this process will end somewhere in the neighborhood of full price and interest rate equalization.

This line of argument introduces some new elements to the discussion of why dollarization has persisted in Bolivia regardless of recent stable macroeconomic conditions. The basic theoretical model of dollarization permits the possibility of reversing the process as the fundamental macroeconomic problems are solved. However, there must be a level of dollarization beyond which the domestic variables stop having an important role and foreign variables become increasingly the dominating driving forces. This is an expression of dollarization irreversibility in a multivariate time series framework. That level of dollarization was apparently reached in Bolivia right after the 1989 political regime change. According to our theory, the degree of dollarization caused by the political regime change may have introduced a scale effect, after which there were more efficiency gains to be

realized in the form of reduced transaction costs in consumption and production when operating in foreign currency. It is important, however, to realize that dollarization could not have proceeded so strongly in such a short period if international interest rates had not been as low as they have been during 1989.

In terms of policy options, if in fact dollarization is irreversible, then the Bolivian government not only lost the battle for its currency, but also lost the monetary policy instrument and the inflation tax revenue. Alternative policy actions must now be based on this framework. In this sense, probably the greatest problem a fully dollarized economy has in this case is the protection of Bolivian savers and the domestic banking system, for example, from sudden changes in international financial markets. As a dollarized economy, the Bolivian Central Bank cannot come to the rescue of a collapsing banking system, but it could ask for higher reserve requirements or expect domestic banks be protected by larger foreign banks. In a situation of increasing international interest rates, the domestic banking system could face an outflow of capital. However, this could be avoided in a climate of political stability and fiscal management free of domestic debt. In this sense, privatization of state-run firms and government decentralization may no longer be a policy option, but rather a requirement to guarantee the government's full commitment to preserve a stable dollarized system.

Even under well designed economic policy, a dollarized economy is more vulnerable to external shocks relative to an economy with less dominant currency substitution conditions. If money supply in terms of the domestic currency includes a large portion of foreign currency balances, changes in exchange rate will affect the composition of the money

stock, thereby rendering weak control over money supply. Currency substitution makes the demand for money unstable. There is some evidence that this has been the case in Bolivia as well as in Mexico and Peru (Savastano, 1992).

The effectiveness of fiscal policy is reduced by currency substitution through the reduced ability of the government to raise revenues via taxation. If dollarization is accompanied by an increase in the informal sector, the base for international trade income and profit taxes is eroded. If transactions and deposits in foreign exchange are authorized by the government, then government revenue from taxation is, *ceteris paribus*, reduced. Seigniorage resulting from the inflation tax is reduced as well.

The welfare effects caused by dollarization can only be characterized in a general way. This characterization of effects can only be made by comparing alternative imperfect conditions rather than comparing the conditions of perfect macroeconomic processes with imperfect conditions. In Bolivia, the switch to dollar currency has enabled economic agents to avoid the inflation tax, reduce transaction costs in the consumption process, and reduce political and exchange risks associated with holding domestic currency-denominated financial assets. The process of substitution in this sense has allowed agents to avoid a reduction in welfare. Holding foreign exchange allows the agents to economize in foreign trade in addition to reduced costs in consumption. In a macroeconomic sense, this suggests that the savings in transaction costs increase with the proportion of goods that are purchased in the domestic economy with foreign currency and with the degree of openness of the economy. We can expect the welfare gains from currency substitution to be higher the more advanced the economy is in the process of dollarization and the more open the economy is

to foreign trade. Bolivia appears to fall in this category, but must watch what happens to price levels of trading partners. Certainly more information is needed on the extent of impacts from the outside in order to formulate policy such as exchange rate policy.

Currency substitution appears to be a more efficient means of financial adaptation to inflation relative to financial indexation. Several arguments support this view as summarized by Rostowski (1992) and as evidence of irreversibility of substitution indicates in the case of Bolivia. The adaptation of indexation to unforeseen price changes can only take place discretely, whereas continual adaptation can take place with use of the dollar in the Bolivian case. Indexation also promotes price rigidity whereby relative prices react sluggishly to market forces. That is, prices lose some of their signaling role in the economy. A dollarized economy is less susceptible to an acceleration of inflation through supply shocks. Probably the only advantage indexation has relative to dollarization is that it does not induce seigniorage losses to governments, but even this advantage has to be weighted against losses of resources that have to be devoted to the expansion of the financial services sector.

Dollarization that is agent driven rather than government authorized would appear to be correlated with the expansion of the informal sector. A greater part of financial intermediation can be expected to take place outside the formal banking sector. Recourse to the informal sector is a means of reducing risks created by the effects of government policy. However, we would have to have information on the extent of informal activities and how they are related to efficiency gains to make a more detailed statement about this aspect of welfare analysis for the Bolivian case.

In the long run, nonauthorized dollarization in Bolivia is likely to induce monetary

discipline in the dollarized Bolivia as mentioned previously. In the short run, governments have the tendency to increase inflation to maintain a level of seigniorage receipts. In the long run, the cycle of increasing inflation and dollarization can only be broken by a sharp shift to monetary and fiscal discipline, or ever increasing dollarization to allow financial intermediation in the dollar.

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CHAPTER 5

FINAL REMARKS

Some final comments are needed with respect to the development and use of the combined methods of time domain time series analysis with bootstrap hypothesis testing. In the first essay, use of the bootstrap in evaluating the Augmented Dickey-Fuller test questioned the stylized facts of difficulty to reject the null of the existence of a unit root and low power associated with this type of tests. Moreover, computation of the contribution of squared bias and variance to total mean square error, which were based on the bootstrapped empirical distributions, may be pointing to the main problem of the Dickey-Fuller test in that it is not necessarily a problem of bad inference methods, but rather a problem of estimation methods.

Use of the bootstrap in the second essay revealed that Johansen's asymptotic critical values can be an acceptable first approximation in testing for cointegration in small samples. This result was also verified in the third essay, which produced a higher number of cointegrating relationships among variables. Similarly, once contamination is considered (which is essential), use of t-table values can be an acceptable first approximation in testing for the significance of parameters in a multivariate time series system. Except for one case (in second essay), in general a high t-statistic is associated with a low bootstrapped p-value.

The practice of bootstrapping proved also quite useful in a hidden way. Model estimation is not a one-time exercise, but rather a process in which problems are found and solved before the final model is produced. In this sense, a model in which many parameters

appear significant (or nonsignificant) by their *t*-statistic but not significant (or significant) by their bootstrapped *p*-values can be an indication of multicollinearity problems, non-white noise residuals, heteroskedasticity in the residuals, biased estimates due to contamination, the need to transform variables to avoid scale effects, or wrong number of leads and lags in the variables. These were some of the problems experienced during its application.

An essential feature of the bootstrap emphasized by Freedman and Peters is that the estimated model and its estimated parameters are the true model of the phenomenon of interest. This condition is difficult to achieve *a priori*, but this is the place where economic theory plays an important role. Economic theory will help not only in identifying the relevant variables, but also in their treatment as exogenous or endogenous, in explaining the possible sources of observed shocks, in model interpretation, and even in theory criticism.

One more benefit of the bootstrap is that it permits a better understanding of estimation methods, not only relative to their performance in terms of contribution of squared bias and variance to total mean square error, but also with respect to specific features in the estimation procedures. Johansen's maximum likelihood approach to cointegration is in essence an application of canonical correlation. The objective of this method is to find linear combinations within two sets of variables such that correlation of linear combinations between sets is the largest possible. Estimation of canonical correlation is performed by maximizing this correlation subject to fixing the variance of each linear combination to one. The feature of fixing variances to one translates to standardizing residuals in the bootstrap algorithms. This correction is essential in the context of error correction models, where the two sets of variables are just the levels and differences of one set of time series variables, one

representing the long-term and the other representing the short-term. This is why the estimated error correction model already contains the short-term, long-term, and short-term-long-term interactions. Random samples from residuals without this correction generate exploding simulated series in the bootstrap replications. That is, the bootstrap would produce biased parameter estimates at each replication. Understanding of this feature gives a hint of why there could be a tendency to biased parameter estimates when testing for unit roots with the Augmented Dickey-Fuller test. This test is a single equation version of Johansen's maximum likelihood approach, but where estimation is performed without fixing the variance of its short-term and long-term components.

APPENDICES

APPENDIX A

UNIT ROOT TESTING BY BOOTSTRAPPING
RESIDUALS FROM OLS ESTIMATION OF
THE ADF EQUATION.

```

ols.strap<-function(x, p, s){
  n_length(x)
  dx_diff(x)
  a_p+1
  b_n-1
  x1_x[a:b]
  nn_n-(p+1)
  y_uroot.ols(x,p)
  time_c(1:nn)
  res_y$res
  res.standardized_(res-mean(res))/sqrt(var(res))
  betas_matrix(0, ncol=p+3, nrow=s)
  std.dev_matrix(0, ncol=p+3, nrow=s)
  tau_rep(0, s)
  zeta_rep(0, s)
  lx_rep(0, nn+2)
  ndx_rep(0, nn+1)
  lx[1]_x[1]
  ndx[1]_dx[1]
  lx[2]_lx[1]+ndx[1]
  for(j in 1:s){
    nres_sample(res.standardized, nn,
      replace=TRUE)
    q_nn+1
    for(i in 2:q){
      ndx[i]_y$stable[1,]%*%t(c(1,i-1,lx[i],ndx[i-1])))+
      nres[i-1]
      lx[i+1]_lx[i]+ndx[i]
    }
    beta.hat_y$stable[1,]
    z_ols.beta(lx, p, beta.hat)
    betas[j,]_z$betas
    std.dev[j,]_z$std.dev
    tau[j]_z$tvalue
    zeta[j]_z$fstat
  }
  table_y$stable
  r2_y$r2
  Qstat_y$Qstat
  fstat_y$fstat
  von.neumann_y$von.neumann
  return(betas, std.dev, tau, zeta, table, r2, Qstat,
    fstat, von.neumann, res)
}

```

}

OLS ESTIMATION OF ADF EQUATION

```

uroot.ols<-function(x, p){
  n_length(x)
  a_p+1
  b_n-1
  x1_x[a:b]
  nn_n-(p+1)
  time_c(1:nn)
  xx_cbind(time, x1)
  dx_diff(x)
  m_length(dx)
  dx0_dx[a:m]
  for(i in 1:p){
    a_p-i+1
    b_m-i
    xx_cbind(xx, dx[a:b])
  }
  y_lsfit(xx, dx0)
  dy_ls.diag(y)
  res_y$residuals
  tss_sum((dx0-mean(dx0))^2)
  rss_u_sum(res^2)
  r2_(tss-rss.u)/tss
  tvalues_y$coef/dy$std.err
  table_rbind(y$coef, t(tvalues))
  xx_cbind(rep(1,nn), xx)
  w_solve(t(xx)%*%xx)
  A_matrix(0, nrow=2, ncol=p+3)
  A[1,2]_1
  A[2,3]_1
  bhat_y$coef
  f_t(A%*%bhat)%*%solve(A%*%w%*%t(A))%
  **/(A%*%bhat)
  fstat_(nn-p-3)*f/(2*rss.u)
  Qstat_Q.stat(res)
  von.neumann_neumann(res)
  return(table, fstat, r2, Qstat, von.neumann, res))
}

```

UNIT ROOT TESTING BY BOOTSTRAPPING
RESIDUALS FROM REG ESTIMATION OF
THE ADF EQUATION

```

nreg.strap<-function(x, p, s){
  n_length(x)
  dx_diff(x)
}

```

```

a_p+1
b_n-1
x1_x[a:b]
nn_n-(p+1)
y_uroot.rreg(x,p)
time_c(1:nn)
rres_y$rrres
res_y$res
res.standardized_(res-mean(res))/sqrt(var(res))
rbetas_matrix(0, ncol=p+3, nrow=s)
rstd.dev_matrix(0, ncol=p+3, nrow=s)
rtau_rep(0, s)
rzeta_rep(0, s)
lx_rep(0, nn+2)
ndx_rep(0, nn+1)
lx[1]_x[1]
ndx[1]_dx[1]
lx[2]_lx[1]+ndx[1]
for(j in 1:s){
  nres_sample(res.standardized, nn,
  replace=TRUE)
  q_nn+1
  for(i in 2:q){
    ndx[i]_y$rtable[1,]%*%t(c(1,i-1,lx[i],ndx[i-1]))
    +nres[i-1]
    lx[i+1]_lx[i]+ndx[i]
  }
  beta.hat_y$rtable[1,]
  rz_rreg.beta(lx, p, beta.hat)
  rbetas[j,]_rz$betas
  rstd.dev[j,]_rz$std.dev
  rtau[j]_rz$tau
  rzeta[j]_rz$zeta
}
weights_y$weights
rtable_y$rtable
rr2_y$rr2
Qrstat_y$Qrstat
rfstat_y$rfstat
rvon.neumann_y$rvon.neumann
return(rbetas, rstd.dev, rtau, rzeta, rtable, rr2,
Qrstat,
rfstat, weights, rvon.neumann, rres, res)
}

```

RREG ESTIMATION OF ADF EQUATION

```

uroot.rreg<-function(x, p){
  n_length(x)
  a_p+1
  b_n-1

```

```

  x1_x[a:b]
  nn_n-(p+1)
  time_c(1:nn)
  xx_cbind(time, x1)
  dx_diff(x)
  m_length(dx)
  dx0_dx[a:m]
  for(i in 1:p){
    a_p-i+1
    b_m-i
    xx_cbind(xx, dx[a:b])
  }
  ry_rreg(xx, dx0)
  weights_sqrt(ry$w)
  res_ry$residuals
  rres_weights*rres
  rtss_sum((dx0*weights-mean(dx0*weights))^2)
  rrss.u_sum(rres^2)
  rr2_(rtss-rrss.u)/rtss
  rs2_sum(rres^2)/(nn-p-3)
  v_diag(weights)
  xx_cbind(rep(1,nn), xx)
  w_solve(t(xx)%*%v%*%xxx)
  rcovar_rs2*w
  rvar_diag(rcovar)
  rtvalues_ry$coef/sqrt(rvar)
  rtable_rbind(ry$coef, rtvalues)
  Qrstat_Q.stat(rres)
  rvon.neumann_neumann(rres)
  A_matrix(0, nrow=2, ncol=p+3)
  A[1,2]_1
  A[2,3]_1
  bhat_ry$coef
  rf_t(A%*%bhat)%*%solve(A%*%w%*%t(A))
  %*%(A%*%bhat)
  rfstat_(nn-p-3)*rf/(2*rrss.u)
  return(rtable, rfstat, rr2, Qrstat, rvon.neumann,
  weights, rres, res)
}

```

DISTRIBUTION OF TAU BY BOOTSTRAPPING RESIDUALS FROM A TS PROCESS

```

ts.boot<-function(x,p,s){
  n_length(x)
  a_p+1
  x0_x[a:n]
  nn_n-p
  xx_c(1:nn)
  for(i in 1:p){

```

```

a_p-i+1
b_n-i
xx_cbind(xx, x[a:b])
}
y_lsfit(xx, x0)
dy_ls.diag(y)
res_y$residuals
tss_sum((x0-mean(x0))^2)
rss_sum(res^2)
r2_(tss-rss)/tss
tvalues_y$coef/dy$std.err
table_rbind(y$coef, t(tvalues))
Qstat_Q.stat(res)
von.neumann_neumann(res)
res.standardized_(res-mean(res))/sqrt(var(res))
tau_rep(0, s)
zeta_rep(0, s)
nx_rep(0, nn+2)
nx[1]_x[1]
nx[2]_x[2]
for(j in 1:s){
  nres_sample(res.standardized, nn,
  replace=TRUE)
  q_nn+2
  for(i in 3:q){
    nx[i]_table[1,]%%*(1,i-2,nx[i-1],nx[i-2])+nres[
    i-2]
  }
  z_uroot.ols.aux(nx, 1)
  tau[j]_z$tvalue
  zeta[j]_z$fstat
}
return(tau,zeta,table,r2,Qstat,von.neumann,res)
}

```

APPENDIX B

ML ESTIMATION OF A GAUSSIAN
COINTEGRATED VAR(p) PROCESS

```

koint<-function(x, p, r){
  dx_diff(x)
  n_nrow(dx)
  m_ncol(x)
  a_p
  b_n
  yt_dx[a:b.]
  a_l
  b_nrow(x)-p
  ypt_x[a:b.]
  nr_n-p+1
  nc_m*(p-1)
  z1_matrix(0, nrow=nr, ncol=nc)
  for(i in 1:p-1){
    for(j in 1:m){
      a_p-i
      b_n-i
      k_i*m-(m-j)
      z1[k_l, dx[j]][a:b]
    }
  }
  zt_cbind(1,z1)
  z_t(zt) # RHS variables
  y_t(yt) # LHS variables
  yp_t(ypt)
  w_solve(z%*%t(z))
  hat_diag(ncol(z))-t(z)%*%w%*%z
  ro_y%*%hat
  r1_yp%*%hat
  n_nrow(x)-p
  soo_(ro%*%t(ro))/n
  s11_(r1%*%t(r1))/n
  slo_(r1%*%t(ro))/n
  sol1_(ro%*%t(r1))/n
  chol.s11_chol(s11)
  g_t(solve(chol.s11))
  gsg_g%*%s1o%*%solve(soo)%*%so1%*%t(g)
  eigen.gsg_eigen(gsg)
  coint.vec_matrix(0, ncol=r, nrow=m)
  for(i in 1:r){
    coint.vec[i]_eigen.gsg$Vectors[i]
  }
  eigen_l-eigen.gsg$values
  log.eigen_t(log(eigen))

```

```

max.eigen.test_matrix(0, nrow=1, ncol=m)
trace.test_matrix(0, nrow=1, ncol=m)
for(i in 1:m){
  max.eigen.test[i]_n*log.eigen[i]
  le_0
  for(j in i:m){
    le_le+log.eigen[j]
  }
  trace.test[i]_n*le
}
Chat_t(coint.vec)%*%g
Hhat_-sol%*%t(Chat)%*%solve(Chat%*%s11
%*%t(Chat))
Dhat_(y+Hhat%*%Chat%*%yp)%*%t(z)%*%w
U_y-Dhat%*%z+Hhat%*%Chat%*%yp
Uhat_(U%*%t(U))/n
A0_Dhat[,1]
mm_m+1
A1_diag(m)+Dhat[,2:mm]
A2_-Hhat%*%Chat-Dhat[,2:mm]
d1_(z%*%t(z))/n
d2_(z%*%t(yp)%*%t(Chat))/n
d3_(Chat%*%yp%*%t(z))/n
d4_(Chat%*%yp%*%t(yp)%*%t(Chat))/n
Mhat2_rbind(cbind(d1, d2), cbind(d3, d4))
nn_m*p-m+1
h1_diag(nn)
h4_t(Chat)
h2_matrix(0, nrow=nrow(h1), ncol=ncol(h4))
h3_matrix(0, nrow=nrow(h4), ncol=ncol(h1))
left_rbind(cbind(h1, h2), cbind(h3, h4))
q1_h1
q4_Chat
q2_matrix(0, nrow=nrow(q1), ncol=ncol(q4))
q3_matrix(0, nrow=nrow(q4), ncol=ncol(q1))
right_rbind(cbind(q1, q2), cbind(q3, q4))
lio_left%*%solve(Mhat2)%*%right
Col_kronecker(lio,Uhat)
std.err1_sqrt(diag(Col1))/sqrt(n)
std.err1_matrix(std.err1, nrow=m, ncol=m*p+1)
l_diag(m)
W_rbind(cbind(1,-1),cbind(matrix(0,nrow=m,ncol=
l=m),I))
a_m+1
b_m*(m*p+1)
Col_Col1[a:b,a:b]

```

```

Co2_(kronecker(t(W),I))%*%Co1%*%(kroneck
er(W,I))
std.err2_sqrt(diag(Co2))/sqrt(n)
std.err2_matrix(std.err2, nrow=m, ncol=m*p)
return(y, yp, coint.vec, g, s11, Chat, Hhat, Dhat,
Uhat, U, max.eigen.test, trace.test,
Co1, z, ro, r1, std.err1, std.err2, A0, A1, A2)
}

```

CRITICAL VALUES FOR THE TRACE TEST
AND MAXIMUM EIGENVALUE TEST BY
BOOTSTRAPPING RESIDUALS FROM AN
ESTIMATED ECM AND FOLLOWING
JOHANSEN'S ASYMPTOTIC PROCEDURE.

```

johansen.bootstrap<-function(b){
s_73
ss_s+1
trace_matrix(0, nrow=b, ncol=5)
max_matrix(0, nrow=b, ncol=5)
y_dkoint(mus,2,5)
res_y$U
x_matrix(res,nrow=365, ncol=1)
x_(x[,1]-mean(x[,1]))/sqrt(var(x[,1]))
# DIMENSION 1
z1_rep(0,ss)
for(j in 1:b){
a1_sample(x, s, replace=T)
saz_0
szz_0
dyn.load("boot1.o")
storage.mode(saz)_"single"
storage.mode(szz)_"single"
tr_.Fortran("boot1",
as.single(a1),as.integer(s),as.single(z1),
as.integer(ss),saz=saz,szz=szz)
saz_tr$saz
szz_tr$szz
A_(saz)%*%solve(szz)%*%t(saz)
trace[j,1]_sum(diag(A))
B_eigen(A)
max[j,1]_max(B$values)
}
# DIMENSION 2
z1_rep(0,ss)
z2_rep(0,ss)
for(j in 1:b){
a1_sample(x, s, replace=T)
a2_sample(x, s, replace=T)
saz_matrix(0, nrow=2, ncol=2)
szz_matrix(0, nrow=2, ncol=2)

```

```

dyn.load("boot2.o")
storage.mode(saz)_"single"
storage.mode(szz)_"single"
tr_.Fortran("boot2",
as.single(a1),as.single(a2),as.integer(s),
as.single(z1),as.single(z2),as.integer(ss),
saz=saz,szz=szz)
saz_tr$saz
szz_tr$szz
A_(saz)%*%solve(szz)%*%t(saz)
trace[j,2]_sum(diag(A))
B_eigen(A)
max[j,2]_max(B$values)
}
# DIMENSION 3
z1_rep(0,ss)
z2_rep(0,ss)
z3_rep(0,ss)
for(j in 1:b){
a1_sample(x, s, replace=T)
a2_sample(x, s, replace=T)
a3_sample(x, s, replace=T)
saz_matrix(0, nrow=3, ncol=3)
szz_matrix(0, nrow=3, ncol=3)
dyn.load("boot3.o")
storage.mode(saz)_"single"
storage.mode(szz)_"single"
tr_.Fortran("boot3",
as.single(a1),as.single(a2),as.single(a3),
as.integer(s),as.single(z1),as.single(z2),
as.single(z3),as.integer(ss),saz=saz,szz=szz)
saz_tr$saz
szz_tr$szz
A_(saz)%*%solve(szz)%*%t(saz)
trace[j,3]_sum(diag(A))
B_eigen(A)
max[j,3]_max(B$values)
}
# DIMENSION 4
z1_rep(0,ss)
z2_rep(0,ss)
z3_rep(0,ss)
z4_rep(0,ss)
for(j in 1:b){
a1_sample(x, s, replace=T)
a2_sample(x, s, replace=T)
a3_sample(x, s, replace=T)
a4_sample(x, s, replace=T)
saz_matrix(0, nrow=4, ncol=4)
szz_matrix(0, nrow=4, ncol=4)

```

```

dyn.load("boot4.o")
storage.mode(saz) "single"
storage.mode(szz) "single"
tr_Fortran("boot4",
  as.single(a1),as.single(a2),as.single(a3),
  as.single(a4),as.integer(s),as.single(z1),
  as.single(z2),as.single(z3),as.single(z4),
  as.integer(ss),saz=saz,szz=szz)
saz_tr$ saz
szz_tr$ szz
A_(saz)%*%solve(szz)%*%t(saz)
trace[j,4]_sum(diag(A))
B_eigen(A)
max[j,4]_max(B$values)
}
# DIMENSION 5
z1_rep(0,ss)
z2_rep(0,ss)
z3_rep(0,ss)
z4_rep(0,ss)
z5_rep(0,ss)
for(j in 1:b){
  a1_sample(x, s, replace=T)
  a2_sample(x, s, replace=T)
  a3_sample(x, s, replace=T)
  a4_sample(x, s, replace=T)
  a5_sample(x, s, replace=T)
  saz_matrix(0, nrow=5, ncol=5)
  szz_matrix(0, nrow=5, ncol=5)
  dyn.load("boot5.o")
  storage.mode(saz) "single"
  storage.mode(szz) "single"
  tr_Fortran("boot5",
    as.single(a1),as.single(a2),as.single(a3),
    as.single(a4),as.single(a5),as.integer(s),
    as.single(z1),as.single(z2),as.single(z3),
    as.single(z4),as.single(z5),as.integer(ss),
    saz=saz,szz=szz)
  saz_tr$ saz
  szz_tr$ szz
  A_(saz)%*%solve(szz)%*%t(saz)
  trace[j,5]_sum(diag(A))
  B_eigen(A)
  max[j,5]_max(B$values)
}
return(trace, max)
}

```

FORTRAN SUBROUTINES BOOT2 TO BOOT5 FOLLOW THE SAME STRUCTURE OF SUBROUTINE BOOT1.

```

subroutine boot1(a1,s,z1,ss,saz,szz)
  integer s,ss,i,j,k,n
  real a1(s),z1(ss),salz1,saz,szz,w1
  do i = 1,s
    z1(i+1) = z1(i)+a1(i)
  end do
  w1 = z1(1)
  do k = 2,s
    w1 = w1 + z1(k)
  end do
  do n = 1,s
    z1(n) = z1(n) - w1/s
  end do
  salz1 = a1(1)*z1(1)
  szl1 = z1(1)*z1(1)
  do j = 2,ss
    salz1 = salz1 + a1(j)*z1(j)
  end do
  szl1 = szl1 + z1(j)*z1(j)
  end do
  saz = salz1
  szz = szl1
end

```

BOOTSTRAPING A COINTEGRATED VAR(p) PROCESS

```

koint.strap<-function(x, p, r, s){
  islm_koint(x, p, r)
  A0_islm$A0[,1]
  A1_islm$A1
  A2_islm$A2
  U_islm$U
  Chat1_matrix(0, nrow=s, ncol=5)
  Hhat1_matrix(0, nrow=s, ncol=5)
  Dhat1_matrix(0, nrow=s, ncol=6)
  Dhat2_matrix(0, nrow=s, ncol=6)
  Dhat3_matrix(0, nrow=s, ncol=6)
  Dhat4_matrix(0, nrow=s, ncol=6)
  Dhat5_matrix(0, nrow=s, ncol=6)
  se1_matrix(0, nrow=s, ncol=10)
  se2_matrix(0, nrow=s, ncol=10)
  se3_matrix(0, nrow=s, ncol=10)
  se4_matrix(0, nrow=s, ncol=10)
  se5_matrix(0, nrow=s, ncol=10)
  sep1_matrix(0, nrow=s, ncol=11)
  sep2_matrix(0, nrow=s, ncol=11)
  sep3_matrix(0, nrow=s, ncol=11)
}

```

```

sep4_matrix(0, nrow=s, ncol=11)
sep5_matrix(0, nrow=s, ncol=11)
x1_rep(0, nrow(x))
x2_rep(0, nrow(x))
x3_rep(0, nrow(x))
x4_rep(0, nrow(x))
x5_rep(0, nrow(x))
x1[1]_x[1,1]
x1[2]_x[1][1]+0.01
x2[1]_x[1,2]
x2[2]_x[2][1]+0.01
x3[1]_x[1,3]
x3[2]_x[3][1]+0.01
x4[1]_x[1,4]
x4[2]_x[4][1]+0.01
x5[1]_x[1,5]
x5[2]_x[5][1]+0.01
for(i in 1:s){
  res1_sample(U[1,], length(U[1,]), replace=T)
  res2_sample(U[2,], length(U[2,]), replace=T)
  res3_sample(U[3,], length(U[3,]), replace=T)
  res4_sample(U[4,], length(U[4,]), replace=T)
  res5_sample(U[5,], length(U[5,]), replace=T)
  nres1_(res1-mean(res1))/sqrt(var(res1))
  nres2_(res2-mean(res2))/sqrt(var(res2))
  nres3_(res3-mean(res3))/sqrt(var(res3))
  nres4_(res4-mean(res4))/sqrt(var(res4))
  nres5_(res5-mean(res5))/sqrt(var(res5))
  dyn.load("loop.o")
  nrowx_nrow(x)
  nrowa_length(A0)
  ncola1_ncol(A1)
  ncola2_ncol(A2)
  lres1_length(res1)
  lres2_length(res2)
  lres3_length(res3)
  lres4_length(res4)
  lres5_length(res5)
  storage.mode(x1)_"single"
  storage.mode(x2)_"single"
  storage.mode(x3)_"single"
  storage.mode(x4)_"single"
  storage.mode(x5)_"single"
  zfort_.Fortran("loop",
    x1=x1,x2=x2,x3=x3,x4=x4,x5=x5,
    as.integer(nrowx),as.integer(A0),
    as.integer(A1),as.integer(A2),
    as.integer(nrowa),as.integer(ncola1),
    as.integer(ncola2),as.integer(nres1),
    as.integer(nres2),as.integer(nres3),
    as.integer(nres4),as.integer(nres5),
    as.integer(lres1),as.integer(lres2),
    as.integer(lres3),as.integer(lres4),
    as.integer(lres5))
  x1_zfort$x1
  x2_zfort$x2
  x3_zfort$x3
  x4_zfort$x4
  x5_zfort$x5
  x_cbind(x1,x2,x3,x4,x5)
  dx_diff(x)
  n_nrow(dx)
  m_ncol(x)
  a_p
  b_n
  yt_dx[a:b.]
  a_l
  b_nrow(x)-p
  ypt_x[a:b.]
  nr_n-p+1
  nc_m*(p-1)
  z1_matrix(0, nrow=nr, ncol=nc)
  for(i in 1:p-1){
    for(j in 1:m){
      a_p-i
      b_n-i
      k_i*m-(m-j)
      z1[,k]_dx[,j][a:b]
    }
  }
  zt_cbind(1,z1)
  z_t(zt)
  y_t(yt)
  yp_t(ypt)
  w_solve(z%*%t(z))
  hat_diag(ncol(z))-t(z)%*%w%*%z
  ro_y%*%hat
  r1_yp%*%hat
  n_nrow(x)-p
  soo_(ro%*%t(ro))/n
  s11_(r1%*%t(r1))/n
  slo_(r1%*%t(ro))/n
  sol_(ro%*%t(r1))/n
  chol.s11_chol(s11)
  g_t(solve(chol.s11))
  gsg_g%*%s1o%*%solve(soo)%*%so1%*%t(g)
  eigen.gsg_eigen(gsg)
  coint.vec_matrix(0, ncol=r, nrow=m)
  for(i in 1:r){
    coint.vec[,i]_eigen.gsg$vectors[,i]
  }

```



```

}
Chat_t(coint.vec)%*%g
Hhat_-so1%*%(Chat)%*%solve(Chat%*%s11
%*%(Chat))
Dhat_(y+Hhat%*%Chat%*%yp)%*%t(z)%*%w
U_y-Dhat%*%z+Hhat%*%Chat%*%yp
Uhat_(U%*%(U))/n
d1_(z%*%t(z))/n
d2_(z%*%t(yp)%*%t(Chat))/n
d3_(Chat%*%yp)%*%t(z))/n
d4_(Chat%*%yp)%*%t(yp)%*%t(Chat))/n
Mhat2_rbind(cbind(d1,d2), cbind(d3,d4))
nn_m*p-m+1
h1_diag(nn)
h4_t(Chat)
h2_matrix(0, nrow=nrow(h1), ncol=ncol(h4))
h3_matrix(0, nrow=nrow(h4), ncol=ncol(h1))
left_rbind(cbind(h1,h2), cbind(h3,h4))
q1_h1
q4_Chat
q2_matrix(0, nrow=nrow(q1), ncol=ncol(q4))
q3_matrix(0, nrow=nrow(q4), ncol=ncol(q1))
right_rbind(cbind(q1,q2), cbind(q3,q4))
lio_left%*%solve(Mhat2)%*%right
Col_kronecker(lio,Uhat)
std.err1_sqrt(diag(Col1))/sqrt(n)
std.err1_matrix(std.err1, nrow=m, ncol=m*p+1)
l_diag(m)
W_rbind(cbind(1,l),cbind(matrix(0,nrow=m,ncol=m),l))
a_m+1
b_m*(m*p+1)
Col_Col[a:b,a:b]
Co2_(kronecker(t(W,l))%*%Co1%*%(kronecker(W,l))
std.err2_sqrt(diag(Co2))/sqrt(n)
std.err2_matrix(std.err2, nrow=m, ncol=m*p)
Chat1[i,] Chat[1,]
Hhat1[i,] Hhat[1,]
Dhat1[i,] Dhat[1,]
Dhat2[i,] Dhat[2,]
Dhat3[i,] Dhat[3,]
Dhat4[i,] Dhat[4,]
Dhat5[i,] Dhat[5,]
sep1[i,]_std.err1[1,]
sep2[i,]_std.err1[2,]
sep3[i,]_std.err1[3,]
sep4[i,]_std.err1[4,]
sep5[i,]_std.err1[5,]
se1[i,]_std.err2[1,]

```

```

se2[i,]_std.err2[2,]
se3[i,]_std.err2[3,]
se4[i,]_std.err2[4,]
se5[i,]_std.err2[5,]
}
return( Chat1, Hhat1,
        Dhat1, Dhat2, Dhat3, Dhat4, Dhat5,
        se1, se2, se3, se4, se5,
        sep1, sep2, sep3, sep4, sep5)
}

```

SUBROUTINE LOOP

```

subroutine loop(x1,x2,x3,x4,x5,nrowx,
+ A0,A1,A2,nrowa,ncola1,ncola2,nres1,
+ nres1,nres2,nres3,nres4,nres5,lres1,
+ lres2,lres3,lres4,lres5,j,k
+ integer nrowx,nrowa,ncola1,ncola2,
+ lres1,lres2,lres3,lres4,lres5,j,k
+ real A0(nrowa),A1(nrowa,ncola1),
+ A2(nrowa,ncola2),nres1(lres1),
+ nres2(lres2),nres3(lres3),nres4(lres4),
+ nres5(lres5),x1(nrowx),x2(nrowx),
+ x3(nrowx),x4(nrowx),x5(nrowx),
+ temp1(10),temp2(10)
do j = 3,nrowx
temp1(1) = x1(j-1)
temp1(2) = x2(j-1)
temp1(3) = x3(j-1)
temp1(4) = x4(j-1)
temp1(5) = x5(j-1)
temp2(1) = x1(j-2)
temp2(2) = x2(j-2)
temp2(3) = x3(j-2)
temp2(4) = x4(j-2)
temp2(5) = x5(j-2)
x1(j) = A0(1) + nres1(j-2)
x2(j) = A0(2) + nres2(j-2)
x3(j) = A0(3) + nres3(j-2)
x4(j) = A0(4) + nres4(j-2)
x5(j) = A0(5) + nres5(j-2)
do k = 1,5
x1(j) = x1(j) + A1(1,k)*temp1(k) +
+ A2(1,k)*temp2(k)
x2(j) = x2(j) + A1(2,k)*temp1(k) +
+ A2(2,k)*temp2(k)
x3(j) = x3(j) + A1(3,k)*temp1(k) +
+ A2(3,k)*temp2(k)
x4(j) = x4(j) + A1(4,k)*temp1(k) +
+ A2(4,k)*temp2(k)
x5(j) = x5(j) + A1(5,k)*temp1(k) +

```

```
      A2(5,k)*temp2(k)
    end do
  end do
end
```

APPENDIX C

Table C1. Significance of HC_{1j} on the GDP Equation

H_0	t-value	Critical values				mean	var
		90%	95%	97.5%	99%		
$HC_{11} = 0$	3.533	1.798	2.137	2.476	2.911	0.880	0.457
$HC_{12} = 0$	3.533	1.799	2.106	2.459	2.945	0.873	0.440
$HC_{13} = 0$	3.533	1.781	2.100	2.473	2.872	0.861	0.429
$HC_{14} = 0$	3.533	1.757	2.086	2.453	2.885	0.861	0.429
$HC_{15} = 0$	3.533	1.770	2.060	2.474	2.877	0.862	0.427

Table C2. Significance of HC_{2j} on the Money Equation

H_0	t-value	Critical values				mean	var
		90%	95%	97.5%	99%		
$HC_{21} = 0$	0.908	1.816	2.152	2.482	2.905	0.891	0.444
$HC_{22} = 0$	0.908	1.818	2.163	2.486	2.853	0.895	0.444
$HC_{23} = 0$	0.908	1.820	2.134	2.450	2.869	0.888	0.442
$HC_{24} = 0$	0.908	1.809	2.143	2.459	2.877	0.888	0.443
$HC_{25} = 0$	0.908	1.808	2.149	2.492	2.862	0.889	0.441

Table C3. Significance of HC_{3j} on the Inflation Equation

H_0	t-value	Critical values				mean	var
		90%	95%	97.5%	99%		
$HC_{31} = 0$	3.414	1.916	2.326	2.651	3.049	0.925	0.523
$HC_{32} = 0$	3.414	1.888	2.245	2.644	2.960	0.913	0.499
$HC_{33} = 0$	3.414	1.855	2.222	2.520	2.829	0.890	0.474
$HC_{34} = 0$	3.414	1.854	2.197	2.509	2.876	0.885	0.469
$HC_{35} = 0$	3.414	1.834	2.180	2.498	2.781	0.876	0.458

Table C4. Significance of HC_{4j} on the Interest Rate Equation

H_0	t-value	Critical values				mean	var
		90%	95%	97.5%	99%		
$HC_{41} = 0$	3.516	1.870	2.249	2.542	2.956	0.912	0.474
$HC_{42} = 0$	3.516	1.853	2.223	2.518	2.995	0.911	0.468
$HC_{43} = 0$	3.516	1.806	2.195	2.528	2.853	0.893	0.449
$HC_{44} = 0$	3.516	1.802	2.203	2.490	2.861	0.891	0.446
$HC_{45} = 0$	3.516	1.815	2.213	2.491	2.887	0.896	0.449

Table C5. Significance of HC_{5j} on the Budget Deficit Equation

H_0	t-value	Critical values				mean	var
		90%	95%	97.5%	99%		
$HC_{51} = 0$	3.952	10.968	14.252	18.944	26.792	5.314	33.569
$HC_{52} = 0$	3.952	10.696	13.670	17.537	25.432	5.116	29.530
$HC_{53} = 0$	3.952	3.908	4.744	5.534	6.835	1.871	2.308
$HC_{54} = 0$	3.952	3.775	4.697	5.418	6.770	1.747	2.210
$HC_{55} = 0$	3.952	4.918	6.168	7.591	8.934	2.375	4.600

APPENDIX D

Table D1. Significance of B_{1j} on the GDP Equation

H_0	t-value	Critical values					var
		90%	95%	97.5%	99%	mean	
$B_{11} = 0$	1.968	1.788	2.103	2.427	2.647	0.857	0.418
$B_{12} = 0$	1.329	1.727	2.042	2.375	2.784	0.838	0.408
$B_{13} = 0$	1.968	1.698	2.016	2.293	2.712	0.835	0.394
$B_{14} = 0$	2.538	1.789	2.113	2.469	2.823	0.863	0.434
$B_{15} = 0$	0.037	1.711	2.083	2.358	2.825	0.842	0.411

Table D2. Significance of B_{2j} on the Money Equation

H_0	t-value	Critical values					var
		90%	95%	97.5%	99%	mean	
$B_{21} = 0$	1.371	1.774	2.078	2.412	2.932	0.848	0.469
$B_{22} = 0$	0.104	1.777	2.081	2.365	2.722	0.847	0.415
$B_{23} = 0$	0.269	1.719	2.097	2.386	2.775	0.840	0.406
$B_{24} = 0$	0.437	1.757	2.093	2.451	2.858	0.853	0.428
$B_{25} = 0$	1.786	1.745	2.166	2.506	2.775	0.859	0.419

Table D3. Significance of B_{3j} on the Inflation Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{31} = 0$	0.382	1.724	2.107	2.378	2.726	0.849	0.401	
$B_{32} = 0$	1.010	1.796	2.162	2.436	2.779	0.861	0.436	
$B_{33} = 0$	4.983	1.696	2.072	2.407	2.731	0.823	0.397	
$B_{34} = 0$	0.451	1.741	2.084	2.434	2.791	0.826	0.422	
$B_{35} = 0$	2.322	1.762	2.095	2.453	2.751	0.848	0.419	

Table D4. Significance of B_{4j} on the Interest Rate Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{41} = 0$	1.357	1.695	2.102	2.422	2.725	0.832	0.414	
$B_{42} = 0$	0.189	1.752	2.079	2.360	2.772	0.860	0.415	
$B_{43} = 0$	1.060	1.717	2.051	2.323	2.624	0.832	0.401	
$B_{44} = 0$	0.415	1.689	2.011	2.366	2.748	0.839	0.398	
$B_{45} = 0$	3.434	1.723	2.063	2.368	2.792	0.845	0.414	

Table D5. Significance of B_{5j} on the Budget Deficit Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{51} = 0$	1.047	2.092	2.489	2.906	3.289	0.978	0.594	
$B_{52} = 0$	0.384	2.019	2.397	2.787	3.193	0.972	0.552	
$B_{53} = 0$	2.099	2.196	2.695	2.005	3.612	1.059	0.688	
$B_{54} = 0$	1.912	1.870	2.259	2.736	3.125	0.912	0.510	
$B_{55} = 0$	0.384	1.955	2.352	2.764	3.200	0.946	0.537	

APPENDIX E

Table E1. Significance of HC_{1j} on the Dollarization Equation

H_0	t-value	Critical values					
		90%	95%	97.5%	99%	mean	var
$HC_{11} = 0$	5.646	2.559	3.098	3.492	4.123	1.264	1.041
$HC_{12} = 0$	1.356	2.103	2.543	2.991	3.512	1.028	0.641
$HC_{13} = 0$	3.845	2.504	2.927	3.290	3.857	1.211	0.842
$HC_{14} = 0$	2.652	2.248	2.617	2.985	3.548	1.078	0.708
$HC_{15} = 0$	0.121	2.121	2.589	2.945	3.184	1.001	0.601

Table E2. Significance of HC_{2j} on the Inflation Equation

H_0	t-value	Critical values					
		90%	95%	97.5%	99%	mean	var
$HC_{21} = 0$	0.025	2.305	2.997	3.462	4.259	1.174	0.918
$HC_{22} = 0$	4.556	2.215	2.642	2.991	3.616	1.062	0.685
$HC_{23} = 0$	3.198	2.315	2.732	3.154	3.686	1.162	0.789
$HC_{24} = 0$	3.005	2.022	2.375	2.840	3.145	0.976	0.565
$HC_{25} = 0$	1.893	1.846	2.186	2.456	2.830	0.873	0.447

Table E3. Significance of HC_{3j} on the Dollar Deposits Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$HC_{31} = 0$	0.617	2.128	2.539	2.911	3.419	1.044	0.605	
$HC_{32} = 0$	1.023	2.003	2.368	2.789	3.118	0.911	0.520	
$HC_{33} = 0$	1.139	1.953	2.326	2.879	3.352	0.989	0.559	
$HC_{34} = 0$	0.560	2.133	2.486	2.830	3.323	1.012	0.605	
$HC_{35} = 0$	0.002	1.964	2.337	2.609	2.909	0.939	0.499	

Table E4. Significance of HC_{4j} on the Boliviano Deposits Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$HC_{41} = 0$	0.939	2.088	2.461	2.298	3.452	1.019	0.671	
$HC_{42} = 0$	3.128	1.948	2.362	2.810	3.201	0.963	0.562	
$HC_{43} = 0$	3.641	2.223	2.735	2.183	3.581	1.080	0.733	
$HC_{44} = 0$	3.942	2.054	2.472	2.735	3.017	0.970	0.559	
$HC_{45} = 0$	2.177	1.894	2.248	2.580	3.079	0.920	0.513	

Table E5. Significance of HC_{5j} on the Depreciation Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$HC_{51} = 0$	1.754	2.018	2.404	2.754	3.383	0.979	0.582	
$HC_{52} = 0$	2.628	1.942	2.443	2.784	3.245	0.950	0.561	
$HC_{53} = 0$	0.543	2.063	2.498	2.824	3.312	0.992	0.586	
$HC_{54} = 0$	0.899	2.077	2.460	2.803	3.430	1.015	0.600	
$HC_{55} = 0$	6.419	2.122	2.480	2.881	3.213	1.020	0.611	

APPENDIX F

Table F1. Significance of B_{1j} on the Dollarization Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{11} = 0$	2.384	2.214	2.564	2.862	3.158	1.008	0.612	
$B_{12} = 0$	0.019	1.881	2.353	2.643	3.023	0.949	0.509	
$B_{13} = 0$	1.608	2.065	2.497	2.755	3.058	1.014	0.572	
$B_{14} = 0$	0.293	1.866	2.242	2.621	3.016	0.919	0.497	
$B_{15} = 0$	2.526	1.929	2.297	2.591	3.051	0.940	0.509	

Table F2. Significance of B_{2j} on the Inflation Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{21} = 0$	0.516	1.843	2.297	2.523	2.770	0.900	0.478	
$B_{22} = 0$	2.710	1.776	2.136	2.428	2.873	0.833	0.445	
$B_{23} = 0$	3.145	1.990	2.387	2.868	2.413	0.971	0.596	
$B_{24} = 0$	1.325	1.886	2.187	2.514	2.836	0.905	0.461	
$B_{25} = 0$	2.157	1.761	2.088	2.381	2.710	0.862	0.414	

Table F3. Significance of B_{3j} on the Dollar Deposits Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{31} = 0$	0.212	1.785	2.138	2.503	2.932	0.882	0.448	
$B_{32} = 0$	0.046	1.734	2.180	2.444	2.628	0.875	0.410	
$B_{33} = 0$	3.225	1.888	2.286	2.622	2.971	0.942	0.487	
$B_{34} = 0$	0.115	1.923	2.314	2.722	3.056	0.939	0.511	
$B_{35} = 0$	0.004	1.901	2.230	2.517	3.033	0.897	0.492	

Table F4. Significance of B_{4j} on the Boliviano deposits Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{41} = 0$	0.520	1.844	2.173	2.551	3.047	0.922	0.494	
$B_{42} = 0$	3.411	1.739	2.094	2.366	2.813	0.858	0.418	
$B_{43} = 0$	0.448	2.005	2.380	2.745	3.137	0.954	0.526	
$B_{44} = 0$	3.260	1.914	2.293	2.614	2.798	0.922	0.491	
$B_{45} = 0$	1.245	1.747	2.085	2.461	2.882	0.880	0.427	

Table F5. Significance of B_{5j} on the Depreciation Equation

H_0	t-value	Critical values					mean	var
		90%	95%	97.5%	99%			
$B_{51} = 0$	0.159	1.948	2.374	2.688	3.137	0.952	0.537	
$B_{52} = 0$	1.676	1.780	2.107	2.475	2.938	0.875	0.431	
$B_{53} = 0$	1.677	1.909	2.296	2.708	3.132	0.946	0.503	
$B_{54} = 0$	1.220	1.899	2.154	2.500	2.875	0.915	0.470	
$B_{55} = 0$	0.590	1.883	2.259	2.660	2.949	0.917	0.488	

CURRICULUM VITAE

Gover Barja Daza
(September, 1995)

CAREER OBJECTIVE:

Policy formulation for sectorial economic, social and institutional development.

Special areas of interest: Economics and statistics research, forecasting, feasibility studies, project management.

EDUCATION:

BS in Economics, Bolivian Catholic University, La Paz, Bolivia, 1983. Project Analysis Program, Arthur D. Little, Boston, Massachusetts, 1986. First place award.

MS in Statistics, Utah State University, Logan, Utah, 1994. PhD in Economics, Utah State University, Logan, Utah (expected December, 1995). Phi Kappa Phi Honor Society, Dean's List Student, John Seymour Memorial Scholarship, President of the Economics Graduate Students.

EXPERIENCE:

ANALYST SPECIALIST IN ECONOMIC POLICY, Bolivian Government's Unit of Economic Policy Analysis (UDAPE), La Paz, Bolivia (two occasions: 1986 and 1989). Team work in data gathering, economic analysis, writing and document presentation to government institutions and international organizations.

FINANCE MANAGER, International Pipeline Services of Bolivia (INTERPIPE), Santa Cruz, Bolivia, 1987-88. Management of 3.5 million Dollars during a 100 km gas pipeline construction with 170 employees. Part of a public bid won by the

Bolivian-Mexican Consortium Protexa-Interpipe for a total of 23 million dollars and 438 km gas pipeline construction to the highlands. Project financed by the Interamerican Development Bank (IDB) and Yacimientos Petroliferos Fiscales Bolivianos (YPFB). Supervised the establishment of a computerized accounting system; Established a daily progress report on itemized construction activities for technical and financial analysis; Prepared cash flow projections based on progress reports; Obtained loans and negotiated debts based on projected cash flows; Prepared monthly technical progress reports and billings in accordance with contract clauses; Technical and financial follow-up of several sub-contracting parties; Closing of technical information with sub-contracting parties, Protexa and YPFB; Closing of accounts with sub-contracting parties, Protexa, and shareholders; Prepared proposals for new public bidding; Supervised the investment of retained earnings in the creation of Monteflor Ltda., a new business in production, processing and export of coffee grain.

PRIVATE PROJECT CONSULTANT, La Paz and Santa Cruz, Bolivia, 1985. Team organizer and leadership in the preparation of feasibility studies and promotion of business development in the private sector.

ADVISER ON AGRICULTURAL ECONOMICS, Chemonics International Consulting Division, La Paz, Santa Cruz and Chuquisaca, Bolivia, 1984. Team work in the promotion and establishment of a seed production system in the Santa Cruz and Chuquisaca areas of Bolivia.

FIRST TECHNICIAN, Bolivian National Investment Institute, Foreign Investment

Department, La Paz, Bolivia, 1983. Benefit-cost analysis of contracts on technology transfer; Promotion of contract negotiations and re-negotiations according to Decision 24 requirements with tax deferral and tariff reduction compensations.

MEMBERSHIP:

The American Economic Association and The American Statistical Association.

PAPERS AND STUDIES:

- Barja, G. "Statistical Properties of the Bolivian Foreign Exchange Market." Research submitted to Utah State University in partial fulfillment of the requirements for an MS degree in statistics, 1994.
- Ayala, V.; and Barja, G. "The Macroeconomic Impact of the Emergency Fund Program." Presented to the Emergency Fund Program and the international community funding programs. *UDAPE*, 1989.
- Requena, J.C.; Barja, G.; Cupe, E.; and Huarachi, G. "Determinants of the Bolivian Budget Deficit: 1980-87." Presented to The Latin American Economic Commission (CEPAL). *UDAPE*, 1989.
- Barja, G.; and Ramirez, J. "A Short Valuation of the Bolivian Experience on Agricultural Credit Programs." Presented to the newly created Peasant Development Fund. *UDAPE*, 1989.
- Requena, J.C.; Barja, G.; and Montaño, O. "The Bolivian Government Budget: 1980-87." Presented to the World Bank. *UDAPE*, 1989.
- Barja, G.; Justiniano, M.; and Arce, S. "A Computer Program for Project Appraisal." Presented to the FOCAS/USAID credit program. *UDAPE*, 1987.
- Zeballos, H.; Barja, G.; and Ramirez, J. "Agricultural Policy and Investment Requirements for the Period 1987-90." Presented to the Planning Ministry. *UDAPE*, 1986.
- Heinrich, T.; and Barja, G. "The Bolivian Agro-Associative System: Valuation and Perspectives." Presented to USAID as an initial study for a new credit program, 1985.
- Barja, G.; and Decormis, E. "Production and processing of coffee grain for Export," "Milk and Poultry Production for the Santa Cruz Market," "Angora wool Production for

Export," "A Private Warehouse System as complement to Agricultural Credit Programs." Feasibility studies presented to private investors, 1985.

Barja, G.; and Cabrera, E. "Feasibility for a Seed Processing Facility in Northern Chuquisaca." Presented to the Chuquisaca Regional Seed Council and the Agricultural Ministry. *Chemomics International*, 1985.

Barja, G.; and Pattie, P. "A Flour Production and Marketing Program for the Charcas Milling Facility." Presented to the Charcas Agroindustrial Cooperative and the Agricultural Ministry. *Chemomics International*, 1984.

Barja, G.; and Cabrera, E. "Feasibility for the Production of Graded Seed." Presented to the Chuquisaca Regional Corporation. *Chemomics International*, 1984.

Barja, G.; and Cabrera, E. "Feasibility for Wheat, Barley and Potato Basic Seed Production." Presented to the Chinoli Experiment Station. *Chemomics International*, 1984.

Minot, N.; and Barja, G. "Feasibility of a Seed Processing Facility in the CIAT Experiment Station." Presented to the Santa Cruz Regional Seed Council and the Agricultural Ministry. *Chemomics International*, 1984.

Barja, Gover. "Subsidized Interest Rates, Resource Transfers and Income Distribution: The Bolivian Agricultural Bank Experience." Thesis submitted to the Bolivian Catholic University in partial fulfillment of the requirements for a BS degree in economics, 1983.