From Rationality to Irrationality : Dynamic Interacting Structures

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14 February 2015

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February 2015

Abstract

This article presents a general method to solve dynamic models of interactions between multiple strategic agents that extends the static model studied previously by the authors. It describes a general model of several interacting agents, their domination relations as well as a graph encoding their information pattern. It provides a general resolution algorithm and discusses the dynamics around the equilibrium.

Our model explains apparent irrational or biased individual behaviors as the result of the actions of several goal-specific rational agents. Our main example is a three-agent model describing "the conscious", "the unconscious", and "the body". We show that, when the unconscious strategically dominates, the equilibrium is unconscious-optimal, but body and conscious-suboptimal. In particular, the unconscious may drive the conscious towards its goals by blurring physical needs.

Our results allow for a precise account of agents' time rate preference. Myopic behavior among agents leads to oscillatory dynamics : each agent, reacting sequentially, adjusts its action to undo other agents' previous actions. This describes cyclical and apparently inconsistent or irrational behaviors in the dual agent. This cyclicality is present when agents are forward-looking, but can be dampened depending on the conscious sensitivity to other agents' actions.

Key words: dual agent; conscious and unconscious, rationality; multi-rationality; emotions; choices and preferences; multi-agent model; consistency; game theory; strategical advantage.

JEL Classification: B41,D01, D81, D82.

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Introduction

Irrational actions are usually attributed to an unconscious agent whose actions would, at times, supersede those of the conscious. Economically, this unconscious activity should be modeled as the action of a single, permanent and fully rational agent acting alongside the conscious. Yet this obvious path to modeling the unconscious as an economic agent has systematically been disregarded by the literature, for a single and no less obvious reason : two rational agents sharing the same information and acting through one individual should have two perfectly identical actions, boiling down to a single rational action. No irrationality would emerge from such a setup.

Yet to make sense of the unconscious' actions, one has to suppose it to be rational. Yet one must also supposed it to be permanent, in the sense that it should not be possible to "dismiss" it at will. Besides, unconscious and conscious must have identical information. Consequently, a difference in actions between these two rational agents can only emerge from different grids of lecture, i.e. from the way they process information.

The question therefore is not to know whether we should model the unconscious as a permanent and rational agent, but to infer how its interpretation of the reality can differ from the conscious' one.

This approach has been developed in [L], who dubbed the combination of these two - conscious and unconscious - fully economically rational agents, "the dual agent". In this setup, irrational behaviors do not arise from the existence of an irrational unconscious, but from the mere fact that two agents, "conscious" and "unconscious", evaluate events through different grids, though acting through one single body.

True, economic models of inconsistent behaviors have already considered the person as a set of interacting sub-agents[1]. They model dual or multiple selves acting "synchronically" and/or "diachronically" on a similar set of information to achieve different objectives[2]. To Picoeconomics, in particular, this situation is a bargaining game between several selves[3]. In this framework, the role of the unconscious - inasmuch as such a notion can be attributed to the sub-structures they consider - usually appears as a mere unknown random modification on the short-term utility. The authors of the present paper believe, on the contrary, that patterns of behavior display some persistency and must consequently be active at all stages of the decision and action processes. They see the human psyche as several agents in complex, simultaneous and possibly strategic interactions.

The notion of dual agent proposed by [L] must specifically be distinguished from the "dual self" considered by Fudenberg and Levin [FL]. In their setup, long-run and short-run sub-structures model behaviors of self-control. The long-run self imposes costs on short-run selves and control their behaviors, so that an agent will successively identify to several selves, each disappearing in turn. Whereas [FL] and Picoeconomics model unknown mechanisms inducing changes in the agent's utility, our goal is to describe these very processes, along with the reason of their incoherence and inconsistency. Only by describing some unconscious rational behavior can we hope to achieve this.

[GL, GLW], two subsequent papers, formalized and generalized the dual agent approach of [L]. They model individual behavior as a system of interacting rational *structures*, or *rational agents*, embedded in a network of interactions. External signals activate structures that react by sending signals throughout the network, in turn activating other structures, and so on[4].

[GL] presented a dynamic model where one agent dominates, and showed that such a model can, depending on its parameters, present stable or unstable oscillations, as well as periodic cycles. More importantly, the actions of the dominating agent induce apparent change of objective, or switch in preferences in other agents. This led us to introduce the notion of effective utility, and propose an alternative interpretation of the independence of irrelevant alternatives.

However, this model did not take into account more complex situations, such as several competing dominant agents, or tripartite games, where a structure is used to manipulate another one. In a given individual, conscious objectives regarding the body may encounter unconscious resistance. In social relations, one group may manipulate an other group against a third one.

[GLW] designed a general pattern of static and strategic interactions among several agents to model

[1] See [Ross].
[2] For example cases of procrastination such as Akerlof's [Akerlof].
[3] [Ainslie92]. see [Ross] and references therein for an account.
[4] See [GL] for a precise account.

such configurations. A graph describing the agents' strategic relations and their interactions command the resolution of the model. We used this pattern to solve a three-agents model, respectively a body, a conscious and an unconscious. These three terms are only generic terms and should be taken as archetype behaviors, where the unconscious has a strategic advantage over the conscious and the body, and the conscious dominates the body. We show that, depending on the parameters of the model, the unconscious can twist the conscious' action via the manipulation of the body's objectives. It may result in an unstable pattern, such as anorexia or bulimia, the body's action being used to deter or reinforce, given the unconscious objective, the task of the conscious agent.

This static one-period model encompasses, as a particular case, the "long-run self" and "short-run selves" as well as the "successive-selves" models of Pico-economics [FL]. Successive agents' actions induce a dynamics allowing to anticipate long-term patterns such as apparently irrational changes of goals or cyclic behavior. However no precise study of their dynamic interactions was provided.

The present paper fills this gap by generalizing [GLW] setup in a dynamic framework, replacing static utilities by intertemporal ones. A one-period delay between one agent's action and its perception by the other agents induces the dynamic of the system. As in [GLW], quadratic utilities model agents' interactions around an initial perturbed equilibrium. The frame of interactions between agents is described by a graph whose vertices are the agents and the oriented arrows represent their pairwise relative strategic advantages.

A method of resolution for this general dynamic model with an arbitrary number of agents is presented. The domination graph gives the order in which each agent's optimization is performed. The solutions are power series in the discount rates, and are obtained recursively by means of usual linear algebra techniques. Contrary to [GLW], since utilities and actions are forward-looking, each agent's pattern of information is presented. We assume that each agent knows the domination relations between the structures he strategically dominates in the graph. It also knows the parameters characterizing the agents' interactions, but ignores the influence of the structures he does not dominate.

This pattern of information is both realistic and general enough to describe the dynamic version of [GLW]: some plans "forecasted" by a first agent, and involving some substructures, may be impaired by the actions of competing agents, leading to a biased equilibrium from the first agent perspective. If this first agent is misleadingly identified with the individual as a whole, one can conclude to an irrational behavior, through a bias, or a change of utility function, as already spotted in [GL] for two agents interacting dynamically.

We then use this resolution method to explicitly solve a dynamic version of our basic archetypal three-agent model. Despite some differences, the long-run equilibrium is similar to the static solution, and the interpretations of [GLW] are preserved. The dynamic model allows a full interpretation of the stability and dynamics of the solution : if the unconscious strategically dominates, the equilibrium will be unconscious-optimal, but both body- and conscious-suboptimal.

Our result allow for a precise account of agents' time rate preference. Myopic behaviors among agents lead to an oscillatory dynamics. Each agent, reacting sequentially, adjusts its present action to other agents' previous actions to undo their work leading to cyclical and apparently inconsistent or irrational behaviors in the dual agent.

This cyclicality is present when agents are forward-looking. However, it may be dampened by the conscious' sensitivity to other agents' actions. When it is mild, a stable but conscious-suboptimal equilibrium may be reached. Otherwise, unstability will appear. A divergent and spiraling behavior reduces the dual agent's welfare.

Inconsistent preferences and apparent switches in behaviors can be ascribed to the unknown action of some strategic structures behind conscious choices. They describe the nature of the structures' utilities as well as their interactions, and give a hint about internal sub-structures' capacity to plan their actions.

The paper is organized in four sections. The first section is a reminder of [GLW] dynamic version of the three-agent static model. It presents, develops and interprets its results. The second section develops a general dynamic model of several interacting structures. It provides a resolution method to find the equilibrium and the fluctuations around it. The third section applies this formalism to section one and gives a detailed solution of three-agent model. The last section concludes.

1 A dynamic three-agent model

This section presents a dynamic version of [GLW] the static three agent model. For the sake of clarity, we will give a detailed reminder of the static model and briefly recall its solutions.

[GLW] described the apparent individual irrational behavior as resulting from the combined actions of three interacting sub-structures - or agents - within the psyche of that individual.

These three interacting agents typically represent two mental processes - "conscious" and "unconscious", along with a third - physical - process, the body. This setting may alternately apply to every configuration in which a global instance may be divided into several goal- and information-specific interacting sub-structures.

Suppose an individual whose conscious wants to perform a specific task. However an other independent agent, his unconscious, has other goals and hence considers this task as suboptimal. Moreover, a third agent, the body, has specific needs that have to be satisfied. We assume that the unconscious partly influences both the body and the conscious by sending signals, such as discomforts or strains.

Consider a student which experiences difficulties in concentrating. Each time he starts studying, he is suddenly overwhelmed by sleepiness, boredom or compulsions, such as smoking or eating. Being both rested and fed, these cannot possibly obey objective physical needs.

In our perspective, these bouts of compulsion encapsulate the action of an other, independant agent, the unconscious, whose priorities and goals differ from those of the conscious, mainly because its interpretation grid differs from the one of the conscious[5]. The conscious actions are read and interpreted by the unconscious according to a specific and completely different grid. To him, these conscious actions are suboptimal or potentially dangerous, and he will react accordingly by deterring the conscious actions.

In the case of our student, we can infer that his unconscious perceives learning as being harmful, and reacts by thwarting the conscious' action. Ultimately, the individual visible behavior will merely be the result of the three agents' interactions, which, in a dynamical perspective, may result in some incoherent patterns, such as an inconsistency in an agent's goal.

1.1 Reminder: the static three-agent model

The body, denoted *Agent B,* or *B,* is modeled as an automaton whose sole action, n, signals either a physical need, when $n > 0$, or a satiety signal, when $n < 0$.

The conscious, *Agent C*, or C, can either perform a task w, whose optimum is arbitrarily normalized to 0, or react to *Agent B*'s signal by satisfying its need with a second action f (feeding). Both actions being exclusive and complementary, the agent's time period will be optimally divided between the two actions, under the constraint :

$$w + f = 1$$

To the unconscious, *Agent U*, or U, the variable $w's$ optimum is not zero, but $\tilde{w}$. To reach its goal, the unconscious will weigh on the conscious' optimization problem by means of three strains[6]. Two of them, s_w and s_f, directly affect the conscious' actions w and f. The third one, s_n influences the body by modifying its need, n. Neither *Agent C* nor *Agent B* are aware of *Agent U* presence, goals, and actions.

The agent utilities The utilities of the body, unconscious and conscious, are denoted U_{B}, U_{U}, U_{C} respectively and set:

$$\begin{aligned} U_{\mathrm{B}} &= -\frac{1}{2}(n+f)^2 - \alpha n s_n \quad (1) \\ U_{\mathrm{U}} &= -\frac{1}{2}\left(\rho\left(f-\tilde{f}\right)^2 + \gamma(w-\tilde{w})^2 + s_n^2 + s_f^2 + s_w^2\right) \\ U_{\mathrm{C}} &= -\frac{1}{2}\left((w-w_0)^2 + \delta n^2\right) - \nu n w - \kappa s_f\left(f-\tilde{f}\right) - \eta s_w(w-\tilde{w}). \end{aligned}$$

[5] See [GL] for more details.
[6] Actually, this would look like changing the conscious'optimum. See [GL].

under the constraint: $w + f = 1$.Utilities are quadratic and normalized so that the terms containing the square control variables have coefficients of $-\frac{1}{2}$ or 0.[7]

The utility of the body The body, being an automaton, has no specific goals, and its utility function U_{B} merely describes its reaction to other agents' actions[8]. Without any interaction with the unconscious U, the body would, in first approximation, react linearly to the conscious C action, "feeding" :

$$-\frac{1}{2}(n+f)^2$$

The unconscious influences the body by perturbating its signal

$$-\alpha n s_n$$

Whereas in the absence of the unconscious, the body's optimum would be reached for

$$n = -f = 0$$

This result being suboptimal for *Agent U*, he will tilt the equilibrium toward its own goal $\tilde{f}$.

Recall that the task performed by the conscious w is not physically demanding, and has no impact on the body's response n. Indeed, we do not model physical efforts per se, but rather seek to understand how the unconscious can manipulate an existing equilibrium between the body and the conscious, i.e. the use of body signals by the unconscious to reach its own goals. By convention α is positive, so that a positive strain will respond to a positive feeding.

The utility of the conscious In the absence of both the unconscious and the body, the conscious' utility would be :

$$-\frac{1}{2}(w-w_0)^2$$

so that in the absence of any constraint set on w, *Agent C* would optimally choose $w = w_0 > 0$.

Body needs affect *Agent C* through

$$-\frac{1}{2}\delta n^2 - \nu n w$$

so that the higher is the need, the more painful is the task.

In the absence of *Agent U*, *Agent C* sets $w = 0$ by adjusting the feeding to the anticipated need. The need is in itself painful since:

$$-\frac{1}{2}\delta n^2$$

so we set

$$\delta > 0$$

The above assumption is a direct consequence of dismissing any cost to the feeding f. Here we depart from standard models where costs, or constraints, are imposed to an agent's tasks.Without *Agent U*, *Agent B* and

[7] Note that, in order to use later dynamic models standard notations, we modified some of the notations of [GLW].

[8] In this setting, endowing the body with specific goals would have allowed it to manipulate the conscious, which was not our purpose here.

f could be discarded from *Agent C*'s equilibrium. Once *Agent U* is included in the system, it indirectly manipulates *Agent C* through *Agent B* by assigning a strategic role to f. However we impose a binding constraint on the feeding by considering f and w as complementary activities within a given time span, and set $f+w=1$, as previously mentioned. The unconscious imposes its goals $\tilde{f}$ and $\tilde{w}$ on the conscious through perturbation terms:

$$-\kappa s_f(f-\tilde{f})-\eta s_w\,(w-\tilde{w})$$

driving *Agent C*'s actions away from 0 and towards $\tilde{f}$ and $\tilde{w}$.

Some additional technical conditions on U_{C} will prove convenient. We will ensure that U_{C} is negative definite and has an optimum by setting :

$$\delta-\nu^2>0$$

Furthermore, excessive working combined with unsatisfied needs should induce a loss in *Agent C* utility. This is implemented by imposing:

$$\nu>0 \text{ for } n>0 \text{ and } w>0$$

.

The utility of the unconscious Agents, conscious or unconscious, build their interpretation of a situation - and thus its utility function - through an own, specific, grid of lecture[9].
Agent U and *Agent C* will therefore have two completely different interpretations of a single situation. And while *Agent C* will consider f and w as optimal, *Agent U* will consider other levels of the conscious' activity, $\tilde{f}$, $\tilde{w}$ as optimal.
Agent U's goals with respect to *Agent C*'s activity are:

$$-\frac{1}{2}\rho(f-\tilde{f})^2-\frac{1}{2}\gamma(w-\tilde{w})^2$$

To insure that U_{U} can have an optimum, we further impose ρ and γ to be positive.

Since the three agents are sub-structures of one single individual, a strain inflicted by one agent ends up being painful for all. The costs incurred are :

$$-\frac{1}{2}\left(s_n^2+s_f^2+s_w^2\right)$$

Additional constraints To have a realistic model, some additional constraints on the coefficients and variables are required.

Since *Agent U* inflicts a pain if his goals are not reached, s_f et s_w should be assumed positive. However such a condition does not exist for s_n. *Agent U* has no optimum value for n. For the unconscious, s_n is a mere adjustment variable, and can be alternately a pain or a reward to the body.

To model the pain imposed by the unconscious over the conscious, the terms $-\kappa s_f(f-\tilde{f})$ and $-\eta s_w\,(w-\tilde{w})$ should be negative. The equilibrium values of f and w will respectively lie between 0 and $\tilde{f}$ and between 0 and $\tilde{w}$. We therefore impose $\tilde{f}$ and κ (respectively $\tilde{w}$ and η) to have opposite signs.

The hierarchy of strategical advantages. In our setting, some agents are assumed to have a strategical advantage over others.
Agent U has a strategic advantage over both *Agent C* and *Agent B*. It is strategically dominant. *Agent C* is strategic only with respect to *Agent B*, and *Agent B* has no strategic advantage. The dynamic model will

[9] See [GL] for further details.

be solved in the order of this hierarchy of strategical advantages [10].

We first implement the time binding constraint $w + f = 1$ for *Agent C*'s actions and replace directly $f = 1 - w$ in the utility functions:

$$
\begin{aligned}
U_{\mathrm{B}} &= -\frac{1}{2}(n+1-w)^2 - \alpha n s_n \\
U_{\mathrm{U}} &= -\frac{1}{2}\rho(1-w-\tilde{f})^2 - \frac{1}{2}\gamma(w-\tilde{w})^2 - \frac{1}{2}\ s_n^2 - \frac{1}{2}s_n^2 - \frac{1}{2}s_w^2 \\
U_{\mathrm{C}} &= -\frac{1}{2}(w-w_0)^2 - \frac{1}{2}\delta n^2 - \nu n w - \kappa s_f(1-w-\tilde{f}) - \eta s_w(w-\tilde{w})
\end{aligned}
$$

Recall that *Agent U* influences *Agent C* in two competing ways. Directly, through the strains s_f and s_w, and indirectly, by triggering a need in *Agent B* through s_n. The relative strengths of the parameters will determine *Agent U*'s preferred channel of action, and lead to different behaviors in the individual.

Hidden and patent manipulation Two types of unconscious manipulation emerge.
In the first, the unconscious does not mask its action. The individual is driven away from $w's$ conscious optimum, and physical needs n may consequently be under or over-satisfied, depending on the unconscious' optimum. However, the individual will be aware of this disequilibrium. If physical needs are under-satisfied, the situation may not be sustainable in the long run. It will be less so if they are over-satisfied.
In the second type of manipulation, the individual is driven away from the conscious optimum w, but the unconscious can manipulate physical needs to mask the disequilibrium. The unconscious may, for instance, sustain phase of hyperactivity by reducing physical needs, so that the individual will not be conscious of the disequilibrium. This situation may be more sustainable, but may lead to a breakdown in the longer run.

1.2 Dynamic model

Summing weighted expectations of future utilities transforms the above static utilities into intertemporal ones, and turn this static model into a dynamic one. However, delays between consecutive actions will induce some adaptations and, more importantly, the information setup needs to be precisely described.

1.2.1 Presentation of the three-agent model.

Given it's own information set, each agent optimizes a forward-looking intertemporal utility function of the form:

$$(V_i t) = \sum_{m \geq 0} \beta_i^m E_i U_i(t+m)$$

where the discount factor β_i is *Agent i* rate of preference for the present, and E_i its expectation operator at time t. *Agent i* forecasts of future quantities will be computed given its information set.

The information setup follows the order of domination among agents[11]. *Agent B,* the less informed of all agents, is only aware of the strains he's affected by. *Agent C* is aware of it's own influence on *Agent B*, and of the strains *Agent U* puts on him. *Agent U,* the most informed of all agents, knows the utilities function of both *Agent C* and *Agent B.*
The instantaneous utility $U_i(t+m)$ at time $t+m$ reproduces the static utility of section 1, but introduces a time dependency in each action variable. We moreover assume that each action taken at time t by any

[10] For a complete resolution of the model and the discussion of its solutions, see [GLW].
[11] For the sake of clarity we do not present here the information set up. It will be fully described in the resolution of the general model.

agent will only be perceived by the other agents at time $t+1$.
Taking into account all of the above, utilities take the following dynamic form:

$$
\begin{aligned}
U_B(t) &= -\frac{1}{2}\left(n(t)+1-w(t-1)\right)^2-\alpha n(t)\, s_n(t-1) \qquad (2)\\
U_U(t) &= -\frac{1}{2}\rho\left(1-w(t-1)-\widetilde{f}\right)^2-\frac{1}{2}\gamma\left(w(t-1)-\widetilde{w}\right)^2-\frac{1}{2}s_n^2(t)-\frac{1}{2}s_f^2(t)-\frac{1}{2}s_w^2(t)\\
U_C(t) &= -\frac{1}{2}\left(w(t)-w_0\right)^2-\frac{1}{2}\delta n^2(t-1)-\upsilon n(t-1)\, w(t)-\kappa s_f(t-1)\left(1-w(t)-\widetilde{f}\right)-\eta s_w(t-1)\left(w(t)-\widetilde{t}\right)
\end{aligned}
$$

Note that in each of the above utilities, the agent own action variables appear with a time index t, as expected for utility at time t, whereas other agents' action variables appear with a time index $t-1$.
The full resolution of the dynamical model being presented in the general resolution method, we postpone the description of the information setup and directly present the results of the dynamic three-agent model.

1.2.2 Solution of the dynamic model

Assuming that the discount rates β_i are identical and equal to a given β, we obtain the following result.

The long-run equilibrium is given by the action variables values :

$$
\begin{aligned}
\bar{n} &= \beta^2\left(\frac{K-\nu w_0}{\nu+1}-\beta\frac{\nu\alpha^2+2K\Delta}{\nu+1}\right)\frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}\\
\bar{w} &= \beta^2\left(\frac{K-\nu w_0}{\nu+1}+\beta\frac{\nu^2\alpha^2-2K\Delta}{\nu+1}\right)\frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}+w_0\\
\bar{s}_n &= \beta^3\alpha\nu\frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}\\
\bar{s}_w &= -\beta^2\eta\left(1-\Delta\beta\right)\frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}\\
\bar{s}_f &= \beta^2\kappa\left(1-\Delta\beta\right)\frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}
\end{aligned}
$$

where we set:

$$\Delta=\delta-\nu^2, \qquad K=\eta^2+\kappa^2, \qquad \text{and} \qquad \zeta=\rho+\gamma.$$

The results are very similar to the static case[12]. Here again, two different general patterns appear for the equilibrium. *Agent U* can drive *Agent C* toward its own objectives by implementing a stress on the need of *Agent B*.
However, the reaction of Agent B is ambiguous, and depends on the relative strength of the parameters. Two different regimes appear in the dynamic setting.

Under the condition :

$$K>\frac{\nu w_0+\alpha^2\nu\beta}{1-2\Delta\beta}$$

both $\bar{w}$ and $\bar{n}$ are positively correlated to Agent U's goal.

To explain this, recall that *Agent U* has two channels of actions on *Agent C*.

[12] See [GLW] for comparison.

The *direct channel* acts through the strains s_f and s_w. It's efficiency on *Agent C*'s utility is measured by the coefficients η and κ and thus by

$$K = \eta^2 + \kappa^2$$

The *indirect channel* influences the needs of *Agent B*, that will in turn impact *Agent C*'s action. Its strength α measures the sensitivity of *Agent B*'s to the strain s_n

The above inequality therefore compares the relative strengths of the direct versus the indirect channel. If the direct channel is more efficient, it is optimal for U to drive *Agent C*'s activity by acting on the direct strains $\bar{s}_w$ and $\bar{s}_f$. Higher feeding will satisfy the higher needs induced by a higher activity for *Agent C*.

On the other hand, under the condition :

$$K < \frac{\nu w_0 + \alpha^2 \nu \beta}{1 - 2\Delta\beta}$$

the indirect channel is relatively more efficient than the direct one. *Agent U* will impose a strain on *Agent B*, inducing a decrease of *Agent B*'s needs, and indirectly allowing *Agent C* to increase its task. This will be achieved at a comparatively lower feeding cost than the direct channel. As a result, the equilibrium values $\bar{w}$ and $\bar{n}$ will respectively be positively and negatively correlated to *Agent U*'s goal.

The difference in patterns of behavior appears clearly in the feeding of the individual. Alongside a normal pattern, where higher feeding is associated with increased work, we find an "anorexic" pattern, where reduced feeding is associated with increased activity.

Fluctuations around the equilibrium. The dynamics and stability of the system may be fully understood by computing the fluctuations around the equilibrium. The utility functions being quadratic, these fluctuations are given by a linear first order dynamic system of the kind

$$[Y(t)] = M\,[Y(t-1)] + \left[\hat{W}(t)\right] \tag{3}$$

where $[Y(t)]$ is the - five dimensions[13] - vector of action variable fluctuations around their equilibrium.

$$[Y(t)] = \begin{pmatrix} n(t) - \bar{n} \\ w(t) - \bar{w} \\ s_n(t) - \bar{s}_n \\ s_w(t) - \bar{s}_t \\ s_f(t) - \bar{s}_f \end{pmatrix}$$

The coefficients of the matrix M depend on the parameters of the model and $\left[\hat{W}(t)\right]$ is a - five components - vector of random perturbation at time t[14].

The effects of an initial shock on the system are computed using Equation (3). Assume a system in equilibrium with $[Y(0)] = 0$ at time $t = 0$, and consider a single initial shock $\left[\hat{W}(t)\right]$ such that $\hat{W}(0) \neq 0$ and $\hat{W}(t) = 0$ for $t > 0$. Given these assumptions, the solution of Equation (3) is :

$$[Y(m)] = M^m \hat{W}(0)$$

[13] The number of action variables.

[14] The explicit forms are given in Section 3.

Stability of the system The stability of the system depends on the nature of the matrix M eigenvalues. Section 3 will show that 0 is a triple eigenvalue, and that the two other eigenvalues are the complex conjugates:

$$\lambda_{\pm} = \pm i\sqrt{\nu}\left(1 - \frac{1}{2}\beta^2\left(\xi K + \Delta\right) + \frac{1}{2}\beta^3\xi\left(-\alpha^2\nu^2 + 2\Delta K\right)\right) + O(\beta^4) \tag{4}$$

at the third order approximation in β.

$\lambda_{\pm}$ being imaginary, the system is oscillating. This is a direct consequence of our model, where non collaborating agents alternately try to stir the equilibrium towards their respective goals by detering other agents actions.

The system is stable if the modulus $|\lambda_{\pm}|$ of $\lambda_{\pm}$ is lower than 1, stably cyclical if it is equal to 1, and unstable if $|\lambda_{\pm}|>1$.

Let us consider, as a benchmark case, the eigenvalues for $\beta = 0$, where all agents only care about the present. The eigenvalues are $\pm i\sqrt{\nu}$, and the dynamic is diverging if $\nu > 1$, cyclic if $\nu = 1$, and converging if $\nu < 1$.

We can interpret these results in the following way. *Agent B* reacting to *Agent C*'s feeding in a 1 to 1 ratio, and *Agent C*'s w reacting to *Agent B*'s need with a ratio ν, both agents' actions will be multiplied by ν over a two-period horizon. *Agent U*'s action paying only over a two to three-periods horizon, it is irrelevant when $\beta = 0$, and prevents *Agent U* from taking it. Myopic behavior among agents leads to an oscillatory dynamics. Each agent, reacting sequentially, adjusts its action to undo other agents' previous actions, describing cyclical and apparently inconsistent or irrational behaviors in the dual agent. These oscillations may diverge or fade away with time, depending on the value of ν.

When β is different from 0 but relatively small, the system is still oscillatory. However the time concern β will have an ambiguous effect on its stability. Decomposing Equation (4) in it's second and third order contributions will illustrate this point.
Because of the delay in the actions' impact, the feedback on the originating agent will be seen in two periods - under the direct channel - or possibly three periods, under the indirect channel[15].

At the second order in β, Equation (4) shows that the agents' concern about the future has a stabilizing effect and reduces the magnitude of the oscillations. At this order, the eigenvalues' module is

$$|\lambda_{\pm}| = \sqrt{\nu}\left|1 - \frac{1}{2}\beta^2\left(\xi K + \Delta\right)\right| + O(\beta^3)$$

which is lower than $\sqrt{\nu}$.

The stabilization effect appears at the two-period horizon[16], since *Agent U* cannot use the indirect channel in time scale. The stabilization is obtained by *Agent U*'s direct action on *Agent C*, and by *Agent C*'s direct action on *Agent B*.
At the three-period horizon (the third order in β), the stability is more ambiguous. *Agent U* can use the indirect channel to stabilize the system around an equilibrium to its own advantage. On the other hand, *Agent C* can react to *Agent U*'s action, by destabilizing *Agent U*'s optimum. Because of theses antinomic actions, the stability of the system depends on the parameters' relative strength.
Neglecting higer order contributions, an overall interpretation of the system can be reached by combining these two different horizon effects. The time concern total correction to the eigenvalues is :

$$-\frac{1}{2}\beta^2\left(\xi K + \Delta\right) + \frac{1}{2}\beta^3\xi\left(-\alpha^2\nu^2 + 2\Delta K\right)$$

[15] i.e. from *Agent U* to *Agent B*, then from *Agent B* to *Agent C* and ultimately from *Agent C* to *Agent U*.
[16] the second-order in β

Recall that ξ measures the overall concern of the unconscious for the conscious activity, K the sensitivity of the conscious agent to the strains imposed by the unconscious, and Δ the relative strength of the direct effect of the need on the conscious agent compared to it's effect via the effort term $-\nu nw$ of U_{C} in equations 1.

The above quantity is positive if

$$K\left(2\Delta\beta-1\right)-\alpha^2\beta\nu^2>0$$

and

$$\xi>\frac{\Delta}{K\left(2\Delta\beta-1\right)-\alpha^2\beta\nu^2}\equiv\xi_{\text{threshold}}$$

Since $\Delta>0$, if Δ and K are large enough, i.e. if they are such that :

$$2\Delta\beta-1>0$$

and

$$K\left(2\Delta\beta-1\right)-\alpha^2\beta\nu^2>0$$

there exists a threshhold $\xi_{\text{threshold}}$ such that when $\xi>\xi_{\text{threshold}}$, the instability of the system is increased by the forward-looking nature of its agents, and its stability reinforced otherwise. As a result, given a time rate preference β, if the conscious is highly sensitive to the other agents' actions, and if conscious and unconscious goals are strongly divergent, an instability in the system may arise. The system is driven away from its equilibrium, leading to an explosive and suboptimal pattern for all agents.

Moreover, the higher the time concern β, the lower the sensitivity Δ can be to make the unstability case arise. In other words, the more the system is forward looking, the more can we expect some unstability.

These results provide a pattern of the nature of structures' utilities as well as their interactions, and give a hint about internal sub-structures' capacity to plan their actions. Inconsistent preferences and apparent switches in behaviors can be ascribed to the unknown action of some strategic structures behind conscious choices. The system's cyclicality, the striking differences emerging between dynamic patterns of behaviors of forward and non-forward looking structures, may, through empirical observations, support forward-looking decisions in hidden partial processes, and give a better understanding of the sequence of mental processes.

2 A general model of several interacting structures

Let us now present and solve the general dynamic model of several interacting structures that encompasses the three-agents example presented above. We will first briefly recall the general static formalism of [GLW], on which the dynamic version will be developed.

2.1 Static several interacting agents.

This static version introduces the various agents' utility functions and the domination graph that commands the resolution algorithm.

Strategic relations between agents The agents' strategic relations define the model setup. An oriented graph Γ whose vertices are labelled by the agents involved describe these relations.

When *Agent* i has a strategic advantage over *Agent* j, we draw an oriented edge from i to j and write $i\rightarrow j$. If there exists an oriented path from i to j, we write the relation $i\rightarrowtail j$, and state that *Agent* i dominates directly or indirectly Agent j or, equivalently, that Agent j is subordinated to Agent i. If there is no oriented path from i to j, we write $j\not\leftarrowtail i$, where it is always understood that $i\neq j$. In the following, we merely consider connected graphs without loops.

Under these conventions, the *situation* is modeled by an oriented graph whith vertices labelled after B,C, and U, and whose edges orientation summarize the agents' strategic advantages

$$C\rightarrow B,\qquad U\rightarrow B,\qquad U\rightarrow C$$

Matricial formalism Agents' utilities are described by the following matricial formalism. Agents' actions are encompassed in a vector of actions, or control variables. The number of possible actions determine the size of the vector. Utilities being quadratic, matrices may be associated with them.

Let $X_i \in R^{n_i}$ be Agent i's vector of control variables, and $\tilde{X}_j^{(i)} \in R^{n_i}$ the vector of goals associated with the variables X_j, as expected by agent i. We normalize $\tilde{X}_j^{(i)}$ to 0, so that Agent i wishes to achieve $X_i = 0$ and $X_j = \tilde{X}_j^{(i)}$.

Agent $i's$ utility is given by:

$$\begin{aligned} U_i &= -\frac{1}{2}{}^t X_i A_{ii}^{(i)} X_i - \frac{1}{2}\sum_{j \leftarrow i} {}^t\left(X_j - \tilde{X}_j^{(i)}\right) A_{jj}^{(i)} \left(X_j - \tilde{X}_j^{(i)}\right) \\ &\quad - \sum_{j \leftarrow i} {}^t X_i A_{ij}^{(i)} X_j - \sum_{j \not\leftarrow i} {}^t\left(X_i - \tilde{X}_i^{(j)}\right) A_{ij}^{(i)} X_j - {}^t X_i P_i \end{aligned}$$

In the absence of any interaction, Agent $i's$ utility is given by the term

$$-\frac{1}{2}{}^t X_i A_{ii}^{(i)} X_i$$

The variables X_i are normalized so that $A_{ii}^{(i)}$ is a $f_i \times n_i$ diagonal matrix whose coefficients are 1 or 0.

If Agent $i's$ subordinate agents' actions X_j depart from $\tilde{X}_j^{(i)}$, Agent $i's$ will experience a loss of utility of the form :

$$\sum_{j \leftarrow i} {}^t\left(X_j - \tilde{X}_j^{(i)}\right) A_{jj}^{(i)} \left(X_j - \tilde{X}_j^{(i)}\right)$$

The $f_j \times n_j$ matrix $A_{jj}^{(i)}$ of parameters is of course symmetric.
The impact of *Agent* j's action on *Agent* i's utility is

$$\sum_{j \leftarrow i} {}^t X_i A_{ij}^{(i)} X_j - \sum_{j \not\leftarrow i} {}^t\left(X_i - \tilde{X}_i^{(j)}\right) A_{ij}^{(i)} X_j$$

where $j \leftarrow i$ can be seen as the impact of Agent $j's$ action on Agent i. In our model, Agent j does not know the agents to whom he is subordinated, and processes their signals as external ones. The second term models the strain imposed on Agent i by Agent j to achieve its own objectives for X_i.

Remark 1 *Since the linear term in X_j disappears during the resolution,*

$$\sum_{j \not\leftarrow i} {}^t X_i A_{ij}^{(i)} X_j$$

is equivalent to

$$\sum_{j \not\leftarrow i} {}^t\left(X_i - \tilde{X}_i^{(j)}\right) A_{ij}^{(i)} X_j$$

Notation 2 *By convention, for the $n_i \times n_j$ parameters matrices $A_{ij}^{(i)}$, we will write ${}^t A_{ij}^{(i)} = A_{ji}^{(i)}$.*

The last term ${}^t X_i P_i$ models the change in utility caused by external perturbations summarized by the vector $P_i \in R^{n_i}$. For instance, the utilities of our three-agent model can be rewritten in the matricial formalism as:

$$\begin{aligned} U_{\mathrm{B}} &= -\frac{1}{2}{}^t X_{\mathrm{B}} X_{\mathrm{B}} - {}^t X_{\mathrm{B}} A_{\mathrm{BU}}^{(\mathrm{B})} X_{\mathrm{U}} - {}^t X_{\mathrm{B}} A_{\mathrm{BC}}^{(\mathrm{B})} X_{\mathrm{C}} - \frac{1}{2}{}^t X_{\mathrm{C}} A_{\mathrm{CC}}^{(\mathrm{B})} X_{\mathrm{C}} + ({}^t X_{\mathrm{C}} - {}^t X_{\mathrm{B}}) E_{\mathrm{B}} \\ U_{\mathrm{U}} &= -\frac{1}{2}{}^t X_{\mathrm{U}} A_{\mathrm{UU}}^{(\mathrm{U})} X_{\mathrm{U}} - \frac{1}{2}{}^t\left(X_{\mathrm{C}} - \tilde{X}_{\mathrm{C}}^{(\mathrm{U})}\right) A_{\mathrm{CC}}^{(\mathrm{U})} \left(X_{\mathrm{C}} - \tilde{X}_{\mathrm{C}}^{(\mathrm{U})}\right) \\ U_{\mathrm{C}} &= -\frac{1}{2}{}^t X_{\mathrm{C}} A_{\mathrm{CC}}^{(\mathrm{C})} X_{\mathrm{C}} - \frac{1}{2}{}^t\left(X_{\mathrm{B}} - \tilde{X}_{\mathrm{B}}^{(\mathrm{C})}\right) A_{\mathrm{BB}}^{(\mathrm{C})} \left(X_{\mathrm{B}} - \tilde{X}_{\mathrm{B}}^{(\mathrm{C})}\right) - {}^t X_{\mathrm{C}} A_{\mathrm{CB}}^{(\mathrm{C})} X_{\mathrm{B}} - {}^t X_{\mathrm{U}} A_{\mathrm{UC}}^{(\mathrm{C})} X_{\mathrm{C}} - {}^t X_{\mathrm{U}} \breve{A}_{\mathrm{UC}}^{(\mathrm{C})} \check{X}_{\mathrm{C}} \end{aligned}$$

where the control variables for the agents are concatenated in the following vectors:

$$X_{\mathrm{B}} = \begin{pmatrix} n \end{pmatrix}, \quad X_{\mathrm{U}} = \begin{pmatrix} s_n \\ s_t \\ s_f \end{pmatrix}, \quad X_{\mathrm{C}} = \begin{pmatrix} w - w_0 \end{pmatrix}.$$

where the left upper-script ${}^t(\cdot)$ denotes the usual transposition of matrices. The goals of *Agent U* are condensed in the vector :

$$\tilde{X}_{\mathrm{C}}^{(\mathrm{U})} = \begin{pmatrix} \frac{\gamma(\tilde{w}-w_0)+\rho\left(1-\left(\tilde{f}+w_0\right)\right)}{\rho+\gamma} \end{pmatrix}$$

and those of *Agent C* are described by:

$$\tilde{X}_{\mathrm{B}}^{(\mathrm{C})} = \begin{pmatrix} \frac{-\nu w_0}{\delta} \end{pmatrix}$$

All others goals are normalized to 0 . We also set :

$$\check{X}_{\mathrm{C}} = \begin{pmatrix} -\left(\tilde{w} - w_0\right) \\ 1 - \left(\tilde{f} + w_0\right) \end{pmatrix}$$

The utilities quadratic relations are commanded by the parameters matrices :

$$\begin{aligned}
A_{\mathrm{BU}}^{(\mathrm{B})} &= \begin{pmatrix} \alpha & 0 & 0 \end{pmatrix}, \qquad A_{\mathrm{BC}}^{(\mathrm{B})} = \begin{pmatrix} -1 \end{pmatrix}, \quad A_{\mathrm{CC}}^{(\mathrm{U})} = (\xi) \\
A_{\mathrm{CC}}^{(\mathrm{C})} &= \begin{pmatrix} 1 \end{pmatrix}, \quad A_{\mathrm{BB}}^{(\mathrm{C})} = \begin{pmatrix} \delta \end{pmatrix}, \quad A_{\mathrm{CB}}^{(\mathrm{C})} = \begin{pmatrix} \nu \end{pmatrix} \\
A_{\mathrm{CC}}^{(\mathrm{B})} &= \begin{pmatrix} 1 \end{pmatrix}, \quad E_{\mathrm{B}} = \begin{pmatrix} 1 \end{pmatrix}, \\
A_{\mathrm{UU}}^{(\mathrm{U})} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A_{\mathrm{UC}}^{(\mathrm{C})} = \begin{pmatrix} 0 \\ \eta \\ -\kappa \end{pmatrix}, \quad \check{A}_{\mathrm{UC}}^{(\mathrm{C})} = \begin{pmatrix} 0 & 0 \\ \eta & 0 \\ 0 & \kappa \end{pmatrix}
\end{aligned}$$

By convention, we set $A_{ji}^{(i)} = {}^t\left(A_{ij}^{(i)}\right)$.
Moreover, since *Agent B* is dominated by both *Agent C* and *Agent U*, and *Agent C* is dominated by *Agent U*, the following matricial utilities will lead to the same equilibrium as $U_{\mathrm{B}}, U_{\mathrm{U}}, U_{\mathrm{C}}$.

$$\begin{aligned}
U'_B &= -\frac{1}{2}{}^t X_{\mathrm{B}} X_{\mathrm{B}} - {}^t X_{\mathrm{B}} A_{\mathrm{BU}}^{(\mathrm{B})} X_{\mathrm{U}} - {}^t X_{\mathrm{B}} A_{\mathrm{BC}}^{(\mathrm{B})} X_{\mathrm{C}} - \frac{1}{2}{}^t X_{\mathrm{C}} A_{\mathrm{CC}}^{(\mathrm{B})} X_{\mathrm{C}} - {}^t X_{\mathrm{B}} E_{\mathrm{B}} \\
U'_U &= -\frac{1}{2}{}^t X_{\mathrm{U}} A_{\mathrm{UU}}^{(\mathrm{U})} X_{\mathrm{U}} - \frac{1}{2}\,{}^t\left(X_{\mathrm{C}} - \tilde{X}_{\mathrm{C}}^{(\mathrm{U})}\right) A_{\mathrm{CC}}^{(\mathrm{U})} \left(X_{\mathrm{C}} - \tilde{X}_{\mathrm{C}}^{(\mathrm{U})}\right) \\
U'_C &= -\frac{1}{2}{}^t X_{\mathrm{C}} A_{\mathrm{CC}}^{(\mathrm{C})} X_{\mathrm{C}} - \frac{1}{2}\,{}^t\left(X_{\mathrm{B}} - \tilde{X}_{\mathrm{B}}^{(\mathrm{C})}\right) A_{\mathrm{BB}}^{(\mathrm{C})} \left(X_{\mathrm{B}} - \tilde{X}_{\mathrm{B}}^{(\mathrm{C})}\right) - {}^t X_{\mathrm{C}} A_{\mathrm{CB}}^{(\mathrm{C})} X_{\mathrm{B}} - {}^t X_{\mathrm{U}} A_{\mathrm{UC}}^{(\mathrm{C})} X_{\mathrm{C}}
\end{aligned}$$

2.2 Presentation of the dynamic version

This section describes the general model for dynamics interacting structures.The procedure is identical to that described in the three-agents case[17]. We adapt the matricial static utilities to a dynamic context, and assume that each agent optimizes a forward-looking intertemporal utility function, given it's own information set.

The intertemporal utility is of the form :

$$V_i(t) = \sum_{m \geq 0} \beta_i^m \mathbf{E}_i U_i\left(t + m\right)$$

[17] See Section 1.2.1

where β_i is *Agent* i's discount factor, and E_i his conditional expectation at time t. We give the information pattern allowing an agent to compute it's own expectations. $U_i\left(t+m\right)$ is period $t+m$ utility.

$$\begin{aligned} U_i\left(t+m\right) &= -\frac{1}{2}X_i^t\left(t+m\right)A_{ii}^{(i)}X_i\left(t+m\right) \\ & -\frac{1}{2}\sum_{j\leftarrowtail i}X_j^t\left(t+m-1\right)A_{jj}^{(i)}X_j\left(t+m-1\right) \\ & -\sum_{j\leftarrowtail i}X_i^t\left(t+m\right)A_{ij}^{(i)}X_j\left(t+m-1\right) \\ & -X_i^t\left(t+m\right)P_i\left(t+m\right) \\ & +\sum_{j\leftarrowtail i}X_j^t\left(t+m-1\right)A_{jj}^{(i)}\tilde{X}_j^{(i)} \\ & -\sum_{j\not\leftarrowtail i}\left(X_i^t\left(t+m\right)-\tilde{X}_i^{(j)}\right)A_{ij}^{(i)}X_j\left(t+m-1\right) \end{aligned} \tag{5}$$

This utility function is,up to some constant irrelevant term, a straightforward generalization of the particular three-agents model[18]. Recall that external and other agents' signals are perceived by *Agent* i with a one period delay[19].

2.3 Resolution of the dynamic model

The resolution proceeds in the reverse order of the domination graph, starting from the bottom of the graph with the optimization for the less strategic agents, and proceeding to the top of the graph.

2.3.1 Optimization problem and form of the solution

At time t, the optimization problem for *Agent* i is :

$$\frac{\partial}{\partial X_i(t)}V_i(t)=0$$

For every *Agent* j such that $i\rightarrowtail j$, the optimization has already been performed, so that, $X_j\left(t+m\right)$ for $m\geq 1$ are functions of $X_i(t)$. We thus obtain :

$$\begin{aligned} & A_{ii}^{(i)}X_i(t)+\mathbf{E}_i\sum_{m\geq 1}\left(\sum_{j\leftarrowtail i}\beta_i^m\left(A_{ij}^{i}\frac{\partial X_j\left(t+m-1\right)}{\partial X_i(t)}\right)^t\right)X_i\left(t+m\right) \\ & +\mathbf{E}_i\sum_{j\leftarrowtail i}\left(A_{ij}^{(i)}X_j\left(t-1\right)+\sum_{m\geq 1}\beta_i^m\left(\frac{\partial X_j\left(t+m-1\right)}{\partial X_i(t)}\right)^tA_{jj}^{(i)}X_j\left(t+m-1\right)\right) \\ & +\sum_{j\not\leftarrowtail i}A_{ij}^{(i)}X_j(t-1) \\ = \; & \varepsilon_i(t) \end{aligned} \tag{6}$$

where

$$\varepsilon_i(t)=-P_i(t)+E_i\sum_{j\leftarrowtail i}\sum_{m\geq 1}\beta_i^m\left(\frac{\partial X_j\left(t+m-1\right)}{\partial X_i(t)}\right)^tA_{jj}^{(i)}\tilde{X}_j^{(i)}$$

Each agent optimizes an intertemporal utility function that only depends on present and future variables.

[18] See Equations 2 and explanations below.

[19] In (5) we have developed the quadratic terms, to isolate the terms linear in $\tilde{X}_j^{(i)}$.

The model is thus forward looking. Each structure reacts to its own forecasts of future variables. These forecasts are built from the signals $X_j(t-1)$, the information set, where j runs over the set of agents. The solution of the model is found by identification. Utilities being quadratic, the linear solution is of the form:

$$X_i(t) = M_i\left[X(t-1)\right] + W_i(t)$$

where $[X(t)]$ is a one-column concatenation of the vectors $X_i(t)$ in an arbitrary chosen order. The coefficients of matrix M_i depend on the parameters of the model, and must be identified by way of the optimization equations. The size of M_i is determined by the dimensions of $X_i(t)$ and $[X(t-1)]$. The constant vectors $W_i(t)$ depend on the perturbation parameters $P_i(t)$ and the objectives $\tilde{X}_j^{(i)}$.

For later purpose we need a concatenated form of the previous set of equations :

$$[X(t)] = M\left[X(t-1)\right] + [W(t)] \tag{7}$$

Its iteration describes the dependency between the variables at time t and $t+m$:

$$\left[X(t+m)\right] = M^m\left[X(t)\right] + \sum_{k=1}^{m} M^{m-k}\left[W(t+k)\right] \tag{8}$$

where $[W(t)]$ is the concatenation of the $W_i(t)$, and the square matrix M is the concatenation of the matrices M_i.

We use the postulated solution (8) to compute $\frac{\partial X_j(t+m)}{\partial X_i(t)}$ for $i \rightarrowtail j$ in the optimization problem for Agent i. This yields:

$$\frac{\partial X_j(t+m)}{\partial X_i(t)} = \left(M^m\right)_{ji} \tag{9}$$

where $\left(M^m\right)_{ji}$ is the block ji in the m the power of the concatenated matrix M.
Inserting (9) in (6) we rewrite the optimization problem at time t for the agent i:

$$\begin{aligned}
& A_{ii}^{(i)} X_i(t) + \mathbf{E}_i \sum_{m \geq 1} \left(\sum_{j \leftarrow i} \beta_i^m \,{}^t\!\left(A_{ij}^i \left(M^{m-1}\right)_{ji} \right) \right) X_i(t+m) \\
& + \mathbf{E}_i \sum_{j \leftarrow i} \left(A_{ij}^{(i)} X_j(t-1) + \sum_{m \geq 1} \beta_i^m \,{}^t\!\left(M^{m-1}\right)_{ji} A_{jj}^{(i)} X_j(t+m-1) \right) \\
& + \sum_{j \not\leftarrow i} A_{ij}^{(i)} X_j(t-1) \\
= \; & \varepsilon_i(t)
\end{aligned}$$

which can be rewritten in a more compact form :

$$\begin{aligned}
A_{ii}^{(i)} X_i(t) = & -\mathbf{E}_i \left\{ \sum_{m \geq 1} \left(\sum_{j \leftarrow i} \beta_i^m \,{}^t\!\left(A_{ij}^i \left(M^{m-1}\right)_{ji} \right) \right) X_i(t+m) \right. \\
& \left. + \sum_{j \leftarrow i} \left(\sum_{m \geq 1} \beta_i^m \,{}^t\!\left(M^{m-1}\right)_{ji} A_{jj}^{(i)} X_j(t+m-1) \right) \right\} \\
& - \sum_{j} A_{ij}^{(i)} X_j(t-1) + \varepsilon_i(t)
\end{aligned} \tag{10}$$

2.3.2 Pattern of information

The full resolution of the model relies on the expectations $E_i M^{m-1}$ which itself relies on each agent's information set, i.e. it's knowledge of other agents parameters.
We propose a pattern of information over the domination graph, which describes the way an agent performs its forecasts.
Each agent knows the domination relations of the subtree he strategically dominates, but ignores the reactivity of the subtree's agents to external, non dominated agents. In other words, Agent i knows the values of the $A_{k\ell}^{(k)}$ for $i \rightarrowtail k$ and $i \rightarrowtail \ell$. The remaining coefficients $A_{k\ell}^{(k)}$ are forecasted to 0 for this agent. Remark that, as a consequence of our assumptions, agents do not attribute a probability to the coefficients they forecast, but rather a fixed value.
Moreover, we assume that, at each period t, Agent i knows the signals $X_j(t-1)$ for $i \rightarrowtail j$ and for the $X_j(t-1)$ $j \not\leftarrowtail i$ by which he is affected.
From our hypotheses we can infer some results about the agents' forecasts.
First, Agent i forecasts to 0 the actions of all agents he does not dominate. That is for $j \not\leftarrowtail i$ and $m \geq 0$:

$$\mathbf{E}_i X_j\left(t+m\right) = 0$$

Consider the optimization equation (10) of Agent j, and compute it's expectation by Agent i. Agent's i forecasts of all coefficients in the equation equal 0 which implies $E_i X_j(t) = 0$.
In the same way,

$$E_i X_j\left(t+m\right) = 0 \text{ for } m \geq 0.$$

Using Equation (7) the last relation can be translated in an expectation formula about matrix M :

$$\mathbf{E}_i M_j = 0 \text{ for } j \not\leftarrowtail i$$

Moreover, Agent i knowing it's own parameters, and it's own reaction to perceived signals, one set :

$$\mathbf{E}_i M_i = M_i$$

More precise forecasts of the remaining blocks of matrix M will be needed. We set:

$$\begin{aligned}\left(\mathbf{E}_i M\right)_{\ell,j} &= 0 \text{ for } j \not\leftarrowtail i \text{ and any } \ell \\ \left(\mathbf{E}_i M\right)_{\ell,j} &= \left(M\right)_{\ell,j} \text{ for } i \rightarrowtail j \text{ and } i \rightarrowtail \ell\end{aligned}$$

Having defined the forecasts of the matrix M of parameters by the various agents, we have to complete the information scheme by fixing the forecasts of the random perturbations as well as the goals vectors.
Future shocks are unknown, and random shocks P_i are assumed to be :

$$E_i P_j\left(t+m\right) = 0 \text{ for } m \geq 1 \text{ and } E_i P_j(t) = 0 \text{ for } i \neq j \text{ and } E_i P_i(t) = P_i(t)$$

At time t, agent i is only aware of the shock he is affected by, $P_i(t)$.
Agent i is aware of the goals of the agents he is strategically dominating, so that :

$$E_i \tilde{X}_j^{(i)} = \tilde{X}_j^{(i)} \text{ for } j \leftarrowtail i$$

and

$$E_i \tilde{X}_k^{(j)} = \tilde{X}_k^{(j)} \text{ for } k \leftarrowtail j \leftarrowtail i.$$

Given this scheme of forecasts and the dynamical equation (8) , we can infer for $m \geq 0$ that, for $i \rightarrowtail j$:

$$\begin{aligned}\mathbf{E}_i\left[X_j\left(t+m\right)\right] &= \mathbf{E}_i\left(\left(\left(M\right)^{m+1}\right)_j\left[X(t-1)\right]\right) \\ &= \left(\left(\mathbf{E}_i M\right)^{m+1}\right)_j \mathbf{E}_i\left[X(t-1)\right],\end{aligned}$$

and for $j = i$:

$$\mathbf{E}_i \left[X_i \left(t+m \right) \right] = \left(\left(\mathbf{E}_i M \right)^{m+1} \right)_i \mathbf{E}_i \left[X(t-1) \right]$$

The information setup and the agents' parameters forecasts being detailed, we can turn to the resolution of the optimization equations. This is done in the most general way, in the form of an algorithm.

2.3.3 The resolution algorithm

The postulated form of the dynamic (Equation (8)) is:

$$\left[X \left(t+m \right) \right] = M^m \left[X(t) \right] + \sum_{k=1}^{m} M^{m-k} \left[W \left(t+k \right) \right]$$

or under the form of its i block line :

$$X_i \left(t+m \right) = \sum\nolimits_{j \in \Gamma} \left(\left(M^m \right)_{ij} X_j(t) + \sum_{k=1}^{m} \left(M^{m-k} \right)_{ij} W_j \left(t+k \right) \right)$$

To identify the matrix M and the vectors $W \left(t+k \right)$, this relation is inserted in Equation (10) for each i.[20] Since we have a set of equation, one for each agent, the resolution follows the order of the graph. We proceed upstream, solving for the less to the most strategical agent.
Rewriting Equation (10) for Agent i, and using the relation

$$E_i \left(M^m \right) = \left(\mathbf{E}_i \left(M \right) \right)^m$$

yields

$$\begin{aligned} A_{ii}^{(i)} X_i(t) \quad = \quad & -\Bigg\{ \sum_{m \geq 1} \left(\sum_{j \leftharpoondown i} \beta_i^m \, {}^t\!\left(A_{ij}^i \left(\left(\mathbf{E}_i M \right)^{m-1} \right)_{ji} \right) \right) \mathbf{E}_i X_i \left(t+m \right) \\ & \quad + \sum_{j \leftharpoondown i} \left(\sum_{m \geq 1} \beta_i^m \, {}^t\!\left(\left(\mathbf{E}_i M \right)^{m-1} \right)_{ji} A_{jj}^{(i)} \mathbf{E}_i X_j \left(t+m-1 \right) \right) \Bigg\} \\ & - \sum_j A_{ij}^{(i)} X_j(t-1) + \varepsilon_i(t). \end{aligned}$$

Implement the *a priori* form of the solution :

$$\left[X \left(t+m \right) \right] = M^m \left[X(t) \right] + \sum_{k=1}^{m} M^{m-k} \left[W \left(t+k \right) \right] \text{ for } m \geq 0$$

The variables $X_i(t-1)$ and $W_i(t)$ being independent, the expectation pattern[21] leads to two systems of equations defining both M_i and $W_i(t)$.

$$\begin{aligned} A_{ii}^{(i)} M_i \left[X(t-1) \right] \quad = \quad & -\Bigg\{ \sum_{m \geq 1} \left(\sum_{j \leftharpoondown i} \beta_i^m \, {}^t\!\left(A_{ij}^i \left(\left(\mathbf{E}_i M \right)^{m-1} \right)_{ji} \right) \right) \left(\left(\mathbf{E}_i M \right)^{m+1} \right)_i \left[X(t-1) \right] \\ & \quad + \sum_{j \leftharpoondown i} \left(\sum_{m \geq 1} \beta_i^m \, {}^t\!\left(\left(\mathbf{E}_i M \right)^{m-1} \right)_{ji} A_{jj}^{(i)} \left(\left(\mathbf{E}_i M \right)^m \right)_j \left[X(t-1) \right] \right) \Bigg\} \\ & - \sum_j A_{ij}^{(i)} X_j(t-1) \end{aligned} \tag{11}$$

[20] The expectations in (10) are computed with the rules given in Section 2.3.2.
[21] See Section 2.3.2.

$$A_{ii}^{(i)}W_i(t) = -\left\{\sum_{m\geq 1}\left(\sum_{j\leftarrow i}\beta_i^{m}\ {}^t\!\left(A_{ij}^{i}\left((\mathbf{E}_iM)^{m-1}\right)_{ji}\right)\right)\sum_{k=1}^{m+1}\left((\mathbf{E}_iM)^{m+1-k}\right)_i\mathbf{E}_i\left[W\left(t-1+k\right)\right]\right.$$
$$\left.+\sum_{j\leftarrow i}\left(\sum_{m\geq 1}\beta_i^{m}\ {}^t\!\left((\mathbf{E}_iM)^{m-1}\right)_{ji}A_{jj}^{(i)}\sum_{k=1}^{m}\left((\mathbf{E}_iM)^{m-k}\right)_j\mathbf{E}_i\left[W\left(t-1+k\right)\right]\right)\right\}$$
$$+\varepsilon_i(t). \tag{12}$$

Since $\left((\mathbf{E}_iM)^{m-1}\right)_{ji}$ is the (ji) block of the concatenated matrix $(\mathbf{E}_iM)^{m-1}$, and M appears in both Equations (11) and (12), we will first solve Equation (11) to find M, then solve Equation (12).

Equation (11) can be written equivalently

$$\left\{A_{ii}^{(i)}M_i+\sum_{m\geq 1}\beta_i^m\sum_{j\leftarrow i}{}^t\!\left((\mathbf{E}_iM)^{m-1}\right)_{ji}\left(A_{ji}^{(i)}\left((\mathbf{E}_iM)^{m+1}\right)_i+A_{jj}^{(i)}\left((\mathbf{E}_iM)^{m}\right)_j\right)\right\}\left[X\left(t-1\right)\right]$$
$$= -\sum_{j\neq i}A_{ij}^{(i)}X_j\left(t-1\right)$$

which leads to the defining relation for M_i:

$$A_{ii}^{(i)}M_i+\sum_{m\geq 1}\beta_i^m\sum_{j\leftarrow i}{}^t\!\left((\mathbf{E}_iM)^{m-1}\right)_{ji}\left(A_{ji}^{(i)}\left((\mathbf{E}_iM)^{m+1}\right)_i+A_{jj}^{(i)}\left((\mathbf{E}_iM)^{m}\right)_j\right)=-\left[A_i^{(i)}\right] \tag{13}$$

where $\left[A_i^{(i)}\right]$ is the concatenation in line of the matrices $A_{ij}^{(i)}$ with 0 in the i-th place.
Since one can always rescale the action variables $X_i(t-1)$ so that the matrix $A_{ii}^{(i)}$ is diagonal with eigenvalues 1,we will, starting from here and without any lack of generality, normalize the $A_{ii}^{(i)}$ to be equal to 1. A null eigenvalue would correspond, in this case, to a redundant action variable**, since it** could be expressed as a function of others. It is therefore excluded.
Equation (13) cannot be solved analytically in a general way. However, it can be solved recursively as series of β_i. To do so, write the matrix M_i as a power series of β_i, that is

$$M_i=\sum_{m\geq 0}\beta_i^m M_i^{(m)}$$

Inserting this expression in Equation (13) leads directly to the recursive system for the matrices $M_i^{(m)}$:

$$M_i^{(0)} = -\left[A_i^{(i)}\right]$$
$$M_i^{(k)} = -\sum_{j\leftarrow i}\sum_{m=1}^{k}\left[{}^t\!\left((\mathbf{E}_iM)^{m-1}\right)_{ji}\left(A_{ji}^{(i)}\left((\mathbf{E}_iM)^{m+1}\right)_i+A_{jj}^{(i)}\left((\mathbf{E}_iM)^{m}\right)_j\right)\right]_{k-m}$$

where the bracket $[A]_{k-m}$ denoting the $k-m$-th term in the power expansion in β_i of any quantity A.

This can be computed for every model given any information pattern. The parameter β_i being lower than 1, the matrices $M_i^{(m)}$ represent some corrections to the case $\beta_i=0$, when agents have no interest in the future. To get an insight of the relevant dynamical mechanisms at stake, it will be sufficient to cut the series in β_i at some fixed order.

Equation (11) being solved for E_iM, we turn to Equation (12) to find the perturbation vector $[W(t)]$. Recall that we set

$$\varepsilon_i(t)=-P_i(t)+\mathbf{E}_i\sum_{j\leftarrow i}\sum_{m\geq 1}\beta_i^{m}\ {}^t\left(M^{m-1}\right)_{ji}A_{jj}^{(i)}\tilde{X}_j^{(i)}.$$

Given the form of the expectations[22], we infer $E_i P_j\,(t+m) = 0$, so that the vector $W_i(t)$ in Equation (12) depends only on the random perturbation $P_i(t)$ and the constants $\tilde{X}_j^{(i)}$ for $j \leftarrowtail i$. Moreover, since (12) is linear, its solution is of the form :

$$W_i(t) = -\left(A_{ii}^{(i)}\right)^{-1}\left(\hat{P}_i(t) + \hat{X}_i\right) \tag{14}$$

where $\hat{P}_i(t)$ is proportional to $P_i(t)$, and $\hat{X}_i$ is a linear function of the $\tilde{X}_j^{(i)}$ with $j \leftarrowtail k$ to be identified. Since we assume the information pattern :

$$E_i \tilde{X}_j^{(i)} = \tilde{X}_j^{(i)} \text{ for } j \leftarrowtail i.$$

we have also

$$E_i \hat{X}_j = \hat{X}_j \text{ for } j \leftarrowtail i \text{ and } j = i$$

.

and

$$E_i W_j\,(t+m) = -\left(A_{jj}^{(j)}\right)^{-1}\hat{X}_j \text{ for } j \leftarrowtail i \text{ and } j = i \text{ for } m \geq 0$$

and finally

$$E_i\left[W\,(t+m)\right] = \left[\hat{W}^{(i)}\,(t+m)\right] \text{ for } m \geq 0$$

where $\left[\hat{W}^{(i)}\,(t+m)\right]$ denotes the concatenated vector whose components are:

$$\hat{W}_i^{(i)}(t) = W_i(t)$$

and

$$\hat{W}_j^{(i)}\,(t+m) = -\left(A_{jj}^{(j)}\right)^{-1}\hat{X}_j$$

for $j \leftarrowtail i$ and 0 otherwise.

Replacing Equation (14) in Equation (12) leads to the following expressions for $\hat{X}_i$ and $\hat{P}_i(t)$:

$$\begin{aligned}
\hat{X}_i \quad = \quad & \sum_{\ell \leftarrowtail i}\left\{\sum_{m\geq 1}\left(\sum_{j \leftarrowtail i}\beta_i^{m\ t}\left(A_{ij}^{i}\left((\mathbf{E}_i M)^{m-1}\right)_{ji}\right)\right)\sum_{k=1}^{m+1}\left((\mathbf{E}_i M)^{m+1-k}\right)_{i\ell}\right. \\
& \left. + \sum_{j \leftarrowtail i}\left(\sum_{m\geq 1}\beta_i^{m\ t}\left((\mathbf{E}_i M)^{m-1}\right)_{ji} A_{jj}^{(i)}\sum_{k=1}^{m}\left((\mathbf{E}_i M)^{m-k}\right)_{j\ell}\right)\right\}\left(A_{\ell\ell}^{(l)}\right)^{-1}\mathbf{E}_i \hat{X}_\ell \\
& - \sum_{j \leftarrowtail i}\sum_{m\geq 1}\beta_i^{m\ t}\left((\mathbf{E}_i M)^{m-1}\right)_{ji} A_{jj}^{(i)}\mathbf{E}_i \tilde{X}_j^{(i)}
\end{aligned} \tag{15}$$

[22] See Section 2.3.2.

and

$$\begin{aligned}\hat{P}_i(t) &= \left\{ \sum_{m\geq 1} \left(\sum_{j \leftarrow\!\!\prec i} \beta_i^{m\ t}\left(A_{ij}^{i} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right) ((\mathbf{E}_i M)^m)_{ii} \right. \\ &\left. + \sum_{j \leftarrow\!\!\prec i} \left(\sum_{m\geq 1} \beta_i^{m\ t}\left((\mathbf{E}_i M)^{m-1} \right)_{ji} A_{jj}^{(i)} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right\} \left(A_{ii}^{(i)} \right)^{-1} \hat{P}_i(t) \\ &+ P_i(t). \end{aligned} \tag{16}$$

The first equation is a recursive relation which can only be solved in the inverse order of domination of the graph. The vectors $\hat{X}_i$ are determined by the vectors $E_i \hat{X}_j$ for $j \leftarrow\!\!\prec i$. Once the $\hat{X}_j$ for $j \leftarrow\!\!\prec i$ are computed, the expectations $E_i \hat{X}_j$ can be calculated, and ultimately the vectors $\hat{X}_i$ are obtained.
The second equation yields directly $\hat{P}_i(t)$ as a function of $P_i(t)$. One gets

$$\hat{P}_i(t) = (1 - C_i)^{-1} P_i(t)$$

where the matrix C_i is given by

$$\begin{aligned} C_i = &\left\{ \sum_{m\geq 1} \left(\sum_{j \leftarrow\!\!\prec i} \beta_i^{m\ t}\left(A_{ij}^{i} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right) ((\mathbf{E}_i M)^m)_{ii} \right. \\ &\left. + \sum_{j \leftarrow\!\!\prec i} \left(\sum_{m\geq 1} \beta_i^{m\ t}\left((\mathbf{E}_i M)^{m-1} \right)_{ji} A_{jj}^{(i)} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right\} \left(A_{ii}^{(i)} \right)^{-1}. \end{aligned}$$

which ends the resolution method of the general case. More explicit formulas are impossible at this level of generality[23]. Nevertheless, we provide a short account of the equilibrium and the fluctuation around it.

2.3.4 Equilibrium and dynamics around the equilibrium

Let us now briefly present the long run equilibrium of the system. Starting from the dynamic equation :

$$X_i(t) = M_i [X(t-1)] + W_i(t).$$

Agent i long run equilibrium $\bar{X}_i$ can be defined by equating $X_i(t) = X_i(t-1) = \bar{X}_i$ for each i, and canceling all random perturbations $P_i(t)$. Concatenating all $\bar{X}_i$ in the system long run equilibrium $\left[\bar{X}\right]$, we get

$$\left[\bar{X}\right] = M \left[\bar{X}\right] + \left[\bar{W}\right]$$

likewise, $\left[\bar{W}\right]$ is the vector $[W]$ where all external perturbations $P_i(t)$ have been set to 0 :

$$\left[\bar{X}\right] = (1 - M)^{-1} \left[\bar{W}\right] = \sum_{m\geq 0} M^m \left[\bar{W}\right].$$

This last expression only exists if $1 - M$ is invertible. Otherwise multiple equilibria or no equilibrium could arise, a possibility that is neither the generic case, nor has any practical interest here[24].
To describe the fluctuation around $\left[\bar{X}\right]$, recall that the vectors $[W(t)]$ has $W_i(t) = -\left(A_{ii}^{(i)}\right)^{-1}\left(\hat{P}_i(t) + \hat{X}_i\right)$ for components. Given the dynamic equation :

$$[X(t)] = M [X(t-1)] + [W(t)]$$

[23] The eigenvalues of M relevant to the dynamic evolution of the system, for instance, can only be computed for particular cases.
[24] Besides, it does not appear in the three-agent model example presented below.

and the equilibrium relation

$$\left[\bar{X}\right] = M\left[\bar{X}\right] + \left[\bar{W}\right],$$

we define the fluctuations vectors :

$$\left[Y(t)\right] = \left[X(t) - \bar{X}\right]$$

and :

$$\left[\hat{W}(t)\right] = \left[W - \bar{W}\right] = -\left[\left(A_{ii}^{(i)}\right)^{-1}\hat{P}_i(t)\right]$$

Finally the relation :

$$\left[X(t) - \bar{X}\right] = M\left[X(t-1) - \bar{X}\right] + \left[W(t) - \bar{W}\right]$$

leads to

$$\left[Y(t)\right] = M\left[Y(t-1)\right] + \left[\hat{W}(t)\right].$$

Assume that the system, in equilibrium at time $t = 0$ at $Y(0) = 0$, is perturbed by one single initial shock $\hat{W}(0)$, i.e. $\hat{W}(t) = 0$ for $t > 0$. In this case, one has

$$\left[Y(m)\right] = M^m \hat{W}(0).$$

and the eigenvalues of M will determine the stability of the system.

Now that we have all the apparatus to solve explicitly any model, analytically in some cases, we will apply it in for the three-agent model[25].

3 Application of the general model: full resolution of the three-agent model

This section details the resolution of the three-agent model presented in Section 2.2.2. The optimization problems are solved in the inverse order of strategic domination, respectively here for *Agent B*, then for *Agent C* and ultimately for *Agent U*. Solving the model amounts to finding the matrices M_i and the vectors W_i. We proceed with the following steps.

1. Determination of the matrices M_i

- **Solve the optimization problem for Agent B** The body has no information about the conscious and the unconscious so that its expectations about their present and future actions are set to zero. And since it ignores the influence of the unconscious on the conscious and reciprocally, equation (11) reduces to

$$M_{\mathrm{B}} = -\left[A_{\mathrm{B}}^{(\mathrm{B})}\right] = (0, 1, -\alpha, 0, 0)$$

- **Solve the optimization problem for Agent C** Recall that we have set $\beta_i = \beta$ for all agents. Thus Equation (11) for M_{C} is

$$M_{\mathrm{C}} = -\left[A_{\mathrm{C}}^{(\mathrm{C})}\right] - \sum_{m\geq 1}\beta^m\ {}^t\!\left(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m-1}\right)_{\mathrm{BC}}\left(\nu\left(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m+1}\right)_{\mathrm{C}} + \delta\left(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m}\right)_{\mathrm{B}}\right)$$

with

$$\left[A_{\mathrm{C}}^{(\mathrm{C})}\right] = (\nu, 0, 0, \eta, -\kappa)$$

[25] For more complex situations and numerically defined parameters, this resolution algorithm allows finding solutions through numerical computations.

and

$$\mathbf{E}_{\mathrm{C}}M = \begin{pmatrix} M_{\mathrm{BB}} & M_{\mathrm{BC}} & 0 \\ M_{\mathrm{CB}} & M_{\mathrm{CC}} & M_{\mathrm{CU}} \\ 0 & 0 & 0 \end{pmatrix}$$

Under our hypotheses, *Agent C* knows *Agent B*'s reaction to its own action, i.e. the reaction coefficient M_{BC}. However it does not know the impact of *Agent U*'s action on *Agent B*. This implies that $E_{\mathrm{C}}M_{\mathrm{BC}} = 0$. Similarly *Agent C* is neither aware of the impact of it's own action over *Agent U,* nor of the impact of *Agent B's* action over *Agent U.* The last row of blocks is therefore set to 0 in $E_{\mathrm{C}}M$. The upper-right zero reflects *Agent C*'s ignorance of *Agent U*'s action over *Agent B*[26].

To solve Equation (11) for $i = C$, we postulate the following general form for $E_{\mathrm{C}}M$:

$$\begin{aligned}
E_{\mathrm{C}}M &= \begin{pmatrix} M_{\mathrm{BB}} & M_{\mathrm{BC}} & 0 \\ M_{\mathrm{CB}} & M_{\mathrm{CC}} & M_{\mathrm{CU}} \\ 0 & 0 & 0 \end{pmatrix} \\
&= \begin{pmatrix} (0) & (1) & \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \\ (x) & (y) & \begin{pmatrix} 0 & z & u \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ x & y & 0 & z & u \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

where the coefficients x, y, z, u have to be identified. To do so, we diagonalize $E_{\mathrm{C}}M$ to determine its powers. Its eigenvalues are 0 as a triple root, and

$$\frac{1}{2}y \pm \frac{1}{2}\sqrt{y^2 + 4x}$$

.

All computations done, this yields for $m \geq 1$

$$(\mathbf{E}_{\mathrm{C}}M)^m = \frac{1}{\sqrt{y^2+4x}} \begin{pmatrix} -x\delta_{m-1} & -\delta_m & 0 & -z\delta_{m-1} & -u\delta_{m-1} \\ -x\delta_m & -\delta_{m+1} & 0 & -z\delta_m & -u\delta_m \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

with:

$$\delta_m = \left(\frac{1}{2}y - \frac{1}{2}\sqrt{y^2+4x}\right)^m - \left(\frac{1}{2}y + \frac{1}{2}\sqrt{y^2+4x}\right)^m$$

Coming back to (11), we have :

$$M_{\mathrm{C}} = -\left[A_{\mathrm{C}}^{(\mathrm{C})}\right] - \sum_{m \geq 1} \beta^m \, {}^t\!\left((\mathbf{E}_{\mathrm{C}}M)^{m-1}\right)_{\mathrm{BC}} \left(\nu\left((\mathbf{E}_{\mathrm{C}}M)^{m+1}\right)_{\mathrm{C}} + \delta\left((\mathbf{E}_{\mathrm{C}}M)^m\right)_{\mathrm{B}}\right).$$

The generic term of the sum over m can thus be expressed as :

[26] For more details see 2.3.2.

$$
\begin{aligned}
&-\sum_{m\geq 1}\beta^m\, {}^t\!\Big(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m-1}\Big)_{\mathrm{BC}}\Big(\nu\left(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m+1}\right)_{\mathrm{C}}+\delta\left(\left(\mathbf{E}_{\mathrm{C}}M\right)^{m}\right)_{\mathrm{B}}\Big)\\
&=\beta\sum_{m\geq 0}\beta^m\frac{\delta_m}{y^2+4x}\left(\begin{array}{c}\nu\left(\begin{array}{ccccc}-x\delta_{m+2} & -\delta_{m+3} & 0 & -z\delta_{m+2} & -u\delta_{m+2}\end{array}\right)\\ +\delta\left(\begin{array}{ccccc}-x\delta_{m} & -\delta_{m+1} & 0 & -z\delta_{m} & -u\delta_{m}\end{array}\right)\end{array}\right)
\end{aligned}
$$

Moreover we have :

$$
-\left[A_{\mathrm{C}}^{(\mathrm{C})}\right]=\left(\begin{array}{ccccc}-\nu & 0 & 0 & -\eta & \kappa\end{array}\right)
$$

By identification, the coefficients of the row $M_{\mathrm{C}}=\left(\begin{array}{ccccc}x & y & 0 & z & u\end{array}\right)$ satisfy the following equations:

$$
\begin{aligned}
x &= -\nu+\frac{1}{y^2+4x}\left(-x\delta\beta\sum_{m\geq 0}\beta^m\delta_m^2-x\nu\beta\sum_{m\geq 0}\beta^m\delta_{m+2}\delta_m\right)\\
y &= \frac{1}{y^2+4x}\left(-\nu\beta\sum_{m\geq 0}\beta^m\delta_m\delta_{m+3}-\delta\beta\sum_{m\geq 0}\beta^m\delta_{m+1}\delta_m\right)\\
z &= \frac{1}{y^2+4x}\left(-z\delta\beta\sum_{m\geq 0}\beta^m\delta_m^2-z\nu\beta\sum_{m\geq 0}\beta^m\delta_{m+2}\delta_m\right)\\
u &= \frac{1}{y^2+4x}\left(-u\delta\beta\sum_{m\geq 0}\beta^m\delta_m^2-u\nu\beta\sum_{m\geq 0}\beta^m\delta_{m+2}\delta_m\right)
\end{aligned}
$$

The sums over β can be explicitly computed using the relations

$$
\begin{aligned}
\frac{\beta}{y^2+4x}\sum_{m\geq 0}\beta^m\delta_m^2 &= \beta\frac{1-\beta x}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)},\\
\frac{\beta}{y^2+4x}\sum_{m\geq 0}\beta^m\delta_m\delta_{m+1} &= \beta^2\frac{y}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)},\\
\frac{\beta}{y^2+4x}\sum_{m\geq 0}\beta^m\delta_{m+2}\delta_m &= \beta\frac{x(1-\beta x)+y^2}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)},\\
\frac{\beta}{y^2+4x}\sum_{m\geq 0}\beta^m\delta_m\delta_{m+3} &= y\beta^2\frac{-x^2\beta+2x+y^2}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)},
\end{aligned}
$$

and

$$
\sum_{m\geq 1}\beta^m\delta_{m-1}\delta_{m+2}=\beta\sum_{m\geq 0}\beta^m\delta_m\delta_{m+3}=\beta\left(\left(\frac{-a^3}{1-\beta a^2}\right)+\frac{a^3(a+y)^3}{1+\beta a(a+y)}+\frac{(a+y)^3}{1-\beta(a+y)^2}\right).
$$

The equations for x and y are then

$$
\begin{aligned}
x &= -\nu+x\beta^2\frac{-(\delta+x\nu)(1-\beta x)-y^2\nu}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)}\\
y &= y\beta^2\frac{-\delta-2x\nu-y^2\nu+x^2\beta\nu}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)}
\end{aligned}
$$

The equation for y implies

$$
y=0\text{ or }1=\beta^2\frac{-\delta-2x\nu-y^2\nu+x^2\beta\nu}{(\beta x+1)\left((1-\beta x)^2-\beta y^2\right)}
$$

The second solution $y \neq 0$ implies a diverging behavior for x when $\beta \to 0$. This contradicts agents' myopic behavior when $\beta \to 0$. We must therefore discard this solution, and merely keep the case $y = 0$, which leads directly to the following equation for x:

$$-x - \nu + x^3\beta^2 - x\beta^2\delta = 0.$$

We systematically refer to the benchmark case $\beta = 0$. An analytic solution for x is therefore useless. Rather, we will look for a solution as a series of β and its explicit expression to the third order.
Indeed, the action of *Agent U* on *Agent B,* it's consequence on *Agent C* and ultimately on *Agent U*, needs three periods to be effective. By identification :

$$x = -\nu + \nu\beta^2\Delta + O\left(\beta^4\right).$$

The parameters z and u depending x and y can then be computed, and are given by:

$$z = -\eta - z\left(\beta\frac{\delta + x\nu - x\beta\delta - x^2\beta\nu}{(x\beta + 1)\left(x^2\beta^2 - 2x\beta + 1\right)}\right)$$

and

$$u = \kappa - uz\left(\beta\frac{\delta + x\nu - x\beta\delta - x^2\beta\nu}{(x\beta + 1)\left(x^2\beta^2 - 2x\beta + 1\right)}\right).$$

It leads to the third order to the following series expansion:

$$\begin{aligned} z &= -\eta + \beta\eta\Delta - \beta^2\eta\Delta^2 + \beta^3\eta\Delta\left(\Delta^2 + 2\nu^2\right) + O\left(\beta^4\right) \\ u &= \kappa - \beta\kappa\Delta + \beta^2\kappa\Delta^2 - \beta^3\kappa\Delta\left(\Delta^2 + 2\nu^2\right) + O\left(\beta^4\right) \end{aligned}$$

• **Solve the optimization problem for Agent U** M_{U} and M can now be computed. Since agent U is the most informed agent, $E_{\mathrm{U}}M = M$. For *Agent U*, Equation (11) is :

$$\left\{M_{\mathrm{U}} + \sum_{m\geq 1}\beta^m \sum_{j\leftarrow U} {}^t\left(M^{m-1}\right)_{jU}\left(A_{ji}^{(\mathrm{U})}\left(M^{m+1}\right)_{\mathrm{U}} + A_{jj}^{(\mathrm{U})}\left(M^m\right)_j\right)\right\} = -\left[A_{\mathrm{U}}^{(\mathrm{U})}\right].$$

which, using $E_{\mathrm{U}}M = M$, reduces to

$$M_{\mathrm{U}} + \xi\sum_{m\geq 1}\beta^m\, {}^t\left(M^{m-1}\right)_{\mathrm{U}}\left(M^m\right)_{\mathrm{C}} = -\left[A_{\mathrm{U}}^{(\mathrm{U})}\right]. \tag{17}$$

We seek M as a series expansion to the third order in β and proceed by a recursive computation.

Write M as the matrix

$$M = \left(\begin{array}{ccccc} 0 & 1 & -\alpha & 0 & 0 \\ x & 0 & 0 & z & u \\ & & M_{\mathrm{U}} & & \end{array}\right)$$

Set the power series expansions

$$M = \sum_{m\geq 0}\beta^m M_m$$

and

$$M_{\mathrm{U}} = \sum_{m\geq 0} \beta^m \left(M_{\mathrm{U}}\right)_m$$

Note that M_m is defined by

$$M_m = \left(\begin{array}{c} \left(\begin{array}{ccccc} 0 & 1 & -\alpha & 0 & 0 \\ x & y & 0 & z & u \end{array} \right)_m \\ \left(M_{\mathrm{U}}\right)_m \end{array} \right)$$

where

$$\left(\begin{array}{ccccc} 0 & 1 & -\alpha & 0 & 0 \\ x & y & 0 & z & u \end{array} \right)_m$$

denotes the matrix

$$\left(\begin{array}{ccccc} 0 & 1 & -\alpha & 0 & 0 \\ x & y & 0 & z & u \end{array} \right)$$

truncated at order 3 in β.

As a consequence, Equation (17) leads to the recursive relation:

$$\left(M_{\mathrm{U}}\right)_{m+1} = -\xi \sum_{k=1}^{m+1} \left({}^t\!\left(M^{k-1}\right)_{\mathrm{U}} \left(M^k\right)_{\mathrm{C}} \right)_{m+1-k}.$$

Starting with $\left(M_{\mathrm{U}}\right)_0 = -\left[A_{\mathrm{U}}^{(\mathrm{U})}\right] = 0$, the above equation yields directly the successive terms $\left(M_{\mathrm{U}}\right)_k$ and $\left(M\right)_k$.

Ultimately, we get

$$\begin{aligned} M \quad = \quad & \left(\begin{array}{ccccc} 0 & 1 & -\alpha & 0 & 0 \\ -\nu & 0 & 0 & -\eta & \kappa \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \\ & +\beta \left(\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta\boldsymbol{\Delta} & -\kappa\boldsymbol{\Delta} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \\ & +\beta^2 \left(\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ \nu\boldsymbol{\Delta} & 0 & 0 & -\eta\boldsymbol{\Delta}^2 & \kappa\boldsymbol{\Delta}^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\nu\xi\eta & \alpha\nu\xi\eta & 0 & 0 \\ 0 & \kappa\nu\xi & -\alpha\kappa\nu\xi & 0 & 0 \end{array} \right) \\ & +\beta^3 \left(\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Delta\eta\left(\Delta^2+2\nu^2\right) & \Delta\kappa^3\left(\Delta^2+2\nu^2\right) \\ \alpha\nu^3\xi & 0 & 0 & \alpha\nu^2\xi\eta & -\alpha\kappa\nu^2\xi \\ 0 & -\Delta\nu\xi\eta & \Delta\alpha\nu\xi\eta & 0 & 0 \\ 0 & \Delta\kappa\nu\xi & -\Delta\alpha\kappa\nu\xi & 0 & 0 \end{array} \right) \\ & +O\left(\beta^4\right) \end{aligned}$$

with

$$\Delta = \delta - \nu^2 \text{ and } K = \eta^2 + \kappa^2$$

The computation of the eigenvalues is straightforward. One is equal to 0, as a triple root, and, the two other ones are complex conjugates

$$\pm i\sqrt{\nu}\left(1-\frac{1}{2}\beta^2\left(\xi K+\Delta\right)+\frac{1}{2}\beta^3\xi\left(-\alpha^2\nu^2+2\Delta K\right)\right)+O\left(\beta^4\right).$$

2. Determination of the vectors W_i. The vectors W_i are linear combinations of the constant vectors $\hat{X}_i$ and the random vectors $\hat{P}_i(t)$[27].

• Computation of the constant part $\hat{X}_i$ The computation of $\hat{X}_i$ yields the constant part of the W_i and are given by equation (15). We have:

$$\begin{aligned}\hat{X}_i &= \sum_{\ell\leftarrow i}\left\{\sum_{m\geq 1}\left(\sum_{j\leftarrow i}\beta_i^{m\ t}\left(A_{ij}^i\left(\left(\mathbf{E}_iM\right)^{m-1}\right)_{ji}\right)\right)\left(\left(\mathbf{E}_iM\right)^m\right)_{i\ell}\right. \\ &\quad \left.+\sum_{j\leftarrow i}\left(\sum_{m\geq 1}\beta_i^{m\ t}\left(\left(\mathbf{E}_iM\right)^{m-1}\right)_{ji}A_{jj}^{(i)}\left(\left(\mathbf{E}_iM\right)^{m-1}\right)_{j\ell}\right)\right\}\left(A_{\ell\ell}^{(l)}\right)^{-1}\hat{X}_\ell \\ &\quad -\sum_{j\leftarrow i}\sum_{m\geq 1}\beta_i^{m\ t}\left(\left(\mathbf{E}_iM\right)^{m-1}\right)_{ji}A_{jj}^{(i)}\tilde{X}_j^{(i)}.\end{aligned}$$

For $i=B$, there are no agents dominated by *Agent B* and $\hat{X}_{\mathrm{B}}=0$.
For $i=C$, since *Agent C* has a goal for *Agent B*, i.e. $\tilde{X}_{\mathrm{B}}^{(\mathrm{C})}\neq 0$, replacing directly in the previous equation yields $\hat{X}_{\mathrm{C}}=\left(\ -\beta^2\nu w_0\ \right)$.
For $i=U$, it is easy to check that the contribution of $\hat{X}_{\mathrm{C}}$ in the previous equation is nul at the third order. As a consequence, since *Agent U* has a goal $\tilde{X}_{\mathrm{C}}^{(\mathrm{U})}\neq 0$ for *Agent C*, Equation (15) for $i=U$ rewrites:

$$\begin{aligned}\hat{X}_{\mathrm{U}} &= \sum_{m\geq 1}\beta^{m\ t}\left(\left(\mathbf{E}_{\mathrm{U}}M\right)^{m-1}\right)_{\mathrm{CU}}\tilde{X}_{\mathrm{C}}^{(\mathrm{U})} \\ &= \sum_{m\geq 1}\beta^{m\ t}\left(M^{m-1}\right)_{\mathrm{CU}}\tilde{X}_{\mathrm{C}}^{(\mathrm{U})}.\end{aligned}$$

which at our order, and all calculations done, reduces to

$$\hat{X}_{\mathrm{U}}=\left(\beta^2\begin{pmatrix}0\\-\eta\\\kappa\end{pmatrix}+\beta^3\begin{pmatrix}\alpha\nu\\\Delta\eta\\-\Delta\kappa\end{pmatrix}\right)\tilde{X}_{\mathrm{C}}^{(\mathrm{U})}+O\left(\beta^4\right).$$

• Computation of the random part $\hat{P}_i(t)$. Following Equation (16), the random part of $\hat{P}_i(t)$ is given by

$$\hat{P}_i(t)=\left(1-C_i\right)^{-1}P_i(t)$$

[27] See Equations (15) and (16)

where C_i is the matrix

$$C_i = \left\{ \sum_{m\geq 1} \left(\sum_{j\leftarrow i} \beta_i^{m\ t} \left(A_{ij}^{i} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right) ((\mathbf{E}_i M)^m)_{ii} + \sum_{j\leftarrow i} \left(\sum_{m\geq 1} \beta_i^{m\ t} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} A_{jj}^{(i)} \left((\mathbf{E}_i M)^{m-1} \right)_{ji} \right) \right\} \left(A_{ii}^{(i)} \right)^{-1}. \quad (19)$$

For $i = B$**,** no agent is dominated by Agent B and we have $C_{\mathrm{B}} = 0$ and $\hat{P}_{\mathrm{B}}(t) = P_{\mathrm{B}}(t)$.

For $i = C$**,** one has

$$\begin{aligned} C_{\mathrm{C}} &= \sum_{m\geq 1} \left(\beta^{m\ t} \left(A_{\mathrm{CB}}^{(\mathrm{C})} \left((\mathbf{E}_{\mathrm{C}} M)^{m-1} \right)_{\mathrm{BC}} \right) \right) ((\mathbf{E}_{\mathrm{C}} M)^m)_{\mathrm{CC}} \\ &\quad + \sum_{m\geq 1} \beta_i^{m\ t} \left((\mathbf{E}_{\mathrm{C}} M)^{m-1} \right)_{\mathrm{BC}} A_{\mathrm{BB}}^{(\mathrm{C})} \left((\mathbf{E}_{\mathrm{C}} M)^{m-1} \right)_{\mathrm{BC}}. \end{aligned}$$

Using $A_{\mathrm{CB}}^{(\mathrm{C})} = \nu$, $A_{\mathrm{BB}}^{(\mathrm{C})} = \delta$ and the expression for $E_{\mathrm{C}} M$ computed previously, Equation (19) reduces to

$$C_{\mathrm{C}} = \beta^2 \Delta + O\left(\beta^4\right)$$

and, we get

$$\hat{P}_{\mathrm{C}}(t) = (1 - C_{\mathrm{C}})^{-1} P_{\mathrm{C}}(t) = \left(1 - \beta^2 \Delta\right) P_{\mathrm{C}}(t)$$

For $i = U$**,** $E_{\mathrm{U}} M = M$ and, in the same way, Equation (19), leads to

$$C_{\mathrm{U}} = \left(\sum_{m\geq 1} \beta^{m\ t} \left((M)^{m-1} \right)_{\mathrm{CU}} \xi \left(M^{m-1} \right)_{\mathrm{CU}} \right)$$

where

$$\xi = \beta + \gamma$$

At our order, it is equal to

$$C_{\mathrm{U}} = \beta^2 \xi \begin{pmatrix} 0 & 0 & 0 \\ 0 & \eta^2 & \kappa\eta \\ 0 & \kappa\eta & \kappa^2 \end{pmatrix} + \beta^3 \xi \begin{pmatrix} \alpha^2\nu^2 & 0 & 0 \\ 0 & -2\eta^2\Delta & -2\kappa\eta\Delta \\ 0 & -2\kappa\eta\Delta & -2\kappa^2\Delta \end{pmatrix} + O\left(\beta^4\right).$$

We find the vectors $W_i(t)$ by gathering the results for the $\hat{X}_i$ and C_i, and using Equation (14)

$$W_i(t) = -\left(A_{ii}^{(i)} \right)^{-1} \left(\hat{P}_i(t) + \hat{X}_i \right).$$

Ultimately, we get :

$$\begin{aligned} W_{\mathrm{B}}(t) &= -P_{\mathrm{B}}(t) \\ W_{\mathrm{C}}(t) &= -\left(1 - \beta^2 \Delta\right) P_{\mathrm{C}}(t) - \beta^2 \nu w_0 + O\left(\beta^4\right) \\ W_{\mathrm{U}}(t) &= -\begin{pmatrix} \alpha\beta^3\nu \\ \Delta\beta^3\eta - \beta^2\eta \\ \kappa\beta^2 - \Delta\kappa\beta^3 \end{pmatrix} \tilde{X}_{\mathrm{C}}^{(\mathrm{U})} - (1 + C_{\mathrm{U}}) P_{\mathrm{U}}(t) + O\left(\beta^3\right) \end{aligned}$$

and

$$\tilde{X}_{\mathrm{C}}^{(\mathrm{U})} = \frac{\gamma\left(\tilde{w}-w_0\right)+\sigma\left(1-\left(\tilde{f}+w_0\right)\right)}{\zeta}.$$

The matrix M and the vectors W_i being now known, they are used to determine the equilibrium.

3. Determination the equilibrium $\left[\bar{X}\right]$ The equilibrium is defined by the equation :

$$\left[\bar{X}\right] = M\left[\bar{X}\right] + \left[\bar{W}\right]$$

Its solution is

$$\left[\bar{X}\right] = \left(1-M\right)^{-1}\left[\bar{W}\right]$$

where

$$\left[\bar{W}\right] = \begin{bmatrix} 0 \\ 0 \\ \hat{X}_{\mathrm{U}} \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{X}_{\mathrm{C}} \\ \hat{X}_{\mathrm{U}} \end{bmatrix}$$

The expansion of $\left[\bar{W}\right]$ starting with a term of order 2, we need only to compute $\left(1-M\right)^{-1}$ to the first order:

$$\left(1-M\right)^{-1} = \begin{pmatrix} \frac{1}{\nu+1} & \frac{1}{\nu+1} & -\frac{\alpha}{\nu+1} & -\frac{1}{\nu+1}\left(\eta-\Delta\beta\eta\right) & \frac{1}{\nu+1}\left(\kappa-\Delta\kappa\beta\right) \\ -\frac{\nu}{\nu+1} & \frac{1}{\nu+1} & \alpha\frac{\nu}{\nu+1} & -\frac{1}{\nu+1}\left(\eta-\Delta\beta\eta\right) & \frac{1}{\nu+1}\left(\kappa-\Delta\kappa\beta\right) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} + O\left(\beta^2\right)$$

to find the equilibrium vector to the third order:

$$\begin{aligned} \left[\bar{X}\right] &= \beta^2 \tilde{X}_{\mathrm{C}}^{(\mathrm{U})} \begin{pmatrix} \frac{1}{\nu+1}\left(\kappa-\Delta\kappa\beta\right)^2 + \frac{1}{\nu+1}\left(\eta-\Delta\beta\eta\right)^2 - \alpha^2\beta\frac{\nu}{\nu+1} \\ \frac{1}{\nu+1}\left(\kappa-\Delta\kappa\beta\right)^2 + \frac{1}{\nu+1}\left(\eta-\Delta\beta\eta\right)^2 + \alpha^2\beta\frac{\nu^2}{\nu+1} \\ \alpha\beta\nu \\ \Delta\beta\eta-\eta \\ \kappa-\Delta\kappa\beta \end{pmatrix} + \frac{1}{\nu+1}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tilde{X}_{\mathrm{B}}^{(\mathrm{C})} + O\left(\beta^4\right) \\ &= \beta^2 \tilde{X}_{\mathrm{C}}^{(\mathrm{U})} \begin{pmatrix} \frac{K}{\nu+1} - \beta\frac{\nu\alpha^2+2K\Delta}{\nu+1} \\ \frac{K}{\nu+1} + \beta\frac{\nu^2\alpha^2-2K\Delta}{\nu+1} \\ \alpha\beta\nu \\ -\eta\left(1-\Delta\beta\right) \\ \kappa\left(1-\Delta\beta\right) \end{pmatrix} - \frac{\beta^2\nu w_0}{\nu+1}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + O\left(\beta^4\right). \end{aligned}$$

leading directly to the values presented in section 1:

$$
\begin{aligned}
\bar{n} &= \beta^2 \left(\frac{K - \nu w_0}{\nu + 1} - \beta \frac{\nu \alpha^2 + 2K\Delta}{\nu + 1} \right) \frac{\gamma \left(\tilde{w} - w_0 \right) + \sigma \left(1 - \left(\tilde{f} + w_0 \right) \right)}{\zeta} \\
\bar{w} &= \beta^2 \left(\frac{K - \nu w_0}{\nu + 1} + \beta \frac{\nu^2 \alpha^2 - 2K\Delta}{\nu + 1} \right) \frac{\gamma \left(\tilde{w} - w_0 \right) + \sigma \left(1 - \left(\tilde{f} + w_0 \right) \right)}{\zeta} + w_0 \\
\bar{s}_n &= \beta^3 \alpha \nu \frac{\gamma \left(\tilde{w} - w_0 \right) + \sigma \left(1 - \left(\tilde{f} + w_0 \right) \right)}{\zeta} \\
\bar{s}_w &= -\beta^2 \eta \left(1 - \Delta\beta \right) \frac{\gamma \left(\tilde{w} - w_0 \right) + \sigma \left(1 - \left(\tilde{f} + w_0 \right) \right)}{\zeta} \\
\bar{s}_f &= \beta^2 \kappa \left(1 - \Delta\beta \right) \frac{\gamma \left(\tilde{w} - w_0 \right) + \sigma \left(1 - \left(\tilde{f} + w_0 \right) \right)}{\zeta}
\end{aligned}
$$

4 Conclusion

This paper developed a dynamic model of interactions between several interacting agents (structures) and solved as an example the dynamic version of the three-agent model presented in [GLW]. This model allows to study the action of a “hidden” agent on the stability of a system, and the biases its presence introduces in an equilibrium. It allows to model apparent irrational behaviors or switches in some agents’ goals and cyclic behaviors. It can explain paradoxes such as the so-called “independence of irrelevant alternatives” or sudden changes in goals when external signals, in a given situation, activate structures that will drive the equilibrium toward an unexpected outcome.

A future, more complete treatment will allow for "unstable" structures switching on and off alternately via the intrinsic dynamics of the model.

Our results allow to ascribe inconsistent preferences and apparent switches in behaviors to the unknown action of some strategic structures behind conscious choices. They provide a pattern for the structures’ utilities and interactions. Besides, such a model is a framework in which the capacity of internal sub-structures to plan their actions could be tested. The cyclicality of the system and the differences between the behaviors of forward and non-forward looking structures could confirm, through empirical observation, the existence of forward-looking decisions in some unconscious processes. It should provide a better understanding of the mechanisms at stake in mental processes.This is left for future research.

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