Banks exposure to market risks

Galy, Michel

Banque de France, 39 rue Croix des Petits Champs, 75001 Paris

10 February 1989

BANQUE DE FRANCE February 10, 1989

EUROBANKING SEMINAR ON RISK ANALYSIS
UTRECHT, MAY 26-20, 1989

BANKS EXPOSURE TO MARKET RISKS

M. GALY (•)

Maturity transformation coupled with open foreign exchange positions expose financial intermediaries to unexpected changes in interest and exchange rates. This paper proposes to measure the degree of banks exposure to market risks by taking the variance of the total differential of the bank net-wealth against these prices.

+
+ +

(•) Deputy Director at BANQUE DE FRANCE, 39 rue croix des petits champs, 75001 Paris.
The views expressed are those of the author and do not reflect necessarily those of BANQUE DE FRANCE. A first French version of this paper was presented at the GRECO (groupement de recherche du CNRS) Conference, which took place in CLERMONT- FERRAND on June 9-10, 1988.

1.0 INTRODUCTION

Maturity transformation and open foreign exchange positions constitute the two main prongs of strategies that are meant to increase the profitability of international and domestic financial intermediaries. They are not immune, however, to sizable market risks since rationally anticipated changes in interest and exchange rates represent only a small fraction of their total variances. With the large price volatility plaguing today's domestic and foreign capital markets, it means that a bank caught by unexpected movements in interest and exchange rates might have its net wealth totally wiped out while its profit-and-loss account is still in the black.

In this context, it is tempting for bank managers and regulators alike to look for a global indicator able to provide an instantaneous and comprehensive picture of the bank exposure to market risks [1]. In that perspective, this paper attempts to evaluate the sensitivity of the bank present net wealth to interest and exchange rate variations by taking its total differential against these prices. The variability of this measure of vulnerability can then be established on the basis of a variances/covariances matrix of changes in interest and exchange rates and provide the management with an estimate of its maximum exposure.

The definition of the bank net wealth and the simplifications on which it relies are presented in section 2. Section 3 deals with the various approaches available to measure the present value of non-marketable fixed rate assets denominated in domestic currency. Section 4 extends this framework to any currency. The present net wealth and its exposure to market risks are determined accordingly in section 5. The last section offers an illustration based on the 1976-1986 variances/covarinces matrix of changes in interest and exchange rates.

---

1 In a perfect capital market it is not clear why bankers and regulators should have to worry about market risks. their surveillance, however, may be justified in the real world by institutional and systemic imperfections; on that point see GUREL and PYLE (1984).

2.0 DEFINITION OF THE BANK NET WEALTH

The net wealth of a financial intermediary can be obtained by adding up the present value of its future cash flows ad infinitum or by measuring the imbalance between its assets and liabilities valued at their actual or estimated market prices. For reasons which would appear clear later, the second solution has been retained. To implement it, lending and borrowing transactions are transformed into fixed rate operations under the following assumptions and simplifications :

- Floating rate instruments, the value of which is supposed to be impervious to interest rate changes [2] are turned into fixed rate assets by conveniently shortening their maturity. For example, a five year eurodollar loan which is rolled over every six months on the basis of the libor will be considered as a six months fixed rate loan.

- Banking operations recorded off-balance sheet are reinterpreted in terms of loans and deposits and included in the balance sheet. For instance, a 6-months forward purchase of U.S. dollars against French franc will be appended to the balance sheet as a combination of a borrowing in French franc and an investment in U.S. dollar for the same maturity. By the same token, the purchase of a 3-months interest rate future contract would translate on the asset side into a purchase of an equivalent security and on the liability side into a 3-months borrowing. Option contracts on interest or exchange rates are first expressed in terms of underlying assets or liabilities according to their hedge ratios. They will be entered in the balance sheet either by using the same transformation as above for options on futures or accompanied by consistent changes in the cash position in the case of spots operations.

- Negotiable instruments are marked to market and the valuation of other claims and debts can be obtained by discounting their future cash flows according to suitable functions of interest and exchange rates.

Given these assumptions, a bank present net wealth (W) -including transactions in all currencies can be defined by the following budget constraint :

$$(1) \qquad W = \sum_{i=1}^{k} W_i = \sum_{i=1}^{k} A_i(r_0, r_i, e_i) - L_i(r_0, r_i, e_i)$$

where the present values of assets A_i and liabilities L_i in currency i [3], expressed in domestic currency, are function of three variables : r_i and r_0 the foreign and domestic interest rates and e_i the exchange rate. [4].

Having laid down this equation and the assumptions on which it relies, it is necessary to specify more accurately the functional form of the valuation process based on r_0, r_i, e_i. This is the object of the next two sections.

---

[2] Note that the assumption that variable rate assets should sell at par is not always consistent with empirical evidence as pointed out for instance by RAMASWAMY and SUNDARESAN (1986).

[3] When the index underlying the variable is equal to 0, it represents the domestic currency with $e_0 = 1$.

[4] The exchange rate is quoted in terms of one unit of foreign currency per units of domestic currency.

3.0 VALUATION OF A NON-MARKETABLE FIXED RATE ASSET IN DOMESTIC CURRENCY

The current value of a fixed rate asset is determined by discounting its stream of future cash flows. The difficulty here is that there are several ways to do it with, as may be expected, different results. These various methods can be classified into three main categories :

- The traditional form of discounting based on the assumption that the interest rate is similar over the whole range of maturities;
- The valuation based on the yield curve;[5]
- The stochastic approach proposed by COX and al. (1979,1985).

3.1 THE TRADITIONAL METHOD OF VALUATION

If A_n is the certain value of a cash flow to be received after n years and r the equilibrium interest rate, the present value (V) of this claim consistent with market equilibrium should be such that :

(2) $$(1+r)^n V = A_n$$

If it were not the case, arbitragists would be in a position to collect riskless profits, which is excluded by assumption in a perfect capital market. Extending this property to an asset generating a stream of payments at regular intervals is straightforward :

$$(1+r)\ V_1 = A_1$$

$$(1+r)^2 V_2 = A_2$$

"

"

$$(1+r)^n V_n = A_n$$

Summing up the V_i for all maturities allows the present value of a fixed rate asset to be written as :

(3) $$V = \sum_{t=1}^{n} \frac{A_t}{(1+r)^t}$$

When this formula is applied to a security with a face value of (K) paying a constant coupon (C), it can be solved and written as follows :

(4) $$V = \frac{C}{r} + \frac{Kr - C}{r(1+r)^n}$$

which shows that the present value is merely the ratio of the coupon to the interest rate when n tends toward infinity. This result should be kept in mind when it comes to value bank assets and liabilities that have no defined terminal maturity. The presentation of equation (3) confirms also the

---

[5] For a general presentation see BIERWAG (1987).

well known characteristic of a fixed rate asset, that is, its present value is equal to its face value when the coupon-rate (C/K) is identical to the current interest rate.

The simplicity of this approach makes it the most frequently used for pricing fixed interest rate instruments. Its conception hinges, however, on the very strong and unrealistic assumption that the interest rate is the same for all maturities.

3.2 VALUATION BASED ON THE TERM STRUCTURE OF INTEREST RATES

In order to provide results more consistent with empirical evidence, this second method substitutes to the single interest rate in equation (3) the yield curve represented by the variable h(0,t) where the first index denotes the time -with 0 for the present- and the second stands for the maturity. Under the same assumptions of market efficiency than previously, the present value of a fixed interest rate asset becomes :

$$(5) \qquad V = \sum_{t=1}^{n} \frac{A_t}{(1+h(0,t))^t}$$

The theoretical advantage of this approach over the traditional method of valuation lies in the fact that it takes implicitly into account the rationally anticipated evolution of interest rates. Indeed , the yield curve should constitute in an efficient capital market with risk neutral economic agents an unbiased predictor of future interest rates in order to prevent the emergence of riskless profits of arbitrage. Basically, it means that an investment V_n over n periods at the interest rate h(0,n) yielding a single cash flow A_n , that is :

$$(6) \qquad (1+h(0,n))^n \; V_{n=}A_n$$

is equivalent, in terms of expectations, to the same investment rolled over every year (t) at the future rate $\tilde{h}(t,1)$ during n years, that is :

$$(7) \qquad E\prod_{t=1}^{n}(1+\tilde{h}(t,1))V_n = A_n$$

The same reasoning can be extended to all maturities on the yield curve and justifies the assertion that the evolution of interest rates is imbedded in their current term structure. The present values stemming from this approach are equal to those of the traditional method for a flat yield curve. they are lower (higher) if the yield curve is sloping upward (downward).

3.3 THE COX AND AL. APPROACH

Their objective is to determine how several random shocks affecting the yield curve can alter the present value of a fixed rate asset. In that perspective, they have replaced the term structure by a stochastic differential equation governing the interest rate, such that :

$$(8) \qquad dr = \beta(\mu - r)dt \; + \sigma\sqrt{r}\,dz$$

where μ is the steady state interest rate, β the speed of adjustment toward equilibrium, σ^2 the instantaneous variance of the interest rate and dz a GAUSS-WIENER process. In this framework, COX and al. show that in their model, specified in continuous time, the usual discount factor $P(r,t) = (1+r)^{-t}$ takes on the following form :

$$(9) \qquad P(r,t) = F(t)\exp(-rG(t))$$

where F and G are defined as :

$$F(t) = \left[\frac{\phi_1 \exp(\phi_2 t)}{\phi_2(\exp(\phi_1 t) - 1) + \phi_1} \right]^{\phi_3}$$

$$G(t) = \frac{\exp(\phi_1 t) - 1}{\phi_2(\exp(\phi_1 t) - 1) + \phi_1}$$

$$\phi_1 = ((\beta + \pi)^2 + 2\sigma^2)^{0.5}$$

$$\phi_2 = \frac{(\beta + \pi + \phi_1)}{2}$$

$$\phi_3 = \frac{2\beta\mu}{\sigma^2}$$

Under the hypothesis of a constant interest rate, it is worth noting that equation (9) collapses into $P(r,t) = \exp(-rt)$ which is nothing more than the usual discount factor of the traditional form of valuation expressed in continuous time.

In order to illustrate the different outcomes one can get by using those three methods of valuation, they were applied to a fixed rate asset issued with a 10 year maturity and a coupon rate of 10% under the assumption that the current interest rate may move from 8% to 10 and 12%. Results detailed in appendix indicate that the present value can vary by more than 15% according to the method chosen. Such an uncertainty should be a matter of concern because it impinges not only on the valuation of the bank net wealth but also on the measure of its vulnerability to interest rate changes.

In the following, this problem would be ignored and the remainder of the paper will focus on the traditional method of valuation.

4.0 VALUATION OF A FIXED RATE ASSET DENOMINATED IN ANY CURRENCY

So far, I have dealt only with the pricing of assets in domestic currency. This approach can be enlarged to an asset denominated in foreign currency by taking into account the change in the exchange rate expected to materialize until the asset matures. If the foreign exchange market is efficient and economic agents are risk neutral, the actual variation of the exchange rate happens to be equal [6], in terms of expectations, to the discount or premium on the forward exchange market or identically to the spread between domestic and foreign interest rates.

Formally, this proposal can be characterized by the two following relationships :

$$(10) \qquad F_{it} = e_{i0} \left[\frac{1+r_0}{1+r_i} \right]^t \simeq e_{i0} \, (1 + r_0 - r_i)^t$$

$$(11) \qquad E(\frac{\tilde{e}_{it}}{e_{i0}}) = \frac{F_{it}}{e_{i0}}$$

Equation (10) specifies the interest rate parity condition which says that the forward exchange rate (F_{it}) of currency i for period t is a function of the spot rate (e_{i0}) and of the interest rate differential ($r_0 - r_i$). Equation (11) defines the rational expectation hypothesis according to which actual changes in the future spot rate ($\tilde{e}_{it}$) cannot differ on average from those imbedded in the forward margin. Combining equations (10) and (11) and ignoring error terms provide us with a formula for the futur spot rate expressed in terms of the current spot rate and the interest differential,that is :

$$(12) \qquad \tilde{e}_{it} = e_{i0} \left[\frac{1+r_0}{1+r_i} \right]^t$$

It is now possible to determine the present value V_i of a fixed rate foreign asset by using equation (12) to translate the value of future cash flows as specified in equation (3) in terms of domestic currency. This new valuation relationship takes on the following form :

$$(13) \qquad V_{i0} = \sum_{t=1}^{n} \frac{a_{it}\tilde{e}_{it}}{(1+r_i)^t}$$

Substituting to $\tilde{e}_{it}$, its expression in equation (12) yields :

$$(14) \qquad V_{i0} = \sum_{i=1}^{n} \frac{e_{i0}a_{it}(1+r_0)^t}{(1+r_i)^{2t}}$$

Equation (14) indicates that the present value of a foreign asset is a function not only of current foreign interest and exchange rates but depends also on the domestic interest rate.

---

[6] a host of studies have dealt with international parity conditions.See for example GAAB and al.(1986).

5.0 BANKS EXPOSURE TO MARKET RISKS AND ITS VOLATILITY

The functionnal form of equation (1) that has been set up in the previous sections can now be used to assess the sensitivity of the bank's net wealth to changes in market prices.To do it, one needs only to calculate the total differential of this relationship against interest and exchange rates. Assuming that those changes are independent, the total differential of a given asset should read :

$$(15) \qquad dV_i = V'_{r_0}dr_0 + V'_{r_i}dr_i + V'_{e_{i0}}de_{i0}$$

where the partial derivatives of V_i vis a vis, respectively, r_0 ,r_i ,e_{i0} are expressed as :

$$(16) \qquad V'_{r_0} = \sum_{t=1}^{n} ta_{it}(1 + r_i)^{-2t}(1 + r_0)^{t-1}e_{i0}$$

$$(17) \qquad V'_{r_i} = -\sum_{t=1}^{n} 2ta_{it}(1 + r_0)^{t}(1 + r_i)^{-2t-1}e_{i0}$$

$$(18) \qquad V'_{e_{i0}} = \sum_{t=1}^{n} a_{it}(1 + r_0)^{t}(1 + r_i)^{-2t} = \frac{V_i}{e_{i0}}$$

Multiplying equations (16) and (17), respectively, by $\frac{1 + r_0}{V_i}$ and $\frac{1 + r_i}{V_i}$ and defining $\dot{D}, \ddot{D}$ as :

$$(19) \qquad \dot{D}_i = \sum_{t=1}^{n} \frac{ta_t(1 + r_i)^{-2t}(1 + r_0)^{t}e_{i0}}{V_i}$$

$$(20) \qquad \ddot{D}_i = 2\dot{D}_i$$

equation (15) can be written as follows :

$$(21) \qquad dV_i = \frac{\dot{D}_i V_i}{1 + r_0}dr_0 - \frac{\ddot{D}_i V_i}{1 + r_i}dr_i + \frac{V_i}{e_{i0}}de_{i0}$$

Checking that equation (21) is relevant for all currencies, including the domestic one, is straightforward. In this case, one gets the following identities : $r_i = r_0$; $e_0 = 1$ so that equation (21) collapses into :

$$(22) \qquad dV_0 = \frac{(\dot{D}_0 - \ddot{D}_0)V_0}{1 + r_0}dr_0 = -\frac{D_0 V_0}{1 + r_0}dr_0$$

where D_0 is the usual form of the MACAULAY duration [7] expressed as :

$$(23) \qquad D_0 = \sum_{t=1}^{n} t a_{0t} \frac{(1+r_0)^{-t}}{V_0}$$

It is worth noting that this measure of sensitivity to price changes can be applied either to non-marketable or marketable instruments. In this case, the market price being given, equation (14) is solved for the yield to maturity (r_i) which can then be used to determine the asset sensitivity in equation (21).

We are now in a position to extend this approach to the bank's balance sheet as a whole, first by determining the sensitivity of its net position in a given currency and then by summing up on all currencies. For a given currency, the vulnerability of a bank can be written as :

$$(24) \qquad dW_i = \frac{(\dot{D}_{ai}A_i - \dot{D}_{li}L_i)}{1+r_0} dr_0 + \frac{(\ddot{D}_{li}L_i - \ddot{D}_{ai}A_i)}{1+r_i} dr_i + (A_i - L_i)\frac{de_{i0}}{e_{i0}}$$

where $(\dot{D}_{ai},\ddot{D}_{ai})$ and $(\dot{D}_{li},\ddot{D}_{li})$ are the durations, respectively, of assets and liabilities in currency i. This relationship means that the sensitivity of the bank exposure in a given currency stems from the combination of duration gaps between assets and liabilities and an open foreign exchange position. It is worth noting that this result is more general than that obtained by GRAMMATIKOS and al. (1986) who are concerned only by foreign exchange positions and cannot therefore cope with the measurement of global market risks. This can be done here by aggregating equation (24) on all currencies, including the domestic one, which yields the total sensitivity of the bank's net wealth as defined in equation (1), that is :

$$(25) \qquad \sum_{i=1}^{k}(dW_i) = \sum_{i=1}^{k}(W'_{ir_0}dr_0) + \sum_{i=1}^{k}(W'_{ir_i}dr_i) + \sum_{i=1}^{k}(e_{i0}W'_{ie_{i0}}\frac{de_i}{e_{i0}})$$

After substituting to $(dr_0, dr_i, \frac{de_i}{e_{i0}})$ their expected values, the ex-ante change of the bank net wealth can be written in matrix form as :

$$(26) \qquad E\,dW = Y\overline{Z}'$$

where the symbols are specified as follows :

- Y is the (1 ; 3(k + 1)) vector of duration gaps and foreign exchange positions with k defining the number of foreign currencies;
- Z is a (m ,3(k + 1)) matrix of a sample of historical changes in interest and exchange rates with m representing the number of obsevations;
- $\overline{Z}$ is the mathematical expectation for Z.

The average sensitivity of the bank net wealth so obtained is not sufficient, however, to appreciate the potential risk undergone by the bank. One needs also to take into account, the

---

[7] MACAULAY (1938) first proposed to determine the average life of a fixed rate security by weighing the dates of its future cash flows by the ratio of these cash flows to the present value of the security.

variability of interest and exchange rates. This can be achieved by estimating the variance of the sensitivity of the bank's net wealth which takes the following matrix form :

$$E(dW^2) = \frac{1}{m}\left(Y(Z-\overline{Z})'(Z-\overline{Z})Y'\right) \tag{27}$$

where $(Z-\overline{Z})'(Z-\overline{Z})$ is the variances/covariances matrix of changes in interest and exchange rates. Note that this equation would be totally consistent with our initial assumption that changes in prices are independent only if covariances were null. This issue that can be settled at the empirical level is raised in the last section.

6.0 SIMULATION OF A BANK'S EXPOSURE TO MARKET RISKS (PERIOD 1976-1986)

To illustrate the impact of changes in market prices on the bank net wealth, suppose that a French financial intermediary has a balance sheet -already adjusted according to the assumptions of section 2- made up of assets and liabilities denominated in five currencies : the French franc, the U.S. dollar, the German mark,the Japanese yen and the pound sterling denoted, respectively, as frf, usd, dem, jpy and gbp. Moreover, assume that the bank practices a significant transformation in French franc and a more modest one in foreign currencies while it carries out long exchange positions in mark and yen, short ones in U.S. dollar and pound sterling with a global net exposure on all currencies slightly in favor of the French franc. Such an hypothetical balance sheet may take on the following structure :

	ASSETS			LIABILITIES		
currency	amount (nns frf)	maturity (year)	cash flow frequency	amount (mns frf)	maturity (year)	cash flow frequency
frf	1000	5	5	800	0.25	1
usd	200	0.5	1	300	0.25	1
dem	150	2	2	100	0.5	1
jpy	150	2	2	100	0.5	1
gbp	100	1	1	200	0.5	1
				100	(equity capital)	
	1600			1600		

Starting from this framework and assuming that these operations were initiated on the 02/19/88 under the following conditions :

currency	interest rates	exchange rates (/frf)
frf	7.4375 %	1.0
usd	6.75	5.77
dem	3.4375	3.38
jpy	3.75	0.004436
gbp	8.875	10.084

the sensitivity of the net position in each currency was measured according to equation (24). To obtain the risk imbedded in the bank exposure the last step consisted in estimating the variances/covariances matrix of interest and exchange rates. This matrix was set up on the basis of quarterly data originating from the I.M.F data base for the period 1976-1986. A distinction was made also between two intermediate periods : 1976-1980 and 1980-1986. Two kinds of calculation were implemented. The first relied on the total variances/covariances matrix while the second assumed that covariances were null. The results obtained are presented in the table below :

Variance and Standard-deviation of the bank's net wealth sensitivity (mns frf)

periods	1-total var/covar matrix variance	st-deviation	2- null covariances variance	st-deviation
1-1976 4-1986	2137.8	46.2	11323.3	106.4
1-1976 4-1980	1643.2	40.5	8627.8	92.9
1-1980 4-1986	2211.9	47.0	11628.2	107.8

Their perusal suggests the following remarks :

- The choice of different periods to estimate the variances/covariances matrix has no sizable influence on the measure of the standard deviation of the sensitivity. Of course, it does not mean that the matrix is stable. It means only that ,all in all, impacts on the sensitivity of changes that affected the variances/covariances of interest and exchange rates tended to offset each other. That such a result can be reproduced in the future is not warranted. Therefore, to implement this approach in the real world it would seem advisable to update the matrix on a regular basis.

- If one takes into account the total variances/covariances matrix, it appears that the existence of the bank would not be jeopardized even in the face an extreme situation, which can be characterized here by a variation of the sensitivity by 2 standard-deviations, since such a variation would not mop up all the initial net wealth.

- This is not so, if one neglects the covariances. It seems important, therefore, to take them into account even if they are unstable otherwise the bank might tend to over-estimate its actual degree of exposure.

*
*   *

CONCLUSION

In this paper, a general framework was laid down to measure the sensitivity of the bank's net wealth to changes in market prices. It seems that it might be of interest for bankers and regulators as well who are looking for instruments designed to evaluate the probability of a bank failure. Moreover, if banks are not allowed to invest significant amounts in equities, our approach does not lend itself to the critic addressed to the GRAMMATIKOS (1986) paper according to which the mean/variance analysis cannot be apply meaningfully to a fraction of the bank portofolio.

M. GALY

BIBLIOGRAPHY

- G. BIERWAG, 1987, Duration analysis, managing interest rate risks, Ballinger publishing company.
- J. COX, J. INGERSOLL and S. ROSS, 1979, Duration and the measurement of basis risk, Journal of business, vol 52 no 1, and 1985, A theory of the term structure of interest rates, Econometrica vol 53, no 2.
- W. GAAB, M. GRANZIOL and M.HORNER, 1986, On some international parity conditions, European economic review vol 30, no 3.
- T. GRAMMATIKOS, A. SAUNDERS and I. SWARY, 1986, Returns and risks of U.S. bank foreign currency activities, Journal of finance vol 41, no 3.
- E. GURDEL, D. PYLE, 1984, Banks income taxes and interest rates risks management :a note, Journal of finance vol 39, no 4.
- F. MACAULAY, 1938, Some theoretical problems suggested by the movements of interest rates, bond yields, stock prices in the U.S. since 1856, New-York, columbia press university.
- K. RAMASWAMY and S. SUNDARESAN, 1986, The valuation of floating rate instruments", Journal of financial economics 17.

APPENDIX
COMPARISON OF VALUATION METHODS

ASSUMPTIONS

- The security has been issued with a 10 year maturity and a annual coupon rate of 10%;
- The current interest rate may take on three values : 8%, 10%, 12%;
- Three yield curves with positive slopes are associated with those interest rates. They have been generated by a function of the following form : $1 + r_t = \exp(at + bt^2 + ct^3 + d)$ and their values are ranging, respectively, from 8 to 9.9%, 10 to 11.9% and 12 to 13.9%
- For the COX method, the parameters are set at the following levels : $\beta = 0.692$ $\mu = 0.08$ 0.10 0.12 $\pi = -0.01$ $\sigma^2 = 0.0004$. Note here that the coefficient β is the one estimated in COX and al. (1979) for U.S. treasury bills over the period 1967-1976.

RESULTS OF THREE METHODS OF VALUATION

yield to maturity	8%			10%			12%		
(*)	mac	f-w	cox	mac	f-w	cox	mac	f-w	cox
present value	113,4	102,4	103,3	100	90,8	88,9	88,7	81.5	76.9
Duration	6,97	6,73	3,88	6,76	6,52	3,60	6,55	6,31	3,53

(*) mac : Traditional form of valuation and MACAULAY duration.
(*) f-w : Valuation accounting for the yield curve and a FISHER-WEIL stochastic process.
(*) cox : COX and al. valuation and stochastic duration.