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Equilibrium Dynamics in a Matching Theoretic-Model of the Housing Market

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Abstract - In light of the recent and growing literature which has extended the use of search and matching models even to the housing market, this paper introduces dynamic analysis to a simple stationary state equilibrium model. Contrary to what occurs in the labour market, the dynamic adjustment to equilibrium depends on the level of matching frictions present in the market. Precisely, if matching frictions are high, sellers bear in mind future expectations regarding total vacancies when deciding how many vacancies to post on the market; as a consequence, the market tensions respond quickly to any changes, immediately reaching the equilibrium value. Instead, with low matching frictions any dynamic adjustment path leads to equilibrium without the need for "forward looking" behaviour on behalf of sellers.

Keywords - Housing Markets, Matching Theory, Equilibrium Dynamics

1. Introduction

Recently, there has been much focus on formulising the behaviour of the housing market through the search and matching theoretic-models usually used for the labour market. The housing market is in fact a "matching market" like the labour market that clears not only through price but also through time and money that the parties spend on the market.

Matching between buyers and sellers, i.e. the number of contracts traded during a given period, is not ensured by a "walrasian" housing market but is achieved by a costly and time-consuming search and matching process. In short, as in the labour market, matching on the housing market is a result of a decentralised and uncoordinated search process for buyers and sellers. Furthermore, the number of agents on the housing market affects the matching probability of other agents on both sides of the market (the so-called "search externality").

In labour market matching models, the analysis is usually limited to the equilibrium conditions of the stationary state, since the associated dynamics are relatively simple and involve a unique adjustment path to equilibrium. Precisely, market tightness responds instantly to any change in parameters or expectations and it immediately achieves its equilibrium value (see Pissarides, 2000).

In light of the recent and growing literature which has extended the use of search and matching models even to the housing market, this paper introduces dynamic analysis to a simple stationary state equilibrium model of the housing market.

The main result of this analysis is that the dynamic adjustment to equilibrium depends on the level of matching frictions present in the market. Precisely, if matching frictions are high, sellers bear in mind future expectations regarding total vacancies when deciding how many vacancies to post on the market; as a consequence, the dynamic adjustment to equilibrium is very similar to that of the labour market. Instead, contrary to what occurs in the labour market, with low matching frictions the dynamic adjustment always leads to equilibrium, without the need for "forward looking" behaviour on behalf of sellers.

The rest of the paper is organised as follows: the next section briefly presents the related literature, while section 3 outlines a simple macroeconomics model of the housing market, thus showing the main results of the dynamic analysis.

2. Related Literature

Starting from the first search model of the housing market (Wheaton, 1990), several papers have developed search and matching models to analyse the formation process of prices in housing markets with trading frictions. Precisely, recent search and matching models of the housing market (Diaz and Jerez, 2009; Novy-Marx, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012) adopt an aggregate matching function and focus on the role of the ratio between vacant houses/sellers and home seekers/buyers (the so-called 'market tightness') in determining the probability of matching between the parties. This is in line with the standard matching approach (Pissarides, 2000).

Nevertheless, unlike the quoted studies, we closely follow the basic matching model *à la* Pissarides. Indeed, we show that the standard matching framework extended to the real estate market is able to analyse the house price formation process without any significant deviation from the baseline model adopted for the labour market.

As far as we are aware, this paper is the first attempt to develop a dynamic analysis of the housing market where the market tightness plays a key role.

3. A Matching Theoretic-Model of the Housing Market

We adapt a standard matching framework *à la* Pissarides (2000) to the housing market analysis. Thus, search is random and prices are determined by Nash bargaining. Also, the market of reference in this model is the homeownership market.

We normalise the number of households in the housing market to the unit, i.e. $1 = b + h$, where b is the share of home-seekers (buyers), i.e. households who need to change their home (for business reasons or family needs). Thus, the vacant houses on the market (v) and the home-seekers (b) are "input" of a matching function (m) which gives the number of contracts traded in each instant of time, namely $m = m(\underset{+}{v}, \underset{+}{b})$.

The matching function is expressed by the functional form commonly used in matching models, i.e. the *Cobb-Douglas* specification, i.e. $m = m(v,b) = v^{1-\alpha} \cdot b^{\alpha}$, where $0 < \alpha < 1$ is the (constant) elasticity of the matching function with respect to the home-seekers (buyers). It follows that the (instantaneous) probability of a match for a home-seeker (buyers) is $\frac{m(v,b)}{b} = \frac{v^{1-\alpha}}{b^{1-\alpha}} = \vartheta^{1-\alpha}$, with $\frac{\partial\{m(v,b)/b\}}{\partial\vartheta} = (1-\alpha)\cdot\vartheta^{-\alpha} > 0$, where $\vartheta \equiv v/b$ is a measure of "tightness" of the housing market; whereas, the (instantaneous) probability of a match for a seller with a vacancy is $\frac{m(v,b)}{v} = \frac{v^{-\alpha}}{b^{-\alpha}} = \vartheta^{-\alpha}$, with $\frac{\partial\{m(v,b)/v\}}{\partial\vartheta} = (-\alpha)\cdot\vartheta^{-\alpha-1} < 0$. Also, standard technical assumptions are assumed: $lim_{\vartheta\to 0}\vartheta^{-\alpha} = lim_{\vartheta\to\infty}\vartheta^{1-\alpha} = \infty$, and $lim_{\vartheta\to 0}\vartheta^{1-\alpha} = lim_{\vartheta\to\infty}\vartheta^{-\alpha} = 0$. The dependence of the instantaneous probabilities on market tightness identifies the so-called search externalities.

3.1. Home-seekers (buyers) dynamics

In the housing market is more interesting to study the transition from buyer to seller rather than the dynamic in and out of the homelessness.[1]

The evolution of home-seekers (buyers) over the course of time (t) is the following:

$$db(t)/dt \equiv \dot{b}(t) = \delta\cdot(1-b(t)) - \vartheta(t)^{1-\alpha}\cdot b(t) \quad (1)$$

where $\delta\cdot(1-b(t))$ represents the home-seekers (buyers) inflows, i.e. at the exogenous rate δ other households (h) need to change their home; whereas, $\vartheta(t)^{1-\alpha}\cdot b(t)$ describes the home-seekers (buyers) outflows, i.e. the home-seekers (buyers) that find a home. Therefore, in steady state we get:[2]

$$\dot{b}(t) = 0 \Rightarrow b = \frac{\delta}{\delta + \vartheta(t)^{1-\alpha}} \quad (2)$$

we obtain that the evolution over time of b depends negatively on the level of b itself:

$$\frac{\partial\dot{b}(t)}{\partial b(t)} = -\left(\delta + \vartheta(t)^{1-\alpha}\right) < 0$$

This implies a converging dynamic of b : in fact, for any initial value of b , the share of home-seekers (buyers) always converges to its equilibrium value of steady state. Instead, the steady state relationship of b with respect to ϑ is negative (cf. *Figure 1*):

$$b = \frac{\delta}{\delta + \vartheta(t)^{1-\alpha}} \Rightarrow \vartheta = \left[\frac{\delta\cdot(1-b)}{b}\right]^{\frac{1}{1-a}}$$

$$\Rightarrow \left.\frac{\partial\vartheta}{\partial b}\right|_{\dot{b}=0} = \left(\frac{1}{1-a}\right)\cdot\left[\frac{\delta\cdot(1-b)}{b}\right]^{\frac{1}{1-a}-1}\cdot\left(\frac{-\delta}{b^2}\right) < 0$$

[1] The "homeless" condition in the housing market is not equivalent to the condition of "being unemployed" in the labour market. Indeed, the two conditions are very different. According to Wheaton (p. 1274, 1990): « […] *homelessness is relatively inconsequential in the housing market, and so moves are like voluntary "quits" in the labour market. Furthermore, moves involve some spell in which the household owns two units, whereas even voluntary job transitions usually carry some period of unemployment. More important, the causes of housing mobility are usually different from those generating job mobility*». Furthermore, the two conditions coincide only in some cases: in fact, many people have a job but not a house (because, for example, they work far from home and pay rent) and many more are unemployed but have a home (because, for example, they live with their parents while searching for a job).

[2] The time reference of the variables can be neglected when we talk about steady state.

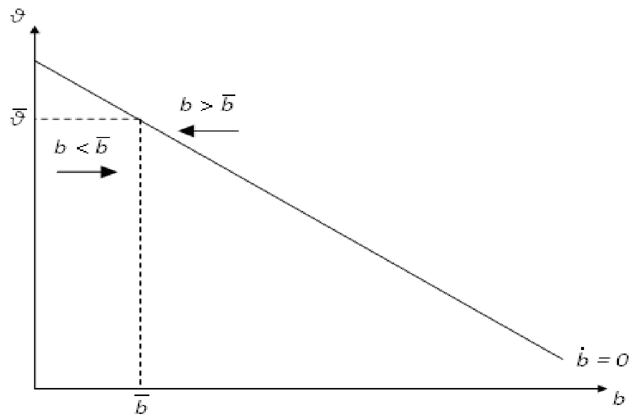


Figure 1. Out-of-steady-state dynamics of homeless-buyers

In Figure 1, for each value of ϑ there is a unique steady state equilibrium value of b. For the points that lie outside of the curve $\dot{b} = 0$, the share of home-seekers (buyers) tends to return to the steady state: in fact, keeping ϑ constant at $\bar{\vartheta}$ in figure 1, a value $b > \bar{b}$ determines an increase in the outflow and a decrease in the inflow from b, as is clear from equation (1), bringing the value of b towards $\bar{b}$. The exact opposite occurs if $b < \bar{b}$.

3.2. *The value functions of the model*

The expected values of a house (vacant V, and occupied J) and of buying a house (H) are given by:[3]

$$rV(t) = -c + \vartheta(t)^{-\alpha} \cdot [J(t) - V(t)] + \dot{V}(t) \tag{3}$$

$$rJ(t) = P(t) + \dot{J}(t) \tag{4}$$

$$rH(t) = -e + \vartheta(t)^{1-\alpha} \cdot [x - H(t) - P(t)] + \dot{H}(t) \tag{5}$$

The terms c and e represent, respectively, the cost flows sustained by sellers and buyers during the search. When a match takes place, the risk neutral buyer gets a linear benefit x from the property and pays the sale price P to the seller. Note that the destruction rate of a specific buyer-seller match does not exist and thus the value of an occupied home is simply given by the selling price.[4]

The endogenous variables of the model are market tightness (ϑ) and house price (P). Assuming profit maximization from the supply side ($V(t) = 0$, $\forall t$) and stationary equilibrium ($\dot{V}(t) = \dot{J}(t) = 0$), we get the first key relationship of the model:

$$\begin{cases} c \cdot \vartheta^{\alpha} = J \\ \dfrac{P}{r} = J \end{cases} \Rightarrow r \cdot c \cdot \vartheta^{\alpha} = P \tag{6}$$

with $\partial\vartheta/\partial P > 0$, $lim_{P \to 0} \vartheta \to 0$, and $lim_{P \to \infty} \vartheta \to \infty$: if the price increases, in fact, more vacancies will be on the market.

3.3. *Price determination*

The sale price in the stationary equilibrium is obtained by the so-called *Nash bargaining solution* usually used for decentralised markets (recall that $V(t) = 0$, $\forall t$):

$$P = argmax \left\{ (J - V)^{\gamma} \cdot (x - H - P)^{1-\gamma} \right\}$$
$$\Rightarrow P = \gamma \cdot (x - H)$$

where $0 < \gamma < 1$ is the share of the bargaining power of sellers. Entering into a contractual agreement obviously implies that $x > H$, $\forall \vartheta$, i.e. the price is always positive. Simple manipulations yield the equation for the selling price:

$$P = \frac{\gamma \cdot (rx + e)}{r + \vartheta^{1-\alpha} \cdot (1 - \gamma) \cdot r^{-1}} \tag{7}$$

with $\partial P/\partial\vartheta < 0$: in fact, if the market tightness increases, the effect of the well-known *congestion externalities* on the sellers' side will lower the price (see Pissarides, 2000).

3.4. *Steady state equilibrium*

Four equations describe the housing market in the steady state equilibrium:

i. $\vartheta \equiv v/b$

ii. $b = \dfrac{\delta}{\delta + \vartheta(t)^{1-\alpha}}$

iii. $r \cdot c \cdot \vartheta(t)^{\alpha} = P$

iv. $P = \dfrac{\gamma \cdot (rx + e)}{r + \vartheta^{1-\alpha} \cdot (1 - \gamma) \cdot r^{-1}}$

Given the recursive structure of the model, it is straightforward to solve this system of four equations in four unknowns. Precisely, equations (6) and (7) give the equilibrium values of P and ϑ, equation (2) gives the equilibrium value of b and finally the definition of tightness allows to obtain the steady state value of v.

3.5. *Market tightness dynamics*

As regards the differential equation for ϑ, the *free-entry* condition for equilibrium is valid even out of the stationary state, i.e. $V(t) = 0 \Rightarrow J(t) = \dfrac{c}{\vartheta(t)^{-\alpha}}$, $\forall t$.

Outside the steady state J changes according to its

[3] Time is continuous and individuals are risk neutral, live infinitely and discount the future at the exogenous interest rate $r > 0$.

[4] In the homeownership market, in fact, if a contract is legally binding it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up.

dynamic equation, i.e. equation (4). Differentiating $J(t) = c \cdot \vartheta(t)^{\alpha}$ with respect to time we obtain: $\dot{J}(t) = c \cdot \alpha \cdot \vartheta(t)^{\alpha - 1} \cdot \dot{\vartheta}(t)$. The rule for subdividing surplus is also valid out of the stationary state. The selling price is, therefore, determined in the same way in both stationary equilibrium and during adjustment. Combining the previous results, the differential equation for ϑ is finally obtained:

$$rJ(t) = P(t) + \dot{J}(t) \Rightarrow$$

$$r \cdot c \cdot \vartheta(t)^{\alpha} = \frac{\gamma \cdot (rx + e)}{r + \vartheta^{1-\alpha} \cdot (1-\gamma) \cdot r^{-1}} + c \cdot \alpha \cdot \vartheta(t)^{\alpha - 1} \cdot \dot{\vartheta}(t)$$

$$\Rightarrow \dot{\vartheta}(t) = \frac{r \cdot \vartheta(t)}{\alpha} - \frac{\gamma \cdot (rx + e) \cdot \vartheta(t)^{1-\alpha}}{\left(r + \vartheta^{1-\alpha} \cdot (1-\gamma) \cdot r^{-1}\right) \cdot c \cdot \alpha} \quad (8)$$

As in the labour market, it is very clear that the steady state relationship of ϑ does not depend (in an independent manner) on the evolution of seekers. Nevertheless, unlike the labour market, the "reaction" of $\dot{\vartheta}$ with respect to ϑ, i.e. the variation over time of ϑ, is *a priori* ambiguous:

$$\frac{\partial \dot{\vartheta}(t)}{\partial \vartheta(t)} = \frac{r}{\alpha} - \mu(t)$$

where $\mu(t) \equiv \dfrac{(1-\alpha) \cdot \vartheta(t)^{-\alpha} \cdot \gamma \cdot (rx + e) \cdot r \cdot c \cdot \alpha}{\left[\left(r + \vartheta^{1-\alpha} \cdot (1-\gamma) \cdot r^{-1}\right) \cdot c \cdot \alpha\right]^2}$.

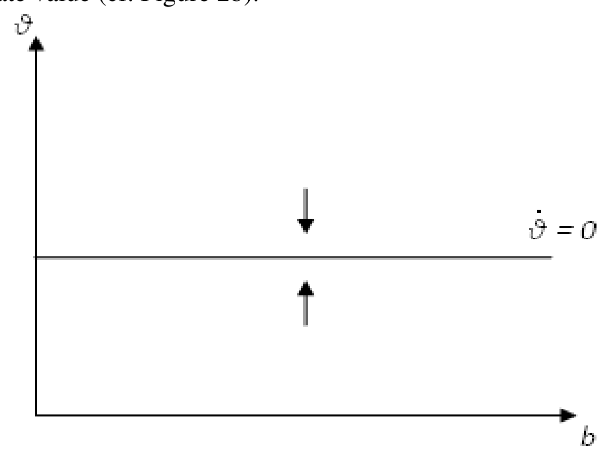
Following Pissarides (2000), the apparent unstable behaviour of ϑ in the case of high matching frictions can be explained by the "forward looking" attitude of sellers which base their decision to post vacant houses on the future expected value of ϑ, and then immediately post more vacancies if they foresee a future increase in ϑ, in order to avoid posting new ones when their opening cost will be higher (the higher ϑ, in fact, the higher the average duration of a filled vacancy). Hence, ϑ responds immediately to changes in parameters or expectations and also immediately achieves its equilibrium value. This implies a very simple adjustment dynamic (see *Figure 3a*), namely the existence of a unique dynamic path (saddle-path) converging at steady state equilibrium (saddle-point), shown by point E[5]. Any other dynamic path, in fact, leads away from the point of stationary equilibrium. With low matching frictions, instead, sellers do not need to have a "forward looking" attitude since today's posted vacancies will be easily filled in the future. Therefore, in this case, any dynamic adjustment path will lead to equilibrium (see Figure 3b), even if not immediately.

In order to clarify the previous expression, we take the limits of ϑ. Knowing the properties of the matching probabilities, we get:

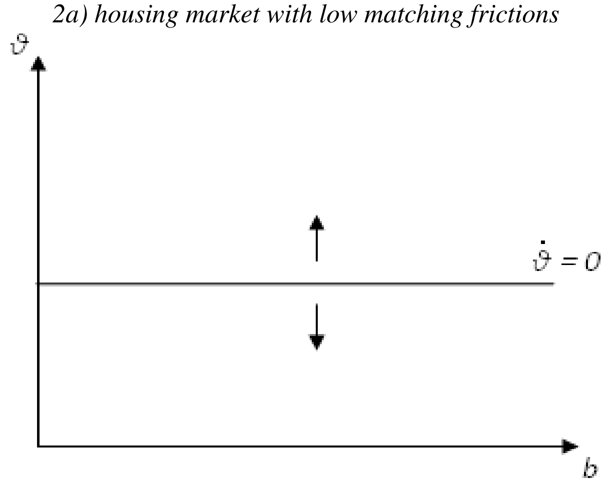
$$\lim_{\vartheta \to 0} \mu(t) = \frac{\infty}{r} \quad \Rightarrow \frac{\partial \dot{\vartheta}(t)}{\partial \vartheta(t)} < 0 \quad ;$$

$$\lim_{\vartheta \to \infty} \mu(t) = \frac{0}{\infty} \Rightarrow \frac{\partial \dot{\vartheta}(t)}{\partial \vartheta(t)} > 0$$

Without loss of generality, it is possible to define a threshold value of market tightness, i.e. $\vartheta = \tilde{\vartheta}$, such that if $\vartheta > \tilde{\vartheta}$, then $\frac{r}{\alpha} > \mu(t)$; whereas if $\vartheta < \tilde{\vartheta}$, then $\mu(t) > \frac{r}{\alpha}$. In short, with low matching frictions ($\vartheta < \tilde{\vartheta}$), market tightness always converges to its equilibrium value of steady state (cf. Figure 2a); whereas, with high matching frictions ($\vartheta > \tilde{\vartheta}$), the dynamic is different and implies that for the points lying above and below the curve $\dot{\vartheta} = 0$, the value of ϑ tends to shift increasingly further from its steady state value (cf. Figure 2b).



2a) housing market with low matching frictions

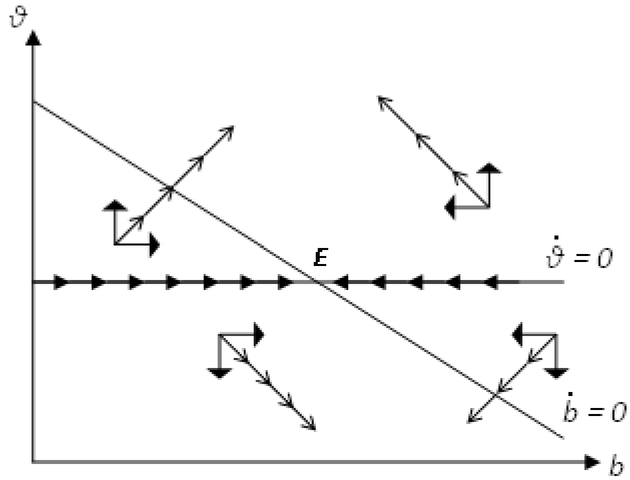


2b) housing market with high matching frictions

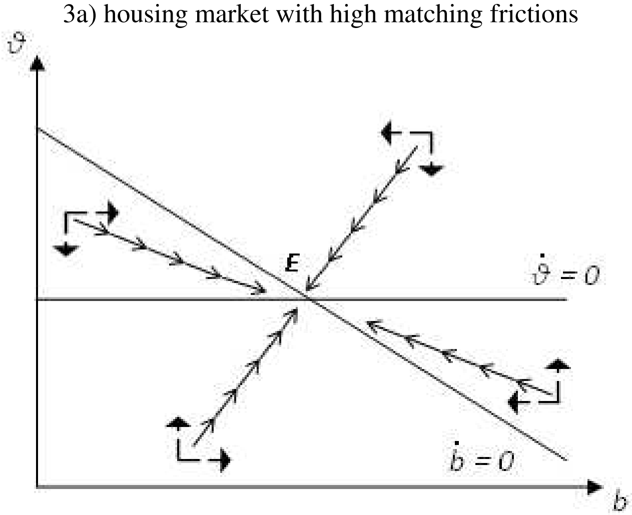
Figure 2. Out-of-steady-state dynamics of market tightness

[5] It is possible to formally verify the nature of an equilibrium *saddlepoint* by linearising the dynamic equations surrounding a generic steady state equilibrium point, i.e. $(\bar{b}, \bar{\vartheta})$: $\begin{pmatrix} \dot{b} \\ \dot{\vartheta} \end{pmatrix} = \begin{pmatrix} - & - \\ 0 & + \end{pmatrix} \cdot \begin{pmatrix} b_n - \bar{b} \\ \vartheta - \bar{\vartheta} \end{pmatrix}$

The negative sign of the determinant of the coefficient matrix confirms the nature of the steady state equilibrium *saddlepoint*. In order to have equilibrium stability, the matrix trace must be negative. In fact, the equilibrium is a node that can be stable or unstable depending on whether the matrix trace is, respectively, smaller than or larger than zero (cf. Bagliano and Bertola, 2004).



3a) housing market with high matching frictions



3b) housing market with low matching frictions

Figure 3. Adjustment paths in market tightness and homeless-buyers space

4. Conclusions

In light of the recent and growing literature which has extended the use of search and matching models even to the housing market, this paper introduces dynamic analysis to a simple stationary state equilibrium model. Precisely, we extend the basic search – matching model, usually used for the labour market, to the housing market analysis. Thus, market tightness and house price are the key variables of the steady state equilibrium, whereas the evolution of home-seekers and market frictions define the dynamic adjustment path to equilibrium.

The main result of this analysis is that the dynamic adjustment to equilibrium depends on the level of matching frictions present in the market. In fact, if matching frictions are high, sellers bear in mind future expectations regarding total vacancies when deciding how many vacancies to post on the market; as a consequence, the market tensions respond quickly to any changes, immediately reaching the equilibrium value. Instead, with low matching frictions any dynamic adjustment path leads to equilibrium without the need for "forward looking" behaviour on behalf of sellers.

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