The Benchmark Macroeconomic Models of the Labour Market

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Abstract: This technical note aims to provide a practical overview of the labour market's benchmark macroeconomic models. The matching models are the primary and most popular theoretical tools used by economists to evaluate various labour market policies and to study the problem of unemployment. These models explain the co-existence in equilibrium of unemployment and vacancies through frictions in matching workers and firms and generate predictions that have the right direction: unemployment goes up in recession and down in boom, while job vacancies shift in the opposite direction. The central role of these models in imperfect labour markets has recently been confirmed by the 2010 Nobel Prize for economy awarded to the founders of this approach: Peter Diamond, Dale Mortensen and Christopher Pissarides.

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JEL Classification: B16; C78; D83; J63; J64

1 Introduction

Nowadays, the matching models of equilibrium unemployment are the primary and most popular theoretical tools used by academic and government economists to evaluate various economic policies and to study the problem of unemployment (Hagedorn and Manovskii, 2008). Indeed, they are the benchmark macroeconomic models of the labour market (Garibaldi, 2006). These models are in fact able to explain the co-existence in equilibrium of unemployment and vacancies through frictions in matching workers and firms. Furthermore, from an empirical point of view, these models appear to satisfactorily explain what occurs in reality: in fact, *« [...] in calibrations, matching models are usually compared with Hansen's calibrated model* (i.e. the benchmark real business cycle model) *and are shown to perform at least as well »* (Pissarides, 2000, p. 36). The central role of these models in imperfect labour markets has recently been confirmed by the 2010 Nobel Prize for economy awarded to the founders of this approach: Peter Diamond, Dale Mortensen and Christopher Pissarides.
The awareness of the fact that modern labour markets are characterised by large flows, both of workers in and out of employment and of job vacancies created and destroyed by firms, has led to this new theoretical approach whose main scope is to derive an empirically realistic equilibrium unemployment theory, in which unemployment persists in equilibrium. The flow of workers between employment, unemployment and inactivity, and the rich dynamics behind them, is a characteristic common to both the American (Blanchard and Diamond, 1990a) and European (Burda and Wyplosz, 1994) labour market. Although these flows are in theory compatible with labour turnover over a fixed number of jobs, the reallocation of workers is actually associated with substantial annual flows in job creation and destruction at the single firm level (Davis and Haltiwanger, 1992). Even in the absence of net changes in employment, the simultaneous creation and destruction of jobs is intense (Bagliano and Bertola, 1999; Andolfatto, 2008).[1]
The acknowledged importance of these flows in the persistence of unemployment, even at equilibrium, substantiates the economic mechanism underlying matching models: the

[1] For example, in Canada during the period 1976 – 1991 a small net change in employment, amounting to 15.000 individuals, was consistent with approximately one million individuals transiting in and out of employment (see Jones, 1993).

matching process between workers and firms. More precisely, employment dynamics are the result of vacancies being created and filled by firms, and the activity of job-seekers, particularly the unemployed.[2] The matching between a firm and worker results in a filled, and thus active, job that therefore produces income and is able to pay wages (Bagliano and Bertola, 1999). However, matching takes time to finalise since the process is characterised by a decentralised, uncoordinated and costly (in terms of both time and money) search conducted by job-seekers and firms (Bagliano and Bertola, 1999).[3] Worker-firm matching is not instantaneous due to the existence of frictions (i.e. search externalities, heterogeneity of individuals and jobs, incomplete information etc.). Search externalities, also known as congestion externalities, are particularly relevant in matching models (see Pissarides, 2000). In fact, every firm that creates new jobs produces externalities that are positive for job-seekers (since the probability of finding a job increases) and negative for other firms (since the probability of filling existing vacancies is reduced); *vice versa*, an increase in job-seekers produces positive externalities for firms and negative externalities for other job-seekers, for precisely the opposite reasons.[4]

It should be specified that the idea that labour market frictions exist and are significant is not unique to matching models and was already present in Hutt (1939) and Hicks (1963). The latter, in particular, claimed that the short-term disequilibrium in the labour market was due to the fact that wages were slow to adjust in the wake of economic shocks, and that this was attributable to existing frictions. This view has essentially been confirmed by more recent studies (cf. Petrongolo and Pissarides, 2001). Keynes (1936), on the other hand, basically coined the term "frictional unemployment", and believed that this type of unemployment was not particularly significant and as a consequence disagreed that frictions played a major role in the slow adjustment of wages.

The work carried out in the '60s and '70s (e.g. Alchian, 1969; Phelps, 1968, 1970, 1972; Mortensen, 1970) successively emphasised the key role played by search frictions and led to today's *search theory*, i.e. an unemployment theory based on the assumption that labour-market search is an economically costly activity. Basically, in models where the individual must choose how to optimally divide his time between work and leisure, a third option is introduced: the option of searching for a new and/or better job. The *search equilibrium* has two key properties: 1) search frictions that introduce monopoly revenue, subdivided between firm and worker through wage determination once a match has been made; 2) indifference to the so called congestion externalities in individual optimisation problems. In short, individuals ignore the effects their actions have on the aggregate probability of finding a job and filling a vacancy.

Starting from the late '70s – early '80s, more analytically sophisticated models were constructed, now commonly known as *search and matching* models. Amongst these, a distinction can be made between those that focus on the entire economy, in particular on the presence of multiple equilibria (Diamond, 1982a, 1982b, 1984), and those whose main focus is on the labour market (Pissarides, 1979, 1984, 1985a, 1985b, 1986, 2000; Mortensen, 1987; Mortensen and Pissarides, 1994, 1998 and 1999; and Pissarides, 2000). The first models in which the matching function is not only present but is also the main economic mechanism underlying unemployment, basically replacing the reservation wage,[5] are

[2] Furthermore, in these models the definition of unemployment is consistent with that typically used in national job-force surveys: individuals are considered unemployed when they do not have a job but are actively searching for one (Andolfatto, 2008).

[3] This differs from the traditional neoclassic model in which the matching process is centralised and coordinated, and work demand and offer are instantly balanced by variations in wages.

[4] In the matching framework, firms and workers have completely rational expectations, i.e. they are fully aware of the matching process. Nonetheless, they act independently, without attempting to coordinate their actions (Pissarides, 2000).

[5] The reservation wage is the wage that leaves an individual indifferent to working or not. It is deduced by

those of Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980).

The matching function is conceptually equivalent to the production function: the result of the "productive process" is the creation of jobs and the "productive factors" are job-seekers (unemployed worker)[6] and vacancies (Bagliano and Bertola, 1999). As a consequence, the use of an aggregate (macroeconomic) function is justified by its empirical relevance and ability to capture the main characteristics of the matching process (Pissarides, 2000). In this sense, the matching function is a useful modelling tool, as it can describe the job formation process without having to clarify the reasons that make this process challenging and costly. Moreover, the matching function is able to grasp (as will become apparent in the next paragraph) variations in both the optimal behaviour of firms and workers and the degree of mismatch present in the labour market.[7]

From an empirical point of view, it is common in the literature to resort to the constant returns to scale hypothesis and utilise a *Cobb-Douglas* type function to describe the matching process. Both of these assumptions are empirically supported (Blanchard and Diamond, 1989, 1990b; Pissarides, 2000; Petrongolo and Pissarides, 2001; Stevens, 2007). However, although the choice of a *Cobb-Douglas* type function is common in the literature, its application lacks a convincing theoretical explanation. It is, in fact, employed mainly due to empirical evidence and not because of consensus at the theoretical (microeconomic) level. Despite its importance, in fact, few attempts have been made at microfounding the matching function and, above all, no microfoundation is better than another (Pissarides, 2000). The aggregate-type matching function is, in fact, usually described as a *"black-box"* (cf. Petrongolo and Pissarides, 2001).[8]

2 The basic matching framework

This paragraph will introduce the baseline matching model commonly used in theoretical analyses. It is common practice to consider a match between job and worker as a firm, in other words to assume that each firm only employs one worker (*one-job-firm assumption*). The following approach essentially focuses on analysing the match rather than the firm.[9]

As previously mentioned, the main element underlying these models is the matching function, which expresses the number of jobs created in any given moment in time ($M = m \cdot L$) as a function of the total number of unemployed workers ($U = u \cdot L$) and of vacancies ($V = v \cdot L$):

$$M = m(U,V) \Rightarrow m \cdot L = m(u \cdot L, v \cdot L) \qquad [1]$$

where m, u and v are, respectively, the rate of matching, unemployment and vacancy, whereas L is the labour force (generally normalised to 1 and assumed to be constant in time). The

equalling the benefit of being employed and the opportunity cost of being employed. Economies with a lower reservation wage have a higher level of employment; however this does not necessarily imply a greater social wellbeing. There is, in fact, no *a priori* reason for believing that higher levels of employment necessarily correspond to higher levels of social wellbeing (Andolfatto, p. 84, 2008).

[6] In the case where on-the-job search (employed individuals searching for a job) is not possible, the only job-seekers are the unemployed.

[7] The degree of mismatch is an empirical concept. Its increase (decrease) indicates that the matching process, under the same conditions of vacancies and unemployment, has become more difficult (easier).

[8] An alternative to the *Cobb-Douglas* matching function, which has received important and recent consensus, is the *stock-flow matching* model (Coles and Smith, 1998; Coles and Muthoo, 1998; Lagos, 2000; Gregg and Petrongolo, 2005; Shimer, 2007; Ebrahimy and Shimer, 2010). The idea behind this approach is the following: when a job-seeker enters the market searching for a job, s/he considers all the available vacancies and applies for the job position s/he deems most adequate. If the response is positive, i.e. s/he is hired, s/he becomes employed and stops searching, whereas in the case of a negative response s/he remains in the market awaiting new vacancies, having already discarded the old ones. As a consequence, job-seekers are initially flows and vacancies are stock, while successively job-seekers are stock and vacancies are flows.

[9] Matching models that disregard the commonly accepted *one-job-firm* hypothesis are those of Bertola and Caballerro (1994) and Garibaldi (2006).

matching function basically describes the efficiency of the matching process, highlighting the importance of the two inputs (vacancies and unemployed workers) in the creation of jobs (Petrongolo and Pissarides, 2001). Assuming, as is common in the literature, that the matching function is increasing and concave in both arguments and degree 1 homogeneous, i.e. characterised by constant returns to scale, equation [1] can be simplified and rewritten as:

$$m \cdot L = L \cdot m(u,v) \Rightarrow m = m(u,v) \qquad [2]$$

Resorting to the commonly used *Cobb-Douglas* functional form, the matching function becomes:[10]

$$m = u^{\alpha} \cdot v^{1-\alpha} \qquad [3]$$

where $0 < \alpha < 1$ is the (constant) elasticity of the matching function with respect to the unemployment rate, namely $\varepsilon_{m,u} = \frac{\partial m}{\partial u} \cdot \frac{u}{m} \Rightarrow \alpha \cdot u^{\alpha-1} \cdot v^{1-\alpha} \cdot \frac{u}{u^{\alpha} \cdot v^{1-\alpha}} = \alpha$. Furthermore, the constant returns to scale hypothesis allows attention to be focalised on a single variable, known as "market tightness" θ, which expresses the relationship between vacancies and unemployment, i.e. $\theta \equiv v/u$. The matching function can be used to calculate both the rate with which an unemployed worker finds a job:

$$\frac{m}{u} = \frac{u^{\alpha} \cdot v^{1-\alpha}}{u} \Rightarrow \left(\frac{v}{u}\right)^{1-\alpha} \equiv \theta^{1-\alpha} \qquad [4]$$

and the rate with which a vacant position is filled:

$$\frac{m}{v} = \frac{u^{\alpha} \cdot v^{1-\alpha}}{v} \Rightarrow \left(\frac{v}{u}\right)^{-\alpha} \equiv \theta^{-\alpha} \qquad [5]$$

$\theta^{1-\alpha}$ and $\theta^{-\alpha}$ are the two rates that characterise the matching process and express, respectively, the instantaneous probability of finding a job and of filling a vacancy. It immediately follows that the instantaneous probability of finding a job is positive-concave with regards to the vacancies-unemployment ratio, whereas the probability of filling a vacancy is negative-convex. Furthermore, these instantaneous probabilities can (theoretically) tend to infinity in an infinitesimal time interval, *dt*. In particular:

$$\lim_{\theta \to 0} \theta^{1-\alpha} = \lim_{\theta \to \infty} \theta^{-\alpha} = 0;$$
$$\lim_{\theta \to \infty} \theta^{1-\alpha} = \lim_{\theta \to 0} \theta^{-\alpha} = \infty.$$

It must be pointed out that these properties hold true independently of whether a *Cobb-Douglas* functional form is used.

Employment (*n*), evolves over time in accordance to inflows (filled vacancies, unemployed workers finding a job) and outflows (existing jobs destroyed with exogenous rate δ). Consequently, the change in employment over time can be expressed as both a function of the firm's transition rate ($\theta^{-\alpha}$): $\dot{n} \equiv \frac{dn}{dt} = v \cdot \theta^{-\alpha} - n \cdot \delta$; and as a function of the worker's transition rate ($\theta^{1-\alpha}$): $\dot{n} \equiv \frac{dn}{dt} = u \cdot \theta^{1-\alpha} - n \cdot \delta$. As a result, it must be true that:

$$v \cdot \theta^{-\alpha} - n \cdot \delta = u \cdot \theta^{1-\alpha} - n \cdot \delta \Rightarrow \theta = \frac{v}{u} \qquad [6]$$

The relationship between the vacancy rate and the unemployment rate represents a measure of

[10] In order to simplify the explanations and for greater clarity, the *Cobb-Douglas* functional form will be used throughout this work. In empirical calibrations, it is common practice to introduce a multiplicative factor to the matching function, i.e. $m = \mu\, u^{\alpha}\, v^{(1-\alpha)}$, in order to account for the degree of *mismatch* which, under the same conditions of vacancies and unemployment, makes the matching process more or less difficult. It follows that the larger μ is, the more efficient the matching process and therefore the smaller the degree of *mismatch*. Moreover, as the search intensity and the posting of vacancies may be seen as parameters of technological change in the matching function (see Pissarides, p. 124, 2000), an increase in the search intensity of workers and/or a higher publicising of vacancies by firms makes more efficient the matching process. For a broader discussion on this subject see Pissarides (2000, chapter 5).

labour market tightness, and as already seen, the probability of finding a job and of filling a vacancy depends on this. The chosen reference point is of utmost importance in understanding how this variable describes labour market frictions: indeed, for the firm, an increase in θ makes filling a vacancy more difficult due to the so called congestion externalities; *vice versa* the situation is improved for the worker since it becomes easier to find a job (the so called positive externalities derived from a "denser" market). In matching models it is common practice to take the firm's point of view as reference, in other words an increase in labour market tensions (or tightness) is associated with an increase in θ.

Another fundamental labour market analysis tool, often associated with the matching function, is the *Beveridge Curve*, i.e. the inverse relationship between unemployment and vacancy rate.[11] This relationship can be easily obtained from the following expression, which describes how the unemployment rate changes over time:

$$\dot{u} = (1-u)\cdot\delta - u\cdot\theta^{1-\alpha} \qquad [7]$$

where $(1-u)\cdot\delta$ represents unemployment inflows, i.e. existing jobs destroyed at rate δ, $1 = n + u$ is in fact the normalised labour force, whereas $u\cdot\theta^{1-\alpha}$ describes the unemployment outflows, i.e. unemployed workers that find a job. In steady state equilibrium, where unemployment is constant over time, i.e. $\dot{u} = 0$, it follows that:

$$u = \frac{\delta}{\delta + \theta^{1-\alpha}} \qquad [8]$$

this equation expresses the reverse relationship between unemployment and the measure of labour market frictions and, therefore, between u and v (since $\partial\theta/\partial v > 0$).

In order to calculate the equilibrium value of θ, it is necessary to introduce the so called *Bellman equations*, named after the mathematician Richard Bellman who originally presented them in the '50s. The *Bellman equations* describe the expected marginal values (from which the interest rate r has been deducted) associated with the differing conditions of labour market participants, basically comparing them to financial securities. Formally, and very generally, the *Bellman equations* associated with the employment value (W), with the unemployment value (U), with the vacancy value (V) and the filled job value (J), are the following:[12]

$$r\cdot W = w + \delta\cdot(U - W) + \dot{W} \qquad [9]$$

$$r\cdot U = b + \theta^{1-\alpha}\cdot(W - U) + \dot{U} \qquad [10]$$

$$r\cdot V = -c + \theta^{-\alpha}\cdot(J - V) + \dot{V} \qquad [11]$$

$$r\cdot J = y - w + \delta\cdot(V - J) + \dot{J} \qquad [12]$$

the terms on the right hand side of the expressions are, respectively, the "dividends" associated with the different conditions (w = wage rate, b = employment opportunity cost, c = cost of opening a vacancy and y = productivity) and the "capital gains or losses", in other words the transition from one condition to the other, influenced by the probability of finding a job, of filling a vacancy and by the job destruction rate.[13] Finally, $\dot{X} \equiv dX/dt$ (where $X = W, U, V, J$) indicates the change over time of the presently considered deducted value. The equilibrium usually characterised by these models is the "ideal" stationary state, in which the values of the variables are not subject to further changes over time. It therefore follows that $\dot{X} = 0\ \forall X$. The condition which allows the equilibrium value θ to be determined is known

[11] This relationship is empirically proven and fully intuitive, since an increase in vacancies corresponds to a decrease in unemployment, and *vice versa*. The *Beveridge Curve* was discovered by, and is named after, the British social economist William Beveridge (1944).

[12] It is common practice in the literature to make use of linear utility functions. Assuming that individuals are risk neutral not only simplifies the analysis, but also allows to focus on the consequences of the search and matching process rather than on the deficiencies of the insurance markets.

[13] Intuitively, the transition from unemployed (vacancy) to employed (filled vacancy) is profitable for the worker (firm). In fact, necessary conditions for non trivial equilibria are $W \geq U$ and $J \geq V$.

as the *zero-profit* or *free-entry* condition: a firm will continue to open new vacancies until the value of a further vacancy becomes equal to zero. In equilibrium, in fact, all the profit opportunities derived from opening new vacancies have been exploited, therefore the value of an additional vacancy is equal to zero.[14] Setting $V=0$ in the *Bellman equations* [11] and [12], the following is obtained:

$$\begin{cases}(r+\delta)\cdot J = y-w \\ \dfrac{c}{\theta^{-\alpha}} = J\end{cases} \Rightarrow c\cdot\theta^{\alpha} = \frac{y-w}{r+\delta} \Rightarrow \theta = \left[\frac{y-w}{c\cdot(r+\delta)}\right]^{\frac{1}{\alpha}} \quad [13]$$

The former expression, which shows an inverse relationship between θ and w, is known as the *Job Creation Condition* (*JCC*).[15] Essentially, the net gain deducted by the firm must cover the expected costs associated with opening a vacancy. The reciprocal probability of filling a vacancy $1/\theta^{-\alpha} \equiv \theta^{\alpha}$ is, in fact, the average length of time for which a vacancy is filled.[16]

With regards to w, wages can be determined in several ways,[17] however it is common practice in the literature to use the generalised *Nash bargaining rule*.[18] Based on this rule, the wage is determined by dividing, between firm and worker, the surplus generated by their matching. The optimisation problem which must be resolved is the following:

$$w = arg\,max_{w}(J-V)^{1-\beta}\cdot(W-U)^{\beta}$$

where $\beta\in(0,1)$ is a measure of the workers' bargaining power, namely the surplus quota owed to the job factor. The relative first-order condition for optimal surplus subdivision is given by:

$$(W-U) = \frac{\beta}{1-\beta}\cdot(J-V)$$

from which the following final expression is obtained (see *Appendix A* for the mathematical details), the so called *Wage Setting* (WS):

$$w = (1-\beta)\cdot b + \beta\cdot y + \beta\cdot c\cdot\theta \quad [14]$$

with $\partial w/\partial\theta > 0$, since an increase in θ increases the probability that an unemployed worker finds a job, thereby improving his/her external opportunities and hence bargaining power.

We now have the three key equations (*Beveridge Curve, Job Creation Condition* and *Wage Setting*) – plus the definition of market tightness, i.e. $\theta \equiv v/u$ – for representing the stationary state equilibrium reached in a labour market with frictions, characterised by four endogenous variables (θ, w, u and v). The equilibrium value of θ and w is determined by the *Job Creation Condition* (equation 13) and the *Wage Setting* (equation 14). The *Wage Setting* is increasing in market tightness, whereas the *Job Creation Condition* can be re-write as $w = y - c\cdot\theta^{\alpha}\cdot(r+\delta)$, thus obtaining a negative relationship between wage and market tightness. Therefore, sufficient condition for the existence of an interior equilibrium is that $\lim_{\theta\to 0}\{w = y - c\cdot\theta^{\alpha}\cdot(r+\delta)\} >$ $\lim_{\theta\to 0}\{w = (1-\beta)\cdot b + \beta\cdot y + \beta\cdot c\cdot\theta\}$, namely $y > b$, which is a necessary condition for a non trivial equilibrium. Finally, the equilibrium value of θ allows the equilibrium values of u and v to be determined (by using the *Beveridge Curve* and the definition of market tightness).

[14] To be more precise, *"at any given instant, in both stationary equilibrium and adjustment, firms take advantage of all profit opportunities that arise due to the opening of a vacancy: $V(t)=0,\ \forall t$. Therefore, even out of stationary equilibrium, $\dot{V}(t)=0,\ \forall t$ "* (Bagliano and Bertola, p.274, 1999). The application of the *zero-profit* condition, which ensures a closed-form solution of the model, was discussed for the first time by Pissarides (1979).

[15] The *Job Creation Condition* can be seen as a "special" job demand curve. Indeed, if the cost of opening a vacancy were zero, it would become a standard work demand, i.e. *y* = *w*.

[16] Similarly, the reciprocal probability of finding a job is the average duration of unemployment.

[17] See Mortensen and Pissarides (1999) for an overview.

[18] The *Nash rule* is appropriate in this context, since it is assumed that both sides of the labour market implement costly search activities and that, therefore, a successful match is in their best interest.

3 Endogenous job destruction rate

It is often not completely realistic to assume that the job destruction rate δ is exogenous. In some cases, in fact, the job destruction rate is more sensitive to economic shocks than the job creation rate (Pissarides, 2000).[19]

When a shock affects job productivity, the firm can decide whether to continue using the labour factor at the new productivity or whether to destroy it.[20] The choice is made by the firm in accordance with the so called "reserve productivity", R: if the shock that affects the labour factor reduces productivity below this threshold, the firm will destroy the job, *vice versa* it will keep it open. In order to derive the reserve productivity, the overall productivity of the labour factor is now indicated by $y \cdot x$, where y is a general productivity parameter, whereas x is the idiosyncratic (or specific) component that describes the change in productivity following the shock. Moreover, it is hypothesised that x is drawn from a known continuous distribution function $G(x)$ and that its value is between 0 and 1. [21] As a consequence, $J(x)$ now represents the value of a filled vacancy with idiosyncratic productivity x, with R satisfying the condition $J(R)=0$. Following a shock, the firm's best choice is to continue producing if and only if $J(x) \geq J(R)$.

In this case, the *Beveridge Curve* of the model will have to account for the fact that not all negative shocks destroy jobs:

$$u = \frac{\delta \cdot G(R)}{\delta \cdot G(R) + \theta^{1-\alpha}} \quad [15]$$

where $G(R) = 1 - \int_R^1 x\, dG(x)$ is the probability that a shock lowers productivity below R, thus destroying the job. Moreover, the threshold value of R must also satisfy the condition $W \geq U$. The rule for determining wages (i.e. the subdivision of surplus) basically excludes voluntary unilateral separations, therefore, in order for the job to be destroyed, it is necessary that firms prefer to do without the labour, i.e. $J(x) < J(R)$, but also that workers prefer to be unemployed, i.e. $W < U$.

The value of a filled vacancy, with idiosyncratic productivity x, and of a vacancy are essentially similar to those described previously (it is assumed that all newly created jobs are characterised by maximum productivity, namely $x = 1$):

$$r \cdot J(x) = y \cdot x - w(x) - \delta \cdot J(x) + \delta \cdot \int_R^1 J(s)\, dG(s) \quad [16]$$

$$r \cdot V = -c + \theta^{-\alpha} \cdot [J(1) - V] \quad [17]$$

when the negative shock hits at the rate δ, the firm must discard the value $J(x)$ for another value, $J(s)$, as long as $J(s) \geq J(R)$. Equations [16] and [17] allow the "new" *Job Creation Condition* and the *Job Destruction Curve* (*JD*) to be obtained (see *Appendix B* for the mathematical details):

$$JCC \Rightarrow c \cdot \theta^{\alpha} = \frac{(1-\beta) \cdot y}{(r+\delta)} \cdot (1-R),$$

with $\frac{d\theta}{dR} < 0$;

$$JD \Rightarrow 0 = R - \frac{b}{y} - \frac{\beta \cdot c \cdot \theta}{(1-\beta) \cdot y} + \frac{\delta}{(r+\delta)} \cdot \int_R^1 (s-R)\, dG(s)$$

, with $\frac{d\theta}{dR} > 0$.

Therefore, sufficient condition for the existence of an interior equilibrium is that

$$\lim_{\theta \to 0} \left\{ c \cdot \theta^{\alpha} = \frac{(1-\beta) \cdot y}{(r+\delta)} \cdot (1-R) \right\} \Rightarrow R = 1 \quad \text{is}$$

higher than

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[19] It must be pointed out that this is, however, mainly empirical evidence relative to the US and not European economy (Boeri, 1996). It is probable that this depends on the restrictions present in the European context that make job closing difficult (Garibaldi, 1998).

[20] In the presence of exogenous job destruction, instead, the immediate destruction of the job was hypothesised following a negative shock.

[21] This hypothesis can be generalised by indicating a positive value x_{max} as the maximum value of idiosyncratic (or specific) component x.

$$\lim_{\theta \to 0} \left\{ 0 = R - \frac{b}{y} - \frac{\beta \cdot c \cdot \theta}{(1-\beta) \cdot y} + \frac{\delta}{(r+\delta)} \cdot \int_R^1 (s-R)\, dG(s) \right\}$$

$$\Rightarrow R = \frac{b}{y} - \frac{\delta}{(r+\delta)} \cdot \int_R^1 (s-R)\, dG(s) < 1 .$$

These results are completely intuitive: the "new" *Job Creation Condition* has a negative slope even in the (θ, R) interval since if R increases, the average duration of a job is reduced, and it is for this reason that the firm opens fewer vacancies, thereby decreasing θ. By inverse reasoning, the *Job Destruction Curve* increases in R and therefore has a positive slope in the (θ, R) interval.

4 Out-of-steady-state dynamics

This paragraph focuses on the behaviour of unemployment rate and market tightness *out-of-steady-state*, namely during the adjustment period that leads to equilibrium.

One of the two main differential equations needed to study the dynamic of the model was introduced in the previous paragraph, i.e. $\dot{u}(t) = (1-u(t)) \cdot \delta - u(t) \cdot \theta(t)^{1-\alpha}$. From the dynamic equation for the unemployment rate, it immediately follows that the "reaction" (i.e. the variation over time) of $\dot{u}$ with respect to u is negative: an increase in u, in fact, reduces the inflows and increases the outflows. This implies that for the points to the left and right of the curve $\dot{u} = 0$, the value of u tends to get increasingly closer to its steady state equilibrium value, i.e. for any initial value of u_0 , unemployment always converges to its equilibrium value (*stable locus* – see Figure 1). Due to the properties of the function $\theta^{1-\alpha}$, the relationship of $\dot{u}$ with respect to θ is also negative. Intuitively, if the probability of finding a job increases, the unemployment rate decreases.

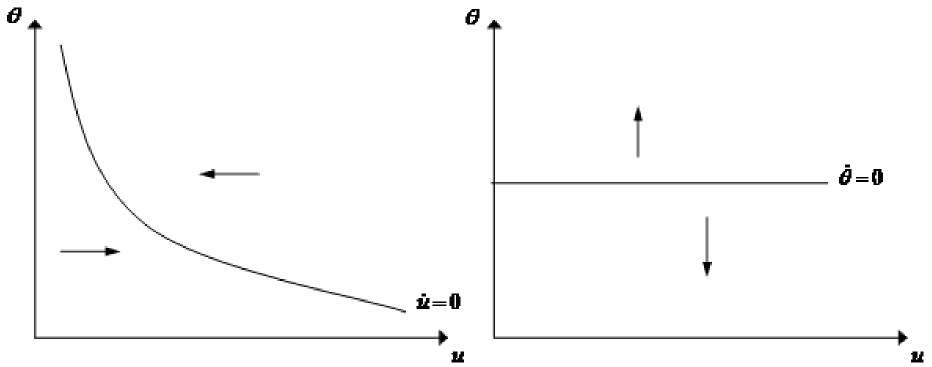

Figure 1. Out-of-steady-state dynamics of unemployment and market tightness

On the other hand, it can be formally proven (cf. *Appendix C* for the mathematical details) that the variation of θ over time does not depend (in an independent manner) on the rate of unemployment, but only on the level of θ and on the model's parameters. The variations in u are mediated by the matching process: in fact, as v (and therefore θ) varies, unemployment also varies due the change in the probability of finding a job.

Furthermore, it is possible to show that $\frac{\partial \dot{\theta}(t)}{\partial \theta(t)} > 0$ (see again *Appendix C*). This implies that for the points lying above and below the curve $\dot{\theta} = 0$, the value of θ tends to shift increasingly further from its steady state value (*unstable locus* – cf. *Figure 1*).

The apparently unstable behaviour of θ is due to the fact that firms base their decision to create vacancies on the future expected value of θ, and immediately create more vacancies if they foresee a future increase in vacant jobs in order to avoid creating new ones when their opening cost will be higher. In fact, the higher θ , the lower the probability of filling a vacancy, whereas the average duration of a filled vacancy increases. This "forward looking" attitude of firms, with regards to vacancies, makes v and θ "jump" variables, i.e. variables which respond immediately to changes in parameters or expectations. For this reason, labour market tightness immediately becomes long term and remains present throughout the entire adjustment period. The presence of a "backward looking" variable, i.e. a predetermined variable (the unemployment rate), and of a "forward looking" variable (the vacancy rate), implies a very simple adjustment dynamic that in turn implies the existence of a unique dynamic path (*saddlepath*) converging at steady state (*saddlepoint*), shown by point E in Figure 2.

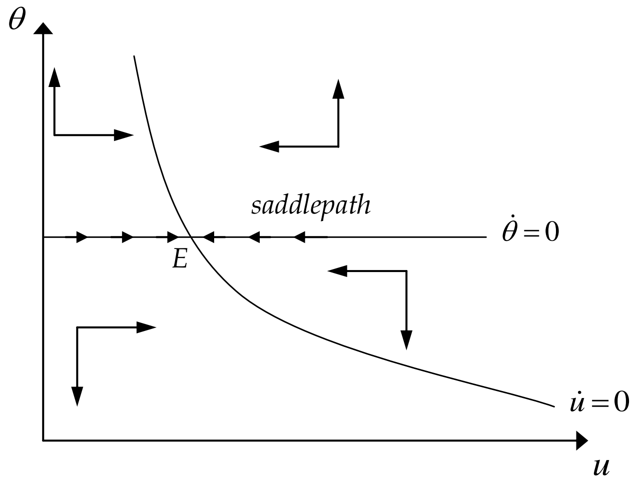

Figure 2. Adjustment paths in labour-market tightness and unemployment space

It is possible to formally verify the nature of an equilibrium *saddlepoint* by linearising the dynamic equations surrounding a generic steady state equilibrium point $(\bar{u},\ \bar{\theta})$:

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} - & - \\ 0 & + \end{pmatrix} \cdot \begin{pmatrix} u-\bar{u} \\ \theta-\bar{\theta} \end{pmatrix}$$

The negative sign of the determinant of the coefficient matrix confirms the nature of the steady state equilibrium *saddlepoint*.[22]

5 The problem of social efficiency in the decentralised equilibrium

The existence of externalities, and the fact that they are not taken into account by individual optimisation problems, immediately questions the social efficiency of the decentralised equilibrium. As shown in Pissarides (chapter 8, 2000) and Bagliano-Bertola (paragraph 5.4, 1999), the decentralised market equilibrium achieved in the matching models coincides with the socially efficient equilibrium solution (in other words, it is efficient) when the surplus quota owed to the labour factor (β) is equal to the elasticity (with respect to θ) of the average duration of a vacancy (α).[23] Formally, the condition $\beta=\alpha$ can be derived by comparing the decentralised solution, put in place by a representative firm, and the socially efficient solution, put in place by a social planner. The solutions of the respective optimisation problems are the following (cf. *Appendix D* for the mathematical details):

- Decentralised solution: $\dfrac{y-w}{r+\delta}=\dfrac{c}{\theta^{-\alpha}}$;
- Socially efficient solution: $\dfrac{y-b}{\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)}=\dfrac{c}{(1-\alpha)\cdot\theta^{-\alpha}}$.

By comparing the two optimality conditions it is deduced that:

a) The "social" discount rate is larger than the "individual" rate $\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)>(r+\delta)$. In fact, in the socially efficient solution, congestion externalities created by an increase in vacancies, and therefore θ, are taken into account by a social planner. Therefore, in the socially efficient solution, the marginal value of a filled vacancy is discounted at a higher rate.

[22] In order to have equilibrium stability, the matrix trace must be negative. In fact, *"The equilibrium is a node that can be stable or unstable depending on whether the matrix trace is, respectively, smaller than or larger than zero"* (cf. Bagliano and Bertola, p.259, 1999).

[23] The average duration of a vacancy is the reciprocal of the probability of filling a vacancy, i.e. θ^{a}. The hypothesis of constant returns to scale implies that the elasticity with respect to θ of the average duration of a vacancy is equal to the elasticity of the matching function with respect to the unemployment rate. According to *Cobb-Douglas*, this elasticity is equal to α.

b) The decentralised solution attributes a lower net productivity to a filled job than the socially efficient solution, since $w \geq b$.[24]
c) The expected cost of a filled vacancy evaluated by the socially efficient solution is larger than the estimated provided by the decentralised solution, namely $\frac{c \cdot \theta^{\alpha}}{(1-\alpha)} > c \cdot \theta^{\alpha}$, since $1 > (1-\alpha)$. This means that, with respect to the decentralised solution, the social planner will open a smaller number of vacancies so as not to further increase the average duration, and therefore the expected cost of a vacancy.
Basically, the two solutions differ due to interest in congestion externalities in the centralised solution and the presence of wages in the decentralised solution. For this reason, the decentralised equilibrium will most probably be inefficient, since the rule for determining wages by subdividing the surplus between matched workers and firms neglects those (vacancies and unemployed) that are still engaged in search activities. The decentralised market equilibrium coincides with the socially efficient solution and, consequently, the wage determined by the *Nash rule* "internalises" the research externalities, when the following is true (in equilibrium $V = 0$):

$$\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} = J + W - U \qquad [18]$$

the efficiency condition requires that the expected cost of a filled vacancy, evaluated by the socially efficient solution, be equal to the surplus created by a match. Combining the former expression with the optimisation condition $W - U = \frac{\beta}{1-\beta}\cdot J$, it follows that:

$$\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} = \frac{1}{1-\beta}\cdot J$$

$$\Rightarrow \frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} = \frac{1}{1-\beta}\cdot\frac{c}{\theta^{-\alpha}}$$

where $J = \frac{c}{\theta^{-\alpha}}$ is the expected cost of a filled vacancy obtained from the optimisation condition in the decentralised equilibrium. The efficiency condition is therefore:[25]

$$1-\beta = 1-\alpha \Rightarrow \alpha = \beta \qquad [19]$$

It should be stressed that social efficiency is most influenced by the allocation of resources, and whether or not an efficient decentralised equilibrium is reached. Unemployment is, in fact, probably the most significant result of the chosen mechanism for resource allocation, but it is not the cause of a non efficient allocation. When $\beta \neq \alpha$ the allocation of resources is not efficient since:

- if $\beta > \alpha$, firms create fewer jobs and workers search with less intensity since the reserve wage is excessively high (result: high unemployment);
- if $\beta < \alpha$, the reserve wage is too low and, as a consequence, workers accept a job too easily (result: underemployment).

Therefore, very generally, equilibrium unemployment is greater than the socially efficient rate if $\beta > \alpha$, whereas the reverse is true for $\beta < \alpha$.[26]

6 The main extension of the basic matching framework: the model with career choice

Since the deliberate focus of these models is on the labour market, the matching literature wouldn't be complete without the formalisation of an individual's fundamental economic choice: the decision between entering the market as an entrepreneur or as a worker.

[24] The socially efficient solution disregards wages (since it simply constitutes a transfer of income between firms and workers) and considers the utility flows due to unemployed workers.

[25] It must be pointed out that this is the efficiency condition only when the matching function displays constant returns to scale.

[26] For a broader discussion on this subject see Pissarides (2000).

However, the formalisation of this choice within a matching framework, is relatively recent (cf. Fonseca *et al*., 2001; Pissarides, 2002; Uren, 2007).

In matching models, the economic decision of an individual to become entrepreneur or worker is based on the comparison of the two values expected from labour market entry, i.e. the unemployment value and the vacancy value. Indeed, in Uren (2007), the equality condition $rV(\theta) = rU(\theta)$ allows the equilibrium value of labour market tensions to be determined, using the already discussed *Bellman equations* (see *Appendix E*):[27]

$$\theta^{1-\alpha} \cdot \beta \cdot (y - 2z + 2c) - \theta^{-\alpha} \cdot (1-\beta) \cdot (y - 2z) + c \cdot (r + \delta) = 0 \qquad [20]$$

The existence and the uniqueness of the value of θ that satisfy this former expression is guaranteed by the condition $y - 2 \cdot z > 0$ (see again *Appendix E*). This condition arises since a job match generates y units of output but requires the input of a worker and an entrepreneur. Each individual may receive a flow utility of z when unemployed. Hence, for gain from production to exist, $y - 2 \cdot z > 0$ is necessary.

Unlike the standard case of the basic model, the *free-entry* condition ($V = 0$) is no longer used to determine the equilibrium value of θ. Intuitively, in a model in which there is a fixed total number of firms, there is no need to apply the *zero-profit* condition when creating vacancies. In brief, if the number of firms is constant, the unrealistic possibility of infinite vacancy openings can never be true due to the fact that each firm only has one job/worker (*one-job firm*) Indeed, in the models that offer a *career choice*, the total population (not the labour force) is normalised to one, i.e. $1 = (1-l) + l = n + v + n + u$, where $(1-l) = n + v$ and $l = n + u$ represent, respectively, the overall quota of entrepreneurs and of workers in the total population. The number of entrepreneurs ($1-l$) and of workers (l) is obtained from the equations describing how vacancies and unemployment evolve over time:

$$\dot{v} = \delta \cdot [(1-l) - v] - \theta^{-\alpha} \cdot v \qquad [21]$$

$$\dot{u} = \delta \cdot (l - u) - \theta^{1-\alpha} \cdot u \qquad [22]$$

where $(1-l) - v$, the difference between the total number of firms and of vacancies, are the filled jobs, whereas $(l - u)$ represents employed workers, i.e. the difference between the labour force and unemployed workers. It is interesting to note that, unlike the basic model analysed previously (in which, given u and θ, the equilibrium level of vacancies is determined by the relationship $v = u \cdot \theta$), this model also uses a dynamic equation for the vacancies. This is due to the fact that the new expression also makes explicit reference to the quota of entrepreneurs/firms in the total population. Finally, by applying the definition of labour market tensions, the values of u and v, obtained through use of the steady state condition $\dot{v} = \dot{u} = 0$, are used to find the equilibrium value of l, which completes the model (see *Appendix E* for the mathematical details):

$$\theta = \frac{v}{u} \Rightarrow l = \frac{\delta + \theta^{1-\alpha}}{\delta \cdot (1+\theta) + 2 \cdot \theta^{1-\alpha}} \qquad [23]$$

From an economic point of view, a clearer distinction between entrepreneurs and workers can be found in Fonseca *et al*. (2001). The authors, in fact, introduce the entrepreneurial ability, ϑ, which follows a known distribution function, $F(\vartheta)$, in the population. This ability is comprised between a positive minimum value, $\vartheta_{min} > 0$, and a finite maximum value, ϑ_{max}. The model's solution is similar to that proposed by Uren, since the threshold value of entrepreneurial ability (S) is obtained from the following inequality:

[27] Uren (2007) uses the z notation to identify the free-time value that essentially replaces the utility flow due to unemployed workers, i.e. the unemployment benefit b. An entrepreneur that places a vacancy deducts the cost of opening a vacancy from the free-time value. Therefore, in the surplus calculation shown in *Appendix A*, the dividend associated to the vacancy value in Uren (2007) is $z - c$. As for the rest, the *Bellman equations* are analogous to those already seen.

$$\vartheta \cdot rV(\theta) - K \geq rU(\theta) \quad [24]$$

where K is a fixed cost (*start-up* cost). Since $V(\theta)$ and $U(\theta)$ are both assumed to be independent of ϑ (entrepreneurial ability is, in fact, a simple multiplicative parameter), the inequality satisfies the so called "reservation of entrepreneurial ability property": i.e. a reservation entrepreneurial ability, S, exists, such that an individual becomes entrepreneur if $\vartheta \geq S$; *vice versa*, for $\vartheta < S$, s/he enters the market as a worker. Consequently, $F(S) = u + n = 1 - \int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)$ is the quota of individuals that become workers, while $1 - F(S) = v + n = \int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)$ is the quota of entrepreneurs. Formally, the threshold value is given by:

$$S = \frac{rU(\theta) + K}{rV(\theta)} \quad [25]$$

with $S'(\theta) > 0$, since $V'(\theta) < 0$ and $U'(\theta) > 0$.[28] These properties can be very simply illustrated through the use of the *Bellman equations* introduced earlier (see *Appendix F* for the mathematical details). Intuitively, instead, the *Job Creation Condition* is decreasing in S, since if the threshold value is higher, then fewer individuals become entrepreneurs and, as a consequence, fewer vacancies are opened (see again *Appendix F*). As illustrated graphically (see *Figure 3*), the function $S(\theta)$ assumes a small but positive value ($S = \vartheta_{min}$) for $\theta = 0$, and tends to infinity for sufficiently large values of θ where $V(\theta) = 0$. *Vice versa*, the *Job Creation Condition* tends to zero for $S = \vartheta_{max}$ (the whole population chooses to become workers), whereas for $S = \vartheta_{min}$ it tends to its maximum value $(\bar{\theta} < \infty)$.[29]

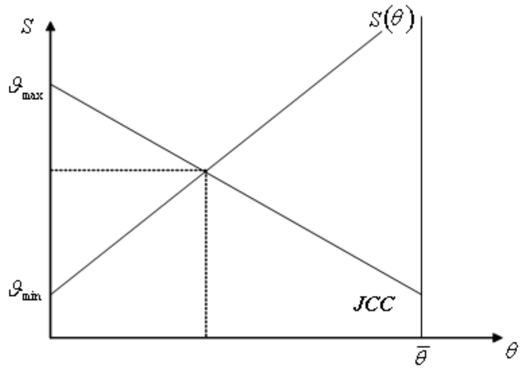

Figure 3. Equilibrium reservation (entrepreneurial) ability and market tightness.

The $\bar{\theta}$ shown in *Figure 3* is the value of θ that satisfies the condition $V(\bar{\theta}) = 0$, i.e. the equilibrium value of θ obtained from the standard matching model in the absence of entrepreneur-worker choice. Essentially, values of θ higher than $\bar{\theta}$ are excluded since if $\theta > \bar{\theta}$ then $V(\theta) < 0$ (see *Appendix F*).

As in the Uren (2007) model, the number of entrepreneurs in the total population is fixed; therefore the key role of the *zero-profits* condition in creating vacancies is lost. More precisely, in the Fonseca *et al*. (2001) model, the *cut-off* condition (from which the threshold value of entrepreneurial ability S is derived) determines – along with the *Job Creation Condition* – the total number of firms (filled jobs and vacancies) and of workers (employed and unemployed).

Finally, Pissarides (2002) basically enhances the former model. Indeed, the choice is now more detailed since the potential new entrepreneur also decides the number of job vacancies to be created and managed (γ), based on the following maximisation condition:

$$\max_{\gamma} \{\gamma \cdot rV(\theta) - \vartheta \cdot g(\gamma)\} \geq rU(\theta) \quad [26]$$

where $g(\gamma)$, with $g'(\gamma) > 0$, is the cost of managing a job which depends on the entrepreneurial ability ϑ. It follows that,

$$\Rightarrow \vartheta \cdot g'(\gamma) = rV \quad [27]$$

i.e. the marginal cost of managing a job is equal to the marginal revenue from the posting of one more job vacancy. The maximisation condition

[28] Intuitively, this is straightforward to understand since the greater θ, the smaller the probability of a firm filling a vacancy, and the greater θ, the higher the probability of the worker finding a job.

[29] Fonseca *et al*. (2001) exclude the value $\theta = \infty$ since in this case a vacancy is never filled.

is also used to obtain the threshold value that determines the entrepreneur-worker decision:

$$S = \frac{\gamma \cdot rV(\theta) - rU(\theta)}{g(\gamma)} \qquad [28]$$

However, unlike the previous model, individuals now become entrepreneurs when $\vartheta \le S$, since the increase in entrepreneurial ability decreases the management costs. Basically, the most able entrepreneurs have a lower ϑ, and therefore a lower management cost $g(\gamma)$.[30]

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[30] Indeed at the limit, when $\vartheta = 0$, the management costs are null, $g(\gamma) = 0$.

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Mathematical Appendixes

Appendix A

Wages are determined starting from the first order condition for the optimal subdivision of surplus, namely

$$w = arg\,max(J-V)^{1-\beta}\cdot(W-U)^{\beta}$$

$\Rightarrow (W-U)=\frac{\beta}{1-\beta}\cdot(J-V)$. By using the *Bellman equations*, it immediately follows that:

$$\Rightarrow \frac{w+\delta\cdot U}{r+\delta}-U=\frac{\beta}{1-\beta}\cdot\frac{y-w+\delta\cdot V}{r+\delta}-V$$

$$\Rightarrow w-rU=\frac{\beta}{1-\beta}\cdot(y-w-rV)$$

$$\Rightarrow w=(1-\beta)\cdot rU+\beta\cdot(y-rV)$$

since the *free-entry* condition ($V=0\Rightarrow J=c\cdot\theta^{\alpha}$) is valid, it is possible to deduce

$$rU=b+\theta^{1-\alpha}\cdot[W-U]\Rightarrow rU=b+\frac{\beta}{1-\beta}\cdot c\cdot\theta\ ,$$

from which the final expression is easily obtained:

$$w=(1-\beta)\cdot b+\beta\cdot y+\beta\cdot c\cdot\theta \qquad \text{[A.1]}$$

The surplus of a job, S, is defined as the sum of the worker's and firm's value of being on the job, net of the respective external options, so that: $S=J+W-V-U$. Applying basic algebra and using the *Bellman equations* the following is obtained:

$$rS=y+\delta\cdot(V-J+U-W)-\left[b+\theta^{1-\alpha}\cdot(W-U)\right]-\left[-c+\theta^{-\alpha}\cdot(J-V)\right]$$

finally, knowing that $(W-U)=\beta\cdot S$ and $(J-V)=(1-\beta)\cdot S$, we obtain:[31]

$$S=\frac{y-b+c}{r+\delta+\beta\cdot\theta^{1-\alpha}+(1-\beta)\cdot\theta^{-\alpha}} \qquad \text{[A.2]}$$

Appendix B

In order to obtain the "new" *Job Creation Condition* (JCC), the equation for determining wages is substituted into the expression for $r\cdot J(x)$:

$$r\cdot J(x)=y\cdot x-\left[\underbrace{(1-\beta)\cdot b+\beta\cdot y\cdot x+\beta\cdot c\cdot\theta}_{wage}\right]-\delta\cdot J(x)+$$

$$\Rightarrow (r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s) \qquad \text{[B.1]}$$

The value of the equation [B.1] is found for $x=R$, with $J(R)=0$:

$$0=(1-\beta)\cdot(y\cdot R-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s) \qquad \text{[B.2]}$$

The value of the equation [B.2] is subtracted from the equation [B.1], obtaining:

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-y\cdot R)$$

$$\Rightarrow J(x)=\frac{(1-\beta)\cdot(y\cdot x-y\cdot R)}{(r+\delta)} \qquad \text{[B.3]}$$

Considering equation [B.3] for $x=1$, (since the firm creates new jobs with maximum productivity), and using the expression for $J(1)$ obtained through the *zero-profit* condition, i.e. $r\cdot V=-c+\theta^{-\alpha}\cdot[J(1)-V]\Rightarrow J(1)=\frac{c}{\theta^{-\alpha}}$, the "new" *Job Creation Condition* (JCC) is obtained:

$$JCC\ \Rightarrow\ c\cdot\theta^{\alpha}=\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot(1-R) \qquad \text{[B.4]}$$

from which it immediately follows that $\frac{d\theta}{dR}<0$.

The *Job Destruction Curve* is determined in the following way. Starting with equation [B.1]:

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s)$$

$J(s)$ is substituted with [B.3], where, obviously, $x=s$

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot(s-R)\,dG(s) \qquad \text{[B.5]}$$

[B.5] is evaluated for $x=R$, which is the threshold productivity value of a job, below which the job itself is destroyed:

$$0=(1-\beta)\cdot y\cdot R-(1-\beta)\cdot b-\beta\cdot c\cdot\theta+\delta\cdot\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot\int_R^1(s-R)\,dG(s) \qquad \text{[B.6]}$$

[31] These rates can be obtained very simply from the first order condition for determining wages.

Finally, in order to obtain a clearer expression, all the members of [B.6] are divided by $(1-\beta)\cdot y$, obtaining:

$$JD \Rightarrow 0 = R - \frac{b}{y} - \frac{\beta \cdot c \cdot \theta}{(1-\beta)\cdot y} + \frac{\delta}{(r+\delta)} \cdot \int_R^1 (s-R)\,dG(s) \quad \text{[B.7]}$$

Completely differentiating this equation, we obtain:

$$\frac{\beta \cdot c}{(1-\beta)\cdot y} \cdot d\theta = dR \cdot \left\{ 1 - \left(\frac{\delta}{(r+\delta)} \right) \cdot [1-G(R)] \right\},$$

with $\frac{d\theta}{dR} > 0$, since the last term between brackets is a product of two numbers, both smaller than one.

Appendix C

The *free-entry* condition for equilibrium is valid even out of the stationary state, i.e. $V(t)=0 \Rightarrow J(t) = \frac{c}{\theta(t)^{-\alpha}}\ \forall\ t$. The dynamic of $J(t)$ out of the steady state is given by:

$$r \cdot J(t) = (y-w) + \delta \cdot [0 - J(t)] + \dot{J}(t)$$

$$\Rightarrow \dot{J}(t) = (r+\delta) \cdot c \cdot \theta(t)^{\alpha} - (y-w) \quad \text{[C.1]}$$

Differentiating $J(t) = c \cdot \theta(t)^{\alpha}$ with respect to time we obtain:

$$\dot{J}(t) = c \cdot \alpha \cdot \theta(t)^{\alpha-1} \cdot \dot{\theta}(t) \quad \text{[C.2]}$$

Substituting [C.2] into [C.1], we obtain the differential equation for θ:

$$\Rightarrow c \cdot \alpha \cdot \theta(t)^{\alpha-1} \cdot \dot{\theta}(t) = (r+\delta) \cdot c \cdot \theta(t)^{\alpha} - (y-w)$$

$$\dot{\theta}(t) = \frac{(r+\delta)}{\alpha} \cdot \theta(t) - \frac{y}{c\cdot\alpha} \cdot \theta(t)^{1-\alpha} + \frac{w(\theta)}{c\cdot\alpha} \cdot \theta(t)^{1-\alpha} \quad \text{[C.3]}$$

From which we get:

$$\left.\frac{\partial \dot{\theta}(t)}{\partial \theta(t)}\right|_{\dot{\theta}(t)=0} = \frac{(r+\delta)}{\alpha} - (1-\alpha)\cdot \frac{y}{c\cdot\alpha}\cdot\theta(t)^{-\alpha} + \frac{\partial w/\partial\theta}{c\cdot\alpha}\cdot\theta(t)^{1-\alpha} + (1-\alpha)\cdot\frac{w}{c\cdot\alpha}\cdot\theta(t)^{-\alpha}$$

Dividing both sides by $\frac{\theta(t)^{-\alpha}}{c}$,

$$\frac{\partial \dot{\theta}(t)}{\partial \theta(t)} / \left[\theta(t)^{-\alpha}/c\right] =$$

$$\frac{c\cdot(r+\delta)\cdot\theta(t)^{\alpha}}{\alpha} - \frac{(1-\alpha)\cdot(y-w)}{\alpha} + \frac{\partial w \partial \theta}{\alpha} \cdot \theta(t) > 0$$

, since we know that $\partial w / \partial \theta > 0$, $\dot{\theta}(t) = 0 \Rightarrow (r+\delta)\cdot c \cdot \theta(t)^{\alpha} = y - w$, and $(1-\alpha) < 1$. Firms' decisions on vacancies to open makes dynamics of θ outside the steady state "unstable".

Appendix D

The representative firm *i* solves the following optimisation problem:[32]

$$\max_{v_i} \int_0^{\infty} \left[y \cdot n_i - w \cdot n_i - c \cdot v_i \right] \cdot e^{-r\cdot t}\, dt$$

subject to the constraint given by employment's evolution over time:

$$\dot{n}_i = \theta^{-\alpha} \cdot v_i - \delta \cdot n_i$$

The representative firm, that can open more than one vacancy at a time, takes the value of labour market tension as given, ignoring the effects that their own decisions will have on the aggregate conditions of labour market tension. Setting up the *Hamiltonian* we have that:

$$H(t) = \left\{ y \cdot n_i - w \cdot n_i - c \cdot v_i + \Lambda(t) \cdot \left[\theta^{-\alpha} \cdot v_i - \delta \cdot n_i \right] \right\} \cdot e^{-r\cdot t}$$

where v_i is the control variable, n_i is the state variable and $\Lambda(t)$ is the so called *"shadow value"* that specifically expresses the marginal value of a filled job for the firm. The optimisation solutions to the problem are the following:[33]

$$\frac{\partial H(t)}{\partial v_i} = 0 \Rightarrow \left[-c + \Lambda(t) \cdot \theta^{-\alpha} \right] \cdot e^{-r\cdot t} = 0$$

$$\Rightarrow \Lambda(t) = \frac{c}{\theta^{-\alpha}} \quad \text{[D.1]}$$

$$\frac{\partial H(t)}{\partial n_i} = -\frac{d\left(\Lambda(t)\cdot e^{-r\cdot t}\right)}{dt} \Rightarrow \left[y - w - \Lambda(t)\cdot\delta \right] \cdot e^{-r\cdot t} = -\left[\dot{\Lambda}(t) - r \cdot \Lambda(t) \right] \cdot e^{-r\cdot t}$$

[32] For simplicity, as is common in matching models, it is assumed that the marginal productivity of labour is a linear function of employment.

[33] The optimisation solutions also include the necessary trasversality condition: $\lim_{t\to\infty} \Lambda(t) \cdot e^{-r\cdot t} \cdot n_i = 0$.

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$$-\frac{d\left(\Lambda(t)\cdot e^{-r\cdot t}\right)}{dt}=-\left[\dot{\Lambda}(t)\cdot e^{-r\cdot t}+\Lambda(t)\cdot(-r)\cdot e^{-r\cdot t}\right]$$

$$\Rightarrow (y-w)=(r+\delta)\cdot\Lambda(t)-\dot{\Lambda}(t) \qquad \text{[D.2]}$$

[D.1] is a standard optimality condition: in equilibrium, the marginal value of a filled job must be equal to the expected cost of a vacancy. [D.2] on the other hand expresses the evolution in time of the marginal value of a filled vacancy. In the steady state, with $\dot{\Lambda}(t)=0$, combining the two solutions, the standard equilibrium condition is obtained for the job demand side, i.e. the *Job Creation Condition*:

$$\frac{y-w}{r+\delta}=\frac{c}{\theta^{-\alpha}} \qquad \text{[D.3]}$$

which is exactly the same as the *JCC* obtained in the standard basic model.

As regards the socially efficient solution, the maximisation problem is the following:

$$\max_{v}\int_{0}^{\infty}\left[y\cdot n+b\cdot(1-n)-c\cdot v\right]\cdot e^{-r\cdot t}\,dt$$

$$\dot{n}=v\cdot\left(\frac{v}{1-n}\right)^{-\alpha}-\delta\cdot n$$

In this case, the value of labour market tension is endogenous. Moreover, the socially efficient solution ignores the wage and considers the utility flows obtained from unemployed workers, i.e. $b\cdot(1-n)$, where the labour force is, for simplicity, normalised to 1; hence, $(1-n)$ is the unemployment rate. As before, the optimisation solutions are obtained by formulating the *Hamiltonian*:

$$H(t)=\left[y\cdot n+b\cdot(1-n)-c\cdot v+\Lambda(t)\cdot\left[v\cdot\left(\frac{v}{1-n}\right)^{-\alpha}-\delta\cdot n\right]\right]\cdot e^{-r\cdot t}$$

$$\frac{\partial H(t)}{\partial v}=0\Rightarrow\left\{-c+\Lambda(t)\cdot\left[\left(\frac{v}{1-n}\right)^{-\alpha}+v\cdot(-\alpha)\cdot\left(\frac{v}{1-n}\right)^{-\alpha-1}\cdot\frac{1}{1-n}\right]\right\}\cdot e^{-r\cdot t}=0$$

$$\Rightarrow\Lambda(t)=\frac{c}{\theta^{-\alpha}-\alpha\cdot\theta^{-\alpha}}=\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} \qquad \text{[D.4]}$$

$$\frac{\partial H(t)}{\partial n_i}=-\frac{d\left(\Lambda(t)\cdot e^{-r\cdot t}\right)}{dt}$$

$$\left\{y-b-\Lambda(t)\cdot\delta+\Lambda(t)\cdot\left[v\cdot v^{-\alpha}\cdot\alpha\cdot(1-n)^{\alpha-1}\cdot(-1)\right]\right\}\cdot e^{-r\cdot t}=$$
$$-\left[\dot{\Lambda}(t)-r\cdot\Lambda(t)\right]\cdot e^{-r\cdot t}$$

$$\Rightarrow (y-b)=\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)\cdot\Lambda(t)-\dot{\Lambda}(t) \,\text{[D.5]}$$

Combining [D.4] with [D.5] at the steady state $\dot{\Lambda}(t)=0$, the marginal value of a filled job is obtained, i.e. the *Job Creation Condition*:

$$\frac{y-b}{\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)}=\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} \qquad \text{[D.6]}$$

Appendix E

With the possibility that an entrepreneur can deduct the cost of opening a vacancy from the free-time value *z*, the surplus is now given by $S=\frac{y-2z+c}{r+\delta+(1-\beta)\cdot\theta^{-\alpha}+\beta\cdot\theta^{1-\alpha}}$. Hence, by using the *Bellman equations*, it is possible to solve the equality condition $rV(\theta)=rU(\theta)$:

$$\Rightarrow(z-c)+\theta^{-\alpha}\cdot[J-V]=z+\theta^{1-\alpha}\cdot[W-U]$$

$$\Rightarrow -c+\theta^{-\alpha}\cdot(1-\beta)\cdot S=\theta^{1-\alpha}\cdot\beta\cdot S$$

$$0=\theta^{1-\alpha}\cdot\beta\cdot(y-2z+2c)-\theta^{-\alpha}\cdot(1-\beta)\cdot(y-2z)+c(r+\delta) \qquad \text{[E.1]}$$

which is defined in the following way,

$$C(\theta)=\theta^{1-\alpha}\cdot\beta\cdot(y-2z+2c)-\theta^{-\alpha}\cdot(1-\beta)\cdot(y-2z)+c(r+\delta) \qquad \text{[E.2]}$$

given the so called *Inada* conditions: $\lim_{\theta\to 0}\theta^{1-\alpha}=\lim_{\theta\to\infty}\theta^{-\alpha}=0$, $\lim_{\theta\to\infty}\theta^{1-\alpha}=\lim_{\theta\to 0}\theta^{-\alpha}=\infty$, under the condition $y-2z>0$, we obtain:

$$C'(\theta)=(1-\alpha)\cdot\theta^{-\alpha}\cdot\beta\cdot(y-2z+2c)-(-\alpha)\cdot\theta^{-\alpha-1}\cdot(1-\beta)$$

; $\lim_{\theta\to 0}C(\theta)=-\infty$; $\lim_{\theta\to\infty}C(\theta)=+\infty$; as a consequence, the *intermediate value theorem* implies the existence of a solution and the monotonic nature of $C(\theta)$ guarantees uniqueness. Once the uniqueness of the equilibrium value of the vacancy-unemployment relationship is guaranteed, it is possible to describe the allocation of the individuals between entrepreneurship and labour force (i.e. to know the equilibrium values of l). In steady state we have: $v=\frac{\delta\cdot(1-l)}{\delta+\theta^{-\alpha}}$, $u=\frac{\delta\cdot l}{\delta+\theta^{1-\alpha}}$. For the steady level of vacancies and unemployment to be consistent with the equilibrium value of labour market tensions, the $\theta=\frac{v(l)}{u(l)}$ relationship must be respected. Solving the former expression for l, it is possible to obtain the equilibrium value of workers (l) and, as a consequence, of entrepreneurs ($1-l$):

$$\Rightarrow \theta = \frac{\delta \cdot (1-l)}{\delta + \theta^{-\alpha}} \cdot \frac{\delta + \theta^{1-\alpha}}{\delta \cdot l} \Rightarrow \theta = \frac{(1-l)}{l} \cdot \frac{\delta + \theta^{1-\alpha}}{\delta + \theta^{-\alpha}}$$

$$\Rightarrow \theta \cdot l \cdot (\delta + \theta^{-\alpha}) = (1-l) \cdot (\delta + \theta^{1-\alpha})$$
$$\Rightarrow \theta \cdot l \cdot (\delta + \theta^{-\alpha}) + l \cdot (\delta + \theta^{1-\alpha}) = \delta + \theta^{1-\alpha}$$
$$l = \frac{\delta + \theta^{1-\alpha}}{\delta \cdot (1+\theta) + 2 \cdot \theta^{1-\alpha}} \qquad \text{[E.3]}$$

Appendix F

From the *Bellman equations*,

$$r \cdot V = -c + \theta^{-\alpha} \cdot (J - V)$$
$$r \cdot J = y - w + \delta \cdot (V - J)$$
$$r \cdot W = w + \delta \cdot (U - W)$$
$$r \cdot U = z + \theta^{1-\alpha} \cdot (W - U)$$

very simple algebra gives:

$$[J - V] = \frac{y - w + c}{r + \delta + \theta^{-\alpha}} ; [W - U] = \frac{w - z}{r + \delta + \theta^{1-\alpha}}$$

Hence, it is straightforward to get:

$$rV = \frac{\theta^{-\alpha} \cdot (y - w) - c \cdot (r + \delta)}{r + \delta + \theta^{-\alpha}}$$

[F.1]

$$rU_r = \frac{\theta^{1-\alpha} \cdot w + z \cdot (r + \delta)}{r + \delta + \theta^{1-\alpha}} \qquad \text{[F.2]}$$

with $\frac{\partial rV}{\partial \theta} = -(y - w + c) \cdot (r + \delta) < 0$ and $\frac{\partial rU}{\partial \theta} = (w - z) \cdot (r + \delta) > 0$, since it must be true that $w > z$. Furthermore, $lim_{\theta_r \to 0} rV_r = y - w$, by the l'*Hôpital rule*; $lim_{\theta_i \to 0} rU = z$; $lim_{\theta_i \to \infty} rV = -c$; $lim_{\theta_i \to \infty} rU = w$, by the l'*Hôpital rule*.

The evolution of employment can be expressed in terms of both firm's transition rates ($\theta^{-\alpha}$) and worker's transition rates ($\theta^{1-\alpha}$), i.e.:

$$\dot{n} = \left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) - n \right) \cdot \theta^{-\alpha} - \delta \cdot n ;$$
$$\dot{n} = [F(S) - n] \cdot \theta^{1-\alpha} - \delta \cdot n$$

Hence, in steady-state ($\dot{n}_r = 0$), we get:

$$n = \frac{\left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) \right) \cdot \theta^{-\alpha}}{\theta^{-\alpha} + \delta} ; \; n = \frac{F(S) \cdot \theta^{1-\alpha}}{\theta^{1-\alpha} + \delta}$$

It follows that for any level of employment *n*,

$$\frac{\left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) \right) \cdot \theta^{-\alpha}}{\theta^{-\alpha} + \delta} = \frac{F(S) \cdot \theta^{1-\alpha}}{\theta^{1-\alpha} + \delta}$$

Straightforward algebra gives:

$$\frac{\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)}{F(S)} = \frac{\theta^{1-\alpha} + \delta \cdot \theta}{\theta^{1-\alpha} + \delta} \qquad \text{[F.3]}$$

By the properties of the matching function, the right-hand side is increasing in θ; whereas, the left-hand side is decreasing in S. Therefore, total differentiation gives $\frac{d\theta}{dS} < 0$.