Model Uncertainty, the Spirit of Capitalism and Asset Pricing

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Abstract

This paper examines how a preference for robustness affects optimal consumption-portfolio rules as well as the equilibrium asset returns when investors care about their social status (or they have the spirit of capitalism). It is shown that the interaction of these two preferences leads to higher equity premium by enhancing investors's effetive risk aversion and making them more conservative in risk-taking. In addition, we find that they also lead to greater precautionary savings and lower risk-free rate in general equilibrium. We then show that the interaction of the two preferences has the potential to resolve the equity premium puzzle and the risk-free rate puzzle for plausible parameter values.

Keywords: the Spirit of Capitalism; Robustness; the Equity Premium Puzzle; the Risk-free Rate Puzzle.

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1 Introduction

In their seminal paper, Mehra and Precott (1985) showed that in order to replicate the empirical risk premium in the representative agent paradigm, the investor must display astronomically high levels of risk aversion. This theoretical difficulty was documented as the equity premium puzzle. Since the high risk premium implies either a high average equity return or a low average risk-free rate, Weil (1989) documented another risk-free rate puzzle: with the desire for consumption-smoothing and a low risk-free rate, individuals still save enough that per capita consumption grows rapidly. In order to solve these asset pricing puzzles, two solutions have been put forward by Baskin and Chen (1996), Smith (2001) and Maenhout (2004), respectively. On one hand, Baskin and Chen (1996) and Smith (2001) suggest that with higher effective risk avesion driven by the spirit of capitalism (henceforth SOC), the individual will be more conservative in risk-taking and more frugal in consumption spending and hence stock prices tend to be more volatile than when the SOC is absent. On the other hand, Maenhout (2004) shows that robustness leads to environment-specific effective risk aversion and hence dramatically decreases the demand for equities and raises precautionary savings simultaneously.[1] Therefore, robustness helps to resolve both the equity premium and the risk-free rate puzzles in the equilibrium setting. Since the implications for asset pricing of these two modelling strategies are examined separately, high preference parameter values for the SOC or robustness might be needed implausibly.

By combining these two modelling strategies into the standard Merton-type model, the paper shows how the SOC and robustness interact in determining asset prices and how the asset pricing puzzle can be resolved. On one hand, both the preference for SOC and for robustness increase the effective risk aversion, make investors more pessimistic in risk-taking and hence generate the high risk premium. On the other hand, these two channels lead to more precautionary savings in general equilibrium and reduce the equilibrium risk-free rate. The qualitative and quantitative analysis show that the combination of these two channels can resolve the risk premium puzzle and the risk-free rate puzzle.

The organization of the paper is as follows. Section 2 presents the Merton-type model with the SOC. Section 3 derives robust consumption and portfolio rules by incorporating model uncertainty into the basic model. Section 4 examines asset pricing rules in a robust equilibrium and presents the calibration exercise. Section 5 concludes.

[1] Luo (2015) studies how robustness affects the intertemporal hedging demand for the risky asset in a constant-absolute-risk-averse (CARA) model with uninsurable labor income.

2 The Merton-type Model with the SOC

In this paper, we follow Bakshi and Chen (1996) and Smith (2001) and consider a continuous-time version of the Merton-type model (1969, 1972) with the SOC, where the typical consumer maximizes "expected" lifetime utility from consumption of two "good": consumption c_t and "status" w_t, and has assess to two financial assets: one riskless, paying an instantaneous return r and one risky (equities), paying a constant intantaneous expected excess return of $\mu - r$. The objective function to maximize (in the absence of a preference for robustness) is

$$E_0\left[\int_{t=0}^{T} \exp\left(-\delta t\right) u\left(c_t, w_t\right) dt\right],$$

where $u\left(c_t, w_t\right) = \left(c_t^a w_t^b\right)^{1-\gamma} / \left(1-\gamma\right)$, $\gamma\,(>0)$ denotes the coefficient of relative risk aversion[2], δ is the discount rate, a and b are positive parameters satisfying $a + b \leq 1$[3], and b measures the degree of the SOC.

The price of the risky asset evolves according to the standard geometric Brownian motion with constant drift coefficients driven by a standard Wiener process B_t:

$$dS_t = \mu S_t dt + \sigma S_t dB_t. \tag{1}$$

Therefore the state equation for wealth is

$$dw_t = \left[w_t\left(r + \alpha_t\left(\mu - r\right)\right) - c_t\right] + \alpha_t \sigma w_t dB_t, \tag{2}$$

where α_t is the fraction of wealth invested in the risky asset at time t. Both controls (c_t, α_t) are nonanticipating and suitably adapted to the σ-algebra generated by the underlying Brownian motion.

In this benchmark model with the SOC, we assume that the consumer trust the model totally, i.e., no model uncertainty. The value function is denoted by $V\left(w_t, t\right)$. The Hamilton-Jacobi-

[2] The calculations in the paper hold well for the case of $\gamma \in (0, +\infty)$. However, in the following discussion, we assume that coefficient of relative risk aversion γ is larger than one for two reasons. One is that we can guarantee that optimal consumption is nonnegative with the assumption of $\gamma > 1$. The other one is empirically relevant. In most of the simulation or estimation excercises, the coefficient of relative risk aversion is larger than (or equals) one. Baskin and Chen (1996) also impose this assumption in their research.

[3] Notice that $\left(c_t^a w_t^b\right)^{1-\gamma} / \left(1-\gamma\right) = \left(c_t^{a/(a+b)} w_t^{a/(a+b)}\right)^{(a+b)(1-\gamma)} / \left(1-\gamma\right)$. Since the utility function is ordinally meaningful, the relative values of a and b are important. Then, we impose the assumption of $a + b \leq 1$ for simplification.

Bellman (henceforth HJB) equation for this optimizing problem can be written as

$$0 = \sup_{\alpha_t, c_t} \left[\frac{\left(c_t^a w_t^b\right)^{1-\gamma}}{1-\gamma} - \delta V(w,t) + \mathcal{D}^{(\alpha,c)} V(w_t,t) \right], \tag{3}$$

where

$$\mathcal{D}^{(\alpha,C)} V(w_t,t) = V_w \left[w \left(r + \alpha_t (\mu - r) \right) - c_t \right] + V_t + \frac{1}{2} V_{ww} \alpha_t^2 \sigma^2 w_t^2, \tag{4}$$

with boundary condition

$$V(w_T, T) = 0. \tag{5}$$

Solving the above HJB subject to (2) and (5) leads to the following portfolio-consumption rules:

$$c_t = \frac{\phi}{1 - e^{-\phi(T-t)}} \frac{a}{a+b} w_t, \tag{6}$$

and

$$\alpha_t = \frac{1}{[1 + (a+b)(\gamma - 1)]} \frac{\mu - r}{\sigma^2}, \tag{7}$$

where

$$\phi \equiv \frac{1}{a(\gamma-1)+1} \left[\delta + r(a+b)(\gamma-1) + \frac{1}{2} \frac{(a+b)(\gamma-1)}{[1+(a+b)(\gamma-1)]} \left(\frac{\mu - r}{\sigma} \right)^2 \right]. \tag{8}$$

Equation (6) tells that the marginal propensity to consume (MPC) of the consumer is an increasing function of the degree of the SOC, b. With stronger degree of the SOC (larger b), the consumer attaches more importance on the "status" good rather than the consumption good and hence the MPC is smaller. It is shown in equation (7) that the optimal share of wealth invested on risky assets is also constant, however, the effective coefficient of relative risk aversion ($[1 + (a+b)(\gamma - 1)]$) depends on the degree of the SOC, b, and the coefficient of the relative risk aversion, γ. Due to the assumption of $\gamma > 1$, then an increase in the degree of the SOC increases the effective risk aversion, $\frac{d\Gamma}{db} = \gamma - 1 > 0$, and hence reduces demand for the risky asset, $\frac{d\alpha_t}{db} = \frac{(1-\gamma)\alpha_t}{R} < 0$. These results are very similar to Bakshi and Chen (1996) and Smith (2001).

3 Robust Portfolio and Consumption Rules

3.1 Incorporating Model Uncertainty

In the above Merton model with the SOC, the consumer knows the exact probability model when they make decisions under rational expectations. In reality, the decision maker accepts the "reference model" as useful, but suspects it to be misspecified. She therefore wants to consider alternative models that are reasonably similar to the reference model when computing her continuation payoff. In a pure diffusion setting, Anderson, Hansen and Sargent (2002; henceforth AHS) show that this adverse alternative model simply adds an endogenous drift $u(W_t)$ to the law of motion of the state variable w_t,

$$dw_t = \mu(w_t)\,dt + \sigma(w_t)\left[\sigma(w_t)\,u(w_t)\,dt + dB_t\right], \tag{9}$$

where $\mu(w_t)$ and $\sigma(w_t)$ are short-hand notation for the drift and diffustion terms in (2). The drift adjustment $u(w_t)$ is chosen endogenously to minimize the sum of the expected (differential) continuation payoff of (4), but adjusted to reflect the additional drift component in (9), and of an entropy penalty, namely,

$$\inf_{u}\left[\mathcal{D}V + u(w_t)\,\sigma(w_t)^2 + \frac{1}{2\widehat{\theta}}u(w_t)^2\,\sigma(w_t)^2\right]. \tag{10}$$

The first two terms in the objective are the expected continuation payoff when the state variable follows (9), that is, the alternative model based on drift distortion u. The third term stands for the entropy penalty incurred when selecting adverse drift distortions in (10) and moving away from the reference model. The parameter $\widehat{\theta} > 0$ measures the strength of the preference for robustness ($\widehat{\theta} = 0$ corresponds to expected utility maximization). Therefore the more robust decision maker ($\widehat{\theta}$ larger) has less faith in the reference model and will consider drift distortions when evaluating her continuation payoff. The parameter $\widehat{\theta}$ is fixed exogenously and state independent in the AHS minimum-entropy robustness model. In this paper, following Maenhout (2004), we impose the "homothetic robustness" property on the model setup, which endogenize $\widehat{\theta}$ by scaling θ by the value function and preference parameters, denoted by $\Psi(w,t) > 0$, namely,

$$\Psi(w_t,t) = \frac{\theta}{(a+b)(1-\gamma)\,V(w_t,t)} > 0. \tag{11}$$

Applying these assumptions to the Merton-type model with the SOC gives us,

$$0 = \sup_{\alpha,c}\inf_{u}\left[\frac{\left(c_t^a w_t^b\right)^{1-\gamma}}{1-\gamma} - \delta V(w,t) + \mathcal{D}^{(\alpha,c)}V(w,t) + u(w_t)\,\sigma(w_t)^2 + \frac{1}{2\Psi(w,t)}u(w_t)^2\,\sigma(w_t)^2\right], \tag{12}$$

where $\mathcal{D}^{(\alpha,c)}V(w,t)$ is given by (4), subject to (5).

3.2 Robust Consumption and Portfolio Rules

The HJB equation (12) will be solved in this section. Solving first for the infimization part of the problem yields

$$u^* = -\Psi V_w. \tag{13}$$

Substituting for u^* in the HJB equation gives

$$0 = \sup_{\alpha,c} \left[\frac{\left(c_t^a w_t^b\right)^{1-\gamma}}{1-\gamma} - \delta V\left(w_t, t\right) + \mathcal{D}^{(\alpha,C)} V\left(w_t, t\right) - \frac{\Psi}{2} V_w^2 \alpha^2 \sigma^2 w_t^2 \right], \tag{14}$$

subject to (5). Plugging equations (4) and (11) into equation (14) leads to

$$0 = \sup_{\alpha,c} \left[\frac{\left(c_t^a w_t^b\right)^{1-\gamma}}{1-\gamma} - \delta V + V_w \left[w_t \left(r + \alpha_t \left(\mu - r\right)\right) - c_t \right] + V_t + \frac{1}{2} \left(V_{ww} - \frac{\theta V_w^2}{(a+b)(1-\gamma) V} \right) \alpha^2 \sigma^2 w_t^2 \right]. \tag{15}$$

The following results can then be obtained.

Proposition 1 *Equation (15) subject to (5) is solved by*

$$V\left(w_t, t\right) = \left(\frac{1 - e^{-\phi(T-t)}}{\phi} \right)^{(a(\gamma-1)+1)} \left(\frac{a}{a+b} \right)^{a(1-\gamma)} \frac{w_t^{(a+b)(1-\gamma)}}{(1-\gamma)}. \tag{16}$$

where $\phi \equiv \frac{1}{a(\gamma-1)+1} \left[\delta + r\left(a+b\right)\left(\gamma-1\right) + \frac{1}{2} \frac{(a+b)(\gamma-1)}{\Gamma} \left(\frac{\mu - r}{\sigma} \right)^2 \right]$. *The optimal portfolio and consumption rules, valid for* $\gamma > 0$, *are given by*

$$c_t = \frac{\phi}{1 - e^{-\phi(T-t)}} \frac{a}{a+b} w_t, \tag{17}$$

$$\alpha_t = \frac{1}{\Gamma} \frac{\mu - r}{\sigma^2}, \tag{18}$$

where $\Gamma \equiv \theta + \left[1 + (a+b)(\gamma - 1)\right]$.

Proof. See Appendix 6.1. ■

This result is remarkably simple, in light of the complexity of the HJB equation (15). The optimal fraction of wealth invested in the risky asset is independent of wealth and time due to the homotheticity. The optimal portfolio weight is also the standard Merton solution, where the effective risk aversion is determined by the coefficient of relative risk aversion, γ, the preference parameter for robustness, θ, and the preference parameter for the status, b. Robustness amounts therefore to an increase in the effective risk aversion. Equation (18) degenerates to the case without the preference for model uncertainty or the one of Meanhout (2004) without the SOC. The consumption rule has the same structure as Merton's solution. The difference is that the key parameter determining the consumption wealth ratio, ϕ, depends on the preferences for robustness and the SOC.

Given that the nonrobust consumer has power utility, she is equally willing to substitute over time as across states (as the coefficient of relative risk aversion is γ, which is also the inverse of the elasticity of intertemporal substitution). What robustness does is to make the consumer less willing to substitute across states (as the coefficient of relative risk aversion becomes $\theta + [1 + (a+b)(\gamma - 1)] > [1 + (a+b)(\gamma - 1)]$), without altering the willingness to subsitute intertemporally (as the elasticity of intertemporal substitution remains $1/\gamma$). In order to make this point more clear, we will extend the model to stochastic differential utility (SDU).

3.3 Stochastic Differential Utility

To further explore the implications on the optimal rules and asset pricing of both robustness and the SOC, we extend the model to the case with stochastic differential utility. Taking (15) and replacing $U(c, w) - \delta V$ by the normalized aggregator of Duffie-Epstein, we obtain

$$0 = \sup_{\alpha, c} \left[\begin{array}{c} \frac{1}{1-\eta} \left\{ \frac{\left(c_t^a w_t^b\right)^{1-\eta}}{((1-\gamma)V)^{\frac{\gamma-\eta}{1-\gamma}}} - \delta (1-\gamma) V \right\} + V_w \left[w_t \left(r + \alpha_t (\mu - r) \right) - c_t \right] \\ + V_t + \frac{1}{2} \left(V_{ww} - \frac{\theta V_w^2}{(a+b)(1-\gamma)V} \right) \alpha_t^2 \sigma^2 w_t^2 \end{array} \right], \tag{19}$$

where η^{-1} denotes the elasticity of intertemporal substitution, γ is risk aversion, and θ is the robustness parameter.

Proposition 2 *Equation (19) subject to equation (5) is solved by*

$$V(w_t, t) = \left[\frac{\psi}{\phi} \left(1 - e^{-\phi(T-t)} \right) \right]^{-\frac{(1-\gamma)(a(1-\eta)-1)}{(1-\eta)}} \frac{w_t^{(a+b)(1-\gamma)}}{(a+b)(1-\gamma)}, \tag{20}$$

where $\phi \equiv \frac{(1-\eta)}{(\gamma-1)(a(1-\eta)-1)} \left[\delta \frac{1-\gamma}{1-\eta} + r(a+b)(\gamma-1) + \frac{1}{2} \frac{(a+b)(\gamma-1)}{\Gamma} \left(\frac{\mu - r}{\sigma} \right)^2 \right]$, $\psi \equiv (a+b) \left(\frac{1}{\alpha} \right)^{\frac{a(1-\eta)}{a(1-\eta)-1}} \left(\frac{1}{a+b} \right)^{\frac{\gamma-\eta}{1-\gamma} \frac{1}{a(1-\eta)-1}}$, *and* $\Gamma \equiv \theta + [1 + (a+b)(\gamma - 1)]$. *The optimal portfolio and consumption rules are given by*

$$c_t = \frac{\phi}{1 - e^{-\phi(T-t)}} \frac{a}{a+b} w_t, \tag{21}$$

$$\alpha_t = \frac{1}{\Gamma} \frac{\mu - r}{\sigma^2}. \tag{22}$$

Proof. See Appendix 2. ■

Since an investor with a homothetic preference for robustness $\Psi = \theta / ((a+b)(1-\gamma)V)$ and CRRA utility is observationally equivalent to a Duffie-Epstein-Zin investor with the EIS $1/\gamma$ and the effective relative risk aversion Γ,[4] the only change in Propostion 3 relative to Propostion 2 concerns the parameters in the consumption rule and value function, which reflects the fact that the elasticity of intertemporal substitution has now been disentangled from the coefficient of relative risk aversion.

To explore the quantitative effect of robustness and the SOC on the optimal portfolio, we present the following useful calculations. The robust investor can be viewed as using an alternative model that adds an endogenous drift term to equation (2)

$$dw_t = [w_t (r + \alpha_t^* (\mu - r)) - c_t] + \alpha_t^* \sigma w_t [\alpha_t^* \sigma w_t u^* dt + dB_t].$$

Because all uncertainty in this budget constrait (i.e., the Brownian motion B_t) stems from the return on the risky asset, this implies that under the modified Markov process, the investor worries that the stock price evolves according to

$$\begin{aligned} \frac{dS_t}{S_t} &= \left[\mu + \alpha_t^* w_t \sigma^2 u^*\right] dt + \sigma dB_t \\ &= \left[\mu - (\mu - r)\frac{\theta}{\Gamma}\right] dt + \sigma dB_t, \end{aligned}$$

where the second equality obtains upon substitution of (13), (22), and (20). Consequently, the investor worries that the excess return on the risky asset is not the "true" equity premium $(\mu - r)$ $(\equiv EP_T)$, but rather EP_P, defined as

$$EP_P \equiv E_t^{u^*}\left[\frac{dS_t}{S_t} - rdt\right] = \frac{1 + (a+b)(\gamma - 1)}{\Gamma}(\mu - r)\, dt, \tag{23}$$

where $E_t^{u^*}[\cdot]$ denotes the expectation according to the alternative model that includes the "optimal drift distortion" u^*. Hence, θ can be then found to be

$$\theta = [1 + (a+b)(\gamma - 1)] \frac{EP_T - EP_P}{EP_P}. \tag{24}$$

[4] The proof is similar to Proposition 2 in Maenhout (2004).

4 Equilibrium Asset Pricing

To explore the equilibrium implication of the robust decision rules with status concern in the previous section, we now consider a simple exchange economy in the style of Lucas (1978). The representative agent recieves an endowment, which he has to consume in equilibrium, and can trade two assets in the economy: a risky asset entitling the agent to the risky endowment (the dividend) and a riskless asset. The returns of these two assets adjust to support a no-trade equilibrium. By utlilizing the above explicit partial-equilibrium result for SDU in the presence of the SOC and robustness, I show in closed form how the different determinants of behavior (EIS η^{-1}, risk aversion γ, SOC b and uncertainty aversion θ) affect the equilibrium equity premium and risk-free rate.

4.1 Robust Equilibrium, Market Price of Risk and Asset Pricing

For simplicity we assume that the dividend or endowment process follows a geometric Brownian motion process,

$$dD_t = \mu_D D_t dt + \sigma_D D_t dB_t, \tag{25}$$

where the expected growth rate μ_D and the standard deviation σ_D are strictyly positive parameters. It is conjectured that the price S_t of the risky asset representing a claim on the divident stream follows an Itô process:

$$dS_t = S_t \left(\mu_S - \frac{D_t}{S_t} \right) dt + \sigma_S S_t dB_t,$$

where the coefficients μ_S and σ_S are to be determined from equilibrium conditions. The conjecture implies that the total return on the risky asset, consisting of both the dividend yield and the capital gain, is simply

$$\frac{dS_t + D_t dt}{S_t} = \mu_S dt + \sigma_S dB_t. \tag{26}$$

Denoting as before the risk-free rate by r, and the fraction of wealth allocated to the risky asset by α, the representative consumer's wealth dynamics are

$$dw_t = \left[w_t \left(r + \alpha_t \left(\mu_S - r \right) \right) - c_t \right] + \alpha_t \sigma_S w_t dB_t. \tag{27}$$

By utilizing the results from Section 2 in the infinite horizion case, we rewritten the HJB for a robust investor with intertemporal substitution elasticity η^{-1}, risk aversion γ, preference for robustness θ and demand for status b as follows:

$$0=\sup_{\alpha,c}\left[\begin{array}{c}\frac{1}{1-\eta}\left\{\frac{\left(c_t^a w_t^b\right)^{1-\eta}}{((1-\gamma)V)^{\frac{\gamma-\eta}{1-\gamma}}}-\delta\left(1-\gamma\right)V\right\}+V_w\left[w_t\left(r+\alpha_t\left(\mu_S-r\right)\right)-c_t\right]\\ +V_t+\frac{1}{2}\left(V_{ww}-\frac{\theta V_w^2}{(a+b)(1-\gamma)V}\right)\alpha^2\sigma_S^2 w_t^2\end{array}\right]. \tag{28}$$

Definition *A robust equilibrium consists of a consumption rule c^*, a portfolio rule α^*, and prices S and r, such that, (1), the agent solves (28) subject to the transversality condition, $\lim_{t\to+\infty} E\left[e^{-\delta t}V\left(w_t\right)\right]=0$; (2), markets clear continuously, namely, $c^*=D$ (good market clears) and $\alpha^*=1$ (asset markets clear).*

Given the closed-form solutions for the partial equilibrium model in Section 3, the optimum of (28) is obtained and summerized explicitly in the following propostion.

Proposition 3 *Equation (28) subject to the transversality condition is solved by*

$$V\left(w,t\right)=\left(\frac{a}{a+b}\right)^{\frac{1-\gamma}{1-\eta}}\phi^{\frac{(1-\gamma)(a(1-\eta)-1)}{(1-\eta)}}\frac{w^{(a+b)(1-\gamma)}}{(1-\gamma)}, \tag{29}$$

where $\phi\equiv-\frac{a(1-\eta)}{(a(1-\eta)-1)}\left[\frac{\delta}{(1-\eta)(a+b)}-r-\frac{1}{2}\frac{1}{\Gamma}\left(\frac{\mu_S-r}{\sigma_S}\right)^2\right]$. *The optimal porfolio and consumption rules are given by*

$$c_t=\phi w_t, \tag{30}$$

$$\alpha_t=\frac{1}{\Gamma}\frac{\mu_S-r}{\sigma_S^2}, \tag{31}$$

Combining the optimal solutions provided in the above propostion and the conditions of market clearing, we can derive the equilibrium risk-free rate, the equity premium and the worst-case scenario for the equity premium supporting the equilibrium.

Proposition 4 *In the robust equilibrium with the SOC, the price of the risky asset is given by $S_t=\frac{D_t}{\phi}$. The excess return on the risky asset follows*

$$\frac{dS_t+D_t dt}{S_t}-rdt=\Gamma\sigma_{CS}dt+\sigma_D dB_t, \tag{32}$$

with $\sigma_{CS}\equiv cov\left(\frac{dC}{C},\frac{dS}{S}\right)$. The equilibrium risk-free rate is given by

$$r=\frac{a}{a+b}\delta+\left[1-a\left(1-\eta\right)\right]\mu_D-\frac{2-a\left(1-\eta\right)}{2}\Gamma\sigma_D^2. \tag{33}$$

The pessimistic scenario for the expected equity premium supporting the equilibrium is

$$EP_P^*=\left[1+\left(a+b\right)\left(\gamma-1\right)\right]\sigma_{CS}. \tag{34}$$

Proof. See appendix 6.2. ∎

Combining equation (31) with the clearing condition of asset market, we obtain the equilibrium equity premium: $\mu_S - r = \Gamma\sigma_S^2$, which tells that compared with Mehra and Prescott (1985), there are two new channels to increase the equilibrium equity premium: one comes from the preference for robustness due to uncertainty aversion (θ), the other stems from the preference for social status. It is obvious that the uncertainty aversion parameter θ is a positive scalar in Γ. Define the effective coefficient of relative risk aversion: $\gamma' \equiv 1 + (a+b)(\gamma - 1)$. Even though we are not certain of the relation $\gamma' > \gamma$ for the simplifying assumption of $a + b \leq 1$, we still know that the stronger demand for social status the larger of the effective risk aversion, i.e., $d\gamma'/db = (\gamma - 1) > 0$.[5] This CCAPM result (Breeden, 1979) follows directly from the fact that consumption growth and equity return are by construction perfectly correlated in the model.

The market price of "risk" denoted by Γ, is determined by market risk, γ, model uncertainty, θ, and the preference for the SOC, b. Except for the empirical prediction of the model is that the market price of risk is higher than what would be expected based on genuine risk aversion alone, the model shows that the higher degree of the desire for status (larger b) will induce the higher price of "risk", since the consumer with higher effective risk aversion will reduces the demand for risky asset.

The robust equilibrium model with the SOC can also explain Weil's (1989) risk-free rate puzzle better. Equation (33) shows that the equilibrium risk-free rate depends on the four determinants of the economy: time preference, intertemporal substitution and growth, model uncertainty and the effective risk aversion adjusted by the SOC. Robustness drives down the equilibrium risk-free rate through the precautionary savings channel. However, the spirit of capitalism reduces the equilibrium risk-free rate through two channels. On one hand, the SOC decreases the effective time preference rate, i.e., $\delta' \left(\equiv \frac{a}{a+b}\delta\right) < \delta$ and hence raises the degree of patience, which will enforce consumers to save more and consume less. On the other hand, the consumer with higher desire for status displays the higher degree of the effective risk aversion and hence enjoys more precautionary savings, that is, the precautionary saving term $(2 - a(1-\eta))\Gamma\sigma_D^2/2$ will be larger. The effective elasticity of intertemporal substitution, defined by $\eta' \equiv 1/[a(\eta - 1) + 1]$, is decoupled from the effective risk aversion γ'. This result is a natural extention of the recursive preference by incorporating the demand for status. It is shown in equation (34) that different

[5] If assuming $a + b > 1$, then we have that $\gamma' > \gamma$. Moreover, if $a + b = 1$ holds, the model degenerates to the case of Maenhout (2004).

from the partial equilibrium result (i.e., $EP_P = [1 + (a+b)(\gamma - 1)](\mu - r)/\Gamma$), the equilibrium worst-case (true equity premium) $EP_P^* (= EP_T^*)$ does not depend on the robustness parameter θ. Hence, θ can be taken as an index measuring the distance between the true and pessimistic equity premium.

There are two facts to be noted. Firstly, the true equilibrium equity premiun is larger than the pessimistic equity premium due to $\alpha_t \in [0,1]$. Secondly, a higher degree of the desire for status leads to a large divergence between these two premiums, namely, $\partial(EP_T^* - EP_P)/\partial b = \partial\left((1-\alpha_t)\Gamma\sigma_S^2\right)/\partial b > 0$. Therefore, the robust equilibrium with the SOC can be interpreted as follows: robust investors with the demand for status worry heavily that the observed premium is too high to be true and invest cautiously and pessimistically. And this conservative behavior generates a high equity premium. Simultaneously, the SOC and robustness makes investors more potient and more conservative, the resulted precautionary savings keep the equilibrium risk-free rate low.

4.2 Calibration and Empirical Implications

In this section, given the estimated parameter values for $\mu_D (= \mu_c)$, $\sigma_{cS} = \sigma_c \sigma_S \rho$, $\sigma_D (= \sigma_c)$, and ρ, the preference parameters δ, γ, η, a, b, and θ are chosen to match the observed equity premium and risk-free rate according to equations (32) and (33). Firs of all, we take the estimated consumption and return parameters from Campbell (1999) based on a long annual sample from 1891 to 1994: $\mu_c = 0.01742$, $\sigma_c = 0.03257$, $\sigma_S = 0.18534$, $\rho = 0.497$, $r = 1.955\%$, and $\mu_S - r = 6.258\%$. For our purpose, we take some preference parameters from Maenhout (2004) as follows: $\delta = 0.02$, $\gamma = 7$, and $\eta^{-1} = 0.6$. The calibrated preference parameter for robustness θ equals 14 in Maenhout (2004), which seems too large. In our model, the combinations between the SOC and robustness can plausably decrease the degree of robustness demanded to generate the high equity premium and the low risk-free rate. To see this, we rewritten equaiton (33) as follow:

$$\theta = \frac{\frac{a}{a+b}\delta - r + [1 - a(1-\eta)]\mu_D}{\frac{1}{2}[2 - a(1-\eta)]\sigma_D^2} - [(a+b)(\gamma - r) + 1], \tag{35}$$

which implies that the preference for robustness, θ, is a strictly decreasing function of the preference for the SOC, b, i.e.,

$$\frac{\partial\theta}{\partial b} = -\left\{\frac{2a\delta}{(a+b)^2\sigma_D^2[2 - a(1-\eta)]} + (\gamma - 1)\right\} < 0.$$

Figure 1 shows that the tradeoff of between the SOC and robustness for different value of a.

5 Concluding Remarks

By introducing both model uncertainty and the spirit of capitalism in the standard Merton model, the paper derives the closed-form solutions for the optimal consumption and portfolio rules of a robust investor with the desire for the SOC. Even though the two channels operate differently, the preference for both the SOC and robustness raises the investor's effective risk aversion, $\theta + [1 + (a + b)(\gamma - 1)]$. The preference for the SOC acts in concert with the coefficient of the relative risk aversion, $[1 + (a + b)(\gamma - 1)]$, while model uncertainty generates uncertainty aversion, θ. Both channels make the investor more pessimistic in risk-taking and help to generate high risk premium. Meanwhile, both an increase of the SOC and the desire for robustness lead to more precautionary savings in equilibrium and the equilibrium interest rate will be lower. Therefore, combining the preference for the SOC and robustness helps to resolve both the equity premium and the risk-free rate puzzle.

6 Appendix (Not for Publication)

6.1 Proof of Proposition 1

Assmue that the value function takes the following form

$$V(w,t) = \frac{A(t)}{(a+b)(1-\gamma)} w^{(a+b)(1-\gamma)}, \tag{36}$$

where $A(t)$ is an undetermined function. Then, we know that $V_w = A(t)\, w^{(a+b)(1-\gamma)-1}$, $V_{ww} = A(t)\,[(a+b)(1-\gamma)-1]\, w^{(a+b)(1-\gamma)-2}$, $V_t = \frac{A'(t)}{(a+b)(1-\gamma)} w^{(a+b)(1-\gamma)}$. Substituting this guess into the HJB equation (15) and taking the associated FOCs lead to the optimal conditions for consumption and portfolio choice:

$$c_t = \left[\frac{1}{a}\left(\frac{1}{a+b}\right)^{\frac{\gamma-\eta}{1-\gamma}} V_w w^{-b(1-\gamma)}\right]^{\frac{1}{a(1-\gamma)-1}}, \tag{37}$$

$$\alpha_t = \frac{1}{\theta + [1-(a+b)(1-\gamma)]} \frac{\mu - r}{\sigma^2}. \tag{38}$$

Substituting (37) and (38) into the HJB equation (15) and arranging it, we can obtain a Bernoulli equation about $A(t)$, namely,

$$A'(t) + \frac{(1-\gamma)(a(1-\eta)-1)}{1-\eta}\phi A(t) = \frac{(1-\gamma)(a(1-\eta)-1)}{1-\eta}\psi A(t)^{1+\frac{1-\psi}{1-\gamma}\frac{1}{a(1-\gamma)-1}},$$

where ϕ and ψ are given by Proposition 2. The change of variable $x(t) = A(t)^{-\frac{1-\psi}{1-\gamma}\frac{1}{a(1-\gamma)-1}}$ leads to a first order linear differential equation of $x(t)$:

$$x'(t) - \phi x(t) = -\psi,$$

whose general solution is $x(t) = \frac{\psi}{\phi} + \kappa e^{\phi t}$, where κ is an arbitrary constant. By utilizing the boundary condtion (5), we can find $\kappa = -\frac{\psi}{\phi} e^{-\phi t}$, and hence the undetermined function in the value function

$$b(t) = \left[\frac{\psi}{\phi}\left(1 - e^{-\phi(T-t)}\right)\right]^{-\frac{(1-\gamma)(a(1-\eta)-1)}{1-\eta}}.$$

6.2 Proof of Proposition 3

The equilibrium can be constructed as follows. The optimality condition are given by (30) and (31). Substituting these two equations into the stochastic differential equation for wealth (27) gives us the equilibrium dynamics of wealth accumulation

$$\frac{dw_t}{w_t} = \left[\frac{r}{1-a(1-\eta)} - \frac{a}{a+b}\frac{\delta}{1-a(1-\eta)} + \frac{2-a(1-\eta)}{2[1-a(1-\eta)]}\frac{1}{\Gamma}\left(\frac{\mu_S - r}{\sigma_S}\right)^2\right]dt + \frac{1}{\Gamma}\frac{\mu_S - r}{\sigma_S}dB_t. \tag{39}$$

Combining the clearing condition of the security market with (31) immediately leads to a CCAPM result: $\mu_S - r = \Gamma\sigma_S^2$. Substituting the result into (39) leads to

$$\frac{dw_t}{w_t} = \left[\frac{r}{[1-a(1-\eta)]} - \frac{a}{(a+b)}\frac{\delta}{[1-a(1-\eta)]} + \frac{2-a(1-\eta)}{2[1-a(1-\eta)]}\Gamma\sigma_S^2\right]dt + \sigma_S dB_t. \tag{40}$$

Equilibrium in the good market implies that $c_t = D_t = \phi w_t$. Moreover $S_t = \frac{1}{\phi}D_t$, so that in equilibrium $S_t = W_t$. In addition, $S_t = \frac{1}{a}D_t$ implies that $\frac{dS_t}{S_t} = \mu_D dt + \sigma_D dB_t$. Combining these results with (40) yields the following equilibrium condition

$$w_0 \exp\left\{\left(\begin{array}{c}\frac{r}{[1-a(1-\eta)]} - \frac{a}{(a+b)}\frac{\delta}{[1-a(1-\eta)]} + \\ \frac{2-a(1-\eta)}{2[1-a(1-\eta)]}\Gamma\sigma_S^2 - \frac{1}{2}\sigma_S^2\end{array}\right)t + \sigma_S\int_{\tau=0}^{t} dB_\tau\right\} = S_0 \exp\left\{\left(\mu_D - \frac{1}{2}\sigma_D^2\right)t + \sigma_D\int_{\tau=0}^{t} dB_\tau\right\}.$$

This results in

$$S_0 = w_0,$$

$$\mu_D - \frac{1}{2}\sigma_D^2 = \left\{ \frac{r}{[1-a(1-\eta)]} - \frac{a}{(a+b)} \frac{\delta}{[1-a(1-\eta)]} + \frac{2-a(1-\eta)}{2[1-a(1-\eta)]}\Gamma\sigma_S^2 - \frac{1}{2}\sigma_S^2 \right\}, \quad (41)$$

$$\sigma_D = \sigma_S. \quad (42)$$

Rearranging the last two equations immediately produces equilibrium risk-free rate, i.e, (33). Combining (26), (23) and the CCAPM result leads to the formula of equilibrium equity premium, i.e., (32) and the pessimistic scenario for the expected equilibrium premium supporting the equilibrium, i.e., (34).

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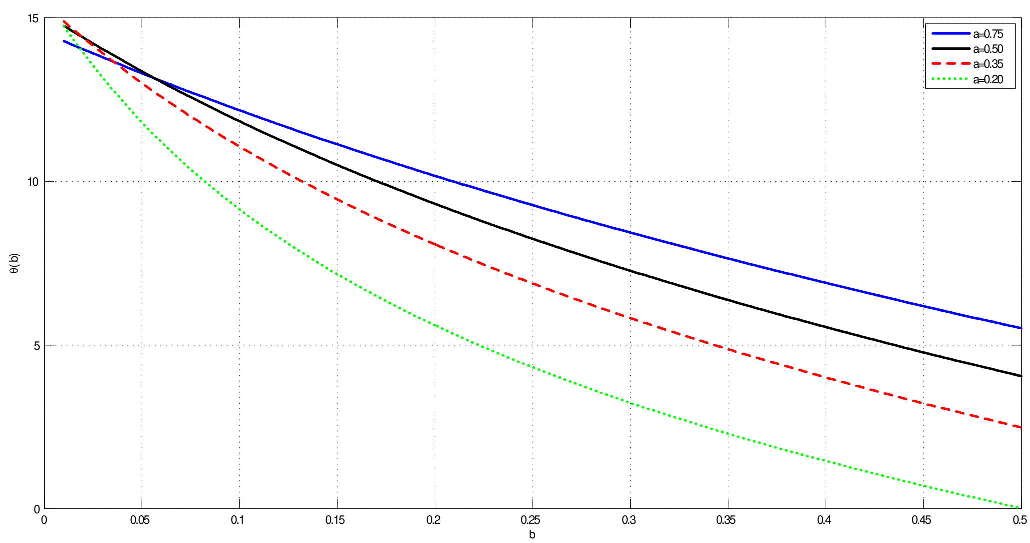

Figure 1: Tradeoffs of the preferences for robustness and the SOC