A synthetic indicator for the currencies of the G5 countries

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A SYNTETIC DIVERGENCE INDICATOR FOR THE CURRENCIES OF THE G5 COUNTRIES

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In their effort to increase the effectiveness of monetary and economic cooperation, the G5 countries, following a suggestion of the April 1986 Interim Committee, are relying on a set of economic indicators to put in an international perspective their respective economic performances and to assess accordingly the degree of misalignment of their currencies. It is tempting, however, for central bankers to look for a global index able to provide a comprehensive description of the various factors affecting the relative position of their currencies as the European Monetary System members have done with their divergence indicator (e.g., Salop 1981; Spaventa 1982).

In order to provide an index more suitable in the context of managed floating adopted by the G5 countries, this paper presents a synthetic indicator that exhibits the influence of domestic and external imbalances for each G5 country on its currency position.

The paper begins by the definition of the divergence indicator and shows that its fluctuations are equal to the deviations of the real effective exchange rate from its long term equilibrium path. Section II stresses the main differences existing between this concept of divergence and those adopted either in Williamson (1983,1985) recent attempts to measure the degree of currency misalignment or in the E.M.S. approach already mentioned. Section III deals with the empirical aspects of the specification of the equilibrium exchange rate model, which determines the long run equilibrium path of real effective exchange rates. Some stringent assumptions are adopted in partitular with respect to the real effective exchange rates, which are endowed with aggregative properties in order to ensure consistency between multilateral and bilateral indexes. Section IV provides an estimate of the synthetic divergence indicator for the period 1974/1987. The last section summarizes the main conclusions.

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I - METHODOLGY AND DEFINITIONS

A country cannot for ever maintain a current account deficit -ignoring the possibility of default-. In the long run, domestic saving and investment should be equal and the current account in equilibrium. The widely held view, implicit in trade theory, is that the real exchange rate should move to restore the external equilibrium. This linkage which is a one-way relationship in the short term is turned into a simultaneous determination in the long term. In this perspective, the equilibrium real exchange rate (ERER) can be defined as the rate which is consistent with current account equilibrium when real growth in the domestic economy and abroad are sticking to their long term equilibrium paths.

Under some restrictive assumptions (1) it is shown in appendix I that deviations of the real exchange rate from its ERER value can be expressed as a weigthed average of domestic and external imbalances which allows the synthetic divergence indicator (2) proposed in this paper to be written as

$$q - \bar{q} = \frac{\beta_m (d - \bar{d}) - \beta_x (d^* - \bar{d}^*) + ca}{\alpha_x - \alpha_m + 1}$$

where q, d, d* and ca are the logarithms of the real exchange rate, real domestic and foreign incomes and the ratio between exports and imports of goods and services. A bar above the variable concerned denotes its long term equilibrium value. The β_x and β_m parameters are long term income elasticities for the demand of export and import respectively while α_x and α_m are the corresponding long term price elasticities.

This measure of exchange rate misalignement exhibits the main following characterictics :

- It is a reflection of the fundamental disequilibria affecting the economy at home and abroad.

- It is mostly impervious to the short term volatility of nominal exchange rates and sensitive to their contemporaneous fluctuations to the extent that they impinge on the current account imbalance. That is to say, short term variations in nominal exchange rates would affect the divergence indicator only through the channel of valuations effects.

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(1) In particular, interest income flows stemming from previous current account imbalances are ignored.

(2) A positive value of the divergence indicator points to an overvaluation of the given currency.

- Its determination is independent of any reference to an hypothetical base period where the external and/or the domestic sectors would have been in equilibrium.

- Once trade elasticities have been estimated, the calculation of the divergence indicator is not influenced by the choice of price indexes as it is the case when misalignements are appreciated through various measures of purchasing power parity.

- Due to the usual delay in the availability of statistical data, the divergence indicator can be measured with certainty only after a lag of at least 2 quarters. Updating and forecasting however are possible by simulating the current account model which has been used to obtain the parameter estimates.

II - COMPARISON WITH OTHER MEASURES OF MISALIGNEMENT

This section emphasizes the main differences existing between this measure of exhange rate misalignement and those adopted either by WILLIAMSON or by the E.M.S. members.

1) - WILLIAMSON defines the Fundamental Equilibium Exchange Rate (FEER) as the rate which ensures during an economic cycle that the current account imbalance would be matched by underlying capital flows consistent with the domestic equilibrium that the country concerned would get in the absence of trade restrictions.

Formally, this equilibrium exchange rate is expressed as a function of the optimal imbalance of the current account, the optimal domestic growth and structural factors. Deviations of the exchange rate from its FEER value measure the degree of over or undervaluation of the currency.

Although the WILLIAMSON approach is close to the concept presented in section I, since they both rely on fundamental economic factors and on an attempt to define equilibriumm, they differ however under three important headings :

- In WILLIAMSON, the fundamental current account could drift far away from equilibrium as long as such a move is consistent with the optimal growth of the domestic economy while this is not possible in our proposal where the fundamental current account is always in balance.

- To determine the fundamental current account equilibrium which is compatible with the desired flows of saving and investment, WILLIAMSON had to choose a base period (1976-1977). This is, for the most part, a normative exercice since it is unlikely that the same period will do for all currencies. Moreover, the choice of such a reference having important consequences on the measure of misalignement should not be left to the appreciation of an economist but should constitute a central element in the negociation between the GV countries.

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- The WILLIAMSON'S FEER can be altered to take into account some structural changes in particular those related to the huge increase in oil prices. However, it is not always so easy to differentiate between cyclical and structural factors. In the specific case of oil, it is worth emphasizing that the measure of the impact of oil price increases on the terms of trade of the industrial countries seems very tricky.

2) - The E.M.S. divergence indicator provides a measure of a currency's position in terms of its Ecu central rate. Since the Ecu can be assimilated to an effective nominal exchange rate the divergence indicator describes the relative movements of a currency in relation to a reference value of an effective exchange rate, this value being considered as an equilibrium value by the monetary authorities on the date of the last realignement.

The main differences between the E.M.S. divergence indicator and our proposal can be summarized as follows :

- The E.M.S. index retains nominal and not real effective exchange rates,

- The equilibrium value which remains constant between two realignements is determined at the discretion of E.M.S. Governments and might therefore be strongly influenced by short term considerations,

- By its very nature, the divergence indicator reflects -in the absence of intramarginal interventions- the influence of short term portofolio adjustments on the E.M.S. currencies's position. It is not conceived to account for the impact of fundamental imbalances on currency misalignements and therefore would be of little interest in the context of the coordination process implemented by the GV countries since this process will, most likely, keep the characteristics of a managed float.

III - A MULTINATIONAL MODEL FOR THE DETERMINATION OF EQUILIBRIUM EXCHANGE RATES

The model specification adopted to estimate equilibrium real exchange rates has been confined to a matrix encompassing the quaterly trade flows between the nine main industrial countries (1). Export and import demands in volume are endogenously determined according to the formulation defined in appendix I where prices, incomes and nominal exchange rates are exogenous variables.

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(1) FRANCE, BELGIUM, GERMANY, NETHERLAND, ITALY, UNITED-KINGDOM, UNITED-STATES, CANADA, JAPAN

1) - Statistical data

International trade series are originating from a matrix of bilateral trade flows established on the basis of OECD export data denominated in US dollar for the period 1965 to 1986. Adding up respectively the rows and columns of this matrix provides for a consistent set of exports and imports for the nine industrial countries. In the absence of price indexes specific to this subset of international trade, GNP implicit deflators have been retained to express nominal flows in volume. It has been assumed also that exports are always denominated in domestic currency while imports are invoiced in foreign currencies whose respective shares depend for each country on the geographic structure of its imports.

Quarterly real income variables are represented by GNP data published by the IMF. When only yearly series are available, quarterly data have been simulated by using industrial production as a benchmark. All nominal exchange rate series come out from the IMF data base.

In order to alleviate agregation problems, weighted variables --import prices, effective exchange rates, world real income-- have been set up using the same following weighting pattern,

FR	BE	DE	NL	IT	UK	EU	CA	JP	Total
11.1	8.0	16.5	9.1	7.3	8.7	21.5	9.9	7.8	100

which is representative of the average international trade structure (exports + imports) observed during the period 1965 to 1985. From this simplification, it results that:

-Each country is facing the same world demand.

-The import price index of a given country is equal to the weighted average of the price indexes of the nine industrial countries expressed in terms of the currency of the country concerned --that is including its own price index--.

-Specific assumptions and constraints have been retained to determine effective nominal and real exchange rates. In particular, nominal and real effective exchange rate are endowed with aggregative properties in order to ensure consistency between multilateral and bilateral indexes (Galy 1983). By definition, real effective exchange rates evolve in line with export market shares in volume. Appendix II describes the theoretical framework adopted for the construction of these indexes. It is worth stressing that for the major currencies and over the period 1974/1987 this approach has generated effective exchange rate indexes that have exhibited basically the same pattern as those stemming from the Multilateral Model of Exchange Rates (MERM) introduced in the early 70's by the IMF (Artus and Rhomberg 1973; Artus and McGuirk 1981)as suggests a perusal of the charts presented in appendix III.

Potential growth for each country is defined -in the most unsatisfactory way- as the GNP exponential trend measured on the period 1965/1984. Table I hereafter presents the equilibrium growth rates so obtained.

At the empirical stage, the specification of the relationship which determines the real equilibrium exchange rate has been altered to take into account the fact that the model retains a subset of international trade instead of the current account data. In the equation (8) appendix I, the equilibrium value of the current account variable which is null has been replaced for each country by the logarithm of its average ratio of exports to imports measured on the period 1965/1984 (see table I below).

TABLE I

	Equilibrium annual growth rate of real income	Equilibrium ratio of exports to imports with the 8 main industrial countries
France	3,61 %	85,6 %
Belgium	3,21 %	94,- %
Germany	2,80 %	110,7 %
Netherland	3,26 %	91,8 %
Italy	3,22 %	97,6 %
United Kingdom	1,93 %	89,6 %
United States	2,53 %	91,5 %
Canada	3,85 %	117,3 %
Japon	5,63 %	161,8 %
All nine countries	3,19 %	-

2) - Empirical results

Export and import equations have been estimated on the period 1965-1985 using the O.L.S. method applied to seasonally adjusted series. Agregation constraints have not been imposed between the equations to ensure consistency in the estimates of income and price elasticities. Therefore, when simulating the international trade model there is no guarantee that total exports would be equal to total imports. However, the strong simplifications imbedded in the specification of the weighted variables have kept the relative discrepancies between total exports and imports below 1.5 per cent.

Income and price elasticities estimates are presented in table II below.

TABLE II

Export and import elasticities

	Exports				Imports			
	Income		Price		Income		Price	
	short term	long term	short term	long term	short term	long term	short term	long term
FR	.54	2.00	- .23	- .86	.89	2.05	+ .25	+ .57
BE	.68	1.70	- .22	- .56	.65	1.89	+ .14	+ .41
DE	.92	1.94	- .36	- .75	.93	2.04	+ .23	+ .51
NL	.81	2.13	- .26	- .68	.51	1.58	+ .05	+ .15
IT	1.05	1.93	- .61	-1.12	.63	1.97	- .00	- .00
UK	.50	2.08	- .08	- .34	.26	3.28	- .00	+ .06
EU	.35	1.31	- .24	- .88	1.15	2.68	+ .43	+1.00
CA	.58	1.84	+ .03	+ .09	.51	1.22	+ .35	+ .84
JP	.36	3.60	- .23	-2.33	.13	.70	+ .19	+1.02

A perusal of these results suggests the following remarks :

- Long term income elasticities in the case of exports are around 2.0 except for the United States and Japan which get respectively 1.3 and 3.6. This implies that an upsurge in world demand would benefit much more to Japan than to the United States. On the import side, this is the contrary which means that the US external balance is more sensitive to the orientation of domestic demand than it is the case for Japan.

- Regarding long term price elasticities, it is worth pointing out that the sum of their import and export absolute values is not generally lesser than one, except for energy exporting countries such as United Kingdom and Netherland.

Starting from this set of parameters, it is now possible to determine the synthetic divergence indicator according to the reduced form equation proposed in section I and to express the apparent sensitivity of this indicator to changes in domestic ($D - \bar{D}$) and external ($D^* - \bar{D}^*$) demand pressures and in the actual current account level (CA). These parameters calculated for the GV countries are displayed in table III below.

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TABLE III
Sensitivity of the Divergence Indicator
to Changes in its components

	Domestic demand pressure	External demand pressure	External Imbalance
USD	- 3,04	+ 1,49	- 1,14
DEM	- 7,85	+ 7,46	- 3,85
JPY	- 0,30	+ 1,53	- 0,43
FRF	- 4,76	+ 4,65	- 2,33
GBP	5,47	- 3,46	1,67

It is worth emphasizing the relative inertia of the divergence indicator of the yen, in particular vis-à-vis its domestic demand and current account position.

IV - EVOLUTION OF THE DIVERGENCE INDICATORS OF THE GV COUNTRIES BETWEEN 1974/1987

Calculations of the divergence indicators of the GV countries have been carried out according to the previous assumptions and simplifications for the period covering the first quarter 1974 to the second quarter 1986. A projection up to the fourth quarter 1988 has been also implemented under the hypothesis that nominal exchange rates would remain constant at their January 1987 level (1) from the first quarter 1987 to the end of the year 1988 and in adopting the last OECD forecasts in terms of real growth and inflation.

The evolution of the divergence indicators are depicted in appendix IV and some of their values are presented in table IV for certain periods.

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(1) USD/FRF = 5.957 USD/BEC = 36.995
USD/DEM = 1.785 USD/NLG = 2.014
USD/JPY = 152.- USD/ITL = 1,271.75
USD/GBP = 0.649 USD/CAD = 1.3445

TABLE IV

Divergence Indicators for the currencies of the GV countries (*)

	USD	DEM	JPY	FRF	GBP
1974-1	- 4,- %	0 %	+ 10,7 %	- 1,2 %	+ 10,7 %
1980-1	- 3,5	+ 1,4	+ 3,-	0	- 5,5
1987-1	+ 15,8	- 5,9	- 14,2	+ 4,2	+ 1,2
1988-4	+ 11,-	- 1,9	- 12,-	+ 7,1	- 2,6

* A positive value points to an overvaluation of the currency concerned

A perusal of these results which should be contemplated only as rough estimates of historical misalignements suggest the following observations :

- Deviations of effective real exchange rates from their equilibrium path were relatively limited during the period 1976/1979 but started increasing after 1980.

- The dollar and the yen have recorded the largest misalignements since 1982. Their indicators reached their peak and trough respectively at the end of 1985. The rapid depreciation of the dollar which began at the outset of 1985 in particular vis-à-vis the yen appear lately to have brought about an incipient reduction in their divergence indicators. A more significant return towards equilibrium should be expected toward the end of 1988. However, the overvaluation of the dollar would be still close to 11 % while the undervaluation for the yen would reach 12 %.

- The three other GV currencies have remained since 1976 as a whole relatively close to their equilibrium path although the pound sterling indicator underwent at times some hectic movements in relation with those of oil prices. For the currencies participating in the European Exchange Rate Mechanism, one can notice that the french franc from 1980 onwards has been systematically overvalued on average by 5 % -with a peak to 9 % on the third quarter 1982- while the german mark has benefited from a slight undervaluation which culminated to 4 % in 1986. For the most part this undervaluation is expected to be mopped up by the end of 1988.

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V - Concluding remarks

This paper proposes a synthetic divergence indicator retracing the influence of domestic and external imbalances for each G5 country on its currency position. This index happens to be equal to the weighted sum of domestic demand pressures and current account disequilibrium, with the weights stemming from a suitable combination of price and income elasticities estimated from a set of international trade equations.

Under some restrictive assumptions, it is shown that the variations of this indicator are tantamount to the deviations of the real effective exchange rate from its long term equilibrium path. In order to determine the divergence indicator for the period 1974/1987, an empirical attempt has been carried out that resorts to a matrix of international trade encompassing the nine main industrial countries and retains a real effective exchange rate concept endowed with aggregative properties that replicates closely the effective exchange rates of the IMF MERM model at least for the major currencies.

A perusal of the empirical results for the years 1985/1987 suggests that the G5 European currencies are presently close to their equilibrium position, whereas the overvaluation of the US dollar and undervaluation of the yen are still substantial.

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REFERENCES

Artus, J.R., and Rhomberg, R.R., "A Multilateral Exchange Rate Model", IMF Staff Papers, Vol. 20, No. 3, Nov. 1973.

Artus, J.R., and McKuirk, A.K., "A Revised Version of the Multilateral Exchange Rate Model", IMF Staff Papers, Vol. 28, No. 2, June 1981.

Galy, M., "Regional Exchange Rate Arrangements in a World of Floating Rates", in "Exchange Rate Determination: Analysis and Policy Issues", Bank of International Settlement; CB382; Pages 80-113; 1983.

Salop, J., "The Divergence Indicator: A Technical Note", IMF Staff Papers, Vo. 28,No. 4 1981.

Spaventa, L., "Algebraic Properties and Economic Properties of the Indicator of Divergence in the European Monetary System", in R. Cooper et al. (eds),"The International Monetary System under Flexible Exchange Rates-- Essays in Honor of Robert Triffin", Cambridge, Mass., Balinger, 1982

Williamson., J. "The Exchange Rate System", Policy Analysis in International Economics 5, Institute of International Economics, Washington, D.C., 1983 revised in 1985.

APPENDIX I

DETERMINATION OF THE REAL EQUILIBRIUM EFFECTIVE EXCHANGE RATE

Symbols : The variables are expressed in logarithms

X_j , M_j : Exports and imports of goods and services and transferts denominated in domestic currency

CA_j : Ratio of exports to imports as defined here-above

x_j , m_j : Exports and imports in volume terms

s_j : Nominal effective exchange rate -quoted in terms of units of foreign currencies for one unit of domestic currency- expressed as

$s_j = \sum_i c_i s_{ij}$ with $s_{jj} = o$, $\sum_i c_i = 1$ where c_i represents the weight of country i

P_j , P_j^* : domestic and foreign price indexes with $P_j^* = \sum_i c_i (P_i - s_{ij})$

D_j , D^* : domestic and world demands in volume

$\bar{D}_j , \bar{D}^*$: potential demands

$Q_j , \bar{Q}_j$: Actual and equilibrium real effective exchange rates defined as $Q_j = P_j + s_j - P_j^*$

To keep notations at their minimum, index j will be omitted in the following.

Assuming that prices and incomes are exogenous and that export supply is infinite, the equations determining the volume of export and import demands of trade in goods and services -ignoring the interest payment item- can be written as

- 1 - $x_t = a_x Q_t + b_x D^*_t + x_{t-1} + C_x$

- 2 - $m_t = a_m Q_t + b_m D_t + m_{t-1} + C_m$

Taking long term elasticities, expressing Q, D, D* and D as weighted averages of their respective past values and finally getting rid of the intercepts, equations (1) and (2) take the simplify following form.

$$- 3 - \quad x_t = \alpha_x Q_t + \beta_x D^*_t$$

$$- 4 - \quad m_t = \alpha_m Q_t + \beta_m D_t$$

Since the ratio of nominal exports to imports is defined by :

$$- 5 - \quad CA_t = X_t - M_t = (x_t + P_t) - (m_t + P^*_t - s_t)$$

It can be transformed using equations (3) and (4) as :

$$- 6 - \quad CA_t = (\alpha_x - \alpha_m + 1) Q_t + \beta_x D^*_t - \beta_m D_t$$

The real effective exchange rate obtained from the previous formula can be expressed as :

$$- 7 - \quad Q_t = \frac{\beta_m D_t - \beta_x D^*_t + CA_t}{\alpha_x - \alpha_m + 1}$$

This reduced form equation can be written in replacing the variables by their deviations from their long term equilibrium values to yield the divergence indicator proposed in this paper.

$$- 8 - \quad (Q_t - \bar{Q}_t) = \frac{\beta_m (D_t - \tilde{D}_t) - \beta_x (D^*_t - \bar{D}^*_t) + CA_t}{\alpha_x - \alpha_m + 1}$$

Appendix II

Derivation of effective exchange rate indices from an aggregatable system of export demand equations

The theoretical framework used to construct these indices is based on the following hypotheses:

(a) The world is composed of "n" countries and each country produces a "composite", exchangeable commodity, the price of this commodity being expressed in domestic currency.

(b) The consumer divides his expenditure among commodities of the exposed and sheltered sectors in accordance with utility functions with the characteristics of group independence as defined by Theil (1980). This hypothesis permits the utility function of the commodities exchanged to be treated independently.

(c) The analysis assumes that all the commodities exchanged may be defined as "specific substitutes" in accordance with Theil's interpretation (1980). This hypothesis is tantamount to assuming that the composite commodities are for the most part industrial goods. It is indeed probable that raw materials and industrial goods are more in the nature of "specific complementary goods".

(d) All consumers are rational and their demand function parameters are stable; they have the same system of preferences. Furthermore, the income elasticity of the demand for goods is equal to unity for all agents. In this way it is possible to aggregate the behaviour of consumers and to consider them as a single agent who divides his expenditure among the "composite" goods of the ""n" countries under the following budget constraint:

$$M_i = \sum_j \frac{P_j}{l_{ij}} Q_j \quad , \qquad j = 1, \ldots n \tag{1}$$

where M_i is the total amount of world exports expressed in currency i, Q_j is the volume of goods exported by country j at the price P_j expressed in currency j, and l_{ij} is the bilateral exchange rate between the currencies i and j[25]. It is further assumed that the world consumer divides his expenditure between the "n" composite commodities so as to maximise a preference function V_i (M_i, Pw_i) expressed as a function of prices and income or as an indirect utility function[26] in the form:

$$V_j(M_i, Pw_i) = \frac{\prod_j Pw_{ij}^{\alpha_j}}{M_i} \quad \text{with} \quad 0 < \alpha_j < 1 \quad \forall_j, \sum_j \alpha_j = 1 \tag{2}$$

$$Pw_{ij} = \frac{P_j}{l_{ij}} \qquad i \neq j \text{ with } Pw_{ii} = P_i \tag{3}$$

$$Pw_i = \pi_j Pw_{ij}^{\alpha_j} \tag{4}$$

where (3) and (4) define, respectively, the price of commodity j and a world price index for exported goods expressed in currency i.

By applying the Roy theorem (1942) to equation (2) it is possible to obtain a consistent set of demand equations[27] for the "n" composite commodities. The demand equation for commodity i (Q_i) derived from the indirect utility function thus becomes:

$$Q_i = -\frac{\partial V_i}{\partial Pw_{ii}} \Big/ \frac{\partial V_i}{\partial M_i} \qquad \text{(ROY identity)} \tag{5}$$

In the present case the two terms of this equation will be equal to:

$$\frac{\partial V_i}{\partial Pw_{ii}} = \alpha_i \, Pw_{ii}^{(\alpha_i - 1)} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{\alpha_j} \, M_i^{-1} \tag{6}$$

$$\frac{\partial V_i}{\partial M_i} = -\frac{1}{M_i^2} \prod_j Pw_{ij}^{\alpha_j} = -\frac{Pw_i}{M^2} \tag{7}$$

By combining (5), (6) and (7) it is possible to obtain the demand equation for commodity i in which, in accordance with hypothesis (d), income elasticity is equal to unity.

$$Q_i = \alpha_i \, Pw_{ii}^{(\alpha_i - 1)} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{\alpha_j} \, \frac{M_i}{Pw_i} \quad \text{with} \quad \sum_{\substack{j \\ j \neq i}} \alpha_j = 1 - \alpha_i \tag{8}$$

Note, here, that this equation is a simplified form of the classic export demand function. It comprises, in fact, a variable representing the volume of world demand (M_i/Pw_i), a price index for the exported commodity (Pw_{ii}) with negative elasticity ($\alpha_i - 1$) and an aggregated price index for competing commodities ($\prod_{\substack{j \\ J \neq i}} Pw_{ij}^{\alpha_j}$) with positive elasticity.

The market share of each commodity can be obtained by substituting Q_j in equation (1) by its expression in equation (8):

$$w_i = \frac{Pw_{ii} Q_i}{M_i} = \alpha_i \tag{9}$$

This result signifies that two particularly strong hypotheses are included in the indirect utility function (2):

(i) The substitution elasticity of each commodity is equal to its market share in world trade.

(ii) The system of demand equations, thus defined, assumes stability of market shares in value terms.

On the basis of this theoretical framework it is possible to define a consistent set of indices of nominal and real effective exchange rates. To do this it is sufficient to divide the two sides of the equation (8) by the quantity $\frac{M_i}{\alpha_i Pw_i}$ thus giving:

$$Q_i \frac{Pw_i}{\alpha_i M_i} = Pw_{ii}^{\alpha_i - 1} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{j} \quad \text{with} \quad \sum_{\substack{j \\ j \neq i}} \alpha_j = 1 - \alpha_i \tag{10}$$

Equation (10) shows that the market share in volume terms[25] simply depends on price indices, exchange rates and substitution elasticities between the various composite commodities. This equation defines the real effective exchange rate concept (PPP_i) adopted in this note:

$$PPP_i = Pw_{ii}^{\alpha_i - 1} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{\alpha_j} = \frac{\prod_j Pw_{ij}^{\alpha_j}}{Pw_{ii}} = \frac{Pw_i}{P_i} \tag{11}$$

α_i, being constant, can be ignored in the analysis; the market share is normally equal to $\frac{Q_i Pw_i}{M_i}$

Equation (11) shows that the real effective exchange rate of a given country is equal to the ratio of an index of world prices - covering all countries, including the country concerned - to the price index of that country.

To obtain the nominal effective exchange rate index it is sufficient to rewrite (11) using (3) and (4) as follows:

$$PPP_i = \frac{\prod_j P_j^{\alpha_j}}{\prod_{\substack{j \\ j \neq i}} 1_{ij}^{\alpha_j} \cdot P_i} = \frac{\bar{P}}{\bar{1}_i \cdot P_i} \tag{12}$$

where Q_i is precisely the nominal effective exchange rate and $\bar{P}$ a weighted index of world prices not deflated by exchange rate variations:

$$\bar{1}_i = \prod_{\substack{j \\ j \neq i}} 1_{ij}^{\alpha_j} \quad \text{with} \quad \sum_{\substack{j \\ j \neq i}} \alpha_j = 1 - \alpha_i \tag{13}$$

$$\bar{P} = \prod_j P_j^{\alpha_j} \quad \text{with} \quad \sum_j \alpha_j = 1 \tag{14}$$

The interest of the effective exchange rate indices (PPP_i and Q_i) is twofold:

- they are interpreted directly in terms of the development of the market share of the country concerned;
- they are also directly consistent with the bilateral exchange rate indices (PPP_{ij}, I_{ij}).

This latter point can be illustrated as follows:

- let PPP_i and PPP_j be the real effective exchange rates of countries i and j, and PPP_{ij} their bilateral index, so that:

$$PPP_{ij} = \frac{P_j}{I_{ij} P_i} \tag{15}$$

Knowing that the relationship of indices PPP_i and PPP_j is:

$$\frac{PPP_i}{PPP_j} = \frac{P_j}{P_i} \cdot \frac{\bar{I}_j}{\bar{I}_i} \tag{16}$$

definition (13) makes it possible to write:

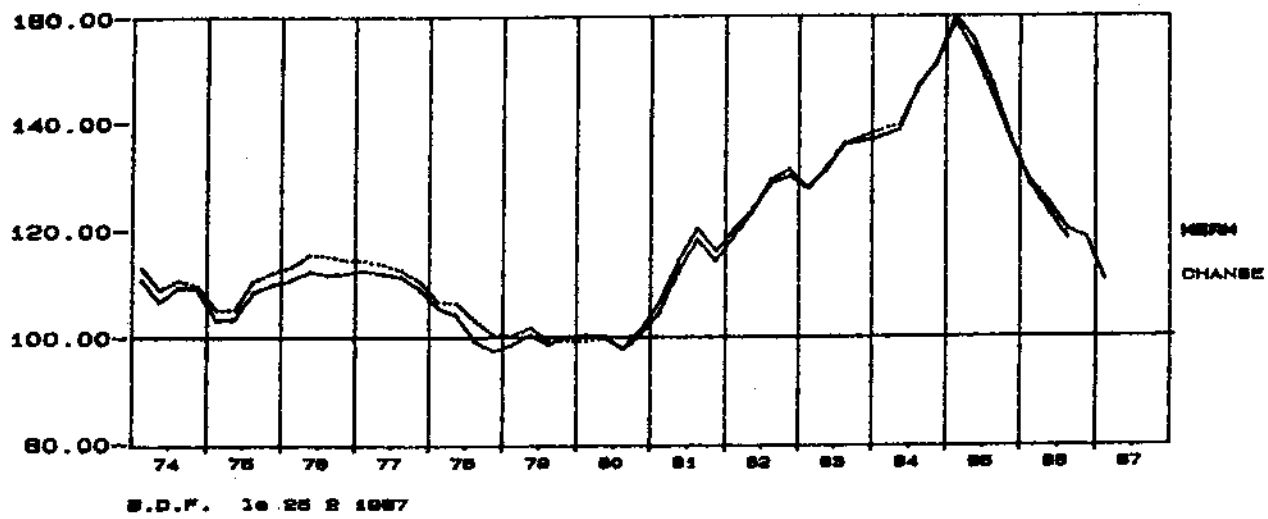
$$\frac{\bar{I}_i}{\bar{I}_j} = I_{ij}^{(\alpha_i+\alpha_j)} \cdot \prod_{\substack{k\neq l \\ k\neq i \\ k\neq j}} \left(\frac{I_{ik}}{I_{jk}}\right)^{\alpha_k} = I_{ij}^{(\alpha_i+\alpha_j)} \cdot I_{ij}^{(1-\alpha_i-\alpha_j)} = I_{ij} \tag{17}$$

Application of this result to equation (16) shows that the bilateral exchange rate is equal to the ratio of the effective exchange rates:

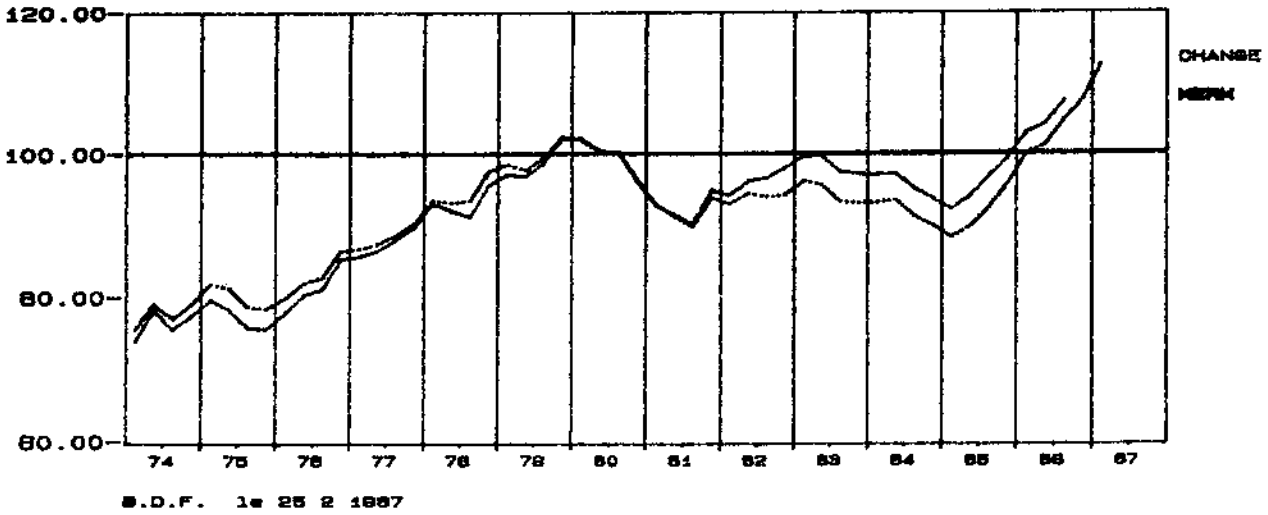
$$\frac{PPP_i}{PPP_j} = \frac{P_j}{I_{ij} P_i} = PPP_{ij} \tag{18}$$

APPENDIX III

Dollar effective exchange rate
dgse-direction des changes
base 100 en 1980



German mark effective exchange rate
dgse-direction des changes
base 100 en 1980



Yen effective exchange rate
dgse-direction des changes
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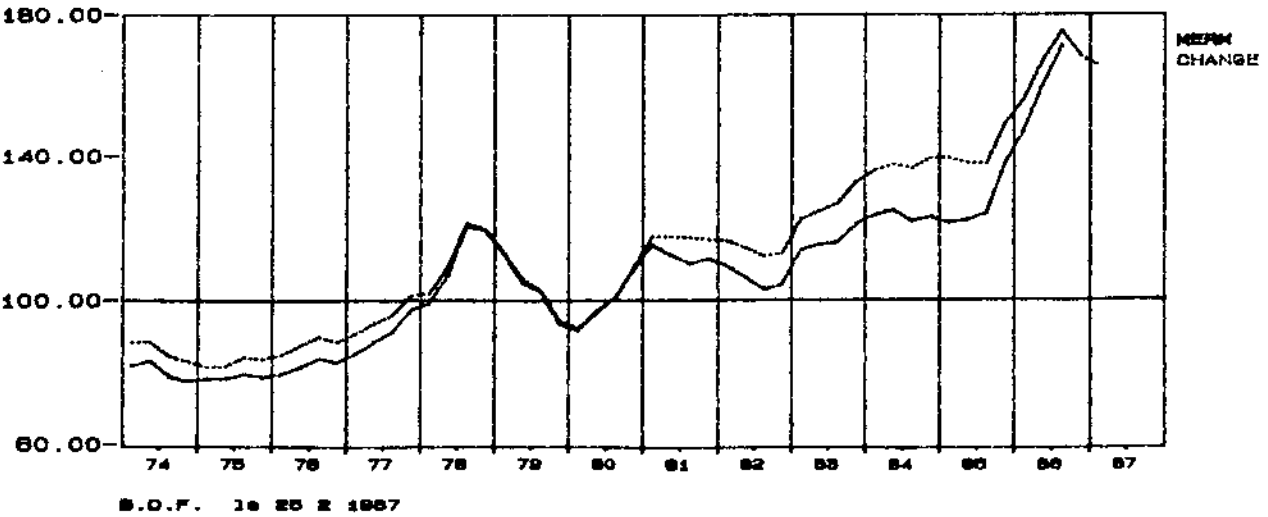
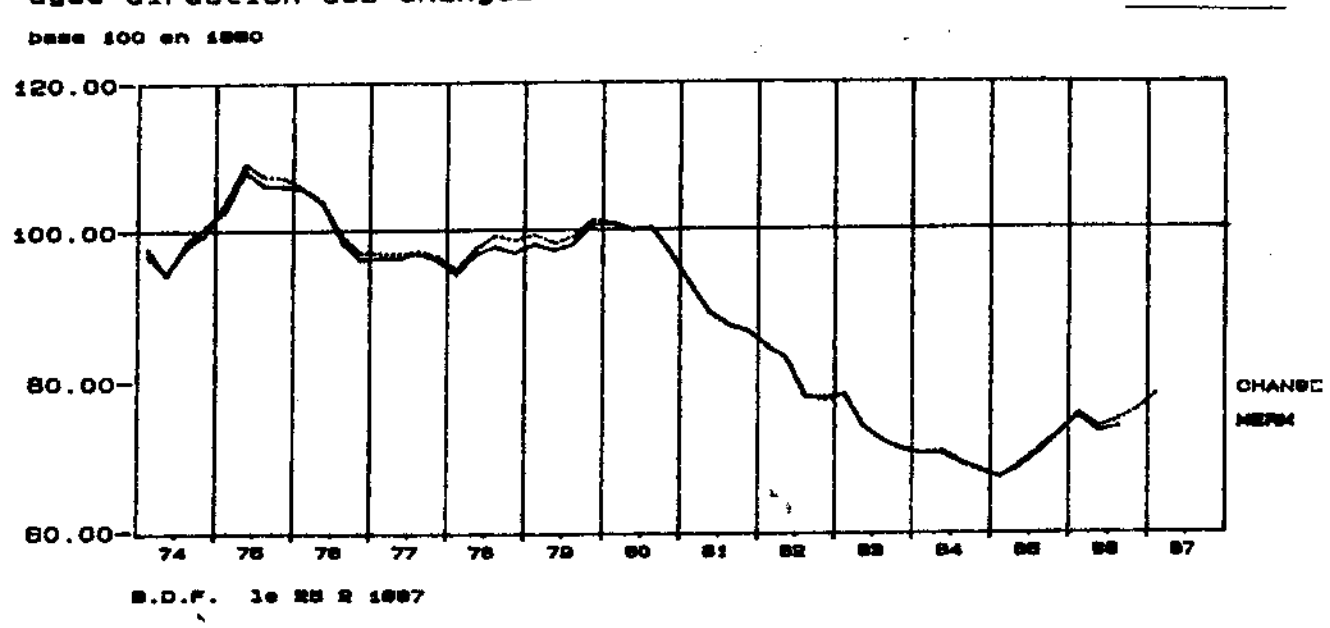
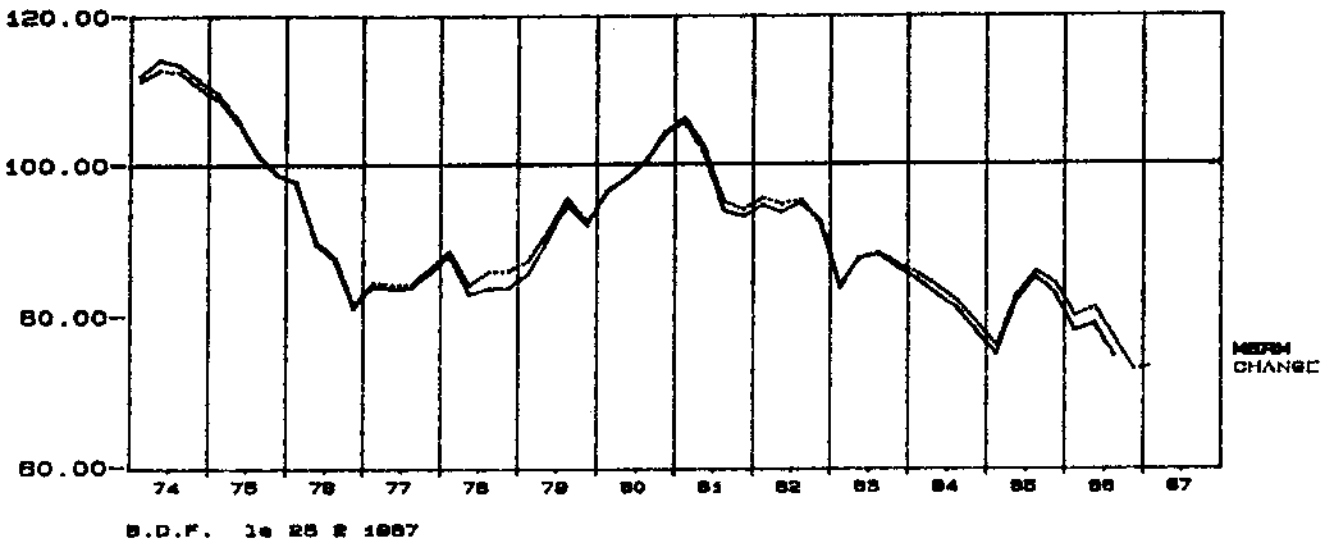


French franc effective exchange rate

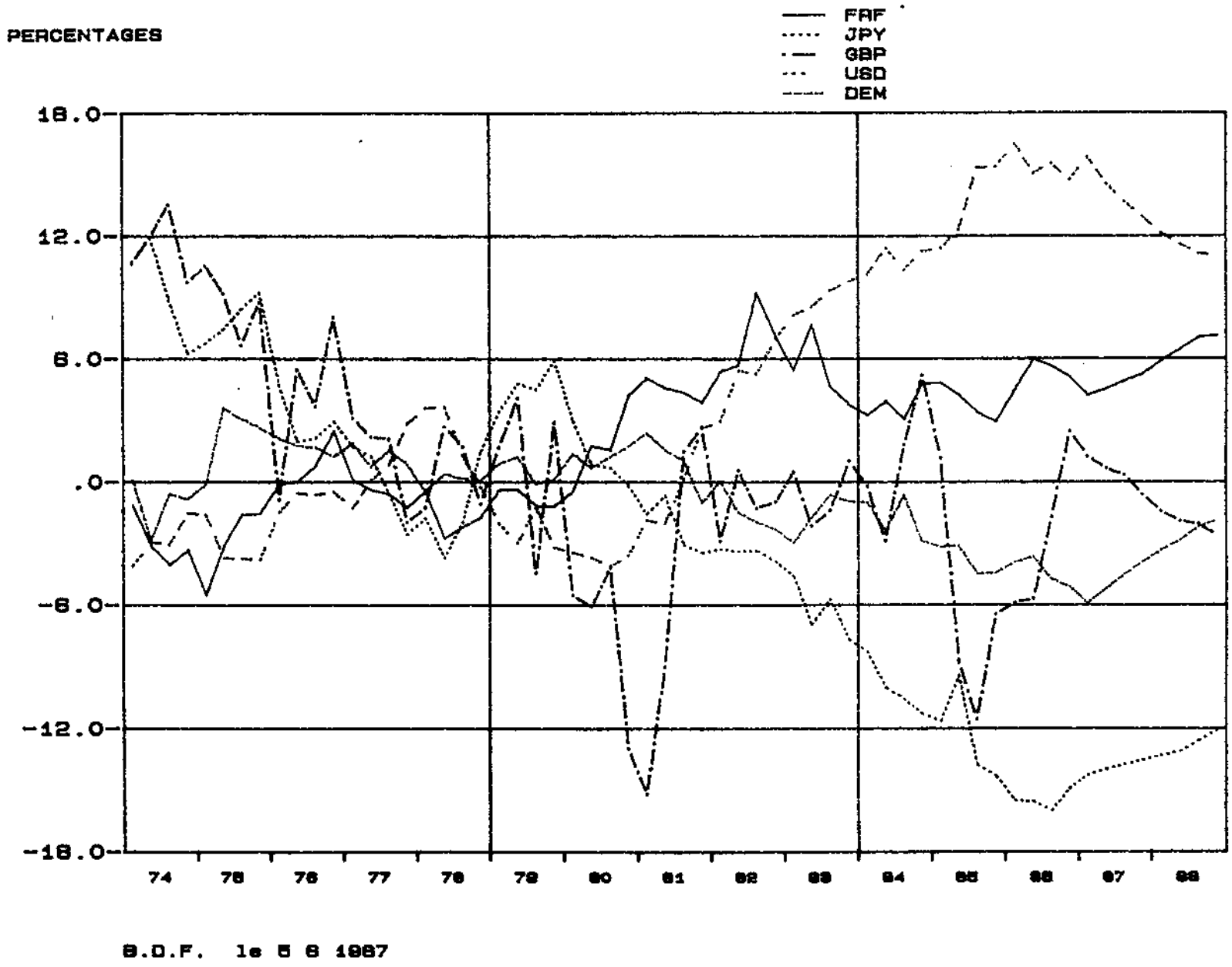


Pound sterling effective exchange rate



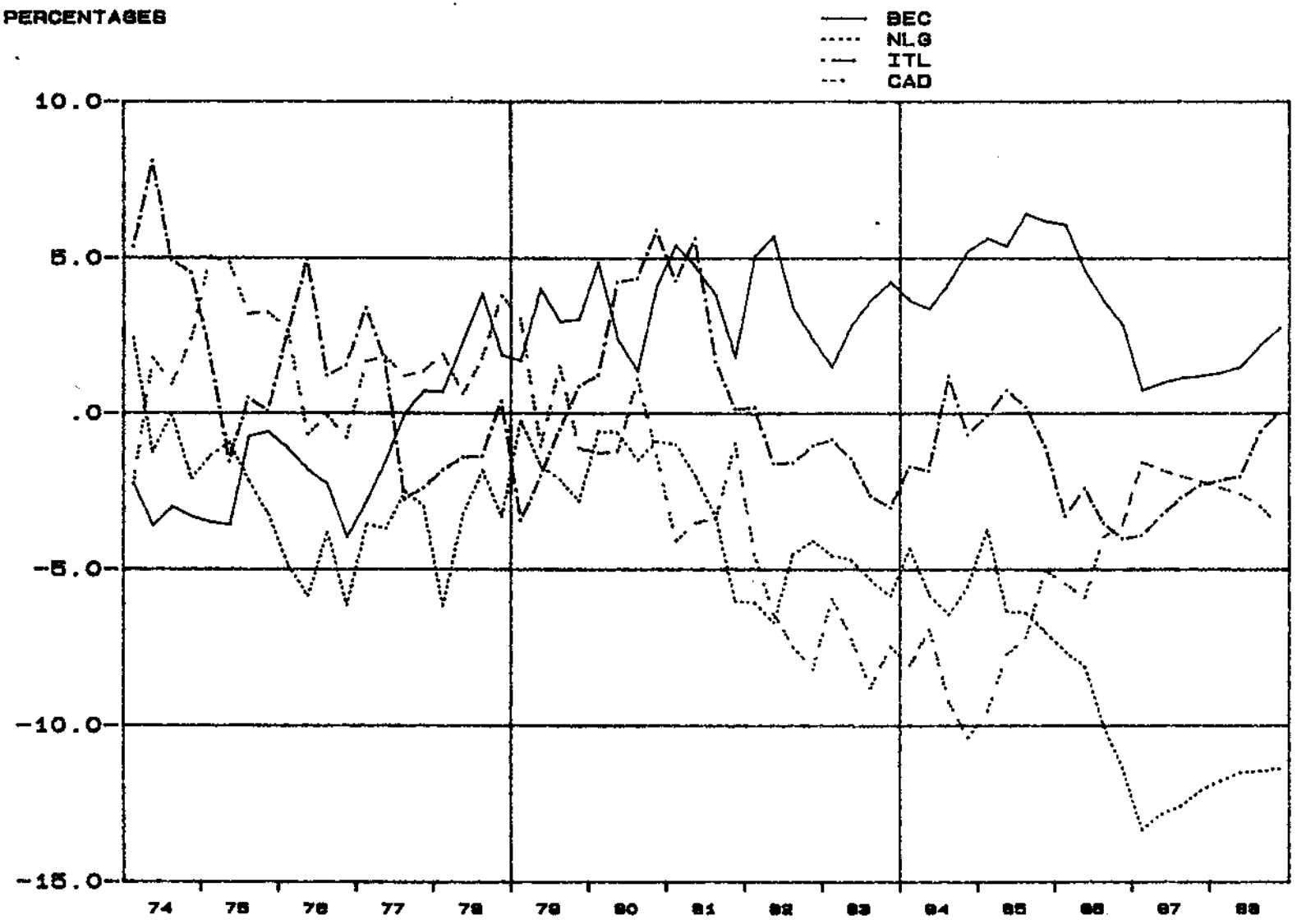
DIVERGENCE INDICATOR FOR THE MAIN CURRENCIES BASED ON A CONCEPT OF EQUILIBRIUM EXCHANGE RATE DGSE-DIRECTION DES CHANGES

APPENDIX IV



DIVERGENCE INDICATOR FOR THE MAIN CURRENCIES BASED ON A CONCEPT OF EQUILIBRIUM EXCHANGE RATE DGSE-DIRECTION DES CHANGES

APPENDIX IV



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