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Identification and Estimation of Auctions with Incomplete Contracts: A Structural Analysis of California Highway Construction Projects

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Abstract

We introduce a structural model of procurement auctions with incomplete contracts, where a procurer chooses an initial project specification endogenously. The contract between the procurer and the winner of the auction is incomplete in that the two parties may agree to adopt a new feasible specification later, and negotiate an additional transfer via Nash Bargaining where both parties' disagreement values depend on the auction price. In a Perfect Bayesian Equilibrium, contractors competing in the auction take account of such incompleteness while quoting prices.

We show that the model primitives are non-parametrically identified and propose a feasible estimation procedure. Using data from highway procurement auctions in California, we estimate the structural elements that determine the hold-up due to incompleteness, and infer how a contractor's bargaining power and the mark-up in the price quoted vary with its characteristics and the features of the construction project. We also find that ignoring the existence of contract incompleteness in the structural analysis of the bidding data leads to substantial over-estimation of the mark-ups in the prices.

Keywords: Identification, estimation, incomplete contracts, procurement auctions

JEL: C14, C57, D44

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1 Introduction

This paper proposes an empirical framework for analyzing the strategic interaction between the buyer (procurer) and the sellers (contractors) in procurement auctions with incomplete contracts. In particular, we focus on the situation where the contractual incompleteness is associated with the procurer’s endogenous choice of an initial specification for the contract. Our work is partly motivated by the seminal work in Tirole (2009), which introduced a model where contractual incompleteness arises as the buyer chooses an optimal level of cognitive effort to learn about the appropriate design. The optimal level of effort is determined by weighing the marginal costs of the effort against the marginal benefits (e.g., those from avoiding a hold-up in the post-contractual negotiation).

Summary of the Model and Contribution We extend the model in Tirole (2009) to an environment where the initial contract price is determined via the competitive bidding among contractors. In our model, a procurer announces an initial specification for the contract, and the interested contractors compete via a first-price sealed-bid auction. Both the contractors and the procurer are aware that a new feasible specification is to be realized *after* the auction, which may supplant the initial design if it leads to a positive net incremental surplus. In case a new design is adopted, the procurer negotiates with the winner and makes a transfer in addition to the auction price based on the initial design. The size of the transfer is determined by a Nash Bargaining solution. Furthermore, the auction price based on the initial specification affects the negotiation outcome through its impact on the disagreement values of both parties. Both the procurer and the contractors take this into account in their strategic decisions prior to and during the auction. The hold-up on the procurer in this model is defined as the negotiated share of the net incremental surplus paid to the winner of the auction.¹

Apart from the procurer’s endogenous choice of the initial specification, our model includes some distinctive features not present in the existing empirical literature (e.g., Bajari, Houghton, and Tadelis (2014)). First, we use a Nash Bargaining solution to account for post-auction negotiation outcomes. Second, we maintain a flexible assumption on the information available to the agents. Prior to the auction, a procurer observes a private noisy signal correlated with the feasible new design in the future and chooses an initial specification to maximize its ex ante payoff. The contractors also do not have perfect foresight of the new design, and quote prices strategically based on the initial specification by taking account of their potential gain from the hold-up due to the adoption of the new design. Third, due to the sequential moves of the procurer and contractors, we adopt a new equilibrium concept of Perfect Bayesian Equilibrium (PBE) in the structural anal-

¹Our model is in line with Tirole (2009) in the sense that the incompleteness of a contract originated in the buyer’s pre-contract choices. In Tirole’s case it is the level of cognitive effort; in ours it is the initial specification. However, unlike in Tirole (2009), the cause for contractual incompleteness in our model lies in the procurer’s uncertainty about the new feasible design (as it only observes a noisy signal). We do not adopt Tirole’s approach to endogenize the procurer’s cognitive effort and tie it to incompleteness. This is mostly due to empirical and identification concerns. In procurement auctions, the data typically do not report any proxy measure of the procurer’s cognitive effort or its costs.

ysis. We characterize and establish the existence of symmetric monotone pure-strategy Perfect Bayesian Equilibria (psPBE) in such a model.

To fill the gap between theory and empirical analyses, we provide original constructive arguments to non-parametrically identify the structural elements in the model. We also propose a feasible estimation procedure that is based on the identification argument and mild parametrization. We then use it to estimate the determinants of the hold-up problem inflicted on the procurer in post-contractual (post-auction) negotiation, and to infer how a contractor’s mark-up varies with its characteristics as well as the features of the contract.

Overview of the Identification Approach The model primitives to be recovered from the data include the bargaining power of the procurer against contractors in post-auction negotiation, the incremental cost and surplus which depend on the initial and the new specification, and the contractors’ private cost distribution. We show that under the assumption of psPBE, these elements can be non-parametrically identified from the information reported in a typical empirical environment. That is, the joint distribution of the initial specification, the auction price, the final design adopted, as well as the negotiated post-auction transfer.

Several structural links between the primitives and the data are instrumental for identifying the model: (a) a first-order condition that characterizes the procurer’s optimal choice; (b) an equilibrium condition that delineates contractors’ bidding strategies; and (c) the relation between the negotiated transfer and model elements that determine the size of the hold-up (that is, the bargaining power and the incremental cost and surplus functions). Our identification argument takes several steps. First, we exploit the link in (c) to recast (a) in the form of an ordinary differential equation in the incremental surplus function. We show that the form of the differential equation is fully determined by quantities directly identifiable from the data. This allows us to recover the incremental surplus function using a boundary condition. Then, with a flexible shape restriction on the incremental cost function, we identify the bargaining power and the cost function from the negotiated transfer using the link in (c). Next, using (b) and a standard argument for identifying bidders’ value distribution in a first-price auction by Guerre, Perrigne, and Vuong (2000), we back out the contractors’ inverse bidding strategies up to a “cost adjustment” term. This term reflects how the contractors adjust their strategic behavior in competitive bidding, knowing that additional gains are possible from the post-auction negotiation. With the quantities already recovered from the preceding steps, this adjustment term is identified. It then follows that the contractors’ full bidding strategy, and consequently the private cost distribution, is identified.

Our method is original in that it fully exploits the procurer’s optimization incentives in (a) and the characterization of the negotiated transfer as a Nash Bargaining solution in (c) in our model, where classical arguments based on (b) are insufficient for the full identification due to the existence of contractual incompleteness. We also extend our approach to identify a model of incomplete contracts where there is no pre-contract competition among contractors and the payment to the sole seller is determined via a bilateral negotiation as opposed to an auction.

Preview of Empirical Findings We apply our model to study the strategic incentives of the procurers and contractors in auctions of highway procurement contracts held by California’s Department of Transportation (Caltrans). Our goals are to understand how contractor characteristics and contract heterogeneity affect the bargaining power and the additional transfer in post-auction negotiation; to infer how the incremental surplus and costs depend on the initial and new specifications; to find out the size of the mark-up in the prices quoted by contractors; and to learn about the determinants of the hold-up for the procurer.

While implementing the estimation, we incorporate contractor and project heterogeneity in the incremental cost and surplus and the bargaining power. The parameters in these objects are estimated via a two-stage extremum estimator, using the variation in the negotiated transfers reported in the data. We then use these estimates to infer the cost distribution of contractors from the prices quoted.

Our first set of empirical results reveal the determinants of contractors’ bargaining power. We find that on average a winning contractor’s bargaining power is significantly higher than that of the procurer, and it is higher for the winners from auctions involving more contractors. Furthermore, the winner from an auction has a higher bargaining power against the procurer if the auction has a higher proportion of non-fringe firms among the competing contractors, or if the average utilization rate (defined as the ratio of a contractor’s backlog over its capacity) is higher among the contractors in the auction.

We also classify the contracts into “major” projects that require more substantive reconstruction and relatively “minor” ones. We find that the procurer has a significantly higher bargaining power for major projects (about 12% higher than that in minor projects). This could be because the procurer is more likely to play hardball in post-auction negotiations if the contract involves a major project (either because the winning contractor cares about the long-term relation with the procurer, or because there are more tax-payer’s money and larger social surplus at stake).

Our estimates suggest that the social surplus function is convex, thus offering some evidence for an economy of scale in how larger projects benefit a greater population. The estimates also show that larger transfers are often associated with projects involving more substantial post-auction modification.

We estimate the private costs of contractors and the mark-ups in their quotes. The overall average mark-up is estimated to be around 9%. We find that the mark-ups increase sharply as the number of bidders decreases. Our estimates also show that the auction winners tend to have substantial cost advantage over other competitors, which allow them to win with high mark-ups. Interestingly, the mark-ups are lower for contractors in major projects and this may be due to their smaller bargaining power in these projects as mentioned in the previous results. Last but not least, we find that ignoring how contractors respond strategically to contractual incompleteness in the auctions leads to a substantial over-estimation of actual markups in their quoted prices.

Relation to Existing Literature A wide array of theoretical models of incomplete contracts have been used to study employment relation (Simon (1951), Klein, Crawford,

and Alchian (1978)), ownership and the property-rights of firm (Williamson (1985), Grossman and Hart (1986), Hart and Moore (1990)), and international trade (Spencer (2005)). The model we introduce in this paper differs qualitatively from those in these papers. In particular, we extend the model in Tirole (2009) to an environment where the initial contract price is determined via the competitive bidding between contractors.

This paper also contributes to a broader literature on the identification of structural models in auction and contract theory. Guerre, Perrigne, and Vuong (2000) proposed a structural mapping between bidders' valuations and their bids in first-price auctions and used it as a basis for identifying the distribution of bidders' private values. Some recent papers that study the identification of models related to the contract theory (e.g., d'Haultfoeuille and Février (2007), Aryal, Perrigne, and Vuong (2010), Perrigne and Vuong (2011) and Perrigne and Vuong (2012)) build their argument around the mapping between the unobserved agent type and the features of the contracts reported in the data (e.g., the price and the package offered in the contracts). In comparison, we propose new identification arguments that explore the procurer's optimization incentives and the characterization of the negotiated transfer in a Nash Bargaining solution in a model where classical arguments based on bidders'/contractors' incentives alone would be insufficient for full recovery of model primitives.

In addition, our paper contributes to a fast-growing empirical literature on post-auction bargaining (e.g., Elyakime, Laffont, Loisel, and Vuong (1997) and Larsen (2014)) and contractual incompleteness in a variety of environments (e.g., Crocker and Reynolds (1993), Bajari, McMillan, and Tadelis (2009) and Bajari, Houghton, and Tadelis (2014)). Bajari, Houghton, and Tadelis (2014) used the same source of data as ours to estimate the adaptation costs due to contract incompleteness. In comparison, our model differs from theirs in several fundamental aspects. We model the post-auction negotiation via a Nash Bargaining solution; allow endogenous choices of initial specification which affect ex post contractual incompleteness; and accommodate more flexible assumptions on the information available to agents. (See Section 5 for a more detailed discussion.) Our new model demands a qualitatively different methodology for identification and estimation. Using our model, we are able to infer how the key determinants of the hold-up (incremental costs and surplus, contractor's bargaining power) depend on contractor characteristics and project heterogeneity. Our findings in Section 5 provide new evidence as to how agents respond strategically to the presence of contractual incompleteness, and what determines the size of the hold-up in highway procurement projects.

Roadmap In Sections 2 we introduce our model of procurement auctions of incomplete contracts with an endogenous initial specification, and characterize its pure-strategy Perfect Bayesian Equilibrium. In Section 3 we establish the main identification results, and then extend the argument to the case of incomplete contracts with bilateral negotiations in Section 4. In Section 5 we propose a feasible estimation procedure based on the identification arguments and apply it to analyze California highway procurement contracts. Section 6 concludes. Proofs and tables are collected in the appendix.

2 The Model and Equilibrium

A buyer (or procurer) holds a first-price sealed-bid procurement auction among N sellers (or contractors). First, the procurer announces an initial contract specification $X \in \mathcal{X} \subset \mathbb{R}$. Upon learning X , each contractor i draws a private cost C_i for completing the contract under this specification, and quotes a price (a bid) $P_i \in \mathbb{R}_+$. The procurer awards the contract to the contractor that quotes the lowest price (a.k.a. the “winner”). For any initial specification X , the contractors’ costs are drawn independently from a continuous distribution $F_{C|X}$ with support $\mathcal{C} \subset \mathbb{R}_+$, which may depend on X . For generic random vectors R and R' , let $F_{R'|R}$ and $f_{R'|R}$ denote the conditional distribution and density of R' given R respectively, and write them as $F_{R'|R=r}$ and $f_{R'|R=r}$ in order to be specific with the value conditioned on. A random variable is denoted by upper cases while its realized values are denoted by lower cases.

In contrast to a standard procurement auction, the contract we consider in this model contains an incompleteness pact. The specification of the contract is subject to a possible modification to a new design $X^* \neq X$ *after* the auction is concluded (that is, *after* a winner is chosen to execute the contract under the initial specification X). The new design X^* is not known *ex ante* to the procurer or any contractor, and is thus considered uncertain when the procurer announces X and the contractors quote prices. The private costs C_i for completing the initial specification are independent from X^* once conditional on X .

We model the occurrence of contractual incompleteness as follows. Let $\pi(X) \in \mathbb{R}$ be the social surplus under a specification X , and let $a(X, X^*) \in \mathbb{R}$ be the incremental cost when the specification is changed from X to X^* . The incremental cost may involve additional construction as well as logistic work, and we assume it is non-separable in X and X^* to allow for the possibility that the marginal costs depend on both X and X^* ; on the other hand the incremental surplus by definition is just the difference of surplus between two specifications and it is separable in X and X^* , i.e., $\phi(X, X^*) \equiv \pi(X^*) - \pi(X)$. Once X^* is realized following the auction, the procurer and the winner in the auction will agree to use the incompleteness pact and adopt the new design X^* if and only if this yields a net incremental surplus relative to the initial specification (that is, $s(X, X^*) \equiv \phi(X, X^*) - a(X, X^*) > 0$).² The incompleteness of a contract is measured by the probability that the initial specification X is altered to be X^* such that $s(X, X^*) > 0$.

If X^* is adopted, the procurer and the winner negotiate further transfer between them *in addition to* the contract price initially quoted by the winner. In our model, the auction price affects the post-auction negotiation outcomes through its impact on the disagreement values of both parties in the Nash Bargaining solution. The contractor covers the incremental costs upfront as they arise in construction.³ The incremental surplus is

²More generally, our identification and estimation method applies when the occurrence of incompleteness follows a general rule that depends on X , X^* and other variables reported in the data.

³In some contracts, the procurer, rather than the contractor, covers the incremental costs up front. In this case, the negotiated transfer from the procurer to the contractor equals the contractor’s share of the net incremental surplus $\phi - a$, which must be proportional to the latter’s bargaining power in a Nash Bargaining solution. Our method in this paper can be readily extended to these contracts.

eventually accrued to the procurer. Both parties take these into account as they negotiate the transfers through a Nash Bargaining process. Specifically, let $y(X, X^*) \in \mathbb{R}$ denote the negotiated transfer from the procurer to the winner. To simplify notation we suppress the input argument (X, X^*) in y, ϕ, a, s below. Let $1 - \gamma \in (0, 1)$ be a constant parameter reflecting the procurer’s bargaining power.⁴ The negotiated transfer determined by the Nash Bargaining solution is such that the auction winner eventually obtains a share of the net incremental surplus proportional to its bargaining power (see Appendix A for details). That is,

$$y - a = \gamma s \Leftrightarrow y = \gamma \phi + (1 - \gamma)a.$$

On the other hand, there is no negotiated transfer ($y = 0$) if no new design is adopted.

Prior to announcing the initial specification, the procurer observes a signal $\tilde{X} \in \mathcal{X}$ that is correlated with X^* . Denote the distribution of X^* conditional on \tilde{X} by $F_{X^*|\tilde{X}}$. The model elements $\gamma, \pi, a, F_{C|X}$ and $F_{X^*|\tilde{X}}$ are common knowledge among the procurer and the contractors. On the other hand, realized values for \tilde{X} and C_i remain private information for the procurer and contractor i respectively. At the beginning of the auction, the procurer announces an X that maximizes its ex ante payoff, based on its signal \tilde{X} . A contractor then observes X , draws its private cost C_i from $F_{C|X}$ and quotes a price to maximize its ex ante profit, which includes the negotiated transfer to be realized.

A leading example for auctions of incomplete contract is the highway procurement auctions in California (see Bajari, Houghton, and Tadelis (2014)). Caltrans is a government department in the State of California that is responsible for the planning, construction, and maintenance of public transportation facilities such as highway, bridge, and rails. It awards highway construction projects to contractors through first-price procurement auctions. Before a procurement auction, Caltrans announces an engineers’ estimate X for the cost of the project. These estimates are essentially the initial specification, conditional on which the contractors calculate their initial projected costs. (In the data used in Bajari, Houghton, and Tadelis (2014), these engineers’ estimates are reported in terms of the quantity for each category of inputs and their per-unit prices.)

Once the engineers’ estimate is announced, the set of auction participants who are interested in the contract draw their private costs for completing the project and quote their prices. The contractor bidding the lowest price is awarded the contract. The contracts are typically incomplete, and post-contractual modifications such as extra work, adjustment, and deduction are often reported. Both parties are aware of this incompleteness pact and take it into account while quoting the prices or announcing the engineers’ estimates. In a majority of the cases, Caltrans and the winner of the auction end up adopting a new specification X^* that differs from X . In such cases, additional transfers (in the form of adjustment, deduction or payment for extra work) are made between the two parties through negotiation.

⁴The bargaining power of the procurer against the winner may in general depend on auction-level characteristics. While we focus on homogeneous auctions in this section to simplify exposition, our identification results in this section could be generalized conditional on observed auction-level heterogeneity. In the empirical application, we estimate a model with heterogeneous auctions.

For the rest of this section, we characterize and discuss the existence of symmetric monotone pure-strategy Perfect Bayesian Equilibria (psPBE) in this model. For now we maintain that the number of contractors in an auction N is common knowledge among the procurer and contractors at the bidding stage. Nonetheless, as explained later in the identification section, our results apply to the case where contractors' participation decisions are exogenous and N is not unknown to both the procurer and the contractors.

A contractor i 's pure strategy maps from his information (C_i, X) to a quoted price; a procurer's pure strategy maps from a signal \tilde{X} to an initial announcement X . Let $s_+ \equiv \max\{s, 0\}$. A *symmetric psPBE* in this model is a pair of procurer strategy α^* and contractor strategy β^* such that (a) for all (x, c_i) ,

$$\beta^*(c_i, x) = \arg \max_{p \in \mathbb{R}_+} \Pr \left(\min_{j \neq i} \beta^*(C_j, X) \geq p \mid X = x \right) \{p - c_i + \bar{\delta}(x; \alpha^*)\}, \quad (1)$$

where $\bar{\delta}(x; \alpha^*) \equiv \mathbb{E}[\gamma s_+(X, X^*) \mid X = x; \alpha^*]$ is a contractor's ex ante share of the negotiated net incremental surplus; (b) the expectation in $\bar{\delta}$ is conditional on the initial specification x , and with respect to a belief about X^* that is consistent with $F_{X^*|\tilde{X}}$ and α^* (explained below); and (c) α^* is the procurer's best response when all contractors follow β^* :

$$\alpha^*(\tilde{x}) = \arg \max_{x \in \mathcal{X}} \{\pi(x) - \psi(x; \beta^*) + \mu(x, \tilde{x})\} \quad (2)$$

where $\psi(x; \beta^*)$ is the procurer's ex ante payment (that is, the price quoted by the auction winner) when contractors follow symmetric pure-strategy β^* and $\mu(x, \tilde{x}) \equiv \mathbb{E}[(1 - \gamma)s_+(x, X^*) \mid \tilde{X} = \tilde{x}]$. (See Appendix B for the closed-form of ψ .) The last term $\mu(x, \tilde{x})$ is a procurer's ex ante payoff due to contractual incompleteness. The expectation in $\mu(x, \tilde{x})$ is taken with respect to X^* according to $F_{X^*|\tilde{X}=\tilde{x}}$ when the initial specification X is fixed at x . In a *monotone symmetric psPBE*, α^* is increasing over \mathcal{X} and β^* is increasing over \mathcal{C} for any $x \in \mathcal{X}$. In this case, the consistency of the contractors' belief in equilibrium means $F_{X^*|X=x}(x^*) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(x^*)$ for all x, x^* .

We now provide some heuristic argument for the existence of symmetric monotone psPBE in the model. (Technical matters such as detailed conditions and the proof are presented in the appendix.) First off, for any initial announcement x and procurer strategy α , a contractor's optimization problem is similar to that in a procurement auction without any incompleteness, except that its private cost is drawn from the distribution of "adjusted costs" $C_i - \bar{\delta}(x; \alpha)$ which depend on x and α . By arguments from the existing literature (such as Athey (2001); McAdams (2003); Reny (2011)), symmetric monotone pure-strategy Bayesian Nash equilibria exist in the procurement auction for a given pair of α and x under regularity conditions.

For a fixed x , let β_α denote a contractor's bidding strategy in a symmetric monotone Bayesian Nash equilibrium when the procurer adopts a monotone procurer strategy α . Recall that α affects contractors' bidding strategies via its impact on their equilibrium belief, which enters $\bar{\delta}(x; \alpha)$. For any x and α , let $\varphi(x; \alpha) \equiv \psi(x; \beta_\alpha)$ denote the procurer's expected payment when contractors follow the strategy β_α . Then in a psPBE the

procurer's strategy α^* is characterized by the solution to a fixed-point equation:

$$\alpha^*(\tilde{x}) = \arg \max_{x \in \mathcal{X}} \{\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \alpha^*)\}. \quad (3)$$

A symmetric monotone psPBE exists if this fixed-point mapping admits a solution α^* that is monotone over \mathcal{X} . Assuming smoothness of π, μ, φ in x and existence of interior solutions, the first-order condition for the maximization problem in (3) evaluated at the procurer's equilibrium strategy takes the form of an ordinary differential equation (ODE) in α^* . The form of the ODE is determined by $\{\pi, a, \gamma, F_{C|X}, F_{X^*|\tilde{X}}\}$. Thus proving the existence and uniqueness of symmetric monotone psPBE boil down to proving the existence and uniqueness of the solution to this ODE.

Using existing theory on ODE, we establish the existence and uniqueness of a global solution α^* in (3) that is increasing over \mathcal{X} under appropriate conditions. The existence and uniqueness of the solution require that the function characterizing the derivative of α^* in the ODE satisfy a Lipschitz condition in the initial announcement. The sufficient conditions for a monotone solution include the assumption that $F_{C|X}$ is first-order stochastically increasing in X .

3 Identification

In this section we show how the elements of the model can be identified when the data report the price quoted by the winner in the procurement auction (V), the initial specification (X), as well as the final design adopted (X^*) and the negotiated transfer (Y) if the new design is adopted. We maintain that the data are rationalized by a single symmetric monotone psPBE $\{\alpha^*, \beta^*\}$. Results in this section can be extended to accommodate contract heterogeneity reported in the data. Our argument below is conditional on the number of bidders N , which is suppressed from the notation for simplicity. Let D be a dummy variable such that $D = 1$ if there is a negotiated transfer in a contract due to the adoption of a new design.

First off, we show that at best the model can only be identified up to a monotone transformation of the procurer's signal. We say two models are *observationally equivalent* if they imply the same distribution of V, Y, D, X and X^* in symmetric psPBE. Suppose the actual data-generating process is $\theta \equiv \{\gamma, \pi, a, F_{C|X}, F_{X^*|\tilde{X}}\}$. Consider an alternative model $\theta_0 \equiv \{\gamma_0, \pi_0, a_0, H_{C|X}, H_{X^*|\tilde{X}}\}$ such that (i) $a = a_0$, $\pi = \pi_0$ and $\gamma = \gamma_0$; (ii) $F_{C|X=x} = H_{C|X=x}$ for all $x \in \mathcal{X}$; and (iii) $F_{X^*|\tilde{X}}(x, \tilde{x}) = H_{X^*|\tilde{X}}(x, h(\tilde{x}))$ for all x, \tilde{x} and some increasing and differentiable function $h : \mathcal{X} \rightarrow \mathcal{T} \subseteq \mathbb{R}$. In other words, the procurer's private signal prior to auctions in θ_0 is an increasing and differentiable transformation of the actual signal \tilde{X} in θ . Note the support of the private signal in θ_0 (denoted \mathcal{T}) is allowed to differ from that in θ .

Lemma 1 (Observable Equivalence) θ and θ_0 are observationally equivalent.

Proof of the lemma is presented in the appendix. For a single-agent model with a non-additive monotone outcome function, Matzkin (2003) showed some scale normalization of unobserved shocks is necessary for nonparametric identification.⁵ In comparison, we show in Lemma 1 that normalizing the distribution of procurers' private signals is also necessary for identifying the richer game-theoretic model we introduce, where the vector of actions (X, X^*) and outcome V, D, Y reported in the data are rationalized by the strategic interaction between contractors and the procurer in equilibrium.⁶

Lemma 1 suggests that we need to impose at least some normalization of the signal \tilde{X} in order to identify the model.⁷ In what follows, we normalize the marginal distribution $F_{\tilde{X}}$ to a standard uniform without loss of generality. Let $\mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in \mathcal{X}\}$ denote the support of X in equilibrium. For each x , define $\omega(x) \equiv \{x^* \in \mathcal{X} : s(x, x^*) > 0\}$.

Lemma 2 (Procurer's Strategy) α^* is identified over the support of \tilde{X} . For all $x \in \mathcal{X}_e$, the distribution of X^* conditional on $X^* \in \omega(x)$ and $\tilde{X} = \alpha^{*-1}(x)$ is identified from the distribution of (X^*, D) conditional on $X = x$.

Proof. Let $\{\alpha^*, \beta^*\}$ denote the symmetric, monotone psPBE in the data-generating process; and let x_τ denote the τ -th quantile of the initial specification X that is identifiable from the data. Recall that we normalize $F_{\tilde{X}}$ to be a standard uniform distribution over $(0, 1)$.⁸ Then the procurer's strategy is identified as $\alpha^*(\tau) = x_\tau$. The monotonicity of α^* then implies that $X = x_\tau$ if and only if $\tilde{X} = \tau$. Hence, we have

$$\begin{aligned} F_{X^*|X^* \in \omega(x_\tau), \tilde{X} = \tau}(x^*) &= \Pr\{X^* \leq x^* \mid \tilde{X} = \tau, s(x_\tau, X^*) > 0\} \\ &= \Pr\{X^* \leq x^* \mid X = x_\tau, D = 1\} \end{aligned}$$

for all $x^* \in \mathcal{X}$ and $\tau \in (0, 1)$. \square

More generally, we may normalize $F_{\tilde{X}}$ to any increasing, differentiable C.D.F. F_0 and let \tilde{x}_τ denote the τ -th quantile of F_0 . Then the same argument applies and $\Pr\{X^* \leq$

⁵Matzkin (2003) considered a nonparametric model $Y = m(X, \epsilon)$ where m is monotone in ϵ , and ϵ is independent from X . Lemma 1 in Matzkin (2003) established that, without further restrictions on m , the model is observationally equivalent to another model $Y = \tilde{m}(X, \tilde{\epsilon})$ where $\tilde{\epsilon}$ is any monotone transformation of ϵ .

⁶The proof of observational equivalence in our case is qualitatively different from that in Matzkin (2003). First, we relate the players' equilibrium strategies in θ to those in θ_0 . Then, we show that θ and θ_0 imply the same equilibrium distribution of (V, Y, D, X, X^*) .

⁷Tirole (2009) considered a model of incomplete contracts where both parties may exert cognitive effort to learn about the appropriate design and how to draft the contract accordingly. In his case, the pre-contractual decision is to choose the level of information acquisition effort, which is measured by acquisition costs. In such a model, the full identification of the information structure may be possible if the endogenous choices of the information acquisition costs as well as the signal for appropriate design are both reported in the data.

⁸In the notation leading to Lemma 1, this is equivalent to picking an observationally equivalent model θ_0 by setting h to be the marginal distribution $F_{\tilde{X}}$ in θ .

$x^* \mid \tilde{X} = \tilde{x}_\tau, s(x_\tau, X^*) > 0\}$ is identified as $\Pr\{X^* \leq x^* \mid X = x_\tau, D = 1\}$. Similarly, the procurer's strategy is identified as $\alpha^*(\tilde{x}_\tau) = x_\tau$. Once the procurer's strategy α^* is identified, we apply an argument based on Guerre, Perrigne, and Vuong (2000) to establish an intermediate result on the identification of contractors' cost distribution.

Lemma 3 (Contractors' Strategies) *The inverse strategy for contractors is identified up to $\bar{\delta}(\cdot; \alpha^*)$, the ex ante share of the net incremental surplus in equilibrium.*

Proof. Given an initial specification x and the procurer's equilibrium strategy α^* , a contractor i 's problem in (1) is reparametrized as:

$$\tilde{\beta}_i(\tilde{c}_i, x; \alpha^*) = \arg \max_{p \in \mathbb{R}_+} \Pr \left\{ \min_{j \neq i} \tilde{\beta}_j(\tilde{C}_j, X; \alpha^*) \geq p \mid X = x \right\} (p - \tilde{c}_i), \quad (4)$$

where $\tilde{C}_i \equiv C_i - \bar{\delta}(X; \alpha^*)$ are i.i.d. conditional on x and α^* for all i . Changing variables between quoted prices and private costs, we can recover the inverse bidding strategy $\tilde{\beta}^{-1}(\cdot, \cdot; \alpha^*)$ in equilibrium in (4) as

$$\tilde{\beta}^{-1}(p, x; \alpha^*) = p - \frac{1}{N-1} \frac{1 - F_{P|X=x}(p)}{f_{P|X=x}(p)}, \quad (5)$$

where $F_{P|X=x}(p)$ denotes the conditional distribution of quoted prices. Because the contractors' private costs are i.i.d. given X , the prices they quote in a symmetric monotone psPBE are also i.i.d. given X . Thus

$$1 - F_{V|X} = (1 - F_{P|X})^N \Rightarrow F_{P|X} = 1 - (1 - F_{V|X})^{\frac{1}{N}}.$$

Substituting this into (5) and using $f_{P|X=x}(p) = \frac{\partial}{\partial p} F_{P|X=x}(p)$, we get

$$\tilde{\beta}^{-1}(p, x; \alpha^*) = p - \frac{N}{N-1} \frac{1 - F_{V|X=x}(p)}{f_{V|X=x}(p)}.$$

By construction $C_i = \tilde{C}_i + \bar{\delta}(X; \alpha^*)$ and thus

$$\beta^{*-1}(p, x; \alpha^*) = \tilde{\beta}^{-1}(p, x; \alpha^*) + \bar{\delta}(x; \alpha^*). \quad (6)$$

Hence the inverse bidding strategy for contractors in (1) given a procurer strategy α^* in the symmetric monotone psPBE is identified up to $\bar{\delta}(\cdot; \alpha^*)$. \square

The result in Lemma 3 is based on the following fact: with contractual incompleteness, contractors adopt equilibrium strategies similar to those in a first-price auction, except that the costs need to be adjusted to take account of the ex ante share of net incremental surplus via Nash bargaining.

Next, we show how to identify γ, a and π . This would imply the identification of $\bar{\delta}(X; \alpha^*) = \mathbb{E}[\gamma s_+(X, X^*) \mid X = x; \alpha^*]$, and subsequently $\beta^{*-1}(p, x; \alpha^*)$ by (6). It would then follow that the distribution of C_i given X is also identified.

Assumption 1 (a) The support \mathcal{X} is convex. (b) $F_{X^*|\tilde{x}}$ is continuous with positive density over \mathcal{X} for all \tilde{x} . (c) π and a are bounded and continuously differentiable. (d) The boundary of $\omega(x) \equiv \{x^* \in \mathcal{X} : s(x, x^*) > 0\}$ is differentiable in x over the interior of \mathcal{X} .

Parts (a)-(c) in Assumption 1 are standard regularity conditions. We exemplify the restriction in (d) as follows. Suppose $\omega(x) \subseteq \mathcal{X} \subseteq \mathbb{R}$ is partitioned into two disjoint intervals $(\iota_1(x), \iota_2(x))$, $(\iota_3(x), \iota_4(x))$. Then (d) requires that $\iota_k(x)$ is differentiable in x for $k = 1, 2, 3, 4$ over the interior of \mathcal{X} .

We now sketch the argument leading to the identification of π under Assumption 1. In a symmetric monotone psPBE, the procurer chooses x to maximize his ex ante return $\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \alpha^*)$. An interior solution is characterized by the first-order condition:

$$\frac{\partial}{\partial x} [\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \alpha^*)]_{x=\alpha^*(\tilde{x})} = 0 \text{ for any } \tilde{x}$$

which implies:

$$\varphi'(z; \alpha^*) = \pi'(z) + \frac{\partial}{\partial x} \left[\int_{\omega(x)} (1 - \gamma)s(x, t) dF_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t) \right]_{x=z} \quad (7)$$

for all z on the equilibrium path (i.e. $z = \alpha^*(\tilde{x})$ for some \tilde{x} , or equivalently, $z \in \mathcal{X}_e$). The condition (7) essentially equates the marginal payment (cost) in the auction with the marginal benefit for the procurer at the level of the initial specification z . The right-hand side of (7) shows that an initial specification affects a procurer's payoff both through the gross surplus and through ex ante share of the net incremental surplus.

Recall that the observed transfer from the procurer to the contractor is:

$$y(x, x^*) = (1 - \gamma)a(x, x^*) + \gamma\phi(x, x^*) \quad (8)$$

for all (x, x^*) such that $s(x, x^*) > 0$; and is zero otherwise. Using Assumption 1(d), we combine (7) with (8) to get:

$$\pi'(z) = [1 - p(z, \alpha^{*-1}(z))]^{-1} \left(\varphi'(z; \alpha^*) + \int_{\omega(z)} y_1(z, t) f_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t) dt \right) \text{ for all } z \in \mathcal{X}_e \quad (9)$$

where $p(x, \tilde{x}) \equiv \Pr\{D = 1 \mid X = x, \tilde{X} = \tilde{x}\}$ denotes the probability that the contract is modified to adopt the new design. (See Appendix C.2 for details.) For any $z \in \mathcal{X}_e$, the density $f_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t)$ is identified for all $t \in \omega(z)$ as

$$f_{X^*|X^* \in \omega(z), \tilde{X}=\alpha^{*-1}(z)}(t) \times \Pr(X^* \in \omega(z) \mid X = z)$$

where the first term is identified due to Lemma 2 and the second term is directly identified as $\Pr(D = 1 \mid X = z)$ from data.

Equation (9) shows how contract incompleteness affects a procurer's decision. Like contractors, the procurer also takes into account the post-contractual negotiated transfer while choosing an initial specification. The right-hand side of (9) consists of quantities

that are identified from the joint distribution of V, Y, D, X , and X^* if the new specification is adopted. This means we can recover π up to an innocuous location normalization. That the location of π can not be identified is because the contractor and procurer strategies only depend on $\pi(x^*) - \pi(x)$ and the derivative $\pi'(\cdot)$ respectively. Without loss of generality, we normalize the location of π by setting $\pi(\underline{x}) = 0$ with \underline{x} being the infimum of \mathcal{X}_e . For all $x \in \mathcal{X}_e$, the closed-form solution for π is

$$\pi(x) = \int_{\underline{x}}^x \left\{ [1 - p(z, \alpha^{*-1}(z))]^{-1} \left(\varphi'(z; \alpha^*) + \int_{\omega(z)} y_1(z, t) f_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t) dt \right) \right\} dz.$$

Because the integrand is identified for all z on the equilibrium support of X , this means π is identified over \mathcal{X}_e . To recover γ and a from (8), we maintain the following conditions.

Assumption 2 *There exist (x_1, x_1^*) and (x_2, x_2^*) such that $x_1, x_2 \in \mathcal{X}_e$ and $s(x_1, x_1^*) > 0$, $s(x_2, x_2^*) > 0$; and $a(x_1, x_1^*) = a(x_2, x_2^*)$, $\phi(x_1, x_1^*) \neq \phi(x_2, x_2^*)$.*

This assumption states that in the data-generating process it is possible to witness two pairs of realized specifications that lead to the same incremental adjustment costs a but different incremental surplus ϕ . This restriction holds for many specifications of a and ϕ . For example, suppose $a(X, X^*) = \sum_{k=0}^K a_k (X^* - X)^k$ but $\pi(X)$ is a polynomial $\sum_{k=0}^K a_k X^k$, $K \geq 2$. Then any pairs with $x_1^* - x_1 = x_2^* - x_2$ and $x_1 \neq x_2$ satisfy this assumption.

Under Assumption 2, $y(x_1, x_1^*) - y(x_2, x_2^*)$ equals the contractor's share of the incremental surplus (that is, $\gamma[\phi(x_2, x_2^*) - \phi(x_1, x_1^*)]$). With π (and hence ϕ) already identified, we can recover γ as the ratio between $y(x_1, x_1^*) - y(x_2, x_2^*)$ and $\phi(x_2, x_2^*) - \phi(x_1, x_1^*)$. With γ and π recovered, the incremental cost $a(x, x^*)$ is then identified from (8) for any $x \in \mathcal{X}_e$ and $x^* \in \omega(x)$.

Theorem 1 *Suppose Assumptions 1- 2 hold. Then γ is identified; and $F_{C|X=x}$, $\pi(x)$ and $a(x, x^*)$ are identified for all $x \in \mathcal{X}_e$ and $x^* \in \omega(x)$.*

Note that Assumption 2 only gives one possible condition under which γ and a are recovered. There exist alternative restrictions that are sufficient for identification. For example, suppose for some x_0 and x_0^* the level of the adjustment cost is known: $a(x_0, x_0^*) = a_0$. Let y_0 and ϕ_0 denote the observed transfer $y(x_0, x_0^*)$ and incremental surplus $\phi(x_0, x_0^*)$, respectively. Then the knowledge of the triple (x_0, x_0^*, a_0) allows us to recover $\gamma = (y_0 - a_0)/(\phi_0 - a_0)$. With γ and π recovered, the incremental cost a is then identified from (8).

Another example is when the incremental cost is homogenous of degree one while the incremental surplus has non-constant (diminishing or increasing) returns to scale. In this case, consider any pair (x_1, x_1^*) and $(x_2, x_2^*) \equiv (\rho x_1, \rho x_1^*)$ with a known constant $\rho > 0$. Let y_k, ϕ_k, a_k be shorthands for the functions y, ϕ, a evaluated at (x_k, x_k^*) respectively,

$k = 1, 2$. Because the incremental cost $a(\cdot, \cdot)$ is homogeneous of degree one, we have $a(x_2, x_2^*) = \rho a(x_1, x_1^*)$ and

$$(y_2 - \gamma\phi_2)/(y_1 - \gamma\phi_1) = (1 - \gamma)a_2/[(1 - \gamma)a_1] = \rho,$$

which implies γ is identified as $(y_2 - \rho y_1)/(\phi_2 - \rho\phi_1)$. (The non-constant returns to scale in the incremental surplus $\phi(\cdot, \cdot)$ ensures the denominator is non-zero.) Then a can be recovered using (8) and knowledge of γ, ϕ as before.

The results in Theorem 1 can be readily extended to the incomplete contracts where the procurer, rather than the contractor, covers the incremental costs up front. In such a case, the transfer to the contractor is $y = \gamma s$. The cost distribution of contractors can be identified from (6) because the ex ante share of the negotiated net incremental surplus $\bar{\delta}(x; \alpha^*)$ is identified. Using the relationship $s = y/\gamma$ and (7), we can get an ordinary differential equation of the surplus function that is similar to (9), with the solution being a function of φ and y for a given γ . By combining this solution of π with the restriction in Assumption 2, we are able to identify γ . Then a is recovered from $y = \gamma s$.

The identification results for our model are presented in an environment where the number of bidders N is known to contractors as well as the procurer at the time of the auctions. Athey and Haile (2007) argued (in their Section 6.3.3.) that in some procurement settings the contractors may in fact know which of their competitors have the capability to compete for a given contract or even which firms have been invited to bid. Bajari, Houghton, and Tadelis (2014) maintained this assumption in their analysis of the highway procurement auctions by Caltrans.

In other contexts, the actual number of auction participants N is not public information to the parties involved in the auction. Nevertheless our identification strategy remains valid in such cases as long as the variation in N is exogenous. (Athey and Haile (2007) provides an example of how such exogenous variation arises in a model where bidders' entry decisions are related to costly signal acquisition.) With the actual distribution of N being common knowledge among both parties, the existence of symmetric monotone psPBE $\{\alpha^*, \beta^*\}$ follows from an argument similar to the proof in Appendix B. The only necessary change is that the ex ante return for the contractors and the ex ante payment by the procurer $\varphi(x; \alpha)$ now need to integrate out the number of bidders using the commonly known distribution of N .

As for identification of the model when N is not known to the bidders, the results in Lemma 1 and Lemma 2 are built on the monotonicity of the procurer's strategy and thus remain valid under any symmetric monotone psPBE in the model with bidders' uncertainty about participation. Also, the result about recovering the inverse bidding strategy in Lemma 3 holds when the contractors and the procurer are uncertain about the participation, provided the data report the prices quoted by all contractors in an auction. In this case, Athey and Haile (2007) showed the mark-up in the inverse bidding strategy needs to incorporate the uncertainty about participation. This is done by integrating out N using the actual distribution of the number of bidders, which is common knowledge among all parties and is directly recoverable from the data (see the equations (6.23) and

(6.24) in Athey and Haile (2007)).⁹ The identification of π , a and γ then follows the same argument as the proof of Theorem 1.

It remains an open question how to fully identify a richer model where decisions to participate in auctions are selective in the sense that the entrants' cost distribution differs from the unconditional cost distribution in the population (this would happen if entry is based on a preliminary signal that is correlated with private costs to be drawn in the bidding stage). We leave this topic for future research.

4 Incomplete Contracts in Bilateral Negotiation

In practice, incomplete contracts also exist in bilateral negotiation between a procurer and a single contractor. In this case, the sole contractor does not face any peer competition in the pre-contract stage, and the initial payment from the procurer is also determined via a direct negotiation between both parties. Such a situation could arise as a result of some special matching process whereby the contract is highly customized to match the procurer's special need, or simply as a result of the contractor's monopoly power on the market. We show that the identification results from Section 3 can be extended to such a bilateral-negotiation case.

In the first stage, the procurer observes a private signal $\tilde{X} \in \mathcal{X}$, and announces a specification $X \in \mathcal{X}$ strategically to maximize his total ex ante return from the contract. The contractor is notified of X and then negotiates with the procurer to set a first-stage payment $\psi(X)$ through a Nash bargaining process. The expected cost for delivering the contract at the initial specification X is $c(X)$. In the second stage, a feasible new design X^* is realized, leading to an incremental surplus $\phi(X, X^*)$ with an incremental cost $a(X, X^*)$. The new design will supplant the initial specification if it yields a positive net incremental surplus $s \equiv \phi - a > 0$. In this case, the contractor then re-negotiates with the procurer via Nash Bargaining to set an additional transfer Y from the procurer. The contractor pays incremental costs upfront; the surplus is accrued to the procurer. The bargaining power is fixed and is common knowledge throughout the two stages: $\gamma \in (0, 1)$ for the contractor and $1 - \gamma$ for the procurer. The other elements π , a , c and $F_{X^*|\tilde{X}}$ are also common knowledge for both parties.

With $\psi(x)$ already determined in the first stage, the additional transfer from the procurer to the contractor in the second stage is characterized by the solution to a Nash Bargaining process

$$y(x, x^*) = \arg \max_t [\psi(x) + t - c - a(x, x^*) - d_c]^\gamma [\pi(x^*) - \psi(x) - t - d_p]^{1-\gamma}$$

where $d_c \equiv \psi(x) - c$ and $d_p \equiv \pi(x) - \psi(x)$ are the disagreement values for the contractor and the procurer, respectively, where c is the ex post cost for delivering the contract under the initial specification x . The feasible new specification x^* is realized in the second stage

⁹Futhermore, Song (2006) showed that identification of inverse bidding strategy in first-price procurement auction with uncertain participation is also possible even if only the winning bid V and the second lowest bid are reported in the data.

and therefore is taken as an ingredient for the Nash Bargaining problem. It then follows that the transfer is given by

$$y(x, x^*) = \gamma\phi(x, x^*) + (1 - \gamma)a(x, x^*) \quad (10)$$

for all x, x^* when the new design is adopted (i.e. $s(x, x^*) > 0$); and the transfer is zero otherwise. This takes the same form as in the model with pre-contractual competition in auctions. We maintain that the negotiated price under an initial specification x in the first stage is the solution to a Nash Bargaining problem with the feasible set given by the ex ante surplus from the contract, and thus satisfies:

$$\gamma \left\{ \pi(x) - c(x) + E[s_+(x, x^*) | \tilde{X} = \tilde{x}] \right\} = \psi(x) - c(x) + \gamma E[s_+(x, x^*) | \tilde{X} = \tilde{x}].$$

The expression between the braces on the left-hand side is the ex ante net surplus of the contract; the right-hand side is the ex ante payoff for the contractor. This implies:

$$\psi(x) = \gamma\pi(x) + (1 - \gamma)c(x). \quad (11)$$

A procurer picks an initial specification to maximize its ex ante payoff:

$$\alpha(\tilde{x}) = \arg \max_{x \in \mathcal{X}} \{ \pi(x) + \mu(x, \tilde{x}) - \psi(x) \}, \quad (12)$$

where $\mu(x, \tilde{x}) \equiv (1 - \gamma)E[s_+(x, x^*) | \tilde{X} = \tilde{x}]$ as in the case with pre-contractual competition. Assuming interior solution for all \tilde{x} , the first-order condition for (12) is

$$\psi'(z) = \pi'(z) + \frac{\partial}{\partial x} \left[\int_{t \in \omega(x)} (1 - \gamma)s(x, t) dF_{X^* | \tilde{X} = \tilde{x}}(t) \right]_{x=z}$$

for all $z \in \mathcal{X}_o \equiv \{x : \exists \tilde{x} \in \mathcal{X} \text{ s.t. } x = \alpha^*(\tilde{x})\}$ where $\omega(x)$ is defined as in Section 3.

Using the same argument as in the proof of Theorem 1 in Section 3, we can recover γ , π and a for all $x \in \mathcal{X}_o$ and $x^* \in \omega(x)$ under Assumptions 1- 2. With $\psi(x)$ directly recoverable from the data, it then follows from (11) that the initial cost function $c(x)$ is also identified on \mathcal{X}_o .

5 Empirical Application

In this section, we apply our model above to analyze the auctions of incomplete highway procurement contracts by Caltrans. Bajari, Houghton, and Tadelis (2014) (hereafter BHT (2014)) used the same source of data to estimate the item-specific adaptation costs due to the contract incompleteness, which were assumed to be proportional to each item in the negotiated transfer from the procurer to the contractors. Our model differs fundamentally from BHT (2014) in several aspects.

First, we endogenize the procurer's announcement of the initial specification. BHT (2014) considered the procurer's initial specification (engineers' estimates) as exogenously

given, and focused on strategic interaction between contractors in first-price procurement auctions where bidders’ interim payoffs are adjusted due to contractual incompleteness. In contrast, we consider a strategic environment where the procurer chooses the initial specification optimally based on its information. The contractors quote prices to maximize their expected return in response to the procurer’s choice. Both sides take the contractual incompleteness into account while making strategic decisions.

Second, we maintain a flexible information structure. BHT (2014) assumed all contractors have perfect foresight about the final specification and the subsequent negotiated transfer.¹⁰ The equilibrium concept in their structural analysis is the *Bayesian Nash Equilibrium* (BNE), where contractors maximize interim payoffs based on the common knowledge of uncertainty in private costs. In comparison, we maintain that (a) the procurer, as the first-mover in a dynamic game, picks an optimal initial specification based on a noisy signal about the new design and (b) the contractors, once notified of this initial specification, update their beliefs about the new specification in a way that is consistent with the procurer’s strategy. The appropriate equilibrium concept in such a dynamic context and information structure is the *Perfect Bayesian Equilibrium* (PBE). As a result, the identification and estimation of the structural elements in our model require an original constructive argument.

Last but not the least, we explicitly model a Nash Bargaining process that determines the negotiated transfer between the procurer and the winning contractor if the new specification is adopted. Specifically, the transfer is negotiated so that each party eventually receives a share of the net incremental surplus that is proportional to its bargaining power. In addition to the distribution of contractors’ private costs, we estimate the procurer’s bargaining power as well as the net social surplus and cost functions which depend on the initial as well as the final specification.

5.1 Data

The data includes 5,862 bids submitted by contractors in 1,293 procurement auctions by Caltrans between 1995 and 2000. We index the contracts by $j = 1, 2, \dots, J$ and the contractors by $i = 1, 2, \dots, n_j$. Over 90% of the contracts in the data receive prices quote from 2 to 7 contractors in the procurement auctions. For each contract, the data report the initial specification (a.k.a. the engineers’ estimate) x , the actual specification adopted x^* (calculated using the Blue Book prices published in the Contract Cost Data Book (CCDB) and the actual quantities of items used for the project), and the post-contractual transfer y (negotiated after the auction) from the procurer to the winning contractor.¹¹ It records zero transfer for some minority cases where no new specification is adopted after the auction. The data also reports the bids submitted by all participating contractors in each auction as well as several cost-related characteristics for the contractors. These include

¹⁰See, for example, the second paragraph in Section II.A and equation (1) in Section II.B in BHT (2014).

¹¹Following BHT (2014), we calculate the total post-contractual transfer by adding up transfers under three categories in the data: “adjustment”, “deduction” and “extra work”.

the distance between the contractor’s location and the work site for the project (dis), a dummy variable that equals one if the contractor is a “fringe” competitor (fri), and the contractor’s utilization rate defined as the ratio of its backlog and capacity (uti).¹²

In addition, we classify the contracts into two types based on the nature of the work according to the project description. Type one contracts ($job = 1$) involve major construction or rebuilding (e.g., replace bridge or widen highway, etc.). Type zero contracts ($job = 0$) only require relatively minor or decorative tasks (e.g., realign curves, install traffic signals or other accessories, etc.). The data report little correlation between the engineers’ estimates and the job type we define (with correlation coefficient < 0.01), and thus it is unlikely that the latter is just a classification of the former.

[Insert Table 1 here]

Table 1 presents the summary statistics for our data. The average post-contractual transfer due to incompleteness is about \$172,000 per contract, which is about 1/10 of the actual size of the project. Among all the contracts, 94.04% (or 1216 cases) reported non-zero transfers negotiated after the auction while 5.96% (or 77 cases) report zero transfer, suggesting the contracts are complete. Type one (major construction) jobs account for 38.0% of all contracts with an average transfer of \$203,478; type zero (minor project) jobs have an average transfer of \$152,979. For each auction, we report the average characteristics such as fringe, utilization, and distance for all participating contractors (labeled as $afri$, $auti$ and $adis$ below). We model such average characteristics as a measure of the factors that potentially affect the procurer’s bargaining power against the contractors. For each contractor in an auction, we also record the minimum distance to the job site ($rdis$) and the minimum utilization rate among its competitors (rut). As in BHT (2014), we model a contractor’s bidding strategy as a function of these contract- and contractor-level variables which are common knowledge among the contractors.

5.2 Estimation Strategy

Our estimation strategy takes two steps. We first estimate the surplus (π), the incremental costs (a) and the bargaining power parameters using the variation in observed transfers and contract characteristics in the data. To do so, we introduce a two-stage extremum estimator. The idea is to construct an objective function that consists of two components that are minimized at the true parameter values. The first is based on the conditional likelihood of observed transfers, and the second on the first-order condition characterizing the procurer’s optimal choice of initial specification. Next, we estimate the cost distribution of contractors using prices quoted by contractors. To do so, we need to use the estimates from the first step to disentangle the original cost distribution based on initial specification and the contractors’ ex ante adjustment in their return due to the contractual incompleteness.

¹²A fringe competitor is defined as a firm that has won less than 1 percent of the value of contracts awarded in the data.

In Section 3, we present the identification argument for the simple case with homogeneous contracts and no structural observational errors. In comparison, we consider in the current section a general environment that allows rich heterogeneity among the contract and contractors. In order to make estimation feasible, we now adopt a parametric specification of model primitives.

5.2.1 Surplus, incremental costs and bargaining power

To simplify exposition, we drop the indices of contract j and contractor i in the notation. Intuitively, contractors' bargaining power should depend on features such as the nature of the task, the intensity of competition among interested bidders and contractor characteristics. Therefore we specify the bargaining power of the contractor in a negotiation as

$$\gamma(w; \lambda) = \exp(w'\lambda) / [1 + \exp(w'\lambda)]$$

with $w \equiv [afri, auti, adis, nbid, job]$ and $nbid$ being the number of contractors who submit bids. The vector w essentially consists of contract-level variables that are known to the procurer and the bidders at the time of their decisions respectively. Such a specification is also motivated by the fact that the bargaining power as perceived by the procurer and contractors in the auction does not depend on ex post individual-specific characteristics of the winner, whose identity is not known before the auction is concluded.

We also introduce some structural errors in the model explained in Section 3 to allow flexibility of the outcome conditional on the explanatory variables. Specifically, the negotiated transfer y is modeled as

$$y = d[\gamma\phi + (1 - \gamma)a + \varepsilon] \tag{13}$$

where

$$d = 1\{\phi - a + \eta > 0\} \tag{14}$$

is the dummy for whether the contract is incomplete with a positive transfer from the post-auction negotiation. For now we suppress all arguments in γ, ϕ, a above to simplify the notation. To understand the selection equation in (14), recall that 5.9% of the contracts are not incomplete (i.e. do not involve any post-auction transfer) and that our model states the incompleteness takes place only when the net incremental surplus is positive. The structural errors (ε, η) are bi-variate zero-mean normal with a standard deviation $(1, \sigma)$ and a correlation coefficient ρ . The structural errors account for measurement errors, or idiosyncratic factors that affect the transfer or contract incompleteness but are orthogonal to the initial and new specification (e.g., additional compensation for delay in payment due to bureaucracy).

Let θ denote the vector of parameters in the surplus π and incremental cost a . We specify the surplus π as

$$\pi(x, job; \theta) \equiv \theta_0 + \theta_1 x + \theta_2(x \times job) + \theta_3 x^2. \tag{15}$$

Thus the incremental surplus is $\phi(x, x^*, job; \theta) \equiv \pi(x^*, job; \theta) - \pi(x, job; \theta)$. This specification captures the fact that the social surplus depends on the job type and specification,

but not the characteristics of the contractor to whom the contract is awarded. The location parameter θ_0 is identifiable with a boundary condition on π . The incremental costs are given by:

$$a(x, x^*, job; \theta) \equiv \theta_4(x^* - x) + \theta_5(x^* - x) \times job + \theta_6(x^* - x)^2. \quad (16)$$

We do not include contractor characteristics in the specification of the incremental costs a . This is because the size of specification changes is mostly small relative to the initial specification. Therefore, a contractor's characteristics have a much more pronounced impact on the costs conditional on initial specification $F_{C|X}$ than on the incremental costs. Thus the specification in (16) offers a reasonable approximation of incremental costs that can be identified. The specifications in (15) and (16) satisfy the identifying condition in Assumption 2. We show in Appendix D that the parametric model is identified.

[Insert Table 2 here]

In order to see how post-auction transfers are related to the contract specification and characteristics and to motivate our specification above, we report the results from several linear regressions of the observed transfer on X , X^* and w in Table 2. In all specifications, the type of work (*job*) has a significant positive effect on the observed transfer, and the effect becomes less pronounced as the size of the new specification X^* increases. The number of bidders, the proportion of fringe bidders in an auction, and the average utilization rate of participants (*nbid*, *afri*, *auti*) all affect the transfer through their interaction with the initial and the new specification. The location of the work site also seems to have significant positive impact on the negotiated transfer. These patterns are consistent with our specification of the bargaining power as well as the surplus and incremental costs.

We estimate $(\theta, \lambda, \rho, \sigma)$ via two stages. In the first stage, we apply the probit algorithm to estimate a subvector of θ from (16). That is, the probit procedure using the explanatory variables $[x^* - x, (x^* - x) \times job, x^{*2} - x^2, (x^* - x)^2]$ returns estimates for $(\hat{\theta}_1 - \hat{\theta}_4, \hat{\theta}_2 - \hat{\theta}_5, \hat{\theta}_3, \hat{\theta}_6) \equiv \hat{\theta}_{-1,2}$. (See Appendix D for details.) To simplify notation, denote the remaining parameters by $\tau \equiv (\theta_1, \theta_2, \lambda, \rho, \sigma)$.

In the second stage, we construct an extremum estimator for τ , using an objective function that consists of two components \mathcal{L}_J and \mathcal{M}_J . The first component \mathcal{L}_J is the log-likelihood for the truncated distribution of negotiated transfer:

$$\begin{aligned} \mathcal{L}_J(\tau) &\equiv J^{-1} \sum_j \hat{l}_j(\tau) \text{ where} \\ \hat{l}_j(\tau) &= \log \Phi \left(\frac{\hat{I}_{s,j} + \frac{\rho}{\sigma}(y_j - \hat{I}_{o,j})}{\sqrt{1-\rho^2}} \right) + \log \phi_{norm} \left(\frac{y_j - \hat{I}_{o,j}}{\sigma} \right) - \log \sigma - \log \Phi(\hat{I}_{s,j}) \end{aligned}$$

where j indexes contracts, Φ and ϕ_{norm} are standard normal CDF and pdf, and

$$I_{o,j} \equiv \gamma_j \hat{\phi}_j + (1 - \gamma_j) \hat{a}_j \text{ and } I_{s,j} \equiv \hat{\phi}_j - \hat{a}_j.$$

with $\gamma_j \equiv \gamma(w_j; \lambda)$, $\hat{\phi}_j \equiv \phi_{norm}(x_j, x_j^*, w_j; \tau, \hat{\theta}_{-1,2})$ and $\hat{a}_j \equiv a(x_j, x_j^*, w_j; \tau, \hat{\theta}_{-1,2})$.

The second component in the objective function \mathcal{M}_J is based on a set of moments derived from the first-order condition in the procurer's maximization problem in equilibrium. The condition, summarized in (7), requires that the marginal impact of the initial specification on the expected price of auction to be equal to its marginal impact on the initial surplus π and the ex ante share of net incremental surplus μ . We construct \mathcal{M}_J as a sample analog for

$$E \left\{ [\varphi_1(x, w) - \pi_1(x, job; \tau) - \mu_1(x, \tilde{x}, job; \tau)]^2 \right\}, \quad (17)$$

where φ_1 , π_1 and μ_1 respectively are derivatives of $\varphi(x, w)$, $\pi(x, job)$ and $\mu(x, \tilde{x}, job)$, which is defined as $\mathbb{E}[(1 - \gamma(w; \lambda))s_+(x, X^*) \mid \tilde{X} = \tilde{x}]$ where the expectation is with respect to the new specification conditional on the signal \tilde{x} . The procurer's ex ante payment $\varphi(x, w)$ depends on the contract heterogeneity reported in the data, and is directly identifiable. By a standard argument for parametric identification in nonlinear least squares, we know that the true parameter τ uniquely minimizes (17) as long as for any $\tau' \neq \tau$, $\pi_1(x, job; \tau) + \mu_1(x, \tilde{x}, job; \tau) \neq \pi_1(x, job; \tau') + \mu_1(x, \tilde{x}, job; \tau')$ at least for some \tilde{x} , job and $x = \alpha^*(\tilde{x})$.

Specifically, we construct the second component in the objective function as

$$\mathcal{M}_J \equiv J^{-1} \sum_j \hat{\xi}_j(\tau)$$

where

$$\hat{\xi}_j(\tau) \equiv [\hat{\varphi}_1(x_j, w_j) - \pi_1(x_j, job_j; \tau) - \hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)]^2$$

and $\tilde{x}_j \equiv r$ if x_j is the 100* r -th percentile of the empirical distribution of the initial specification (recall that we normalize the marginal distribution of \tilde{x} to a standard uniform). To calculate $\hat{\xi}_j$, we first estimate the regression model

$$\varphi(x, w; \kappa) \equiv E[y \mid x, w] = \tilde{w}'\kappa \quad (18)$$

where $\tilde{w} \equiv [1, x, x^2, afri, auti, adis, nbid, job, x \times afri, x \times auti, x \times adis, x \times nbid, x \times job]$. This is the ex ante payment by the procurer and thus does not depend on the new specification x^* which is not known at the time of the auctions. Then we estimate φ_1 by plugging in the OLS estimates from the regression in (18), and denote the estimate by $\hat{\varphi}_1$.¹³ To calculate $\hat{\mu}_1$, we exploit the independence between w and (x^*, ε) given job as well as the fact that

$$\mu_1(x, \tilde{x}, job; \tau) = E[(1 - \gamma(w; \lambda) \mid job) \times E \left[\Phi(s(X^*, x, job)) \frac{\partial s(X^*, x, job)}{\partial x} \Big| \tilde{x}, job \right]] \quad (19)$$

¹³That is, $\hat{\varphi}_1(x, w)$ is calculated as

$$\hat{\kappa}_1 + 2\hat{\kappa}_2x + \hat{\kappa}_8afri + \hat{\kappa}_9auti + \hat{\kappa}_{10}adis + \hat{\kappa}_{11}nbid + \hat{\kappa}_{12}job$$

where $\hat{\kappa}_1, \hat{\kappa}_2, \hat{\kappa}_8, \hat{\kappa}_9, \hat{\kappa}_{10}, \hat{\kappa}_{11}, \hat{\kappa}_{12}$ are regression estimates for the coefficients in front of x, x^2 and the last five product terms of \tilde{w} in (18).

under our parametric specification. The second conditional expectation on the r.h.s. of (19) is taken with respect to the new specification X^* conditional on the procurer's noisy signal \tilde{X} . For each τ , we calculate $\hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)$ by plugging in the first-stage estimates $\hat{\theta}_{-1,2}$ in the expression for s and $\partial s/\partial x$, taking the conditional sample average to estimate the first expectation, and evaluating the second expectation using simulated observations of x^* that are drawn from the estimated distribution of X^* given $\tilde{X} = \tilde{x}_j$.¹⁴ Finally, our estimator $\hat{\tau}$ in the second stage is:

$$\hat{\tau} = \max_{\tau} \{ \mathcal{L}_J(\tau) - \mathcal{M}_J(\tau) \}. \quad (20)$$

The true parameters $\theta_{1,2}$ and τ are identified in this two-stage approach of estimation. First off, $\theta_{-1,2}$ is identified in the probit procedure in the first-stage by standard arguments. Next, as we show in Appendix D, under the parametrization we use, the true τ is a unique maximizer of the population counterpart for \mathcal{L}_J (that is, the uniform probability limit of \mathcal{L}_J over the parameter space given consistent first-stage estimator). Also, the equilibrium first-order condition also implies that τ is the unique minimizer of the uniform probability limit of \mathcal{M}_J under the mild parametric condition mentioned above. Hence τ is identified as the unique maximizer of the uniform probability limit of $\mathcal{L}_J - \mathcal{M}_J$.

Because the estimand in (20) is a smooth function of the sample analogs, the two-stage extremum estimator is asymptotically normal under usual regularity conditions. (If we were to following the asymptotic plug-in approach for inference, then the asymptotic variance of $\hat{\tau}$ needs to include a term that accounts for the first-stage estimation error from the probit procedure.) In practice, we use the bootstrap procedure to estimate the standard error of our estimator. To implement the maximization routine in the second stage, we pick an initial value for τ by estimating a two-stage MLE that maximizes \mathcal{L}_J alone (which produces a consistent estimator for τ).

5.2.2 Distribution of contractors' private costs

Recall that $\bar{\delta}(X; \alpha^*) = \mathbb{E}[\gamma s_+(X, X^*) | X = x; \alpha^*]$ and $y - a = \gamma s$ whenever $s > 0$. Let $q(x, job)$ denote the probability that the contract is incomplete with observed transfer ($d = 1$) conditional on the information available to the contractors while bidding in the auction (that is, engineers' estimates and the job type). It then follows from Lemma 3 that for a contractor i in auction j ,

$$c_{j,i} = p_{j,i} - \frac{1}{n_j - 1} \frac{1 - F_{P|Z}(p_{j,i} | z_{j,i})}{f_{P|Z}(p_{j,i} | z_{j,i})} + E[Y - a(x_j, X^*, job_j) | x_j, z_{j,i}, d_j = 1] q(x_j, job_j), \quad (21)$$

¹⁴Recall that we normalize the marginal distribution of \tilde{X} to a standard uniform, which is without loss of generality due to Lemma 2. Also monotone strategy in equilibrium implies that $F_{X^*|\tilde{X}=\alpha^{*-1}(x)}$ equals $F_{X^*|X=x}$. The empirical density of X^* given X in auctions with an observed transfer is approximated well by a normal distribution. Thus we specify the relationship between X^* and X as $(X^* - X)/X \sim N(\nu, \tilde{\sigma}^2)$ for incomplete auctions with observed transfers, and then use a maximum likelihood formula to estimate $\nu, \tilde{\sigma}$. The second expectation is then calculated for each τ using simulated draws of x^* based on these estimates.

where $z_{j,i} \equiv [fri_{j,i}, uti_{j,i}, dis_{j,i}, job_j, ruti_{j,i}, rdis_{j,i}]$ and $ruti_{j,i}, rdis_{j,i}$ are contractor-level variables recording the minimum distance and utilization rate among the competitors that a contractor i faces, respectively. The expectation of $y - a$ conditions only on x_j and $z_{j,i}$ because the contractor's ex ante share of the net incremental surplus depends on contractor characteristics revealed prior to the auction.

For simplicity, we suppress the auction index j in $(c_{j,i}, p_{j,i}, z_{j,i})$ and (n_j, x_j) and only keep the contractor index i when there is no ambiguity. We adopt a logit specification for the probability for contractual incompleteness q :

$$q(x, job; \vartheta) = \exp(v'\vartheta)/[1 + \exp(v'\vartheta)] \quad (22)$$

where $v \equiv [1, x, job, x \times job, x^2]$. Denote the maximum likelihood estimator by $\hat{\vartheta}$.¹⁵

Following the specification in BHT (2014), we estimate bidders' bidding strategies (normalized by the initial specification or engineers' estimates) via the following regression:

$$\frac{p_i}{x} = f(z_i; \omega) + u + e_i, \quad (23)$$

where u is a contract-specific fixed-effect that accounts for unobserved contract characteristics, f is specified to be linear in the parameter ω and the idiosyncratic error e_i is independent from z and u . As BHT (2014) mentioned, the specification allows for heterogeneity in the structural error via the contract size in x . Recall that, in our model, neither the contractors or the procurer know or have perfect foresight of the negotiated transfer (unknown and stochastic ex ante). Thus, the bidding strategy can not depend on any ex post observation of these transfers.¹⁶

For each contract indexed by j , we estimate the coefficient $\hat{\omega}$ and the fixed effect \hat{u} using a standard fixed-effect approach conditional on the number of bidders. These estimates are then used to calculate the residual \hat{e}_i . The distribution of e_i depends on the number of contractors bidding for a contract, but is orthogonal to z and u . By construction, for each number of bidders, $F_{P|Z}$ equals:

$$F_{P|Z}(p_i | z_i) = \Pr(f(z_i; \omega) + u + e_i \leq p_i/x) = F_e(p_i/x - f(z_i; \omega) - u),$$

where $F_e(\cdot)$ is the CDF of the e_i (we suppress the dependence of $F_{P|Z}$ and F_e on n to simplify notation). The corresponding conditional density of bids is

$$f_{P|Z}(p_i | z_i) \equiv \frac{\partial}{\partial p} F_e(p_i/x - f(z_i; \omega) - u) = f_e(p_i/x - f(z_i; \omega) - u)/x.$$

We estimate $F_{P|Z}$ and $f_{P|Z}$ by plugging in the estimates \hat{u} , $\hat{\omega}$, and the empirical distribution and kernel density of \hat{e}_i to the right-hand sides above. Let $\hat{F}_{P|Z}$ and $\hat{f}_{P|Z}$ denote the estimates of $F_{P|Z}$ and $f_{P|Z}$, respectively.

¹⁵Alternatively, we can also estimate $q(x, job)$ nonparametrically via kernel regression.

¹⁶Note that the inverse bidding strategy for contractors is recovered nonparametrically due to (21). Hence in principle the reduced-form linear specification in (23) is not necessary for estimating bidding strategies in large samples. We implement this reduced-form regression to estimate bidding strategies mostly due to data constraints.

Using the specification (21), the cost for bidder i in contract j is estimated as:

$$\hat{c}_{j,i} = p_{j,i} - \frac{1}{n_j - 1} \frac{1 - \hat{F}_{P|Z}(p_{j,i} | z_{j,i})}{\hat{f}_{P|Z}(p_{j,i} | z_{j,i})} + \hat{\delta}_{j,i} \hat{q}_j,$$

where $\hat{q}_j \equiv q(x_j, job_j; \hat{\vartheta})$ and

$$\hat{\delta}_{j,i} \equiv \hat{E}[Y_j | x_j, z_{j,i}, d_j = 1] - S^{-1} \sum_{s=1}^S a(x_j, x_{j,s}^*, job_j; \hat{\theta}).$$

The first term in $\hat{\delta}_{j,i}$ is estimated using nonparametric kernel regression; the second term in $\hat{\delta}_{j,i}$ is a simulation-based estimate for the expectation of the incremental costs a conditional on contractual incompleteness, initial specification and contract characteristics. Specifically, $x_{j,s}^*$ are independent draws from the estimated density of x^* given x_j and $d_j = 1$ (which is estimated in the previous step).

We use a bootstrap procedure to calculate the standard error of parameters in the surplus and incremental cost functions and the average marginal effect of contract characteristics on the contractor's bargaining power.

5.3 Results

Table 3 reports the regression estimates for the expected payment in (18) under three nested specifications. In all specifications, the initial specification (engineers' estimate) has a significant positive marginal effect on the expected auction price (the lowest price quoted by contractors in an auction). This suggests the engineers' estimate on average is a reasonable predictive factor for the auction price. That the marginal effect is statistically greater than one is due to the mark-ups added by the contractors in their quotes. There is also evidence in the third specification that this effect diminishes for contracts with larger initial specifications. On average, a higher utilization rate among the contractors tends to lower the auction price. This may indicate that there is an economy of scale in the construction costs for contractors working on multiple contracts simultaneously. The job type, the number of bidders and the other average contractor characteristics all have a statistically significant impact on the auction prices, especially via their positive impacts on the marginal effect of engineers' estimates.

[Insert Table 3 here]

Table 4 reports the estimates for the average marginal effects of contract and contractor characteristics on the bargaining power. The number of bidders (*nbid*), the average utilization rate among bidders (*auti*), the proportion of fringe firms (*afr*) and the type of job (*job*) all have significant bearings on the bargaining power. For example, consider the third specification. On average, the bargaining power for a winning contractor is 10.2% higher if the winner had to defeat an additional competing contractor in the auction. This indicates that the intensity of the pre-contractual competition in the auction

is positively related to the winner’s subsequent bargaining power against the procurer in post-contractual negotiation. This may be because the strength of a contractor (e.g., cost or logistic efficiency), which enables it to defeat other competitors in an auction, also gives it more leverage in the negotiation with the procurer.

The average utilization rate and the proportion of fringe contractors are both important in explaining the bargaining power. Our estimates show that if the average utilization rate in an auction is increased by 10%, then the bargaining power of the contractor increases by 1.86%. This conforms with the intuition that a contractor has more leverage over the procurer when its competitors are on average more occupied or committed to other projects. Our estimate also suggests that, in an auction where all contractors are major competitors (non-fringe firms) in the industry ($afri = 0$), the bargaining power for the winner of the auction in post-contractual negotiation is 29.6% higher than in an auction that only receives quotes from fringe contractors ($afri = 1$). This result reveals that a contractor who defeats major competitors has a greater bargaining power than one who wins the competition with fringe competitors.

[Insert Table 4 here]

There are two important observations regarding the relation between the job type and the post-contractual negotiation. First, the contractor has a greater bargaining power than the procurer in subsequent negotiations. Our estimates show the contractor’s mean bargaining power is significantly greater than 50% regardless of job types. One explanation for this is that the procurer is more subject to the possible high costs to extend project deadlines and to solicit quotes from other contractors if the negotiation breaks down. Second, other things being equal, the procurer’s bargaining power is 11.7% greater in contracts that require substantial work ($job = 1$) than in those involving less onerous tasks. These patterns persist in all three specifications presented in Table 4. This could be ascribed to the possibility that procurers (municipal government departments) are more likely to play hardball in post-auction negotiations if the contract involves a major project and more tax-payer’s money and larger social surplus at stake. Another interpretation is that the winners in major projects are top firms in the industry who participate in and win the contracts more often than the smaller firms. Thus, they have the incentive to maintain a good and long-term relationship with the procurer by making concessions to the procurer in negotiation.

[Insert Table 5 here]

Table 5 reports the estimates of the social surplus and the incremental cost functions. The estimates suggest that the social surplus function is convex in the size of the specification. This is consistent with the fact that highway construction / renovation is essentially public service and a larger project generally benefits a greater population, thus resulting in increasing returns to scale in the surplus it induces. Our estimates for the incremental costs conform with the following pattern in the data: Larger transfers are

often reported in projects that involve more substantial post-contractual modification in the specification.

We test the null hypothesis that the incremental surplus s is linear in the post-contractual change $(X^* - X)$ (that is, the coefficients for x^2 in π and for $(x^* - x)^2$ in a). The result rejects the null hypothesis at the 5% significance level with a p -value of 2.3% in the third (the most general) specification. In addition, we use a Wald statistic to test the joint significance of x and x^* in s . This yields a p -value of 0.2%. These results validate our specification that the incremental cost and surplus depend on the initial and the final specifications as well as the job type.

To quantify the job type's effect on the post-contractual transfer, we plug in the structural estimates above to calculate the average difference between the truncated mean of the transfer (expectation of transfer given that the contract is incomplete with new specification adopted) conditional on $job = 1$ (major projects) and conditional on $job = 0$ respectively. This provides us with estimates for the ex ante marginal effect of the job type on the truncated means, with other contract characteristics in w , the initial specification x and the final specification x^* integrated out in the expectation. For the most general specification, the estimated difference is 0.052 with a standard error (calculated using a bootstrap procedure) of 0.020. The tests under the other two nested specifications also report statistical significance of job at the 5% level. Our estimates from Table 4 and 5 also indicate that the type of job affects the negotiated transfer mainly through its effect on the bargaining power.

[Insert Table 6 here]

Table 6 reports estimates in the logit model for contract incompleteness (22), which are used for estimating the cost distribution of contractors. By definition $q(x, job; \vartheta)$ is the probability of contractual incompleteness in equilibrium in the data. Our estimates in Table 6 illustrate that the engineers' estimate is a main determinant for the probability of contractual incompleteness whereas the job types do not play an important role in this regard. This empirical evidence is in the line with the observation in Tirole (2009) that the hold-up problem occurs under the incomplete contracts where the probability of incompleteness is determined by the endogenous choice of the procurer.

[Insert Table 7 here]

Table 7 reports the estimates of the markup as a percentage of contractors' costs. That is, it reports the ratio of markups defined as $(p_{j,i} + \hat{\delta}_{j,i}\hat{q}_j - \hat{c}_{j,i})/\hat{c}_{j,i}$. The estimates for contractors' ex ante payoff consists of the initial payment $p_{j,i}$ from the procurer following the auction and a negotiated share of the expected net incremental surplus $(\hat{\delta}_{j,i}\hat{q}_j)$. The markup ratio is estimated by $\hat{\omega}_{j,i}/\hat{c}_{j,i}$ where $\hat{\omega}_{j,i} \equiv [1 - \hat{F}_{P|Z}(p_{j,i}|z_{j,i})]/[(n_j - 1)\hat{f}_{P|Z}(p_{j,i}|z_{j,i})]$ for contractor i in auction j . On average, a contractor obtains 9% markup in a highway construction project. The table demonstrates that contractors' markups are affected by both their characteristics and the contract-level heterogeneity.

We observe that the markups in contracts involving substantial work ($job = 1$) are lower than those involving relatively minor ones. This may be due to our finding that the contractors' bargaining power on major projects are smaller, and hence they obtain a smaller share of the incremental surplus from negotiation. To investigate the impact of competition on contractors' markups, we classify the number of bidders in four groups: 2, {3, 4, 5}, {6, 7, 8}, and $\{> 8\}$. Within each group, the markups for different number of bidders are very close. Our estimates reveal a clear pattern on the decreasing markups in the number of competitors. In particular, when there are only two competitors, the winning contractor benefits from substantially higher markups than in auctions with more competitors. This also offers some evidence supporting the assumption that contractors are aware of the number of competitors in an auction. On average, the auctions in the data have five bidders, which suggests a moderate level of competition. For auctions with five contractors, the average markup is 9.94%.

The winning contractors appear to enjoy much higher markups than those who lose in auctions. An explanation for this pattern is that the cost for a winner is usually much lower than competitors. Hence the winner could quote a competitive price even with a high markup. Besides, our estimates show that the markup of a non-fringe firm is slightly higher than that from a fringe firm at the 25-th, median and 75-th quantiles.

Another factor that affects contractors' markups is the utilization rate. In our data, about 50% of the contractors have a zero utilization rate (that is, they are not committed to other ongoing projects simultaneously). These contractors' markups on average are higher than those with positive utilization rates. Nonetheless the impact does not seem to be substantial, especially because the estimates do not suggest any significant relation between the markups and the level of positive utilization rates. The estimated markups for contractors with different distances from the job sites also do not appear to vary substantially.

Notice that our estimates of markups rely on the contractors' negotiated share of the net incremental surplus. Without taking into account the contractors' additional revenue due to the incompleteness of a contract, we would estimate the markups as $\hat{\omega}_{j,i}/(p_{j,i} - \hat{\omega}_{j,i})$, which is strictly greater than $\hat{\omega}_{j,i}/\hat{c}_{j,i}$ because the estimates for ex ante net incremental surplus are positive. This means if we were to estimate the markups in these procurement auctions without accounting for the strategic incentives due to contractual incompleteness, we would end up over-estimating these markups substantially. To further illustrate this point, we estimate the markups by assuming away the negotiated share of incremental surplus and the results are presented in Table 8. The estimates demonstrate that the average markups can be over-estimated to be two times as high as the correct estimates (17.60% v.s. 9.03%). The effects of contractors' characteristics and the project-level heterogeneity on markups are all exaggerated.

6 Concluding Remarks

This paper studies incomplete contracts in procurement auctions. We set up a model that endogenizes a procurer's choice of initial specification. The model also rationalizes

the post-auction transfers as Nash Bargaining outcomes from the negotiation between the procurer and the auction winner, and maintains a flexible assumption on the information structure. We show that the model components (the incremental cost and surplus functions, the bargaining power for contractors and the cost distribution for contractors) are non-parametrically identified from the contract prices and negotiated transfers. The identification results are also extended to bilateral incomplete contracts without pre-contractual competition. We apply the model to analyze the data on the Caltrans auctions of highway procurement contracts. Our estimates provide new evidence as to how agents respond strategically to the presence of contractual incompleteness, and what determines the size of the hold-up in highway procurement projects.

One of the goals of this paper is to provide the literature with a new methodology to empirically study the vast field data of incomplete contracts. It will be interesting to explore whether the methods used here can be useful for evaluating the impact of incompleteness on the efficiency of mechanisms, e.g., whether revenue equivalence still holds once incompleteness occurs. The exposition of our method is in the context of procurement auctions but it might be extended to other formats of pre-contractual competition, e.g., English auctions. We are considering these possibilities in future work.

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Appendix

A The Nash Bargaining Solution

This part of the appendix characterizes the Nash Bargaining solution in the post-auction negotiation. Let v denote the auction price and c denote the auction winner's ex post cost for delivering the contract under the initial specification x . The disagreement value (a.k.a. reservation value) for the auction winner (the contractor quoting the lowest price) is

$$d_c \equiv v - c$$

while the disagreement value for the procurer is

$$d_p \equiv \pi - v$$

with π being the social surplus under the initial specification x . With a realized new feasible design x^* , the ex post total net social surplus to be shared among the procurer and the auction winner is

$$u_0 \equiv \pi^* - a - c$$

where π^* is the social surplus under x^* and a is the incremental costs for delivering the contract under x^* (in addition to the costs c for delivering the contract under x). With γ denoting the bargaining power of the contractor, the Nash Bargaining solution is characterized by

$$\max (u_c - d_c)^\gamma (u_p - d_p)^{1-\gamma}$$

subject to $u_p + u_c \leq u_0$. By a standard argument,

$$u_c \equiv \gamma(u_0 - d_p) + (1 - \gamma)d_c = \gamma s + v - c$$

and

$$u_p \equiv u_0 - u_c = (1 - \gamma)s + \pi - v$$

with $s \equiv \pi^* - \pi - a$ defined as the net incremental surplus.

As stated in the text, we maintain that the contractor covers the incremental costs a as they arise in construction while the incremental surplus $\pi^* - \pi$ is eventually accrued to the procurer. Then the negotiated transfer y needs to satisfy:

$$d_c + y - a = u_c \text{ and } d_p + \pi^* - \pi - y = u_p$$

which is equivalent to

$$y - a = \gamma s \text{ and } \pi^* - \pi - y = (1 - \gamma)s.$$

This proves that the post-auction negotiated transfer characterized in Section 2.

B Existence and Uniqueness of PsPBE

Let $G(\cdot | x; \alpha)$ denote the C.D.F. of adjusted costs $\tilde{C}_i \equiv C_i - \bar{\delta}(x; \alpha)$ conditional on $X = x$ and a procurer strategy α . That is,

$$\begin{aligned} G(t | x; \alpha) &\equiv \Pr(C_i - \bar{\delta}(x; \alpha) \leq t | X = x) \\ &= \Pr(C_i \leq t + \bar{\delta}(x; \alpha) | X = x). \end{aligned}$$

For any given α , the standard arguments such as that in Krishna (2009) show that a symmetric monotone pure-strategy Bayesian Nash equilibrium exists in such procurement auctions where contractors' private costs are drawn from $G(\cdot | X; \alpha)$. Let $\tilde{G} \equiv 1 - G$ denote a survival function, so that $\tilde{G}(c | x; \alpha)^{N-1} \equiv \Pr\{\min_{j \neq i} C_j \geq c | x; \alpha\}$ and $1 - \tilde{G}(c | x; \alpha)^{N-1} \equiv \Pr\{\min_{j \neq i} C_j \leq c | x; \alpha\}$. For all $i = 1, 2, \dots, N$ and any $c \in \mathcal{C}$ and $x \in \mathcal{X}$,

$$\beta_i(c, x; \alpha) = \frac{\int_c^{\bar{c}} s d[1 - \tilde{G}(s | x; \alpha)^{N-1}]}{\tilde{G}(c | x; \alpha)^{N-1}},$$

where \bar{c} denotes the upper bound of \mathcal{C} and it can also be infinity.

We prove the existence of pure-strategy PBE through two steps. First, we express the solution to a procurer's optimization problem as the solution to an ordinary differential equation (ODE). Then, we show a solution to this ODE exists.

First, recall that a procurer's optimization problem is

$$\alpha(\tilde{x}) = \arg \max_{x \in \mathcal{X}} \{\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \alpha)\},$$

where $\varphi(x; \alpha)$ is a shorthand of the procurer's expected payment $\psi(x; \beta_\alpha)$. By definition,

$$\begin{aligned} \varphi(x; \alpha) &= N \int_0^{\bar{c}} \beta_i(c, x; \alpha) \Pr\left(\min_{j \neq i} C_j \geq c\right) dG(c | x; \alpha) \\ &= N \int_0^{\bar{c}} s d\left[1 - \tilde{G}(s | x; \alpha)^{N-1}\right] - N \int_0^{\bar{c}} s \tilde{G}(s | x; \alpha) d\left[1 - \tilde{G}(s | x; \alpha)^{N-1}\right] \end{aligned}$$

where the first term is $N\mathbb{E}\left[\min_{j \neq i} \tilde{C}_j | X = x\right]$; and the second term is

$$\begin{aligned} &N \int_0^{\bar{c}} s \tilde{G}(s | x; \alpha) (N-1) \tilde{G}(s | x; \alpha)^{N-2} g(s | x; \alpha) ds \\ &= (N-1) \int_0^{\bar{c}} s d\left[1 - \tilde{G}^N(s | x; \alpha)\right] = (N-1) \mathbb{E}\left[\min_i \tilde{C}_i | X = x\right]. \end{aligned}$$

Hence we can write

$$\varphi(x; \alpha) = N\mathbb{E}\left[\min_{j \neq i} \tilde{C}_j | X = x\right] - (N-1) \mathbb{E}\left[\min_i \tilde{C}_i | X = x\right].$$

By construction, $\mathbb{E}[\min_i \tilde{C}_i \mid X = x] = \mathbb{E}[\min_i C_i \mid X = x] - \bar{\delta}(x; \alpha)$ and similarly we have that $\mathbb{E}[\min_{j \neq i} \tilde{C}_j \mid X = x] = \mathbb{E}[\min_{j \neq i} C_j \mid X = x] - \bar{\delta}(x; \alpha)$. Therefore

$$\varphi(x; \alpha) = \sigma(x) - \bar{\delta}(x; \alpha)$$

with $\sigma(x) \equiv N\mathbb{E}[C^{(1:N-1)} \mid X = x] - (N-1)\mathbb{E}[C^{(1:N)} \mid X = x]$; and $C^{(m:n)}$ being the m -th smallest out of n independent draws from $F_{C \mid X}$. It then follows that a symmetric monotone psPBE of our model exists if the fixed-point mapping

$$\alpha(\tilde{x}) = \arg \max_{x \in \mathcal{X}} [\pi(x) + \mu(x, \tilde{x}) + \bar{\delta}(x; \alpha) - \sigma(x)] \quad (\text{B.1})$$

admits a monotone solution α^* .

Let $\delta(x, \tilde{x}) \equiv \mathbb{E}[\gamma_{s_+}(x, X^*) \mid \tilde{X} = \tilde{x}]$, where the expectation is taken with respect to X^* according to $F_{X^* \mid \tilde{X}}$ with the initial specification X fixed at x . By construction $\bar{\delta}(x; \alpha) = \delta(x, \alpha^{-1}(x))$ for any increasing procurer strategy α . The functions π, μ, σ, δ are all determined by model primitives and do not involve any endogenous equilibrium object. Suppose π, μ, σ, δ are all differentiable; and interior solutions to (B.1) exist for all $\tilde{x} \in \mathcal{X}$. Then for any α that solves (B.1):

$$\begin{aligned} & \left. \frac{\partial[\pi(X) + \mu(X, \tilde{X}) + \bar{\delta}(X; \alpha) - \sigma(X)]}{\partial X} \right|_{X=\alpha(\tilde{x}), \tilde{X}=\tilde{x}} \\ &= \pi'(\alpha(\tilde{x})) + \mu_1(\alpha(\tilde{x}), \tilde{x}) - \sigma'(\alpha(\tilde{x})) + \delta_1(\alpha(\tilde{x}), \tilde{x}) + \delta_2(\alpha(\tilde{x}), \tilde{x}) \frac{1}{\alpha'(\tilde{x})} = 0, \forall \tilde{x} \in \mathcal{X}, \end{aligned}$$

where μ_l and δ_l denote partial derivatives of μ and δ w.r.t. their l -th arguments respectively. In what follows, we focus on showing that the first-order ordinary differential equation below admits a monotone solution:

$$\alpha'(\tilde{x}) = \frac{\delta_2(\alpha(\tilde{x}), \tilde{x})}{\sigma'(\alpha(\tilde{x})) - [\pi'(\alpha(\tilde{x})) + \mu_1(\alpha(\tilde{x}), \tilde{x}) + \delta_1(\alpha(\tilde{x}), \tilde{x})]} \equiv \lambda(\tilde{x}, \alpha(\tilde{x})). \quad (\text{B.2})$$

To show the existence of a unique symmetric monotone psPBE, we need to show the differential equation (B.2) has a unique monotone solution. We present the regularity conditions below and then a lemma for existence. Let $\mathcal{D} \equiv \mathcal{X} \times \mathcal{X}$ be a rectangle on which $\lambda(\tilde{x}, x)$ is defined.

Assumption B.1. (Global Lipschitz Condition) $\exists M > 0$ such that for all (\tilde{x}, x) and (\tilde{x}, x') in \mathcal{D} , $|\lambda(\tilde{x}, x) - \lambda(\tilde{x}, x')| \leq M|x - x'|$.

Assumption B.2. (Initial Point) There exists (\tilde{x}_0, x_0) in the interior of \mathcal{D} such that $\alpha(\tilde{x}_0) = x_0$.

Assumption B.3. (Monotonicity) For any pair $x_1 > x_2$ in \mathcal{X} , the conditional distribution of cost $F_{C \mid X=x_1}$ first order stochastically dominates $F_{C \mid X=x_2}$. The function $\delta(x, \tilde{x})$ is increasing in \tilde{x} ; and $\pi(x) - \mathbb{E}[s_+(x, X^*) \mid \tilde{X} = \tilde{x}]$ is non-increasing in x for each \tilde{x} .

Lemma B.1 *Under Assumptions B.1-B.3 there exists a unique and monotone solution α^* to (B.2).*

Proof of lemma B.1. The existence and uniqueness of a solution on \mathcal{X} to the differential equation (B.2) is a variant of Picard-Lindelöf Theorem, which states a result of existence and uniqueness of local solutions for a given initial condition if $\lambda(\tilde{x}, x)$ is continuous in \tilde{x} and locally Lipschitz continuous in x . The global Lipschitz continuity of $\lambda(\tilde{x}, x)$ in Assumption B.1 strengthens Picard-Lindelöf theorem and guarantees a unique solution globally defined on \mathcal{X} for a given initial condition in Assumption B.2. The detailed discussion on the results above can be found in, e.g., Theorem 10 of Chapter 1 in Adkins and Davidson (2012).

To show the monotonicity of the solution α^* , it suffices to prove that $\lambda(x, \tilde{x}) > 0$ on \mathcal{D} , which can be described using model primitives as:

$$\begin{aligned} \lambda(x, \tilde{x}) &= \frac{\partial \mathbb{E} \left[\gamma s_+(x, X^*) | \tilde{X} = \tilde{x} \right] / \partial \tilde{x}}{\sigma'(x) - \pi'(x) - \partial \mathbb{E} \left[(1 - \gamma) s_+(x, X^*) | \tilde{X} = \tilde{x} \right] / \partial x - \partial \mathbb{E} \left[\gamma s_+(x, X^*) | \tilde{X} = \tilde{x} \right] / \partial x} \\ &= \frac{\partial \mathbb{E} \left[\gamma s_+(X, x^*) | \tilde{X} = \tilde{x} \right] / \partial \tilde{x}}{\sigma'(x) - \pi'(x) - \partial \mathbb{E} \left[s_+(x, X^*) | \tilde{X} = \tilde{x} \right] / \partial x}. \end{aligned}$$

We first investigate the term $\sigma'(x)$. Recall

$$\sigma(x) \equiv N \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^{N-1} ds - (N-1) \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^N ds.$$

Taking the derivative of $\sigma(x)$ with respect to x , we obtain

$$\sigma'(x) = -N(N-1) \int_0^{\bar{c}} \frac{\partial F_{C|X}(s|x)}{\partial x} [1 - F_{C|X}(s)]^{N-2} F_{C|X}(s) ds > 0,$$

where the last line is due to the assumption of stochastic dominance on $F_{C|X}$. The properties in Assumption B.3 then lead to the conclusion that $\alpha'(\cdot) > 0$. \square

The proof of existence in this section is derived under the simplification that the procurer's price ceiling is not binding in the auction. The result can be extended to the case with a binding price ceiling. Let the price ceiling be $r \in (0, \bar{c})$, we present the expression of $\beta(c, x; \alpha)$ and $\varphi(x; \alpha)$ while omitting the details for the proof of existence because it is qualitatively the same as the case with a non-binding price ceiling.

$$\beta(c, x; \alpha) = \begin{cases} c - \delta(x; \alpha) + \frac{\int_c^r [1 - F_{C|X}(s)]^{N-1} ds}{[1 - F_{C|X}(c)]^{N-1}}, & c \leq r \\ 0, & c > r. \end{cases}$$

$$\begin{aligned} \varphi(x; \alpha) &= -r[1 - F_{C|X=x}(r)]^N + \bar{\delta}(x; \alpha) [(1 - F_{C|X=x}(r))^N - 1] \\ &+ N\mathbb{E}[C^{(1:N-1)}|X=x] - (N-1)\mathbb{E}[C^{(1:N)}|X=x]. \end{aligned}$$

C Identification Proofs

C.1 Proof of Observational Equivalence

In this section we provide a detailed argument for the observational equivalence of the two models θ and θ_0 mentioned in Section 3. The first step is to relate the equilibrium strategy in θ to that in θ_0 . Let $\alpha(\cdot; \theta) : \mathcal{X} \rightarrow \mathcal{X}$ denote the procurer strategy under θ in a symmetric monotone psPBE; and likewise define $\alpha(\cdot; \theta_0)$. Let $\alpha, \dot{\alpha}$ be shorthands for $\alpha(\tilde{x})$ and $\alpha'(\tilde{x})$ respectively. Recall that in equilibrium the procurer's strategy $\alpha(\cdot; \theta)$ solves the ordinary differential equation (ODE):

$$\dot{\alpha} = \frac{\delta_2(\alpha, \tilde{x}; \theta)}{\sigma'(\alpha; \theta) - \pi'(\alpha) - \mu_1(\alpha, \tilde{x}; \theta) - \delta_1(\alpha, \tilde{x}; \theta)} \equiv \lambda(\alpha, \tilde{x}) \quad (\text{C.1})$$

for some initial condition $\alpha(\tilde{x}_0; \theta) = x_0$, where μ, δ, σ are defined above. We highlight the dependence of μ, δ, σ on θ throughout this subsection. While μ and δ depend on $\{\gamma, \pi, a, F_{X^*|\tilde{X}}\}$, $\sigma : \mathcal{X} \rightarrow \mathbb{R}$ only depends on $F_{C|X}$ in θ . Likewise, $\alpha(\cdot; \theta_0)$ solves an ODE similar to (C.1), only with θ therein replaced by θ_0 :

$$\dot{\alpha}_0 = \frac{\delta_2(\alpha_0, t; \theta_0)}{\sigma'(\alpha_0; \theta_0) - \pi'_0(\alpha_0) - \mu_1(\alpha_0, t; \theta_0) - \delta_1(\alpha_0, t; \theta_0)} \quad (\text{C.2})$$

and a corresponding initial condition. By construction, $F_{C|X} = H_{C|X}$ and $F_{X^*|\tilde{X}=\tilde{x}} = H_{X^*|\tilde{X}=h(\tilde{x})}$ for all $\tilde{x} \in \mathcal{X}$. Hence $\sigma'(\cdot; \theta) = \sigma'(\cdot; \theta_0)$ and

$$\mu(\cdot, h^{-1}(t); \theta) = \mu(\cdot, t; \theta_0) \text{ and } \delta(\cdot, h^{-1}(t); \theta) = \delta(\cdot, t; \theta_0) \quad (\text{C.3})$$

for any $t \in \mathcal{T}$. Substitute (C.3) into the ODE for θ_0 in (C.2) gives:

$$\dot{\alpha}_0 = \frac{\delta_2(\alpha_0, h^{-1}(t); \theta)}{\sigma'(\alpha_0; \theta) - \pi'(\alpha_0) - \mu_1(\alpha_0, h^{-1}(t); \theta) - \delta_1(\alpha_0, h^{-1}(t); \theta)} \left(\frac{dh^{-1}(t)}{dt} \right) \quad (\text{C.4})$$

for any $t \in \mathcal{T}$. To establish the relation between $\alpha(\cdot; \theta)$ and $\alpha(\cdot; \theta_0)$, we need to show the following auxiliary lemma.

Lemma C.1 *Let $\zeta : \mathcal{T} \rightarrow \mathcal{X}$ be an increasing differentiable function, where \mathcal{T}, \mathcal{X} are open sets in a Euclidean space. If $\alpha(\tilde{x})$ solves the ODE “ $\dot{\alpha} = \lambda(\alpha, \tilde{x})$, $\tilde{x} \in \mathcal{X}$ ” with an initial condition $\alpha(\tilde{x}_0) = x_0$, then $\alpha_0(t) \equiv \alpha(\zeta(t))$ solves “ $\dot{\alpha}_0 = \lambda(\alpha_0, \zeta(t))\zeta'(t)$, $t \in \mathcal{T}$ ” with an initial condition $\alpha_0(\zeta^{-1}(\tilde{x}_0)) = x_0$.*

Proof of Lemma C.1. By the definition of $\alpha(\tilde{x})$ as a solution to the first ODE, we have $\alpha'(\tilde{x}) = \lambda(\alpha(\tilde{x}), \tilde{x})$ for all $\tilde{x} \in \mathcal{X}$. If $\alpha_0(t) \equiv \alpha(\zeta(t))$ for all $t \in \mathcal{T}$, then $\alpha'_0(t) = \alpha'(\zeta(t))\zeta'(t)$ for all $t \in \mathcal{T}$. Combining the two equalities, we have $\alpha'_0(t) = \lambda(\alpha(\zeta(t)), \zeta(t))\zeta'(t)$ for all $t \in \mathcal{T}$. With $\alpha_0(t) \equiv \alpha(\zeta(t))$, this equality can be written as $\alpha'_0(t) = \lambda(\alpha_0(t), \zeta(t))\zeta'(t)$ for

all $t \in \mathcal{T}$. Finally, note $\alpha(\tilde{x}_0) = x_0$ and $\alpha_0(t) = \alpha(\zeta(t))$ implies $\alpha_0(\zeta^{-1}(\tilde{x}_0)) = \alpha(\tilde{x}_0) = x_0$. This proves the lemma. \square

An application of Lemma C.1 to (C.4) with $\zeta \equiv h^{-1}$ and $\lambda(\alpha, \tilde{x})$ defined on the r.h.s. of (C.1) implies the procurer equilibrium strategies under θ_0 and θ are related as $\alpha(t; \theta_0) = \alpha(h^{-1}(t); \theta)$ for all $t \in \mathcal{T}$.

Next, note that the joint distribution of (X^*, X) at (x^*, x) according to θ is $F_{X^*, \tilde{X}}(x^*, \alpha^{-1}(x; \theta))$ while that under θ_0 is $H_{X^*, \tilde{X}}(x^*, \alpha^{-1}(x; \theta_0))$. The relation between $\alpha(\cdot; \theta)$ and $\alpha(\cdot; \theta_0)$ implies that $\alpha^{-1}(x; \theta_0) = h(\alpha^{-1}(x; \theta))$ for all $x \in \mathcal{X}$. Hence

$$H_{X^*, \tilde{X}}(x^*, \alpha^{-1}(x; \theta_0)) = H_{X^*, \tilde{X}}(x^*, h(\alpha^{-1}(x; \theta))) = F_{X^*, \tilde{X}}(x^*, \alpha^{-1}(x; \theta))$$

where the second equality is due to the specified relation between $F_{X^*, \tilde{X}}$ and $H_{X^*, \tilde{X}}$. Therefore the two joint distributions of (X^*, X) in equilibrium under θ and θ_0 are identical. Let $\phi(x, x^*) \equiv \pi(x^*) - \pi(x)$ and $\phi_0(x, x^*) = \pi_0(x^*) - \pi_0(x)$. Then that $a = a_0$, $\pi = \pi_0$ and $\gamma = \gamma_0$ in θ and θ_0 imply $\gamma\phi + (1 - \gamma)a = \gamma_0\phi_0 + (1 - \gamma)a_0$ for all x, x^* . Furthermore, a pair (X, X^*) leads to positive net surplus $\phi - a$ under θ if and only if it does so under θ_0 . Hence the two models θ and θ_0 imply the same probability of $D = 1$ given (X, X^*) , and the same distribution of Y given $D = 1$ and (X, X^*) in equilibrium.

Finally, the distribution of prices quoted by contractors at equilibrium under θ is determined by the distribution of costs $F_{C|X}$ as well as the expectation $\bar{\delta}(X; \alpha(\cdot; \theta)) \equiv \mathbb{E}[\gamma s_+(X, X^*) \mid X; \alpha(\cdot; \theta)]$. By construction, $F_{C|X} = H_{C|X}$ and $\gamma(\phi - a) = \gamma_0(\phi_0 - a_0)$. Furthermore, the argument above already shows that the joint distribution of (X, X^*) in psPBE is identical under θ and θ_0 . It then follows that θ and θ_0 can generate the same joint distribution of V, Y, D, X and X^* in psPBE.

C.2 Proof of Theorem 1.

We first show the identification of π . Rewrite (7) as:

$$\begin{aligned} & \varphi'(z; \alpha^*) - \pi'(z) \\ &= -\frac{\partial}{\partial x} [(1 - \gamma)\pi(x)p(x, \tilde{x})]_{x=z} + \frac{\partial}{\partial x} \left[\int_{\omega(x)} (1 - \gamma)[\pi(t) - a(x, t)] dF_{X^*|\tilde{x}}(t) \right]_{x=z} \end{aligned} \quad (\text{C.5})$$

for any \tilde{x} , where $z \equiv \alpha^*(\tilde{x})$ as in the text. Recall for any $t \in \omega(x)$, $y(x, t) = (1 - \gamma)a(x, t) + \gamma[\pi(t) - \pi(x)]$. Therefore, the second term on the r.h.s. above is

$$-\frac{\partial}{\partial x} [\gamma\pi(x)p(x, \tilde{x})]_{x=z} + \frac{\partial}{\partial x} \left[\int_{\omega(x)} \{\pi(t) - y(x, t)\} dF_{X^*|\tilde{x}}(t) \right]_{x=z}. \quad (\text{C.6})$$

Hence (C.5) can be written as

$$\begin{aligned} & \varphi'(z; \alpha^*) - \pi'(z) \\ &= -\pi'(z)p(z, \tilde{x}) - p_1(z, \tilde{x})\pi(z) + \frac{\partial}{\partial x} \left[\int_{\omega(x)} \{\pi(t) - y(x, t)\} dF_{X^*|\tilde{x}}(t) \right]_{x=z}, \end{aligned} \quad (\text{C.7})$$

for all $z \in \mathcal{X}_e$, where $p_1(z, \tilde{x})$ is the derivative of $p(x, \tilde{x})$ with respect to the first argument evaluated at $x = z$ and \mathcal{X}_e is the equilibrium support for initial specifications $\{x : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in \mathcal{X}\}$. Recall that $\omega(x) \subseteq \mathcal{X} \subseteq \mathbb{R}$. For now, suppose it can be partitioned into three intervals $(-\infty, \iota_1(x))$, $(\iota_2(x), \iota_3(x))$ and $(\iota_4(x), \infty)$. Then by Assumption 1-(d),

$$\begin{aligned} p_1(z, \tilde{x}) &= f_{X^*|\tilde{x}}(\iota_1(z))\iota_1'(z) - f_{X^*|\tilde{x}}(\iota_4(z))\iota_4'(z) \\ &\quad + f_{X^*|\tilde{x}}(\iota_3(z))\iota_3'(z) - f_{X^*|\tilde{x}}(\iota_2(z))\iota_2'(z). \end{aligned}$$

Applying the Leibnitz Rule to the last term on the right-hand side of (C.7), we can write (C.7) as:

$$\begin{aligned} \varphi'(z; \alpha^*) &= [1 - p(z, \tilde{x})]\pi'(z) + \pi(z) [f_{X^*|\tilde{x}}(\iota_4(z))\iota_4'(z) - f_{X^*|\tilde{x}}(\iota_1(z))\iota_1'(z) \\ &\quad + f_{X^*|\tilde{x}}(\iota_2(z))\iota_2'(z) - f_{X^*|\tilde{x}}(\iota_3(z))\iota_3'(z)] + \\ &\quad 0 - [\pi(\iota_4(z)) - y(z, \iota_4(z))] f_{X^*|\tilde{x}}(\iota_4(z))\iota_4'(z) - \int_{\iota_4(z)} y_1(z, t) f_{X^*|\tilde{x}}(t) dt + \\ &\quad [\pi(\iota_1(z)) - y(z, \iota_1(z))] f_{X^*|\tilde{x}}(\iota_1(z))\iota_1'(z) - 0 - \int^{\iota_1(x)} y_1(z, t) f_{X^*|\tilde{x}}(t) dt + \\ &\quad [\pi(\iota_3(z)) - y(z, \iota_3(z))] f_{X^*|\tilde{x}}(\iota_3(z))\iota_3'(z) - [\pi(\iota_2(z)) - y(z, \iota_2(z))] f_{X^*|\tilde{x}}(\iota_2(z))\iota_2'(z) \\ &\quad - \int_{\iota_2(z)}^{\iota_3(z)} y_1(z, t) f_{X^*|\tilde{x}}(t) dt \end{aligned} \quad (\text{C.8})$$

for any $z \in \mathcal{X}_e$. Next, note that for all x, x^* ,

$$\begin{aligned} \pi(x^*) - y(x, x^*) &= \pi(x^*) - \gamma[\pi(x^*) - \pi(x)] - (1 - \gamma)a(x, x^*) \\ &= \pi(x) + (1 - \gamma)[\pi(x^*) - \pi(x) - a(x, x^*)] \\ &= \pi(x) + (1 - \gamma)s(x, x^*). \end{aligned}$$

With $s(x, x^*) = 0$ at $x^* = \iota_k(x)$ for $k = 1, 2, 3, 4$, we have $\pi(\iota_k(z)) - y(z, \iota_k(z)) = \pi(z) + (1 - \gamma) \cdot 0$ for all $z \in \mathcal{X}_e$. Substitute this into (C.8). Then for all z on the equilibrium support of initial announcement \mathcal{X}_e , we have

$$\pi'(z) = [1 - p(z, \tilde{x})]^{-1} \left(\varphi'(z; \alpha^*) + \int_{\omega(z)} y_1(z, t) f_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t) dt \right). \quad (\text{C.9})$$

The r.h.s. of (C.9) only contains quantities directly identifiable from the data. To see this, first note that by the monotonicity of α^* and the normalization of the distribution of \tilde{X} we can identify $\tilde{x} = \alpha^{*-1}(z)$. (That is, if z is the τ -th quantile of X , then $\alpha^{*-1}(z)$ is recovered as the τ -th quantile of \tilde{X}) For any $z \in \mathcal{X}_e$,

$$f_{X^*|\tilde{X}=\alpha^{*-1}(z)}(t) = f_{X^*|D=1, \tilde{X}=\alpha^{*-1}(z)}(t) \times \Pr(D = 1|X = z) \text{ for all } t \in \omega(z),$$

because $D = 1$ if and only if $X^* \in \omega(z)$. Also, by construction, both $y(z, t)$ and $\varphi(z; \alpha^*)$ are directly identifiable for all $z \in \mathcal{X}_e$ and $t \in \omega(z)$. Thus $\pi'(\cdot)$ is identified over the

interior of \mathcal{X}_e . With a local normalization $\pi(\underline{x}) = 0$ where \underline{x} is the infimum of \mathcal{X}_e , we obtain a closed-form solution for π : That is, for all $x \in \mathcal{X}_e$,

$$\pi(x) = \int_{\underline{x}}^x \left\{ [1 - p(z, \alpha^{*-1}(z))]^{-1} \left(\varphi'(z; \alpha^*) + \int_{\omega(z)} y_1(z, t) f_{X^* | \tilde{X} = \alpha^{*-1}(z)}(t) dt \right) \right\} dz.$$

With π recovered over \mathcal{X}_e , we can recover the contractor's bargaining power as:

$$\gamma = \frac{y(x_2, x_2^*) - y(x_1, x_1^*)}{\phi(x_2, x_2^*) - \phi(x_1, x_1^*)}$$

using any two pairs (x_1, x_1^*) and (x_2, x_2^*) that satisfy Assumption 2. Then a is recovered as

$$a(x, x^*) = \frac{y(x, x^*) - \gamma \phi(x, x^*)}{1 - \gamma}$$

for all $x \in \mathcal{X}_e$ and $x^* \in \omega(x)$.

Finally, with γ, π, a identified above, we can recover $\bar{\delta}(x; \alpha^*) \equiv \mathbb{E}[\gamma s_+(x, X^*) \mid X = x; \alpha^*]$ for any $x \in \mathcal{X}_e$. This in turn implies we can recover the contractors' inverse bidding strategy in equilibrium $\beta^{*-1}(\cdot, x; \alpha^*)$, and consequently the cost distribution $F_{C|X=x}$ for any $x \in \mathcal{X}_e$.

D Further Details for Parametric Estimation

This model in Section 5.2 accommodates both contract and contractor heterogeneity and includes additive structural errors ε, η . The following proposition establishes the identification of this parametric model under mild support conditions of the explanatory variables. We assume structural parameters are non-zero (that is, $\theta_r \neq 0$ for $r = 1, 2, \dots, 6$, $\sigma \neq 0$ and $\lambda \neq 0$.)

Proposition D.1. *In the model (13)-(14) with the specification (15)-(16), the structural parameters $\{\theta_j\}_{j=1,\dots,6}, \lambda, \rho$ and σ^2 are identified if the support of $[(x^* - x), x^2, x^{*2}, x^*x, w]$ has full rank.*

We now sketch a proof of this identification result. To simplify presentation, we first condition on the other explanatory variables in w (other than job) and suppress them in the notation below. Let γ_k be shorthand for the bargaining power when $job = k \in \{0, 1\}$. To begin with, expand the latent variable $\gamma_k \phi + (1 - \gamma_k)a + \varepsilon$ in the outcome equation (13) into:

$$\begin{aligned} & \gamma_k [\theta_1(x^* - x) + \theta_2(x^* - x) \times k + \theta_3(x^{*2} - x^2)] \\ & + (1 - \gamma_k) [\theta_4(x^* - x) + \theta_5(x^* - x) \times k + \theta_6(x^* - x)^2] + \varepsilon \\ = & \beta_1 \times (x^* - x) + \beta_2 \times x^2 + \beta_3 \times x^{*2} + \beta_4 \times (x^* - x) \times k + \beta_5 \times (x^* \times x) + \varepsilon \end{aligned}$$

where

$$\begin{aligned}\beta_1 &\equiv [\gamma_k\theta_1 + (1 - \gamma_k)\theta_4]; \beta_2 \equiv [(1 - \gamma_k)\theta_6 - \gamma_k\theta_3]; \beta_3 \equiv [\gamma_k\theta_3 + (1 - \gamma_k)\theta_6]; \\ \beta_4 &\equiv [\gamma_k\theta_2 + (1 - \gamma_k)\theta_5]; \beta_5 \equiv -2(1 - \gamma_k)\theta_6.\end{aligned}$$

Likewise, also expand the latent variable in the selection equation (14) into

$$\tilde{\beta}_1 \times (x^* - x) + \tilde{\beta}_2 \times x^2 + \tilde{\beta}_3 \times x^{*2} + \tilde{\beta}_4 \times (x^* - x) \times job + \tilde{\beta}_5 \times (x^* \times x) + \eta$$

where

$$\begin{aligned}\tilde{\beta}_1 &\equiv \theta_1 - \theta_4; \tilde{\beta}_2 \equiv -\theta_3 - \theta_6; \tilde{\beta}_3 \equiv \theta_3 - \theta_6; \\ \tilde{\beta}_4 &\equiv \theta_2 - \theta_5; \tilde{\beta}_5 \equiv 2\theta_6.\end{aligned}$$

Recall the coefficients in a probit model are identified, provided the support of the explanatory variables has full rank. Thus, in our model the variation in x and x^* on the equilibrium path helps us recover $\{\tilde{\beta}_j : j = 1, \dots, 5\}$ (and therefore $\theta_1 - \theta_4$, $\theta_2 - \theta_5$ and θ_3 , θ_6) from the conditional probability of incompleteness given x , x^* and job .

Next, note the expectation of the negotiated transfer conditional on x , x^* , $job = 1$, $d = 1$ is

$$(\beta_1 + \beta_4)(x^* - x) + \beta_2 \times x^2 + \beta_3 \times x^{*2} + \beta_5 \times (x^* \times x) + E[\varepsilon \mid x, x^*, job = 1, d = 1] \quad (D.1)$$

where the last conditional expectation is the inverse mill's ratio evaluated at

$$(\tilde{\beta}_1 + \tilde{\beta}_4) \times (x^* - x) + \tilde{\beta}_2 \times x^2 + \tilde{\beta}_3 \times x^{*2} + \tilde{\beta}_5 \times (x^* \times x)$$

where $\{\tilde{\beta}_j : j = 1, \dots, 5\}$ are already identified from the selection equation (14). It then follows that we recover $\beta_1 + \beta_4, \beta_2, \beta_3, \beta_5$ from (D.1) as long as the typical rank condition holds.¹⁷ It then follows that $\gamma_1 = 1 + \beta_5/\tilde{\beta}_5$ is identified. (In fact γ_1 is over-identified because we can also express γ_1 as a function of θ_3, θ_6 and β_3 .)

By repeating this argument conditioning on $job = 0$, we can show that γ_0 is also over-identified. It only remains to show that $\theta_1, \theta_2, \theta_4, \theta_5$ are identified. To do this, note that we have already recovered: (a) $\gamma_1\theta_1 + (1 - \gamma_1)\theta_4 + [\gamma_1\theta_2 + (1 - \gamma_1)\theta_5]$ from the outcome equation conditional on $job = 1$; (b) $\gamma_0\theta_1 + (1 - \gamma_0)\theta_4$ from the outcome equation conditional on $job = 0$; and (c) $\theta_1 - \theta_4$ and $\theta_2 - \theta_5$ from the selection equation (14). Thus we can construct a linear system of four equations and four unknowns

$$\begin{pmatrix} \gamma_0 & 1 - \gamma_0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \gamma_1 & 1 - \gamma_1 & \gamma_1 & 1 - \gamma_1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_4 \\ \theta_2 \\ \theta_5 \end{pmatrix} = r.h.s.$$

¹⁷The rank condition requires the support of $x^* - x, x^2, x^{*2}, x^*x$ and the mill's ratio to have full rank. This condition can be satisfied under mild conditions due to the nonlinearity of the mill's ratio (even when the selection equation does not involve any exogenous variable that is excluded from the outcome equation). See the last but one paragraph on page 806 in Wooldridge (2010) (which starts with "As a technical point, we do not ...") for more detailed discussion.

where the right-hand side consists of quantities that are already identified. The coefficient matrix on the l.h.s. is non-singular for all $\gamma_0, \gamma_1 \in [0, 1]$. It then follows $\theta_1, \theta_2, \theta_4, \theta_5$ are all identified. Finally, recall that the identification result above was shown by conditioning on w , which means the bargaining power, as a function of characteristics w and the job type, is identified for all values w and job . The identification of λ then follows from the parametric form of $\gamma(w, job; \lambda)$ and the full-rankness of the support of w .

E Tables

Table 1: Summary statistics

Variable	Mean	Std. Dev.	10-th pctile	median	90-th pctile
Contract (Project)-level					
number of observations	1293				
number of bidders	4.531	2.272	2	4	7
engineer estimate [◊]	2.393	4.268	.304	.929	5.914
actual project size [◊]	2.33	4.206	.345	1.017	4.928
winning bid [◊]	2.29	4.421	.269	.872	5.339
transfer [◊]	.172	.356	0	.037	.469
fringe of the winning bidder	.489	.5	0	0	1
distance of the winning bidder	97.569	143.618	9.78	46.44	254.73
utilization of the winning bidder	.125	.218	0	.005	.406
average fringe [†]	.567	.292	0	.6	1
average distance [†]	107.66	98.743	23.717	81.2	213.429
Contractor (Auction)-level					
number of observations	5862				
submitted bids [◊]	2.623	16.276	.287	.9	5.537
fringe	.622	.485	0	1	1
distance	102.894	143.431	12.78	55.4	255
utilization	.112	.224	0	0	.387
minimal distance among rivals	43.161	57.341	4.5	25.32	97.43
minimal utilization among rivals	.017	.075	0	0	.012
submitted bids [◊]	3.055	17.185	.288	.912	5.876

[◊] The unit is million dollars.

[†] The average is taken for all bidders within an auction.

Table 2: Regression Results of Transfer

	(1)	(2)	(3)
constant	-0.0778** (0.0351)	-0.0709* (0.0383)	-0.112*** (0.0424)
act. size (X^*)	-0.151*** (0.0372)	-0.249*** (0.0411)	-0.230*** (0.0416)
eng. estimate (X)	0.266*** (0.0332)	0.368*** (0.0371)	0.364*** (0.0372)
afri	0.0662 (0.0468)	0.0376 (0.0463)	0.0612 (0.0467)
nbid	0.00613 (0.00585)	0.00518 (0.00571)	0.00580 (0.00566)
job	0.138*** (0.0247)	0.147*** (0.0241)	0.142*** (0.0240)
eng. estimate* afri	-0.180*** (0.0428)	-0.229*** (0.0423)	-0.236*** (0.0443)
eng. estimate* nbid	-0.0224*** (0.00698)	-0.0243*** (0.00681)	-0.0197*** (0.00685)
eng. estimate * job	0.0300** (0.0146)	0.0194 (0.0146)	-0.00513 (0.0204)
act. size * afri	0.112** (0.0457)	0.169*** (0.0451)	0.168*** (0.0477)
act. size * nbid	0.0202*** (0.00742)	0.0228*** (0.00725)	0.0178** (0.00728)
act. size * job	-0.0954*** (0.0159)	-0.0890*** (0.0158)	-0.0615*** (0.0226)
eng. estimate * eng. estimate	0.00131*** (0.000184)	0.000687*** (0.000205)	0.000625*** (0.000224)
act. size * act. size	0.000559** (0.000230)	0.00128*** (0.000251)	0.00151*** (0.000278)
auti		0.0653 (0.0948)	0.0804 (0.0945)
eng. estimate * auti		-0.297*** (0.0796)	-0.274*** (0.0790)
act. size * auti		0.225*** (0.0833)	0.197** (0.0827)
adis			0.000222* (0.000130)
eng. estimate * adis			-0.0000504 (0.0000661)
act. size * adis			-0.0000310 (0.0000718)
N	1224	1224	1224
R^2	0.889	0.896	0.898
adj. R^2	0.888	0.894	0.896

Standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Regression results of expected payment

	(1)	(2)	(3)
constant	-0.0460 (0.0872)	0.0583 (0.0972)	0.104 (0.104)
eng. estimate	1.105*** (0.0160)	1.080*** (0.0165)	1.029*** (0.0170)
eng. estimate ²	0.0000175 (0.000133)	0.000107 (0.000133)	-0.000228* (0.000135)
afri	-0.0465 (0.117)	-0.0574 (0.118)	-0.108 (0.115)
nbid	0.0151 (0.0146)	0.0126 (0.0145)	0.0121 (0.0140)
job	-0.137** (0.0620)	-0.145** (0.0614)	-0.121** (0.0596)
eng. estimate * afri	0.128*** (0.0198)	0.120*** (0.0197)	0.150*** (0.0194)
eng. estimate * nbid	-0.0490*** (0.00314)	-0.0484*** (0.00312)	-0.0487*** (0.00302)
eng. estimate * job	0.0622*** (0.00842)	0.0648*** (0.00835)	0.0539*** (0.00818)
auti		-0.631** (0.246)	-0.613** (0.239)
eng. estimate * auti		0.150*** (0.0283)	0.142*** (0.0274)
adis			-0.0000780 (0.000302)
eng. estimate * adis			0.000299*** (0.0000347)
<i>N</i>	1306	1306	1304
<i>R</i> ²	0.977	0.978	0.979
adj. <i>R</i> ²	0.977	0.978	0.979

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Estimates of Average Marginal Effects on Bargaining Power

	(1)	(2)	(3)
nbid	0.139*** (0.027)	0.095*** (0.033)	0.102*** (0.026)
afri	-0.447*** (0.156)	-0.356** (0.201)	-0.296*** (0.081)
auti		0.507** (0.215)	0.186** (0.076)
adis			-0.0002 (0.0014)
job type 1 [◇]	0.624*** (0.086)	0.789*** (0.094)	0.809*** (0.063)
job type 0 [♣]	0.915*** (0.061)	0.898*** (0.063)	0.926*** (0.031)
job [†]	-0.291*** (0.099)	-0.109 (0.099)	-0.117** (0.068)

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

[◇] Average bargaining power for job type 1.

[♣] Average bargaining power for job type 0.

[†] Difference of average bargaining power for two job types.

Table 5: Estimates of Surplus and Cost Functions

	(1)	(2)	(3)
Surplus function $\phi(\cdot)$			
$X^* - X$	-2.837** (1.169)	-2.942** (1.160)	-2.916** (1.214)
$(X^* - X) * \text{job}$	0.050 (0.169)	0.905 (1.002)	0.175 (0.933)
$X^{*2} - X^2$	0.101*** (0.038)	0.101*** (0.038)	0.101*** (0.038)
Cost function $a(\cdot)$			
$(X^* - X)$	-0.837 (0.798)	-0.943 (0.794)	-0.916 (0.843)
$(X^* - X) * \text{job}$	0.413 (1.216)	-1.368 (1.114)	0.637 (1.037)
$(X^* - X)^2$	1.565*** (0.589)	1.565** (0.589)	1.565** (0.589)
observations	1224	1224	1224
log-likelihood	-3007.286	-2795.490	-2665.141

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 6: Regression results of probability of incompleteness

	(1)	(2)	(3)
constant	0.997*** (0.249)	1.179*** (0.273)	1.161*** (0.275)
eng.estimate	2.149*** (0.381)	1.798*** (0.410)	1.846*** (0.417)
job	0.226 (0.256)	-0.403 (0.512)	-0.391 (0.512)
eng.estimate* job		1.309 (0.968)	1.287 (0.967)
eng.estimate*eng.estimate	-0.0198*** (0.00529)		-0.0202*** (0.00675)
observations	1306	1306	1306
correctly classified \diamond	94.10%	94.10%	94.10%
log-likelihood	-246.72	-245.81	-245.70
pseudo R^2	0.157	0.160	0.161

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

\diamond Correctly classified: $\Pr(\text{predicted } D = j | \text{observed } D = j) \geq 0.5, j = 0, 1$.

Table 7: Estimates of markups

		25-th qtile	median	75-th qtile	mean
job	1	1.07%	1.90%	3.71%	6.21%
	0	1.17%	2.08%	4.17%	10.83%
winning bidders	Yes	3.41%	6.87%	14.25%	29.13%
	No	0.99%	1.63%	2.69%	3.22%
fringe	Yes	1.07%	1.86%	3.67%	9.16%
	No	1.23%	2.27%	4.47%	8.83%
utilization	0	1.03%	1.83%	3.49%	10.60%
	(0, 0.3)	1.35%	2.48%	4.90%	7.07%
	[0.3, 1]	1.09%	1.97%	4.12%	6.29%
distance	[0.1, 11.9]	1.15%	2.17%	4.91%	7.23%
	[12, 247.9]	1.14%	1.99%	3.96%	9.26%
	> 247.9	0.94%	1.89%	3.93%	9.04%
number of bidders	2	3.42%	5.36%	9.85%	25.35%
	3, 4, 5	1.58%	2.45%	4.61%	9.94%
	6, 7, 8	0.87%	1.30%	2.23%	5.09%
	8+	0.66%	1.01%	1.81%	3.25%
overall		1.12%	2.00%	4.04%	9.03%

Table 8: Estimates of markups (assuming away incompleteness)

		25-th qtile	median	75-th qtile	mean
job	1	2.14%	3.83%	7.85%	13.09%
	0	2.39%	3.82%	7.77%	20.47%
winning bidders	Yes	7.69%	13.92%	27.76%	61.70%
	No	1.96%	3.08%	4.95%	5.16%
fringe	Yes	2.05%	3.36%	6.90%	17.01%
	No	2.76%	4.58%	8.89%	18.52%
utilization	0	2.01%	3.35%	6.67%	18.51%
	(0, 0.3)	2.93%	4.87%	9.56%	18.17%
	[0.3, 1]	2.47%	4.30%	8.24%	12.19%
distance	[0.1, 11.9]	2.36%	4.17%	9.48%	48.17%
	[12, 247.9]	2.27%	3.76%	7.40%	14.65%
	> 247.9	2.32%	4.16%	8.78%	10.70%
number of bidders	2	6.48%	10.35%	19.84%	29.10%
	3, 4, 5	3.07%	4.74%	8.97%	21.84%
	6, 7, 8	1.74%	2.50%	4.30%	11.29%
	8+	1.02%	1.64%	3.27%	5.67%
overall		2.29%	3.82%	7.78%	17.60%