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Shintaku, Koji

Graduate School of Economics, Kyoto University

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Koji Shintaku*
Graduate School of Economics, Kyoto University

Abstract

By constructing an intra-industry trade model with the division of labor within firms, this study shows that opening up to trade improves firm productivity through promoting the division of labor. The division of labor is limited not by the size of each market but by the number of export markets that firms enter. Reallocation of labor across firms based on free-entry condition, fixed export costs, and constant markup rate plays a key role behind this result. Firms enter the export markets in the ascending order of entry costs. As trade costs decrease, firms enter more export markets if the number of markets does not reach the upper bound. Hence, the division of labor is essentially limited by trade costs. This implication brings new insight to Adam Smith's theorem.

Keywords: the number of export markets that firms enter; division of labor within firms; labor reallocation

JEL classification numbers : F12

1 Introduction

Adam Smith (1776) indicated the importance of productivity improvement induced through the division of labor within a firm using the example of a pin factory; "As firms input more labor, the firms' productivity increase." Then, Adam Smith suggested a famous proposition called *Adam Smith's theorem*: *"The division of labor is limited by the extent*

*Corresponding author. Graduate School of Economics, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail address: shintaku.shitanku@gmail.com. I am grateful to Naoto Jinji and Kenn Ariga for their support and advice. I also thank Keita Kamei, Tadashi Morita, Ryuhei Wakasugi, Yoichi Sugita and Mitsuo Inada for their helpful comments. I also thank all the participants of Asia Pacific Trade Seminars (APTS).

of the market." Many economists have justified Adam Smith's theorem theoretically and empirically.[1)2)] In particular, these empirical studies justify Adam Smith's theorem for both the manufacturing and non-manufacturing sectors.[3)]

These studies address the autarky economy. How can this theorem be extended to an open economy? Little is known of this; however, one exception is Chaney and Ossa (2013) who indicated that an increase in *market size* (labor force) theoretically promotes the division of labor within firms. Their model implicitly indicates that an increase in the market size of each country promotes the division of labor; i.e., the division of labor is limited by *the size of each market.* This is a new insight for the division of labor within firms in the context of intra-industry trade.

Chaney and Ossa (2013) do not treat fixed costs, and they evoke labor reallocation across firms based on a pro-competition effect, such as Krugman (1979). Under fixed costs, constant markup rate, and free entry, considering a mechanism by which firm productivity increases according to the extent of the division of labor and excess profits causes new entrants; to what extent firms choose the division of labor and what that extent depends on? Furthermore, when firms must pay fixed export costs, how do these costs and the number of export markets that firms enter affect the firm's decision? This study shows a simple trade model that investigates these problems.

We construct a model that is quite similar to standard intra-industry trade models presented by Krugman (1980), except for the division of labor, fixed export costs, and symmetric multi-countries. We treat the division of labor within firms similar to that by Chaney and Ossa (2013). Chaney and Ossa (2013) succeeded in formalizing Adam Smith's (1776) pin factory story. We assume that firms enter export markets in the ascending order of entry costs. This assumption plays a key role in determining the number of export markets that firms enter.

This study's main results are as follows. Under positive fixed export costs and free-entry condition without pro-competition effect, opening up to trade makes surviving firms

1) This proposition is interpreted often as the division of labor not only within firms but also across firms within an industry and across industries. For example, Ethier (1982) treats the division of labor as an expansion of the varieties of intermediate goods. In this study, the division of labor is treated as a narrower task set in which each worker engages.

2) For example, Stigler (1951) interprets Adam Smith's theorem as the relation between market size and vertical integration (an increase in market size raises the number of specialized firms). The study of the division of labor reported by Stigler (1951) is supported by Levy (1984). Levy (1984) supports Stigler's (1951) hypothesis using data from 38 manufacturing industries. Borghans and Weel (2006) suggested firm productivity improvements induced by the division of labor within firms through communication technology adoption, which reduces the coordination cost within firms. Edwards and Starr (1987), Swanson (1999), and Becker and Murphy (1992) present theoretical analyses.

3) Baumgardner (1988) indicated that the more populous counties have more medical specialists. Garcano and Hubbard (2008) presented similar results for law firms.

promote the division of labor. The division of labor becomes stronger as the number of export markets that firms enter increases; i.e., the division of labor is limited by *the number of export markets that firms enter* but not by the size of each market. As trade costs decrease, firms enter more export markets if the number of markets does not reach the upper bound. Hence, the division of labor is essentially limited by *trade costs*.

There are quite few papers that analyze international trade by explicitly incorporating the division of labor within firms. Kamei (2013), Francois (1987), and Zadef (2013) are exceptions. Similar to this study, Kamei (2013) incorporated Chaney and Ossa's (2013) division of labor into a general oligopolistic equilibrium model with a variable markup rate and demonstrated an increase in the number of trading countries promotes the division of labor. Francois (1987) adopted Edwards and Starr's (1987) division of labor and analyzed trade in services and its effect on the division of labor. Zadeh (2013) presented a model in which there are two types of workers and heterogeneity of firms. Zadeh (2013) focused on relative specialization and skill premium through trade liberalization. Unlike these studies, our study focuses on the relation between optimal entry for export markets and the division of labor within firms. Our results described above are in contrast to Adam Smith's theorem and the result of Chaney and Ossa (2013) and Kamei (2013) that the division of labor is limited by *the size of each market*. In Chaney and Ossa (2013) and Kamei (2013), the division of labor is promoted through labor reallocation across firms *pro-competition effect*. However, our model does not include the pro-competition effect. Furthermore, in our model, when trade costs decrease, firms enter more export markets. Hence, the division of labor is essentially limited by trade costs. This implication brings new insight to Adam Smith's theorem. This result can not be given by the model of Chaney and Ossa (2013) because their markup rate does not depend on the number of firms. Our study's results are different from Zadef (2013). Zadef (2013) indicated that trade liberalization promotes relative specialization but has nothing special to imply about the relation between the optimal number of export markets and the division of labor. Our model is simpler and more analytical.

The topic of this study–the division of labor within firms promoted by trade–can be associated with the restructuring of firm organization by trade.[4)5)] These studies reveal a

4) In more general, this research line is included in trade-induced firm productivity improvement. Yeaple (2005) and Bustos (2011) studied technology adoption. McLaren (2000) studied productivity improvement through vertical restructuring. Clerides, Lach, and Tybout (1998) and Martins and Yang (2009) empirically studied trade-induced firm productivity improvement. There is no consensus on whether improvements to firm productivity are induced by trade. A survey by Wagner (2007) indicated that this effect is mixed and unclear. However, Martins and Yang (2009) indicated that many empirical studies recognize firm productivity improvements induced by trade, considering more than 30 papers.

5) This research line is different from the trade-induced industry productivity improvement of Melitz (2003). He focused on average industry productivity with heterogeneous firms. In contrast to this, our

composition of firm productivity from the viewpoint of firm organization.[6)]

The rest of this paper is organized as follows. Section 2 analyzes autarkic equilibrium. Section 3 analyzes how opening up to trade promotes the division of labor and increases welfare. Section 4 analyzes how trade liberalization promotes the division of labor. Finally, we present the conclusion and Appendix.

2 The model

We introduce the division of labor into the trade model of monopolistic competition with fixed export costs. The setup of the model is based on the idea of Chaney and Ossa (2013). In this section, we set up the model of an autarky economy.

2.1 Households

There are L units of household, and each household supplies one unit of labor inelastically at wage rate w. The preference of each consumer is given by a constant elasticity of substitution (CES) utility function over a continuum of goods indexed by θ: $U = \left[\int_{\theta\in\Theta} c(\theta)^{\rho} d\theta\right]^{1/\rho}, 0 < \rho < 1$, where the measure of the set Θ represents the mass of available differentiated goods, and $c(\theta)$ represents the consumption of variety θ. From standard utility maximization, the price index can be obtained as follows: $P = \left[\int_{\theta\in\Theta} (p(\theta))^{1-\sigma} d\theta\right]^{1/(1-\sigma)}$, where $\sigma = 1/(1-\rho) > 1$ is the elasticity of the substitution between any two varieties and also represents the price elasticity of demand for each variety.

2.2 Firm organization

We introduce the division of labor within firms in a similar manner to that done by Chaney and Ossa (2013) for the reason as follows.[7)] Traditional production management, which promotes the division of labor and production on a large scale, is the scientific management

study focuses on firm productivity with homogeneous firms in productivity.

6) Caliendo and Rossi-Hansberg (2012) indicated that trade liberalization increases the number of layers of management and raises firm productivity by calibrating the model to the U.S. economy. Cuñat and Guadalupe (2009) indicated that higher foreign competition leads to more incentive provision in various ways, which is compatible with recent trends in compensation structures of U.S. executives. Davidson et al. (2013) provided empirical evidence by using the matched employer-employee data in Sweden that multinational enterprises (MNEs) possess greater skill intensity than local firms and non-MNEs. Then, they present a model explaining the empirical results.

7) In addition to Chaney and Ossa (2013), some papers formalized the division of labor. Edwards and Starr (1987) presented a model in which the division of labor is not a sufficient condition for increasing returns to scale. Swanson (1999) presented a quite simple model that analyzes the relation among human capital investment, the division of labor, and firm productivity. Becker and Murphy (1992) showed explicitly that the cost of promoting the division of labor is coordination cost.

advocated by Frederick Taylor. However, today, team production is important in many industries, as reported by Daft (2000). Chaney and Ossa (2013) allowed such a production management approach explicitly, and hence, we also adopt it.

Each firm produces a variety of differentiated final goods. As for the production of goods, we modify the model developed by Chaney and Ossa (2013). Many tasks are sequentially distributed over the set $[0,2]$ for each firm. One unit of final good is produced by inputting one unit of preliminary good for task set $[0,2]$. A firm assigns these tasks to t teams, where $t \in R_+$. Because the teams are symmetric, an identical range of task subsets is assigned to each team. One unit of preliminary good for a certain task set $[\underline{\omega}, \overline{\omega}]$ is produced by inputting the following units of labor:

$$l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \times \underbrace{\left(\int_{\underline{\omega}}^{\overline{\omega}} \gamma |\omega_c - \omega| d\omega \right)}_{\textbf{Area of two right-angled triangles}}, \quad \omega_c \in [0,2],\ \gamma > 0, \tag{1}$$

where $(\underline{\omega} + \overline{\omega})/2$ denotes this team's core competency, and γ denotes the team's burden parameter.[8] Core competency is a task that the team is most suited to undertake. As γ is high, certain task sets need more labor force. γ can be interpreted as the difficulty of multitasking.

This implies that the larger γ is, the less efficient is the assigning of many task sets to one team: a decrease in γ raises the team's performance. Figure 1 illustrates this feature for task set $[0, 4/t]$ when t is a positive integer. The integral term in (1) corresponds to the area of two right-angled triangles formed in linear symmetry with respect to the vertical direction shown in Figure 1.[9]

Let $l_{pre|unit}$ denote labor requirements for producing one unit of preliminary good for the task set $[0,2]$. By combining (1) for each team, $l_{pre|unit}$ can be obtained as follows :[10]

$$l_{pre|unit} = t \times \underbrace{\left(\int_0^{1/t} \gamma \omega d\omega \right)}_{\textbf{labor input per one team}} = \frac{\gamma}{2t}. \tag{2}$$

(2) indicates that as the number of teams increases, labor input per one team converges

8) In Chaney and Ossa (2013), the firm assigns a core competency to each team; i.e., the core competency is endogenously determined. We also derive the core competency endogenously in Appendix A.

9) For the assumption of $l(\underline{\omega}, \overline{\omega})$, Chaney and Ossa (2013) adopt a more general form, $l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega}+\overline{\omega}}{2} - \omega \right)^{\beta} d\omega$, where $\beta >$ is a positive parameter. By formulating $l(\underline{\omega}, \overline{\omega})$ the way as (1), we can make the model highly tractable. See Appendix A for the generality and validity of the technology in (1).

10) On the right hand side of (1), by dividing the integral term by two, we get a quite simple form for the units of labor. See Appendix A for the derivation.

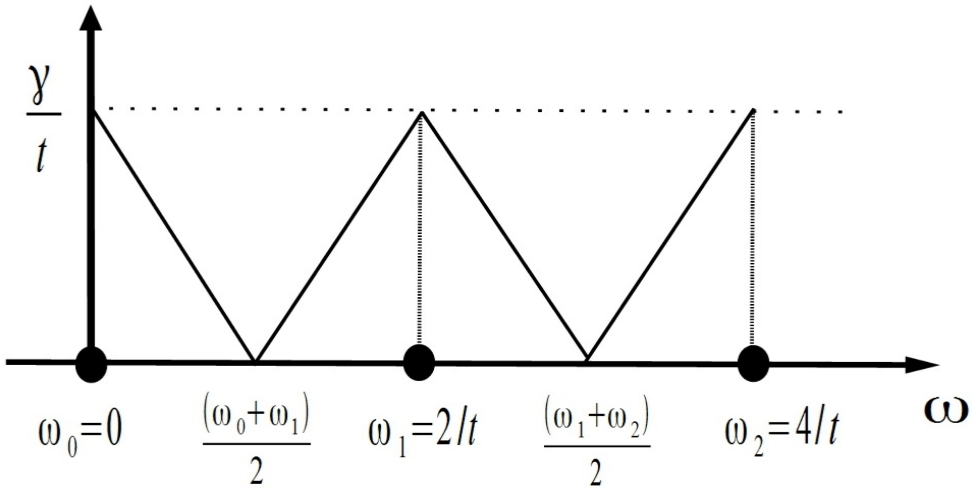

Figure 1: sequential task structure

with order 2 to 0 from $\int_0^{1/t} \gamma\omega d\omega = \gamma/(2t^2)$, while the number of teams diverges with order 1 to $+\infty$. Hence, as the number of teams increases, $l_{pre|unit}$ decreases.

Organizing one team requires $f(> 0)$ units of labor, which is interpreted as coordination costs.[11] Then, y units of final goods is produced for a given number of teams, t, by inputting the following units of labor on *production divisions*, $l(t, y) = tf + yl_{pre|unit} = tf + \gamma y/(2t)$.

Each firm selects the number of teams t such that the above labor input $l(t, y)$ is minimized. In this problem, the firm experiences a trade-off among productivity improvements by increasing the number of teams and costs of organizing teams. The optimal number of teams t is $t(y) = [\gamma y/(2f)]^{1/2}$.

Each firm inputs labor into the production divisions and a further $f_d(> 0)$ units of labor into the management division, where $f_d(> 0)$ is fixed and wf_d represents overhead production costs. Total labor input is $l + f_d$.

Combining $l(t, y)$ and $t(y)$ gives the total cost function under the optimal organization as follows:

$$TC(y) = wl(y) + wf_d = w(2\gamma fy)^{1/2} + wf_d. \tag{3}$$

This cost function shows that the firm's technology exhibits increasing returns to scale and that marginal cost is decreasing at all levels of output.

From $l(t, y)$, we can obtain the production function as follows: $y = l^2/(2\gamma f)$. The marginal productivity of labor $MPL(l)$ is given by $MPL(l) = dy/dl = l/(\gamma f)$. This shows that the expansion of labor input increases marginal productivity. Furthermore, from $t(y)$, $MPL(l)$, and the production function, we can obtain $MPL = \frac{2t}{\gamma}$. We can confirm that

11) f can be interpreted as mid-level management costs. Because each team specializes in a certain task set, the firm needs coordinators. Becker and Murphy (1992) emphasized that coordination cost is the brake for the division of labor.

the division of labor raises firm productivity. In the same manner as above, we can get $MC = w\gamma/(2t)$. The division of labor reduces marginal cost.

2.3 Equilibrium allocation

We analyze the firm's profit maximization problem in a monopolistic competitive market. Each firm experiences a residual demand curve with constant elasticity σ and therefore sets $p = \mu MC(y)$, where $\mu \equiv \sigma/(\sigma - 1)$ and $MC(y) \equiv dTC(y)/dy$. Using $l(t, y)$, this optimal pricing rule is written by the PP_A schedule as follows:

$$PP_A : \frac{p}{w} = \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2}. \tag{4}$$

Firms can enter and exit freely. This gives zero profit $\pi = 0$; this is written by $p = AC(y)$, where $AC(y) \equiv TC(y)/y$. Using $l(t, y)$, this free-entry condition is written by the FE_A schedule as follows:

$$FE_A : \frac{p}{w} = \left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}. \tag{5}$$

(4) and (5) characterize $(y, p/w)$ at equilibrium as follows: $y_A = f_d^2/(2\gamma f B^2)$, and $(p/w)_A = B(B + 1)2\gamma f/f_d$, where $B \equiv \mu/2 - 1$ and subscript "A" represents variables in autarkic equilibrium.

Hereafter, we assume Assumption 1 given below to ensure the unique internal solution.

Assumption 1. [12] *$0 < B < \infty$, i.e., $2 < \mu < \infty$ $(1 < \sigma < 2)$ and $f_d > 0$ hold.*

We can immediately obtain the next proposition from y_A and $(p/w)_A$.

Proposition 1. *Under Assumption 1, a unique internal solution in which $y > 0$ and $p/w > 0$ exists.*

Note that if $f_d = 0$ holds, then the internal solution does not exist.[13] Hence, we need to assume $f_d > 0$. Even if $f_d > 0$, under $\sigma \geq 2$, $y \to \infty$; i.e., the internal solution requires a sufficiently low elasticity of substitution among varieties (consumers value variety strongly).

Figure 2 illustrates the features of an autarkic equilibrium. The figure has a unique intersection between the FE_A and PP_A curves at point E_A where $(y, p/w) = (y_A, (p/w)_A)$. The PP_A curve is cut by the FE_A curve only once. This ensures a unique internal solution.[14]

12) This internal condition makes us reconsider firm technology as represented by (1). See Appendix A for details. However, we will adopt technology in (1) and Assumption 1 for analytical simplicity.
13) When $f_d = 0$ and $B = 0$, equilibrium output y is not determined. When $f_d = 0$ and $B \neq 0$, equilibrium output y is zero or approaches positive infinity.
14) The characteristic of Figure 2 is supported by Appendix B.

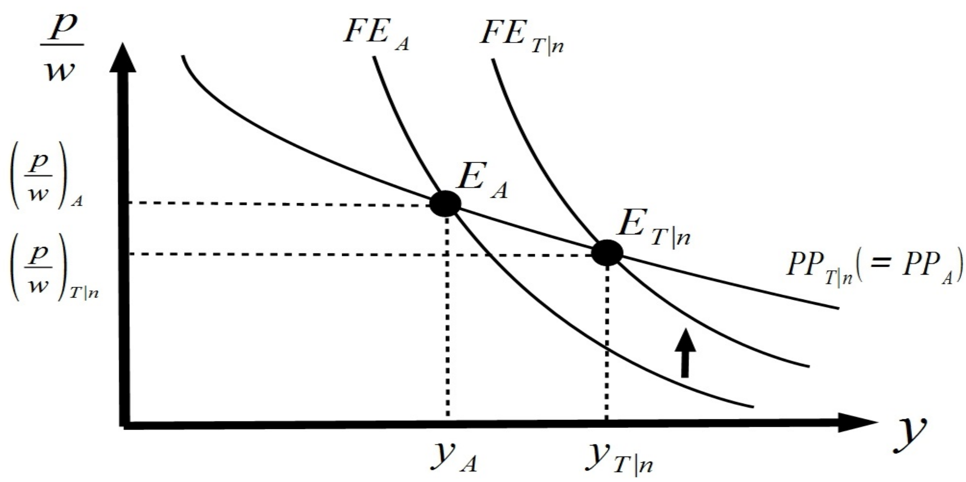

Figure 2: Autarky and Tarding equilibrium in $(y, p/w)$ space.

Substitute y_A into $t(y)$ to yield the equilibrium level of t: $t_A = f_d/(2fB)$. The equilibrium level of l is obtained by substituting y_A and t_A into $l(t,y)$: $l_A = f_d/B$. Then, substitute l_A into $MPL(l)$ to yield $MPL_A = f_d/(\gamma f B)$. This equation implies that $MPL_A = 2t_A/\gamma = l_A/(\gamma f)$. Further, $(w/p)_A = t_A/[\gamma(B+1)] = l_A/[2(B+1)\gamma f]$ holds. At equilibrium, labor productivity and real wages are proportional to the number of teams and the labor input on production divisions.

Now, we can completely characterize the equilibrium allocation by determining the number of varieties. Labor market clearing condition $L = M(l + f_d)$ gives the following equilibrium number of varieties M_A using l_A: $M_A = [B/(B+1)](L/f_d)$.[15)]

From MPL_A and M_A, the following proposition is immediately obtained.

Proposition 2. *Under Assumption 1, an expansion of aggregate labor force does not promote the division of labor and thus does not raise firm productivity and only raises the number of firms.*

Proposition 2 means that the division of labor is *not* limited by the *size* of the market.[16)] Remember that output y_A is completely characterized by PP and FE conditions. These conditions do not depend on market size L. Hence, y_A also does not depend on market size. This result is in contrast to Chaney and Ossa (2013) and Kamei (2014) in which PP conditions depend on market size L, and then, labor reallocation across firms based on *pro-competition effect* occurs similar to Krugman (1979).

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15) To obtain M_A, we use labor market clearing condition and do not use income-expenditure clearing condition of each household because this condition is redundant on this equilibrium.

16) Proposition 2 implies that Proposition 1 holds even if $L \to \infty$. This is because all the effects of an increase in labor forces are not absorbed into an increase in demand for each variety but in the number of firms.

The mechanism behind Proposition 2 is explained as follows. In the short run, the number of firms cannot be adjusted. An expansion of labor force increases employed workers in each firm and hence improves firm productivity; then, firms obtain excess profits. However, in the long run with free entry and exit, new firms enter and recruit some workers from incumbent firms. Therefore, the effect of productivity improvements is just outset entirely.

3 Opening up to international trade

We extend the model reported in the previous section to the case of trade among identical $\bar{n}+1$ countries with fixed export costs. The assumption of fixed export costs is essential for the division of labor promoted by trade. We assume $\bar{n} \in R_{++}$ for analytical simplicity. Without the loss of generality, we focus on the home country's allocation.

3.1 Two types of firm decision processes

Each firm's decision process has two stages. The first stage is the market entry process. The second stage is a choice of optimal quantity and price. This firm's optimization problem can be solved using backward induction. We begin with the second problem. In the second stage, the number of export markets that firms enter is given.

Each firm experiences two types of trade costs. First, firms must export $\tau \in [1, \infty)$ units of product to send one unit (iceberg trade cost) to a foreign market. Second, to enter export markets, firms must pay fixed costs.[17)]

Some empirical studies indicate that most firms export to a few markets.[18)] Hence, it is natural to think that firms enter export markets in the ascending order of entry costs. Then, we assume that firms entering n export markets must pay fixed costs $wn^{\alpha}f_x$, where $\alpha \geq 1$; i.e., these costs vary across markets and can be interpreted as marker-specific fixed export costs.[19)] This assumption plays a key role in determining optimal entry.

17) The examples are as follows: collecting information about foreign markets and consumer tastes, adapting their products to foreign administrative standards, establishing a distribution network, and standardizing products to fulfill market-specific regulations.

18) Gullstrand (2011) used data from the Swedish food sector and indicated that *"only two firms export to 50 countries or more in a single year while roughly half export to just ten countries or fewer"* and *"On average, an exporter sells to 13 countries."*

19) Maurseth and Medin (2013) used a survey of Norwegian sea food firms and found that "*having exported to a particular market the previous periods doubles the probability of to the same market in the current period*". They interpret this result as the existence of market-specific sunk export costs. Gullstrand (2011) used data from the Swedish food sector and indicated that for firms' export decisions, firm destination effects are more important than unobserved firm characteristics. In particular, they emphasized exchange rate stability. Blanes-Cristobal et al. (2008) used a survey of Spanish manufacturing firms and indicated that the costs to enter developed markets are higher (especially the EU) than those of the rest of the world.

We focus on firms entering n export markets. These firms' output for the home market is denoted by y_d, and each of the foreign markets is denoted by y_x. Then, we can define the total output of firms as $y = y_d + ny_x$.

The firm production function is given by $y = l^2/(2\gamma f)$, where l represents labor inputs in production divisions to sell in $n+1$ markets. This firm's total labor input is $l+f_d+n^\alpha f_x$.

This gives the marginal product of labor as follows: $MPL = l/(\gamma f)$. The total cost function is given by

$$TC(y) = w\left[(2\gamma fy)^{1/2} + f_d + n^\alpha f_x\right].$$

Note that under these technologies, $TC(y) < w[(2\gamma fy_d)^{1/2} + f_d] + w[(2\gamma fy_x)^{1/2} + n^\alpha f_x]$ holds. This implies that each firm's total profits cannot be decomposed into profits from the home market and those from export markets $\pi \neq \pi_d + n\pi_x$.

Price for the home market is denoted by p_d and that for the export market as p_x. Mill price in the export market is $p_x = \tau p_d$, from the assumption.

Home consumers buy goods from n foreign countries as the trade-balanced condition is satisfied. Home consumers experience all countries' brands and $(n/\bar{n})M$ brands, on average, per one foreign country. Hence, the price index is given by

$P_{T|n} = \left[\int_{\theta\in\Theta}(p_d(\theta))^{1-\sigma}d\theta + \bar{n}\int_{\theta^*\in\Theta^*}[\tau p_d(\theta^*)]^{1-\sigma}d\theta^*\right]^{1/(1-\sigma)}$, where an asterisk represents foreign brands.

Accounting for a final goods market-clearing condition, firm profit maximization is characterized by optimal price setting (See Appendix C).

3.2 Trading equilibrium and the division of labor promoted by trade

We define trading equilibrium in almost the same manner as autarkic equilibrium. However, we need to account for firms' decisions of export market entry. Subscript " T " represents variables in trading equilibrium. Then, we define trading equilibrium as follows.

Definition 1. *We define trading equilibrium as an equilibrium that satisfies the conditions as follows.*

(I) Optimal price-setting rules, free-entry conditions, goods market-clearing conditions, labor market-clearing conditions, and trade-balanced conditions are satisfied.

(II) No firms have an incentive to deviate from the equilibrium for the number of export markets that firms enter.

European norms may be quite specific and homogeneous among members. Bugameli and Infante (2003) used a survey of Italian manufacturing firms and emphasized informational barriers.

We consider firms' decisions at the second stage (optimal price and output), treating the positive number of export markets that firms enter $n > 0$ as given. Firm equilibrium allocation is characterized by the optimal pricing rule $PP_{|n} : p_d/w = (\mu/2)\,(2\gamma f/y)^{1/2}$ and free-entry condition $FE_{|n} : p_d/w = (2\gamma f/y)^{1/2} + (f_d + n^\alpha f_x)/y$. Subscript " $|n$ " represents variables being conditional on n. Figure 2 illustrates the features of the trading equilibrium as point $E_{T|n}$.

As shown in Figure 2, positive fixed export costs $n^\alpha f_x$ shift the FE curve upward. Note that the free-entry condition holds for the world market as a whole and that the only difference between autarky and trading equilibrium conditions is the fixed costs term. This implies that we can obtain $y_{T|n}$ by replacing f_d with $f_d + n^\alpha f_x$ in y_A: $y_{T|n} = (f_d + n^\alpha f_x)^2/(2\gamma f B^2)$. In the similar manner, we have $l_{T|n} = (f_d + n^\alpha f_x)/B$, $t_{T|n} = (f_d + n^\alpha f_x)/(2fB)$, and $M_{T|n} = L/[(2\gamma f y)^{1/2} + f_d + n^\alpha f_x]$. M represents the number of home country firms.

We find $y_{T|n} > y_A$, $(w/p_d)_{T|n} > (w/p_d)_A$, $t_{T|n} > t_A$, $l_{T|n} > l_A$, $MPL_{T|n} > MPL_A$ and $M_{T|n} < M_A$. $M_{T|n} < M_A$ means that some firms exit. $t_{T|n} > t_A$ and $MPL_{T|n} > MPL_A$ mean that the division of labor is promoted by opening trade.

Those results are summarized in the proposition as follows.

Proposition 3. *Under Assumption 1 and given $n > 0$, opening up to trade with positive fixed export costs promotes the division of labor.*

Proof. From $t_{T|n} - t_A = (f_d + n^\alpha f_x)/(2fB) - f_d/(2fB)$, for all $n \in R_{++}$, $t_{T|n} > t_A$. Q.E.D.

We can explain a mechanism behind this result from a viewpoint of labor reallocation across firms. In Figure 2, the point E_A satisfies the optimal pricing rule, $PP_{T|n}$, and not free entry condition, $FE_{T|n}$. Each of firms has negative profit at the point E_A because the average cost is higher than the price. Hence, on opening up to trade, some firms try to enter export markets just to survive. These firms must pay fixed export costs. To pay these costs, these firms recruit workers. Firms that succeed in recruiting workers can promote the division of labor. This recruiting competition raises the real wage rate. This causes firms that do not succeed in recruiting to exit.

Note that this selection mechanism is different from that of Chaney and Ossa (2013) and Kamei (2014). Their selection mechanism is driven by *pro-competition effect* similar to Krugman (1979), in which PP condition depends on market size.

An allocation in a trading equilibrium without trading costs is accorded with that of an integrated economy's equilibrium because this model does not have a pro-competition effect. Therefore, Proposition 3 implies Corollary 1 as follows.

Corollary 1. *Under Assumption 1 and given $n > 0$, opening up to trade without fixed export costs does not raise firm productivity.*

Proof. Let f_x be zero. For all $n \in R_{++}$, $t_{T|n} = t_A$. Q.E.D.

We find that positive fixed export costs are essential for Proposition 3. This mechanism is parallel to the result in Melitz (2003).[20)]

From $t_{T|n}$, we obtain Corollary 2 as follows.

Corollary 2. *Under Assumption 1 and given $n > 0$, as the number of export markets that firms enter increases, the division of labor becomes stronger.*

Proof. From $dt_{T|n}/dn = \alpha(f_d + n^{\alpha-1} f_x)/(2fB)$, for all $n \in R_{++}$, $dt_{T|n}/dn > 0$. Q.E.D.

This corollary means that the division of labor is limited by *the number of export markets that firms enter*; i.e., firms select the optimal division of labor according to the number of export markets they enter. Free-entry conditions play a key role behind the results. [21)]

3.3 Optimal entry for export markets

Next, we consider firms' decisions on the first stage: the entry process. Firms select the number of export markets to enter while fixing the number of export markets the other firms enter.

The number of export markets that firms should enter depends on the parameter set. The optimal number is uniquely determined under certain assumptions, as shown in Proposition 4. To clarify those assumptions, we introduce a function $G(n)$.

Definition 2. *We define function $G(n)$, which is a function of $n \in R$ as follows: $G(n) \equiv I(n)/H(n)$, where is, $I(n) \equiv 1 + \tau^{1-\sigma} n$, $H(n) \equiv (1 + n^\alpha f_x/f_d)^{2-\sigma}$.*

In addition, We define values $n_c \in R_{++}$ and $n_e \in R_{++}$ that satisfy the conditions as follows: $G(n_c) = 1$, $G'(n_e) = 0$.

For analytical simplicity, we focus on an equilibrium in which all firms enter the same number of markets. To focus on such an equilibrium, we impose restrictions on $\bar{n}$.

Assumption 2. *We assume the condition as follows:*

(I) $(2-\sigma)\alpha \geq 1 \Rightarrow G'(\bar{n}) < 0$.

(II) $[(2-\sigma)\alpha < 1 \;\wedge\; \alpha = 1 \;\wedge\; f_d < f_x \tau^{\sigma-1}(2-\sigma)] \Rightarrow \bar{n} > n_c$,

20) Melitz's (2003) footnote 24 says "*In the absence of such costs (...), opening to trade will not induce any distributional changes among firms, and heterogeneity will not play an important role.*"

21) We can interpret a mechanism of this corollary in the same manner as Proposition 3.When n is high, much of the labor force is concentrated in surviving firms through opening up to trade.

(III) $\left[(2-\sigma)\alpha < 1 \;\wedge\; \alpha > 1 \;\wedge\; f_d = f_x \left[\tau^{\sigma-1}(2-\sigma)\right]^{\alpha} \left[(\alpha-1)/[1-(2-\sigma)\alpha]\right]^{\alpha-1}\right] \Rightarrow$ $\bar{n} \neq n_e$,

(IV) $\left[(2-\sigma)\alpha < 1 \;\wedge\; \alpha > 1 \;\wedge\; f_d < f_x \left[\tau^{\sigma-1}(2-\sigma)\right]^{\alpha} \left[(\alpha-1)[1-(2-\sigma)\alpha]\right]^{\alpha-1}\right] \Rightarrow$ $[\bar{n} < n_e \;\vee\; G(\bar{n}) > G(n_e)]$.

(V) There is a unique $\delta^* \in (0,1)$ *such that satisfies:*
$\delta^{*a-1} = [I'(n)/I(n)][H'(n)/H(n)]\left[(f_d + n^{\alpha} f_x \delta^{*\alpha})/(f_d + n^{\alpha} f_x)\right]^{\sigma-1}$.

We introduce Lemmas 1 and 2 to determine n_T, as shown in Appendices I and J.[22] Then, we can obtain Proposition 4.

Proposition 4. *The following properties hold.*

(I) If and only if Assumption 1, $(2-\sigma)\alpha \geq 1$, *and (I) of Assumption 2 hold, there is a unique equilibrium in which all firms enter* $n_T + 1$ *markets, where* n_T *satisfies* $I'(n)/I(n) = H'(n)/H(n)$*: partial entry regime.*

(II) If and only if Assumption 1, $(2-\sigma)\alpha < 1$, *and (II) – (V) of Assumption 2 hold, there is a unique equilibrium in which all firms enter* $\bar{n}+1$ *markets; i.e.,* $n_T = \bar{n}$*: full entry regime.*

Proof. See Appendix L. Q.E.D.

We investigate what each regime demands.[23] Whether the full entry or partial entry regime is satisfied highly depends on the property of fixed export costs distribution. Full entry regime demands $(2-\sigma)\alpha < 1$, and the partial entry regime demands $(2-\sigma)\alpha \geq 1$. In particular, $\alpha = 1$ holds only in the case of $(2-\sigma)\alpha < 1$; i.e., when there is no market specificity for export fixed costs, a partial entry regime cannot be achieved. Conversely, when export fixed costs are highly specific, a partial entry regime is achieved.

By using n_T in Proposition 4, we can completely characterize the trading equilibrium n_T, c_T, c'_T, y_T, $(w/p_d)_T$, t_T, l_T, and M_T.[24]

---

22) Lemma 1 indicates that the numbers of export markets that firms enter are not distributed in a certain condition. This lemma holds when $G(n) > 1$ holds; i.e., n is sufficiently high relative to trade costs τ and f_x/f_d. Lemma 2 indicates that the equilibrium condition *(II)* in Definition 1 is satisfied if all firms enter n export markets. This lemma holds when an optimal n maximizes $G(n)$. We can interpret this immediately. The numerator of $G(n)$ $I(n) = (1 + n\tau^{1-\sigma})$ and the denominator of $G(n)$ $H(n) = (1 + n^{\alpha} f_x/f_d)^{2-\sigma}$ can be interpreted as entry gain and entry loss, respectively, as explained in Appendix K. Therefore, it is natural that n maximizes $G(n)$.

23) Both regimes demand $G(n_T) > 1$ and the maximization of $G(n)$ by n_T. $G'(\bar{n}) < 0$ in *(I)* of Assumption 2 certifies $G'(n_T) = 0$ for $n_T < \bar{n}$. All conditions in *(II) – (IV)* of Assumption 2 are bound when a combination of variable and fixed trade costs are sufficiently high relative to domestic fixed costs. n_T of partial regime demands $I'(n_T)/I(n_T) = H'(n_T)/H(n_T)$. This means that the elasticity of entry gains for n is equal to the elasticity of entry loss for n.

24) We have already imposed final goods market-clearing conditions and labor market-clearing conditions for $n+1$ countries but have not imposed the income-expenditure clearing condition and the trade-balance conditions for $n+1$ countries because these conditions are redundant on this equilibrium.

Though we account decision on the number of export markets firms enter in trading equilibrium, the following result similar to Proposition 2 holds.

Corollary 3. *Under Assumption 1 and 2, uniform expansion of aggregate labor force of all countries does not promote the division of labor in both regimes.*

Proof. From $t_{T|n} = (f_d + n^\alpha f_x)/(2\gamma f B)$, $t_{T|n}$ does not directly depend on L. Since G(n) does not depend on labor force L, optimal n_T, which maximizes $G(n)$ also does not depend on labor force L. Hence, $t_{T|n}$ does not depend on L. Q.E.D.

That is, the division of labor is *not* limited by the size of each market in trading equilibrium. The free-entry condition has a key role in this result.[25)]

3.4 Gains from trade

At the trading equilibrium, the real wage rates are identical in all countries, and hence, the indirect utility function is given by $V_T = (w/P)_T$. This leads to the following proposition.

Proposition 5. *Under Assumptions 1 and 2, $V_T > V_A$.*

Proof. We can obtain $V_T > V_A \leftrightarrow G(n_T) > 1$ under Assumption 1 (see Appendix D for the proof). Assumption 2 certifies $G(n) > 1$ (see Appendix L for the proof). Q.E.D.

The necessary condition for optimal entry $G(n) > 1$ is equivalent to condition $V_T > V_A$; i.e., the optimal entry condition certifies gains from opening up to trade.

We next decompose gains from trade. At trading equilibrium, the welfare of each country V_T is given by $V_T = (w/p_d)_T[(1 + n_T\tau^{1-\sigma})M_T]^{1/(\sigma-1)}$ shown in Appendix D. We define the effective number of varieties as $M_W \equiv [(1+n_T\tau^{1-\sigma})M_T]^{1/(\sigma-1)}$, and this represents the number of varieties consumers buy that are discounted by variable trade costs τ.[26)] Then, we can decompose gains from trade into changes in real wages (*productivity effect*) and change in the effective number of varieties (*effective variety effect*).

Productivity effect is positive from Proposition 3. In contrast to this, the effective variety effect is ambiguous; i.e., even if the effective variety effect is negative, the positive productivity effect dominates this effect on the equilibrium (See Appendix E for details).

25) Proposition 8 of Appendix G indicates that an increase in the size of each market promotes the division of labor if the free-entry condition is not imposed.

26) Note: a decrease in σ raises the effective number of varieties; i.e., as consumers values variety stronger the effective number of varieties increases.

4 Trade liberalization

We define trade liberalization as a decrease in variable trade cost τ or in fixed export cost f_x or in specificity of fixed export costs α, or an increase in the number of trading partners, $\bar{n}$.[27] Note that these changes are worldwide because all countries are symmetric. Then, we consider an increase in $\bar{n}$ only in the case of $\alpha = 1$, for analytical simplicity.[28]

We can implement a comparative statistical analysis for trade liberalization as follows.

Proposition 6. *Under Assumption 1 and 2, trade liberalization has impacts on equilibrium allocation and social welfare as follows.*

(I) Full entry regime

1. A decrease in τ does not change n_T, y_T, t_T, M_T, and M_W and raises V_T.

2. A decrease in f_x does not change n_T, reduces y_T and t_T, and raises M_T, M_W, and V_T.

3. A decrease in α does not change n_T, reduces y_T and t_T, and raises M_T, M_W, and V_T.

4. Under $\alpha = 1$, an increase in $\bar{n}$ raises n_T, y_T, t_T, and V_T, and reduces M_T. Then, whether M_W increases is ambiguous.

(II) Partial entry regime

Each of a decrease in τ, f_x, and α raises n_T, y_T, t_T, and V_T and reduces M_T. Then, whether M_W increases is ambiguous.

Proof. See Appendix F for details. Q.E.D.

Theses results indicate that the division of labor is limited by *the number of export markets firms enter*, no matter which regime is achieved. Under partial entry regime, that the division of labor is essentially limited by *trade costs*.

The mechanism behind the above results is described as follows. To begin with, we consider the effect on n_T. Under full entry regime, a decrease in τ, f_x, and α raises entry gain relative to entry loss but does not change n_T because n_T is bound at $n_T = \bar{n}$. In contrast to this, an increase in $\bar{n}$ raises n_T.[29] Under a partial entry regime, all these

---

27) For these changes in f_x, α and $\bar{n}$ there are the examples as follows: A decrease in f_x is brought about by export promotion and deregulation. A decrease in α is brought about by the standardization of products, regulations, and administration. An increase in $\bar{n}$ describes a situation in which some rising countries enter an intra-industry trade market or in which some countries enter into a multilateral trade agreement.

28) We introduce an increase in $\bar{n}$ in such a manner that symmetry is maintained among countries. However, this is difficult. For example, symmetry is broken if all firms of incumbent countries must pay identical fixed export costs to enter new markets. For analytical simplicity, we consider an increase in $\bar{n}$ only in the case of $\alpha = 1$.

29) This is not trivial. When firms raise n_T, they face a trade-off between an increase in total revenue $r = r_d + nr_x$ and total fixed costs $f_d + n^{\alpha} f_x$. The former effect dominates the latter. Therefore, an increase in $\bar{n}$ raises n_T, concentrates labor on surviving firms, and promotes the division of labor.

changes raise n_T because n_T does not reach the upper bound. Ascending order of entry costs plays a key role in this result.

Next, we consider channels in which changes in τ, f_x, α, and $\bar{n}$ affect $t_{T|n}$ from $t_{T|n} = l_{T|n}/(2\gamma f)$. A change in $t_{T|n}$ depends on only $l_{T|n}$. From $l_{T|n} = (f_d + n^\alpha f_x)/B$, $l_{T|n}$ directly depends on f_x, and α (direct effect). In addition, $l_{T|n}$ indirectly depends on τ, f_x, and α through n (indirect effect). Under full entry regime, indirect effect is shut down except for change in $\bar{n}$. Hence, the results of Proposition 6 are different between the two regimes.[30)31)]

However, both regimes have the same mechanism in the sense that a direction of labor reallocation effect across firms determines whether the division of labor is promoted or refrained (See Proposition 10 of Appendix H). However, we should note that the direction of labor reallocation across firms depends on firm's entry for export markets. In this sense, ascending order of entry costs determining the number of markets firms enter plays a key role in trade liberalization. The above results under partial regime can not be given by the model of Chaney and Ossa (2013) because they does not assume ascending order of entry costs and their markup rate does not depend on the number of firms.

Whether welfare rises depends on whether $G(n_T)$ rises because $V_T = V_A G(n_T)^{1/(\sigma-1)}$. All changes raise $G(n_T)$ and hence raise V_T (See Appendix F for details).

5 Conclusion

This study analyzes how trade promotes entry into export markets, the division of labor, and changes firm productivity. Under positive fixed export costs, free-entry conditions, and constant markup rate, opening up to trade causes surviving firms to promote the division of labor. The division of labor becomes stronger as the number of export markets that firms enter increases; i.e., the division of labor is limited by *the number of export markets that firms enter* but not by the size of each market. This result is in contrast with Adam Smith's theorem and result of Chaney and Ossa (2013) that the division of labor is limited by *the size of each market.* In Chaney and Ossa (2013), labor reallocation across firms behind the division of labor promoted is based on the *pro-competition effect.*

Firms enter the export markets in the ascending order of entry costs. As trade costs decrease, firms enter more export markets if the number of markets does not reach the upper bound. Hence, the division of labor is essentially limited by *trade costs.* This implication

30) We consider effects of the parameters on l_T and $t_{T|t}$ under full entry regime. A decrease in τ does not affect $l_{T|n}$ directly and indirectly. A decrease in f_x and α shifts the FE curve of Figure 2 downward and reduces l_T and t_T. A increase in $\bar{n}$ shifts the FE curve of Figure 2 upward and raises l_T and t_T

31) We consider effects of the parameters on $t_{T|t}$ under partial entry regime. A decrease in τ raises n and then raises l_T and t_T indirectly. A decrease in f_x and α reduces l_T and t_T directly but raises them indirectly. Indirect effect dominates direct effect and hence, a decrease in f_x and α raises l_T and t_T.

provides a new insight for Adam Smith's theorem.

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Appendix

Appendix A: Firm structure

Derivation of optimal core competency

While we treats core competency as exogenous variable, we treats this as endogenous variable in this appendix.

Firms select core competency, ω_c, on certain task set $[\underline{\omega}, \overline{\omega}]$. Let $l_{|\omega_c}(\underline{\omega}, \overline{\omega})$ denote labor input of task set $[\underline{\omega}, \overline{\omega}]$ for producing one unit of preliminary good for given ω_c. Then, $l_{|\omega_c}(\underline{\omega}, \overline{\omega})$ is given as follows:

$$l_{|\omega_c}(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \gamma |\omega_c - \omega| d\omega, \;\; \omega_c \in [0, 2], \; \gamma > 0.$$

For minimization problem, $l(\underline{\omega}, \overline{\omega}) = \min_{\omega_c \in [\underline{\omega}, \overline{\omega}]} \; l_{|\omega_c}(\underline{\omega}, \overline{\omega})$, we rewrite objective function as follows:

$$\begin{aligned} l_{|\omega_c}(\underline{\omega}, \overline{\omega}) &= \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \gamma |\omega_c - \omega| d\omega \\ &= \frac{\gamma}{2} \left[\int_{\underline{\omega}}^{\omega_c} (\omega_c - \omega) d\omega + \int_{\omega_c}^{\overline{\omega}} (\omega - \omega_c) d\omega \right] \\ &= \frac{\gamma}{2} \left[\frac{-1}{2} \left[(\omega_c - \omega)^2 \right]_{\underline{\omega}}^{\omega_c} + \frac{1}{2} \left[(\omega - \omega_c)^2 \right]_{\omega_c}^{\overline{\omega}} \right] \\ &= \frac{\gamma}{2} \left[\frac{1}{2} (\omega_c - \underline{\omega})^2 + \frac{1}{2} (\overline{\omega} - \omega_c)^2 \right]. \end{aligned}$$

By minimizing $l(\underline{\omega}, \overline{\omega})$ with respect to ω_c, we can obtain the following first order condition:

$$(\omega_c - \underline{\omega}) - (\overline{\omega} - \omega_c) = 0.$$

Let $\omega_{c|[\underline{\omega}, \overline{\omega}]}$ denote optimal core competency for task set $[\underline{\omega}, \overline{\omega}]$. Hence, we have optimal core-competency as follows

$$\omega_{c|[\underline{\omega}, \overline{\omega}]} = \frac{\underline{\omega} + \overline{\omega}}{2}. \tag{A.1}$$

The optimal core competency is certainly the mid-point in the assigned task set. This is because each task set is symmetric with respect to the core competency.

Derivation of $l_{pre|unit}$ of (2)

By substituting $\omega_{c|[\underline{\omega},\overline{\omega}]}$ for ω_c of $l_{|\omega_c}(\underline{\omega},\overline{\omega})$, we can obtain the following equations:

$$
\begin{aligned}
l(\underline{\omega},\overline{\omega}) &= \frac{1}{2}\int_{\underline{\omega}}^{\overline{\omega}} |\omega_{c|[\underline{\omega},\overline{\omega}]} - \omega| d\omega \\
&= \frac{\gamma}{2}\left[\frac{1}{2}\left(\frac{\underline{\omega}+\overline{\omega}}{2} - \underline{\omega}\right)^2 + \frac{1}{2}\left(\overline{\omega} - \frac{\underline{\omega}+\overline{\omega}}{2}\right)^2\right] \\
&= \frac{\gamma}{2}\left(\frac{\overline{\omega}-\underline{\omega}}{2}\right)^2.
\end{aligned}
$$

$l(\underline{\omega}, \omega_{c|[\underline{\omega},\overline{\omega}]})$ can be obtained as follows:

$$
\begin{aligned}
l(\underline{\omega}, \omega_{c|[\underline{\omega},\overline{\omega}]}) =& \frac{\gamma}{2}\int_{\underline{\omega}}^{\omega_{c|[\underline{\omega},\overline{\omega}]}} |\omega_{c|[\underline{\omega},\overline{\omega}]} - \omega| d\omega \\
=& \frac{\gamma}{4}\left[(\omega_c - \omega)^2\right]_{\underline{\omega}}^{\omega_c} \\
=& \frac{\gamma}{4}\left(\frac{\overline{\omega}-\underline{\omega}}{2}\right)^2.
\end{aligned}
$$

Hence, we can get

$$
l(\underline{\omega},\overline{\omega}) = 2l^*(\underline{\omega}, \omega_{c|[\underline{\omega},\overline{\omega}]}). \tag{A.2}
$$

Because the teams are symmetric, identical range of task subset, $[0, 2/t]$, is assigned to each team and then, labor input of each reach is identical.

We can obtain $l_{pre|unit}$ from the following calculation:

$$
\begin{aligned}
l_{pre|unit} =& t \times l(0, 2/t) \\
=& 2t \times l(0, 1/t) \qquad\qquad \text{by (A.2) and (A.3)} \\
=& 2t \times \frac{1}{2}\int_0^{1/t} \gamma\omega d\omega \\
=& t\left(\int_0^{1/t} \gamma\omega d\omega\right).
\end{aligned}
$$

From $\int_0^{1/t}\omega d\omega = 1/(2t^2)$, we can obtain $t\left(\int_0^{1/t}\gamma\omega d\omega\right) = \gamma/(2t)$.

Generality of the technology in (1)

Next, We examine that how general and valid the technology which we adopt in equation (1) is in comparison to the one adopted by Chaney and Ossa (2013).

The technology we adopted is different from the one adopted by Chaney and Ossa

(2013), in two points. Equation (1) in this paper corresponds to the equation of Chaney and Ossa (2013) as follows:

$$l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega} + \overline{\omega}}{2} - \omega \right)^{\beta} d\omega. \tag{A.3}$$

Equation (A.3) and (1) are equal, when $\beta = 1$ in (A.3) and $\gamma = 1$ in (1).

We examine a characteristic of parameter, β by seeing shape of $l(\underline{\omega}, \overline{\omega})$. For simplicity, we assume $\gamma = 1$ and $t = 1$. When $\beta = 1$, the integral term of the right hand side in (A.3) corresponds to the area formed by "Benchmark Line" shown in Figure A.1. When $\beta > 1$, the one corresponds to the area formed by "Curve H" shown in Figure A.1. When $0 < \beta < 1$, the one corresponds to the area formed by "Curve L" shown in Figure A.1 implies that the effect of an increase in β is parallel to the effect of a decrease in γ.

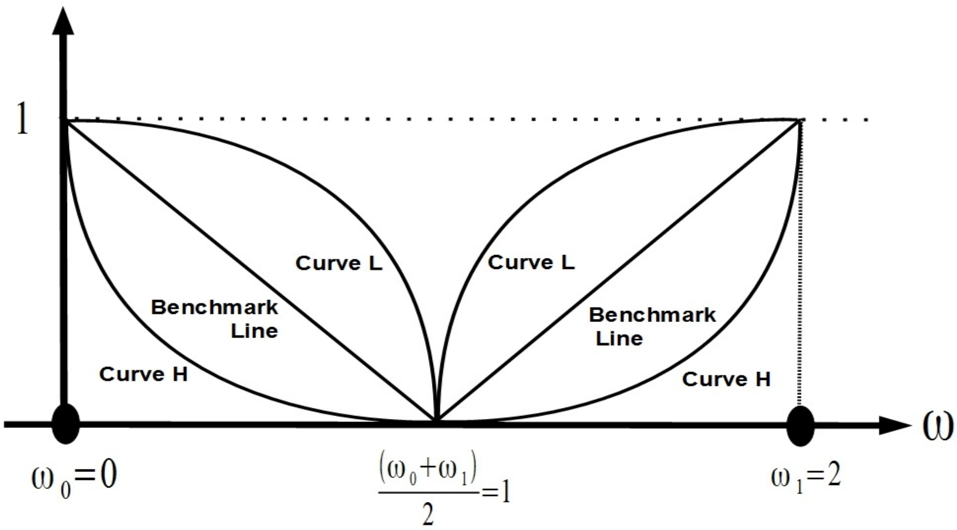

Figure A. 1: comparison between sequential task structures

If we adopts the technology in (A.3), the equilibrium allocation are rewritten by:

$$l_A = \frac{2(\beta+1) - \mu}{\mu - (\beta+1)} f_d,$$

$$y_A = \left(\frac{\beta+1}{\mu - (\beta+1)} f_d \right)^{\beta+1} \left(\frac{\beta}{\beta+1} \frac{1}{f} \right)^{\beta},$$

$$MPL_A = (\beta+1) \left[\left(\frac{\beta}{\beta+1} \right) \left(\frac{\beta+1}{\mu - (\beta+1)} \right) \frac{f_d}{f} \right]^{\beta},$$

$$t_A = \left(\frac{\beta}{\beta+1} \right) \left(\frac{\beta+1}{\mu - (\beta+1)} \right) \frac{f_d}{f}.$$

The next table shows that the effect of an increase in β is parallel to the effect of a decrease in γ on certain conditions.

Table 1

	l_A	y_A	MPL_A
$\beta \uparrow$	0	+ only if $t_A > 1$	+ only if $t_A > 1$

α's amplification effect also occurs on certain conditions. Moreover, effect of f does not change. Therefore, this suggests that the technology which we adopt does not loose generality quite much in comparison to the one adopted by Chaney and Ossa (2013) .

Validity of the technology in (1)

Martins, Scarpetta and Pilat (1996) shows that almost all industries in OECD have markup rate which belongs to set $(1, 2)$. Therefore, the internal solution condition $2 < \mu$ does not seems to have reality. This property highly depends on organization parameter β. If we adopts the technology in (A.3), internal solution condition is

$$\mu > \beta + 1.$$

Therefore, by assuming organization parameter β to be in (0,1), model's mark-up rate μ can be consistent with the empirical studies.

However, assuming β to be in (0,1) makes tractability of the model decrease. For analytical simplicity, we assume β to be 1.

Appendix B: Shape of PP_A curve and FE_A curve in Figure 2

In this section, we examine shape of PP_A curve and FE_A curve in Figure 2.

We define $Z(y)$ as difference between right hand side of PP_A relation and of FE_A relation:

$$Z(y) \equiv \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2} - \left[\left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}\right] = B(2\gamma f)^{1/2}y^{-1/2} - f_d y^{-1}.$$

Certainly, $Z(y_A) = 0$ holds.

The derivative of function $Z(y)$ is given by

$$Z'(y) = -2^{-1}B(2\gamma f)^{1/2}y^{-3/2} + f_d y^{-2}.$$

When $y = y_A^*$, $Z'(y_A^*) = 0$ holds, where y_A^* is given by

$$y_A^* = 2\frac{f_d}{B^2\gamma f} = 4\frac{f_d}{B^2 2\gamma f} = 4y_A.$$

From $B > 0$, when $y < 4y_A$, $Z'(y) > 0$ holds and when $y > 4y_A$, $Z'(y) < 0$ holds. Furthermore, for the second order derivative of function $Z(y)$, $Z''(64y_A/9) = 0$ holds.

The limits of function $Z(y)$ are given by

$$\lim_{y\to\infty} Z(y) = 0,$$

$$\lim_{y\to 0} Z(y) = -\infty.$$

The above relations are proved in the following manner.

Proof.

$$\lim_{y\to\infty} Z(y) = \lim_{y\to\infty} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{0 - f_d}{\infty} \to 0,$$

$$\lim_{y\to 0} Z(y) = \lim_{y\to 0} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{-f_d}{0} \to -\infty.$$

Q.E.D.

According to the above results, the shape of $Z(y)$ is the one as shown in Figure A.2.

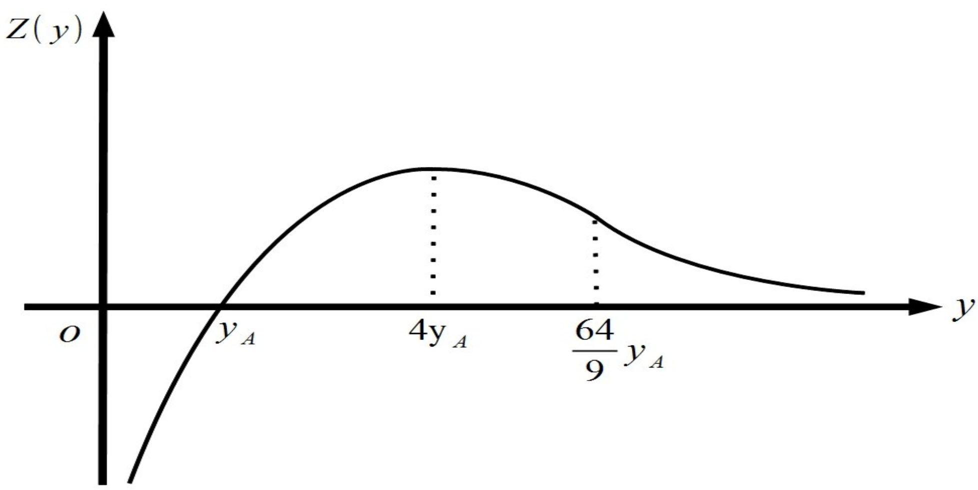

Figure A. 2: the shape of Z(y)

Figure A.2 is consistent to Figure 2 and hence, Figure 2 is supported.

Appendix C: Derivation of optimal quantity-price rule on trading equilibrium

In this section, we assume n as given. In open economy, home country's house holds have preference represented by utility function:

$$U = \left[\int_{\theta\in\Theta} c(\theta)^{\rho} d\theta + \bar{n} \int_{\theta^*\in\Theta^*} c(\theta^*)^{\rho} d\theta \right]^{1/\rho}, \ \ 0 < \rho < 1.$$

Utility maximization drives the following price index:

$$P_{T|n} = \left[\int_{\theta\in\Theta} (p(\theta))^{1-\sigma} \, d\theta + \bar{n} \int_{\theta^*\in\Theta^*} [\tau p(\theta^*)]^{1-\sigma} \, d\theta^* \right]^{1/(1-\sigma)}.$$

On trading equilibrium, all firms' profit are zero and then, each household's income contains the only wage income. Consumption of domestic household for domestic and foreign brand are respectively:

$$c = p_d^{-\sigma}(P_{T|n})^{\sigma-1}w, \ \ c' = (\tau p_d^*)^{-\sigma}(P_{T|n})^{\sigma-1}w. \tag{C.1}$$

Consumption of foreign household for foreign and domestic brand are respectively:

$$c^* = p_d^{*-\sigma}(P^*_{T|n})^{\sigma-1}w^* \ \ c^{*\prime} = (\tau p_d)^{-\sigma}(P^*_{T|n})^{\sigma-1}w^*. \tag{C.2}$$

Prime represents consumption for import brand. The above equations show that the elasticity of demand for price is σ regardless of source countries.

From definition of iceberg cost τ, export revenue is defined as $r_x \equiv p_x y_x/\tau$. Since mill price in export market is $p_x = \tau p_d$, export revenue can be rewritten as $r_x = \tau p_d y_x/\tau = p_d y_x$. Total revenue from all markets $r = r_d + n r_x$ can be rewritten as $r = p_d y_d + p_d n y_x = p_d y$. Total profit from all markets π is

$$\pi = p_d y - TC(y). \tag{C.3}$$

Market clear condition for home country's brand is

$$y = Lc + n\tau L^* c^{*\prime}. \tag{C.4}$$

(C.1), (C.2), and (C.4) derive

$$y = L[p_d^{-\sigma} P_T^{\sigma-1} w] + n\tau L^*[(\tau p_d)^{-\sigma} P_T^{*\sigma-1} w^*]. \tag{C.5}$$

This shows that each firm faces individual demand curve whose elasticity of demand for price is σ. From (C.3), and (C.5), profit maximization problem derives

$$PP_{|n} : p_d = \mu MC(y).$$

Appendix D: Proof of Proposition 5.

Social Welfare in closed economy

We treat representative household's utility as a measure of social welfare. Under the utility maximization, indirect utility function of each household is $V_A = (w/P)_A$. On equilibrium, firms set identical price, p and from the definition of P, the following relation is given:

$$V_A = \left(\frac{w}{p}\right)_A M_A^{\frac{1}{\sigma-1}}. \tag{D.1}$$

Note that the indirect utility can be decomposed to real wage rate and the number of varieties. We substitute $(p/w)_A$ and M_A into (D.1) and consequently, obtain equilibrium social welfare as follows:

$$V_A = (2\gamma f)^{-1} L^{\frac{1}{\sigma-1}} (B+1)^{\frac{-\sigma}{\sigma-1}} B^{\frac{2-\sigma}{\sigma-1}} f_d^{\frac{\sigma-2}{\sigma-1}}. \tag{D.2}$$

Social Welfare in open economy economy

In trading equilibrium, the real wage rates are identical in the all countries and hence, the indirect function is given by $V_T = (w/P)_T$.

Consumers of home country face $(n_T/\bar{n})M$ brands on the average per one foreign country. Then, P_T can be rewritten as follows:

$$\begin{aligned} P_T^{1-\sigma} =& \int_{\theta\in\Theta} (p(\theta))^{1-\sigma}\, d\theta + \bar{n} \int_{\theta^*\in\Theta^*} [\tau p(\theta^*)]^{1-\sigma}\, d\theta^* \\ =& M_T p_{d,T}^{1-\sigma} + \bar{n}\left(\frac{n_T}{\bar{n}}\right) M_T \tau^{1-\sigma} p_{d,T}^{1-\sigma} \\ =& M_T p_{d,T}^{1-\sigma}(1 + n_T\tau^{1-\sigma}). \end{aligned} \tag{D.3}$$

Since countries are symmetric, the social welfare is obtained by

$$V_T = \left(\frac{w}{p_d}\right)_T \left[(1 + n_T\tau^{1-\sigma})M_T\right]^{\frac{1}{\sigma-1}}. \tag{D.4}$$

By substituting $(p_d/w)_{T|n}$ and $M_{T|n}$ into (D.4), we can obtain equilibrium social welfare as follows

$$V_T = (2\gamma f)^{-1} L^{\frac{1}{\sigma-1}} (B+1)^{\frac{-\sigma}{\sigma-1}} B^{\frac{2-\sigma}{\sigma-1}} (f_d + n_T^{\alpha} f_x)^{\frac{\sigma-2}{\sigma-1}} (1 + n_T \tau^{1-\sigma})^{\frac{1}{\sigma-1}}.$$

This expression is rewritten as

$$V_T = V_A G(n_T)^{\frac{1}{\sigma-1}}. \tag{D.5}$$

Comparing V_T of (D.5) to V_A of (D.2), the following relationship is obtained:

$$V_T > V_A \leftrightarrow G(n_T) > 1.$$

Appendix E: Decomposition of gains from trade

Productivity effect is positive from Proposition 3. In the contrast to this, does the number of the effective varieties increase through the opening up to trade ? The change in the number of the effective varieties depends on parameters as follows. Then, we obtain Proposition 7.

Proposition 7. *Under Assumption 1 and 2, gains from opening trade is decomposed to productivity effect and effective variety effect.*

(I) When $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ holds, both productive effect and effective variety effect are positive.

(II) When $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ does not hold, effective variety effect is non-positive and then, positive productivity effect dominates this effect and gains from opening trade exists.

Proof. (I) Productivity effect is positive from Proposition 3. The number of the effective varieties in the autarkic equilibrium is $(M_A)^{1/(\sigma-1)}$. Note that $M_W > (M_A)^{1/(\sigma-1)}$ is equivalent to $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$. In fact, From M_A and the definition of M_W, we can get

$$\frac{M_W}{(M_A)^{1/(\sigma-1)}} = \left[(1 + n_T \tau^{1-\alpha}) \frac{f_d}{f_d + n_T^{\alpha} f_x} \right]^{\frac{1}{\sigma-1}}.$$

From this relation, $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ is equivalent to $M_W > (M_A)^{1/(\sigma-1)}$. Furthermore, $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ is equivalent to $\tau^{1-\sigma} n_T > n_T^{\alpha} f_x / f_d$. Then, we can obtain

$$1 + \tau^{1-\sigma} n_T > 1 + n_T^{\alpha} \frac{f_x}{f_d} > \left(1 + n_T^{\alpha} \frac{f_x}{f_d} \right)^{2-\sigma}.$$

Hence, $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ implies $G(n_T) > 1$. From $G(n_T) > 1 \leftrightarrow V_T > V_A$, $V_T > V_A$ holds.

(II) We show that intersection of set of $G(n) > 1$ and complement set of $f_d > n_T^{\alpha-1}\tau^{\sigma-1} f_x$ is not empty. We assume $\alpha = 1$, $\sigma = 3/2$, $\tau = 4$, and $f_x/f_d = 2$. Then,

$1 < n_T^{\alpha}\tau^{\sigma-1}f_x/f_d$ holds from $1 < 4^{1/2}2 = 4$. $\bar{n} > 4$ is equivalent to the following relation:

$$1 + 4^{-1/2}\bar{n} > (1 + 2\bar{n})^{1/2}.$$

That is, $I(n_T) > H(n_T)$ holds. Hence, there is a pair of $(\alpha, \sigma, \tau, f_x, f_d, \bar{n})$ such that satisfies both $G(n_T) > 1$ and $f_d < n_T^{\alpha}\tau^{\sigma-1}f_x$. Therefore, intersection of set of $G(n_T) > 1$ and complement set of $f_d > n_T^{\alpha-1}\tau^{\sigma-1}f_x$ is not empty. Q.E.D.

The condition $f_d > n_T^{\alpha-1}\tau^{\sigma-1}f_x$ demands that trade costs (combination of τ and f_x) and n are sufficiently low relative to f_d and also that entry gain is sufficiently high and entry loss is sufficiently low.

Appendix F: Proof of Proposition 6.

We present the proof of Proposition 6 by using results of Appendix L.

Property $dy_T/d\bar{n}$, $dl_T/d\bar{n}$, and $dt_T/d\bar{n}$ under full entry regime

We can immediately obtain $dy_T/d\bar{n} > 0$, $dl_T/d\bar{n} > 0$, and $dt_T/d\bar{n} > 0$ by differentiating y_T, l_T, and t_T with respect to $\bar{n}$, respectively.

Property $dM_W/d\bar{n}$ under full entry regime

From $M_W = [(1 + \bar{n}^{\alpha}\tau^{1-\sigma})M_T]^{1/(\sigma-1)}$, we can obtain the following condition:

$$\frac{dM_W}{d\bar{n}} = \frac{M_W^{2-\sigma}}{\sigma - 1}\left[\frac{d(1 + \bar{n}^{\alpha}\tau^{1-\sigma})}{d\bar{n}}M_T + (1 + \bar{n}^{\alpha}\tau^{1-\sigma})\frac{dM_T}{d\bar{n}}\right] = \frac{\alpha n^{\alpha-1}(\tau^{1-\sigma}f_d - f_x)}{(\sigma - 1)(f_d + \bar{n}^{\alpha}f_x)}M_T M_W^{2-\sigma}.$$

This condition implies

$$\frac{dM_W}{d\bar{n}} > 0 \leftrightarrow f_d > \tau^{\sigma-1}f_x. \tag{F.1}$$

Hence, the effective variety effect is ambiguous.

Changes in welfare under full entry regime

We have $V_T = V_A G(n_T)$ from (D.5). Note the following conditions are satisfied:

$$\frac{dV_A}{d\tau} = \frac{dV_A}{df_x} = \frac{dV_A}{d\alpha} = \frac{dV_A}{d\bar{n}} = 0.$$

Hence, changes in V_T depend only on changes in $G(n_T)$.

Change in τ does not change n_T and then, we can get.

$$\frac{dG(n_T)}{d\tau} = \frac{\partial G(n_T)}{\partial \tau} + \frac{dG(n_T)}{dn_T}\underbrace{\frac{dn_T}{d\tau}}_{0} = \frac{\partial G(n_T)}{\partial \tau}.$$

In the similar manner, we can obtain $dG(n_T)/df_x = \partial G(n_T)/\partial f_x$ and $dG(n_T)/d\alpha = \partial G(n_T)/\partial \alpha$. Then, we can obtain the following conditions:

$$\frac{\partial G(n_T)}{\partial \tau} = -(\sigma - 1)\frac{n_T \tau^{-\sigma}}{\left(1 + n_T^{\alpha}\frac{f_x}{f_d}\right)^{2-\sigma}} < 0,$$

$$\frac{\partial G(n_T)}{\partial f_x} = -(2 - \sigma)\frac{(1 + n_T \tau^{-\sigma})\frac{n_T^{\alpha}}{f_d}}{\left(1 + n_T^{\alpha}\frac{f_x}{f_d}\right)^{3-\sigma}} < 0,$$

$$\frac{\partial G(n_T)}{\partial \alpha} = -(2 - \sigma)\frac{(1 + n_T \tau^{-\sigma})\frac{f_x}{f_d}\frac{\partial n_T^{\alpha}}{\partial \alpha}}{\left(1 + n_T^{\alpha}\frac{f_x}{f_d}\right)^{3-\sigma}} < 0,$$

where $\partial n_T^{\alpha}/\partial \alpha = n_T^{\alpha} \log n_T$ holds.

These implies $\partial V_T/\partial \tau < 0$, $\partial V_T/\partial f_x < 0$, and $\partial V_T/\partial \alpha < 0$. In a decrease in τ, positive effective variety effect and no productivity effect leads to an increase in welfare. In a decrease in f_x and α, positive effective variety effect dominates negative productivity effect and this leads to an increase in welfare.

A change in $\bar{n}$ changes n_T. Then, we can obtain the following condition:

$$\frac{dG(n_T)}{d\bar{n}} = \underbrace{\frac{dG(n_T)}{dn_T}}_{+}\underbrace{\frac{dn_T}{d\bar{n}}}_{+} > 0,$$

where $dn_T/d\bar{n} = 1$ holds from $n_T = \bar{n}$ and $dG(n_T)/dn_T > 0$ holds from Appendix L.

The above result is explained as follows. A increase in $\bar{n}$ raises both entry gain and loss. Entry gain dominates entry loss. Hence, welfare increases. In other words, even if the effective variety effect is negative, the positive productivity effect dominates the negative effective variety effect and hence, welfare increases.

The relation between the change in $l_{T|n}$ and changes of the other endogenous variables under partial entry regime

We consider the relation between the change in $l_{T|n}$ and changes of the other endogenous variables in order to investigate an effect of trade liberalization.

From production function, we can get

$$\frac{dy_{T|n}}{dl_{T|n}} = \frac{l_{T|n}}{\gamma f} > 0.$$

Under given n, $t(y) = [\gamma y/(2f)]^{1/2}$ and $y = l^2/(2\gamma f)$ give $t_{T|n} = l_{T|n}/(2f)$. Then, we can get

$$\frac{dt_{T|n}}{dl_{T|n}} = \frac{1}{2f} > 0.$$

Labor market clear conditions give $M_{T|n} = L/(l_{T|n} + f_d + n_T^\alpha f_x)$. Then, we can obtain the following condition:

$$\frac{dM_{T|n}}{dl_{T|n}} = -\frac{L}{(l_{T|n} + f_d + n_T^\alpha f_x)^2} < 0.$$

From the above conditions, we can the impacts of the change in τ as follows:

$$\frac{dy_T}{d\tau} = \underbrace{\frac{dy_{T|n}}{dl_{T|n}}}_{+} \frac{dl_{T|n}}{d\tau},$$

$$\frac{dt_T}{d\tau} = \underbrace{\frac{dt_{T|n}}{dl_{T|n}}}_{+} \frac{dl_{T|n}}{d\tau},$$

$$\frac{dM_T}{d\tau} = \underbrace{\frac{dM_{T|n}}{dl_{T|n}}}_{-} \frac{dl_{T|n}}{d\tau}.$$

We can pin down these directions by determining the direction of $dl_{T|n}/d\tau$. The similar arguments hold the impacts of the changes in f_x and α. Hence, we check the directions of $dl_{T|n}/d\tau$, $dl_{T|n}/df_x$, and $dl_{T|n}/d\alpha$ in the following analysis.

Property of $dl/d\tau$ under partial entry regime

We differentiate $l_{T|n}$ for τ and we can obtain

$$\frac{dl_{T|n}}{d\tau} = \frac{dl_{T|n}}{dn_T}\frac{dn_T}{d\tau}. \tag{F.2}$$

$dl_{T|n}/dn_T$ satisfies

$$\frac{dl_{T|n}}{dn_T} > 0. \tag{F.3}$$

How about $dn_T/d\tau$? By calculating total differentiation for (L.1) of Appendix L which characterize n_T and we can obtain

$$(\sigma-1)\tau^{\sigma-2}d\tau+\frac{(2-\sigma)\alpha-1}{(2-\sigma)a}dn_T=-\frac{\alpha-1}{(2-\sigma)\alpha}\frac{f_d}{f_x}\frac{1}{n_T^{\alpha}}dn_T.$$

Rearranging this equation, we have

$$\frac{dn_T}{d\tau}=-(2-\sigma)\alpha\frac{(\sigma-1)\tau^{\sigma-2}}{[(2-\sigma)\alpha-1]+[(\alpha-1)\frac{f_d}{f_x}n_T^{-\alpha}]}.$$

Partial entry regime demands $\alpha-1>0$ and $(2-\sigma)\alpha-1\geq 0$. Hence, we can obtain $dn_T/d\tau<0$.

From $dn_T/d\tau<0$, (F.2), and (F.3), we can obtain the following condition:

$$\frac{dl_{T|n}}{d\tau}=\underbrace{\frac{dl_{T|n}}{dn_T}}_{+}\underbrace{\frac{dn_T}{d\tau}}_{-}<0.$$

Property of dl/df_x under partial entry regime

We differentiate $l_{T|n}$ for f_x and we can obtain

$$\frac{dl_{T|n}}{df_x}=\frac{dl_{T|n}}{dn_T^{\alpha}f_x}\frac{dn^{\alpha}f_x}{df_x}=\frac{1}{B}\frac{dn^{\alpha}f_x}{df_x}=\frac{1}{B}\left(\frac{\partial n^{\alpha}f_x}{\partial f_x}+\frac{\partial n_T^{\alpha}f_x}{\partial n}\frac{dn_T}{df_x}\right). \tag{F.4}$$

By calculating total differentiation for (L.1) of Appendix L which characterize n_T and we can obtain

$$\frac{(2-\sigma)\alpha-1}{(2-\sigma)\alpha}dn_T=-\frac{1}{(2-\sigma)\alpha}\frac{f_d}{n_T^{\alpha-1}}\frac{1}{f_x^2}df_x-\frac{\alpha-1}{(2-\sigma)\alpha}\frac{f_d}{f_x}\frac{1}{n_T^{\alpha}}dn_T.$$

Rearranging this equation, we have

$$\frac{dn_T}{df_x}=-(2-\sigma)\alpha\frac{\frac{1}{(2-\sigma)\alpha}\frac{f_d}{f_x^2}n_T^{-(\alpha-1)}}{[(2-\sigma)\alpha-1]+[(\alpha-1)\frac{f_d}{f_x}n_T^{-\alpha}]}<0. \tag{F.5}$$

We differentiate $n_T^{\alpha}f_x$ for f_x and we can obtain

$$\begin{aligned}\frac{dn_T^{\alpha}f_x}{df_x}=&n_T^{\alpha}+f_x\alpha n_T^{\alpha-1}\frac{dn_T}{df_x}\\=&n_T^{\alpha}+\alpha f_x(-1)(2-\sigma)\alpha\frac{\frac{1}{(2-\sigma)\alpha}\frac{f_d}{f_x^2}n_T^{-(\alpha-1)}}{[(2-\sigma)\alpha-1]+[(\alpha-1)\frac{f_d}{f_x}n_T^{-\alpha}]}\end{aligned}$$

$$= -\frac{\tau^{\sigma-1}(2-\sigma)\alpha}{[(2-\sigma)\alpha-1]+[(\alpha-1)\frac{f_d}{f_x}n_T^{-\alpha}]} < 0. \tag{F.6}$$

From (F.4), (F.5), and (F.6), we can obtain the following condition:

$$\frac{dl_{T|n}}{df_x} = \frac{1}{B}\left(\underbrace{\frac{\partial n_T^{\alpha} f_x}{\partial f_x}}_{+} + \underbrace{\frac{\partial n_T^{\alpha} f_x}{\partial n_T}}_{+}\underbrace{\frac{dn_T}{df_x}}_{-}\right) < 0.$$

Property of dl/da under partial entry regime

We differentiate $l_{T|n}$ for α and we can obtain

$$\begin{aligned}\frac{dl_{T|n}}{d\alpha} &= \frac{dl_{T|n}}{dn_T^{\alpha} f_x}\frac{dn_T^{\alpha} f_x}{d\alpha}\\ &= \frac{1}{B}\frac{dn_T^{\alpha} f_x}{d\alpha}\\ &= \frac{f_x}{B}\frac{dn_T^{\alpha}}{d\alpha}.\end{aligned} \tag{F.7}$$

By calculating total differentiation for (L.1) of Appendix L which characterize n_T and we can obtain

$$\frac{(2-\sigma)\alpha-1}{(2-\sigma)\alpha}dn_T + \frac{n}{(2-\sigma)\alpha^2}d\alpha = -\frac{\alpha-1}{(2-\sigma)\alpha}\frac{f_d}{f_x}\frac{1}{n_T^{\alpha}}dn_T - \frac{f_d}{f_x}\frac{n_T^{-\alpha}}{(2-\sigma)\alpha}\left[\frac{n_T}{\alpha}+(\alpha-1)\right]d\alpha.$$

Rearranging this equation, we have

$$\frac{dn_T}{d\alpha} = -\frac{n}{(2-\sigma)\alpha}\frac{\frac{1}{\alpha}+\frac{f_d}{f_x}n_T^{-(\alpha+1)}\left[\frac{n_T}{\alpha}+(\alpha-1)\right]}{[(2-\sigma)\alpha-1]+[(\alpha-1)\frac{f_d}{f_x}n_T^{-\alpha}]} < 0. \tag{F.8}$$

Rearranging (L.1) of Appendix L, we have

$$\frac{f_d}{f_x}n_T^{-\alpha} = (2-\sigma)\alpha - 1 + \frac{(2-\sigma)\alpha\tau^{\sigma-1}}{n_T}.$$

By calculating total differentiation for this equation and we can obtain

$$\frac{f_d}{f_x}dn_T^{-\alpha} = (2-\sigma)d\alpha + \frac{(2-\sigma)\tau^{\sigma-1}}{n_T}d\alpha + (-1)n_T^{-2}(2-\sigma)\alpha\tau^{\sigma-1}dn_T.$$

Rearranging this equation, from (F.8) we have

$$\frac{dn_T^{-\alpha}}{d\alpha} = \frac{f_x}{f_d}\left[(2-\sigma)\left(1+\frac{\tau^{\sigma-1}}{n_T}\right)\right] - \frac{dn_T}{da}n_T^{-2}(2-\sigma)\alpha\tau^{\sigma-1} > 0.$$

Then, we can obtain

$$\frac{dn_T^{\alpha}}{d\alpha} = \frac{dn_T^{\alpha}}{dn_T^{-\alpha}}\frac{dn_T^{-\alpha}}{d\alpha} = (-1)(n_T^{-\alpha})^2\frac{dn_T^{-\alpha}}{d\alpha} < 0. \tag{F.9}$$

From (F.7) and (F.9), we can obtain the following condition:

$$\frac{dl_{T|n}}{d\alpha} = \frac{f_x}{B}\underbrace{\frac{dn_T^{\alpha}}{d\alpha}}_{-} < 0.$$

Changes in the number of effective under partial entry regime

A change in τ changes n_T and then, we can obtain

$$\frac{dM_W}{d\tau} = \frac{(M_W)^{2-\sigma}}{\sigma-1}\left(\underbrace{\frac{\partial M_W}{\partial \tau}}_{-} + \frac{dM_W}{dn_T}\underbrace{\frac{dn_T}{d\tau}}_{-}\right),$$

where dM_W/dn_T satisfies as follows:

$$\frac{dM_W}{dn_T} = \underbrace{\frac{d(1+n_T\tau^{1-\sigma})}{dn_T}}_{+}M_T + (1+n_T\tau^{1-\sigma})\underbrace{\frac{dM_T}{dn_T}}_{-}.$$

Because dM_W/dn_T is ambiguous, $dM_W/d\tau$ also is ambiguous.

A change in f_x changes n_T and then, we can obtain the following condition:

$$\frac{dM_W}{df_x} = \frac{(M_W)^{2-\sigma}}{\sigma-1}\frac{dM_W}{dn_T}\underbrace{\frac{dn_T}{df_x}}_{-},$$

Because dM_W/dn_T is ambiguous, dM_W/df_x also is ambiguous. In the similar to this, $dM_W/d\alpha$ also is ambiguous.

Changes in welfare under partial entry regime

The changes in V_T depend only on changes in $G(n_T)$ from $V_T = V_A G(n_T)$ in the similar manner to that of full entry regime.

A change in τ changes n_T and then, we can obtain the following condition:

$$\frac{dG(n_T)}{d\tau} = \frac{\partial G(n_T)}{\partial \tau} + \underbrace{\frac{dG(n_T)}{dn_T}}_{0}\frac{dn_T}{d\tau} = \frac{\partial G(n_T)}{\partial \tau},$$

where $dG(n_T)/dn_T = 0$ holds from Lemma 2.

In the similar manner, we can obtain $dG(n_T)/df_x = \partial G(n_T)/\partial f_x$ and $dG(n_T)/d\alpha = \partial G(n_T)/\partial \alpha$. Then, we can obtain $\partial V_T/\partial \tau < 0$, $\partial V_T/\partial f_x < 0$, and $\partial V_T/\partial \alpha < 0$ in the similar manner to that of full entry regime. Q.E.D.

Appendix G: Trading equilibrium in the short run and the market size effect

Trading equilibrium in the short run

Up to the previous section, we have studied equilibria where firms can enter and exit freely any markets. That is, these equilibria have time span in which entry and exit can be adjusted. We call such a time span long run. In this section, we study trade equilibrium in the short run in which the number of export markets firms enter, n, and the number of firms, M, can not be adjusted. In particular, zero profit condition is not imposed.

We need this short run equilibrium to decompose effects of trade liberalization into short run and long run effect. and prove Lemma 2 of Appendix J.

Market clearing condition for final goods of home country firms is given by

$$\begin{aligned}
y =& Lc + n\tau Lc^{*\prime} \\
=& LIp_d^{-\sigma} P_T^{\sigma-1} + n\tau LI(\tau p_{d,n})^{-\sigma} P_T^{\sigma-1} && \text{by (C.1) and (C.2)} \\
=& LIp_d^{-\sigma} P_T^{\sigma-1}(1 + n\tau^{1-\sigma}) \\
=& LIp_d^{-1} M^{-1}. && \text{by (D.3)} \qquad \text{(G.1)}
\end{aligned}$$

(G.1) derives total revenue of home country:

$$r = p_d y = LIM^{-1}. \qquad \text{(G.2)}$$

Firms input $(2\gamma f y)^{1/2} + (f_d + nf_x)$ units of labor and this derives the following total cost function:

$$TC = w\left[(2\gamma f y)^{1/2} + (f_d + n^{\alpha} f_x)\right]. \qquad \text{(G.3)}$$

This equation and optimal pricing rule gives

$$p_d = w(B+1)(2\gamma f)^{1/2} y^{-1/2}. \qquad \text{(G.4)}$$

By multiplying both sides of equation (G.4) by y, we have

$$p_d y = w(B+1)(2\gamma f y)^{1/2}.$$

Rearranging this equation, we can obtain

$$w(2\gamma f y)^{1/2} = \frac{r}{B+1}. \tag{G.5}$$

(G.3) and (G.5) gives the following optimal total cost function of short run:

$$TC = \frac{r}{B+1} + w(f_d + n^{\alpha} f_x). \tag{G.6}$$

We substitute (G.2) and (G.6) into $\pi = r - TC$ and obtain

$$\begin{aligned}
\pi =& r - TC \\
=& r - \left[\frac{r}{B+1} + w(f_d + n^{\alpha} f_x)\right] \\
=& \frac{B}{B+1} r - w(f_d + n^{\alpha} f_x) \\
=& \frac{B}{B+1} \frac{LI}{M} - w(f_d + n^{\alpha} f_x).
\end{aligned} \tag{G.7}$$

(G.7) and household's income $I = w + M\pi/L$ give the following conditions:

$$\frac{I}{w} = (B+1)\left[1 - \frac{M(f_d + n^{\alpha} f_x)}{L}\right], \tag{G.8}$$

$$\frac{\pi}{w} = \frac{BL}{M} - (B+1)(f_d + n^{\alpha} f_x). \tag{G.9}$$

(G,1) and (G.4) gives

$$y = \frac{[L(I/w)]^2}{(2\gamma f)[(B+1)M]^2}. \tag{G.10}$$

(G.8) and (G.10) gives

$$y_S = \frac{\left[\frac{L}{M} - (f_d + n^{\alpha} f_x)\right]^2}{2\gamma f}. \tag{G.11}$$

where subscript "S" represents variables in the short run trading equilibrium.

(G.4) and (G.11) gives

$$\left(\frac{p_d}{w}\right)_S = \frac{(B+1)(2\gamma f)}{L/M - (f_d + n^{\alpha} f_x)}.$$

From (G.11) and production function $y = l^2/2\gamma f$, we can obtain labor input on production divisions, l as follows:

$$l_S = \frac{L}{M} - (f_d + n^{\alpha} f_x), \tag{G.12}$$

(G.12), production function and optimal team numbers $t(y) = [\gamma y/(2f)]^{1/2}$ gives t in the short run equilibrium as follows:

$$t_S = \frac{l_S}{2f} = \frac{1}{2f}\left[\frac{L}{M} - (f_d + n^{\alpha} f_x)\right]. \tag{G.13}$$

(G.13) gives Proposition 8.

Proposition 8. *In the short run equilibrium of both regimes, an increase in market size L promotes the division of labor within firms.*

As Proposition 2, this gives firms excess profits. However, new firms enter in the long run and then, zero profit condition is achieved.

Appendix H: Labor reallocation across and within firms behind the division of labor promoted in trade liberalization

In this appendix, we consider labor reallocation behind the division of labor promoted in trade liberalization by decomposing the effect of trade liberalization on the the division of labor into the short run effect and the long run effect.

Trade liberalization in the short run

To begin with, we consider the division of labor caused by trade liberalization in the short run.

From (G.13), comparative statics in the short run is obtained immediately in Proposition 9 as follows.

Proposition 9. *In the short run equilibrium of both regimes, comparative statics for t_S is obtained as follows;*

(I) A decrease in τ does not change t_S.

(II) A decrease in f_x raises t_S.

(III) An decrease in α raises t_S.

(IV) Under $\alpha = 1$, an increase in $\bar{n}$ does not change t_S.

We can explain a mechanism behind the above results form a view of labor allocation.

(G.12) is equivalent to the following equation:

$$\overbrace{\underbrace{l_S}_{\textbf{production division}} + \underbrace{(f_d + n^{\alpha} f_x)}_{\textbf{headquarter division}}}^{\textbf{Total labor input per one firm}} = \underbrace{\left(\frac{L}{M}\right)}_{\textbf{constant}}.$$

This means that there is *no* labor reallocation across firms by trade liberalization in the short run because total labor input per one firm is fixed at L/M. All labor reallocations by trade liberalization in the short run are caused *within firms.*

A decrease in f_x and α induce firms to increase labor input in production divisions through the reduction of labor input in headquarter division. Then, they can promote the division of labor. In the contrast with this, a decrease in τ and $\bar{n}$ does not change labor input in headquarter division and then, can not promote the division of labor. Note that a increase in $\bar{n}$ does not change n in the short run under because n is fixed in the short run.

Trade liberalization in the long run under full entry regime

In the long run, M and n also are endogenous. Then, the division of labor promoted by trade liberalization can be decomposed into two effects, *reallocation effect across firms* and *reallocation effect within firms.* Reallocation effect across firms indicates that the division of labor promoted through changes in total labor input per one firms and the number of firms. Reallocation effect within firms indicates that the division of labor promoted through the another channel.

From (G.12), (G.13) and results of Proposition 6, we can obtain the following conditions for full entry regime:

$$\frac{dt_T}{d\tau} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial \tau}}_{0}}^{\textbf{short run (0)}} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{d\tau}}_{0} \right)}_{\textbf{reallocation within firms (0)}} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{d\tau}}_{0}}_{\textbf{reallocation across firms (0)}} = 0,$$

$$\frac{dt_T}{df_x} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial f_x}}_{-}}^{\textbf{short run }(-)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{df_x}}_{0} \right)}_{\textbf{reallocation within firms }(-)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{df_x}}_{-}}_{\textbf{reallocation across firms }(+)} > 0,$$

$$\frac{dt_T}{d\alpha} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial \alpha}}_{-}}^{\textbf{short run }(-)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{d\alpha}}_{0} \right)}_{\textbf{reallocation within firms }(-)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{d\alpha}}_{-}}_{\textbf{reallocation across firms }(+)} > 0,$$

and under $\alpha = 1$,

$$\frac{dt_T}{d\bar{n}} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial \bar{n}}}_{0}}^{\textbf{short run }(0)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{d\bar{n}}}_{+} \right)}_{\textbf{reallocation within firms }(-)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{d\bar{n}}}_{-}}_{\textbf{reallocation across firms }(+)} > 0.$$

Trade liberalization in the long run under partial entry regime

The next, we consider the decomposition for partial entry regime. From (G.12), (G.13) and results of Proposition 6, we can obtain the following conditions:

$$\frac{dt_T}{d\tau} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial \tau}}_{0}}^{\textbf{short run }(0)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{d\tau}}_{-} \right)}_{\textbf{reallocation within firms }(+)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{d\tau}}_{+}}_{\textbf{reallocation across firms }(-)} < 0.$$

This shows that the effect of labor reallocation *across* firms (negative effect) dominates the effect of that *within* firms (positive effect).

We can obtain the similar relation for change in α as follows:

$$\frac{dt_T}{df_x} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial f_x}}_{-}}^{\textbf{short run }(-)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{df_x}}_{-} \right)}_{\textbf{reallocation within firms }(+)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{df_x}}_{+}}_{\textbf{reallocation across firms }(-)} < 0.$$

The term of reallocation within firms is positive from the following reason. This term can be rewritten as follows from (G.12):

$$\frac{dt_S}{dl_S}\frac{\partial l_S}{\partial f_x} + \frac{dt_S}{dl_S}\frac{\partial l_S}{\partial n_T}\frac{dn_T}{df_x} = \frac{dt_S}{dl_S}\left(\frac{\partial l_S}{\partial f_x} + \frac{\partial l_S}{\partial n_T}\frac{dn_T}{df_x}\right) = \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T^{\alpha} f_x}}_{-} \underbrace{\frac{dn_T^{\alpha} f_x}{df_x}}_{-} > 0.$$

Note that $dn_T^{\alpha} f_x / df_x < 0$ is proved at (F.6).

We can obtain the similar relation for change in α as follows:

$$\frac{dt_T}{d\alpha} = \underbrace{\left(\overbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial \alpha}}_{-}}^{\textbf{short run } (-)} + \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T}}_{-} \underbrace{\frac{dn_T}{d\alpha}}_{-} \right)}_{\textbf{reallocation within firms } (+)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial M_T}}_{-} \underbrace{\frac{dM_T}{d\alpha}}_{+}}_{\textbf{reallocation across firms } (-)} < 0.$$

The term of reallocation within firms is positive from the following reason. This term can be rewritten as follows from (G.12):

$$\frac{dt_S}{dl_S}\frac{\partial l_S}{\partial \alpha} + \frac{dt_S}{dl_S}\frac{\partial l_S}{\partial n_T}\frac{dn_T}{d\alpha} = \frac{dt_S}{dl_S}\left(\frac{\partial l_S}{\partial \alpha} + \frac{\partial l_S}{\partial n_T}\frac{dn_T}{d\alpha}\right) = \underbrace{\frac{dt_S}{dl_S}}_{+} \underbrace{\frac{\partial l_S}{\partial n_T^\alpha f_x}}_{-} \underbrace{\frac{dn_T^\alpha f_x}{d\alpha}}_{-} > 0.$$

Note that $dn_T^\alpha f_x/d\alpha < 0$ holds because (F.9) proves $dn_T^\alpha/d\alpha < 0$ and hence, this implies immediately $dn_T^\alpha f_x/d\alpha < 0$.

Summary of the results

We can summarize the above results as follows.

Proposition 10. *Under both regimes, the division of labor promoted by trade liberalization can be decomposed into reallocation effect within and across firms. In all cases of τ, f_x, α, and $\bar{n}$, reallocation effect across firms dominates reallocation effect within firms.*

In this sense, both regimes have the same mechanism for the relation between the division of labor promoted and reallocation of labor.

A graphical intuition for a decrease in f_x under partial regime

These properties seem to be novel. For a decrease in f_x under partial regime, we can get a graphical intuition as shown in Figure A.3 by decomposing the effect on marginal productivity into three effects. Figure A.3 illustrates three production curves, PC 1, PC 2, and PC 3 (PC 4) , in ($l^{\text{total}} - y$) space. l^{total} is firm's total labor inputs. That is, l^{total} is defined as $l^{\text{total}} = l + f_d + n^\alpha f_x$. Note that from this definition, $l_T^{\text{total}} - (f_d + n_T^\alpha f_x)$ means labor input of production division, l_T. Let l^h be labor input of headquarter division. $l_{T,j}^{\text{total}}$ and l_j^h represent variables at j-th stage where $j \in \{1, 2, 3, 4\}$. In the first stage at the initial equilibrium, we have each firm's employment and production which is represented by point A on PC 1. In the second stage after f_x decrease and before n_T and M_T changes, we have that represented by point B. In the third stage after f_x decrease and n_T increases

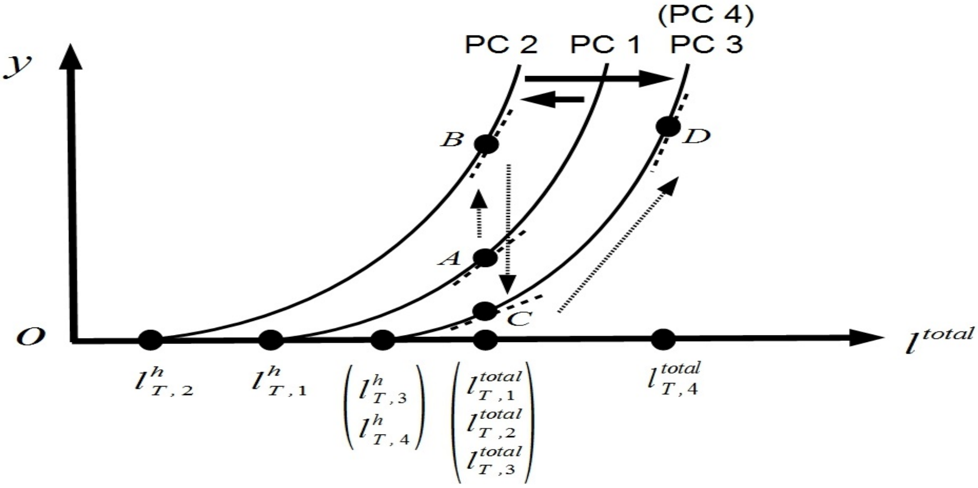

Figure A. 3: production curve and the change in MPL.

and before M_T changes, we have that represented by point C. In the forth stage after f_x decrease and n_T increases, and M_T decreases, we have that represented by point D.

The first effect of a decrease in f_x on marginal productivity is a transition from point A on PC 1 to point B on PC 2. PC 2 means production function with $l^h_{T,2} = f_d + n^\alpha_{T,2} f_{x,2}$ where $f_{x,2} < f_{x,1}$ and $n^\alpha_{T,2} = n^\alpha_{T,1}$. In this transition, $l_{T,1}$ increases by interval $l^h_{T,1} l^h_{T,2}$. This indicates that in the short run, firms reassign labor of interval $l^h_{T,1} l^h_{T,2}$ from the management division to the production division ($l^h_{T,2} < l^h_{T,1}$) while keeping $l^{\text{total}}_{T,1}$ units of total labor ($l^{\text{total}}_{T,2} = l^{\text{total}}_{T,1}$). This reassignment effect on the number of teams and productivity is positive as shown in Figure A.3 where the slope of the tangent increases.

The second effect is a transition from point B on PC 2 to point C on PC 3. PC 3 means production function with $l^h_{T,3} = f_d + n^\alpha_{T,3} f_{x,3}$ where $f_{x,3} = f_{x,3}$ and $n^\alpha_{T,3} > n^\alpha_{T,2}$. Just after f_x decreased, all incumbent firms earn positive profit. This makes these firms enter more markets ($n^\alpha_{T,3} > n^\alpha_{T,2}$) and reassign labor from the production division to the management division ($l^h_{T,3} > l^h_{T,2}$) while keeping total labor input ($l^{\text{total}}_{T,3} = l^{\text{total}}_{T,2}$). This reassignment effect on the number of teams and productivity is negative. This effect dominates the first effect. Hence, the slope of the tangent at point C is less than that at point A. To put it differently, in the long run, labor input of the production division decreases (negative reallocation effect within firms).

The third effect is a transition from point C PC 3 (PC 4) to point D on PC 3 (PC 4). At point C, all firms earn negative profit. This makes some firms exit and concentrates workers on survived firms ($l^{\text{total}}_{T,4} > l^{\text{total}}_{T,3}$). In this transition, l_T increases by interval $l^{\text{total}}_{T,1}$ $l^{\text{total}}_{T,4}$. This indicates that these firms succeed in recruiting new workers and assign them jobs of production division. This concentration effect on the number of teams and productivity is positive as shown in Figure A.3 where the slope of the tangent increases (positive reallocation effect across firms).

A transition from point A to point C reduces labor input of production division by the interval $l_{T,3}^{h} l_{T,1}^{total}$ (negative reallocation effect within firms) while a transition from point C to point D raises labor input of production division by the interval $l_{T,1}^{total} l_{T,4}^{total}$ (positive reallocation effect across firms). Since the interval $l_{T,1}^{total} l_{T,4}^{total}$ is greater than the interval $l_{T,3}^{h} l_{T,1}^{total}$, the slope of the tangent at point D is greater than that at point A.

The above results indicates that an essential source of the division of labor in the long run is labor reallocation across firms (the recruiting competition for survival). However, we should note that the direction of labor reallocation across firms depends on firm's entry for export markets. In this sense, ascending order of entry costs determining the number of markets firms enter plays a key role in trade liberalization.

Appendix I: Lemma 1 and the proof.

Lemma 1 and Lemma 2 derive the following Proposition 4. In this section, we consider Lemma 1.

Introduction of Lemma 1

Can the numbers of export markets that firms enter n are distributed ? The lemma as follows indicates that the numbers of export markets that firms enter are not distributed in a certain condition.

Lemma 1. *All firms enter n export markets if and only if the following each of (I) - (IV) condition is satisfied:*

(I) $(2-\sigma)\alpha \geq 1 \wedge G(n) < 1$,

(II) $(2-\sigma)\alpha \geq 1 \wedge G(n) > 1 \wedge G'(n) = 0$,

(III) $(2-\sigma)\alpha < 1 \wedge G(n) < 1 \wedge G'(n) = 0$,

(IV) $(2-\sigma)\alpha < 1 \wedge G(n) > 1 \wedge \Xi$,

where Ξ is a condition that there is a unique $\delta^* \in (0,1)$ such that satisfies

$$\delta^{*a-1} = [I'(n)/I(n)][H'(n)/H(n)] \left(\frac{f_d + n^{\alpha} f_x \delta^{*\alpha}}{f_d + n^{\alpha} f_x} \right)^{\sigma-1}.$$

Note that $G(n) > 1$ holds if n is sufficiently high relative to trade costs, τ and f_x/f_d.

Proof of Lemma 1

In this appendix, we examine whether n distributes or not on trading equilibrium.

Consider two firm (firm a and firm b). Firm a and firm b enter not only the domestic market and also enter n_a and n_b export markets respectively.

Optimal consumption conditions and optimal pricing rules give

$$\begin{aligned}
\frac{Lc_d + n_a L\tau c^{*\prime}}{Lc_d + n_b L\tau c^{*\prime}} &= \frac{P_T^{\sigma-1}(1+n_a\tau^{1-\sigma})Lwp_{d,a}^{-\sigma}}{P_T^{\sigma-1}(1+n_b\tau^{1-\sigma})Lwp_{d,b}^{-\sigma}} \quad \text{by optimal consumption} \\
&= \frac{1+n_a\tau^{1-\sigma}}{1+n_b\tau^{1-\sigma}}\left(\frac{(\mu/2)[(2\gamma f)/y_{T|a}]^{1/2}}{(\mu/2)[(2\gamma f)/y_{T|b}]^{1/2}}\right)^{-\sigma} \quad \text{by optimal pricing} \\
&= \frac{1+n_a\tau^{1-\sigma}}{1+n_b\tau^{1-\sigma}}\left(\frac{y_{T|a}}{y_{T|b}}\right)^{\sigma/2}. \qquad \text{(I.1)}
\end{aligned}$$

Firm i's good market clearing condition is given by $y_i = Lc_d + n_i L\tau c^{*\prime}$ where i denotes a or b. This good market clearing condition and (I.1) gives

$$\frac{y_{T|a}}{y_{T|b}} = \left(\frac{1+n_a\tau^{1-\sigma}}{1+n_b\tau^{1-\sigma}}\right)^{2/(2-\sigma)}. \qquad \text{(I.2)}$$

From $y_{T|n} = (f_d + n^\alpha f_x)^2/(2\gamma f B^2)$, firms' optimal pricing rules and zero profit conditions give

$$\frac{y_{T|a}}{y_{T|b}} = \left(\frac{f_d + n_a^\alpha f_x}{f_d + n_b^\alpha f_x}\right)^2. \qquad \text{(I.3)}$$

From (I.2) and (I.3), we can get

$$\frac{1+n_a\tau^{1-\sigma}}{1+n_b\tau^{1-\sigma}} = \left(\frac{f_d + n_a^\alpha f_x}{f_d + n_b^\alpha f_x}\right)^{2-\sigma}. \qquad \text{(I.4)}$$

We define $\delta \in [0, \frac{\bar{n}}{n_b}]$ as $n_a = \delta n_b$. Using this δ, we can rewrite (I.4) as follows:

$$\frac{1+n_b\tau^{1-\sigma}\delta}{1+n_b\tau^{1-\sigma}} = \left(\frac{f_d + n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\right)^{2-\sigma}. \qquad \text{(I.5)}$$

Clearly, $\delta = 1$ satisfies $(I.5)$ in any combinations of n_b, τ, f_x, σ, and α. Can the other values of δ satisfy $(I.5)$? If only $\delta = 1$ satisfies $(I.5)$, all firms enter the same number of markets.

Left hand side of (I.5) is linear for δ and this is denoted by $J(\delta)$. Right hand side of (I.5) is nonlinear for δ and this is denoted by $K(\delta)$. A line represented by $J(\delta)$ has an intersection $J(0)$ in (δ, J) space and a curve represented by $K(\delta)$ has the following intersection in (δ, K) space $K(0)$.

When $K(0) = J(0)$ holds, $\delta = 0$ also satisfies $(I.5)$. Hence, in the flowing analysis, we exclude a case of $K(0) = J(0)$.

Property of $K(\delta)$

By differentiating $K(\delta)$, we can obtain

$$\frac{dK(\delta)}{d\delta} = (2-\sigma)\left(\frac{f_d + n_b^\alpha f_x \delta}{f_d + n_b^\alpha f_x}\right)^{1-\sigma} \frac{n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\alpha\delta^{\alpha-1}.$$

Furthermore by differentiating $dK(\delta)/d\delta$, we can obtain

$$\begin{aligned}\frac{d^2K(\delta)}{d\delta^2} =&(1-\sigma)(2-\sigma)\left(\frac{f_d + n_b^\alpha f_x \delta}{f_d + n_b^\alpha f_x}\right)^{-\sigma}\left(\frac{n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\alpha\delta^{\alpha-1}\right)^2 \\ &+(2-\sigma)\left(\frac{f_d + n_b^\alpha f_x \delta}{f_d + n_b^\alpha f_x}\right)^{1-\sigma}\frac{n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\alpha(\alpha-1)\delta^{\alpha-2} \\ =& \underbrace{(2-\sigma)\left(\frac{f_d + n_b^\alpha f_x \delta}{f_d + n_b^\alpha f_x}\right)^{-\sigma}\frac{n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\alpha\delta^{\alpha-2}}_{+} \\ &\times\left[[\alpha(2-\sigma)-1]\frac{n_b^\alpha f_x \delta^\alpha}{f_d + n_b^\alpha f_x}\delta^\alpha + \underbrace{(\alpha-1)\frac{f_d}{f_d + n_b^\alpha f_x}}_{+\text{ or } 0}\right].\end{aligned}$$

When $\alpha(2-\sigma) \geq 1$ holds, $\alpha > 1$ holds from Assumption 1 ($1 < \sigma < 2$). Then, when $\alpha(2-\sigma) \geq 1$ holds, we have $d^2K(\delta)/d\delta^2 > 0$.

The other hand, when $\alpha(2-\sigma) < 1$ holds, $K''(\delta) > 0$ holds for $\delta < \bar{\delta}$ and $K''(\delta) \leq$ holds for $\delta \geq \bar{\delta}$, where δ satisfies the following condition:

$$\bar{\delta}^\alpha = \frac{\alpha-1}{1-\alpha(2-\sigma)}\frac{f_d}{n_b^\alpha f_x}.$$

In a case of $(2-\sigma)\alpha < 1$

We consider a case of $(2-\sigma)\alpha < 1$. δ of (I.5) determines uniquely in the following manner.

When $K(0) < J(0)$ holds, δ has the unique solution only when $J'(1) = K'(1)$ holds. line $J(\delta)$ and curve $K(\delta)$ have the unique cross point, as is shown in Figure A.4. Line J and curve K come in contact with each other at $\delta = 1$ and this point represents the unique solution.

When $K(0) > J(0)$ and $k(\delta^*) > J(\delta^*)$ hold where $\delta^* \in (0,1)$ satisfies $K'(\delta^*) = J'(\delta^*)$, these line and curve have the unique cross point, as is shown in Figure A.5.

In a case of $(2-\sigma)\alpha > 1$

We consider a case of $(2-\sigma)\alpha > 1$. δ of (I.5) determines uniquely in the following manner.

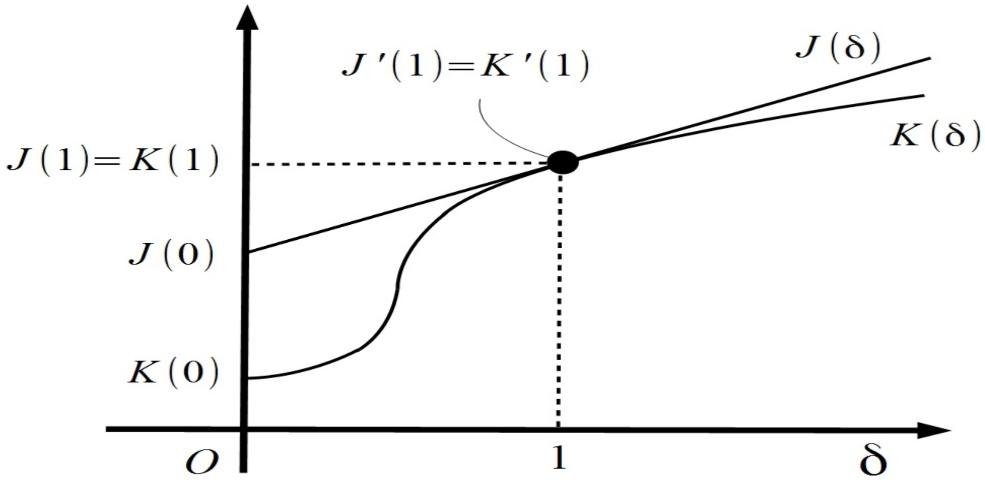

Figure A. 4: n's degeneration in a case of $(2-\sigma)\alpha < 1$: Case 1

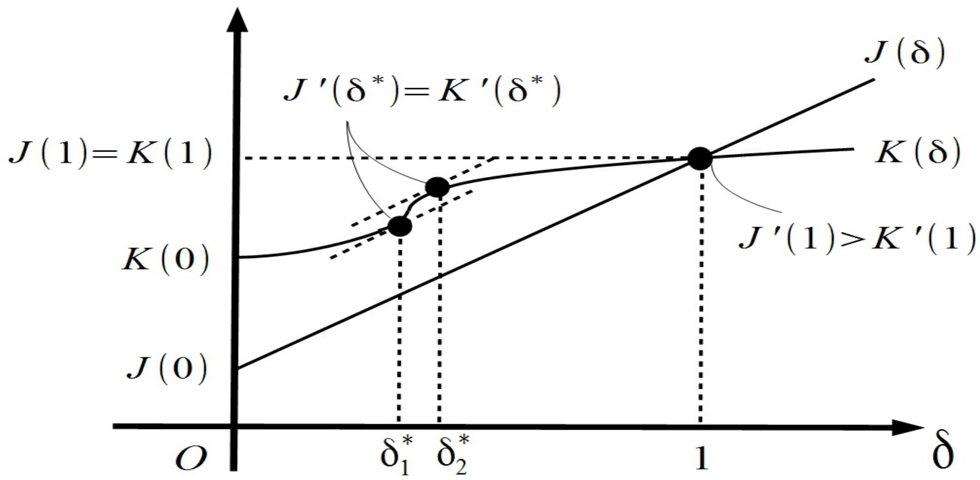

Figure A. 5: n's degeneration in a case of $(2-\sigma)\alpha < 1$: Case 2

When $K(0) < J(0)$ holds, δ has always the unique solution as is shown in Figure A.6.

When $K(0) > J(0)$ and $K'(1) = J'(1)$ hold, these line and curve have the unique cross point, as is shown in Figure A.7.

Rewriting the above results

We rewrite the above results by using I, H, G. Note the following relations hold:

$$J(0) = \frac{1}{I(n_b)},$$

$$K(0) = \frac{1}{H(n_b)}.$$

Hence, $K(0) > J(0)$ is equivalent to $G(n_b) > 1$.

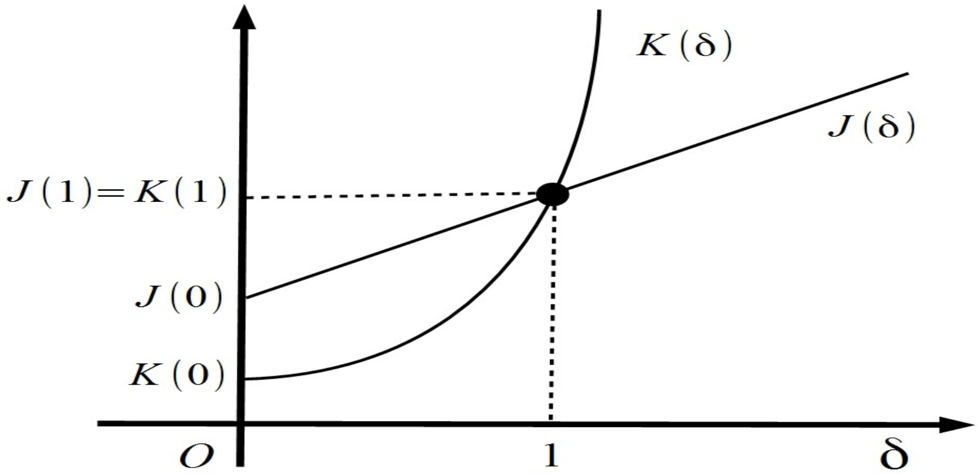

Figure A. 6: n's degeneration in a case of $(2-\sigma)\alpha \geq 1$: Case 1

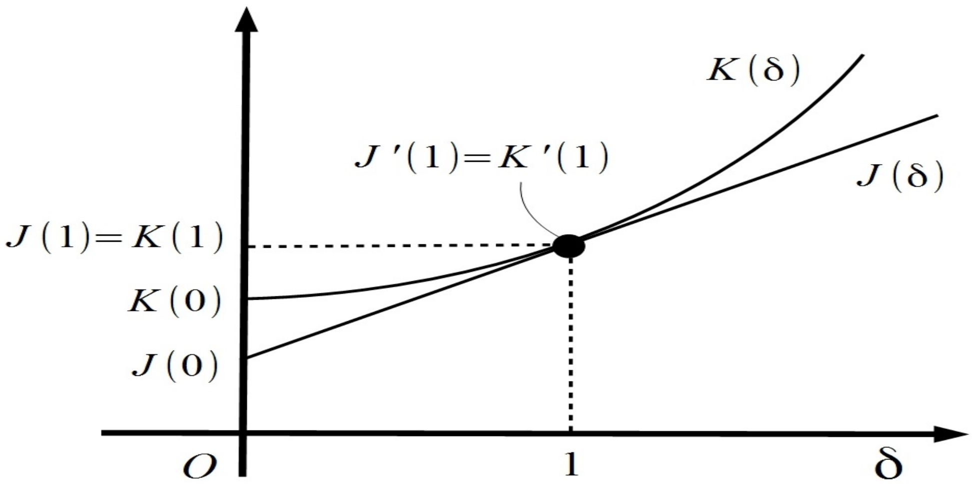

Figure A. 7: n's degeneration in a case of $(2-\sigma)\alpha \geq 1$: Case 2

A simple calculation gives

$$J'(\delta) = n_b \frac{I'(n_b)}{I(n_b)},$$

$$K'(\delta) = n_b \frac{H'(n_b)}{H(n_b)} K(\delta)^{\frac{1-\sigma}{2-\sigma}} \delta^{\alpha-1}.$$

Since δ^* satisfies $K'(\delta^*) = J'(\delta^*)$, δ^* can be characterized as follows,

$$\delta^{*a-1} = \frac{I'(n)/I(n)}{H'(n)/H(n)} \left(\frac{f_d + n^\alpha f_x \delta^{*\alpha}}{f_d + n^\alpha f_x} \right)^{\sigma-1}.$$

By substitute $\delta = 1$ for $J'(\delta)$ and $K'(\delta)$, we can get

$$J'(1) = n_b \frac{I'(n_b)}{I(n_b)},$$

$$K'(1) = n_b \frac{H'(n_b)}{H(n_b)}.$$

Hence, $J'(1) = K'(1)$ is equivalent to $I'(n_b)/I(n_b) = H'(n_b)/H(n_b)$.

We differentiate $G(n_b)$ for n_b, we can obtain

$$\frac{dG(n_b)}{dn_b} = \frac{I'(n_b)H(n_b) - I(n_b)H'(n_b)}{H^2(n_b)}.$$

Hence, $I'(n_b)/I(n_b) = H'(n_b)/H(n_b)$ implies $G'(n_b) = 0$. Q.E.D.

Appendix J: Lemma 2 and the proof.

Introduction of Lemma 2

If all firms enter n export markets, when an equilibrium condition (II) in Definition 1 is satisfied ? The following Lemma 2 answers this question.

Lemma 2. *When Assumption 1 holds and the number of export markets each firm enters is identical, an equilibrium in which all firms enter n export markets uniquely exists if and only if the following condition is satisfied:* $\forall \hat{n} \in [0, \bar{n}]$, $\hat{n} \neq n \;\wedge\; G(n) > G(\hat{n})$.

This lemma indicates that optimal n maximizes $G(n)$. We can interpret this immediately. The numerator of $G(n)$, $I(n) = (1 + n\tau^{1-\sigma})$, and the denominator of $G(n)$, $H(n) = (1 + n^\alpha f_x/f_d)^{2-\sigma}$ can be interpreted as entry gain and entry loss respectively, as explained in Appendix D. Therefore, it is natural that n maximizes $G(n)$.

Proof of Lemma 2

In this appendix, we use subscript n, $\hat{n}$ which represents the number of export markets firms enter.

We call non-deviation condition for a condition that a firm does not have incentive to enter $\hat{n} \neq n$ export markets when all the other firms enter n export markets. This condition certifies existence of trading equilibrium. We shows non-deviation condition is equivalent to $G(n) > G(\hat{n})$. That is, we show the following proposition:

$$\text{For given } n, \forall M, \forall \hat{n}(\neq n), [0 = \pi_n > \pi_{\hat{n}}] \rightarrow [G(n) > G(\hat{n})].$$

From $P_T^{1-\sigma} = Mp_{d,n}^{1-\sigma}(1 + n\tau^{1-\sigma})$, we can obtain the following short run profit of firms entering n export markets in the same manner as (G.7):

$$\pi_n = \frac{B}{B+1}\frac{LI}{M} - w(f_d + n^\alpha f_x).$$

On the other hand, profit of firms entering $\hat{n}(\neq n)$ export markets is derived in the following manner.

Demands for this firms is

$$
\begin{aligned}
y_{\hat{n}} =& Lc + \hat{n}\tau Lc^{*\prime} \\
=& LIp_{d,\hat{n}}P_T^{\sigma-1} + \hat{n}\tau LI(\tau p_{d,\hat{n}})^{-\sigma}P_T^{\sigma-1} \\
=& LIp_{d,\hat{n}}^{-\sigma}P_T^{\sigma-1}(1+\hat{n}\tau^{1-\sigma}) \\
=& p_{d,\hat{n}}^{-1}LIM^{-1}\frac{1+\hat{n}\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\left(\frac{p_{d,n}}{p_{d,\hat{n}}}\right)^{\sigma-1} \\
=& p_{d,\hat{n}}^{-1}LIM^{-1}\frac{1+\hat{n}\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\left(\frac{f_d+\hat{n}^\alpha f_x}{f_d+n^\alpha f_x}\right)^{\sigma-1}.
\end{aligned}
$$

This conditions give

$$
r_{\hat{n}} = p_{d,\hat{n}}y_{\hat{n}} = LIM^{-1}\frac{1+\hat{n}\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\left(\frac{f_d+\hat{n}^\alpha f_x}{f_d+n^\alpha f_x}\right)^{\sigma-1}.
$$

Optimal pricing rule gives $TC_{\hat{n}} = \frac{r_{\hat{n}}}{B+1} + w(f_d+\hat{n}f_x)$ and these conditions give

$$
\pi_{\hat{n}} = \frac{B}{B+1}\frac{LI}{M}\frac{1+\hat{n}\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\left(\frac{f_d+\hat{n}^\alpha f_x}{f_d+n^\alpha f_x}\right)^{\sigma-1} - w(f_d+\hat{n}^\alpha f_x).
$$

$\forall M, \forall\hat{n}(\neq n), [\pi_n > \pi_{\hat{n}}]$ holds for M in which $\pi_n = 0$ holds. That is, for such a M, $0 = \pi_n > \pi_{\hat{n}}$ holds. This condition is equivalent to

$$
f_d + n^\alpha f_x = \frac{B}{B+1}\frac{LI}{wM} < \frac{1+n\tau^{1-\sigma}}{1+\hat{n}\tau^{1-\sigma}}\left(\frac{f_d+n^\alpha f_x}{f_d+\hat{n}^\alpha f_x}\right)^{\sigma-1}(f_d+\hat{n}^\alpha f_x).
$$

This condition is equivalent to $G(n) > G(\hat{n})$. Q.E.D.

Appendix K : Deviation condition and an interpretation of $G(n)$

We can interpret $G(n)$, from deviation condition shown in the following way. In this appendix, we use subscript n, $\hat{n}$ which represents the number of export markets firms enter.

We call *deviation condition* for a condition that a firm has incentive to enter $\hat{n} \neq n$ export markets when all the other firms enter n export markets. We shows deviation condition is equivalent to $G(\hat{n}) > G(n)$. That is, we show the following proposition: When

$P_T^{1-\sigma} = Mp_{d,n}^{1-\sigma}(1+n\tau^{1-\sigma})$ holds, the following condition holds:

$$\text{For given } n, \forall M, [\pi_{\hat{n}} > \pi_n] \rightarrow [G(\hat{n}) > G(n)]$$

As with the manner in proof of non-deviation condition of Appendix J, we can obtain profits of each type of firm as follows

$$\pi_n = \frac{B}{B+1}\frac{LI}{M} - w(f_d + n^\alpha f_x),$$

$$\pi_{\hat{n}} = \frac{B}{B+1}\frac{LI}{M}\frac{1+\hat{n}\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\left(\frac{f_d + \hat{n^\alpha} f_x}{f_d + n^\alpha f_x}\right)^{\sigma-1} - w(f_d + \hat{n^\alpha} f_x).$$

From some M, $\pi_{\hat{n}} > 0 = \pi_n$. This implies $G(\hat{n}) > G(n)$.

Now, we can interpret this as follows. Though Lemma 2 demands $G(n)$ must be maximized *on equilibrium*, the above result demands $G(n)$ must be maximized *off equilibrium*. Therefore, The numerator of $G(n)$, $(1 + n\tau^{1-\sigma})$, can be interpreted as entry gain. The denominator of $G(n)$, $(1 + n^\alpha \frac{f_x}{f_d})^{2-\sigma}$, can be interpreted as entry loss.

Appendix L: Proof of Proposition 4

Lemma 1 and Lemma 2 derive Proposition 4 as follows. To characterize of n_T, we begin with clarifying property of $G(n)$.

Property of $G(n)$

From definition of $G'(n)$, we can obtain following condition:

For $n = 0$, $G'(n)$ satisfies

$$\frac{dG(n)}{dn}|_{n=0} = [I'(0)H(0) - I(0)H'(0)]H(0)^{-2} = \tau^{1-\sigma} > 0.$$

For $n > 0$, the following relations hold:

$$\begin{aligned} G'(n) > 0 \ &\leftrightarrow \ \frac{I'(n)}{I(n)} > \frac{H'(n)}{H(n)} \\ &\leftrightarrow \ \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}n} > (2-\sigma)\frac{\frac{f_x}{f_d}\alpha n^{\alpha-1}}{1+\frac{f_x}{f_d}n^\alpha} \\ &\leftrightarrow \ \frac{1}{(2-\sigma)\alpha}\frac{1+\frac{f_x}{f_d}n^\alpha}{\frac{f_x}{f_d}n^{\alpha-1}} > \frac{1+\tau^{1-\sigma}n}{\tau^{1-\sigma}} \end{aligned}$$

$$\leftrightarrow \underbrace{\frac{1}{(2-\sigma)\alpha}\frac{f_d}{f_x}\frac{1}{n^{\alpha-1}}}_{A(n)} > \underbrace{\tau^{\sigma-1}+\frac{(2-\sigma)\alpha-1}{(2-\sigma)\alpha}n}_{Q(n)}.$$

Let $A(n)$ and $Q(n)$ denote left hand side and right hand side of the above condition respectively. This relation between curve $A(n)$and $Q(n)$ derives sign of $G'(n)$.

A case of $(2-\sigma)\alpha \geq 1$

Figure A.8 describes the relation between $A(n)$ and $Q(n)$ in a case of $(2-\sigma)\alpha \geq 1$.

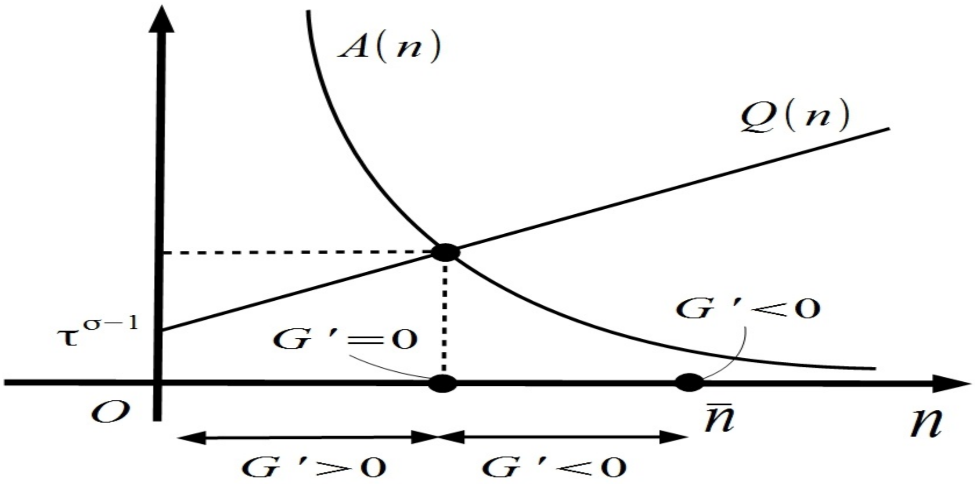

Figure A. 8: The relation between $A(n)$ and $Q(n)$ under partial entry regime – $A(n) \gtreqless Q(n)$ –

We can describe $G(n)$ as shown in Figure A.9 from Figure A.8.

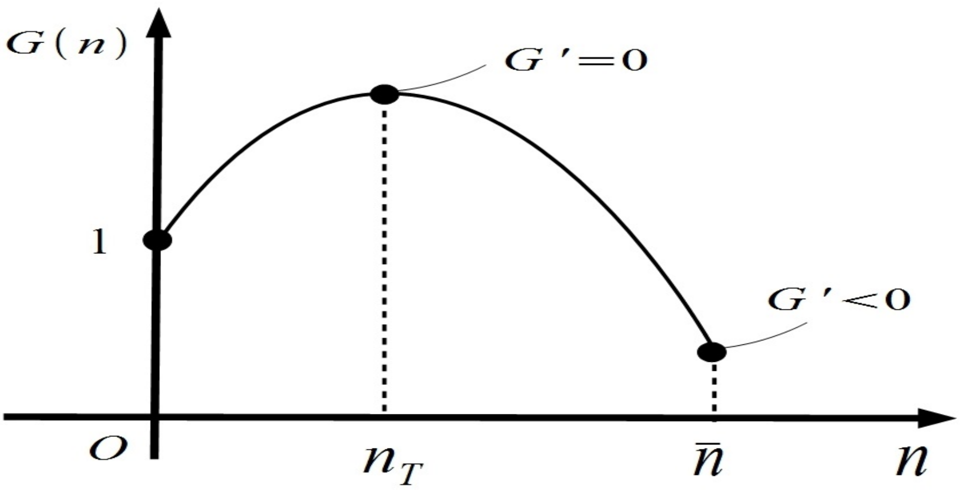

Figure A. 9: Optimal entry n_T under partial entry regime – $G'(n) \gtreqless 0$ –

Then, n_T of Figure A.9 satisfies $G(n) > 1$ of Lemma 1 and maximized G of Lemma 2.

Hence, n_T is characterized by

$$\frac{1}{(2-\sigma)\alpha}\frac{f_d}{f_x}\frac{1}{n_T^{\alpha-1}} = \tau^{\sigma-1} + \frac{(2-\sigma)\alpha - 1}{(2-\sigma)\alpha} n_T. \tag{L.1}$$

Note Figure A.9 has a restriction, $G'(\bar{n}) < 0$ on $\bar{n}$.

A case of $(2-\sigma)\alpha < 1$

In figure A.5, $K'(\delta^*) = J'(\delta^*)$ implies $K'(1) < J'(1)$. From these property and $K(0) > J(0)$, we can obtain $I'(n_b)/I(n_b) > H'(n_b)/H(n_b)$, and $I(n_b) > H(n_b)$. This implies

$$G'(n_T) > 0, G(n_T) > 1.$$

Note that when $\alpha = 1$ holds, $A(n)$ is constant. Hence, in a case of $\alpha(2-\sigma) < 1$, $A(n)$ can be constant. Then, we analyze the case of $\alpha(2-\sigma) < 1$ in two different cases: where $\alpha = 1$ holds and where $\alpha > 1$ holds.

A case of $(2-\sigma)\alpha < 1$ **with** $\alpha = 1$

When $(2-\sigma)\alpha \geq 1$ and $f_d \geq (2-\sigma)\tau^{\sigma-1} f_x$ hold, the relation between $A(n)$ and $Q(n)$ is described as shown in Figure A.10.

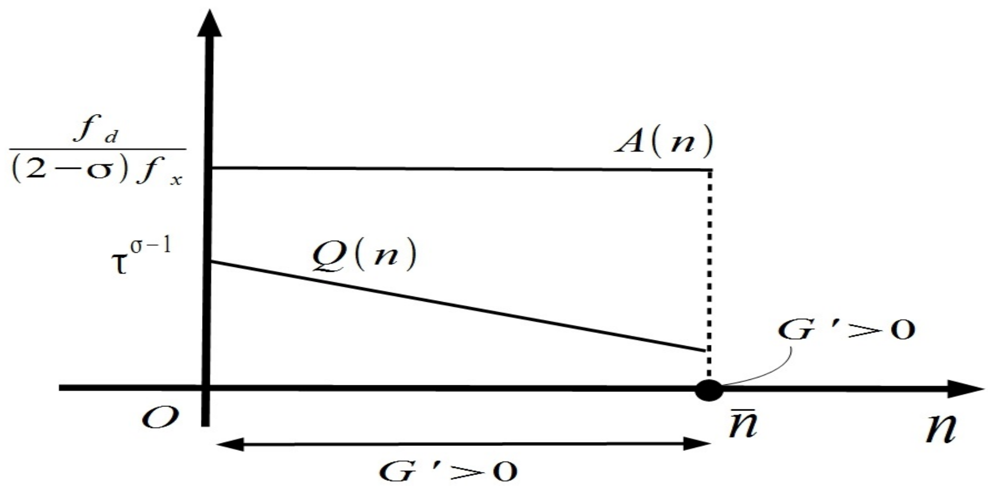

Figure A. 10: The relation between $A(n)$ and $Q(n)$ under full entry regime with $\alpha = 1$: Case 1 – $A(n) \geqq Q(n)$ –

Then, We can describe $G(n)$ as shown in Figure A.11 from Figure A.10. From Figure A.11, We find $n_e \leq 0$. Hence, G is increasing for $n \geq n_e$. This leads $n_T = \bar{n}$.

When $(2-\sigma)\alpha \geq 1$ and $f_d < (2-\sigma)\tau^{\sigma-1} f_x$ hold, the relation between $A(n)$ and $Q(n)$ is described as shown in Figure A.12.

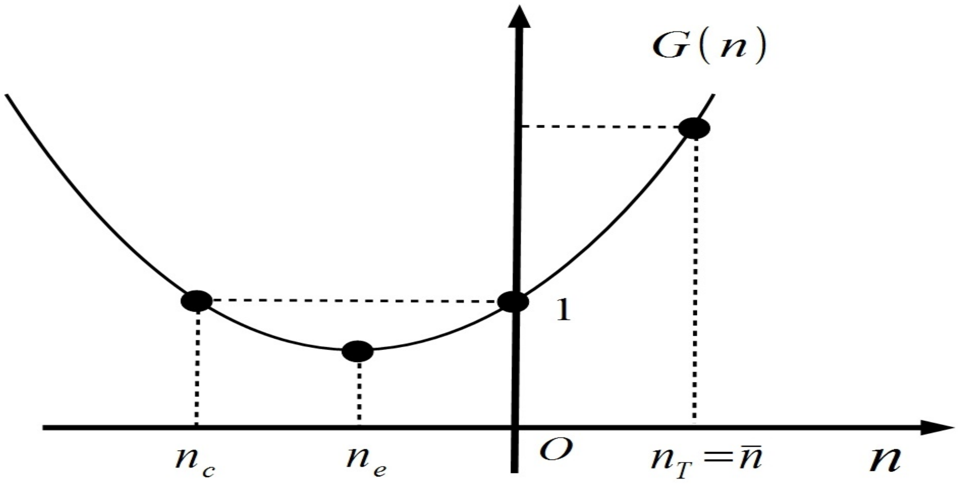

Figure A. 11: Optimal entry n_T under full entry regime with $\alpha = 1$: Case 1 – $n_c > 0$ –

Then, We can describe $G(n)$ as shown in Figure A.13 from Figure A.12. From Figure A.13, We find $n_e > 0$. Hence, G is decreasing for $n \leq \bar{n}$ and increasing for $n > n_e$.

In this case, n_T is characterized in the following manner. When $\bar{n} < n_c$ holds, G is maximized at $n = 0$ for $n \in [0, \bar{n}]$. In this case, $G(0) > 1$ does not holds since $G(0) = 1$. Therefore, When $\bar{n} > n_c$ holds, n_T does not exists.

On the other hand, when $\bar{n} > n_c$ holds, $n_T = \bar{n}$ since G is increasing for $n \geq n_e$ and $G(\bar{n}) > 1$ holds for $n > n_c$. This leads $n_T = \bar{n}$.

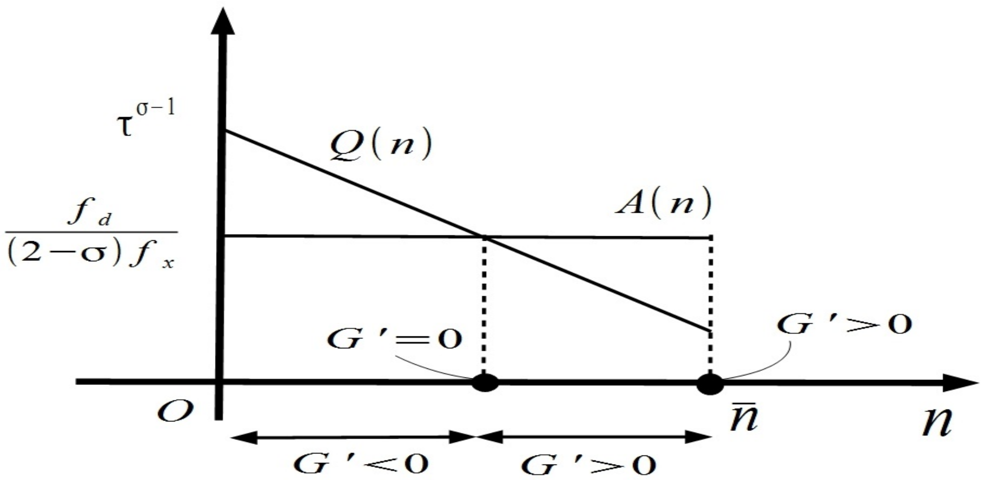

Figure A. 12: The relation between $A(n)$ and $Q(n)$ under full entry regime with $\alpha = 1$: Case 2 – $A(n) \lesseqgtr Q(n)$ –

The above analysis certifies n_T as the unique solution.

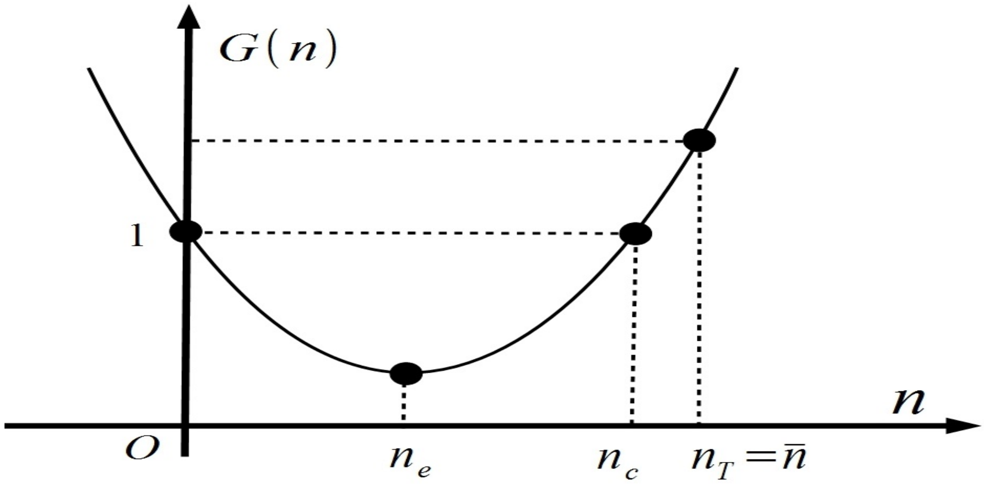

Figure A. 13: Optimal entry n_T under full entry regime with $\alpha = 1$: Case 2 – $n_c < 0$ –

A case of $(2-\sigma)\alpha < 1$ **with** $\alpha > 1$

Let n^* represents a n which satisfies $A'(n) = Q'(n)$. We analyze a case of $(2-\sigma)\alpha < 1$ with $\alpha > 1$ in three different cases : where $A(n^*) > Q(n^*)$ holds and where $A(n^*) = Q(n^*)$ holds and where $A(n^*) < Q(n^*)$ holds.

Figure A.14 describes the relation between $A(n)$ and $Q(n)$ when $A(n^*) > Q(n^*)$ holds. Then, We can describe $G(n)$ as shown in Figure A.15 from Figure A.14. In this case, $\bar{n}$ maximizes G, $G(\bar{n}) > 1$, and $G'(\bar{n}) > 0$ holds. Hence, we obtain $n_T = \bar{n}$.

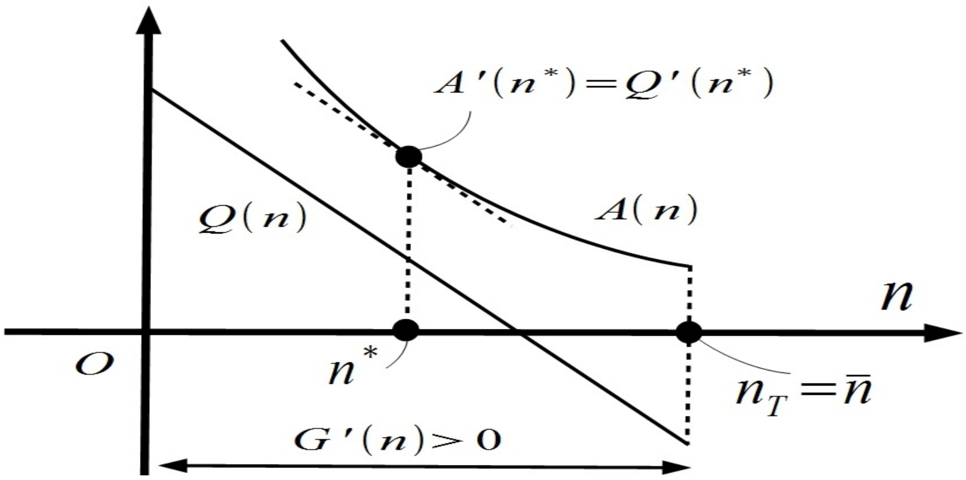

Figure A. 14: The relation between $A(n)$ and $Q(n)$ under full entry regime with $\alpha > 1$: Case 1 – $A(n^*) > Q(n^*)$ –

Figure A.16 describes the relation between $A(n)$ and $Q(n)$ when $A(n^*) = Q(n^*)$ holds.

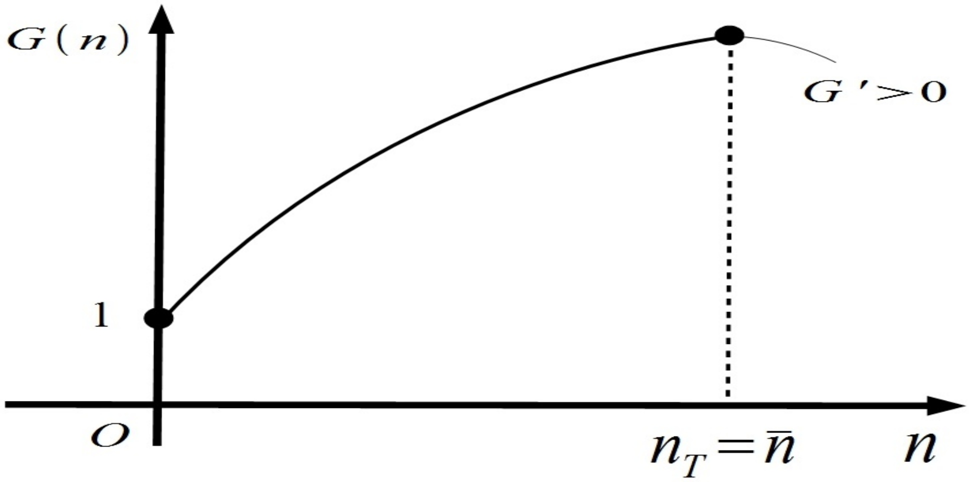

Figure A. 15: Optimal entry n_T under full entry regime with $\alpha > 1$: Case 1 – $G'(n) > 0$ –

Note $A(n^*) = Q(n^*)$ can be rewritten as

$$f_d = f_x \left[(2-\sigma)\tau^{\sigma-1}\right]^{\alpha} \left[\frac{\alpha - 1}{1-(2-\sigma)\alpha}\right]^{\alpha-1}.$$

Then, We can describe $G(n)$ as shown in Figure A.17 from Figure A.16. In this case, if $\bar{n}$ is different from n_e, $\bar{n}$ maximizes G, $G(\bar{n}) > 1$, and $G'(\bar{n}) > 0$ holds. Hence, we obtain $n_T = \bar{n}$ unless $\bar{n} = n_e$.

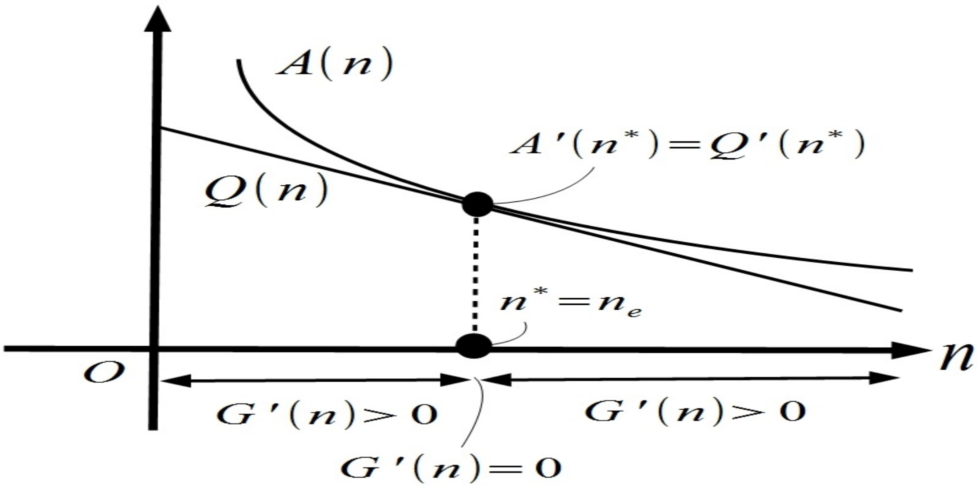

Figure A. 16: The relation between $A(n)$ and $Q(n)$ under full entry regime with $\alpha > 1$: Case 2 – $A(n^*) = Q(n^*)$ –

Figure A.18 describes the relation between $A(n)$ and $Q(n)$ when $A(n^*) < Q(n^*)$ holds.

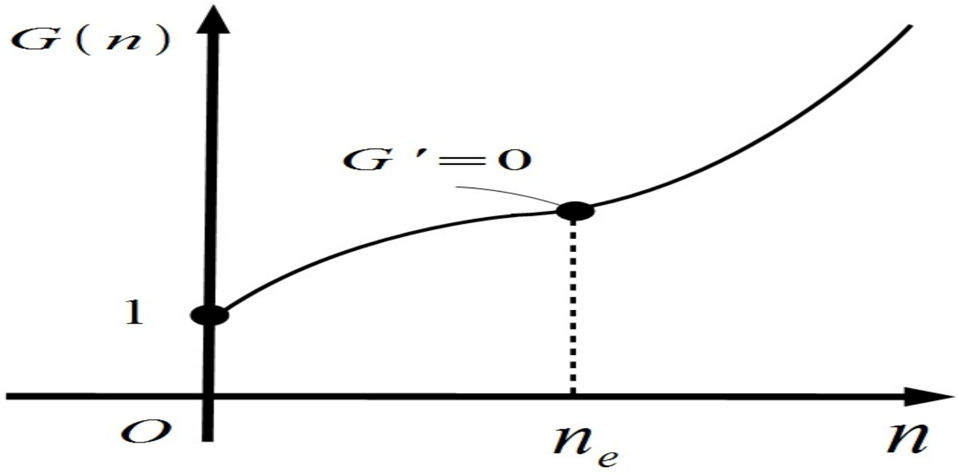

Figure A. 17: Optimal entry n_T under full entry regime with $\alpha > 1$: Case 2 – $G'(n) \geqq 0$ –

Note $A(n^*) < Q(n^*)$ can be rewritten as

$$f_d < f_x \left[(2-\sigma)\tau^{\sigma-1}\right]^{\alpha} \left[\frac{\alpha-1}{1-(2-\sigma)\alpha}\right]^{\alpha-1}.$$

Then, we can describe $G(n)$ as shown in Figure A.19 from Figure A.18.

In this case, if $\bar{n} < n_{e_1}$, we can get $n_T = \bar{n}$ immediately.

If $G(\bar{n}) > G(n_{e_1})$ holds, $\bar{n}$ maximizes G, $G(\bar{n}) > 1$, and $G'(\bar{n}) > 0$ holds. Hence, we obtain $n_T = \bar{n}$ unless $\bar{n} = n_e$. Q.E.D.

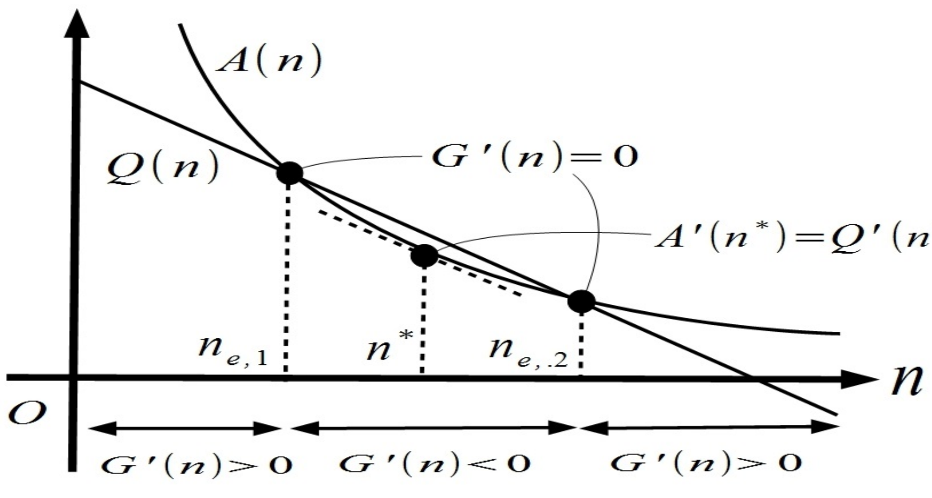

Figure A. 18: The relation between $A(n)$ and $Q(n)$ under full entry regime with $\alpha > 1$: Case 3 – $A(n^*) \gtreqless Q(n^*)$ –

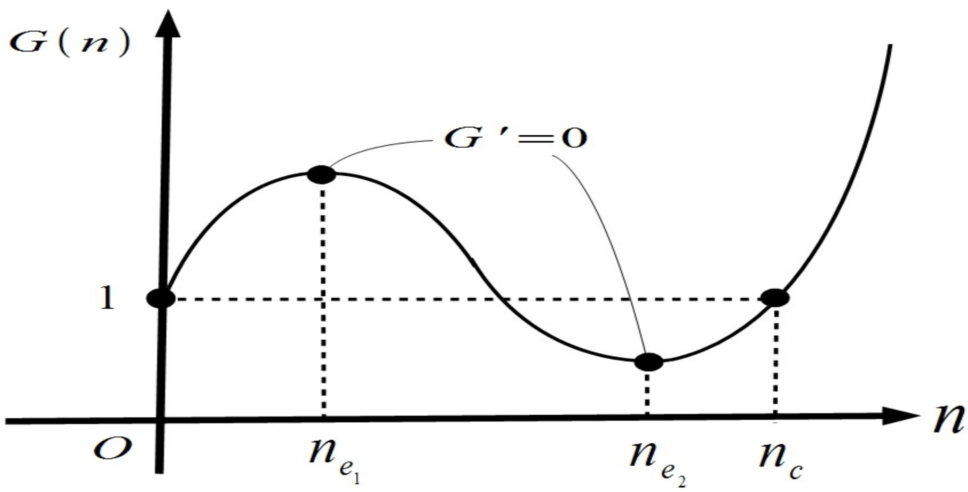

Figure A. 19: Optimal entry n_T under full entry regime with $\alpha > 1$: Case 3 – $G'(n) \gtreqless 0$ –