Change in Fixed Costs and the Division of Labor within Firms through Labor Reallocation

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Abstract

This paper investigates the effects of a decrease in fixed costs on the division of labor within firms. In the constant markup rate model, a decrease in fixed costs curbs the division of labor. In the short run, the division of labor is promoted through labor reallocation within firms while in the long run, the division of labor is curbed through labor reallocation across firms. The latter effect dominates the former effect. In the variable markup rate model whose markup rate depends on the number of firms, the decrease in fixed costs induces labor reallocation across firms which is the opposite direction of that of the constant markup rate model in addition. The direction of labor reallocation across firms based on procompetition is opposite to that of the model of Kamei (2014) which does not impose free-entry and free-exit condition. The free-entry and free-exit condition plays a key role in determining the direction of that reallocation based on procompetition effect.

Keywords: fixed costs; division of labor within firms; labor reallocation

JEL classification numbers: E23; E24; J24; L16; L22

1 Introduction

Many studies have modeled how the promotion of the division of labor raises firm productivity. However, most of these studies focus on the optimal firm structure problem that firms face and do not clarify the relationship between the number of firms and the

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division of labor. When fixed costs decrease, what is the relationship between firms' entry to and exit from markets and the division of labor? Moreover, how is the labor force reallocated? Can these properties be changed by a competitive environment? This paper presents a simple model to investigate these problems.

We construct a model that is quite similar to Chaney and Ossa (2013). Chaney and Ossa (2013) succeed in formalizing Adam Smith's (1776) pin factory story. Although Chaney and Ossa (2013) do not assume fixed costs, we assume *fixed costs*. In addition, we formulate a variable markup rate model following Blanchard and Givazzi (2003). Their variable markup rate depends on the number of firms. We compare the effects of a decrease in fixed costs on the division of labor between both of our models.

This paper's main results are as follows. Under constant markup rate, positive fixed costs, and a free-entry and free-exit condition, a decrease in fixed costs curbs the division of labor. This result can be decomposed into two effects in the short run and long run. In the short run, the division of labor is promoted through labor reallocation within firms while in the long run, the division of labor is curbed through labor reallocation across firms. The latter effect dominates the former effect. Hence, an essential source of the division of labor in the long run is labor reallocation across firms. In the variable markup rate model, the decrease in fixed costs induces labor reallocation across firms which is the opposite direction of that of the constant markup rate model in addition although the direction of labor reallocation within firms is the same as that of the constant markup rate model.

The results of the variable markup rate model are in contrast with Kamei (2014). Kamei (2014) indicates that an increase in the number of firms curbs the division of labor without imposing the free-entry and free-exit condition. On the other hand, our variable markup rate model indicates that labor reallocation based on procompetition effect promotes the division of labor while that reallocation raises the number of firms, real wage rate increase, and firm output. This suggests that the free-entry and free-exit condition plays a key role in determining the direction of that reallocation based on procompetition effect.

This division of labor is interpreted often as the division of labor not only within firms but also across firms within an industry and across industries. For example, Ethier (1982) treats the division of labor as an expansion of the varieties of intermediate goods. In this paper, the division of labor is treated as a narrower task set in which each worker engages.

In addition to Chaney and Ossa (2013), some studies formalize the division of labor *within* firms. Edwards and Starr (1987) present a model in which the division of labor is not a sufficient condition for increasing returns to scale. Swanson (1999) presents a quite simple model that analyzes the relationship among human capital investment, the division of labor, and firm productivity. Becker and Murphy (1992) show explicitly that the cost of promoting the division of labor is coordination cost.

Some empirical studies show a positive relationship between firm productivity and the division of labor *within* firms for both the manufacturing and nonmanufacturing sectors. Baumgardner (1988) indicates that more populous counties have more medical specialists. Garicano and Hubbard (2008) present similar results for law firms. Borghans and Weel (2006) suggests firm productivity improvements induced by the division of labor within firms through communication technology adoption, which reduces the coordination cost within firms by using a survey among Dutch establishments.

The rest of this paper is organized as follows. Section 2 analyzes firm structure. Section 3 analyzes equilibrium allocation. Section 4 analyzes how a decrease in fixed costs affects the division of labor and social welfare. Section 5 analyzes labor reallocation behind the decrease in fixed costs. Section 6 compares the results between constant and variable markup rate models. Finally, we present the Conclusion and Appendix.

2 Firm structure

We introduce the division of labor into a trade model of monopolistic competition with fixed costs. The setup of the model is based on Chaney and Ossa (2013). To begin with, we consider firm structure.

We introduce the division of labor within firms similarly to Chaney and Ossa (2013) for the following reason. Traditional production management, which is the scientific management advocated by Frederick Taylor, promotes the division of labor and production on a large scale. However, today, team production is important in many industries, as reported by Daft (2000). Chaney and Ossa (2013) explicitly allow such a production management approach, and hence, we also adopt it.

2.1 Optimal competency

Each firm produces a variety of differentiated final goods. As for the production of goods, we modify the model developed by Chaney and Ossa (2013). Many tasks are sequentially distributed over the set $[0, 2]$ for each firm. One unit of final good is produced by inputting one unit of preliminary good for task set $[0, 2]$. A firm assigns these tasks to t teams, where $t \in R_+$. One unit of preliminary good for a certain task set $[\underline{\omega}, \overline{\omega}]$ is produced by inputting the following units of labor:

$$l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \times \underbrace{\int_{\underline{\omega}}^{\overline{\omega}} \gamma |\omega_c - \omega| d\omega}_{\text{Area of two right-angled triangles}}, \quad \omega_c \in [0, 2], \ \gamma > 0, \tag{1}$$

where ω_c denotes this team's core competency, and γ denotes the team's burden parameter. Core competency is a task that the team is most suited to undertake. As γ is high, certain task sets require more labor. γ can be interpreted as the difficulty of multitasking.

The firm assigns a core competency to each team; that is, the core competency is endogenously determined. The optimal core competency is a solution of the following cost minimization problem; $l^*(\underline{\omega}, \overline{\omega}) = \min_{\omega_c \in [\underline{\omega}, \overline{\omega}]} l(\underline{\omega}, \overline{\omega})$. The optimal core competency is certainly the midpoint in the assigned task set as follows:

$$\omega_{c|[\underline{\omega}, \overline{\omega}]} = \frac{\underline{\omega} + \overline{\omega}}{2}. \tag{2}$$

This is because each task set is symmetric with respect to the core competency (See Appendix A for a detailed derivation).

2.2 Optimal number of teams

Figure 1 illustrates these features for task set $[0, 4/t]$ when t is a positive integer. The integral term in (1) corresponds to the area of two right-angled triangles formed in linear symmetry with respect to the vertical direction shown in Figure 1.[1)]

(1) and (2) derives labor input per one unit of preliminary good for an arbitrary task set $[\underline{\omega}, \overline{\omega}]$ as follows (See Appendix A for the detailed derivation):

$$l^*(\underline{\omega}, \overline{\omega}) = 2l^*(\underline{\omega}, \omega_{c|[\underline{\omega}, \overline{\omega}]}). \tag{3}$$

Because the teams are symmetric, the identical range of the task subset, $[0, 2/t]$, is assigned to each team and then, the labor input of each reach is identical.

Let $l_{line|unit}$ denote labor requirements on product line for one unit of output. By combining (1) for each team, $l_{line|unit}$ is given as follows (See Appendix A for the detailed

1) For the assumption of $l(\underline{\omega}, \overline{\omega})$, Chaney and Ossa (2013) adopt a more general form, $l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega} + \overline{\omega}}{2} - \omega \right)^{\beta} d\omega$, where $\beta >$ is a positive parameter. By formulating $l(\underline{\omega}, \overline{\omega})$ in the same way as (1), we can make the model highly tractable. See Appendix C for the generality of the technology in (1).

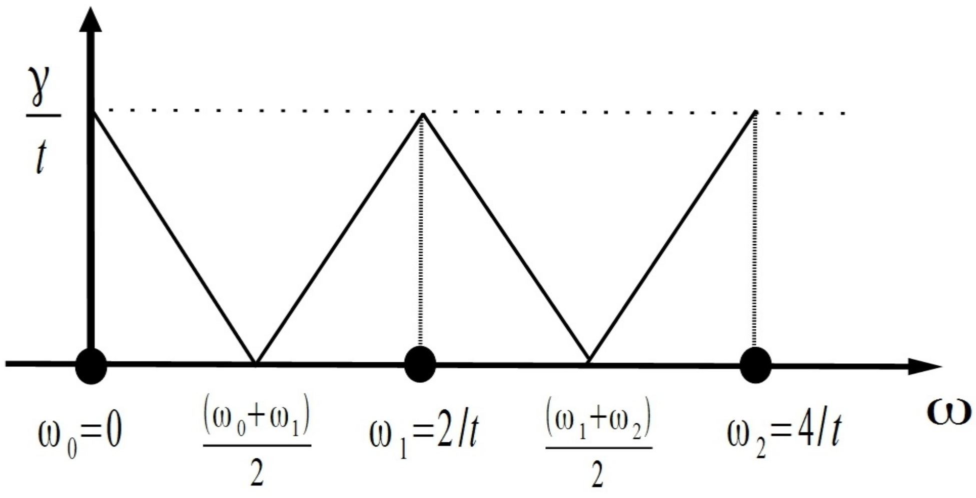

Figure 1: sequential task structure

derivation):

$$l_{line|unit} = t\left(\int_0^{1/t} \gamma\omega d\omega\right). \tag{4}$$

Figure 2 illustrates this features.

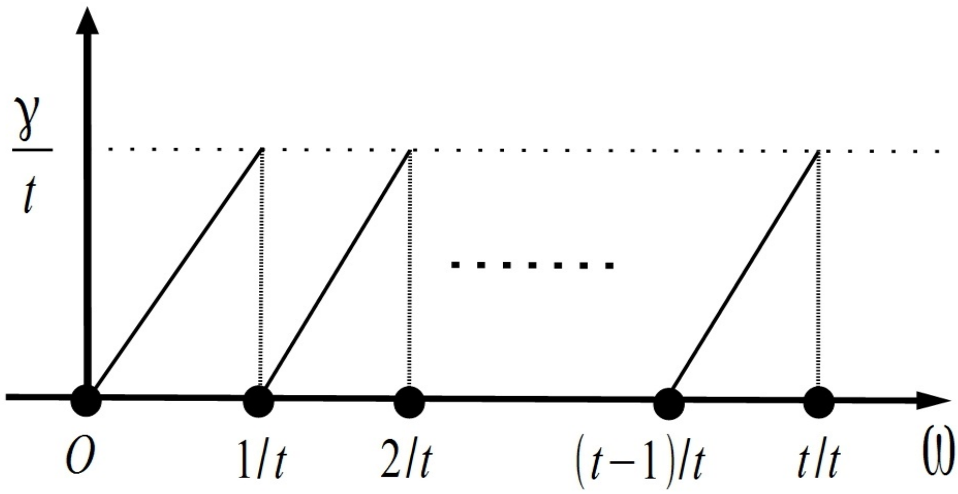

Figure 2: sequential task structure

(4) indicates that as the number of teams increases, labor input per one team converges with order 2 to 0 from $\int_0^{1/t} \gamma\omega d\omega = \gamma/(2t^2)$, while the number of teams diverges with order 1 to $+\infty$. Hence, as the number of teams increases, $l_{pre|unit}$ decreases.

Let l_{line} denote labor requirements on product line for y units of output. From $\int_0^{1/t} \omega d\omega = 1/(2t^2)$, l_{line} is given by

$$l_{line} = y \times l_{line|unit} = \frac{\gamma y}{2t}.$$

Organizing one team requires $f(>0)$ units of labor, which is interpreted as coordination costs.[2] Then, y units of final goods are produced for a given number of teams, t, by inputting the following units of labor:

$$l(t, y) = tf + l_{line} = tf + \frac{\gamma y}{2t}.$$

Each firm selects the number of teams t such that the abovementioned labor input $l(t, y)$ is minimized. In this problem, the firm experiences a tradeoff among productivity improvements by increasing the number of teams and costs of organizing teams. The optimal number of teams t is

$$t(y) = \left(\frac{\gamma y}{2f}\right)^{1/2}$$

This implies that as firm size increases, the extent of the division of labor increases.

2.3 Total cost function and the extent of the division of labor

Each firm inputs labor into the production divisions and a further $f_d(>0)$ units of labor into the management division, where $f_d(>0)$ is fixed and wf_d represents overhead production costs. Total labor input is $l + f_d$.

Combining $l(t, y)$ and $t(y)$ gives the total cost function under the optimal organization as follows (See the Appendix for the detailed derivation):

$$TC(y) = wl(y) + wf_d = w(2\gamma fy)^{1/2} + wf_d. \tag{5}$$

This derives the average cost function, $AC(y) = TC(y)/y = w\left[(2\gamma f)/y\right]^{1/2}$, and the marginal cost function, $MC(y) = dTC(y)/dy = (w/2)\left[(2\gamma f)/y\right]^{1/2}$. Both cost functions are decreasing for y.

On the other hand, $l(t(y), y)$ derives the production function as $y = l^2/(2\gamma f)$ under the optimal division of labor. This indicates that average and marginal labor productivity are increasing for l. This indicate a reverse causal relationship to the proposition, for example, that of Melitz (2003), which indicates that high productivity firms become large firms.

These productivity and cost functions have the following relationship with the extent of the division of labor.

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2) f can be interpreted as midlevel management costs. Because each team specializes in a certain task set, the firm needs coordinators. Becker and Murphy (1992) emphasized that coordination cost acts as a brake for the division of labor.

Proposition 1. *As the extent of the division of labor becomes greater, marginal cost decreases and marginal productivity increases.*

Proof: See Appendix C.

From this proposition, we can use the number of teams as measurements of firm productivity.

3 Equilibrium allocation

3.1 Households

There are L units of households, and each household supplies one unit of labor inelastically at wage rate w. The preference of each consumer is given by a constant elasticity of substitution utility function over a continuum of goods indexed by θ: $U = \left[\int_{\theta\in\Theta} c(\theta)^\rho d\theta\right]^{1/\rho}$, $0 < \rho < 1$, where the measure of the set Θ represents the mass of available differentiated goods, and $c(\theta)$ represents the consumption of variety θ. From standard utility maximization, the price index can be obtained as follows: $P = \left[\int_{\theta\in\Theta} (p(\theta))^{1-\sigma} d\theta\right]^{1/(1-\sigma)}$, where $\sigma = 1/(1-\rho) > 1$ is the elasticity of the substitution between any two varieties and also represents the price elasticity of demand for each variety.

3.2 Equilibrium conditions

We analyze the firm's profit maximization problem in a market of monopolistic competition. Each firm experiences a residual demand curve with constant elasticity σ, and therefore, sets $p = \mu MC(y)$, where $\mu \equiv \sigma/(\sigma-1)$ and $MC(y) \equiv dTC(y)/dy$. Using $l(t, y)$, this optimal pricing rule is written by the PP schedule as follows

$$PP : \frac{p}{w} = \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2}. \tag{6}$$

Firms can enter and exit freely. This gives zero profit $\pi = 0$; this is written by $p = AC(y)$, where $AC(y) \equiv TC(y)/y$. Using $l(t, y)$, this free-entry and free-exit condition is written by the FE schedule as follows:

$$FE : \frac{p}{w} = \left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}. \tag{7}$$

(6) and (7) characterize $(y, p/w)$ at equilibrium as follows: $y_E = f_d^2/(2\gamma f B^2)$, and $(p/w)_E = B(B+1)2\gamma f/f_d$, where B is defined as $B \equiv \mu/2-1$ and subscript E represents variables in equilibrium.

3.3 Internal solution

Hereafter, Assumption 1 holds in order to ensure a unique internal solution.

Assumption 1. [3] $0 < B < \infty$, *that is,* $2 < \mu < \infty$ $(1 < \sigma < 2)$ *and* $f_d > 0$ *hold.*

We can immediately obtain the next proposition from y_E and $(p/w)_E$.

Proposition 2. *Under Assumption 1, a unique internal solution exists in which* $y > 0$ *and* $p/w > 0$.

Note that if $f_d = 0$ holds, then the internal solution does not exist.[4] Hence, we need to assume $f_d > 0$. Even if $f_d > 0$, under $\sigma \geq 2$, $y \to \infty$; that is, the internal solution requires a sufficiently low elasticity of substitution among varieties as consumers value variety strongly).

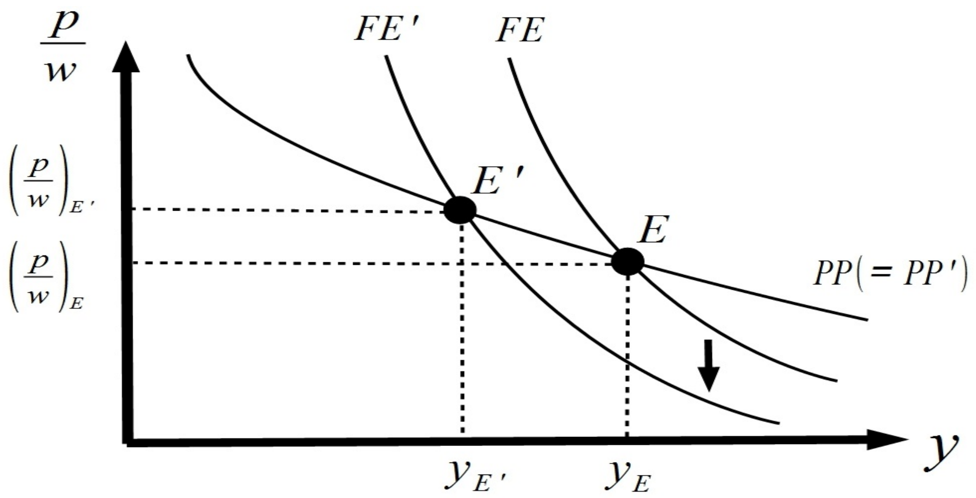

Figure 3: Equilibrium allocation and a decrease in fixed cost

Figure 3 illustrates the features of autarkic equilibrium. The figure has a unique intersection between the FE and PP curves at point E where $(y, p/w) = (y_E, (p/w)_E)$. The PP curve is cut by the FE curve only once. This ensures a unique internal solution.[5]

3) This internal condition makes us reconsider firm technology as represented by (1). See Appendix C for details. However, we adopt technology in (1) and Assumption 1 for analytical simplicity.

4) When $f_d = 0$ and $B = 0$, equilibrium output y is not determined. When $f_d = 0$ and $B \neq 0$, equilibrium output y is zero or approaches positive infinity.

5) The characteristics of Figure 3 are supported by Appendix D.

Substitute y_E into $t(y)$ to yield the equilibrium level of t: $t_E = f_d/(2fB)$. The equilibrium level of l is obtained by substituting y_E and t_E into $l(t, y)$: $l_E = f_d/B$. Then, substitute l_E into $MPL(l)$ to yield $MPL_E = f_d/(\gamma f B)$. This equation implies that $MPL_E = 2t_E/\gamma = l_E/(\gamma f)$. Furthermore, $(w/p)_E = t_E/[\gamma(B+1)] = l_E/[2(B+1)\gamma f]$ holds. In equilibrium, labor productivity and real wages are proportional to the number of teams and the labor input on production divisions.

Now, we can completely characterize the equilibrium allocation by determining the number of varieties. Labor-market clearing condition $L = M(l + f_d)$ gives the following equilibrium number of varieties M_E using l_E: $M_E = [B/(B+1)](L/f_d)$.[6)]

4 Impact of decrease in fixed costs

4.1 Impact on division of labor

Significant changes in management technology have occurred in the post-World War II period (e.g., automatization, the IT revolution, and offshoring). These changes have tended to decrease the fixed labor inputs of head offices, such as clerks. In this section, we analyze the impact of a decrease in fixed costs on the division of labor.

As shown in Figure 3, a decrease in fixed costs (from f_d to f'_d) shifts the FE curve downward. Hence, we have new equilibrium at point E'. Note that the only difference occurs from the fixed costs term. This implies that we can obtain $y_{E'}$ by replacing f_d with f'_d in y_E: $y_{E'} = {f'_d}^2/(2\gamma f B^2)$. In a similar manner, we obtain $l_{E'} = f'_d/B$, $t_{E'} = f'_d/(2fB)$ and $M_{E'} = L/[(2\gamma f y)^{1/2} + f'_d]$.

We find $y_{E'} < y_E$, $(w/p_d)_{E'} < (w/p_d)_E$, $t_{E'} < t_E$, $l_{E'} < l_E$, $MPL_{E'} < MPL_E$, and $M_{E'} > M_E$. $M_{E'} > M_E$ means that some firms enter the market. $t_{E'} < t_E$ means that the division of labor is curbed by a decrease in fixed costs. Hence, we obtain the following proposition.

Proposition 3. *Under Assumption 1, a decrease in fixed costs curbs the division of labor.*

Proof. From $t_{E'} - t_E = (f'_d - f_d)/(2fB) < 0$, we obtain $t_{E'} < t_E$. Q.E.D.

We can explain the mechanism behind this result from the viewpoint of labor reallocation across firms. In Figure 3, the point E satisfies the optimal pricing rule, PP, and not the free-entry and free-exit condition, FE'. Hence, each firm has positive profit at

6) To obtain M_E, we use the labor market-clearing condition and do not use the income–expenditure clearing condition of each household, which is redundant in this equilibrium.

the point E because the average cost is less than the price. Then, some firms try to enter the market. To do so, these firms recruit workers from existing firms. This reallocation across firms curbs the division of labor.

4.2 The impact on social welfare

We treat a representative household's utility as a measure of social welfare. Under the utility maximization, the indirect utility function of each household is $V_E = (w/P)_E$. In equilibrium, firms set the identical price, p, and from the definition of P, the following relationship is given:

$$V_E = \left(\frac{w}{p}\right)_E M_E^{\frac{1}{\sigma-1}}. \tag{8}$$

Note that the indirect utility can be decomposed into the real wage rate and the number of varieties. We substitute $(p/w)_E$ and M_E into (8), and consequently, we obtain equilibrium social welfare as follows:

$$V_E = (2\gamma f)^{-1} L^{\frac{1}{\sigma-1}} (B+1)^{\frac{-\sigma}{\sigma-1}} B^{\frac{2-\sigma}{\sigma-1}} f_d{}^{\frac{\sigma-2}{\sigma-1}}. \tag{9}$$

By differentiating V_E of (9) with respect to f_d, we obtain

$$\frac{V_E}{df_d} = -\frac{2-\sigma}{\sigma-1}\frac{V_E}{f_d} < 0.$$

Under Assumption 1, we obtain $dV_E/df_d < 0$ from $1 < \sigma < 2$. Hence, we have the following proposition.

Proposition 4. *Under Assumption 1, a decrease in fixed costs raise social welfare.*

This effect can be decomposed into a change in $(w/p)_E$ and $M^{1/(\sigma-1)}$. A decrease in fixed costs curbs the division of labor, and then, reduces $(w/p)_E$ (negative productivity effect) but raises $M^{1/(\sigma-1)}$ (positive variety effect). The latter dominates the former effect, and hence, social welfare rises.

5 Labor reallocation within and across firms

In this section, we explicitly consider the labor reallocation behind the division of labor that is promoted by a decrease in f_d by decomposing the effect into a short run effect and a long run effect.

5.1 Labor reallocation in the short run

Previously, we studied equilibrium in which firms can freely enter and exit markets. That is, such equilibrium has a time span in which entry and exit can be adjusted. We call such a time span the long run. In this section, we study trade equilibrium in the short run, in which the number of firms, M, cannot be adjusted. In particular, the zero profit condition is not imposed.

From the labor market-clearing condition, $M(l + f_d) = L$, we can obtain labor input on production divisions as follows

$$l_S = \frac{L}{M} - f_d, \tag{10}$$

where subscript "S" represents variables in the short-run trading equilibrium.

(10), production function $y = l^2/2\gamma f$, and optimal team numbers $t(y) = [\gamma y/(2f)]^{1/2}$ give t in the short-run equilibrium as follows

$$t_S = \frac{l_S}{2f} = \frac{1}{2f}\left[\frac{L}{M} - f_d\right]. \tag{11}$$

(11) implies the following proposition.

Proposition 5. *Under Assumption 1, a decrease in fixed costs promotes the division of labor in the short run.*

This result is in contrast with that in the long run. We can explain a mechanism behind these results form the viewpoint of labor allocation as follows.

(10) is equivalent to the following equation

$$\overbrace{\underbrace{l_S}_{\textbf{production division}} + \underbrace{f_d}_{\textbf{headquarter division}}}^{\textbf{Total labor input per one firm}} = \underbrace{\left(\frac{L}{M}\right)}_{\textbf{constant}}.$$

This means that there is *no* labor reallocation across firms by trade liberalization in the short run because total labor input per firm is fixed at L/M. All labor reallocation by a decrease in f_d in the short run is caused *within firms.*

A decrease in f_d induces firms to increase labor input in production divisions through the reduction of labor input in head offices. This, then, can promote the division of labor.

5.2 Decomposition of labor reallocation

Now, we explicitly decompose the effect of a decrease in f_d on t_E into an effect in the short run and long run as follows

$$\frac{dt_E}{df_d} = \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+}\underbrace{\frac{\partial l_S}{\partial f_d}}_{-}}_{\textbf{reallocation within firms }(-)} + \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+}\underbrace{\frac{\partial l_S}{\partial M_E}}_{-}\underbrace{\frac{dM_E}{df_d}}_{-}}_{\textbf{reallocation across firms }(+)}. \tag{12}$$

From (12), the result of Proposition 3, and $dt_E/df_x > 0$, we obtain the following proposition.

Proposition 6. *Under Assumption 1, a decrease in fixed costs promotes the division of labor in the short run while it curbs that in the long run. The latter effect dominates the former. Labor reallocation across firms is an essential source of the division of labor in the long run.*

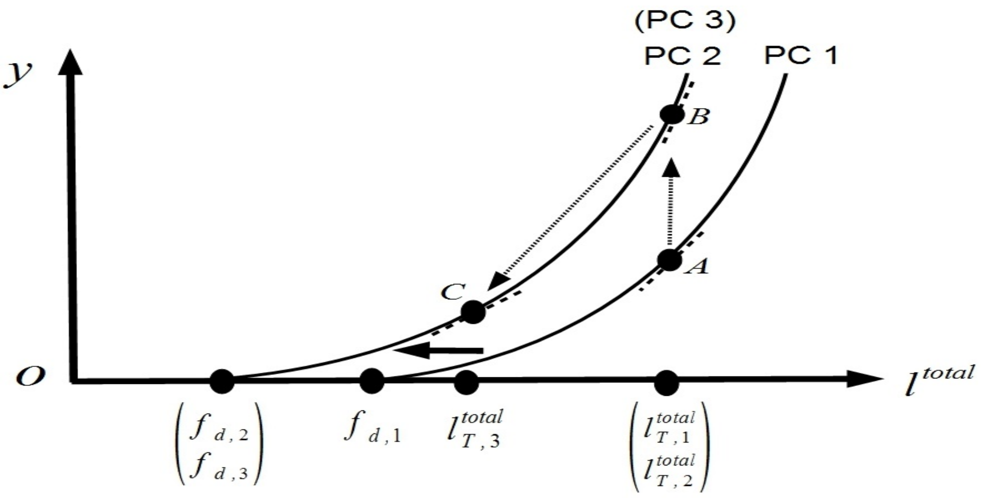

Figure 4: Productivity and labor reallocation in the short run and long run

For a decrease in f_d under a partial regime, Figure 4 shows the decomposition of the effects of a decrease in f_d on marginal productivity into three effects. Figure 4 illustrates two production curves, PC 1 and PC 2 (PC 3) in $(l^{\text{total}} - y)$ space. l^{total} is a firm's total labor inputs. That is, l^{total} is defined as $l^{\text{total}} = l + f_d$. Note that from this definition, $l_E^{\text{total}} - f_d$ refers to the labor input of the production division, l_E. Let l^h be the labor input of the head office. $l_{E,j}^{\text{total}}$ and l_j^h represent variables at the j-th stage where $j \in \{1, 2, 3\}$. In the first stage of the initial equilibrium, we obtain each firm's employment and production, which is represented by point A on PC 1. In the second stage after f_d decrease and before the number of firms changes, this is represented by point B. In the third stage after f_d decrease and before the number of firms changes, it is represented by point C.

The first effect of a decrease in f_d on marginal productivity is a transition from point A on PC 1 to point B on PC 2 after a decrease in f_d. At this stage, $f_{d,2} < f_{d,1}$ hold. In this transition, l_E increases by interval $f_{d,1}f_{d,2}$. This indicates that in the short run, firms reassign labor of interval $f_{d,1}f_{d,2}$ from the management division to the production division ($l^h_{E,2} < l^h_{E,1}$) while retaining $l^{\text{total}}_{E,1}$ units of total labor ($l^{\text{total}}_{E,2} = l^{\text{total}}_{E,1}$). This reassignment effect on the number of teams and productivity is negative, as shown in Figure 4, where the slope of the tangent decreases (positive reallocation effect within firms).

The second effect is a transition from point B on PC 2 (PC 3) to point C on PC 2 (PC 3). At point B, all firms earn positive profit. This causes new entrants and decentralizes workers ($l^{\text{total}}_{E,3} < l^{\text{total}}_{E,1}$). In this transition, l_E decreases by interval $l^{\text{total}}_{E,3}$ $l^{\text{total}}_{E,1}$. This decentralization effect on the number of teams and productivity is negative, as shown in Figure 4, where the slope of the tangent increases (negative reallocation effect across firms).

A transition from point A to point B raises the labor input of the production division by interval $f_{d,2}f_{d,1}$ (positive reallocation effect within firms) while a transition from point B to point C reduces the labor input of the production division by interval $l^{total}_{T,3}l^{total}_{T,1}$ (positive reallocation effect across firms). Since the interval $l^{total}_{T,3}l^{total}_{T,1}$ is greater than the interval $f_{d,2}f_{d,1}$, the slope of the tangent at point C is greater than that at point A. These results indicate that an essential source of the division of labor in the long run is labor reallocation across firms (the decentralization of labor promoted by new entrants).

6 Comparison with variable markup rate model

Can the abovementioned results be changed by a competitive environment? In this section, we compare the effects of a decrease in f_d between a constant markup rate model and a variable markup rate model. We focus on a variable markup rate, such as that depending on the number of firms. The model of Chaney and Ossa is also a variable markup rate model in the sense that the markup rate depends on the aggregate labor force, such as the model of Krugman (1979). However, the effect of a decrease in f_d in the model of Chaney and Ossa is the same as that of the constant markup rate model. We formulate the markup rate as $\mu = g(M)$, where $g'(M) < 0$ follows Blanchard and Givazzi (2003).

We should note that in such a model, we can obtain an equation such as (12). In the short run, the effect of a decrease in f_d in the variable markup rate model is the same as that of the constant markup rate model because the number of firms is fixed in the short run and a procompetition effect does not occur. Hence, the magnitude of the short run effect is the same in both models. However, they are different in the long run. How do they differ?

In order to analyze this, we further decompose the effects of labor reallocation across firms. In terms of these effect, the magnitude of $(dt_S/dl_S)(\partial l_S/\partial M_E)$ is the same in both models but (dM_E/df_d) is different. Then, we decompose dM_E/df_d as follows. M depends on l from the labor market-clearing condition $Ml = L$. We let M_{LMC} denote the number of firms characterized by the labor market-clearing condition. l depends on y from production function $y = l^2/2\gamma f$. We let l_{PF} denote the labor input in the production division characterized by the production function. Hence, we obtain the following condition

$$\frac{dM_E}{df_d} = \underbrace{\frac{dM_{LMC}}{dl_{PF}}}_{-} \underbrace{\frac{dl_{PF}}{dy_E}}_{+} \frac{dy_E}{df_d}. \tag{13}$$

For the effects on the right-hand side, the magnitude of dM_{LMC}/dl_{PF} and dl_{PF}/dy_E are the same in both models. Hence, the only difference in both models is dy_E/df_d.

In the variable model, dy_E/df_d can be decomposed as follows

$$\frac{dy_E}{df_d} = \overbrace{\underbrace{\frac{\partial y_E}{\partial f_d}}_{+}}^{\textbf{shift of FE}} + \overbrace{\underbrace{\frac{\partial y_E}{\partial \mu}}_{-} \underbrace{\frac{d\mu}{dM_E}}_{-} \frac{dM_E}{df_d}}^{\textbf{shift of PP}}, \tag{14}$$

From (13) and (14), we can find $dM_E/df_d < 0$ and $dy_E/df_d > 0$. The second term on the right-hand side of (14) represents the procompetition effect. This effect is negative from the following reason. In the short run, all firms make losses, and hence, some firms exit the market in the long run. This is represented by $dM_E/df_d < 0$. This exit raises the markup rate from the assumption of $g'(M) < 0$. This is explained as represented by $d\mu/dM_{VM} < 0$ in (14). A decrease in the markup rate reduces p keeping y from $PP : p = \mu MC(y)$. This enables firms to raise the output form $FE : p = AC(y)$. Otherwise, $p < AC(y)$ holds and this makes firms exit the market. That is, firms raise output in order to survive. This is explained by $\partial y_E/\partial \mu < 0$.

From $dy_E/df_d > 0$ and $dt_E/df_d = (dt_E/dl_{PF})(dl_{PF}/dy_E)(dy_E/df_d)$, we can obtain $dt_E/df_d > 0$. From $dt_E/df_d > 0$, $dy_E/df_d > 0$, (13), and (14) we can obtain the following conditions.

$$\frac{dt_E}{df_d} = \underbrace{\underbrace{\frac{dt_S}{dl_S}}_{+}\underbrace{\frac{\partial l_S}{\partial f_d}}_{-}}_{\textbf{reallocation within firms } (-)} + \underbrace{\underbrace{\frac{\partial t_S}{\partial M_E}}_{-}\underbrace{\frac{dM_E}{df_d}}_{-}}_{\textbf{reallocation across firms } (+)}$$

$$= \underbrace{\frac{dt_S}{dl_S}\frac{\partial l_S}{\partial f_d}}_{-} + \underbrace{\underbrace{\frac{\partial t_S}{\partial M_E}}_{-}\underbrace{\frac{dM_{LMC}}{dy_E}}_{-}\underbrace{\overbrace{\frac{\partial y_E}{\partial f_d}}^{\textbf{shift of FE}}}_{+}}_{+} + \underbrace{\underbrace{\frac{\partial t_S}{\partial M_E}}_{-}\underbrace{\frac{dM_{LMC}}{dy_E}}_{-}\underbrace{\overbrace{\frac{\partial y_E}{\partial \mu}\frac{d\mu}{df_d}}^{\textbf{shift of PP}}}_{-}}_{-} > 0. \qquad (15)$$

From (15), we can obtain the following proposition.

Proposition 7. *Under Assumption 1 and the variable markup rate, a decrease in fixed costs curbs the division of labor though labor reallocation based on the procompetition effect promotes that.*

We can explain the abovementioned analysis using Figure 5, in which initial equilibrium is shown in point E. A decrease in f_d immediately shifts the FE curves downward. Furthermore, a decrease in f_d reduces the markup rate, μ, and this shifts the PP curve downward. Then, new equilibrium is shown in point E'' in Figure 5.

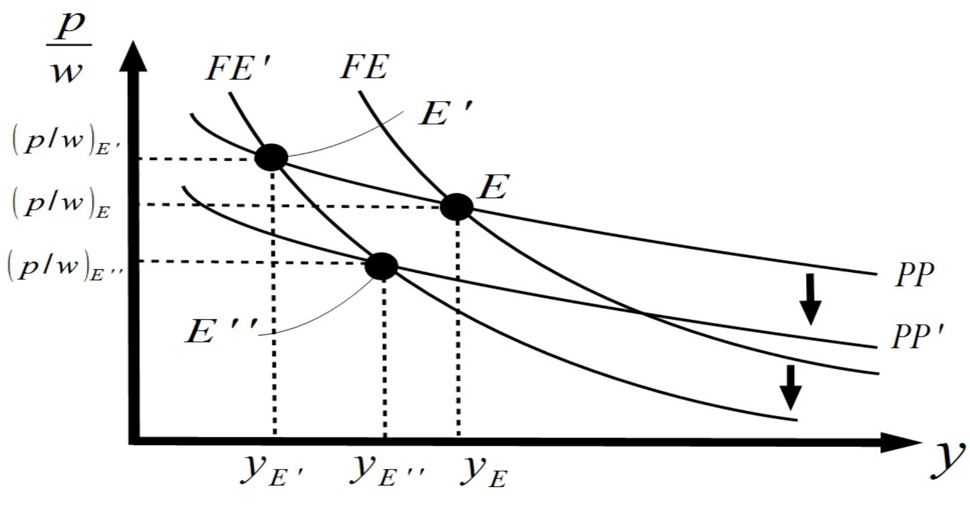

Figure 5: Equilibrium allocation and decrease in fixed costs

Equilibrium point E' accounts for only an effect of a shift of FE and this point is similar to equilibrium point E' of the constant markup rate model in Figure 3.

That is, the procompetition effect represents positive impact on the division of labor while the effect of shift of FE represents negative impact on it. In other words, the procompetition effect represents labor concentration on operating firms while the effect of shift of FE represents labor decentralization. Therefore, labor reallocation across firms that accounts for a decrease in fixed costs differs between both models.

These results are in contrast with Kamei (2014). Kamei (2014) incorporates Chaney and Ossa's (2013) division of labor into a general oligopolistic equilibrium model with a variable markup rate. The variable markup rate of Kamei (2014) depends on not only the number of firms and but also firm output. Kamei (2014) does not impose a free-entry and free-exit condition. In such a model, Kamei (2014) indicates that an increase in the number of firms (exogenous change) curbs the division of labor. That is, labor reallocation across firms based on the procompetition effect has a negative impact on the division of labor. This is because an increase in the number of firms reduces the markup rate, raises the real wage rate, and then, curbs the division of labor.

On the other hand, our variable markup rate model indicates that in a transition from point E' to E'' (procompetition effect), the division of labor is promoted. while the number of firms, real wage rate increase, and firm output increase. This result can be explained as follows. When the markup rate decreases through an increase in the number of firms, firms face an increase in the real wage rate from $PP : p = \mu MC(y)$ and raise output to attain zero profit from $FE : p = AC(y)$. This promotes the division of labor.

These results suggests that the free-entry and free-exit condition plays a key role in determining the direction of labor reallocation across firms based on procompetition effect.

7 Conclusion

This paper investigates the effects of a decrease in fixed costs on the division of labor within firms. We construct a fixed-cost model that is quite similar to Chaney and Ossa (2013). In addition, we formulate a variable markup rate model following Blanchard and Givazzi (2003). Their variable markup rate depends on the number of firms.

In the constant markup rate model, a decrease in fixed costs curbs the division of labor. In the short run, the division of labor is promoted through labor reallocation within firms while in the long run, the division of labor is curbed through labor reallocation across firms. The latter effect dominates the former effect.

In the variable markup rate model, a decrease in fixed costs induces labor reallocation across firms which is the opposite direction of that of the constant markup rate model in addition although the direction of labor reallocation within firms is the same as that of the constant markup rate model.

The direction of labor reallocation across firms based on procompetition is opposite to that of the model of Kamei (2014), which does not impose the free-entry and free-exit condition. The free-entry and free-exit condition plays a key role in determining the direction of that reallocation based on procompetition effect.

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Appendix

Appendix.A: Derivation of some equations

Derivation of optimal core-competency of (2)

For minimization problem, $l^*(\underline{\omega}, \overline{\omega}) = \min_{\omega_c \in [\underline{\omega}, \overline{\omega}]} \; l(\underline{\omega}, \overline{\omega})$, we rewrite objective function as follows:

$$\begin{aligned} l(\underline{\omega}, \overline{\omega}) &= \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \gamma |\omega_c - \omega| d\omega \\ &= \frac{\gamma}{2} \left[\int_{\underline{\omega}}^{\omega_c} (\omega_c - \omega) d\omega + \int_{\omega_c}^{\overline{\omega}} (\omega - \omega_c) d\omega \right] \\ &= \frac{\gamma}{2} \left[\frac{-1}{2} \left[(\omega_c - \omega)^2 \right]_{\underline{\omega}}^{\omega_c} + \frac{1}{2} \left[(\omega - \omega_c)^2 \right]_{\omega_c}^{\overline{\omega}} \right] \\ &= \frac{\gamma}{2} \left[\frac{1}{2} (\omega_c - \underline{\omega})^2 + \frac{1}{2} (\overline{\omega} - \omega_c)^2 \right]. \end{aligned}$$

By minimizing $l(\underline{\omega}, \overline{\omega})$ with respect to ω_c, we can obtain the following first order condition:

$$(\omega_c - \underline{\omega}) - (\overline{\omega} - \omega_c) = 0.$$

Hence, we have core-competency as follows

$$\omega_{c|[\underline{\omega},\overline{\omega}]} = \frac{\underline{\omega}+\overline{\omega}}{2}.$$

Derivation of $l(\omega_i, \omega_{i+1})$ of (3)

By substituting $\omega_{c|[\underline{\omega},\overline{\omega}]}$ for ω_c of $l(\underline{\omega},\overline{\omega})$, we can obtain the following equations:

$$\begin{aligned}
l^*(\underline{\omega},\overline{\omega}) =& \frac{1}{2}\int_{\underline{\omega}}^{\overline{\omega}} |\omega_{c|[\underline{\omega},\overline{\omega}]} - \omega| d\omega \\
=& \frac{\gamma}{2}\left[\frac{1}{2}\left(\frac{\underline{\omega}+\overline{\omega}}{2} - \underline{\omega}\right)^2 + \frac{1}{2}\left(\overline{\omega} - \frac{\underline{\omega}+\overline{\omega}}{2}\right)^2\right] \\
=& \frac{\gamma}{2}\left(\frac{\overline{\omega}-\underline{\omega}}{2}\right)^2.
\end{aligned}$$

$l^*(\underline{\omega},\omega_{c|[\underline{\omega},\overline{\omega}]})$ can be obtained as follows:

$$\begin{aligned}
l^*(\underline{\omega},\omega_{c|[\underline{\omega},\overline{\omega}]}) =& \frac{\gamma}{2}\int_{\underline{\omega}}^{\omega_{c|[\underline{\omega},\overline{\omega}]}} |\omega_{c|[\underline{\omega},\overline{\omega}]} - \omega| d\omega \\
=& \frac{\gamma}{4}\left[(\omega_c - \omega)^2\right]_{\underline{\omega}}^{\omega_c} \\
=& \frac{\gamma}{4}\left(\frac{\overline{\omega}-\underline{\omega}}{2}\right)^2.
\end{aligned}$$

Hence, we can get $l^*(\underline{\omega},\overline{\omega}) = 2l^*(\underline{\omega},\omega_{c|[\underline{\omega},\overline{\omega}]})$.

Derivation of $l_{line|unit}$ of (4)

We can obtain $l_{line|unit}$ of (4) from the following calculation:

$$\begin{aligned}
l_{line|unit} =& t \times l(0, 2/t) \\
=& 2t \times l(0, 1/t) && \text{by (2) and (3)} \\
=& 2t \times \frac{1}{2}\int_0^{1/t} \gamma\omega d\omega \\
=& t\left(\int_0^{1/t} \gamma\omega d\omega\right).
\end{aligned}$$

Derivation of optimal total cost function of (5)

$l(t,y) = tf + l_{line} = tf + (\gamma y)/(2t)$ and $t(y) = [(\gamma y)/(2f)]^{1/2}$ give optimal total cost function of (5) as follows:

$$\begin{aligned}
TC(y) =& wf_d + wl(t^*,y) \\
=& wf_d + wft^* + w\left(\frac{\gamma y}{2t^*}\right) && \text{by } l(t,y) = tf + l_{line} = tf + (\gamma y)/(2t) \\
=& wf_d + wf\left(\frac{\gamma y}{2f}\right)^{1/2} + w\frac{\gamma y}{2}\left(\frac{\gamma y}{2f}\right)^{-1/2} && \text{by } t(y) = [(\gamma y)/(2f)]^{1/2} \\
=& wf_d + wf^{1/2}y^{1/2}\left[\left(\frac{\gamma}{2}\right)^{1/2} + \frac{\gamma}{2}\left(\frac{\gamma}{2}\right)^{-1/2}\right] \\
=& wf_d + w(2\gamma fy)^{1/2}.
\end{aligned}$$

Appendix.B: Proof of Proposition 3

A relation between the number of team and marginal cost

We can obtain a relation between the number of team and marginal cost from the following calculation:

$$\begin{aligned}
MC =& \frac{dTC(y,t)}{dy} \\
=& \frac{\partial TC(y,t)}{\partial y} + \underbrace{\frac{\partial TC(y,t)}{\partial t}}_{0}\frac{dt}{dy} \\
=& \frac{\partial TC(y,t)}{\partial y} \\
=& \frac{w\gamma}{2t}.
\end{aligned}$$

A relation between the number of team and marginal labor productivity

We can obtain a relation between the number of team and marginal labor productivity from the following calculation:

$$\begin{aligned} MPL &= \frac{dy}{dl} \\ &= \left[\frac{dl(y,t)}{dy}\right]^{-1} \\ &= \left[\frac{MC}{w}\right]^{-1} \\ &= \frac{2t}{\gamma}. \end{aligned}$$

Appendix.C: Firm structure

Generality of the technology in (1)

Next, We examine that how general and valid the technology which we adopt in equation (1) is in comparison to the one adopted by Chaney and Ossa (2013).

The technology we adopted is different from the one adopted by Chaney and Ossa (2013), in two points. Equation (1) in this paper corresponds to the equation of Chaney and Ossa (2013) as follows:

$$l(\underline{\omega}, \overline{\omega}) = \frac{1}{2}\int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega}+\overline{\omega}}{2} - \omega\right)^{\beta} d\omega. \tag{C.1}$$

Equation (C.1) and (1) are equal, when $\beta = 1$ in (C.1) and $\gamma = 1$ in (1).

We examine a characteristic of parameter, β by seeing shape of $l(\underline{\omega}, \overline{\omega})$. For simplicity, we assume $\gamma = 1$ and $t = 1$. When $\beta = 1$, the integral term of the right hand side in (C.1) corresponds to the area formed by "Benchmark Line" shown in Figure 6. When $\beta > 1$, the one corresponds to the area formed by "Curve H" shown in Figure 6. When $0 < \beta < 1$, the one corresponds to the area formed by "Curve L" shown in Figure 6 implies that the effect of an increase in β is parallel to the effect of a decrease in γ.

If we adopts the technology in (A.1), the equilibrium allocation are rewritten by:

$$l_E = \frac{2(\beta+1)-\mu}{\mu-(\beta+1)} f_d,$$

$$y_E = \left(\frac{\beta+1}{\mu-(\beta+1)} f_d\right)^{\beta+1} \left(\frac{\beta}{\beta+1}\frac{1}{f}\right)^{\beta},$$

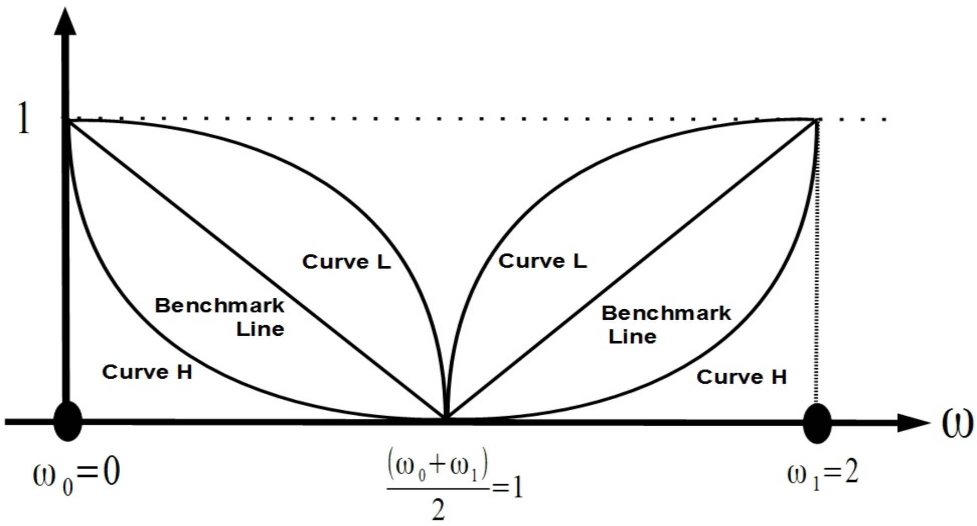

Figure 6: comparison between sequential task structures

$$MPL_E = (\beta+1)\left[\left(\frac{\beta}{\beta+1}\right)\left(\frac{\beta+1}{\mu-(\beta+1)}\right)\frac{f_d}{f}\right]^{\beta},$$

$$t_E = \left(\frac{\beta}{\beta+1}\right)\left(\frac{\beta+1}{\mu-(\beta+1)}\right)\frac{f_d}{f}.$$

The next table shows that the effect of an increase in β is parallel to the effect of a decrease in γ on certain conditions.

Table 1

	l_E	y_E	MPL_E
$\beta\uparrow$	0	+ only if $t_E > 1$	+ only if $t_E > 1$

α's amplification an effect also occurs on certain conditions. Moreover, an effect of f does not change. Therefore, this suggests that the technology which we adopt does not loose generality quite much in comparison to the one adopted by Chaney and Ossa (2013) .

Validity of the technology in (1)

Martins, Scarpetta and Pilat (1996) shows that almost all industries in OECD have markup rate which belongs to set $(1,2)$. Therefore, the internal solution condition $2<\mu$ does not seems to have reality. This property highly depends on organization parameter β. If we adopts the technology in (C.1), internal solution condition is

$$\mu > \beta + 1.$$

Therefore, by assuming organization parameter β to be in (0,1), model's mark-up rate μ can be consistent with the empirical studies.

However, assuming β to be in (0,1) makes tractability of the model decrease. For analytical simplicity, we assume β to be 1.

Appendix D: Shape of PP_E curve and FE_E curve in Figure 3

In this section, we examine shape of PP_E curve and FE_E curve in Figure 3.

We define $Z(y)$ as difference between right hand side of PP_E relation and of FE_E relation:

$$Z(y) \equiv \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2} - \left[\left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}\right] = B(2\gamma f)^{1/2}y^{-1/2} - f_d y^{-1}.$$

Certainly, $Z(y_E) = 0$ holds.

The derivative of function $Z(y)$ is given by

$$Z'(y) = -2^{-1}B(2\gamma f)^{1/2}y^{-3/2} + f_d y^{-2}.$$

When $y = y_E^*$, $Z'(y_E^*) = 0$ holds, where y_E^* is given by

$$y_E^* = 2\frac{f_d}{B^2\gamma f} = 4\frac{f_d}{B^2 2\gamma f} = 4y_E.$$

From $B > 0$, when $y < 4y_E$, $Z'(y) > 0$ holds and when $y > 4y_E$, $Z'(y) < 0$ holds. Furthermore, for the second order derivative of function $Z(y)$, $Z''(64y_E/9) = 0$ holds.

The limits of function $Z(y)$ are given by

$$\lim_{y\to\infty} Z(y) = 0,$$

$$\lim_{y\to 0} Z(y) = -\infty.$$

The above relations are proved in the following manner.

Proof.

$$\lim_{y\to\infty} Z(y) = \lim_{y\to\infty} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{0 - f_d}{\infty} \to 0,$$

$$\lim_{y\to 0} Z(y) = \lim_{y\to 0} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{-f_d}{0} \to -\infty.$$

Q.E.D.

According to the above results, the shape of $Z(y)$ is the one as shown in Figure 7.

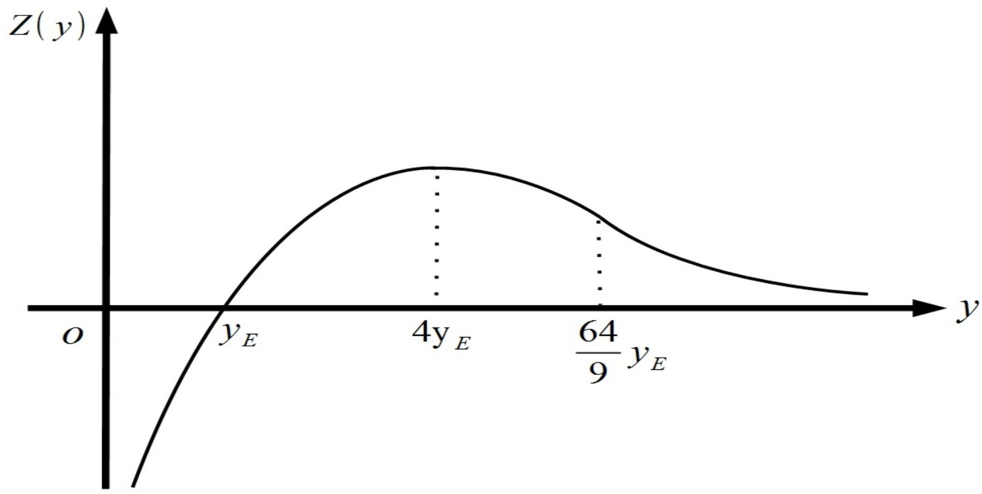

Figure 7: the shape of Z(y)

Figure 7 is consistent to Figure 3 and hence, Figure 3 is supported.