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Venture Capitalists, Asymmetric Information and Ownership in the Innovation Process*

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Abstract

This paper constructs a model where entrepreneurial innovations are sold into oligopolistic industries and where adverse selection problems between entrepreneurs, venture capitalists and incumbents are present. We first show that aggressive development of a basic innovation by better informed venture-backed firms is used as a signaling device to enhance the sale price of the innovation. We then show that incumbents can undertake early, preemptive, acquisitions to prevent such signaling driven overinvestment, despite the risk of buying a non-productive innovation. Therefore, to exist in equilibrium, venture capitalists must be sufficiently more efficient in selecting innovation projects, otherwise preemptive acquisitions will take place.

 $\label{lem:keywords: wenture-backed firm, innovation, signaling, over$ $investment, interim development, \\ M \& A$

JEL classification: C7, D21, D82, G24, L2, M13, O3

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1 Introduction

Venture-backed firms have been shown to be very aggressive in their early stage development of innovations. Increased venture capital activity is associated with relatively higher patenting rates, as shown by Kortum and Lerner (2000) and with a significant reduction in the time required for bringing a product to the market, as in Hellmann and Puri (2000).

The ability to solve moral hazard problems¹ and the exploitation of strategic productmarket effects² have been put forward in the literature aimed at explaining why venture
capitalists are more aggressive and successful in creating commercialized innovations. We
contribute to this literature by showing that an aggressive development of basic innovative
ideas by better informed venture-backed firms may be used as a signaling device to enhance
the sale price of the developed innovation. We also show that incumbents can undertake
early preemptive acquisitions to prevent such signaling driven overinvestment, despite the
risk of acquiring bad basic innovations.

To this end, we construct a model with the following features. There is a product-market, which is served by several incumbent oligopolists. There is also an entrepreneur possessing a basic innovation which, after additional development, may be good or bad for commercial use. In particular, if the nature of the basic innovation is good, investment into its development is assumed to increase a possessing incumbent's product-market profit and decrease the profits of its rivals. If the nature of the basic innovation is bad, it is assumed not to have any effect on the product-market profits of the possessor and its rivals. The entrepreneur cannot develop the basic innovation herself, but may either sell it to one of the

¹ See Gompers and Lerner (2001) for an overview of the empirical literature and Kaplan and Strömberg (2001) and Keuschnigg and Nielsen (2004) for specific models.

² See Norbäck and Persson (2006).

oligopolists or, alternatively, seek support from one of several venture capitalists. We then assume that Venture capitalists might be better in selecting good early stage projects than incumbents.³⁴ More formally, we assume that before making an offer for the basic innovation, the venture capitalists are perfectly informed about the nature of the basic innovation, while the incumbents only have *prior beliefs* about its nature.

If an incumbent firm directly obtains the innovation, the acquiring firm invests in development, and thereafter uses the innovation in the product market. If, on the other hand, the entrepreneur teams up with one of the venture capitalists, a venture-backed firm is founded, which invests in the development. Beyond the normal cost for the venture-backed firm of development, it is assumed that mimicking the development of a good innovation is costly to the venture capitalist owning a bad basic innovation⁵. As the investment in development only has an impact on product-market profits if the basic innovation is good, the size of the investment in development may therefore also serve as a signal⁶ to potential buyers of the

³ There are several reasons as to why this may be the case. One is that venture capitalists have specialized in assessing innovations whereas the management of incumbent firms has several different tasks to handle. Another reason can be that entrepreneurs may be less afraid of revealing information about an idea to a venture capitalist than to an incumbent since the risk of expropriation is lower: Contrary to incumbents, venture capitalists typically lack the ability to pursue the development of the idea without the entrepreneur. See, e.g., Gans et al. (2002, 2007).

⁴ Hellmann and Puri (2000) find that innovator firms are more likely to obtain venture funding than imitator firms; Engel and Keilbach (2007) conclude that venture capital firms have an advantage over incumbents in selecting firms with high innovative potential; and Kortum and Lerner (2000) argue that, as a consequence of intense scrutinizing of business plans, venture capitalists are able to finance many more risky, early-stage, projects than corporate research laboratories. See also Gompers and Lerner (2005, p. 160).

⁵ Reasons for this may lie in the cost of building up a convincing Potemkin village. It may also be a reduced form of modeling reputational concerns of the venture capitalists.

⁶ Technology journals provide evidence that firms in high-tech industries indeed use technology proxies such as the number of R&D personnel to signal the value of their firms to investors. See Megginson, Wang, and Chua, (2001) and references to articles in technology journals therein.

developed innovation. After the development choice the venture-backed firm then exits by selling the *developed* innovation to one of the incumbents.⁷ In the final stage, the acquirer and the non-acquiring incumbents then generate profits in the product market.

We first show that the better informed venture-backed firms have an incentive to use "overinvestment" to signal a good quality of the innovation to be sold to the incumbents, since this increases the sale price of the developed innovation.

Next, we turn to the issue of whether the incumbent firms benefit from waiting for a venture capitalist to signal good and bad basic innovations. Prima facie, it seems reasonable to believe that the incumbents let the better informed venture capitalist signal the quality of the projects and then acquire it. However, due to the interaction between informational and oligopolistic externalities, this does not necessarily hold. Indeed, we show that if the venture capitalists' informational advantage is not too large, it is optimal for the incumbents not to wait, but to use a preemptive acquisition. Key to understanding this result is to see how the signal affects the acquisition price of the developed innovation in this oligopolistic setting. In equilibrium, the acquisition price of the developed innovation is shown to equal an incumbent firm's valuation of obtaining it which, in turn, consists of the firm's profit if it obtains the developed innovation, net of its profit, if the innovation is obtained by a rival firm. As the signal through aggressive investments in development amplifies the acquisition price by more than the increase in the acquirer's profit – the increase in the acquiring the

⁷ Acquisitions are an important mode of exit for venture-backed firms. For instance, Cochrane (2005) uses data over the period 1987 to June 2000 from the VentureOne database and shows that 20 % of the ventures were acquired, 21 % were IPOs, 9% went out of business, while 49% remained private. Cumming and MacIntosh (2003) found similar figures. Other studies show that incumbents acquire innovative targets in order to gain access to their technologies; see Granstrand and Sjölander (1990), Hall (1990), Lerner and Merges (1998) and Bloningen and Taylor (2000).

developed innovation may be very costly.

We then show that *only* if venture-backed firms are sufficiently better informed than the incumbents about the prospect of the basic innovation and if the cost of mimicking a good innovation is sufficiently high, the incumbents wait and let a more informed venture capitalist select the projects before acquiring it. Consequently, to exist in equilibrium, venture capitalists must be sufficiently more efficient in selecting projects.

To our knowledge, this paper is the first in the literature on venture capital that studies the impact of adverse selection problems between entrepreneurs, venture capitalists and incumbents, when entrepreneurial innovations are sold into oligopolistic industries.⁸. This enables us to examine how the emergence of a well informed venture capital market can effect incentives for creating, developing and acquiring entrepreneurial innovations. In this vein, the paper can also be seen as a contribution the literature on entrepreneurship and innovation.⁹.

The existing literature on the pattern of and timing of sales (or licensing) of innovations to incumbents shows that early sales or licensing are more likely when property rights are more secure¹⁰. We add to this literature by showing that early sales are the more likely, the less efficient venture capitalists are in selecting projects, the better is the potential of the

⁸ See Hellmann (2002) for one of the few studies where venture capitalists compete with established firms financing entrepreneurs. The author shows that if and only if the innovation is a complement to the established firm's business, the established firm will finance the project. However, signaling and oligopolistic effects, which are the focus of our paper, are abstracted away.

⁹ For overviews, see Acs and Audretsch (2005) and Bianch and Henrekson (2005). Baumol (2004) stresses the importance of the different roles played by small entrepreneurial firms and large established firms in the innovation process in the USA, where small entrepreneurial firms create a large share of breakthrough innovations and large establish firms provide more routinized R&D.

¹⁰ See, for instance, Anton and Yao (1994), Arora and Ceccagnoli (2006), Gans and Stern (2000, 2003), Gans, Hsu, and Stern (2002, 2007), Hsu (2006) and Teece (1986).

project and the smaller is the cost of mimicking a good project.¹¹

This paper also make a contribution to the signaling literature¹². A crucial feature of the models in that literature, which we share, is that a seller of a good uses some device to signal the quality of the good. We add to this literature by focusing on a productive signal, i.e., a signal that affects the productivity of the asset (good) sold post-signaling in the ensuing product-market interaction¹³. Moreover, we add to the signaling literature by endogenously determining whether the ability to signal will be used in equilibrium. In our model, in order to exist in equilibrium, a signaler (the venture-backed firm) must be sufficiently more efficient than incumbent firms when selecting the innovation; otherwise incumbents will block the signaling through an early preemptive acquisition of the basic innovation.

The model is presented in Section 2. In Section 3, we explore how the magnitude of investments depends on the incentives to signal a good innovation. In Section 4, we determine the ownership pattern of innovations and study the effects of venture capital on the incentives for entrepreneurs to come up with basic innovations. In Section 5, we give a parametric example for our model, using a product market with linear demand and Cournot competition and a quadratic cost function for investments in the development. In Section 6, we explore the empirical implications of our model and discuss its robustness with regard to allowing the venture-backed firm to exit also by means of an initial public offering (IPO) as well as

¹¹ This paper also relates to the literature on patent licensing, where a licence is sold at an auction and the potential buyers are competing in a downstream market. See Katz and Shapiro (1986) or, for an overview, Kamien (1992). However, in those studies, the investment in the size of the sold asset does not affect both the information set and the technological set for the buyer and thus, the focus of those papers differs from ours.

¹² See, for instance, Riley (2001) for an overview.

¹³An exemption is Ben Shahar (2004) who allows for productive signaling in a real estate setting. However, in that paper, no product market effects are present, and no preemptive acquisitions are possible, both of which are crucial to our results.

allowing for both productive and non-productive signals. Section 7 concludes the paper.

2 The Model

Consider the model summarized in figure 1, where an industry is served by a set $\mathcal{I} = \{1, 2, ..., i, ..., N_I\}$ of ex-ante symmetric oligopolistic incumbents. In stage 0 of the model, an entrepreneur, denoted E, invests in a costly research effort e that could lead to the creation of a unique asset, which we refer to as a basic innovation. The basic innovation requires costly additional development for commercial use. Assume that the entrepreneur lacks the financial means to develop the basic innovation himself. Consequently, in stage 1, he may either sell it to one of the incumbents in the industry or, alternatively, seek support from one of the venture capitalist. We assume there to be a set $\mathcal{J} = \{1, 2, ..., j, ..., N_J\}$ of symmetric venture capitalists, who compete to provide expertise and financial support to the entrepreneur in return for equity holdings in the firm. We model the decision of whether to sell to an incumbent or team up with a venture capitalist as a first price sealed bid auction with the ex-ante symmetric oligopolists and the venture capitalists bidding for appropriating the basic innovation. This way of modeling enables us to concentrate on the basic message of our paper: how information and product-market effects determine the ownership of the basic innovation. Denote the Sale price of the Basic innovation by S^B .

We consider two types, θ , of innovations, good and bad ones. For a good basic innovation, $\theta = g$, a costly investment, measured by $k \geq 0$, leads to a successful development for commercialized use. A bad one, $\theta = b$, will never be commercially successful, irrespective of the level of investment into its development. We assume that whether the innovation is good or bad is not verifiable by a court at any point in time and, therefore, it is not contractible.

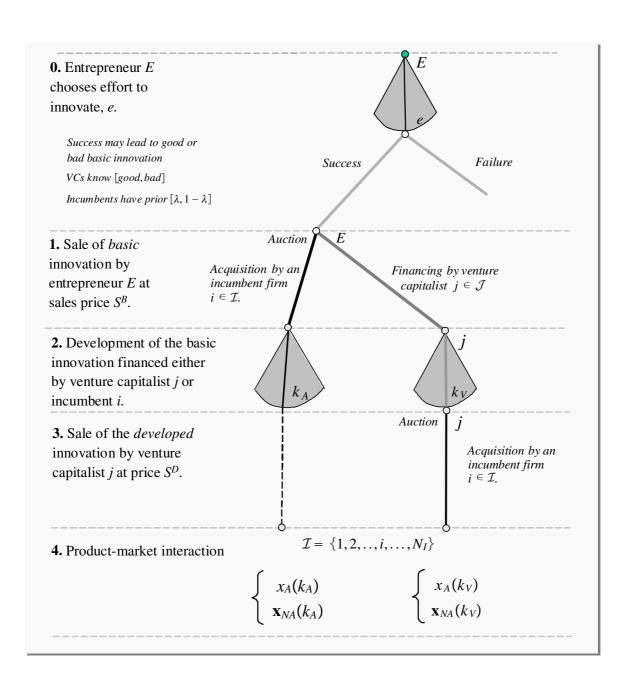


Figure 1: The structure of the game.

Instead, any investment made into its development is costlessly verifiable. Capturing the venture capitalists' superior information about the type of a basic innovation, we assume them to be perfectly informed about an innovation's type, while the incumbents initially only have prior beliefs about its type. In particular, incumbents assign a probability $\lambda \in]0,1[$ to the event that the innovation is good, and $1-\lambda$ to the event that it is bad. This prior is common knowledge. It is only after obtaining an innovation that an incumbent can inform himself perfectly regarding its nature. We finally assume that the nature of the innovation is revealed to all incumbents prior to their product-market interaction.

If an incumbent firm i obtains the innovation in stage 1, the acquiring firm invests k_i in its development in stage 2. We assume that the acquiring *incumbent* faces an investment cost function C(k) which, independent of the nature of the project, has the following properties:

Assumption 1 C(0) = 0, C'(k) > 0.

If the entrepreneur, on the other hand, obtains financing and support from a venture capitalist j in stage 1, the venture-backed firm can, in stage 2, invest a costlessly verifiable and irreversible amount k_j in the development of the basic innovation. To stress the role played by asymmetric information in the innovation process, we assume the development of an innovation to be equally costly for the incumbents and a venture backed firm. Venture-backed firms, however, incur an extra cost in addition to this development cost, $\Delta(k)$, if

¹⁴ As noted above, reasons why this may be the case are that venture capitalists have specialized in assessing innovations whereas the management of incumbent firms has several different tasks to handle; or that entrepreneurs may be less afraid of revealing information about an idea to a venture capitalist than to an incumbent since the risk of expropriation is lower.

¹⁵ Note that λ can be interpreted as the degree of asymmetric information between incumbents and venture capitalists. If the innovation is good, the higher the λ , the lower is the asymmetry of information; if the innovation is bad, the higher is the λ , the higher is the degree of asymmetry of information.

they develop a bad basic innovation¹⁶. In particular, we assume that the venture-backed firm faces an investment cost function

$$C(k|\theta) = \begin{cases} C(k) & \text{for } \theta = g \\ C(k) + \Delta(k) & \text{for } \theta = b \end{cases}$$

with the following properties:

Assumption 2
$$C(0|\theta) = 0, C'(k|g) < C'(k|b)$$
.

An implication of assumption 2 is that for a venture-backed *firm*, an idea is not only good or bad in its capacity of delivering a successful development, but also in the sense of being *more* or *less resource consuming* during its development.

In stage 3, upon development, the venture-backed firm j exits by selling the developed innovation by means of a first-price perfect information auction, to one of the N_I incumbent firms. We denote the Sale price of this Developed innovation by S^D . Finally, in stage 4, the incumbent firms compete in oligopoly interaction, setting an action x_i , taking into account the investment level, k_i or k_j , respectively.¹⁷ This action may be considered as (a combination of) setting a price, setting a quantity, and/or engaging in advertising, etc.

¹⁶ This captures the cost of building up a convincing *Potemkin village*, i.e., the cost of enabling the venture-backed firm to pretend that the innovation is good if, in fact, it were bad; or a reduced form of modeling reputational concerns of the venture capitalists.

¹⁷ In the degenerate case where $\lambda=1$, it is shown in Norbäck and Persson (2006) that the acquiring firm will never invest sequentially in equilibrium. As this result would carry over to our model setup, we do not lose generality by assuming that the acquiring oligopolist will not add to the venture capitalist's investment before entering the product market competition. We gain, however, in the simplicity of our exposition.

3 Incentives to develop

In this section, we will show how the incentives to develop the basic innovation differ between venture-backed and incumbent firms. We start with the product-market equilibrium. Solving the game backwards, we then determine the sale price S^D in stage 3 and the venture-backed firm's optimal investment decision in stage 2 in a separating perfect Bayesian equilibrium, where venture-backed firms signal the nature of the basic innovation through their investment choice. In case of an early – preemptive – acquisition, we solve for an incumbent's optimal investment.

3.1 Product-market equilibrium

In the product-market interaction, each firm i seeks to maximize its direct product-market profit, $\Pi_i(x_i, \mathbf{x}_{-i}, k | \theta)$, by choosing an action $x_i \in R^+$. Besides its own choice x_i , its profit also depends on the vector of actions taken by rival incumbent firms, \mathbf{x}_{-i} , the amount of development undertaken, k, as well as the nature of the project, θ . Assume the existence of a unique Nash-Equilibrium in actions, $\mathbf{x}^*(k | \theta)$, defined by the first-order conditions

$$\frac{\partial \Pi_{i}}{\partial x_{i}}\left(x_{i}^{*}\left(k|\theta\right), \mathbf{x}_{-i}^{*}\left(k|\theta\right), k|\theta\right) = 0, \ \forall i \in \mathcal{I},\tag{1}$$

and where x_i^* is firm i's equilibrium action; and \mathbf{x}_{-i}^* the vector of its rivals' equilibrium actions. Using the ex-ante symmetry among incumbents, ex-post there are going to be two types of firms; the acquiring firm, denoted A, and the non-acquiring firms, denoted N. Denote by x_A^* the equilibrium action of the acquiring firm and by x_N^* the vector of the equilibrium actions of the non-acquiring incumbent firms. Note that the equilibrium actions x_A^* and x_N^* only depend on k and θ . For this reason, we can directly define the equilibrium

product-market profits R_A of the acquirer and R_N of the non-acquirers as functions of k and θ :¹⁸

$$R_{A}(k|\theta) \equiv \Pi_{A}(x_{A}^{*}(k|\theta), \mathbf{x}_{N}^{*}(k|\theta), k|\theta);$$

$$R_{N}(k|\theta) \equiv \Pi_{N}(\mathbf{x}_{N}^{*}(k|\theta), x_{A}^{*}(k|\theta)).$$
(2)

We denote these reduced form product-market profits by R_A and R_N , respectively, as they have revenue character in the investment stage of the game. We assume that they have the following properties:

Assumption 3 The equilibrium product-market profits, $R_A(k|\theta)$ and $R_N(k|\theta)$, satisfy

1.
$$R_A(0|g) = R_N(0|g) = R_A(k|b) = R_N(k|b);$$

2.
$$\frac{dR_A(k|g)}{dk} > 0$$
, $\frac{dR_N(k|g)}{dk} < 0$, $\frac{dR_A(k|b)}{dk} = \frac{dR_N(k|b)}{dk} = 0$.

This assumption stipulates (1) if a good innovation does not receive investment into its development, or if a basic innovation is bad for development, this does not affect the product market profits. It specifies (2) that the equilibrium product-market profit of the acquirer of a good innovation is strictly increasing in the investments into the innovation's development, whereas such investments strictly decrease the rivals' profits; and that the effect of a bad innovation on the product-market is nil, irrespective of the amount of investment into its development¹⁹.

To save space, we write the arguments in $R_N(k|\theta) \equiv \Pi_N(\mathbf{x}_N^*(k|\theta), x_A^*(k|\theta))$ with a slight abuse of notation. Note that $R_N(k|\theta) = \Pi_N(x_N^*(k|\theta), \underbrace{x_N^*(k|\theta), ..., x_N^*(k|\theta)}_{N_I-2}, x_A^*(k|\theta))$.

To keep the exposition simple, we do not formulate this as an assumption on the primitives of the model but use the derivatives of equilibrium product market profits, $\frac{dR_A(k|\theta)}{dk}$ and $\frac{dR_N(k|\theta)}{dk}$, keeping in mind that these summarize the total effects on the product-market profits. This assumption holds, e.g., in the Linear-Quadratic Cournot model which is presented in section 5 but it is also compatible with other oligopoly models such as that of Farrell and Shapiro (1996).

3.2 Sale of the developed innovation by the venture-backed firm

After having solved for the product market equilibrium, we now turn to stage 3, the sale of a developed innovation by a venture-backed firm. Note that stage 3 is only reached if in stage 1, there was no preemptive acquisition, i.e., if a venture-backed firm has been established in stage 1. As mentioned, we will solve for a separating perfect Bayesian equilibrium in which a venture-backed firm signals the type of the innovation in its possession.

In our model, a perfect Bayesian equilibrium consists of a set of strategies and a belief function, giving the incumbents' common probability assessment that the innovation on sale by a venture-backed firm is good after observing the investment level k_j , such that (i) the venture-backed firm's strategy is optimal given the incumbents' strategies; (ii) the beliefs are derived from the venture-backed firm's strategy using Bayes' rule where possible; and (iii) the incumbents' bids following the venture-backed firm's choice of k_j constitute a Nash equilibrium in the first price sealed-bid auction where the probability that the innovation on sale is good is given by the beliefs following the observation of k_j . The perfect Bayesian equilibrium is separating if the venture-backed firm's investment choice is different for the two types of basic innovations.

Since, in a separating equilibrium, the type of the innovation is correctly inferred by the bidding incumbent firms, it is appropriate to model the acquisition process in stage 3 as a perfect information first price sealed bid auction with the N_I incumbents as the bidders and the venture-backed firm as the seller. After the bids have been announced, the developed innovation is sold to one of the incumbents at the bid price.

In order to solve for the auction, we first consider the incumbents' valuations, w, for the developed innovation. For this purpose, define $w(k|\theta) = R_A(k|\theta) - R_N(k|\theta)$, the first term of which shows the profit for an incumbent firm possessing the innovation, and the second

term of which shows the profit of an incumbent if the innovation has been obtained by a rival incumbent firm.

Let $k_j^*(\theta)$ be the venture-backed firm's equilibrium investment choice as a function of the type of innovation in its possession and $S^D(k)$ the equilibrium sale price of the developed innovation in stage 3 as a function of the venture-backed firm's investment choice. Then, we can state the following:

Lemma 1 In any separating perfect Bayesian equilibrium, in stage 3, a good developed innovation is acquired by an incumbent firm, at a price equal to a rival incumbent firm's valuation of the developed good innovation, i.e., $S^{D}\left(k_{j}^{*}\left(g\right)\right)=w\left(k_{j}^{*}\left(g\right)\middle|g\right)$; whereas a bad innovation receives an equilibrium price of $S^{D}\left(k_{j}^{*}\left(b\right)\right)=w\left(k_{j}^{*}\left(b\right)\middle|b\right)=0$.

Proof. See Appendix A. ■

In the separating equilibrium, the type of the innovation is correctly signaled to the buyers. If the buyers infer from the venture-backed firm's investment decision that the innovation is good, they have a valuation of w(k|g), which corresponds to the acquirer's product market profit less the profit an acquirer would receive from his outside option, i.e. to not acquire. The equilibrium action in a perfect information first price auction is to bid the second highest bidder's valuation, which – as all incumbents are symmetric – equals the highest bidder's valuation. Thus, the sale price equals the incumbents' valuation, w(k|g). If they infer that the innovation is bad, the profits of an acquirer and those of a non-acquirer are the same and thus, w(k|b) = 0, which is also the equilibrium bid.

3.3 Development of the basic innovation and signaling

Solving the game further backwards, we now turn to stage 2 where we determine the optimal investment into the development of the basic innovation an acquiring incumbent (subsection 3.3.1) and a venture-backed firm (subsection 3.3.2) would choose.

3.3.1 The acquiring incumbent's optimal development

Assume first that an incumbent preemptively acquired the innovation in stage 1. After having acquired the innovation, all assets and documentation from the entrepreneur go into his possession. This is in line with the previously made assumption that after obtaining and before investing into a project, the incumbent can perfectly inform himself regarding its nature.

Consider first that the incumbent acquirer i learns that $\theta = b$. As $\frac{dR_A(k|b)}{dk} = 0$, there is nothing to be gained from investing into a bad project and it invests $k_i^*(b) = 0$.

If, on the other hand, it learns that $\theta = g$, it faces the following maximization problem:

$$\max_{k} \left[R_A \left(k | g \right) - C \left(k \right) \right], \tag{3}$$

where $C(k) = \int_0^k C'(k) dk$ is the cost of investing k in developing the basic innovation, and where C'(k) is the associated marginal cost. Assume $R_A(k|g) - C(k)$ to be strictly concave in k. Then, the acquiring incumbent i chooses $k_i^*(g) = k_A$, such that

$$\frac{dR_A}{dk} = C'(k_A) \tag{4}$$

holds. We denote this optimal investment level by k_A as it is an Acquirer's optimal investment level into a good project. Figures 2 and 3 depict this optimality condition in point A. From panels (i), it can be seen that in this point, the marginal product-market profit $\frac{dR_A(k|g)}{dk}$ equals

the marginal cost of investment C'(k). In panels (ii) it can be seen that k_A maximizes the acquiring incumbent's profit, $R_A(k|g) - C(k)$. We will turn to a description of the other parts of these figures and comment upon them in subsection 3.3.2.

Lemma 2 summarizes our findings.

Lemma 2 The acquiring incumbent's optimal investment in stage 2 is $k_i^*(b) = 0$ and $k_i^*(g) = k_A$.

3.3.2 The venture backed firm's optimal development

Assume now that in stage 1, the entrepreneur teamed up with a venture capitalist to develop the basic innovation within a venture-backed firm. As it exits through a sale in stage 3, the venture-backed firm chooses its investment level k_j such that it maximizes the sale price $S^D(k_j)$, derived in Lemma 1, net the cost of that investment. As we are solving for a separating equilibrium, we need to determine an optimal investment choice for each type of basic innovation.

Consider the equilibrium investment level for the venture backed firm owning a bad basic innovation. As in a separating equilibrium, the bidding incumbents correctly infer its type and bid such that the venture-backed firm receives a sale price of $S^D\left(k_j^*\left(b\right)\right) = 0$ (see Lemma 1) so that an investment has no value $k_j > 0$. Therefore, the following Lemma holds:

Lemma 3 In any separating perfect Bayesian equilibrium, $k_j^*(b) = 0$.

Proof. See appendix B.

Using Lemmas 1 and 3, we can now construct the separating equilibrium. For this purpose, first note that assumptions 1 and 2 imply that the isoprofit curves of a venture-backed firm possessing a bad innovation and those of one possessing a good innovation only

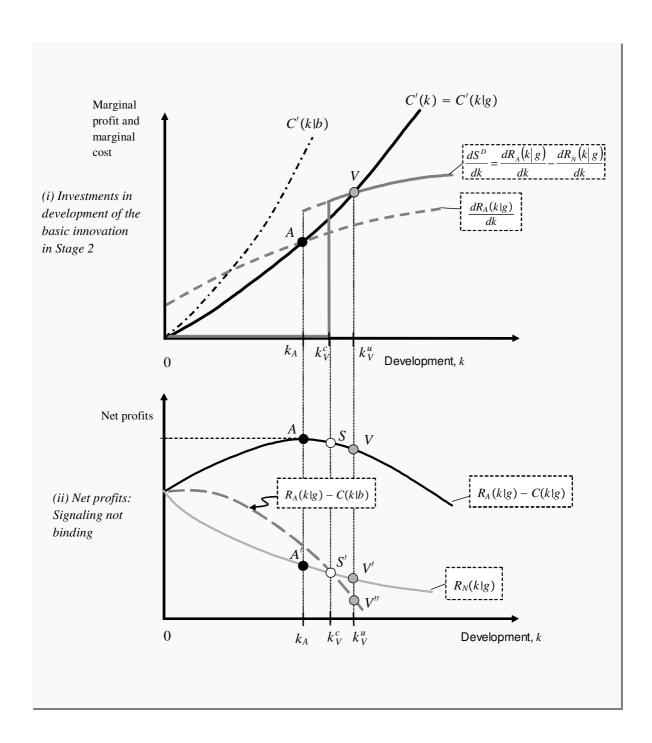


Figure 2: Incentives to develop innovations and net profits when the incentive compatibility constraint is not binding.

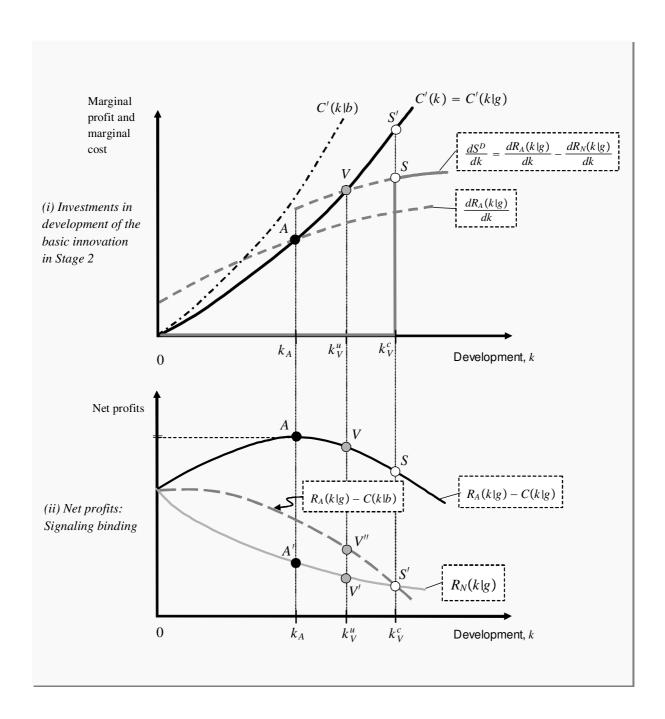


Figure 3: Incentives to develop innovations and net profits when the incentive compatibility constraint is binding.

cross once in the $S^D - k_j$ space, i.e., they fulfill the single-crossing property. Using this property, we can define an incentive compatibility constraint, (IC), that, if it holds, ensures that a venture-backed firm possessing a bad innovation would not mimic a firm possessing a good innovation. Equilibrium investment levels $k_j^*(g)$ and $k_j^*(b)$ fulfill this requirement if and only if

$$S^{D}(k_{i}^{*}(g)) - C(k_{i}^{*}(g)|b) \le S^{D}(k_{i}^{*}(b)) = 0.$$
 (IC)

Using this incentive compatibility constraint and the single-crossing property of the venture-backed firm's isoprofit curves, we can define the minimum investment level needed by a venture-backed firm to undertake to signal the good type of its innovation to the incumbents. Define this investment level with k_V^c .²⁰ This notation anticipates that k_V^c will be a Venture-backed firm's optimal level of investment if it is indeed constrained in its choice by this incentive compatibility constraint.

For a visualization, once more consider figures 2 and 3, in particular panels (ii). In these figures, any investment level larger than or equal to the one in point S', in which the dashed $R_A(k|g) - C(k|b)$ and the grey $R_N(k|g)$ lines intersect, is incentive compatible, as for $k \ge k_V^c$, $S^D(k) - C(k|b) = R_A(k|g) - C(k|b) - R_N(k|g) \le 0$.

Given the incentive compatibility constraint, one consistent equilibrium belief of the incumbents following an investment k is as follows: Incumbents assign a probability one to the project being bad (and bid zero) if they observe $k < k_V^c$ and they assign a probability one to the project being good (and bid w(k|g)) if they observe $k \ge k_V^c$.

Now that we have determined the equilibrium conditions that $k_j^*(g)$ must fulfill, we can solve for it. For this purpose, note that investment in a good basic innovation increases its sale price S^D as the incumbents' valuation of a good developed innovation in stage 3,

That is, k_{V}^{c} is defined by $S^{D}\left(k_{V}^{c}\right) - C\left(k_{V}^{c}|b\right) = 0$.

w(k|g), is an increasing function of k. Therefore, it is possible that, in equilibrium, the incentive compatibility constraint is not binding and the venture-backed firm chooses k such as to solve an *unconstrained* maximization program. Using Lemma 1, this maximization program is given by:

$$\max_{k} \left[S^{D}(k|g) - C(k|g) \right] \Leftrightarrow \max_{k} \left[R_{A}(k|g) - R_{N}(k|g) - C(k|g) \right].$$

Assuming $R_A(k|g) - R_N(k|g) - C(k|g)$ to be strictly concave in k, the unconstrained optimal investment of a Venture-backed firm, k_V^u , satisfies

$$\frac{dS^D}{dk} = \frac{dR_A}{dk} - \frac{dR_N}{dk} = C'(k_V^u | g). \tag{5}$$

Once more, consider figures 2 and 3. Panels (i) depict how the unconstrained optimal investment level, k_V^u , is derived from the optimality condition $\frac{dS^D}{dk} = \frac{dR_A}{dk} - \frac{dR_N}{dk} = C'(k_V^u|g)$. Panels (ii) of these figures then examine whether this unconstrained optimal investment level satisfies the incentive compatibility constraint. It holds for k_V^u in Figure 2, but not in Figure 3.

If k_V^u is insufficient to signal the good nature of the innovation, as in figure 3, then the venture-backed firm needs to invest beyond this level in order to signal. As we assumed $R_A(k|g)-R_N(k|g)-C(k|g)$ to be strictly concave in k, a venture-backed firm's profit beyond k_V^u is strictly decreasing, and therefore it chooses the smallest investment level satisfying incentive compatibility; k_V^c . Thereby, we have derived the following:

$$\textbf{Lemma 4} \ \textit{In the separating perfect Bayesian equilibrium, } k_{j}^{*}\left(g\right) = \left\{ \begin{array}{ll} k_{V}^{c} & \textit{if } k_{V}^{u} \leq k_{V}^{c} \\ \\ k_{V}^{u} & \textit{otherwise} \end{array} \right..$$

While Lemma 3 showed that in a separating perfect Bayesian equilibrium, a venture capitalist does not develop a bad basic innovation, Lemma 4 demonstrates that it develops

good basic innovations as if there were no incentive compatibility problem, as long as the unconstrained optimally chosen level of development suffices to signal. Good basic innovations are developed beyond this level in order to signal the good nature of the innovation. It follows from the construction of the incentive compatibility constraint that an additional investment must be undertaken whenever it is not sufficiently costly for a venture-backed firm possessing a bad innovation to mimic one possessing a good one. Thereby, we have derived the following result:

Proposition 1 In a separating equilibrium, a venture-backed firm signals

- (i) the bad nature of an idea by not investing anything into its interim development;
- (iia) the good nature of an idea by investing as it would under full information, as long as it is sufficiently costly for a venture-backed firm to mimic the development of a good innovation when it is bad; and
- (iib) the good nature of an idea by investing more than it would under full information, otherwise.

Note that the optimal investment level of a venture-backed firm, $k_j^*(g)$, is systematically higher than that of an incumbent firm, $k_i^*(g)$. There are two reasons for that. The first comes from the product-market interaction of the acquiring and non-acquiring incumbents: If (IC) is not binding, the venture-backed firm does not only take into account the positive impact of its investment on the profits of the acquirer, $\frac{dR_A(k|g)}{dk}$, but also the negative impact on the non-acquirers' profits, $\frac{dR_N(k|g)}{dk}$. The second reason comes from the necessity to signal. If (IC) is binding, the venture-backed firm has to invest beyond k_V^u . Therefore, we can state:

Proposition 2 For a good project, the optimal level of development by a venture-backed firm which sells the developed innovation to an incumbent firm exceeds the optimal level of

development by the acquiring incumbent firm, i.e., $k_{j}^{*}(g) > k_{i}^{*}(g)$.

This proposition extends Norbäck and Persson (2006), which states that venture-backed firms would develop basic innovations more aggressively than incumbents in order to internalize the strategic product-market effects. In addition to this internalization, in our framework, they invest aggressively in order to overcome an adverse selection problem.

4 The equilibrium ownership of basic innovations and incentives for basic ideas

In this section, we use the results obtained for the separating equilibrium in order to derive who develops basic innovations and, therefore, how aggressively this development will be pursued in equilibrium. To this end, we will first derive the venture capitalists' and the incumbents' valuations, determine their equilibrium bids, and then characterize ownership patterns. We conclude the section by pointing out implications for the incentives to come up with basic innovations.

4.1 Equilibrium ownership

The first step towards determining the equilibrium ownership and the acquisition price is to derive the stage 1 valuations. In contrast to stage 3 valuations, which we denoted by w, these stage 1 valuations will be denoted by v. Note that, as venture capitalists know the nature of the basic innovation, we need to distinguish between valuations for good and bad basic innovations.

Consider a venture capitalist's valuation for a bad basic innovation, denoted as $v_V(b)$. As

investing into a bad basic innovation does not result in an asset that can be sold at a positive price²¹, a venture capitalist has a valuation of zero for it, i.e., $v_V(b) = 0$. Now, consider a venture capitalist's valuation for a good innovation, denoted as $v_V(g)$. This is the sale price of the developed innovation in stage 3, net the investment costs. From Lemma 1, we have $S^D(k_j^*(g)) = w(k_j^*(g)|g) = R_A(k_j^*(g)|g) - R_N(k_j^*(g)|g)$ and thus, the venture capitalist's valuation of the entrepreneur's basic innovation is:

$$v_{V}(g) = S^{D}(k_{j}^{*}(g)) - C(k_{j}^{*}(g)|g)$$

$$= R_{A}(k_{j}^{*}(g)|g) - R_{N}(k_{j}^{*}(g)|g) - C(k_{j}^{*}(g)|g).$$
(6)

In line with our earlier notation, we denote by $v_V^u(g)$ the venture capitalists' valuation if the venture-backed firm is unconstrained in its choice of k, i.e., if its incentive compatibility constraint is not binding and by $v_V^c(g)$ the one if the venture-backed firm is constrained in its choice of k, i.e., if its incentive compatibility constraint is binding.

Let us turn to the valuation of the incumbents. Denote by v_{II} the ex-ante expected value for an incumbent firm of acquiring the basic innovation, when it would otherwise be obtained by a rival incumbent:

$$v_{II} = \lambda \left[R_A (k_A | g) - C (k_A) - R_N (k_A | g) \right].$$
 (7)

This is the difference in the expected net profit of the acquirer, $\lambda \left[R_A \left(k_A | g \right) - C \left(k_A \right) \right] + (1 - \lambda) R_A \left(0 | b \right)$, and the expected profit of the non-acquirer $\lambda R_N \left(k_A | g \right) + (1 - \lambda) R_A \left(0 | b \right)$; evaluated at the acquiring incumbent's optimal development, $k_i^* \left(g \right)$ and $k_i^* \left(b \right)$; and weighted with the prior probability to preemptively acquire a good project, λ . Note that since v_{II} This is true as we are solving for the separating equilibrium. We will discuss this equilibrium choice and, in particular, the reasons why we do not present pooling equilibria in detail in section

6.

is not a function of $k_j^*(g)$, it is independent of whether the venture-backed firm faces a constrained or an unconstrained problem.

Compare $v_V(g)$ with v_{II} . If the venture-backed firm's problem is unconstrained, the investment choice is k_V^u , which maximizes $R_A(k) - C(k) - R_{NA}(k)$. Therefore, in that case, $v_V(g)$ must exceed v_{II} . This is illustrated in Figures 2 (ii) and 4 (ii). In Figure 2 (ii), $v_V^u(g)$ is shown as the vertical distance between V and V'. In addition, the vertical distance between A and A' gives v_{II} for $\lambda = 1$. Figure 4 (ii) shows both valuations as functions of λ . It demonstrates how for any $\lambda > 0$, $v_{II} < v_V^u(g)$.

If the venture-backed firm's incentive compatibility constraint holds with equality, a venture-backed firm must invest more in order to signal the type of the innovation to the potential acquirers. This is reflected in a lower – constrained maximized – profit from selling the developed innovation and thus, in a lower valuation for the basic innovation. Consider panel (ii) of figure 3. In this figure, the effect of a binding incentive compatibility constraint on the venture capitalists' valuation is reflected by a smaller distance between S and S', as compared to that between V and V'. Now turn to figure 5 (ii). Here, we show the effect of the incentive compatibility constraint by indicating $v_V^c(g)$, which is smaller than $v_V^u(g)$.

In the situation where venture-backed firms incur a very small additional cost for developing a bad idea, a very high level of k_V^c needs to be chosen in order to signal that the innovation is good, which depresses the venture capitalists' valuation by a large amount. If this situation coincides with a sufficiently high λ , which caters to a relatively high valuation of the incumbents, we may have $v_V^c(g) < v_{II}$. Denote the λ for which $v_V^c(g) = v_{II}$ with λ^{II} . In Figure 6 (ii), which depicts this situation, there exists a λ^{II} such that $v_{II} < v_V(g)$ for $\lambda < \lambda^{II}$ and $v_{II} > v_V(g)$ for $\lambda > \lambda^{II}$. Lemma 5 summarizes this finding.

Lemma 5 For $C(k_V^u|b) - C(k_V^u|g)$ sufficiently small, $\exists \lambda^{II} \in]0,1[$, s.t. for $\lambda > \lambda^{II}$,

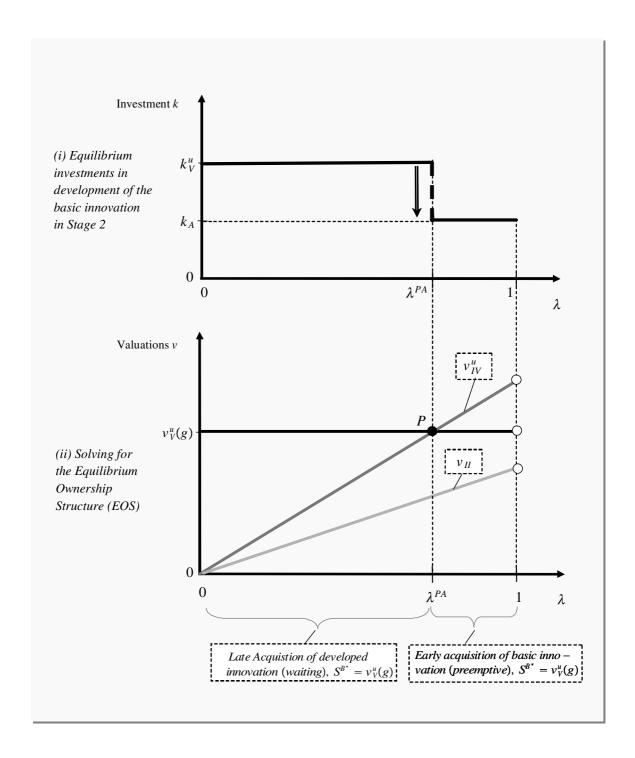


Figure 4: Equilibrium investment and ownership structure with a non-binding incentive compatibility constraint.

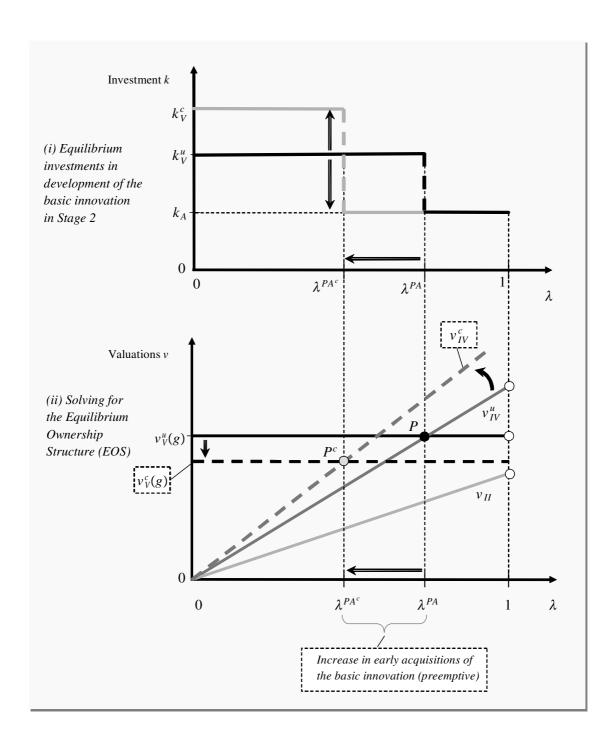


Figure 5: Equilibrium investment and ownership structure with a binding incentive compatibility constraint.

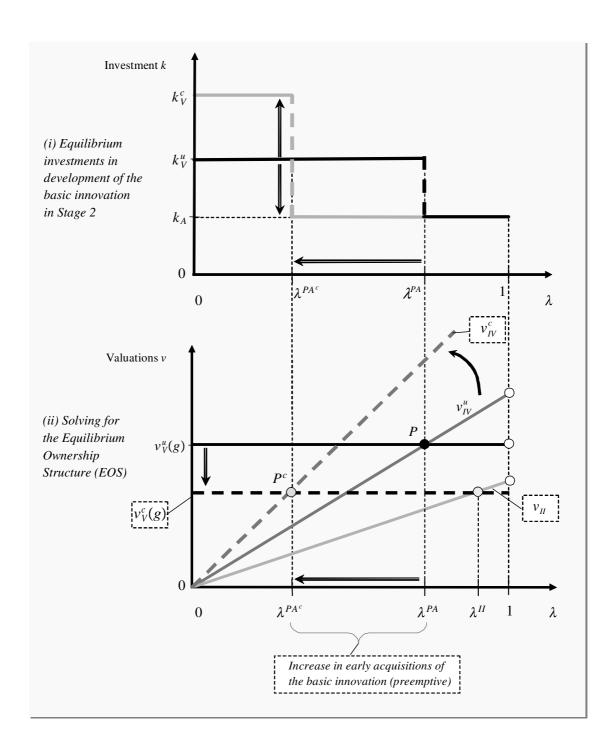


Figure 6: Equilibrium investment and ownership structure with a binding incentive compatibility constraint; for $\lambda > \lambda^{II}$, $v_{II} > v_V^c(g)$.

$$v_{II} > v_V(g)$$
 and for $\lambda < \lambda^{II}$, $v_{II} < v_V(g)$.

Note that the preemptive acquisition of the basic innovation by another incumbent firm is not the only alternative scenario an incumbent must consider when it is bidding for it. The innovation could be obtained by a venture capitalist. Therefore, we need to take into account the incumbent firms' expected value of obtaining the innovation when it would otherwise be obtained, over-developed, and sold by a venture-backed firm:

$$v_{IV} = \lambda \left[R_A(k_A | g) - C(k_A) - R_N(k_i^*(g) | g) \right]. \tag{8}$$

Let us compare this valuation with v_{II} . Recall from Lemma 2 that a venture-backed firm develops a good basic innovation more aggressively than an incumbent, i.e., $k_j^*(g) > k_A$. This implies that a non-acquirer's profit if a venture capitalist obtains a good basic innovation in stage 1, $R_N(k_j^*(g)|g)$, is lower than its profit if it is acquired by a competing incumbent, $R_N(k_A|g)$. Therefore, for all $\lambda \in]0,1[$, $v_{II} < v_{IV}$: the ex-ante expected value for an incumbent firm of acquiring the basic innovation, when it would otherwise be obtained by a rival incumbent, v_{II} , is strictly smaller than its expected value of obtaining the innovation when it would otherwise be obtained, over-developed, and sold by a venture-backed firm, v_{IV} .

Let us now compare v_{IV} with $v_V(g)$. As before, denote by v_{IV}^u the incumbent's valuation if the venture-backed firm is unconstrained in its choice of k, and by v_{IV}^c its valuation if it is constrained by the necessity to signal.

Consider first $\lambda \to 1$ and assume that the venture-backed firm's incentive compatibility constraint is not binding. The valuation for this case, v_{IV}^u , is also shown in Figure 4 (ii). As illustrated by the figure, for $\lambda \to 1$, $v_{IV}^u > v_V^u(g)$: an incumbent is willing to pay more than a venture capitalist to obtain the innovation in order to avoid the overinvestment by the venture capitalist. To see this, consider the difference between these values for $\lambda \to 1$:

 $\lim_{\lambda \to 1} (v_{IV}^u - v_V^u(g)) = R_A(k_A|g) - C(k_A) - [R_A(k_V^u) - C(k_V^u)].$ Since $k_A < k_V^u$ maximizes the acquiring incumbents' net profits $R_A(k) - C(k)$, this difference must be positive.

Still, consider $\lambda \to 1$, but now assume that the venture-backed firm's incentive compatibility constraint is binding. In this case, the venture-backed firm must invest more than k_V^u to signal the good nature of the innovation. This reinforces the result that $\lim_{\lambda \to 1} \left(v_{IV}^u - v_V^u \left(g \right) \right) > 0$ for two reasons. First, due to the extra investment necessary to signal, the incumbents' outside option in case of a binding incentive compatibility constraint is worse than if the constraint is not binding, $R_N\left(k_V^c\left(g\right)|g\right) < R_N\left(k_V^u\left(g\right)|g\right)$, so that $v_{IV}^c > v_{IV}^u$. Second, as described earlier, this extra investment to signal is costly for the venture-backed firm, thus, $v_V^c\left(g\right) < v_V^u\left(g\right)$. This is illustrated in Figure 5 (ii).

Now consider $\lambda \to 0$. In this case, irrespective of whether the venture-backed firm's incentive compatibility constraint were binding, $v_{IV} = 0 < v_V(g)$. This is also illustrated in Figure 5 (ii).

As for $\lambda \to 0$, $v_{IV} < v_V(g)$ and for $\lambda \to 1$, $v_{IV} > v_V(g)$, and as $v_{IV} - v_V(g)$ is continuous and monotonously increasing in $\lambda \in]0,1[$, there must be a $\lambda \in]0,1[$ for which $v_{IV} = v_V(g)$. Denote this λ by λ^{PA} . This notation anticipates that, for $\lambda > \lambda^{PA}$, Preemptive Acquisitions of basic innovation will occur. We summarize these intermediate results in Lemma 6.

Lemma 6 For all cost functions satisfying assumptions 1 and 2, $\exists \lambda^{PA} \in]0,1[$, s.t. for $\lambda > \lambda^{PA}$, $v_{IV} > v_V(g)$ and for $\lambda < \lambda^{PA}$, $v_{IV} < v_V(g)$.

Using Lemmas 5 and 6, we can solve the first price sealed bid auction in stage 1 in order to derive the equilibrium ownership of the basic innovation. First, note that bidding competition among the symmetric venture capitalists implies that the equilibrium price of a good innovation cannot be lower than $v_V(g)$. Also, note that even though there are several

symmetric incumbents bidding for the basic innovation, they will not bid up to v_{IV} . To see this, recall the two purposes of a preemptive acquisition by an incumbent. The first is to avoid that another incumbent preemptively acquires the innovation, the value of which for an incumbent is v_{II} . The second is to avoid that a venture capitalist acquires, overdevelops, and sells the innovation at a high sale price, the value of which to an incumbent is v_{IV} . We have shown that $v_{II} < v_{IV}$. Therefore, once one incumbent outbids the venture capitalists (by an ε), no other incumbent has an incentive to further outbid this incumbent.

Therefore, it can be shown that the unique Nash equilibrium in that auction entails that one of the venture capitalists acquires a good innovation at a price $S^{B^*} = v_V(g)$ if $v_V(g) > v_{IV} > v_{II}$; that no one acquires a bad innovation if $v_V(g) > v_{IV} > v_{II}$; that one of the incumbents acquires the basic innovation at a price $S^{B^*} = v_V(g)$ if $v_{IV} > v_V(g) > v_{II}$; and that one of the incumbents acquires the basic innovation at a price $S^{B^*} = v_{II}$ if $v_{IV} > v_{II} > v_V(g)$. The intuition for the fact that there is no one acquiring the bad innovation for $\lambda < \lambda^{PA}$ is the following: Venture capitalists only bid a positive amount for good basic innovations, and this bid is higher than the expected value of the innovation to the incumbents. Thus, in this interval, incumbents are not able to appropriate good innovations. However, if they were to bid anything positive, they would appropriate the innovation if it is bad. Therefore, they maximize their expected payoff by bidding zero. We can express this result as in proposition 3.

Proposition 3 1. For $\lambda < \lambda^{PA}$ (a) a good basic innovation will be developed by a venture-backed firm, where the venture capitalist paid a price $S^{B^*} = v_V(g)$; and (b) a bad basic innovation will not be acquired by either incumbents or venture capitalists.

2. For $\lambda^{PA} < \lambda < \lambda^{II}$ (a) a good basic innovation will be acquired and developed by

an incumbent that paid a price $S^{B^*} = v_V(g)$; and (b) a bad basic innovation will be acquired but not developed by an incumbent that paid a price $S^{B^*} = v_V(g)$;

3. For $\lambda^{II} < \lambda$ (a) a good basic innovation will be acquired and developed by an incumbent that paid a price $S^{B^*} = v_{II}$; and (b) a bad basic innovation will be acquired but not developed by an incumbent that paid a price $S^{B^*} = v_{II}$.

Proof. See Appendix C. ■

Depending on the extent of the venture capitalists' information advantage and their ability to signal which, in turn, depend on the cost difference for developing good and bad basic innovations, the basic innovation is either acquired by an incumbent that invests k_A^* and pays $S^{B^*} = v_V(g)$ or $S^{B^*} = v_{II}$ or by a venture capitalist that invests k_V^* and pays $S^{B^*} = v_V(g)$. This leads to the following corollaries.

Corollary 1 As long as λ is sufficiently high, incumbents acquire basic innovations to preempt, for them, excessive investments in development that would otherwise be undertaken by a venture-backed firm. The threshold level of λ , for which preemptive acquisitions occur, λ^{PA} , is smaller the less costly it is for a venture-backed firm to mimic the development of a good innovation when it is bad.

Our results predict when we should expect an aggressive development of good basic innovations. This is the case whenever venture capitalists get to develop the basic innovation. Consider panels (i) of figures 4, 5, and 6. If it is unlikely for a basic idea to be good and, thus, to lead to a successful commercializable innovation after its development, i.e., if $\lambda < \lambda^{PA}$, we observe aggressive development by venture-backed firms. In addition, once we compare the three figures, we can see that the less costly it is for a venture-backed firm to mimic

the development of a good idea when it is bad, the more aggressively venture-backed firms owning a good basic innovation will have to develop it in order to signal the good nature of the innovation.

Corollary 2 (1) Investment into the development of good basic innovations is aggressive for low λ as innovations will then be developed by a venture-backed firm. (2) Investment into the development of good basic innovations by venture-backed firms is the more aggressive, the cheaper it is for a venture-backed firm possessing a bad innovation to mimic one possessing a good innovation.

4.2 Incentives for basic innovations

Our results have striking consequences for the incentives to come up with basic ideas in the first place. If there were no venture capitalists, entrepreneurs could only turn to incumbents for the development of their basic innovations. In this case, the incumbents' valuation and winning bid for basic innovations is, as we have shown, v_{II} . As demonstrated in proposition 3, if entrepreneurs could also turn to venture capitalists, the winning bid may be higher, even if incumbents get to develop the innovation. These consequences are outlined in this subsection.

If $\lambda < \lambda^{PA}$, the good basic innovation will be developed by a venture-backed firm. Venture capitalists bid $v_V(g)$ in case the innovation is good and zero in case it is bad and incumbents always bid zero. Consequently, the entrepreneur has an expected payoff for the basic innovation of $\lambda v_V(g)$, which we have shown to be greater than v_{II} .

If $\lambda^{PA} < \lambda < \lambda^{II}$, the basic innovation will be preemptively acquired by an incumbent, which develops it in case it turns out to be good. The incumbent bids $v_V(g)$, which we have

also shown to be greater than v_{II} , and the expected payoff for the entrepreneur is $v_{V}(g)$.

If $\lambda^{II} < \lambda$, once more, the basic innovation will be preemptively acquired by an incumbent, which develops it in case it turns out to be good. The incumbents bid v_{II} and the entrepreneur has exactly the same expected payoff in the presence of venture capitalists as it has in their absence.

Therefore, as long as $\lambda < \lambda^{II}$, the existence of venture capitalists – whether or not they get to develop the basic innovations – increases the expected payoff of the entrepreneurs for coming up with basic innovations.

Proposition 4 The existence of venture capitalists increases the incentives for entrepreneurs to come up with basic innovations for $\lambda < \lambda^{II}$ and it does not change them otherwise.

The existence of venture capitalists may not only generate a more aggressive development of existing basic innovations, which reduces unit production costs and increases competitiveness in the market; but it may also give higher incentives to entrepreneurs to come up with new basic innovations in the first place.

5 Example: Linear-Quadratic Model

In this section, we consider a duopolistic market with linear inverse demand P = a - bX, where a > 0 denotes consumers willingness to pay and b > 0 denotes market size²², P is the price of the product, in which ex-ante symmetric incumbents with unit production costs, c, compete à la Cournot. The strategic variable in the product-market interaction (stage 4) is the quantity x_i , with $\sum_{i=1}^{N_I} x_i = X$, chosen by each firm i. We assume that the development of a good basic innovation will lead to a unit production cost reduction $\frac{1}{2}$ The is higher b, the smaller is the market.

of $k \in]0,c[$ units, whereas that of a bad one does not reduce it, regardless of the level of k. Satisfying assumptions 1 and 2, the cost of development for a venture-backed firm is assumed to be $C(k|\theta) = \frac{\mu_{\theta}k^2}{2}$, where $\mu = \mu_g$ if the basic innovation is good and $\mu = \mu_b$ if the innovation is bad, with $0 < \mu_g < \mu_b < \infty$. The cost of development for an incumbent is assumed to be $C(k) = \frac{\mu_g k^2}{2}$.

For the specific assumptions taken in this section, standard Cournot analysis results in $x_A^*(k|g) = \frac{a-c+2k}{3b}$, $x_N^*(k|g) = \frac{a-c-k}{3b}$, $R_A(k|g) = b\left(\frac{a-c+2k}{3b}\right)^2$, and $R_N(k|g) = b\left(\frac{a-c-k}{3b}\right)^2$ for a good developed innovation and in $x_A^*(k|b) = x_N^*(k|b) = \frac{a-c}{3b}$, and $R_A(k|b) = R_N(k|b) = b\left(\frac{a-c}{3b}\right)^2$ for a bad innovation. It is straightforward to verify that these reduced-form product-market profits fulfill assumption 3.

Remember that an acquiring incumbent can verify the nature of the basic innovation after having obtained it and before investing into it. Thus, its optimal investment into the development of a good basic innovation can be shown to equal $k_i^* = k_A = \frac{4(a-c)}{9b\mu_g-8}$. Similarly, a venture-backed firm's optimal investment into the development of a good basic innovation—if it is unconstrained by (IC)—equals $k_j^* = k_V^u = \frac{2(a-c)}{3b\mu_g-2}$. If it is constrained, the venture-backed firm's optimal investment equals $k_j^* = k_V^c = \frac{4(a-c)}{3b\mu_b-2}$.

The venture-backed firm's profit from auctioning off the good developed innovation in stage 3 corresponds to its willingness to pay for the basic innovation, $v_V(g)$. It equals $v_V^u(g) = \frac{2(a-c)^2}{3b(3b\mu_g-2)}$ for the unconstrained problem and $v_V^c(g) = \frac{\mu_b - \mu_g}{2} \left(\frac{4(a-c)}{3b\mu_b-2}\right)^2$ for the constrained one. The problem is unconstrained as long as k_V^u fulfills the venture-backed firm's incentive compatibility constraint, i.e., as long as $S^D(k_V^u|g) - \frac{\mu_b(k_V^u)^2}{2} < 0 \Leftrightarrow \frac{1}{b\mu_b} < \frac{3}{6b\mu_g-2}$. It can be verified that $v_V^c(g) = v_V^u(g)$ if and only if $\frac{1}{b\mu_b} = \frac{3}{6b\mu_g-2}$ and $v_V^c(g) < v_V^u(g)$ otherwise.

The value for an incumbent firm of acquiring the basic innovation, when it would otherwise be obtained by a rival incumbent equals $v_{II} = \lambda \frac{16(a-c)^2(b\mu_g-1)}{b(9b\mu_g-8)^2}$. The value for an incum-

bent firm of obtaining the innovation when it would otherwise be obtained, over-developed, and sold by a venture capitalist is for the unconstrained case $v_{IV}^u = \lambda \frac{8(a-c)^2}{9b} \left(\frac{1}{9b\mu_g - 8} + \frac{3(b\mu_g - 1)}{2(3b\mu_g - 2)^2} \right)$, and for the constrained case $v_{IV}^c = \lambda \frac{8(a-c)^2}{9b} \left(\frac{1}{9b\mu_g - 8} + \frac{3b\mu_b - 4}{(3b\mu_b - 2)^2} \right)$.

Define $\eta_g = \frac{1}{b\mu_g}$ as the relative return to development of a good project and $\eta_b = \frac{1}{b\mu_b}$ as that of a bad project. A high b is equivalent to a small market size and a high μ to a high cost of development. Therefore, the higher is $\eta_\theta = \frac{1}{b\mu_\theta}$, the higher is the relative return to development, the more worthwhile is the extra-investment into the development of the innovation. Using this transformation, we can represent the equilibrium organization for each λ in an $\eta_g - \eta_b$ graph. We show three of these graphs (for $\lambda = 0.3$, $\lambda = 0.6$, and $\lambda = 0.9$) for a good innovation in figure 7. As $\eta_g > \eta_b$ by assumption, given $\mu_g < \mu_b$, the only region that is economically sensible for our analysis is the one below the diagonal. Note that the closer is a point to the diagonal, the more similar are the costs of developing good and bad basic innovations.

Our example shows the patterns of the model: Close to the diagonal, where the development costs of good and bad innovations are similar, a venture-backed firm would have to choose a very high investment level in order to signal a good innovation. This would increase its sale price in a late acquisition, and would therefore be more likely trigger a preemptive acquisition. Furthermore, the higher is λ , the more preemptive acquisitions will occur. For the linear-quadratic model, this replicates the general result from Corollary 1.

6 Discussion and empirical implications

What are the empirical implications of our model? Will our main findings hold also when we relax some of the assumptions made in the above analysis? In the below subsections, we

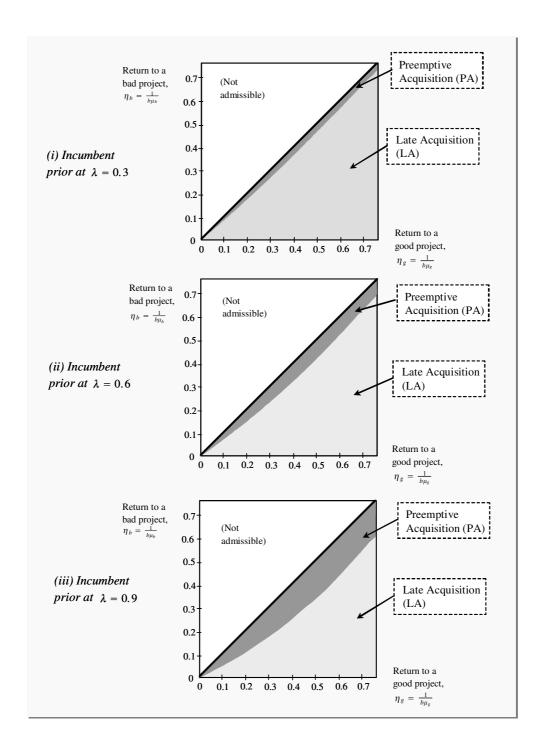


Figure 7: Equilibrium Ownership in the Linear-Quadratic Model; PA: Pre-emptive acquisition and development by an incumbent; LA: Late Acquisition and venture-backed development. η_g and η_b are the relative return to development of good and bad basic innovations, respectively, with $\eta_g=1/\mu_g b$ and $\eta_b=1/\mu_b b_{36}$

first highlight some empirical implications from our model which relate to the literature on finance and the firms' choice to innovate. Then, we explore the effects of allowing for the venture-backed firm exiting by an initial public offering in addition to a sale to incumbents, e.g., through a late M&A, the possibility for the venture-backed firm to not only use a productive signal but also a non-productive signal, and some equilibrium selection issues.

6.1 Empirical implications

The first purpose of this paper was to provide a signaling-based explanation for the observation that venture-backed firms are more aggressive in their development of basic innovations than incumbents – but our results also have other empirical implications.

First, in our model, M&As are observed either in early stages as preemptive acquisitions, or in late development stages after a good idea with potentially high relative returns to development has been over-developed. Early, preemptive, acquisitions do not take advantage of the venture capitalists' ability to select good basic innovations and therefore, bear the risk of turning out unprofitable. Late M&As, on the other hand, take advantage of this ability and are, thus, not subject to such a risk. Therefore, a first empirical implication is that it is important to distinguish between early and late M&As of innovative firms and carefully determine the counter factual in respective stage, when assessing their average profitability in empirical studies.

Second, the higher the asymmetry of information between incumbents and venture capital firms, the more likely is development of a good innovation within a venture-backed firm which will develop it aggressively. If we assume that the source of the asymmetric information is a lack of intellectual property rights protection for basic innovations and the consequent fear of expropriation by incumbents that have both the funds and the know-how for developing

basic ideas without the entrepreneur. Then, the λ in our model might correspond to the maximum amount of information an incumbent would have about a given innovation in its early stage. Consequently, there would be more venture capital funding in industries where it is easier to steal ideas and entrepreneurs are more reluctant to reveal information regarding their basic innovations to incumbents.

Third, if the costs of developing good and bad innovations differ sufficiently, the venture-backed firms' investment choice is unconstrained by the necessity to signal. However, once venture-backed firms are constrained in their investment choice, however, then – all else equal – the more similar are these costs, (1) the more aggressively will good basic innovations be developed, (2) the higher will be the price of good developed innovations, and (3) the lower will be the price of basic innovations.

If we interpret the cost difference in developing good and bad innovations as reflecting the venture capitalists' cost of losing or not obtaining a good reputation. The higher is the value of the reputation at stake, the more different are the costs of developing good and bad innovations. With this interpretation, our model would link the value of the venture capitalists' reputation to the extent of development and the prices for basic and developed innovations.²³ It would be interesting to further study the role of the venture capitalists' reputation within this context. This is left to future research.

²³ In particular, if there is a high value of the reputation at stake, the venture-backed firms' investment choice is likely to be unconstrained. In this case, a slight increase in reputation will not change the prices for good basic and developed innovations and the extent of their development. If, however, there is only a low value of the reputation at stake, the venture-backed firms' investment choice is likely to be constrained. In this case, a slight increase in reputation will increase the prices for good basic innovations, decrease the prices for good developed innovations, and decrease the extent of their development.

6.2 Initial public offerings (IPOs).

Basically all existing literature on venture capital studies exit by IPOs, whereas we study exit by sale to incumbents. What would happen if we allowed both types of exits, i.e. IPOs and sale to incumbents? In principle, the venture-backed firm must then, prior to its choice of development (signaling), consider whether it is more profitable to exit by IPO or by a sale to an incumbent. If it is more profitable to exit by a sale to an incumbent, our model set-up is valid. So, when is exit by a sale to an incumbent more profitable? Gans et al. (2002) and Gans and Stern (2003) show that firms are more likely to act as suppliers of technology when intellectual property rights are secure, investment costs are high and brokers facilitating trade are available. When the opposite applies, start-ups are more likely to commercialize their innovations through entry. However, also taking into account adverse selection problem and product-market effects, we expect several other variables to be important for this choice, such as what type of signaling devices are open to the different exit modes, and the type of innovation: product or process innovation, drastic and non-drastic innovation. A study of this issue is left to future research.

6.3 Productive versus non-productive signals

We have assumed that it is difficult or costly for the venture-backed firm to use classical financial signals, which are typically used in initial public offerings (IPOs), such as capital structure and underpricing of stocks when selling directly to an incumbent. In practice, there is evidence that firms in high tech industries indeed use technology proxies such as the number of R&D personnel to signal the value of their firms to investors.²⁴ There are also studies showing a linkage between R&D spending and investors' expectations about

²⁴ Megginson, Wang, and Chua (2001).

firms' future value. See, for instance, Chan, Martin and Kessinger (1990) who find that high-technology firms experience higher abnormal returns than low technology firms when announcements of increased R&D spending are made, and Doukas and Switzer (1992) who find that firms in high concentration industries experience positive abnormal returns when announcements in R&D are made.

What would happen if we allowed both types of signals, i.e., productive and non-productive ones? In principle, the venture-backed firm must then, prior to its choice of development, consider whether it is more profitable to use productive or non-productive signals. If it is more profitable to use non-productive signals, it will use them and set the overinvestment level equal to the case when no information problem exists. So, when are non-productive signals more profitable? What we can say is that if the cost of signaling or verifying that an innovation is good is equally costly for the productive and non-productive signal, the venture-backed firm will choose the productive signal. The reason is that the productive signal will not only increase the reward (sale price or entry profit) from the signaling effect but also from the direct product-market profit effect whereas the non-productive signals only increase the reward from the signaling effect. More generally, the choice between a productive and non-productive signal will depend on what strategies are available and the underlying cost and demand parameters, and need to be determined within a specific model. A study of this issue is left to future research.

6.4 Equilibrium selection

It is commonly known that there exist multiple separating equilibria in signaling games. We have chosen a commonly used equilibrium selection criterion and solved our model for the *efficient* separating equilibrium: the one where either the unconstrained profit-maximizing

investment, k_V^u , or the minimum necessary investment to signal, k_V^c , is chosen.

However, there also exist pooling equilibria where venture capitalists choose equal levels of investment into the development of good and bad basic innovations. In this case, incumbents would not learn the type of the innovation and could in stage 3 only bid in expectations. As in the separating equilibrium, also in the pooling equilibrium would venture-backed firms choose the amount of development so as to maximize the expected difference between the profit of the acquiring and that of a non-acquiring incumbent. This increases the sale price an acquirer would have to pay and decreases the profit of non-acquirers as they would face an aggressive rival in the market place. In the separating equilibrium, there was an off-setting effect from the transmission of the venture capitalist's superior information to the potential acquirer; with the consequence that incumbents did not prefer to preemptively acquire the basic innovation if the information advantage of the venture capitalist was sufficiently large. This effect does not exist in a pooling equilibrium and, therefore, incumbents would always acquire preemptively if venture-backed firms were to choose equal investments into the development of good and bad basic innovations. For this reason, we chose not to characterize this equilibrium and instead concentrated on the more interesting case of the separating equilibrium.

7 Concluding remarks

In this study we have provided a possible explanation for why venture-backed firms are observed to be more aggressive in the development of innovations than incumbent firms. If venture capitalists are specialized in selecting promising research ideas, they have an incentive to use "overinvestment" to signal a good quality of the innovation when exiting by a sale to

an incumbent. But, we have also shown that incumbents can undertake early, preemptive, acquisitions to prevent such signaling driven overinvestment, despite the risk of buying a bad idea. Consequently, to exist in equilibrium, venture capitalists must be sufficiently more efficient in selecting projects, otherwise preemptive acquisitions by incumbents will take place.

More generally, the paper shows that the emergence of venture capitalists specialized in scrutinizing business plans will not only help the market select projects; it may also create a more aggressive development of innovations in the market, due to the signaling effect, and higher rewards for entrepreneurs to find new basic innovations.

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Appendix

A Proof of Lemma 1

Proof. Denote the equilibrium investment level into a bad project in a separating equilibrium by $k_j^*(b)$ and that into a good basic innovation by $k_j^*(g)$. In any perfect Bayesian

equilibrium, beliefs about the equilibrium path must be correctly derived from the equilibrium strategies using Bayes' rule. This implies that observing $k_j^*(g)$, firms must assign a probability one to the project being good and observing $k_j^*(b)$, firms must assign a probability one to the project being bad.

Assume first that incumbents observe $k_j^*(b)$. As $\frac{dR_h(k|b)}{dk} = 0$, the resulting valuation after observing $k_j^*(b)$ is $w(k|b) = R_A(k|b) - R_N(k|b) = 0$. The resulting maximum (and winning) bid is then $S^D = 0$.

Assume now that incumbents observe $k_j^*(g)$. Denote by b_i incumbent i's bid and by $b \in \mathbb{R}^{N_I}$ the vector of these bids. First, consider the equilibrium candidate where incumbent $\iota \in \mathcal{I}$ acquires the innovation, denoted \mathbf{b}^* . Note that $b_i^* > w - \varepsilon$ is a weakly dominated strategy, since no owner will post a bid over its maximum valuation of obtaining the innovation. If $b_i^* < w - \varepsilon$, firm ι' benefits from deviating to $b_{\iota'}^{**} = b_i^* + \varepsilon$, since it then obtains the innovation and pays a price lower than its valuation of obtaining it. Last, consider candidate $b_i^* = w - \varepsilon$, $b_{\iota'}^* = w - 2\varepsilon$. Then, no owner has an incentive to deviate. Thus, this is a Nash equilibrium and the only NE where firm ι obtains the assets. Second, note that the situation where no incumbent obtains the innovation cannot occur if there is no reservation price at the auction.

B Proof of Lemma 3

Proof. Suppose that when the project is bad, the venture-backed firm chooses some strictly positive equilibrium investment level $k'_j > 0$. According to Lemma 1, it receives a payment equal to zero, which it could receive if it chose $k_j = 0$. Since choosing $k_j = 0$ saved it the cost of development, the venture-backed firm would be strictly better off by doing so, which

C Proof of Proposition 3

Proof. First note that bidding competition among the symmetric venture capitalists implies that the equilibrium price cannot be lower than v_V . Moreover, note that no venture capitalist has an incentive to bid higher.

Assume that $\lambda < \lambda^{PA}$. According to Lemma 6, in this case, $v_{IV} < v_V$, so that no incumbent would outbid the venture capitalist. In fact, given that in this case a venture capitalist wins if the basic innovation is good and all venture capitalists bid zero if it is bad, incumbents always bid zero as long as $\lambda < \lambda^{PA}$. This shows part 1.

Assume that $\lambda^{PA} < \lambda < \lambda^{II}$. According to Lemma 5 and 6, $v_{II} < v_V < v_{IV}$. Let us now consider the equilibrium candidate where one incumbent bids v_V and the second highest bid is by a venture capitalist that bids $v_V - \varepsilon$. Note that the acquiring incumbent will not deviate to a lower bid since it benefits in expectation from an acquisition at $S^B = v_V$ by avoiding the excessive investments by venture capitalists, which would otherwise occur in case the basic innovation turns out to be good. This follows from the acquiring incumbent's net profit being $\pi_A^* = \lambda \left(R_A \left(k_A^* \right) - C \left(k_A^* \right) \right) - v_V = v_{IV} - v_V + \lambda R_N \left(k_V^* \right) > \lambda R_N \left(k_V^* \right)$ by Lemma 6. Clearly, deviating to a higher bid is not profitable for the winning incumbent. Moreover, other incumbents will not challenge an acquisition by a rival firm since they benefit from weaker market competition, while not bearing the cost of the acquisition. This follows from the fact that $\lambda R_N \left(k_A^* \right) = \lambda \left(R_A \left(k_A^* \right) - C \left(k_A^* \right) \right) - v_{II} > \lambda \left(R_A \left(k_A^* \right) - C \left(k_A^* \right) \right) - v_V = \pi_A^*$ holds by Lemma 6. This shows part 2.

Assume $\lambda^{II} < \lambda$. According to Lemma 5 and 6, $v_V < v_{II} < v_{IV}$. In this case, bidding

competition among the symmetric incumbents implies that the equilibrium price cannot be lower than $S^B = v_{II}$. Moreover, note that no incumbent has an incentive to bid higher. This shows part 3. \blacksquare