Local advertising externalities and cooperation in one manufacturer-two retailers channel with exogenous marginal profits

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8 March 2015

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March 2015

Abstract

Game theory is a relevant and powerful tool for analyzing strategic interactions in a supply chain in which the decision of each player affect the payoff of other players. In order to relax the classical two supply chain members' situation to a three supply chain members' situation and to integrate the problem of competition at retail level, we consider a supply chain consisting of a monopolistic manufacturer and two duopolistic retailers. The latter two are geographically related.

Our paper examines the optimal decisions on advertising (local, national and cooperative advertising) in a centralized and a decentralized supply chain using Stackelberg – Cournot game, Stackelberg - Collusion game and Cooperative games, and we investigate the impact of the existing of competition at retail level, the retailer coalition and the cooperation between all supply chain members' on the channel members' optimal decisions, on the sales volume and on the profits.

Applying the equilibrium analysis and using numerical example, comparing results indicates that all advertising, the sales volume of each member and the total profit in the centralized decision-making are larger than those in the decentralized decision-making. Retailer coalition harms themselves (in terms of profit) despite the increasing of sales, but is beneficial to the manufacturer. We identify also the feasible solutions of the best cooperative advertising scheme that members are interesting in cooperation.

Keywords: Game theory; Cooperative advertising; Supply chain coordination; Retail competition, retail coalition.

JEL Classification: C7, M3.

1. Introduction

Over the past two decades, many researchers and practitioners have shown great interest in the application of game theory in marketing and supply chain management. Especially, game theoretic models have proven of considerable interest and useful in the supply chain coordination. The latter has become an effectively mechanism improving the performance of organizations in various industries. The uncoordinated system leads to the classic phenomenon of "double marginalization" identified by Spengler (1950) in which the retailer does not consider the supplier's profit margin when making arbitrary his decisions and to the concept of “bullwhip effect” which occurs when the amplification of demand variability increases as one moves up supply chain and makes decision ignoring the others.

We can distinguish between decentralized and centralized supply chain system. In supply chain with centralized decision making, both manufacturer and retailers are trying to integrate the channel and to determine the optimal solution that improves the total performance of supply chain. In a supply chain with decentralized decision making, each supply chain member acts independently in order to improve its profit share (individual performance).

For effective modeling the supply chain system and analyzing the decision making, game theory provides mathematical background and generates solutions in competitive or conflicting situations (conflicting and antagonistic). Traditionally, game theory, also called strategic situations, is divided into two main approaches according to the interdependence between the actors (members of supply chain) who make interactive procedures to benefit themselves or the entire supply chain. These approaches may be cooperative or non-cooperative. In the non cooperative approach, the actors are individual players. This game approach specifies various actions available to the players and tries to predict the strategies chosen by the other player. By contrast, in the cooperative approach, the actors are able to communicate and cooperate (the coalition players). This game approach describes the set of possible outcomes, studies the payoffs that the players can attain after the coalition, predicts the coalitions that will form and divides the outcomes.

Many important studies in the recent years have been addressed different aspects of supply chain coordination and business decisions, including pricing, advertising, production, purchasing and inventory management, etc. Most of these studies have focused on a bilateral monopoly model by considering a single manufacturer sells its product through an exclusive and independent retailer and they have identified the coordination efforts by implementing a cooperative (co-op) advertising program which is, practically, an arrangement and interactive relationship between the members of distribution whereby the manufacturer pays a portion of the retailer’s local advertising expenditures; the fraction shared by the manufacturer is generally referred to as the manufacturer’s participate rate. The main aim for a manufacturer to use the cooperative advertising is to strengthen the image of the national brand name investment and to motivate immediate sales incentives by local advertisement at the retail level. However, the existing literature based on a bilateral duopoly by considering a competition in retailing level is very scanty.

The research literature related to our paper on supply chain coordination management using the static game theoretic model for cooperative advertising can be divided into two groups according to the numbers of the supply chain members. It is shown that some previous studies have therefore focused on the Stackelberg structure, some others on Nash structure and some others on both structures. The Stackelberg game can be summarized by a manufacturer acts first as a Stackelberg leader and a retailer move sequentially as Stackelberg follower. In the Nash game, the players move simultaneous.

The first group concentrates on supply chain consisting of one manufacturer and one retailer. In this context, the model proposed by Xie and Wei (2009) assumed that demand function is influenced by both pricing and advertising expenditures. The authors founded closed-form unique optimal solutions in both the Stackelberg model and the cooperative model in a single period. Comparing the results of these two models, Xie and Wei explained the importance of adopting two coordination instruments (wholesale price and coop participation rate) and demonstrated that the highest total profit is gained when both players cooperate. This cooperation has led to a decreasing of retail price to consumer and to an increasing of advertising expenditures.

The first paper, which considered not only a leadership of the manufacturer, but also a dominant retailer, was written by Xie and Neyret (2009). They used a static model to propose a model including the cooperative advertising and pricing decisions simultaneously. They developed four different models which are based on three non cooperative games (i.e., Nash, Stackelberg manufacturer and Stackelberg retailer) and one cooperative game. A comparison between the variables, especially the profits, of all cases leads to consider the cooperation case in which profits are the highest for both retailer and the manufacturer.

SeyedEsfahani et al (2011) consider vertical co-op advertising along with pricing decisions in manufacturer-retailer supply chain. This work applied these four games on the model proposed by Xie and Wei (2009), but relaxed the assumption of a linear price demand function. The authors illustrated that both the manufacturer and the retailer reach the highest profit level and the retail price is the lowest when the members' supply chain decide to cooperate, but the advertising expenditures are higher in non-cooperative games.

Aust and Busher (2012) extended the existing research by considering four classical types of relationships between a manufacturer and a retailer using the game theory models .They modified the price and the advertising demand functions of SeyedEsfahani et al (2011) by introduction the retailer margin according to Choi (1991, 1996) in order to relax the restrictive assumption of identical margins. The authors have not limited the ratio between manufacturer's and retailer's margin and they obtained the same results in Xie and Wei (2009).

The second group includes the existing research of game theoretical models for coordinating cooperative advertising (with the presence of competition at retailing level in supply chain) in channel exhibiting competition at retail level.

Karray and Zaccour (2006) considered a model of retail's promotional efforts for distribution channel formed by two manufacturers and two retailers and investigated the effectiveness of cooperative advertising programs. The author demonstrated that cooperative advertising can also have harmful impacts on the retailers.

Wang et al (2011) considered the cooperative advertising issues for a supply chain consisting of one monopolistic manufacturer and two duopolistic retailers. Through a comparison among the four game structures (Stackelberg - Cournot, Stackelberg – Collusion, Nash - Cournot, Nash – Collusion), the authors revealed the impact of duopolistic retailers' competitive behaviors on the local advertising expenditures, brand name investment, local advertising allowance level and the profits of all participants. In addition, the authors developed the centralized decision model and presented a local advertising cost sharing contract based on the utility of risk preference. They determined whether the partners have any motivations to transit to a different behavior cited above and concluded that the joint decision can improve the performance of the entire supply chain.

Another study contributes to extend the studies of cooperative advertising issues of a single manufacturer-single retailer and to provide new insights is the study of Zhang and Xie (2012). Choosing to use a static game, this study developed a multiple-retailer model and examined the impacts of this retailer's multiplicity on channel members' optimal decisions and on total channel efficiencies by measuring explicitly the gain/losses of channel efficiencies under different game considered. The quantitative findings differed according to the type of retailer. With a multiple symmetric retailers in the distribution channel, the channel efficiency increases with the local advertising effectiveness and the manufacturer's (national advertising investment) relative channel power, however, it deteriorates quickly as the number of retailer scales up. With a multiple asymmetric retailers, the distribution channel suffers from the manufacture's uniform participation strategy and benefits with the manufacturer's retailer-specific participation strategy.

Ben Youssef and Dridi (2013) suggest a marketing channel with one manufacturer and two symmetric retailers where the demand function is dependent on pricing and national advertising. Three game theoretic models are established including the non cooperative game, the partial cooperative game and the full cooperation game. The authors propose a new and unusual evaluation of consumers' surplus which positively depends not only on the price-demand function but also on the spending in national advertising.

The paper of Alirezai and KhoshAlhan (2014) considered pricing and cooperative advertising decisions in two-stage supply chain. They developed a monopolistic retailer and duopolistic retailer's model by using Nash, Manufacturer – Stackelberg and cooperative game. The authors modified the demand function of Xie and Wei (2009) by introducing the retailer margin as a new decision variable according to Choi (1991) (like the paper of Aust and Busher (2012) with $v = 1$). In the case of the duopolistic retailers' model, Alirezai and KhoshAlhan suggest that one party's advertising effort will reduce the other's share of the marketing demand (see Luo (2006)). The authors found that the supply chain members' can gain more profits in the cooperative than in non cooperative structure in both models.

Jorgensen S. and Zaccour G. (2013) and Aust G. and Busher U. (2014) surveyed the literature on cooperative advertising in marketing channels using game theoretic models. The paper of Jorgensen S. and Zaccour G. (2013) divided the literature into main approaches: the first treats simple marketing channels (one manufacturer – one retailer channel). The second adopts complex marketing channel (multiple manufacturer – multiple retailer channels). Each approach presented two models: static model and dynamic model.

Nevertheless, researches which explicitly evaluate the competition effect between upstream or downstream members (a supply chain formed by a multiple manufacturers or multiple retailers) are very complex, scarce and sparse. For this reason, we intend to extend, firstly, the existing research illustrated above that has not integrated the problem of downstream competition because the assumption of more one retailer is more pragmatic. In other words, we aim to relax the classical two supply chain members' situation to a three supply chain members' situation. Secondly, we want to enrich, according to our knowledge, the few research studies that considered the channel with duopolistic competitive retailers and we aspire to get new/better insights into the effects of the existing of competition at retail level and the cooperation on the channel members' optimal decisions and on the profits.

The remainder of our paper is organized as follows. Section 2 presents the necessary notations and assumptions of our study and develops the decentralized and centralized decision models. Section 3 compares the three game structures of decision model via simple and general cases. Section 4 summarizes our main findings and proposes possible directions for future research

2. The basic model and assumptions

In this paper, we consider a distribution channel consisting of one manufacturer and two retailers in which the monopolistic manufacturer produces its product and sells it to final consumer who is the last point in the distribution channel through a retailer in competition with another independent retailer.

We assume that the manufacturer invests in the product's national brand name advertising in order to influence potential consumers to consider a particular brand and develops brand knowledge and preference (creating effective brand awareness). The retailers invest in the local advertising effort to boost the consumer demand and to enhance the sales.

Regardless of the local or national levels, we know full well that the advertising efforts influence separately the generating sales and depend strongly on the consumer perception. Also, they generally have positive effects on the ultimate product sales.

Our demand function, extended to the model of Kim and Staelin (1999), Karray and Zaccour (2006) and Xie and Wei (2009), is assumed to be a function based on square roots of the two retailers' local advertising expenditures and the manufacturer's national advertising expenditure. So, we do not take pricing decisions into account in order to simplify our expressions. We assume that the different retailers are geographically related: the local advertising effort benefits both the investing retailer and the other competing retailer located in close physical proximity.

We postulate the expected sales volume function S_i of retailer i to be determined by:

$$S_i = \alpha_i + k\sqrt{a_i} + m\sqrt{a_j} + l\sqrt{A}\,, \quad i = 1, 2, j = 3 - i \tag{1}$$

where the parameters $k, m\ and\ l$ are positive constants; they may be interpreted as follows:

α_i : denotes the demand base with respect to zero advertising input.

k : denotes the measure of sensitivity of retailer1's sales with respect to changes of retailer2's local advertising expenditures.

a_i : the local advertising expenditures invested by retailer i.

m : denotes the measure of sensitivity of retailer 2's sales with respect to changes of retailer 1's local advertising expenditures.

A : the national brand name investment of the manufacturer.

l : denotes the influence degree of manufacturer's brand name investment on each retailer's demand.

We assume that $S_m = \sum_{i=1} S_i$, the demand function of the manufacturer.

We note that the demand function is an increasing and concave function with respect to a_i and A as agreed in marketing literature.

Further, the manufacturer looks for motivated immediate sales at the retail level. For this reason, he adopts the cooperative advertising programs by sharing a proportion of the local advertising expenditures in order to increase the retailers' advertising budgets without spending more of retailers' own funds. In other words, for every unit spent by the retailers for product advertising, we suppose that the manufacturer will compensate them a percentage amount t, where $0 \leq t \leq 1$. This means that if each retailer spends a unit in advertising, he will get t unit back from the manufacturer as a motivation for his advertising efforts.

The profit functions for each retailer i, the manufacturer, the two retailers and the supply chain system are as follows, respectively:

$$\Pi_{ri} = \rho_i \, S_i - (1-t)a_i \tag{2}$$

$$\Pi_m = \rho_m \left(S_i + S_j\right) - t\left(a_i + a_j\right) - A \tag{3}$$

$$\Pi_R = \Pi_{ri} + \Pi_{rj} \tag{4}$$

$$\Pi_t = \Pi_m + \Pi_{ri} + \Pi_{rj} \tag{5}$$

where S_i is the sales volume function of retailer i,. And we have the following notations:

ρ_i : the retailer i's marginal profit per unit.

ρ_m : the manufacturer's marginal profit per unit.

t : the participation rate that is the fraction of local advertising expenditures shared by the manufacturer with each retailer.

Π_{ri}: the profit of retailer i.

Π_m : the profit of the manufacturer.

Π_R : the profit of the two retailers.

Π_t : the profit for the whole supply chain.

Before concluding this section, we specify the necessary assumption used in our paper.

- ✓ **Assumption 1:** the selling prices of the retailers are exogenous.

- ✓ **Assumption 2**: we assume $\alpha_1 = \alpha_2 = 0$ to imply that the demand base level is the same.

- ✓ **Assumption 3**: the manufacturer sets the same proportion of the advertising allowance to two competing retailers because we prefer to treat the duopolistic retailers equally due to the existence of competition at this level.

- ✓ **Assumption 4:** we assume that $k + m > 1$ and $k > m$ to ensure the "existence of the equilibrium solution ". The latter means that the demand of each retailer faces is more sensitive (is greater) to his own local advertising efforts than to his rival's because the local advertising efforts by one retailer can reach consumers who prefer to buy from a competition retailer. Otherwise, no one would be interested in spending money on the local advertising (see Wang et al., (2011)).

To simplify notation, we use $(*), (\sim)$ and $(-)$ to denote, respectively, scenario1 : St-Cournot, scenario2 : St-collusion and scenario3 : Cooperation.

2.1 The decentralized decision models

In the setting of decentralized decision-making structure, the manufacturer and the two competitive retailers act individually to maximize their own profit without considering the profits of others. In the following, we will analyze the interactive relationship between the manufacturer and the duopolistic retailers as game theoretic model in order to examine how the supply chain members make independently their (advertising policies) decisions when the manufacturer moves first as the Stackelberg leader (manufacturer - Stackelberg process), and then the two retailers react as followers by adopting three different competitive behaviors cited below:

(i) In this setting, each retailer moves independently at the same time in the downstream supply chain market by pursuing Cournot behavior and assuming his rival's as a parameter. This game is known as the simultaneous move game.

(ii) In this setting, the retailers work collaboratively against the manufacturer, that's why they obey Collusion behavior in the downstream market of supply chain. This game is known as the cooperative game.

(iii) In this setting, the retailers pursue the Stackelberg behavior. One retailer acts as a leader, while the other acts as a follower. This game is known as the Stackelberg game.

In our study, we only focus on the two first settings because the latest setting has the same results with those obtained in first setting.

2.1.1 Stackelberg - Cournot game

In this subsection, we model a channel members' decision process as a two-stage non cooperative game, with the manufacturer as the leader and the duopolistic retailers as the followers (the manufacturer has the greater power than the retailers). In the first stage, the manufacturer decides its national advertising expenditure A and its co-op participation rate$t$. In the second stage, the two competitive retailers decide, simultaneously and independently, their local advertising expenditures $a_i, i = 1,2$.

This game is solved by backward induction method and its solution is called Stackelberg equilibrium. To find the equilibrium of a Stackelberg game, we begin at the end of this game (the second stage) and we first need to solve the retailers' optimal problems when the brand name investment A and the shared fraction of local advertising expenditure t declared by the manufacturer are given.

Following Cournot behavior, retailer 1 will maximize Π_{r1} with respect to $a_1$while treating a_2, A and t as a parameters, and retailer 2 will maximize Π_{r2} with respect to a_2 while treating a_1 A and t as a parameters[1]:

$$max\ \Pi_{r1} = \rho_i\ (k\ \sqrt{a_1} + m\ \sqrt{a_2} + l\ \sqrt{A}) - (1-t)a_1$$

$$\text{st}\ \ a_1 \geq \frac{(m\sqrt{a_2}+l\sqrt{A})^2}{k^2}. \tag{6}$$

$$max\ \Pi_{r1} = \rho_i\ (k\ \sqrt{a_1} + m\ \sqrt{a_2} + l\ \sqrt{A}) - (1-t)a_1$$

$$\text{st}\ \ a_2 \geq \frac{(m\sqrt{a_1}+l\sqrt{A})^2}{k^2}. \tag{7}$$

Noting that Π_{r1} and Π_{r2} are, respectively, concave functions of a_1 and a_2 and have a unique optimal solution, we can solve the two first order equations $\frac{\partial \Pi_{r1}}{\partial a_1} = 0$ and $\frac{\partial \Pi_{r2}}{\partial a_2} = 0$ to get the optimal values cited below.

Given the decisions variables of manufacturer and under the duopolistic retailers' Cournot behavior, the optimal local advertising for each retailer is:

$$a_1 = \frac{1}{4}\frac{\rho_1^2\ k^2}{(1-t)^2} \tag{8}$$

$$a_2 = \frac{1}{4}\frac{\rho_2^2\ k^2}{(1-t)^2} \tag{9}$$

[1] Second-order conditions are verified because : $\frac{\partial^2 \Pi_{r1}}{\partial a_1^2} = -\frac{1}{4}\frac{\rho_1 k}{a_1^{\frac{3}{2}}} < 0\ and\ \frac{\partial^2 \Pi_{r2}}{\partial a_2^2} = -\frac{1}{4}\frac{\rho_2 k}{a_2^{\frac{3}{2}}} < 0.$

In the first stage, the manufacturer knows the retailers' reaction functions and maximizes his profit function[2]. Hence, the manufacturer's optimal problem can be formulated by substituting (8) and (9) into (3) as:

$$\max \Pi_m = \rho_m\left(\frac{1}{2}\frac{(k+m)k\,(\rho_1+\rho_2)}{(1-t)} + 2\,l\sqrt{A}\right) - \frac{1}{4}\frac{t\,k^2\,(\rho_1^2+\rho_1^2)}{(1-t)^2} - A$$

$$st\ \ 0 \le t \le 1, A \ge 0 \tag{10}$$

In order to solve this problem, we neglect the constraints and we equate the two first order partial derivatives to zero as below:

$$\frac{\partial \Pi_m}{\partial A} = \frac{\rho_m\,l}{\sqrt{A}} - 1 = 0 \tag{11}$$

$$\frac{\partial \Pi_m}{\partial t} = \frac{1}{2}\frac{\rho_m\,(k+m)k\,(\rho_1+\rho_2)}{(1-t)^2} - \frac{1}{4}\frac{k^2(\rho_1^2+\rho_2^2)}{(1-t)^2} - \frac{1}{2}\frac{t\,k^2\,(\rho_1^2+\rho_2^2)}{(1-t)^3} = 0 \tag{12}$$

Proposition 1: the Stackelberg - Cournot game, where the manufacturer behave as leader and the two retailers are Cournot competitors and act as followers, has a unique equilibrium (a_1^*, a_2^*, A^*, t^*)

$$a_1^* = \frac{1}{16}\frac{\rho_1^2\,(2(\rho_1+\rho_2)(k+m)\rho_m + \ k(\rho_1^2+\rho_2^2))^2}{(\rho_1^2+\rho_2^2)^2} \tag{13}$$

$$a_2^* = \frac{1}{16}\frac{\rho_2^2\,(2(\rho_1+\rho_2)(k+m)\rho_m + \ k(\rho_1^2+\rho_2^2))^2}{(\rho_1^2+\rho_2^2)^2} \tag{14}$$

$$A^* = \rho_m^2\,l^2 \tag{15}$$

$$t^* = \frac{2(\rho_1+\rho_2)(k+m)\rho_m - \ k(\rho_1^2+\rho_2^2)}{2(\rho_1+\rho_2)(k+m)\rho_m + \ k(\rho_1^2+\rho_2^2)} \tag{16}$$

See Appendix for the expressions of the sales volume and the profit of each member and the total channel's profit for all cases.

2.1.2 Stackelberg - Collusion game

[2] Second-order conditions are verified because:

$\frac{\partial^2 \Pi_m}{\partial A^2} = -\frac{1}{2}\frac{\rho_{m}l}{(\rho_m^2 l^2)^{\frac{3}{2}}} < 0$, $\frac{\partial^2 \Pi_m}{\partial t^2} = -\frac{1}{32}\frac{(2(\rho_1+\rho_2)(k+m)\rho_m+k(\rho_1^2+\rho_2^2))^4}{k^2(\rho_1^2+\rho_2^2)^3} < 0$, $\frac{\partial^2 \Pi_m}{\partial A \partial t} = 0$. One can simply check the Hessian as: $\begin{vmatrix} \frac{\partial^2 \Pi_m}{\partial A^2} & 0 \\ 0 & \frac{\partial^2 \Pi_m}{\partial t^2} \end{vmatrix} = \frac{1}{64}\frac{(2(\rho_1+\rho_2)(k+m)\rho_m+k(\rho_1^2+\rho_2^2))^4}{(\rho_1^2+\rho_2^2)^3 k^2 \rho_m^2 l^2} > 0.$

In this section, we assume that the manufacturer in the upstream market and the two retailers, in the downstream market, recognize their interdependence and agree to work together in order to maximize their total profit[3]. So, the total profit of the downstream retail market is:

$$\Pi_R = (\rho_1 k + \rho_2 m)\sqrt{a_1} + (\rho_1 m + \rho_2 k)\sqrt{a_2} + (\rho_1 + \rho_2) l\sqrt{A} - (1-t)(a_1 + a_2) \tag{17}$$

This optimization problem can easily be solved by differentiating the total profit of retail market with respect to a_1 and a_2 and equating to zero. The total profit of the duopolistic retailers Π_R achieves its maximum, for any given national advertising and rate participation, at:

$$a_1 = \frac{1}{4}\frac{(\rho_1 k + \rho_2 m)^2}{(1-t)^2} \tag{18}$$

$$a_2 = \frac{1}{4}\frac{(\rho_2 k + \rho_1 m)^2}{(1-t)^2} \tag{19}$$

As a leader, the manufacturer knows the optimal values of retailers' local advertising expenditures defined in (18) and (19), and she substitute it into the manufacturer's profit function defined in (3). The manufacturer's optimization problem can be formulated as[4]:

$$\max \Pi_m = \rho_m \left(\frac{1}{2}\frac{(k+m)^2(\rho_1+\rho_2)}{(1-t)} + 2l\sqrt{A}\right) - \frac{1}{4}\frac{((\rho_1 k + \rho_2 m)^2 + (\rho_2 k + \rho_1 m)^2)t}{(1-t)^2}$$
$$-A$$

$$s.t.\ 0 \le t \le 1, A \ge 0 \tag{20}$$

Then, by ignoring the constraints and solving the first partial derivatives of Π_m with respect to A and t, we can obtain the unique equilibrium solution as follows:

Proposition 2: the unique equilibrium solution for the Stackelberg - Collusion game is given by:

$$\tilde{a}_1 = \frac{1}{16}\frac{1}{((\rho_1^2+\rho_2^2)(k^2+m^2)+4\rho_1 k\rho_2 m))^2}((\rho_1 k + \rho_2 m)^2((\rho_1^2 + 2\rho_m\rho_1$$
$$+\rho_2(2\rho_m+\rho_2)(k^2+m^2) + 4m((\rho_m+\rho_2)\rho_1 + \rho_m\rho_2)k)^2 \tag{21}$$

$$\tilde{a}_2 = \frac{1}{16}\frac{1}{((\rho_1^2+\rho_2^2)(k^2+m^2)+4\rho_1 k\rho_2 m))^2}((\rho_2 k + \rho_1 m)^2((\rho_1^2 + 2\rho_m\rho_1$$

[3] Second-order conditions are verified because : $\frac{\partial^2\Pi_R}{\partial a_1^2} = -\frac{1}{4}\frac{\rho_1 k+\rho_2 m}{a_1^{\frac{3}{2}}} < 0\ and\ \frac{\partial^2\Pi_R}{\partial a_2^2} = -\frac{1}{4}\frac{\rho_2 k+\rho_1 m}{a_2^{\frac{3}{2}}} < 0.$

[4] Second-order conditions are verified because : $\frac{\partial^2\Pi_m}{\partial A^2} < 0, \frac{\partial^2\Pi_m}{\partial t^2} < 0.$

$$+\rho_2(2\rho_m+\rho_2)(k^2+m^2)+4m((\rho_m+\rho_2)\rho_1+\rho_m\rho_2)k)^2 \tag{22}$$

$$\tilde{A}=\rho_m^2\, l^2 \tag{23}$$

$$\tilde{t}=\frac{\left(-\rho_1^2+2\rho_m\rho_1-\rho_2\left(-2\rho_m-\rho_2\right)\right)k^2-4m\left(\left(\rho_2-\rho_m\right)\rho_1-\rho_m\rho_2\right)k-}{\left(\rho_1^2+2\rho_m\rho_1+\rho_2\left(2\rho_m+\rho_2\right)\right)k^2+4m\left(\left(\rho_2+\rho_m\right)\rho_1+\rho_m\rho_2\right)k+}$$

$$\frac{\left(\rho_1^2-2\rho_m\rho_1+\rho_2\left(-2\rho_m+\rho_2\right)\right)m^2}{\left(\rho_1^2+2\rho_m\rho_1+\rho_2\left(-2\rho_m+\rho_2\right)\right)m^2} \tag{24}$$

2.2 The centralized decision model

In this section, we analyze the decisions of the centralized supply chain. We focus on a cooperative game structure in which the manufacturer and the duopolistic retailers cooperate as integrated channel and agree to make decisions together that maximize the total supply chain profit function which is the sum of the manufacturer's profit and the retailers' profit.

Hence, the optimization problem under centralized model can be written as[5] :

$$\max \Pi_t=\rho_m(k+m)\left(\sqrt{a_1}+\sqrt{a_2}\right)+(\rho_1 k+\rho_2 m)\sqrt{a_1}+(\rho_1 m+\rho_2 k)\sqrt{a_2}$$

$$+l\sqrt{A}(2\rho_m+\rho_1+\rho_2)-(a_1+a_2)-A$$

$$s.t\ \ a_1\geq 0, a_2\geq 0\ and\ A\geq 0 \tag{25}$$

The joint profit maximization, described by the above equation, depends only on a_1, a_2, A: when the manufacturer and the two retailers agree to cooperate, the manufacturer neglect the arrangement whereby he pays a percentage of local advertising expenditures for both retailers.

In order to solve this optimization and to find final results, we differentiate the total supply chain profit function with respect to each decision variables and we equate to zero.

Proposition 3: under centralized channel (25), the channel members' equilibrium advertising levels that maximize the total channel profit are:

[5] Second-order are verified because : $\frac{\partial^2\Pi_t}{\partial a_1^2}<0,\ \frac{\partial^2\Pi_t}{\partial a_2^2}<0, \frac{\partial^2\Pi_t}{\partial A^2}<0, \frac{\partial^2\Pi_t}{\partial a_1\partial a_2}=0,\ \frac{\partial^2\Pi_t}{\partial a_1\partial A}=0, \frac{\partial^2\Pi_t}{\partial a_2\partial A}=0$. One can simply check the Hessians as : $\begin{vmatrix}\frac{\partial^2\Pi_t}{\partial a_1^2} & 0\\ 0 & \frac{\partial^2\Pi_t}{\partial a_2^2}\end{vmatrix}>0, \begin{vmatrix}\frac{\partial^2\Pi_t}{\partial a_1^2} & 0 & 0\\ 0 & \frac{\partial^2\Pi_t}{\partial a_2^2} & 0\\ 0 & 0 & \frac{\partial^2\Pi_t}{\partial A^2}\end{vmatrix}<\ 0.$

$$\bar{a}_1 = \frac{1}{4}(\rho_m(k+m) + \rho_1 k + \rho_2 m)^2 \tag{26}$$

$$\bar{a}_2 = \frac{1}{4}(\rho_m(k+m) + \rho_1 m + \rho_2 k)^2 \tag{27}$$

$$\bar{A} = \frac{1}{4}\, l^2(2\rho_m + \rho_1 + \rho_2)^2 \tag{28}$$

3. Comparison of optimal solutions for the three games

3. 1 A special case with $\rho_1 = \rho_2 = \rho$

After having presented the equilibrium solutions of the centralized and decentralized games above, we can now compare them. It is convenient to first consider an important special case in which $\rho_1 = \rho_2 = \rho$. This case can be intuitively explained as the duopolistic retailers having the similar marginal profit. We can utilize the same analyses as in the previous two sections in order to identify the optimal solutions for the three game scenarios Stackelberg-Cournot, Stackelberg collusion and cooperation in this special case. The results obtained are presented in the following table:

Table 1

Summary of the optimal solutions if $\rho_1 = \rho_2 = \rho$.

	Scenario 1: St-Cournot	**Scenario 2: St-collusion**	**Scenario 3 : Cooperation**
$\boldsymbol{a_i}$	$\frac{1}{16}((2k+2m)\rho_m + k\rho)^2$	$\frac{1}{16}(k+m)^2(2\rho_m + \rho)^2$	$\frac{1}{4}(k+m)^2(\rho_m + \rho)^2$
$\boldsymbol{A}$	$l^2\,\rho_m^2$	$l^2\,\rho_m^2$	$l^2(\rho_m + \rho)^2$
$\boldsymbol{t}$	$\frac{2(k+m)\rho_m - k\rho}{2(k+m)\rho_m + k\rho}$	$\frac{2\rho_m - \rho}{2\rho_m + \rho}$	–
$\boldsymbol{S_{ri}}$	$\frac{1}{4}(4km + 2k^2 + 4l^2 + 2m^2)\rho_m + \frac{1}{4}k\rho(k+m)$	$\frac{1}{4}(2m^2 + 4km + 4l^2 + 2k^2)\rho_m + \frac{1}{4}\rho(k+m)^2$	$\frac{1}{2}(k^2 + 2mk + m^2 + 2l^2)(\rho_m + \rho)$

S_m	$\frac{1}{2}(4km + 2k^2 + 4l^2 + 2m^2)\rho_m + \frac{1}{2}k\rho(k + m)$	$\frac{1}{2}(2m^2 + 4km + 4l^2 + 2k^2)\rho_m + \frac{1}{2}\rho(k + m)^2$	$(k^2 + 2mk + m^2 + 2l^2)(\rho_m + \rho)$
Π_{ri}	$\frac{1}{8}\rho((2k^2 + 6km + 4m^2 + 8l^2)\rho_m + k\rho(k + 2m))$	$\frac{1}{8}\rho((2\rho_m + \rho)(k + m)^2) + 8l^2\rho_{m})$	$\frac{1}{2}\rho(\rho_m + \rho)((k + m)^2 + 2l^2) + \frac{1}{4}(t - 1)(\rho_m + \rho)^2(k + m)^2$
Π_m	$\frac{1}{8}(4k^2 + 8km + 8l^2 + 4m^2)\rho_m^2 + \frac{1}{2}k(k + m)\rho\rho_m + \frac{1}{8}k^2\rho^2$	$\frac{1}{8}(2\rho_m + \rho)^2(k + m)^2 + 2\rho_m^2 l^2$	$\rho_m(((k + m)^2 + 2l^2)(\rho_m + \rho)) - \frac{1}{2}t(\rho_m + \rho)^2(k + m)^2 - l^2(\rho_m + \rho)^2$
Π_t	$\frac{1}{8}(4k^2 + 8km + 8l^2 + 4m^2)\rho_m^2 + (k^2 + 2km + 2l^2 + m^2)\rho\rho_m + \frac{3}{8}k(\frac{4}{3}m + k)\rho^2$	$\frac{1}{8}(4k^2 + 8km + 8l^2 + 4m^2)\rho_m^2 + (k^2 + 2km + 2l^2 + m^2)\rho\rho_m + \frac{3}{8}\rho^2(k + m)^2$	$\frac{1}{2}(\rho_m + \rho)^2(k^2 + 2mk + m^2 + 2l^2)$

The expressions cited in table above depend on the three parameters used on our analysis, i.e. k, m and l. These parameters can be interpreted as effectiveness of local and national advertising expenditures. Under the two first cases, one can easily observe that the manufacturer gives a fraction of local advertising expenditures to the two retailers. Also, we remark that if $\rho_1 = \rho_2 = \rho$, we have $a_1 = a_2 = a$ and $\Pi_{r1} = \Pi_{r2}$. That is to say, if the duopolistic retailers have the same marginal profit, they have the same local advertising expenditures and the same incomes.

According to the theoretical analysis above, we find that the effectiveness of compete retailer's local advertising (m) has an important effect on the optimal decisions of the members[6], the profit of each member and the profit of whole supply chain. Here, we will give some numerical examples and then we will compare between the three cases using schemas. We assume that the values of parameters are: $\rho_m = 80, \rho = 55, k = 1, l = 1$[7].

[6] We note that the national advertising A depends only on l.

[7] The rest of the examples show similar results to those of the chosen, so we omit presenting their results (same for the general case).

Fig 1: Local advertising of retailer *i*

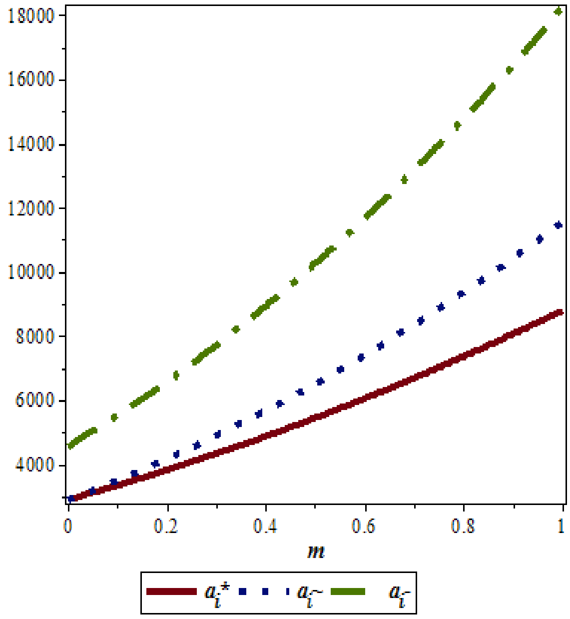


Fig 2: Manufacture's participation rate

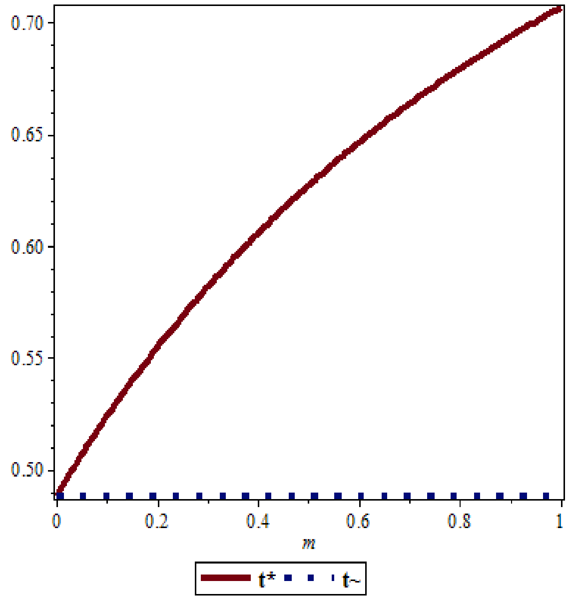


Fig 3: Demand function of retailer *i*

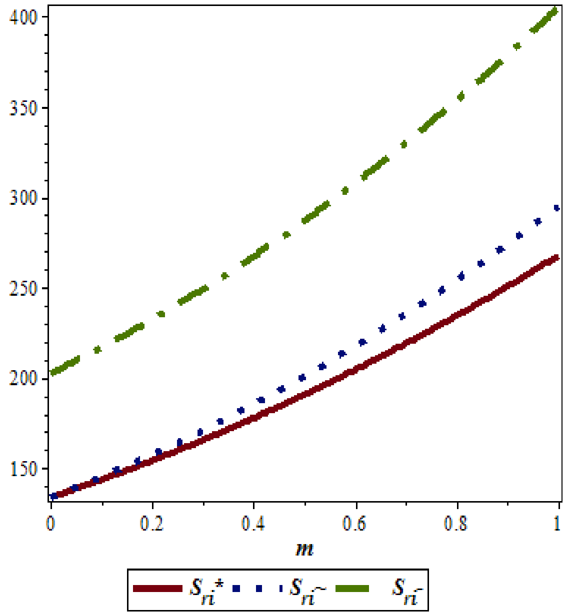


Fig 4: Demand function of manufacturer

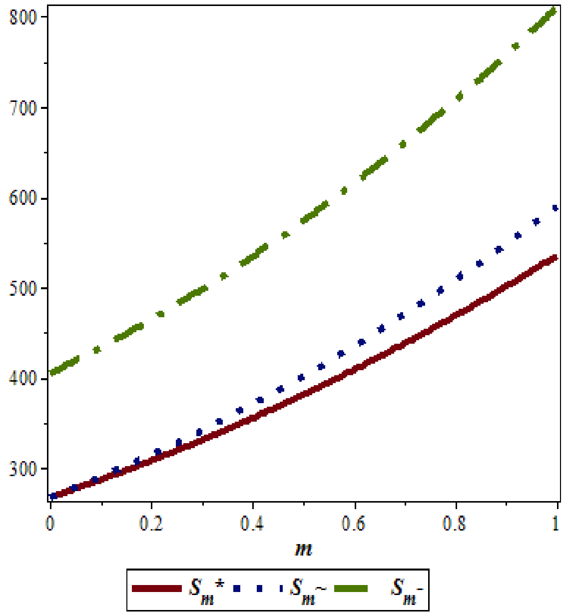


Fig 5: The profit of retailer *i*

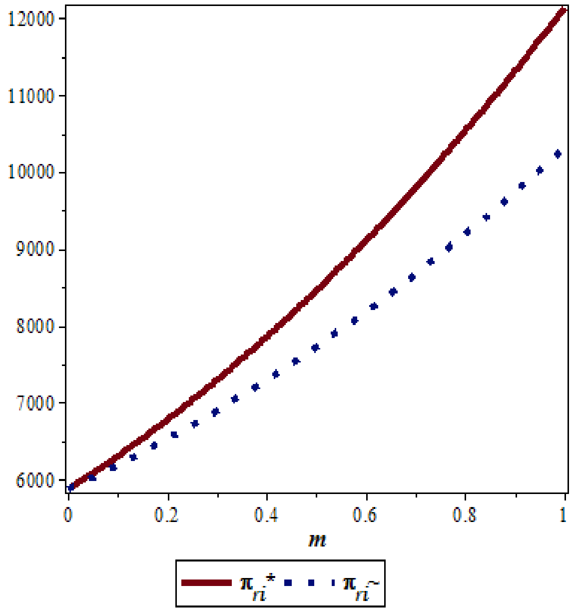


Fig 6: The profit of manufcturer

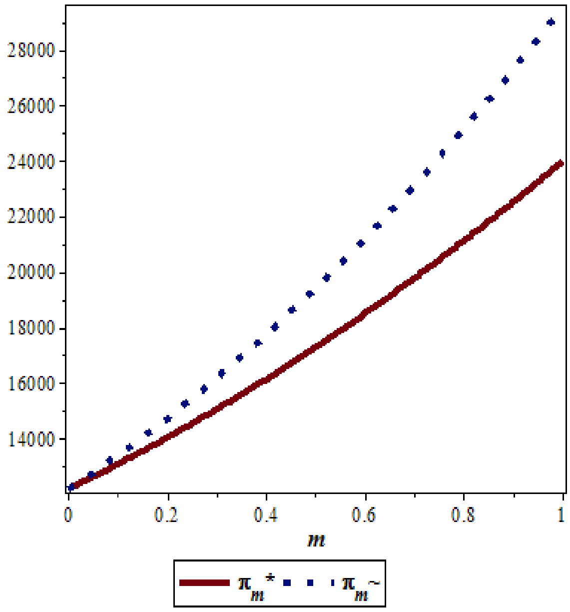
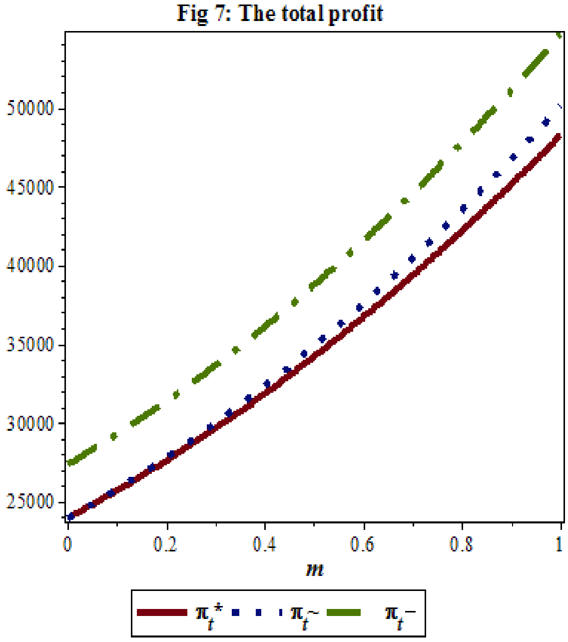


Fig 7: The total profit



From the schemes presented above, we can derive the following proposition:

Proposition 4:

(i) $a^* < \tilde{a} < \bar{a}$

(ii) $A^* = \tilde{A} < \bar{A}$

(iii) $t^* > \tilde{t}$

(iv) $S_{ri}^* < \tilde{S}_{ri} < \bar{S}_{ri}$

(v) $S_m^* < \tilde{S}_m < \bar{S}_m$

Under Stackelberg-Collusion scenario, the two retailers would allocate more money on local advertising rather than under Stackelberg-Cournot scenario. However, the manufacturer keeps the same amount of investment in national advertising and the fraction of advertising expenditures that reimburse by him drops off. This result is unexpected and surprising because the only paper that addresses the problem of the cooperative advertising issues for two echelons supply chain consisting of one manufacturer and two competitive retailers demonstrated that, on one hand, the manufacturer would spend more money on national advertising investment.

On other hand, the equilibrium fraction of local advertising expenditures that shared by manufacturer, when the tow retailers collude, would exceed the counterpart when the two retailers behave Cournot. This may be explained by the fact that if the manufacturer is the leader and the duopolistic retailers pursue Collusion behavior, the manufacturer refrains from increasing the value of his participation in local advertising cost. Further, the manufacturer prefers the lowest participation rate when the effectiveness of local advertising efforts is higher than the effectiveness of national advertising efforts.

Under Cooperation between all the members of supply chain, both the monopolistic manufacturer and the duopolistic retailers will invest more on advertising: the highest national and local advertising expenditures occur in the cooperation. These high investments in advertising yield to increase sales. Consequently, the advertising is a type of an effective investment and has a positive association with the profit.

Proposition 5:

(i) $\Pi_{ri}^* > \widetilde{\Pi}_{ri}$

(ii) $\Pi_m^* < \widetilde{\Pi}_m$

(iii) $\Pi_t^* < \widetilde{\Pi}_t < \overline{\Pi}_t$

Normally, the profit of each retailer improves if they are followers and play Stackelberg-Collusion. Or in our paper, we discover that the cooperation between retailers leads to a decrease in their profit. That is to say, the retailers choosing Stackelberg-collusion will not benefit themselves but they will benefit the manufacturer because the latter obtains the highest profit under Stackelberg-collusion, yet the lowest under Stakelberg-Cournot, which is out of our expectation. This implies that the manufacturer prefers the cooperation situation between retailers and not the conflict situation.

The cooperative model generates the highest channel's total profit than the Stakelberg-Cournot model. So, we confirm the popular result founded in the previous literature that is the highest profit will be attained in the cooperation scenario and the aim of making decision cooperatively is to maximize the total system's profits.

Proposition 6:

(i) $\overline{\Pi}_{ri} > \Pi^{*}_{ri}$

(ii) $\overline{\Pi}_{m} > \widetilde{\Pi}_{m}$

This proposition identifies the conditions under which the profit of each member is the highest in the cooperation situation than in the others situations. We will explain numerically this proposition in the proposition (9) and (10).

3. 2 The general case with $\rho_1 \neq \rho_2$

The purpose of the general case is to find an appropriate answer to the question of whether the results obtained in the special case with $\rho_1 = \rho_2 = \rho$ still remain in general case with $\rho_1 \neq \rho_2$. We can intuitively interpret this case as the two competitive retailers having different marginal profit. We will thus present numerical examples to investigate it because our analytical expressions are complex to provide meaningful insights. We illustrate the theoretical results and explore the differences between the centralized supply chain and the decentralized supply chain. For our numerical example, we assume the following values of the parameters considered: $\rho_m = 80, \rho_1 = 50, \rho_2 = 40, k = 1, l = 1$.

Fig 8: Local advertising of retailer 1

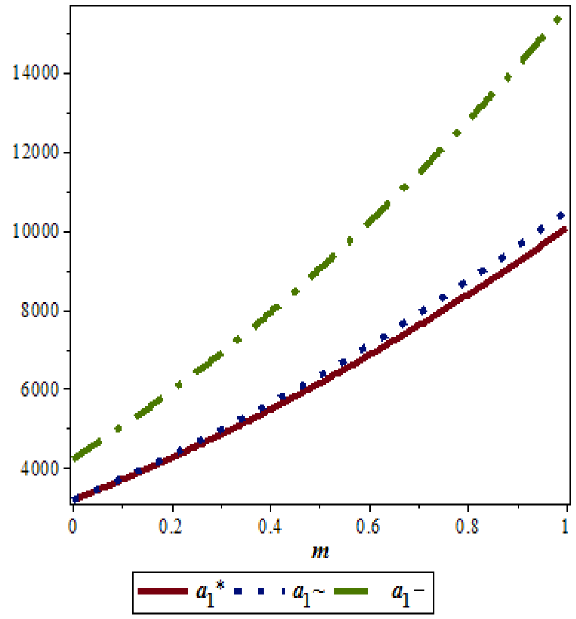


Fig 9: Local advertising of retailer 2

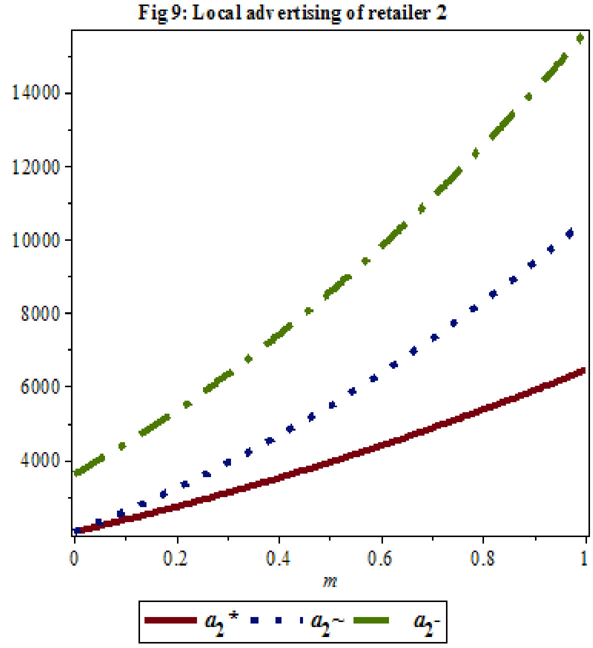


Fig 10: Manufacturer's participation rate

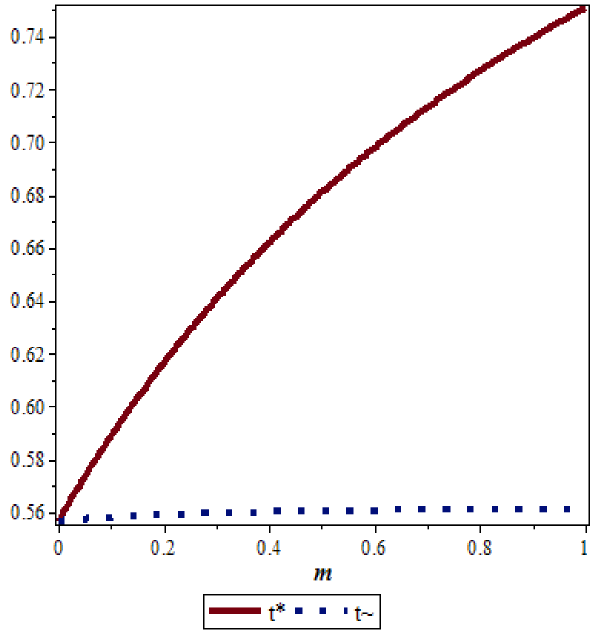


Fig 11: Demand function of retailer 1

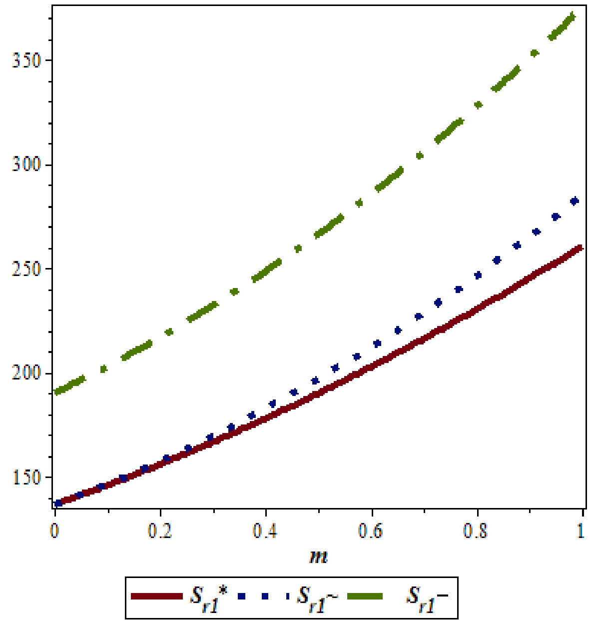


Fig 12: Demand function of retailer 2

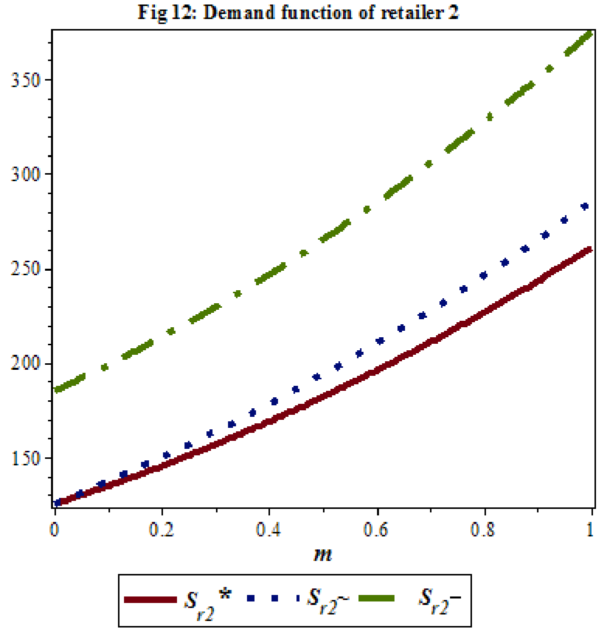
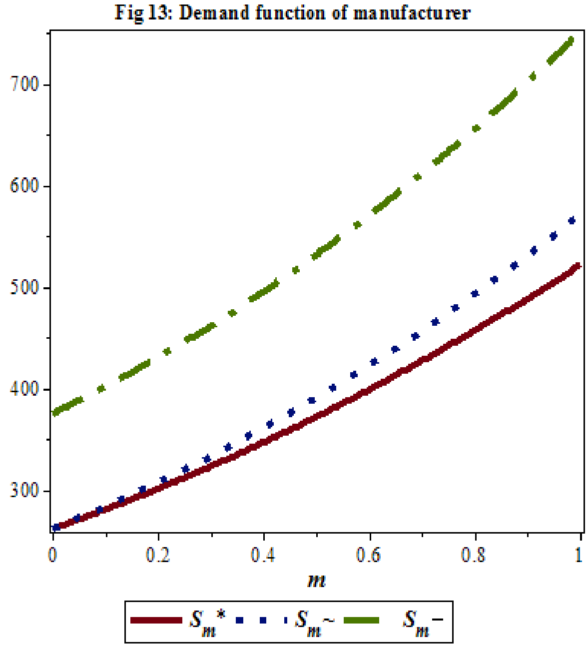


Fig 13: Demand function of manufacturer



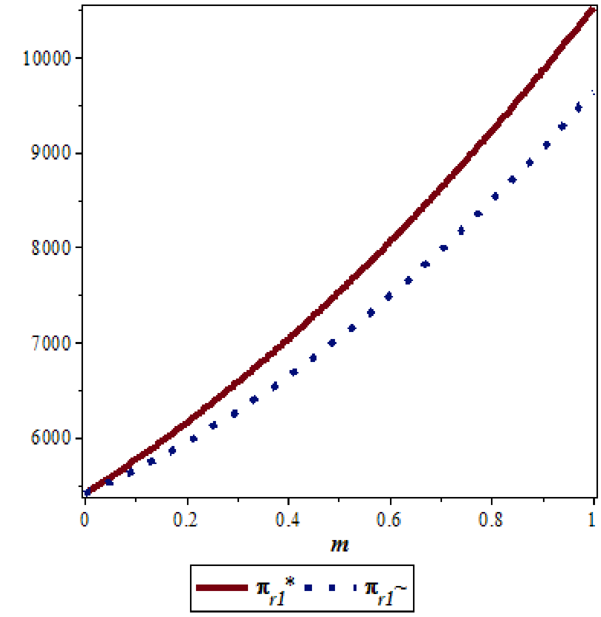


Fig 14: The profit of retailer 1

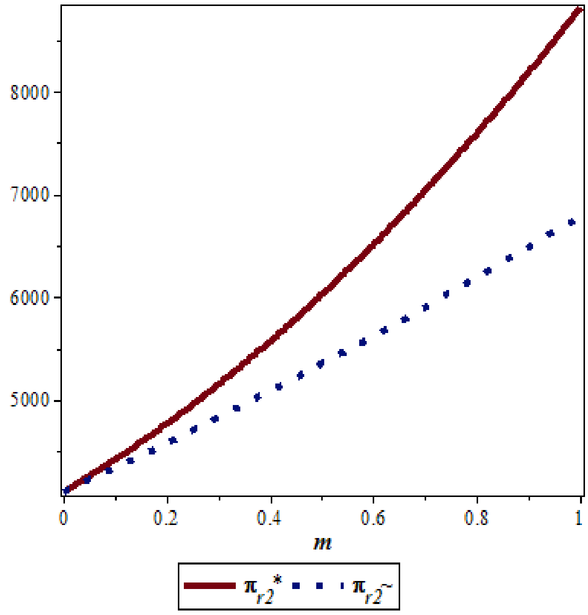


Fig 15: The profit of retailer 2

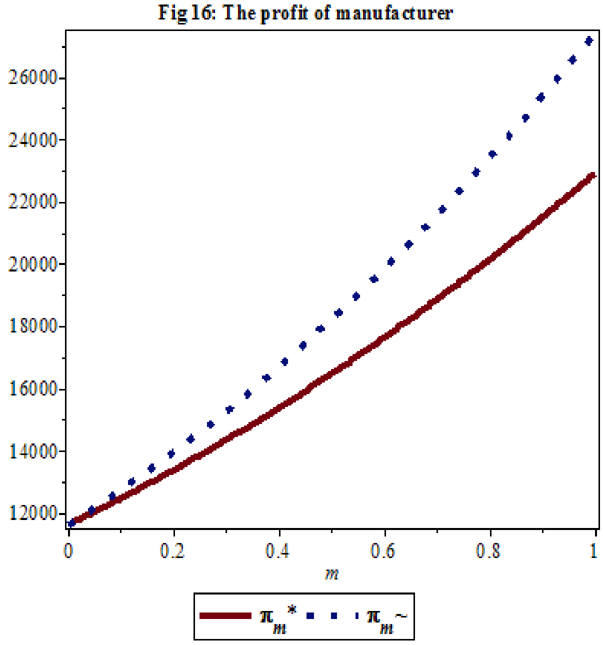


Fig 16: The profit of manufacturer

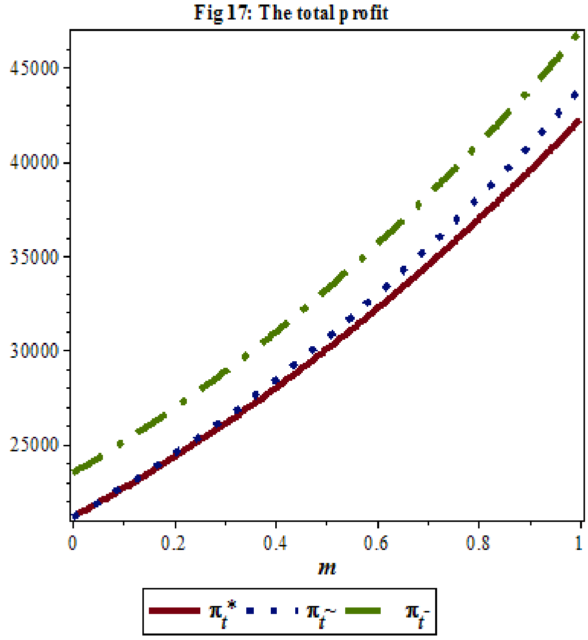


Fig 17: The total profit

As shown in the figures above, the results - concerning the retailers' local advertising, the national advertising, the participate rate, the sales volume function of each member - are similar with those obtained previously in the simple case.

Now we assume that the members of supply chain are willing to engage in the cooperative program. The manufacturer and the two retailers set the national and the local advertising expenditures that maximize the total profit of cooperation. Then, for any arbitrarily given participation rate t, the profits of the manufacturer and the two retailers would be respectively given by:

$$\overline{\Pi}_m = \rho_m \left(\bar{S}_i + \bar{S}_j\right) - t\left(\bar{a}_i + \bar{a}_j\right) - \bar{A} \tag{29}$$

$$\overline{\Pi}_{ri} = \rho_i \bar{S}_i - (1-t)\bar{a}_i \tag{30}$$

The monopolistic manufacturer and the duopolistic retailers agree to make joint decisions if their individual profit is higher in the cooperative game than in non-cooperative ones. So, we need to illustrate that:

$$\overline{\Pi}_m > \max(\Pi_m^*, \widetilde{\Pi}_m) = \widetilde{\Pi}_m \tag{31}$$

and

$$\overline{\Pi}_{ri} > \max(\Pi_{ri}^*, \widetilde{\Pi}_{ri}) = \Pi_{ri}^* \tag{32}$$

By integrating equations (29) and (30), we have:

$$\overline{\Pi}_T = \overline{\Pi}_m + \overline{\Pi}_{ri} > \widetilde{\Pi}_m + \Pi_{ri}^* \tag{33}$$

Proposition 7:

(i) $\overline{\Pi}_{ri} > \Pi_{ri}^* \Leftrightarrow \rho_i \bar{S}_i - (1-t)\bar{a}_i > \Pi_{ri}^*$

$$\Leftrightarrow t > t_{min} = \frac{1}{\bar{a}_i}(\Pi_{ri}^* - \rho_i \bar{S}_i - \bar{a}_i)$$

(i) $\overline{\Pi}_m > \widetilde{\Pi}_m \Leftrightarrow \rho_m \left(\bar{S}_i + \bar{S}_j\right) - t\left(\bar{a}_i + \bar{a}_j\right) - \bar{A} > \widetilde{\Pi}_m$

$$\Leftrightarrow t < t_{max} = \frac{1}{(\bar{a}_i + \bar{a}_j)} (\rho_m \left(\bar{S}_i + \bar{S}_j\right) - \bar{A} - \widetilde{\Pi}_m$$

As shown in proposition 10, the lowest amount of profit for the manufacturer and for the whole supply chain is reached under the Stackelberg - Cournot game. The manufacturer prefers that the two duopolistic retailers play a Stackelberg – Collusion game rather than the Stackelberg – Cournot game. This result is different from this found by Ben Youssef and Dridi (2013) because they have shown that the manufacturer’s profit decrease under the Stackelberg – Collusion.

The manufacturer agrees to cooperate if he offers a fraction of the retailers’ advertising expenditures lower than $\frac{1}{(\bar{a}_i + \bar{a}_j)} (\rho_m \left(\bar{S}_i + \bar{S}_j\right) - \bar{A} - \widetilde{\Pi}_m$. The two retailers will engage in a cooperative program if the manufacturer offers them a participation rate higher than $\frac{1}{\bar{a}_i}(\Pi_{ri}^* - \rho_i \bar{S}_i - \bar{a}_i)$.

From propositions (9) and, all supply chain members are interested in cooperation if the participation rate is between a minimal value and a maximal value ($0 < t_{min} = \frac{1}{\bar{a}_i}(\Pi^*_{ri} - \rho_i \bar{S}_i - \bar{a}_i) < t < t_{max} = \frac{1}{(\bar{a}_i + \bar{a}_j)} (\rho_m (\bar{S}_i + \bar{S}_j) - \bar{A} - \widetilde{\Pi}_m) < 1$).

4. Concluding Remarks

In this present paper, our main motivation is to extend the scarcity of existing researches considered a supply chain formed by one manufacturer - one retailer to supply chain formed by one manufacturer - two competitive retailers with demand function based on square roots of the retailers' local and manufacture's national advertising expenditures. Also, we consider a cooperative advertising program, where the manufacturer agrees to pay a percentage of the retailers' advertising expenditures.

To examine the impacts of the existing competition at retail level and the cooperation on channel members' decisions (local advertising expenditures, national advertising expenditures, cooperative advertising program), we analyze three different relationships within the monopolistic manufacturer – duopolistic retailers supply chain using games theory (i.e., Stackelberg-Cournot game, Stackelberg-collusion game and cooperative game). And then, we identify the optimal advertising decisions.

Through a comparison of the results obtained under three games and resorting to numerical example, this paper shows that: (a) the two competitive retailers agree to engage in the cooperative program if the manufacturer gives them a sufficient participation rate ($t > t_{min} = \frac{1}{\bar{a}_i}(\Pi^*_{ri} - \rho_i \bar{S}_i - \bar{a}_i)$). Otherwise, they prefer working independently. The manufacture, in turn, agrees to cooperate if he offers a participation rate of local advertising expenditures less than $t_{max} = \frac{1}{(\bar{a}_i + \bar{a}_j)} (\rho_m (\bar{S}_i + \bar{S}_j) - \bar{A} - \widetilde{\Pi}_m$. (b) The highest profit of the manufacturer is gained when the two competitive retailers work together and play Stackelberg-Collusion game. However, under this coalition between the retailers, the manufacturer reduces his participation rate in retailers' local advertising expenditures and keeps the same amount of national advertising spending. (c) The total profit of the supply chain is the highest under the cooperation game, while the lowest is under the Stackelberg - Cournot game.

This paper has several directions deserving future research. First, we suppose here that the manufacturer plays Stackelberg leader, but there are practical applications of retailers as Stackelberg leaders. We believe that is an interesting topic for future research if the retailers act as the Stackelberg leaders of the channel. Second, adopting a different form of the sales volume function may yield some more exciting results and could change the conclusions of our paper.

Appendix

The expressions of the sales volume and the profit of each member and the total channel's profit for case 1:

■ $S_{r1} = \frac{1}{4(\rho_1^2+\rho_2^2)}(\rho_1^3 k^2 + \left(\rho_2 mk + 2\rho_m(km + k^2 + 2l^2)\right)\rho_1^2 + (k^2\rho_2 + 2\rho_m(k+m)^2)$

$\rho_2\rho_1 + \rho_2^2(\rho_2 mk + 2\rho_m(m^2 + km + 2l^2)))$

■$S_{r2} = \frac{1}{4(\rho_1^2+\rho_2^2)}(\rho_1^3 mk + \left(k^2\rho_2 + 2\rho_m(m^2 + km + 2l^2)\right)\rho_1^2 + 2(\frac{1}{2}\rho_2 mk + \rho_m$

$(k+m)^2)\ \rho_2\rho_1 + (k^2\rho_2 + 2\rho_m(km + k^2 + 2l^2))\rho_2^2)$

■$S_m = \frac{1}{4(\rho_1^2+\rho_2^2)}(k(k+m)\rho_1^3 + \left(k(k+m)\rho_2 + 2\rho_m(m^2 + 2km + 4l^2 + k^2)\right)\rho_1^2 + ($

$\rho_2 k4\rho_m(k+m)(k+m)\rho_2\rho_1 + (k(k+m)\rho_2 + 2\rho_m(m^2 + 2km + 4l^2 + k^2))\rho_2^2)$

■$\pi_{r1} = \frac{1}{8(\rho_1^2+\rho_2^2)}((k^2\rho_1^3 + \left(2m\rho_2 k + 2\rho_m(km + k^2 + 4l^2)\right)\rho_1^2 + \rho_2(k^2\rho_2 + 2\rho_m(k+2$

$m)(k+m))\rho_1 + 2\rho_2^2(m\rho_2 k + 2\rho_m(m^2 + km + 2l^2)))\rho_1)$

■$\pi_{r2} = \frac{1}{8(\rho_1^2+\rho_2^2)}((k^2\rho_2^3 + \left(2m\rho_1 k + 2\rho_m(km + k^2 + 4l^2)\right)\rho_2^2 + \rho_1(k^2\rho_1 + 2\rho_m(k+2$

$m)(k+m))\rho_2 + 2\rho_1^2(m\rho_1 k + 2\rho_m(m^2 + km + 2l^2)))\rho_2)$

■$\pi_m = \frac{1}{16(\rho_1^2+\rho_2^2)}((4(\rho_1+\rho_2)^2 k^2 + 8m(\rho_1+\rho_2)^2 k + (16l^2 + 4m^2)\rho_1^2 + 8\rho_1 m^2\rho_2$

$+(16l^2 + 4m^2)\rho_2^2)\rho_m^2 + 4k(\rho_1+\rho_2)(\rho_1^2+\rho_2^2)(k+m)\rho_m + k^2(\rho_1^2+\rho_2^2)^2)$

■$\pi_t = \frac{1}{16(\rho_1^2+\rho_2^2)}(((8km + 4k^2 + 16l^2 + 4m^2)\rho_1^2 + 8\rho_2(k+m)^2\rho_1 + 4\rho_2^2(2km + k^2$

$+4l^2 + m^2))\rho_m^2 + 8(\rho_1+\rho_2)((km + 2l^2 + k^2)\rho_1^2 + 2\rho_2 m(k+m)\rho_1 + \rho_2^2(km + 2l^2$

$+ k^2))\rho_m + 3\left(k\rho_1^2 + \frac{8}{3}m\rho_2\rho_1 + k\rho_2^2\right)(\rho_1^2+\rho_2^2)k)$

The expressions of the sales volume and the profit of each member and the total channel's profit for case 2:

■$S_{r1} = \frac{1}{\left((4\rho_1^2 + 4\rho_2^2)k^2 + 16\rho_1 k\rho_2 m + 4m^2(\rho_1^2+\rho_2^2)\right)}(\rho_1(\rho_1^2 + 2\rho_m\rho_1 + \rho_2(2\rho_m + \rho_2))$

$k^4+6m\left(\left(\rho_2+\frac{2}{3}\rho_m\right)\rho_1^2+\frac{4}{3}\rho_1\rho_m\rho_2+\frac{1}{3}\rho_2^2(2\rho_m+\rho_2)\right)k^3+((2\rho_1^3+4\rho_1^2\rho_m+(12\rho_m$

$\rho_2+10\rho_2^2)\rho_1+8\rho_2^2\rho_m)m^2+4l^2\rho_m(\rho_1^2+\rho_2^2))k^2+16m((\left(\frac{1}{4}\rho_m+\frac{3}{8}\rho_2\right)\rho_1^2+\frac{1}{2}\rho_1\rho_m$

$\rho_2+\frac{1}{8}\rho_2^2(2\rho_m+\rho_2))m^2+l^2\rho_1\rho_m\rho_2)k+4(\frac{1}{4}\rho_1(\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2))m^2+l^2$

$\rho_m(\rho_1^2+\rho_2^2))m^2)$

■$S_{r2}=\frac{1}{\left((4\rho_1^2+4\rho_2^2)k^2+16\rho_1k\rho_2m+4m^2(\rho_1^2+\rho_2^2)\right)}(\rho_2(\rho_2^2+2\rho_m\rho_2+\rho_1(2\rho_m+\rho_1))$

$k^4+2\big((2\rho_m+3\rho_1)\rho_2^2+4\rho_2\rho_m\rho_1+\rho_1^2(\rho_1+2\rho_m)\big)mk^3+((2\rho_2^3+4\rho_2^2\rho_m+(12\rho_m\rho_1+$

$10\rho_1^2)\rho_2+8\rho_1^2\rho_m)m^2+4l^2\rho_m(\rho_1^2+\rho_2^2))k^2+16m((\left(\frac{1}{4}\rho_m+\frac{3}{8}\rho_1\right)\rho_2^2+\frac{1}{2}\rho_2\rho_m\rho_1+\frac{1}{8}$

$\rho_1^2(\rho_1+2\rho_m))m^2+l^2\rho_m\rho_1\rho_2)k+4m^2(\frac{1}{4}\rho_2(\rho_2^2+2\rho_m\rho_2+\rho_1(\rho_1+2\rho_m)m^2+l^2\rho_m($

$\rho_1^2+\rho_2^2)))$

■$S_m=\frac{1}{\left((4\rho_1^2+4\rho_2^2)k^2+16\rho_1k\rho_2m+4m^2(\rho_1^2+\rho_2^2)\right)}((\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2))$

$(\rho_1+\rho_2)k^4+2m(\rho_1+\rho_2)^2(\rho_1+\rho_2+4\rho_m)k^3+(2(\rho_1^2+(6\rho_m+4\rho_2)\rho_1$
$+\rho_2(\rho_2+6\rho_m))$

$(\rho_1+\rho_2)m^2+8l^2\rho_m(\rho_1^2+\rho_2^2))k^2+32m(\frac{1}{16}(\rho_1+\rho_2)^2(\rho_1+\rho_2+4\rho_m)m^2+l^2\rho_m\rho_1\rho_2)$

$k+8m^2(\frac{1}{8}\big(\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2)\big)(\rho_1+\rho_2)m^2+l^2\rho_m(\rho_1^2+\rho_2^2)))$

■$\pi_{r1}=\frac{1}{\left((8k^2+8m^2)\rho_1^2+32\rho_1k\rho_2m+8\rho_2^2(m^2+k^2)\right)}((3m^2k^2+2m^4+k^4)\rho_1^4+(4$

$m^4\rho_m+10k\left(\frac{4}{5}\rho_m+\rho_2\right)m^3+(8l^2\rho_m+6\rho_mk^2)m^2+6k^3\left(\rho_2+\frac{2}{3}\rho_m\right)m+2\rho_mk^2(k^2$

$+4l^2))\rho_1^3+\rho_2((\rho_2+4\rho_m)m^4+12m^3k\rho_m+10k^2\left(\rho_2+\frac{7}{5}\rho_m\right)m^2+8k\rho_m(k^2+4l^2)m$

$+(2\rho_m+\rho_2)k^4)\rho_1^2+2(-m^4\rho_m-m^3k\rho_2+(4l^2\rho_m+3\rho_mk^2)m^2+m(2\rho_m+\rho_2)k^3$

$+4l^2\rho_mk^2)\rho_2^2\rho_1-((2\rho_m+\rho_2)m^2+4mk\rho_m+k^2(2\rho_m+\rho_2))\rho_2^3m^2)$

$$\blacksquare\pi_{r2}=\frac{1}{\left((8k^2+8m^2)\rho_2^2+32\rho_1k\rho_2m+8\rho_1^2(m^2+k^2)\right)}((3m^2k^2+2m^4+k^4)\rho_2^4+($$

$$4m^4\rho_m+10k\left(\frac{4}{5}\rho_m+\rho_1\right)m^3+(8l^2\rho_m+6\rho_mk^2)m^2+6k^3\left(\rho_1+\frac{2}{3}\rho_m\right)m+2\rho_mk^2(k^2$$

$$+4l^2))\rho_2^3+((\rho_1+4\rho_m)m^4+12m^3k\rho_m+10k^2\left(\rho_1+\frac{7}{5}\rho_m\right)m^2+8k\rho_m(k^2+4l^2)m$$

$$+(2\rho_m+\rho_1)k^4)\rho_1\rho_2^2+2(-m^4\rho_m-m^3k\rho_1+(4l^2\rho_m+3\rho_mk^2)m^2+m(2\rho_m+\rho_1)k^3$$

$$+4l^2\rho_mk^2)\rho_1^2\rho_2-((2\rho_m+\rho_1)m^2+4mk\rho_m+k^2(2\rho_m+\rho_1))\rho_1^3m^2)$$

$$\blacksquare\pi_m=\frac{1}{\left((16\rho_1^2+16\rho_2^2)k^2+64\rho_1k\rho_2m+16m^2(\rho_1^2+\rho_2^2)\right)}((\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m$$

$$+\rho_2))^2k^4+8m\big((\rho_m+\rho_2)\rho_1+\rho_m\rho_2\big)\big(\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2)\big)k^3+((2\rho_1^4+8\rho_m\rho_1^3$$

$$+(20\rho_2^2+24\rho_m^2+40\rho_m\rho_2)\rho_1^2+40\rho_m\rho_2\left(\frac{6}{5}\rho_m+\rho_2\right)\rho_1+2\rho_2^4+8\rho_2^3\rho_m+24\rho_m^2\rho_2^2)m^2$$

$$+32\rho_m^2l^2(\rho_1^2+\rho_2^2)\rho_1+2\rho_2^4+8\rho_2^3\rho_m+24\rho_m^2\rho_2^2)m^2+32\rho_m^2l^2(\rho_1^2+\rho_2^2))k^2+128m($$

$$\frac{1}{16}((\rho_m+\rho_2)\rho_1+\rho_m\rho_2)(\rho_1^2+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2))m^2+\rho_2\rho_1\rho_m^2l^2)k+32m^2(\frac{1}{32}(\rho_1^2$$

$$+2\rho_m\rho_1+\rho_2(2\rho_m+\rho_2))^2m^2+\rho_m^2l^2(\rho_1^2+\rho_2^2)))$$

$$\blacksquare\pi_t=\frac{1}{\left((16\rho_1^2+16\rho_2^2)k^2+64\rho_1k\rho_2m+16m^2(\rho_1^2+\rho_2^2)\right)}((4(\rho_1+\rho_2)^2k^4+16m(\rho_1+$$

$$\rho_2)^2k^3+\big(24(\rho_1+\rho_2)^2m^2+16l^2(\rho_1^2+\rho_2^2)\big)k^2+64\left(\frac{1}{4}(\rho_1+\rho_2)^2m^2+\rho_1\rho_2l^2\right)mk+$$

$$16(\frac{1}{4}(\rho_1+\rho_2)^2m^2+l^2(\rho_1^2+\rho_2^2))m^2)\rho_m^2+8(\rho_1+\rho_2)((\rho_1^2+\rho_2^2)k^2+4\rho_2\rho_1km+m^2(\rho_1^2$$

$$+\rho_2^2))(k^2+2km+m^2+2l^2)\rho_m+3((\rho_1^2+\rho_2^2)k^2+4\rho_2\rho_1km+m^2(\rho_1^2+\rho_2^2))^2)$$

The expressions of the sales volume and the profit of each member and the total channel's profit for case 3:

$$\blacksquare S_{r1}=\frac{1}{2}(k^2+2mk+m^2+2l^2)\rho_m+\frac{1}{2}\rho_1k^2+k\rho_2m+\frac{1}{2}\rho_1m^2+\frac{1}{2}l^2(\rho_2+\rho_1)$$

$$\blacksquare S_{r2}=\frac{1}{2}(k^2+2mk+m^2+2l^2)\rho_m+\frac{1}{2}\rho_2k^2+k\rho_1m+\frac{1}{2}\rho_2m^2+\frac{1}{2}l^2(\rho_2+\rho_1)$$

$$\blacksquare S_m=\frac{1}{2}(k^2+2mk+m^2+2l^2)(\rho_1+2\rho_m+\rho_2)$$

$$■\pi_{r1} = \rho_1\left(\frac{1}{2}k(k+m)\rho_m + \rho_1 k + \rho_2 m\right) + \frac{1}{2}m\big((k+m)\rho_m + \rho_1 m + \rho_2 k\big) + \frac{1}{2}l^2(\rho_1 + 2$$

$$\rho_m + \rho_2)) - \frac{1}{4}(1-t)((k+m)\rho_m + \rho_1 k + \rho_2 m)^2$$

$$■\pi_{r2} = \rho_2\left(\frac{1}{2}k(k+m)\rho_m + \rho_1 m + \rho_2 k\right) + \frac{1}{2}m\big((k+m)\rho_m + \rho_1 k + \rho_2 m\big) + \frac{1}{2}l^2(\rho_1 + 2$$

$$\rho_m + \rho_2)) - \frac{1}{4}(1-t)((k+m)\rho_m + \rho_1 m + \rho_2 k)^2$$

$$■\pi_m = \frac{1}{4}\big((4-2t)k^2 - 4m(-2+t)k + (4-2t)m^2 + 4l^2\big)\rho_m^2 - \frac{1}{2}(k+m)^2(\rho_2+\rho_1)$$

$$(-1+t)\rho_m - \frac{1}{4}t(\rho_1^2+\rho_2^2)k^2 - t\rho_1 k\rho_2 m - \frac{1}{4}t(\rho_1^2+\rho_2^2)m^2 - \frac{1}{4}l^2(\rho_2+\rho_1)^2$$

$$■\pi_t = \frac{1}{4}(4mk + 2k^2 + 2m^2 + 4l^2)\rho_m^2 + \frac{1}{2}(k^2 + 2mk + m^2 + 2l^2)(\rho_1+\rho_2)\rho_m + \frac{1}{4}(k^2$$

$$+\mathbf{m^2} + l^2)\rho_1^2 + \frac{1}{4}(2l^2 + 4mk)\rho_2\rho_1 + \frac{1}{4}\rho_2^2(k^2 + m^2 + l^2)$$

References

Alirezai, A., and KhoshAlhan, U., (2014). Coordination of pricing and co-op advertising models in supply chain: A game theoretic approach. International Journal of Industrial Engineering Computations, 23 - 40.

Aust, G., and Busher, U., (2012). Vertical cooperative advertising and pricing decisions in a manufacturer-retailer supply chain: A game theoretic approach. European Journal of Operational Research, 223, 473 - 482.

Aust G. and Busher U. (2014). Cooperative advertising models in supply chain management: A review. European Journal of Operational Research 234, 1-14.

Ben Youssef, S., & Dridi, D. (2013). National advertising and cooperation in a manufacturer two retailers channel. International Review of Management and Marketing, 75 - 85.

Choi, S. (1991). Price competition in a channel structure with a common retailer. Marketing Science, 271 - 296.

Jorgensen S. and Zaccour G. (2013). A survey of game-theoretic models of cooperative advertising. European Journal of Operational Research, 237, 1–14.

Hutchins, M. S., 1953. Cooperative Advertising. Roland Press, New York.

Karry, S., & Zaccour, G., 2006. Could co-op advertising be a manufacturer's counterstrategy to store brands?. Journal of Business Research 59 (9), 1008 - 1015.

Kim, S., Staelin, R., 1999. Manufacturer allowances and retailer pass-through rates in a competitive environment. Marketing Science, 59 - 76.

Luo, P. L. (2006). The theory of industrial organization. shanghai: Fundan university Press.

SeyedEsfahani, M. M., Biazaran, M., & Gharakhani, M. (2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer-retailer supply chains. European Journal of Operational Research, 211, 263 - 273.

Spengler, J. J., (1950). Vertical integration and anti-trust policy. The Journal of Political Economy, 58 (4), 347 - 352.

Wang, S. D., Zhou, Y.W., Min, J., & Zhong, Y. G. (2011). Coordination of cooperative advertising models in one-manufacturer two-retailer supply chain system. Computer & Industrial Engineering, 61, 1053 - 1071.

Xie, J., & Neyret, A. (2009). Co-op advertising and pricing models in manufacturer-retailer supply chains. Computer & Industrial Engineering, 56, 1375 - 1385.

Xie, J., & Wei, J. C. (2009). Coordinating advertising and pricing in a manufacturer-retailer channel. European Journal of Operational Research, 197, 785 - 791.

Zhang, J., & Xie, J. (2012). A game theoretical study of cooperative advertising with multiple retailers in a distribution channel. Journal of Systems Science and Systems Engineering, 21 (1), 37 - 55.