Local advertising externalities and cooperation in one manufacturer-two retailers channel

Dridi, Dhouha and Ben Youssef, Slim

8 March 2015

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Dhouha DRIDI and Slim BEN YOUSSEF

Manouba University and ESC de Tunis,

Campus Universitaire, 2010 La Manouba, TUNISIA.

March 2015

Abstract

In this paper, we consider a static model for advertising strategies and pricing decisions in supply chain with one monopolistic manufacturer and two duopolistic retailers. We assume an additive form of the consumer demand which is influenced by retail price and advertising. The manufacturer sets the wholesale price, invests in advertising (at national level) and offers cooperative advertising to boost the advertising expenditures of their retailers. The retailers set the retail price and invest in advertising (at local level). By means of game theory, we discuss three different relationships between the supply chain members: two non cooperative games including the Stackelberg – Cournot and the Stackelberg – Collusion and one cooperative game. The comparison between the three models reveals that the advertising, the pricing, the consumer demand and the profits are affected by various relationships. Furthermore, under the cooperation situation, we propose a channel coordination mechanism through a manufacturer's participation rate in retailers' local advertising cost and wholesale price by using utility function.

Keywords: Game theory, supply chain, cooperative advertising, pricing, retail competition, retail coalition, coordination mechanism.

JEL Classifications: C70, D60, M30.

1. Introduction

Pricing[1] and advertising decisions in a manufacturer-retailer channel gained much attention recently, especially when game theoretical models are used to analyze the interaction of different channel members. These have different advertising objectives. The manufacturer invests in national advertising in order to build brand equity and to promote and lure the

[1] Pricing is a core topic in the marketing research literature on supply chain including retail price and wholesale price.

potential consumers choosing its product (the long-term sales effect). Whereas, the retailer invests in local advertising in order to boost the local demand (the short-term sales effect).

The inconsistency of two types of advertising is assumed of a sufficient magnitude to create a role for cooperative advertising, which is one of the most important issues in marketing programs and plays a significant role in the analysis of supply chain relationships. Cooperative advertising is defined as an interactive relationship and financial arrangement between the members of supply chain, in which the manufacturer shares a part of the retailer's advertising expenditures - commonly known as the manufacture's participation rate - to motivate immediate sales at the retail level where the retailer is not able to invest a sufficient sum of money in the local advertising.

The stream of literature that is related to an application of game theoretical models in marketing channel can be divided, in general, into two main categories according to the nature of the model used: static models or dynamic models. Under each category, we can identify the structure of marketing channel (marketing channel with simple or complex structure), the mechanism of coordination as quantity discount, profit or revenue sharing, advertising allowances, local advertising cost-sharing, two-part tariffs and the game type including Stackelberg, Nash and cooperation.

In static model, the relationships between supply chain members are discussed in a single period. In dynamic model, the interactions among supply chain members are discussed in continuous time as differential games or optimal control problems.

In the following, we summarize the papers that refer only to the first category because our study focuses on the static models. The papers of second category have some assumptions in common: they employed the advertising goodwill model of Nerlove and Arrow (1962) as a description of the dynamics of the game. Examples in this category are Jorgensen et al. (2000, 2001a), Jorgensen and Zaccour (2004), He et al. (2011, 2012), Zhang et al. (2013) and Chutani and Sethi (2012b, 2014).

The papers, that have adopted static models, are divided into two main parts where the first one deals with classical marketing channel having one manufacturer – one retailer. The second one deals with more than one manufacturer – one retailer. Berger (1972) was the first person to propose a primary static model analyzing the cooperative advertising in a marketing channel. He concluded that offering an advertising allowance to the retailer by manufacturer can generate higher profits. Following this research, several authors have addressed this concept by different point of view (see. Crimmins (1985), Berger and Magliozzi (1992), Dant and Berger (1996), Kim and Staelin (1999), Nagler (2006)).

Most studies included in the current review mainly focused on the first part (single manufacturer – single retailer framework). Xie and Ai (2006) extended the paper of Li et al. (2002) who have developed a new game structure called "higher order Stackelberg" equilibrium. The authors considered a model with non linear demand and developed three strategic models for determining equilibrium marketing and investment effort levels.

Furthermore, they compared the results with those of the paper of Huang and Li (2001) which are different. Yue et al. (2006) extended the model of Huang et al. (2002). They studied the coordination of cooperative advertising when the manufacturer offers price deductions to customers. The novelty in this paper is that these authors highlighted the problem of negative demand for certain values of the decisions variables. They suggested two constraints to fix it, but they did not incorporate these constraints into their mathematical analysis. These authors found out that if the upstream and downstream firms cooperate, they achieve cooperative advertising. They showed that, on a cooperative model, the total profit of the supply chain is higher than without cooperation. The constraints suggested by Yue et al. (2006) had been taken into account by Ahmadi-Javid and Hoseinpour (2011). They corrected the demand function of Huang and Li (2001), Huang et al. (2002), Li et al. (2002), Xie and Ai (2006) and Yue et al. (2006).

All the paper mentioned before examine a static game theoretic model for cooperative advertising in supply chain with one manufacturer and one retailer. There are many researches, which have received considerable attention recently, study the same model described earlier and assume that the market demand is influenced by both advertising and pricing. Xie and Wei (2009) considered two game models including a non cooperative game (the Stackelberg-manufacturer) and a cooperative game and then compared between them. They showed that cooperative model achieves better coordination by generating higher channel total profits, lower retail price to consumers, and higher advertising efforts for all channel members than the non-cooperative model. They identified the feasible solutions to a bargaining problem where the channel members can determine how to divide the extra-profits generated by cooperation. Xie and Neyret (2009) followed a similar approach and investigated four games models (cooperative game, Nash game, Stackelberg-Manufacturer and Stackelberg-Retailer) to analyze a cooperative advertising in two-tier marketing channel. Szamerkovsky and Zhang (2009) considered pricing and advertising decisions in a two member supply chain, where costumer demand depends on both retail price and advertisement. They obtained the optimal decisions of each member by solving a Stackelberg game with the manufacturer as the leader and the retailer as the follower. The authors proved that the participation rate of local advertising does not work well, it is better for the manufacturer to advertise nationally and offer the retailer a lower wholesale price.

Further, Seyed Esfahani et al. (2011) discuss the cooperative and non-cooperative game between a manufacturer and a retailer. They introduce a slight modification in the linear price demand function by assuming a relatively general form compared with the classical linear relationship: they introduce a new parameter v which yields convex ($v < 1$), linear ($v = 1$) and concave ($v > 1$) curves. Aust and Busher (2012) extend the work of Seyed Esfahani et al. (2011). They add a retailer margin “m” ($p = m + w$) as a new decision variable according to Choi (1991, 1996). The aim of this modification is to get better idea into the effects of market power on the distribution of channels profits. They prove that both the manufacture and the retailer achieve the highest profits level when they cooperate.

In order to follow this research approach and to relax the classical marketing channel having one manufacturer – one retailer, some studies suggested adding competition, whether between manufacturers or between retailers to enrich the model used. Karray and Zaccour (2007) considered a supply chain formed by two manufacturer and two retailers. The retailers choose their levels of marketing efforts and the manufacturer control their participation rates. The authors assumed that the national advertising efforts are exogenous parameters instead of manufacturer's decisions variables. Wang et al. (2011) and Zhang and Xie (2012) consider a supply chain in which a monopolistic manufacturer sells its product through duopolistic retailers. They assume that the demand function depends only on local and national advertising: they suppose prices as constant parameters. The paper of Wang et al. (2011) discusses four possible game structures: Stackelberg - Cournot, Stackelberg - Collusion, Nash - Cournot and Nash - collusion. The authors reveal how cooperative advertising policies and profits of all participants are affected by various competitive behaviors. However, all their results are made under the heavy assumption that the retailers and the manufacturer's marginal profits are exogenously determined. However, the paper of Zhang and Xie (2012) explore the impacts of the retailer's multiplicity on channel members' optimal decisions and on the total channel efficiency. Ben Youssef and Dridi (2013) propose a supply channel composed of one manufacturer and two symmetric retailers where the demand function is dependent on pricing and national advertising. Three game theoretic models are established including the non cooperative game, the partial cooperative game and the full cooperation game. In addition, the authors suggest a new and unusual evaluation of consumers' surplus which positively depends not only on the price-demand function but also on the spending in national advertising. For more details, one can refer to the papers of Jorgensen S. and Zaccour G. (2013) and Aust G. and Busher U. (2014). These papers surveyed the literature on cooperative advertising in marketing channels using game theoretic models.

Our main motivation is the scarcity of researches that develops game theoretical models with multiple retailers because the assumption of multiple retailers is more realistic. According to our best knowledge, only the papers of Alirezai and KhoshAlhan (2014) and of Karray and Amin (2014) that addressed to study pricing and cooperative advertising at the same time in supply chain formed by monopolistic manufacturer and duopolistic retailer. Alirezai and KhoshAlhan (2014) adopt a multiplicative form of demand function of Aust and Busher (2012) with some simplifications. They consider that the local advertising of each retailer has a negative effect on the other retailer and use the Nash, Manufacturer – Stackelberg and cooperative games for this investigation. Karray and Amin (2014) evaluate the profitability of cooperative advertising in a channel with competing retailers. In this work, the authors proposed the retail price and the local advertising expenditures as decision variables of retailers and the coop participation rate and the wholesale price as decision variables of manufacturer. But, they ignored the national advertising expenditures (decision variable of manufacturer). They assumed a demand function that has been commonly used in the literature by McGuire and Staelin (1983), Choi (1991), Karray and Zaccour (2006) and Karray (2013). This demand function is positively influenced by his local advertising and negatively affected by the local advertising of his competitor. The authors developed two non-cooperative games (one is without cooperative advertising and another is with cooperative advertising) and a cooperative game and they provided equilibrium solutions for each game.

In our paper, we intend to study pricing and advertising decisions in one manufacturer – two competing retailers marketing channel using game theoretic models (Stackelberg – Cournot, Stackelberg – Collusion and cooperation) in order to explore the impacts of retail competition, retail coalition and cooperative on the channel members' decisions, the profit of each member, the surplus consumers and the social welfare.

The rest of this paper is organized as follows. Section 2 first introduces the basic model. The game theoretic analysis is addressed in section 3. Comparisons between three different game theoretic structures are made in section 4. Finally, our conclusions are shown in section 5.

2. The basic model

To model the relationship between monopolistic manufacturer and duopolistic retailers, we consider cooperative advertising issues of a supply chain in which both retailers sell only the manufacturer's product. The manufacturer sets the wholesale price and invests in national advertising in order to create brand awareness and to increase consumer's interest for the product. In addition, the manufacturer utilizes the cooperative advertising programs by sharing a part of retailers' advertising costs in the hope for stimulating potential consumers'. On the other hand, each retailer decides on a retail price and invests also in the local advertising.

Unlike the existing research related to this issue, our paper suggest an additive form of demand function which is influenced by advertising expenditures and retail price. We likewise assume, as in Lal (1990) and Wang et al. (2009), that the local advertising effort not only benefits the investing retailer but also the other competing retailer located in close physical proximity. The consumer demand function V_i for the product proposed by retailer i depends on the retail prices and the advertising level as:

$$V_i = b_i - p_i + a_i + \beta\, a_j + A, \quad i = 1, 2, j = 3 - i \tag{1}$$

where b_i is a positive constant and denotes the maximal potential demand faced by retailer i if prices and advertising expenditures are zero. In the game theoretic analysis, we assume $b_i = 1$ to simplify the expressions;

p_i is the sale price charged by retailer i to consumer;

a_i (a_j) is the local advertising expenditure of retailer i (j);

β is the effectiveness of compete retailer's local advertising $(0 \leq \beta \leq 1)$;

A is the manufacturer's national advertising expenditures.

In Eq. 1, we assume that the demand function is not affected by competitor's price because the retailers' prices are often the same when the retailers are geographically related.

Furthermore, the retailers will set identical prices for the reason that they face the same demand function[2].

The profit of each retailer, the manufacturer, the two retailers and the total system can be expressed as follows, respectively:

$$\pi_{ri} = (p_i - w)V_i - (1 - t)a_i^2 \tag{2}$$

$$\pi_m = w \sum_{i=1,2} V_i - t \sum_{i=1,2} a_i^2 - A^2 \tag{3}$$

$$\pi_r = \sum_{i=1,2} \pi_{ri} \tag{4}$$

$$\pi_t = \pi_m + \pi_r = \pi_m + \sum_{i=1,2} \pi_{ri} \tag{5}$$

where w is the manufacturer's wholesale price $(0 < w < p_i)$[3]. t is the manufacture's participation rate in retailers' local advertising expenditures $(0 \leq t \leq 1)$.

In the previous equations, the local advertising expenditures are quadratic as incremental investments in brand-specific service become increasingly costly. Wang et al (2009) note that assuming cost $\mathrm{c} = \mathrm{a}^2$ in the previous equations is equivalent to assuming diminishing returns to local advertising expenditures $\mathrm{a} = \sqrt{\mathrm{c}}$. Moreover, the national advertising expenditures are equal to A^2 as with local advertising expenditures.

In addition, our important contribution in this paper is the evaluation of the impact of retail competition, retail coalition and cooperation between all members of the supply chain on consumer surplus and social welfare. The consumer surplus engendered by the consumption of quantity V_i of the product sold by retailer i is:

$$CS\,(V_i) = \int_0^{V_i} p_i(t)dt - p_i V_i \tag{6}$$

From (1), we have:

$$p_i(V_i) = 1 + a_i + \beta a_j + A - V_i, i = 1, 2, j = 3 - i \tag{7}$$

Using (7) in (6), we get:

$$CS\,(V_i) = \frac{1}{2} V_i^2 \tag{8}$$

[2] In Eq. (14), we will prove that $p_i = p_j$. For more details, one can see the paper of Wang et al (2009).

[3] $V_i > 0 \Leftrightarrow 1 - p_i + a_i + \beta\, a_j + A > 0 \Leftrightarrow 0 < w < p_i < 1 + a_i + \beta\, a_j + A$

The above expression is a new evaluation of consumer surplus because it not only depends on pricing but also related to the local and the national advertising expenditures.

The total of consumer surplus engendered by the consumption of the two products is:

$$CS_t = CS\,(V_i) + CS\,(V_j) \tag{9}$$

Then, we define the social welfare as the sum of the total of consumer surplus and the total profit of the supply chain:

$$S = CS_t + \pi_t \tag{10}$$

3. Game theoretic analysis

In this section we discus three game theoretic models based on two non cooperative games including Stachelberg – Cournot and Stachelberg – Collusion and one cooperative game.

3.1 The Stackelberg – Cournot game

We assume a relationship among the manufacturer and the retailers when the manufacturer is the leader and the two retailers are followers. At the retail level, we consider that the retailers simultaneously and non-cooperatively maximize their own profits (retailers acting independently).

To determine the equilibrium of the Stackelberg - Cournot game, we firstly solve the optimization problem of retailer i. That is:

$$max\ \pi_{ri} = (p_i - w)V_i - (1-t)a_i^2 \tag{11}$$

$st: 0 < p_i < 1 + a_i + \beta\ a_j + A$ and $a_i > 0$.

To solve these optimization problems, each retailer take the first derivative of formulas described above with respect to p_i and a_i as following:

$$\frac{\partial \pi_{ri}}{\partial p_i} = 1 - 2\ p_i + a_i + \beta\ a_j + A + w = 0 \tag{12}$$

$$\frac{\partial \pi_{ri}}{\partial a_i} = p_i - w - 2(1-t)\ a_i = 0 \tag{13}$$

Solving these equations and substituting the algebraic expressions yield to the following retailers' decision variables:

$$p_i = \frac{2wt + 2At + 2t + \beta w - 2A — w - 2}{4t - 3 + \beta} \tag{14}$$

$$a_i = \frac{1 + A - w}{3 - 4t - \beta} \tag{15}$$

Integrating (14) and (15) with (3), we can get the manufacturer's optimization problem as follows:

$$max\pi_m = \frac{4(-1+t)(-w+A+1)w}{4t+\beta-3} - \frac{2t(-w+A+1)^2}{(4t+\beta-3)^2} - A^2 \quad (16)$$

$st: 0 < w < p_i, 0 \le t < 1$ and $A > 0$.

Taking the first derivatives of formula (16) with respect to A, t and w. Let $\frac{\partial \pi_m}{\partial A} = 0, \frac{\partial \pi_m}{\partial t} = 0, \frac{\partial \pi_m}{\partial w} = 0$, we can get the following expressions after substitution:

$$A = \frac{2(-1+t)w(4t-3+\beta)+2wt-2t}{-22t+16t^2+8\beta t+9-6\beta+\beta^2} \quad (17)$$

$$t = \frac{(-3+\beta)(-2\beta w+1+A-3w)}{4w+8\beta w+4A+A} \quad (18)$$

$$w = \frac{2\beta-13}{4\beta^2+12\beta-15} \quad (19)$$

Proposition 1: the Stackelberg - Cournot game previously described has the following unique equilibrium solution that is given with the parameter $\beta \in [0;\ 0.94]$:

$$w^* = \frac{2\beta-13}{4\beta^2+12\beta-15} \quad (20)$$

$$A^* = \frac{8}{15-12\beta-4\beta^2} \quad (21)$$

$$t^* = \frac{1+2\beta}{3+2\beta} \quad (22)$$

$$p^* = \frac{2\beta-21}{4\beta^2+12\beta-15} \quad (23)$$

$$a^* = \frac{2(2\beta+3)}{15-12\beta-4\beta^2} \quad (24)$$

Proposition 1 prove that the retailers set identical prices and local advertising expenditures as we mentioned earlier. The second order conditions prove that the manufacture's advertising participation rate must be between 0 and $\frac{3}{4}$ (See Appendix for the proof of the second order condition).

3.2 The Stackelberg – Collusion game

Under the Stackelberg – Collusion game, the leader is still the monopolistic manufacturer who acts as the first mover by choosing the wholesale price, the national advertising and the participation rate of cooperative advertising expenditures (the first stage of game). The two duopolistic retailers, acting as the followers, then make decisions together to maximize jointly their profits by choosing the retail prices and the local advertising expenditures (the second stage of game).

First, we analyze the retailers' optimal pricing and advertising, in the second stage of the game. The joint profit function of the two retailers can be expressed as:

$$\pi_r = (p_i - w)\big(1 - p_i + a_i + \beta a_j + A\big) - (1-t)\,a_i^2 + \big(p_j - w\big)\big(1 - p_j + a_j + \beta a_i + A\big)$$
$$-(1-t)\,a_j^2 \qquad (25)$$

To maximize this joint profit function, one can easily solve the retailers' decision problem by equating the first partial derivatives, with respect to p_i and a_i, to zero and taking the manufacturer's decision variables as constants.

$$\frac{\partial \pi_r}{\partial p_i} = 1 - 2p_i + a_i + \beta a_j + A + w = 0 \qquad (26)$$

$$\frac{\partial \pi_r}{\partial a_i} = p_i - w - 2(1-t)a_i + (p_j - w)\beta = 0 \qquad (27)$$

From these equations and after substitution, we obtain the optimal symmetric solutions of the retail price and the local advertisement as shown:

$$p_i = \frac{\beta^2 w + 2\beta w + 2t + 2At + 2wt - w - 2A - 2}{4t + \beta^2 - 3 + 2\beta} \qquad (28)$$

$$a_i = -\frac{(1 - w + A)(1 + \beta)}{4t + \beta^2 - 3 + 2\beta} \qquad (29)$$

Second, as a leader, the manufacturer knows the retailers' reaction functions given in (28) and (29) before setting w, A, t (the first stage of the game). So, the manufacturer's profit function for any w, A and t can be formulated by substituting (28) and (29) into (3) as:

$$\pi_m = \frac{4(-1+t)(1-w+A)w}{4t + \beta^2 + 2\beta - 3} - \frac{2t(1-w+A)^2(1+\beta)^2}{(4t + \beta^2 + 2\beta - 3)^2} - A^2 \qquad (30)$$

Solving $\frac{\partial \pi_m}{\partial A} = 0, \frac{\partial \pi_m}{\partial t} = 0, \frac{\partial \pi_m}{\partial w} = 0$ and substituting lead us to the optimal expressions shown below:

$$A = \frac{-13w + 3\beta^2 w + 6\beta w - 1 - 2\beta - \beta^2}{-23 + 9\beta^2 + 18\beta} \qquad (31)$$

$$t = \frac{1}{2}\frac{3\beta^2 w - \beta^2 + 6\beta w - 2\beta + 3 - 7w}{-2 - 3w} \tag{32}$$

$$w = \frac{3\beta^2 + 6\beta - 13}{9\beta^2 - 18\beta - 15} \tag{33}$$

Proposition 2: The Stackelberg equilibrium solution of the Stackelberg - Collusion game is unique and is given by:

$$\tilde{p} = \frac{\beta^2 + 2\beta - 7}{3\beta^2 + 6\beta - 5} \tag{34}$$

$$\tilde{a} = \frac{2(1+\beta)}{5 - 6\beta - 3\beta^2} \tag{35}$$

$$\tilde{A} = \frac{8}{15 - 18\beta - 9\beta^2} \tag{36}$$

$$\tilde{t} = \frac{1}{3} \tag{37}$$

$$\widetilde{w} = \frac{3\beta^2 + 6\beta - 13}{9\beta^2 + 18\beta - 15} \tag{38}$$

From this proposition, we observe that the decision variables depend on β expect the participation rate which is unaffected of competitor's advertising. β must take any value between 0 and 0.63. (See Appendix for proof of proposition 2).

3.3 The cooperation game

In this subsection, we focus on a cooperative game structure. Under this situation, the monopolistic manufacturer and the two duopolistic retailers agree to make decisions in order to optimize the whole supply chain's profit. These decisions include the optimal retail price, the optimal local and national advertising expenditures. The maximization problem for the supply chain under the cooperation situation is:

$$\begin{aligned} &\max \pi_t = -p_i^2 + \left(1 + a_i + \beta a_j\right)p_i - p_j^2 + \left(1 + a_j + \beta a_i + A\right)p_j - a_i^2 - a_j^2 - A^2 \\ &st: 0 < p_i < 1 + a_i + \beta a_j + A, a_i > 0, A > 0, \end{aligned} \tag{39}$$

We observe that the objective function does not depend on the wholesale price and the participation rate of local advertisement. However, the individual profit of each supply chain member is dependent of these variables. To solve this optimization problem, we determine the five first derivatives and then equate to zero as follows:

$$\frac{\partial \pi_t}{\partial p_i} = -2p_i + a_i + 1 + \beta a_j + A = 0 \tag{40}$$

$$\frac{\partial \pi_t}{\partial a_i} = p_i + \beta p_j - 2a_i = 0 \tag{41}$$

$$\frac{\partial \pi_t}{\partial A} = p_i + p_j - 2A = 0 \tag{42}$$

From (40), (41), and (42), we derive the following expressions:

$$p_i = \frac{1}{2}(1 + a_i + \beta a_j + A) \tag{43}$$

$$a_i = \frac{1}{2}(p_i + \beta p_j) \tag{44}$$

$$A = \frac{1}{2}(p_i + p_j) \tag{45}$$

Proposition 3: Cooperation between a monopolistic manufacturer and duopolistic retailers has the objective to maximize its joint profit which reaches its maximum at the following equilibrium:

$$\bar{p} = \frac{2}{1 - 2\beta - \beta^2} \tag{46}$$

$$\bar{a} = \frac{\beta + 1}{1 - 2\beta - \beta^2} \tag{47}$$

$$\bar{A} = \frac{2}{1 - 2\beta - \beta^2} \tag{48}$$

In the cooperation situation, the local advertising expense of each retailer is less than the national advertising expense. By checking the second-order conditions, the parameter β can take either value between 0 and 0.41. (See Appendix for proof of proposition 3).

4. The results and discussion

In this section, we discuss whether the manufacturer and the retailers would be better off in the cooperation than in the Cournot and Collusion. We derive the differences between the optimal decision variables and the profits in the three games and then we compare between them to develop some managerial guidelines.

Finally, we discuss the question of how the monopolistic manufacturer and the two duopolistic retailers should share the savings that the cooperation offers their members.

4.1 Comparison of results

In the table 1, we summarize the optimal decision variables of wholesale and retail prices, the national and local advertising expenditures and the participation rate founded earlier of three games. We also determine the demand function, the profit of each supply chain member and the overall profit of the system of the three games in Table 2. Given the difficulty of comparing the results analytically, we resort to explanatory schemas.

Table 1: Optimal expressions in each game

	Stackelberg - Cournot	**Stackelberg - Collusion**	**Cooperation**
$\boldsymbol{w}$	$\frac{2\beta-13}{4\beta^2+12\beta-15}$	$\frac{3\beta^2+6\beta-13}{9\beta^2+18\beta-15}$	————
$\boldsymbol{t}$	$\frac{1+2\beta}{3+2\beta}$	$\frac{1}{3}$	————
$\boldsymbol{A}$	$\frac{8}{15-12\beta-4\beta^2}$	$\frac{8}{15-18\beta-9\beta^2}$	$\frac{2}{1-2\beta-\beta^2}$
$\boldsymbol{a_i}$	$\frac{2(2\beta+3)}{15-12\beta-4\beta^2}$	$\frac{2(1+\beta)}{5-6\beta-3\beta^2}$	$\frac{\beta+1}{1-2\beta-\beta^2}$
$\boldsymbol{p_i}$	$\frac{2\beta-21}{4\beta^2+12\beta-15}$	$\frac{\beta^2+2\beta-7}{3\beta^2+6\beta-5}$	$\frac{2}{1-2\beta-\beta^2}$

Fig 1: Wholesale price w

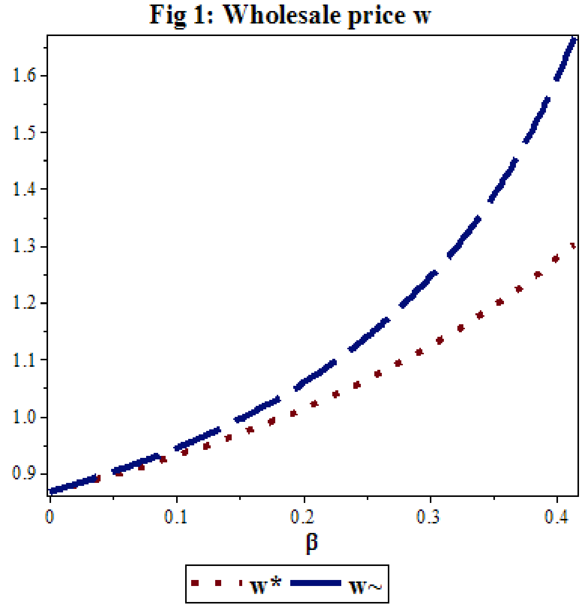


Fig 2: Manufacturer's participation rate t

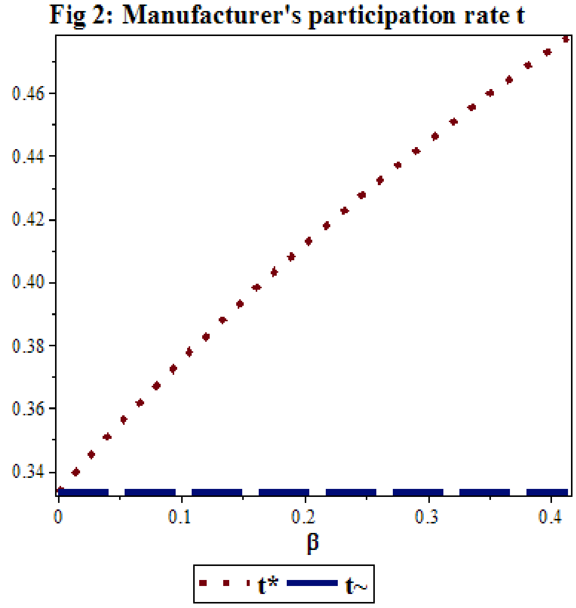


Fig 3: National advertising costs A

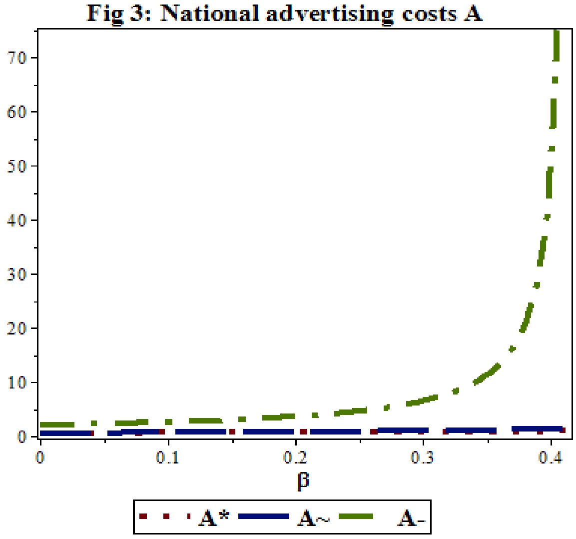


Fig 4: Local advertising costs *a*

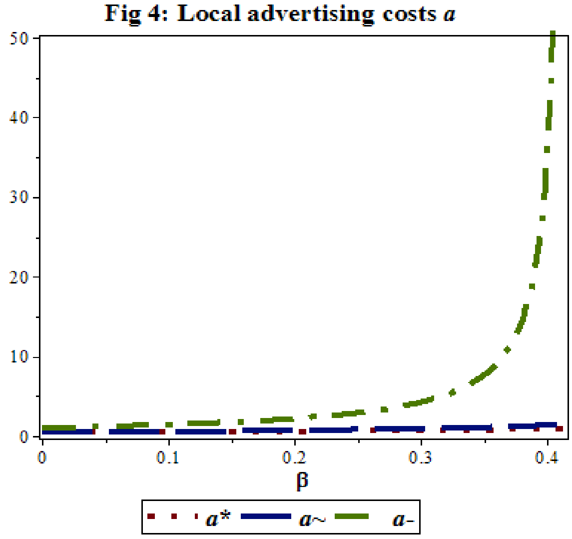
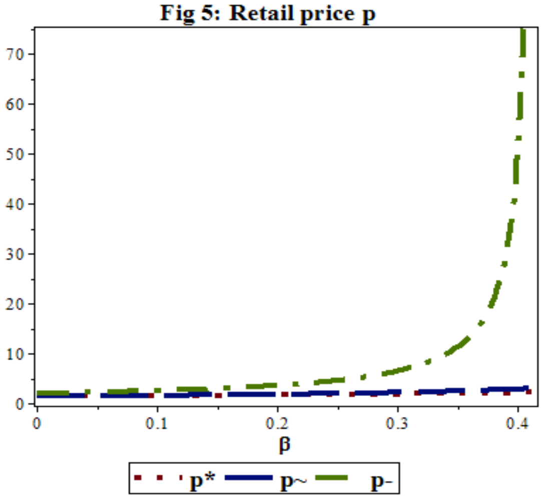


Fig 5: Retail price p



From the figures (1), (2), (3), (4) and (5), we derive the following proposition:

Proposition 4:

i. $w^* < \tilde{w}$

ii. $t^* > \tilde{t}$

iii. $\bar{p}_i > \tilde{p}_i > p_i^*$

iv. $\bar{A} > \tilde{A} > A^*$

v. $\bar{a}_i > \tilde{a}_i > a_i^*$

The results demonstrate that the different behaviors of the supply chain's members make the manufacturer and the two retailers set different decisions variables. As shown in (i) and (ii), we observe that the manufacturer declares a highest wholesale price and reduces the participation rate of retailers' advertising expenditures when the two retailers cooperate. Fig. 2 shows that the manufacture's advertising participation rate does not exceed 48%.

From (iii), the highest retail price can be found in the cooperation, while the Stackelberg - Cournot yields the lowest retail price. This result is new and surprise as because the previous analysis prove that the cooperation between all members of supply chain produces the lowest retail price. Under Stakelberg – Collusion (comparing with Stackleberg – Cournot), the two retailers charge a higher retail price as the manufacturer declares a higher wholesale price. This situation is not favorable for the consumers (the competition between retailers is favorable for consumers as it generates a lower retail price).

Furthermore, one can see that the manufacturer can benefit from the cooperation between retailers as he is able to set a higher wholesale price.

From (iv) and (v), each member of supply chain invests more in advertising if they cooperate and maximize the whole profit together. When the manufacturer is the leader, the duopolistic retailers spend more on local advertising under Stackelberg - Collusion game due to the diminishing of the manufacturer's participation rate.

Table 2: Demand function and profits

	Stackelberg - Cournot	**Stackelberg - Collusion**	**Cooperation**
$\boldsymbol{V_i}$	$\frac{8}{15-12\beta-4\beta^2}$	$\frac{8}{3(5-6\beta-3\beta^2)}$	$\frac{2}{1-2\beta-\beta^2}$
$\boldsymbol{\pi_{ri}}$	$\frac{8(5-2\beta)}{(4\beta^2+12\beta-15)^2}$	$\frac{8}{9(5-6\beta-3\beta^2)}$	$\frac{3+2w\beta^2+4w\beta-2w-\beta^2-2\beta+t\beta^2+2t\beta+t}{(\beta^2+2\beta-1)^2}$
$\boldsymbol{\pi_m}$	$\frac{8}{15-12\beta-4\beta^2}$	$\frac{8}{3(5-6\beta-3\beta^2)}$	$-\frac{2(2w\beta^2+4w\beta-2w+t\beta^2+2t\beta+t+2)}{(\beta^2+2\beta-1)^2}$
$\boldsymbol{\pi_t}$	$\frac{8(25-16\beta-4\beta^2)}{(4\beta^2+12\beta-15)^2}$	$\frac{40}{9(5-6\beta-3\beta^2)}$	$\frac{2}{1-2\beta-\beta^2}$
$\boldsymbol{CS_t}$	$\frac{64}{(4\beta^2+12\beta-15)^2}$	$\frac{64}{(9\beta^2+18\beta-15)^2}$	$\frac{4}{(\beta^2+2\beta-1)^2}$
$\boldsymbol{SW}$	$\frac{8(33-16\beta-4\beta^2)}{(4\beta^2+12\beta-15)^2}$	$\frac{8}{3}\frac{11-10\beta-5\beta^2}{(3\beta^2+6\beta-5)^2}$	$\frac{2(3-2\beta-\beta^2)}{(\beta^2+2\beta-1)^2}$

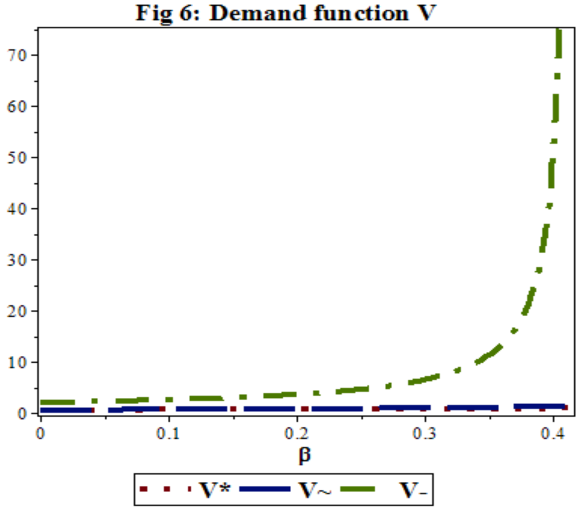


Fig 6: Demand function V

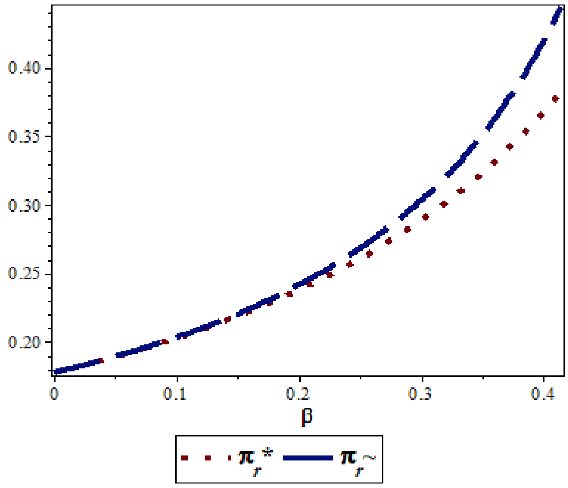


Fig 7: Retailer's profit π_r

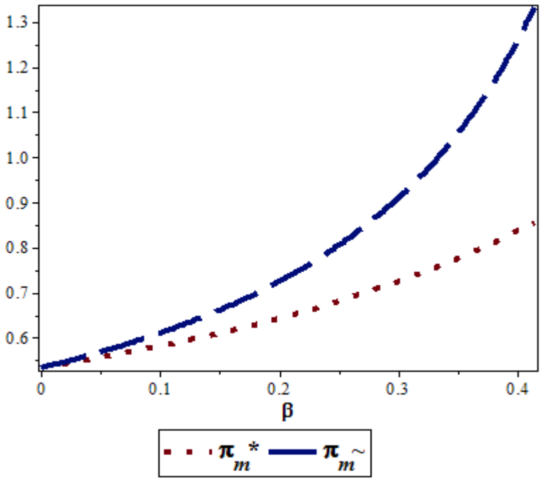


Fig 8: Manufacturer's profit π_m

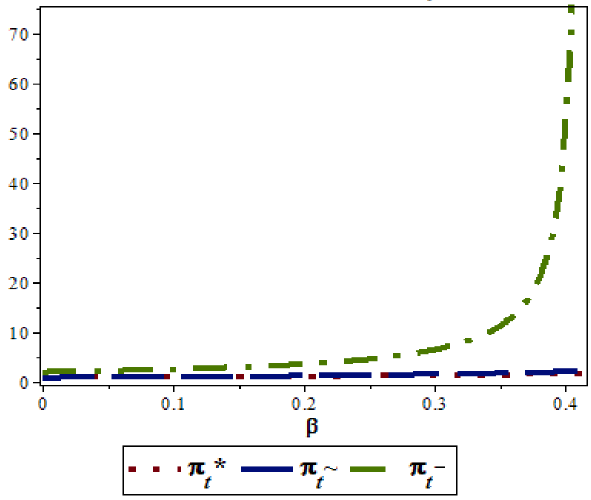


Fig 9: System's profit π_t

From the figures (6), (7), (8) and (9), we reveal the following proposition:

Proposition 5:

i. $\bar{V} > \tilde{V} > V^*$

ii. $\tilde{\pi}_{ri} > \pi^*_{\boldsymbol{ri}}$

iii. $\tilde{\pi}_m > \pi^*_{\boldsymbol{m}}$

iv. $\bar{\pi}_t > \tilde{\pi}_t > \pi^*_{\boldsymbol{t}}$

As we mentioned earlier that the retail price increases under a Cooperation strategy, the demand does not decrease. So, an increase in retail price does not necessarily lead to a decrease in sales. This is largely explained by spending more money on national and local advertising.

When the manufacture is a leader and the two retailers pursue collusive strategy, the quantity demanded increases even though the retail price is high. Our research proves that, as the two retailers collude and maximize the joint profit together, they gain more profits.

The manufacture, as a leader, prefers that the duopolistic retailers act together rather than act separately for the reason that the manufacturer's profit is the higher under the Stackelberg - Collusion than the Stackelberg - Cournot.

Proposition $(5 - iv)$ is consistent with the well-known result of the literature: the supply chain members agree to take decisions together (to cooperate) in order to maximize the entire channel's profit only if they cannot get any higher profits in any other strategies. In this cooperation strategy, the members will develop a coordination mechanism to share the extra-profits[4] by finding $(\bar{w}, \bar{t})$. This issue is addressed in the next subsection as channel coordination mechanism through sharing local advertising $\bar{t}$ and wholesale price $\bar{w}$.

Proposition 6:

i. $\overline{CS}_t > \widetilde{CS}_t > {CS_t}^*$ [5]

ii. $\overline{SW} > \widetilde{SW} > SW^*$

In our research, we provide a new evaluation of the consumer surplus and social welfare that depend on retail pricing, local advertising and national advertising. Retail competition reduces the consumer surplus and social welfare. Nevertheless, the cooperation between all members of supply chain increases them.

4.2 Channel coordination mechanism through the manufacturer's participation rate in retailers' local advertising costs and wholesale price

Neither the monopolistic manufacturer nor the duopolistic retailers would be willing to cooperate if their individual profit is lower than those in a non-cooperative game. We assume that the members of supply chain accept the strongly feasible solution $(\bar{p}, \bar{a}, \bar{A}, \bar{w}, \bar{t})$.

The manufacturer agrees to cooperate only if following condition is satisfied:

$$\bar{\pi}_m\ (\bar{p}, \bar{a}, \bar{A}, \bar{w}, \bar{t}) \geq \max\left(\pi_m^*(p^*, a^*, A^*, w^*, t^*), \tilde{\pi}_m((\tilde{p}, \tilde{a}, \tilde{A}, \tilde{w}, \tilde{t})\right) = \tilde{\pi}_m \tag{49}$$

And the duopolistic retailers cooperate only if following condition is satisfied:

$$\bar{\pi}_{ri}(\bar{p}, \bar{a}, \bar{A}, \bar{w}, \bar{t}) \geq \max\left(\pi_{ri}^*(p^*, a^*, A^*, w^*, t^*), \tilde{\pi}_{ri}(\tilde{p}, \tilde{a}, \tilde{A}, \tilde{w}, \tilde{t})\right) = \tilde{\pi}_{ri} \qquad (i = 1,2) \tag{50}$$

Summing up (49) and (50), equivalently we have:

$$\bar{\pi}_t = \bar{\pi}_m + \bar{\pi}_{ri} + \bar{\pi}_{rj} \geq \tilde{\pi}_t = \tilde{\pi}_m + \tilde{\pi}_{ri} + \tilde{\pi}_{rj} \tag{51}$$

[4] The increased profit gain from the cooperation strategy.

[5] $CS_t = V_i^2$.

If the channel members commit to a cooperative program, they will split the extra-profits according to this bargaining problem:

$\Delta\pi_m = \bar{\pi}_m - \tilde{\pi}_m \geq 0$

$\Delta\pi_{ri} = \bar{\pi}_{ri} - \tilde{\pi}_{ri} \geq 0$

$$st: 0 < w < \bar{p} = \frac{2}{1 - 2\beta - \beta^2}, 0 \leq t \leq 48\%. \tag{52}$$

where $\Delta\pi_m$ and $\Delta\pi_{ri}$ are the extra-profits that can be made by manufacturer and retailer i $(i = 1,2)$ respectively and evidently verify $\Delta\pi_t = \Delta\pi_m + \Delta\pi_{r1} + \Delta\pi_{r2}$.
To find a suitable division of funds between the three partners, we use the approach of Nash bargaining model (Nash, 1950) using a power utility functions of type $u(x) = x^{\lambda}$[6].

We can formulate the Nash bargaining model by:

$$\mathrm{u_t} = u_m(\Delta\pi_m)^{\mu_m}\ u_{ri}(\Delta\pi_{ri})^{\mu_{ri}}\ u_{rj}(\Delta\pi_{rj})^{\mu_{rj}} \tag{53}$$

where μ_m, μ_{ri} and μ_{rj} are positive parameters reflecting each member's bargaining power and verifying $\mu_m + \mu_{ri} + \mu_{rj} = 1$. Next, we derive the following optimization problem:

$$\max u_t = {\Delta\pi_m}^{\lambda_m\mu_m} {\Delta\pi_{r1}}^{\lambda_{ri}\mu_{ri}}\ {\Delta\pi_{rj}}^{\lambda_{rj}\mu_{rj}}$$

$$st: \Delta\pi_t = \Delta\pi_m + \Delta\pi_{ri} + \Delta\pi_{rj} \qquad \Delta\pi_m, \Delta\pi_{ri}, \Delta\pi_{rj} > 0 \tag{54}$$

where λ_m, λ_{ri} and λ_{rj} are positive parameters denoting the risk attitudes of the manufacturer and the two retailers.

Proposition 7: The Nash bargaining model leads to the following division of profits:

$$\Delta\pi_m = \frac{\lambda_m\mu_m}{\lambda_m\mu_m + \lambda_{r1}\mu_{r1} + \lambda_{r2}\mu_{r2}}\Delta\pi_t \tag{55}$$

$$\Delta\pi_{r1} = \frac{\lambda_{r1}\mu_{r1}}{\lambda_m\mu_m + \lambda_{r1}\mu_{r1} + \lambda_{r2}\mu_{r2}}\Delta\pi_t \tag{56}$$

$$\Delta\pi_{r2} = \frac{\lambda_{r2}\mu_{r2}}{\lambda_m\mu_m + \lambda_{r1}\mu_{r1} + \lambda_{r2}\mu_{r2}}\Delta\pi_t \tag{57}$$

After determining $\Delta\pi_m$, $\Delta\pi_{r1}$ and $\Delta\pi_{r2}$, the manufacturer and the two retailers can prepare themselves to make good decisions about $\bar{w}$ ant $\bar{t}$.

From (52), we derive respectively:

[6] Another possible utility function is the exponential function $u(x) = 1 - \exp(-\tau x)$

$$\bar{w} \leq \bar{w}_{max} = \frac{-\frac{1}{2}\tilde{\pi}_m(\beta^2 + 2\beta - 1)^2 - 2 - t(\beta^2 + 2\beta + 1)}{2(\beta^2 + 2\beta - 1)} \tag{58}$$

$$\bar{w} \geq \bar{w}_{min} = \frac{\tilde{\pi}_{ri}(\beta^2 + 2\beta - 1)^2 + \beta^2 + 2\beta - 3 - t(\beta^2 + 2\beta + 1)}{2(\beta^2 + 2\beta - 1)} \tag{59}$$

That is to say, the three partners can coordinate the supply chain if the wholesale price satisfies the next proposition.

Proposition 8: The monopolistic manufacturer and the duopolistic retailers adopt the cooperative strategy if and only if the wholesale should be between a minimal value and a maximal value as described below:

$$\frac{\tilde{\pi}_{ri}(\beta^2 + 2\beta - 1)^2 + \beta^2 + 2\beta - 3 - t(\beta^2 + 2\beta + 1)}{2(\beta^2 + 2\beta - 1)} \leq \bar{w} \leq \frac{-\frac{1}{2}\tilde{\pi}_m(\beta^2 + 2\beta - 1)^2 - 2 - t(\beta^2 + 2\beta + 1)}{2(\beta^2 + 2\beta - 1)} \tag{60}$$

For each $t \in [0; 0{,}48]$.

Still from (52), we rewrite:

$$\bar{\pi}_m = \Delta\pi_m + \tilde{\pi}_m \Leftrightarrow \frac{3 + 2w\beta^2 + 4w\beta - 2w - \beta^2 - 2\beta + t\beta^2 + 2t\beta + t}{(\beta^2 + 2\beta - 1)^2} = \Delta\pi_m + \tilde{\pi}_m \tag{61}$$

Proposition 9: The manufacture's participation rate for local advertising expenditures is:

$$\bar{t} = \frac{-\frac{1}{2}(\beta^2 + 2\beta - 1)^2(\Delta\pi_m + \tilde{\pi}_m) - 2w(\beta^2 + 2\beta - 1) - 2}{(\beta^2 + 2\beta + 1)} \tag{62}$$

5. Conclusion

Our paper investigated a static model for cooperative advertising and pricing decisions in supply chain with one monopolistic manufacturer and two duopolistic retailers. Cooperative advertising has attracted much attention in the academic field and has been widely used in practice as a significant marketing tool that influences the advertising activities and pricing policies in supply chain.

This research is motivated by the scarcity of studies that discussed a marketing channel where a single manufacturer sells its product through two competing retailers. In other words, we want to relax the classical one manufacturer one retailer framework to one manufacture two retailers framework (the supply chain becomes more pragmatic). Also, the little research which deals with one manufacturer two retailers does not take pricing decisions into account directly. For this reason, we propose an additive form of a demand function which is influenced by advertising expenditures and retail price. We likewise assume that the local advertising effort not only benefits the investing retailer but also the other competing retailer located in close physical proximity. The novelty here is that we used an additive form of demand function.

To develop optimal cooperative advertising strategies and pricing decisions, we develop three games including the Stackelberg – Cournot game (retail competition), the the Stackelberg – Collusion game (retail coalition) and the cooperative game (cooperation of the manufacturer and the two retailers). Through a comparison between them, we obtain the following insights:

1) All decision variables (except $\tilde{t}$), the demand and the profits in three games depend on the effectiveness of compete retailer's local advertising (β). Our research indicates that β must be between 0 and 0.41. Also as it increases, the variables decisions, the demand and the profits increase.

2) Retail competition offers the best price to the consumer, but requires the manufacturer to reduce its participation rate in local advertising expenditures. Under the retail competition, the wholesale price, all advertising, the consumer demand, the profit of each member as well as the total profit, the consumer surplus and the social profit are lower than under retail coalition.

3) The highest total profit and the highest local and national advertising expenditures are generated under the cooperation. Although this situation produces the highest retail price and the highest consumer demand. In addition, the surplus consumer and the social welfare increase.

For future researches, there are several possible directions. First, while our model assumes the manufacturer to be Stackelberg leader, one can consider that the retailers have the real channel power to dictate the terms to the manufacturer (the retailers are the Stackelberg leaders). Second, the demand function proposed in this paper is a deterministic in nature; uncertain demand seems to be an interesting topic for future research.

Appendix

Proof of Proposition 1.

● Solving the first partial derivatives of the profit function of each retailer yields to the following expressions:

$$p_i = \frac{1}{2}\left(1 + a_i + \beta a_j + A + w\right)$$

$$a_i = \frac{1}{2}\frac{p_i - w}{1 - t}$$

After substitution, we get the solutions of equations (14) and (15).

To proof the second order conditions of the solutions of retailer i, we calculate the Hessian matrix:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_{ri}}{\partial p_i^2} & \frac{\partial^2 \pi_{ri}}{\partial p_i \partial a_i} \\ \frac{\partial^2 \pi_{ri}}{\partial a_i \partial p_i} & \frac{\partial^2 \pi_{ri}}{\partial a_i^2} \end{pmatrix}$$

The second order partial derivatives are as follows:

$$\frac{\partial^2 \pi_{ri}}{\partial p_i^2} = -2 \qquad \frac{\partial^2 \pi_{ri}}{\partial a_i^2} = -2(1-t) \qquad \frac{\partial^2 \pi_{ri}}{\partial p_i \partial a_i} = 1$$

The first principal minor of H is negative (H^1). The second principal minor of H is: $H^2 = \frac{\partial^2 \pi_{ri}}{\partial p_i^2} \cdot \frac{\partial^2 \pi_{ri}}{\partial a_i^2} - (\frac{\partial^2 \pi_{ri}}{\partial p_i \partial a_i})^2 = -2(1-t)$ is positive if $t < \frac{3}{4}$. This means that the profit function of each retailer π_{ri} is concave at the solution (p^*, a^*), which is a local maximum.

- The first derivatives of formula (15) with respect to A, t and w are:

$$\frac{\partial \pi_m}{\partial A} = \frac{4(-1+t)w}{4t+\beta-3} - \frac{4t(-w+A+1)}{(4t+\beta-3)^2} - 2A = 0.$$

$$\frac{\partial \pi_m}{\partial t} = \frac{4(-w+A+1)w}{4t+\beta-3} - \frac{16(-1+t)(-w+A+1)w}{(4t+\beta-3)^2} - \frac{2(-w+A+1)^2}{(4t+\beta-3)^2} +$$

$$\frac{16t(-w+A+1)^2}{(4t+\beta-3)^3} = 0$$

$$\frac{\partial \pi_m}{\partial w} = -\frac{4(-1+t)w}{4t+\beta-3} + \frac{4(-1+t)(-w+A+1)}{4t+\beta-3} + \frac{4t(-w+A+1)}{(4t+\beta-3)^2} = 0$$

After substitution and algebraic simplification, we obtain the solutions of equations (17), (18) and (19).

To proof the second order conditions of the solutions of manufacturer, we have the following Hessian matrix:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_m}{\partial A^2} & \frac{\partial^2 \pi_m}{\partial A \partial t} & \frac{\partial^2 \pi_m}{\partial A \partial w} \\ \frac{\partial^2 \pi_m}{\partial t \partial A} & \frac{\partial^2 \pi_m}{\partial t^2} & \frac{\partial^2 \pi_m}{\partial t \partial w} \\ \frac{\partial^2 \pi_m}{\partial w \partial A} & \frac{\partial^2 \pi_m}{\partial w \partial t} & \frac{\partial^2 \pi_m}{\partial w^2} \end{pmatrix}$$

The first principal minor of H is $H^1 = -\frac{68\beta-62+26\beta^2-40\beta^3-8\beta^4}{(2\beta^2+5\beta-5)^2}$ and is negative. The second principal minor of H is $H^2 = -\frac{2048\left(\beta+\frac{3}{2}\right)^4(\beta-\frac{23}{8})}{(2\beta^2+5\beta-5)^2(4\beta^2+12\beta-15)^2}$ and is positive. The third principal minor of H is $H^3 = \frac{512(\beta+\frac{3}{2})^4}{(4\beta^2+12\beta-15)(2\beta^2+5\beta-5)^2}$ is negative if $4\beta^2 + 12\beta - 15 < 0$. That means $\beta \in [0,0.94]$ and H is negative definite. So, the manufacturer's profit function π_m is concave at the solution (A^*, t^*, w^*), which is a local maximum.

This completes the proof of Proposition 1.

Proof of Proposition 2.

● Solving the first partial derivatives of the joint retailers' profit function yields to the following expressions:

$$p_i = \frac{1}{2}(1 + w + a_i + \beta a_j + A)$$

$$a_i = \frac{1}{2}\frac{p_i - w + \beta(p_j - w)}{1 - t}$$

After substitution and algebraic simplification, we obtain the solutions of equations (28) and (29).

To proof the second order conditions of the solutions of two retailers, we calculate the following Hessian matrix:

$$H = \begin{pmatrix} \frac{\partial^2\pi_r}{\partial p_i^2} & \frac{\partial^2\pi_r}{\partial p_i\partial p_j} & \frac{\partial^2\pi_r}{\partial p_i\partial a_i} & \frac{\partial^2\pi_r}{\partial p_i\partial a_j} \\ \frac{\partial^2\pi_r}{\partial p_j\partial p_i} & \frac{\partial^2\pi_r}{\partial p_j^2} & \frac{\partial^2\pi_r}{\partial p_j\partial a_i} & \frac{\partial^2\pi_r}{\partial p_j\partial a_j} \\ \frac{\partial^2\pi_r}{\partial p_i\partial p_i} & \frac{\partial^2\pi_r}{\partial a_i\partial p_j} & \frac{\partial^2\pi_r}{\partial a_i^2} & \frac{\partial^2\pi_r}{\partial a_i\partial a_j} \\ \frac{\partial^2\pi_r}{\partial a_j\partial p_i} & \frac{\partial^2\pi_r}{\partial a_j\partial p_j} & \frac{\partial^2\pi_r}{\partial a_j\partial a_i} & \frac{\partial^2\pi_r}{\partial a_j^2} \end{pmatrix}$$

The first principal minor of H is $H^1 = -2$ is negative. The second principal minor of H is $H^2 = 4$ and is positive. The third principal minor of H is $H^3 = 2\beta^2 - \frac{10}{3}$ and is negative. The fourth principal minor of H is $H^4 = \frac{1}{9}(3\beta^2 + 6\beta - 5)(3\beta^2 - 6\beta - 5)$ and is positive if $\beta \in [0, 0.63]$. So, the principal minors of H have alternating algebraic signs at the solution $(\tilde{p}, \tilde{a})$. This means that H is negative definite and the profit of both retailers π_r is concave at this solution, which is a local maximum.

● The three first partial derivatives of the manufacturer's profit function are:

$$\frac{\partial \pi_m}{\partial A} = \frac{4(-1+t)w}{4t+\beta^2-3+2\beta} - \frac{4t(1-w+A)(1+\beta)^2}{(4t+\beta^2-3+2\beta)^2} - 2A = 0$$

$$\frac{\partial \pi_m}{\partial t} = \frac{4(1-w+A)w}{4t+\beta^2-3+2\beta} - \frac{16(-1+t)(1-w+A)}{(4t+\beta^2-3+2\beta)^2} - \frac{2(1-w+A)^2(1+\beta)^2}{(4t+\beta^2-3+2\beta)^2} + \frac{16t(1-w+A)^2(1+\beta)^2}{(4t+\beta^2-3+2\beta)^3} = 0$$

$$\frac{\partial \pi_m}{\partial w} = -\frac{4(-1+t)w}{4t+\beta^2-3+2\beta} + \frac{4(-1+t)(1-w+A)}{4t+\beta^2-3+2\beta} + \frac{4t(1-w+A)(1+\beta)^2}{(4t+\beta^2-3+2\beta)^2} = 0$$

After substitution and algebraic simplification, we find the solutions of equations (31), (32) and (33).

To proof the optimality of the solutions of manufacturer under the Stackelberg- Collusion game, we calculate the Hessian matrix as shown in proof of proposition 1 and we found:

The first principal minor of H is $H^1 = -\frac{12(1+\beta)^2}{(3\beta^2+6\beta-5)^2} - 2$ is negative. The second principal minor of H is $H^2 = -\frac{144(1+\beta^2)(9\beta^2+18\beta-23)}{(3\beta^2+6\beta-5)^4}$ and is positive if $\beta \in [0, 0.88]$. The third principal minor of H is $H^3 = \frac{864(1+\beta)^2}{(3\beta^2+6\beta-5)^3}$ and is negative if $\beta \in [0, 0.63]$. So, the principal minors of H have alternating algebraic signs at the solution $(\tilde{A}, \tilde{t}, \widetilde{w})$. This means that H is negative definite and the profit of manufacturer π_m is concave at this solution, which is a local maximum.

This completes the proof of Proposition 2.

Proof of proposition 3.

To proof the second order conditiond of the solutions of supply chain members, we calculate the following Hessian matrix:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_t}{\partial p_i^2} & \frac{\partial^2 \pi_t}{\partial p_i \partial p_j} & \frac{\partial^2 \pi_t}{\partial p_i \partial a_i} & \frac{\partial^2 \pi_t}{\partial p_i \partial a_j} & \frac{\partial^2 \pi_t}{\partial p_i \partial A} \\ \frac{\partial^2 \pi_t}{\partial p_j \partial p_i} & \frac{\partial^2 \pi_t}{\partial p_j^2} & \frac{\partial^2 \pi_t}{\partial p_j \partial a_i} & \frac{\partial^2 \pi_t}{\partial p_j \partial a_j} & \frac{\partial^2 \pi_t}{\partial p_j \partial A} \\ \frac{\partial^2 \pi_t}{\partial p_i \partial p_i} & \frac{\partial^2 \pi_t}{\partial a_i \partial p_j} & \frac{\partial^2 \pi_t}{\partial a_i^2} & \frac{\partial^2 \pi_t}{\partial a_i \partial a_j} & \frac{\partial^2 \pi_t}{\partial a_i \partial A} \\ \frac{\partial^2 \pi_t}{\partial a_j \partial p_i} & \frac{\partial^2 \pi_t}{\partial a_j \partial p_j} & \frac{\partial^2 \pi_t}{\partial a_j \partial a_i} & \frac{\partial^2 \pi_t}{\partial a_j^2} & \frac{\partial^2 \pi_t}{\partial a_j \partial A} \\ \frac{\partial^2 \pi_t}{\partial A \partial p_i} & \frac{\partial^2 \pi_t}{\partial A \partial p_j} & \frac{\partial^2 \pi_t}{\partial A \partial a_i} & \frac{\partial^2 \pi_t}{\partial A \partial a_j} & \frac{\partial^2 \pi_t}{\partial A^2} \end{pmatrix}$$

The first principal minor of H is $H^1 = -2$ is negative. The second principal minor of H is $H^2 = 4$ and is positive. The third principal minor of H is $H^3 = 2\beta^2 - 6$ and is negative. The fourth principal minor of H is $H^4 = (\beta - 1)\,(\beta + 3)(\beta - 3)(1 + \beta)$ and is positive. The fifth principal minor of H is $H^5 = -2(\beta - 3)(1 + \beta)(\beta^2 + 2\beta - 1)$ and is negative if $\beta \in [0, 0.41]$. So, the principal minors of H have alternating algebraic signs at the solution $(\bar{p}, \bar{a}, \bar{A})$. This means that H is negative definite and the profit of the entire supply chain π_t is concave at this solution, which is a local maximum.

This completes the proof of Proposition 3.

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