Relative Risk Perception and the Puzzle of Covered Call Writing

Hammad Siddiqi

University of Queensland

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Hammad Siddiqi
The University of Queensland
h.siddiqi@uq.edu.au
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Market professionals with decades of experience typically argue that a call option is a surrogate for the underlying asset, indicating that they perceive the risk of a call option as similar to the risk of the underlying asset. Experimental evidence also points to the same conclusion. Such relative risk perception is in sharp contrast with finance theory, which argues that only the absolute quantity of risk contained in a call option should matter for its price. I show that relative risk perception provides a potential explanation for the puzzling performance of covered call writing.

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JEL Classification: G13, G12, G02
Relative Risk Perception and the Puzzle of Covered Call Writing

The profitability of covered call writing (one long stock plus one short call) is puzzling given the predictions of finance theory. Whaley (2002) examines the profitability of BXM (a Buy Write Monthly Index tracking a Covered Call on S&P 500) over a period ranging from 1988 to 2001 and find that BXM has significantly lower historical volatility when compared with the index, however, it has offered nearly the same return as the index.

Informational efficiency requires that markets equilibrate to the point where the risk adjusted returns from all assets are equal. It follows that, in an informationally efficient market, covered call writing should offer lower returns when compared with just holding the underlying stock, as covered call has lower risk than the underlying stock. A pre-requisite for informational efficiency is that all risks are correctly perceived. Correct perception requires that one forms a judgment regarding the total quantity of risk associated with any given asset. However, risk is a highly subjective notion, and the argument that risks are commonly misperceived in various contexts is frequently made. See Rotheli (2010), Bhattacharya, Goldman, & Sood (2009), Akerlof and Shiller (2009), and Barberis & Thaler (2002) among many others. For a broad survey of research on the psychology of risk from behavioral finance perspective, see Ricciardi (2008).

We have been hired-wired to think via analogies and comparisons. In fact, such thinking has been called the core of cognition and the fuel and fire of thinking by cognitive scientists and psychologists. See Hofstadter and Sander (2013).

Weber (2004) writes, “First, perceived risk appears to be subjective, and in its subjectivity, causal. That is, people’s behavior is mediated by their perception of risk. Second, risk perception, like all other perception, is relative. We seem to be hard-wired for relative rather than absolute evaluation. Relative judgments require comparisons, so many of our judgment are comparative in nature even in situations where economic rationality would ask for absolute judgments.” (p. 172).
There is strong field evidence suggesting that relative risk perception matters for call options. Specifically, it is common to find market professionals with decades of experience who argue that a call option is a surrogate for the underlying stock, and who typically consider call risk to be quite similar to and often no greater than the risk of the underlying stock. Furthermore, findings from controlled laboratory experiments also indicate that participants consider call option risk as similar to the underlying stock risk, and expect similar returns from them. See Rockenbach (2004), Siddiqi (2011), and Siddiqi (2012). It must be noted that such analogy making is very tempting as call’s payoffs are strongly related to the underlying stock’s payoffs.

In this article, I show that if the risk of a call option is perceived as similar to the risk of the underlying stock, then covered call writing is a lot more profitable strategy than what the theory (based on absolute risk judgments) suggests. Hence, relative risk perception provides a potential explanation for the puzzling historical performance of covered call writing.

The central prediction of asset pricing theory is:

\[ E[R_t] = R_F - \frac{1}{E[U'(\gamma_t+1)']} \text{Cov}[U'(\gamma_t+1), R_t] \]  

(0.1)

Where \( R_t \) and \( R_F \) denote the return on a risky asset and the return on the risk free asset respectively. Equation (0.1) shows that the return that a subjective expected utility maximizer expects from a risky asset depends on his belief about the covariance of the asset’s return with his marginal utility of consumption. Hence, one is required to form judgment about the total risk of a risky asset, which is given by the covariance of one’s marginal utility of consumption with the return of the risky asset. Theory requires that such judgments are formed for all available assets individually. It is difficult to see how one can go about doing that individually for all assets. It’s more likely that such judgments are formed for a more familiar asset, and then by analogy, extended to another similar asset. A call option by definition is strongly related to the underlying stock, and it seems reasonable to think that

\[ A \] A few examples of investment professionals arguing that a call option is a surrogate for the underlying asset are:

http://finance.yahoo.com/news/sto...
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm
risk judgments are extrapolated from stocks to call options, which are instruments derived from stocks.

In this article, relative risk perception means that one uses the risk of the corresponding underlying stock as a reference point for forming judgments regarding the risk of a call option. In contrast, absolute risk perception implies that one does not use such a reference point in forming risk judgments.

Relative judgment matters because it is typical for one to fail to adjust fully when starting from a reference point. That is, there is an automatic inclination to latch onto the reference point, and only deviate slightly from it. This is known as the “anchoring” heuristic. Anchoring effect is one of the most robust cognitive heuristics. 40 years of extensive research on the topic has shown it to be applicable to a wide variety of decision contexts. See Furnham, A., and Boo, H. C. (2011) for a literature review on anchoring.

Anchoring implies that using the risk of the corresponding underlying stock as a reference point for forming beliefs about the risk of a call option leads to an underestimation of risk. Such underestimation increases the price of the call option. Hence, covered call writing becomes more profitable.

If one forms a judgment about call option risk in comparison with his judgment about the underlying stock risk (relative risk perception), then one may write:

\[
\text{Cov}(u'(c_{t+1}), R_c) = \text{Cov}(u'(c_{t+1}), R_s) + \epsilon
\]  

(0.2)

Where \(R_c\) and \(R_s\) are call and stock returns respectively, and \(\epsilon\) is the (small) adjustment used to arrive at call option risk from the underlying stock risk. Almost always, assets pay more (less) when consumption is high (less), hence, the covariance between an asset’s return and marginal utility of consumption is negative. That is, I assume that \(\text{Cov}(u'(c_{t+1}), R_s) < 0\). So, in order to make a call option at least as risky as the underlying stock, I assume \(\epsilon \leq 0\).

In contrast, as call option is a leveraged position in the underlying stock, if the risk of call option and the underlying stock are separately considered (absolute risk perception) in isolation as assumed in theory, then one should expect:

\[
\text{Cov}(u'(c_{t+1}), R_c) = \pi \cdot \text{Cov}(u'(c_{t+1}), R_s)
\]  

(0.3)
Where $\pi > 1$ and typically takes very large values. That is, typically $\pi \gg 1$. To appreciate, the difference between (0.3) and (0.2), note that under the Black Scholes assumptions, $\pi = \Omega$, which is call option elasticity w.r.t the underlying stock price. $\Omega$ takes very large values, especially for out-of-the-money call options. That is $\text{Cov}(u'(c_{t+1}), R_C)$ is a far bigger negative number with absolute risk judgment than with relative risk perception. Hence, a comparison of (0.2) and (0.3) indicates that, with relative risk perception, one remains anchored to the risk of the underlying stock while forming risk judgments about the call option leading to underestimation of its risk.

Shouldn’t rational investors who form absolute risk judgments arbitrage away investors prone to relative risk perceptions? Barberis and Thaler (2002) argue that the absence of “free lunch” does not imply that prices are right. Relative risk perception creates a clear “free lunch” only if the underlying stock price follows geometric Brownian motion and there are no transaction costs. That is, if we are living in the Black Scholes world. Neither the assumption of “geometric Brownian motion” nor the assumption of “having zero transaction costs’ is defensible. Moving away from geometric Brownian motion to a more realistic process such as jump diffusion implies that an option cannot be replicated by other assets (see Merton (1976)). And, dropping the assumption of zero transaction cost implies that the total cost of replication grows without bound in continuous time (see Soner, Shreve, and Cvitanic (1995)). Hence, realistically, relative risk perception does not create a “free lunch”. Hence, relative risk perception, which seems to be an in-built human trait, is unlikely to go away.

1. **Relative Risk Perception and the Return from Covered Call Strategy**

Covered call writing is a long position in the underlying stock combined with a short position in a call option (typically, out-of-the-money) on the stock:

$$V = S - C$$ (1.1)

Where $S$ denotes the underlying stock price, and $C$ is the price of a call option on the stock.

Over a small time interval, $dt$, the portfolio value changes to:

$$dV = dS - dC$$ (1.2)
The expected return on the portfolio over a time interval $dt$ is given by:

$$E\left(\frac{dV}{V}\right) = \frac{1}{V} \cdot \{E(dS) - E(dC)\} \quad (1.3)$$

To proceed further, the dynamics of the underlying stock need to be specified. Following Merton (1976), I assume that the underlying stock price follows a jump diffusion process, which is a mixture of geometric Brownian motion and occasional Poisson jumps:

$$dS = [dS]_{Brownian} + [dS]_{jump} \quad (1.4)$$

$$[dS]_{Brownian} = \mu Sdt + \sigma SdZ \quad (1.5)$$

Where $dZ$ is a standard Gauss-Weiner process. Parameters $\mu$ and $\sigma$ capture drift and volatility respectively.

$$[dS]_{jump} = (J - 1)Sdq \quad (1.6)$$

Where $J$ is the size of the jump, and:

$dq = 1; \text{with probability } \lambda dt$

$dq = 0; \text{with probability } (1 - \lambda)dt$

That is, $E[dq] = \lambda dt$

In other words, the stock price dynamics are assumed to be as described by geometric Brownian motion interspersed with occasional jumps. When a jump happens, which happens with a probability $\lambda dt$, the stock price jumps to $JS$. For simplicity, I assume that jumps are symmetrically distributed around the current stock price. That is, $E[J] = 1$.

Substituting (1.5) and (1.6) into (1.4), one gets:

$$dS = \mu Sdt + \sigma SdZ + (J - 1)Sdq \quad (1.7)$$

In the same fashion as discussed above, one can write for the value of the covered call portfolio:

$$dV = [dV]_{Brownian} + [dV]_{jump} \quad (1.8)$$
\[ \text{[dV]}_{\text{Brownian}} = \text{[dS]}_{\text{Brownian}} - \text{[dC]}_{\text{Brownian}} \]

\[ \Rightarrow \text{[dV]}_{\text{Brownian}} = (\mu S dt + \sigma S dZ) - \left( \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2} \right) dt - \sigma S \frac{\partial C}{\partial S} dZ \tag{1.9} \]

(1.9) uses Ito’s lemma and makes a substitution from (1.5).

\[ \text{[dV]}_{\text{jump}} = \text{[dS]}_{\text{jump}} - \text{[dC]}_{\text{jump}} \]

\[ \Rightarrow \text{[dV]}_{\text{jump}} = (f - 1)Sdq - \{C(JS, t) - C(S, t)\}dq \tag{1.10} \]

Substituting (1.9) and (1.10) into (1.8) and taking expectations (assumption: probability of jump and jump size are independent), one gets:

\[ E[dV] = \mu S dt - \left( \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2} + \lambda [E[C(JS, t) - C(S, t)]] \right) dt \tag{1.11} \]

Hence, (1.3) can be expressed as:

\[ E \left[ \frac{dV}{V} \right] = \frac{\mu S dt - \left( \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2} + \lambda [E[C(JS, t) - C(S, t)]] \right) dt}{S - C} \tag{1.12} \]

The above equation is a general formulation for the expected return from covered call writing under jump diffusion. Up to this point, nothing has been assumed about how one forms risk judgments (either relative or absolute), and consequently, nothing has been said about the expected return from the call option.

To set up a benchmark for assessing the implications of relative risk perception, I consider the following two cases:

1) If absolute risk judgments (correct risk judgments) are formed, what is the expected return from covered call under the Black Scholes assumptions?

2) What is the expected return from covered call under the Jump Diffusion model with absolute risk judgments?

To see how the expected return from covered call changes with relative risk judgments, I consider relative risk judgments both in the Black Scholes and in Jump Diffusion settings separately.
1.1 Absolute Risk Judgment and Covered Call: The Black Scholes Case

If jump intensity is zero, that is, $\lambda = 0$, the jump diffusion model reduces to the Black Scholes model, so (1.12) becomes:

\[
E \left[ \frac{dv}{v} \right] = \frac{\mu Sdt - \frac{\partial C}{\partial t} + \mu \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}}{2}}{s - c} dt
\]

(1.13)

The Black Scholes PDE is:

\[
\frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}}{2} = rC
\]

(1.14)

Substituting (1.14) into (1.13) and realizing that call option elasticity w.r.t. the underlying stock price is $\Omega = \frac{s dC}{C dS}$, leads to the following:

\[
E \left[ \frac{dv}{v} \right] = \frac{\mu Sdt - C(\Omega + r)dt}{s - c}
\]

(1.15)

As $\Omega > 1$, the expected return from covered call is smaller than the expected return from the underlying stock. In fact, $\Omega$ takes very large values for out-of-the-money calls. This means that covered call writing should have substantially low return when compared with the underlying stock, as covered call writing typically involves out-of-the-money calls.

The empirical finding that covered call returns are similar to the underlying stock returns then implies either of the following:

a) The Black Scholes assumption regarding the underlying stock dynamics are wrong.

b) Actual risk perception is different from absolute risk judgment as assumed in theory.

c) Both a and b are true.

Next, I keep the assumption of absolute risk judgment, and show what happens to the expected return from covered call writing under the more realistic assumption of jump diffusion.
1.2 Absolute Risk Judgment and Covered Call: Jump Diffusion

If jump risk is priced, then the jump diffusion PDE is given by:

\[
\frac{\partial c}{\partial t} + \mu S \frac{\partial c}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} + \gamma E[C(SY, t) - C(S, t)] = g(S, t)c
\]  (1.16)

Where \(g(S, t)\) is the expected return from the call option when the jump risk is priced.

If the jump risk is not priced, then the expected return from call with jump diffusion should be equal to \(g(S, t) = \Omega(\mu - r) + r\). That is, it should equal the expected return under the Black Scholes case. It follows that if additional risk caused by jumps is priced, then:

\[
g(S, t) > \Omega(\mu - r) + r
\]  (1.17)

Substituting (1.16) in (1.12) and given (1.17), it follows:

\[
E \left[ \frac{dv}{V} \right] = \frac{\mu Sdt - g(S, t)Cdt}{S-C} < \frac{\mu Sdt - \Omega(\mu - r) + r dt}{S-C}
\]  (1.18)

Hence, adding jumps worsens the covered call puzzle, as the expected return from covered call with jump diffusion is even lower than the expected return under the Black Scholes assumptions. Historical covered call performance is consequently a major challenge to theory, as it cannot be explained by relaxing Black Scholes assumptions regarding the stock dynamics. This indicates that perhaps relative risk perception is at play here.

Next, the implications of relative risk perception for covered call performance are examined both under the Black Scholes as well as Jump Diffusion frameworks.

1.3 Relative Risk Perception and Covered Call Writing

With relative risk perception, the risk of a call option is assessed in comparison with the risk of the underlying stock. From (0.1) and (0.2) (assuming negative covariance between stock’s return and marginal utility of consumption, and that a call option is considered at least as risky the underlying stock), the expected return from a call option over a small interval \(dt\) is:

\[
E \left[ \frac{dc}{C} \right] = \mu + |\varepsilon|
\]  (1.19)
If the underlying stock follows geometric Brownian motion, then over a small interval $dt$:

$$E[dC] = \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2}$$

(1.20)

From (1.20) and (1.19), it follows:

$$\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2} = (\mu + |\epsilon|) C$$

(1.21)

Substituting (1.21) in (1.13) yields the expected return from covered call writing with relative risk perception if the underlying stock follows geometric Brownian motion:

$$E \left[ \frac{dv}{V} \right] = \frac{\mu Sdt - C(\mu + |\epsilon|)dt}{S - C}$$

(1.22)

If $\epsilon = 0$, then the expected return from covered call writing with relative risk perception is exactly equal to the expected return from the underlying stock. If $\epsilon \neq 0$, then the expected return is slightly lower. Hence, the empirical finding regarding return from covered call writing is no puzzle with relative risk perception.

If the underlying follows jump diffusion, then, with relative risk perception, it follows:

$$\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \gamma E[C(SY, t) - C(S, t)] = (\mu + |\epsilon|) C$$

(1.23)

Substituting (1.23) in (1.12) leads to the expected return from covered call writing with relative risk perception if the underlying stock follows jump diffusion:

$$E \left[ \frac{dv}{V} \right] = \frac{\mu Sdt - C(\mu + |\epsilon|)dt}{S - C}$$

(1.24)

(1.24) is the same as (1.22).

Hence, the empirical findings regarding covered call writing returns is no puzzle with relative risk perception irrespective of the whether the underlying follows geometric Brownian motion or jump diffusion.

Next, I examine the impact of relative risk perception on the volatility of covered call writing returns.
2. Relative Risk Perception and the Volatility of Covered Call Returns

The variance of returns from the covered call strategy is given by:

\[ E \left[ \frac{dv}{v} - E \left[ \frac{dv}{v} \right] \right]^2 \]  \hspace{1cm} (1.25)

Where

\[ dV = (\mu Sdt + \sigma SDZ) - \left( \frac{\partial c}{\partial t} + \mu S \frac{\partial c}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 c}{\partial S^2} \right) dt - \sigma S \frac{\partial c}{\partial S} dZ \]

\[ + (J - 1)SDq - \{C(JS, t) - C(S, t)\} dq \]  \hspace{1cm} (1.26)

And, \(E[dV]\) is given in (1.11).

So, the variance is equal to:

\[ E \left[ \left( \sigma S - \sigma \frac{\partial c}{\partial S} \right) dZ + \left( (J - 1)S - C(JS, t) + C(S, t) \right) dq + \lambda E[C(JS, t) - C(S, t)] dt \right]^2 \]

\[ = \frac{S - \Omega_c}{S - C} \]  \hspace{1cm} (1.27)

Note, that \(dZ\) and \(dq\) are assumed to be independent. Also, the probability of jump and jump size are independent. Expanding the square, and realizing that as \(dt \to 0\), higher powers of \(dt\) converge to zero at a faster rate, so higher powers of \(dt\) can be ignored, and also noting that \(Var(dq) = E[dq] = \lambda dt \Rightarrow E[dq^2] = 2\lambda dt\), one arrives at the following:

\[ E \left[ \frac{dv}{V} - E \left[ \frac{dv}{V} \right] \right]^2 = (\sigma^2 + 2\lambda E[(J - 1)^2]) \left[ \frac{S - \Omega_c}{S - C} \right]^2 dt \]  \hspace{1cm} (1.28)

Where \(\Omega_c\) is call price elasticity w.r.t the underlying stock price. As \(\Omega_c > 1\), and \((\sigma^2 + 2\lambda E[(J - 1)^2])\) is the volatility of the underlying stock returns, it is easy to see that covered call writing returns have lower volatility when compared with the volatility of underlying stock returns.

(1.28) holds for both relative as well as absolute risk perception. Compared to absolute risk perception, relative risk perception lowers \(\Omega_c\) slightly, however, it increases \(C\). Hence, moving to relative risk perception from absolute risk perception can potentially lead to both an increase as well
as a decrease in volatility, when compared with the volatility under absolute risk perception. However, the volatility remains substantially smaller than the volatility of the underlying stock returns in either case as $\Omega_c > 1$ with both relative as well as absolute risk perception.

3. Conclusion

The superior historical performance of covered call writing is an important puzzle in option pricing. Covered call writing is clearly a risk reducing strategy, so it should offer a lower return than the underlying stock in an informationally efficient market. However, empirically, it offers nearly the same return as the underlying while volatility of its returns is substantially lower. I show that if the risk of the underlying stock is used as a reference point for forming judgments about the risk of a call option, then covered call writing becomes substantially more profitable, while volatility of its returns remain substantially lower than the volatility of underlying stock returns. Hence, relative risk perception provides an explanation for the covered call writing puzzle.
References


