Can the Law of One Price be tested?

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Abstract

A simple stylised model, that incorporates transaction costs, is developed. The Law of One Price (LOP) is assumed to hold with regard to a reference market that is not taken into account in the empirical testing of the Law. It is shown that under these assumptions the empirical tests of the LOP will fail.

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One of the simplest and most intuitive market efficiency arguments with regard to price dynamics is incorporated into the purchasing power parity (PPP) and the law of one price (LOP). In simple words they state that the price levels (in the PPP case) or individual product prices (in the LOP case) should move together. In this paper we will be concerned specifically with the LOP. This avoids some complications related to the aggregation problems that can arise in a PPP context as well as the presence of non-tradable goods. Additionally, the validity of the LOP is essentially a necessary requirement for the PPP to hold.

Moreover, the LOP is such a fundamental and intuitive proposition that Lamont and Thaler (2003) define it as the 'Second law of economics'. It is often implicitly or explicitly postulated in quantitative models. There is a huge literature concerned with testing both the LOP and the PPP. A simple formal test on the validity of the LOP will consist of simply testing the stationarity of the price differential ΔP_t between two markets. If it is found to be stationary, the two prices are moving together in the long term, validating the LOP. If however the price differential contains unit root, this would reject the LOP. The above presents a strong version of the LOP. A somehow weaker version can be implemented by testing for stationarity the relative price. This weaker version allows for different preferences across the countries.

Recently transaction costs have become the major explanations for the empirical rejection of the purchasing power parity and the law of one price. Following Heckscher (1916), some authors have considered the possibility of an inaction region (or "inaction band") where the real exchange rate (RER) (or simply the price differential in the LOP case (or the relative price)) may behave like a random walk. The rationale for this is that in this case the price differential would be smaller than the transaction costs for trade. No arbitrage would then take place. Outside the inaction band defined by the transaction costs the excessive price differential would compensate these costs and the resulting trade (arbitrage) activities will bring the real exchange rate (price differential/relative price) back onto these bands (Obstfeld and Taylor, 1997; Bec *et al*., 2004). Microeconomic foundations for such behaviour are laid down in the "iceberg" model presented in Sercu *et al*. (1995) or its simplified version due to by Bec *et al*. (2004).

Note that although the above-described process is non-stationary in the inner inaction band, it is stationary in the outer bands and is thus globally stationary.

Real markets however are often characterised by numerous imperfections. In particular, information asymmetries and barriers to entry can create segmented markets (Shiha and Chavas, 1995). With segmented markets the inner inaction band model may not hold. Furthermore segmented markets create conditions for price discriminations, which may lead to asymmetric adjustment process, such as for example stationary process if the price differential is positive (probably after taking into account the transaction costs band, but a non-stationary process when the price differential is positive. Additionally there may be cases, such as neighboring countries, or countries which are both net exporters (importers) sharing the same main destination (source) market, in which the inner inaction band may not exist. Another reason for via lotion of the LOP may be price discrimination. Alessandria and Kaboski (2004) found "evidence of systematic international price discrimination based on the local wage of consumers in the destination market". Additionally institutional arrangements may cause the LOP to fail even in highly liquid financial markets (Lamont and Thaler, 2003). Barzel (2005) goes even further in arguing that the LOP would generally fail due to the unobservable informational content of the goods subject to the exchange. His informal theoretical model resembles closely the latent price process model introduced in Clark (1973).

Even if unobservable informational content of the goods is not an issue, the transaction cost models assume direct trade between two reference markets. This is obviously a very restrictive assumption. One can imagine situation where this cannot be the case, as for example both these markets can be only indirectly linked, by e.g. exporting onto a third reference market. This setup can have surprising implications for the tests of the LOP.

Let us define three markets with prices x_t, y_t and z_t.

Also let $\Delta_{1t} = y_t - x_t$; $\Delta_{2t} = y_t - z_t$; $\Delta_{3t} = x_t - z_t$. For brevity, we will omit the time subscript from most of our further notation.

Now let us assume that we observe only the *x* and *y* prices, but the 'true' reference market is the one for which we do not observe the price (i.e. *z*). This simply means that in a test of the LOP one does not use this price *z*, although it may be available in principle. In this case an econometrician would be interested in testing Δ_{1t} for unit roots.

Let us further assume that a transaction cost version of the LOP holds with regard to the reference market. The question is what would be the implications for the tests of the LOP, based on data on the other two markets.

For simplicity let us further assume with no loss of generality symmetric transaction cost bands.

Thus

$$\Delta_{2t} = \begin{cases} p_1 \Delta_{2,t-1} & \Delta_{2,t-1} > t \\ \Delta_{2,t-1} & -t \le \Delta_{2,t-1} \le t \\ p_2 \Delta_{2,t-1} & \Delta_{2,t-1} < -t \end{cases} \tag{1}$$

and

$$\Delta_{3t} = \begin{cases} q_1 \Delta_{3,t-1} & \Delta_{3,t-1} > s \\ \Delta_{3,t-1} & -s \le \Delta_{3,t-1} \le s \\ q_2 \Delta_{3,t-1} & \Delta_{3,t-1} < -s \end{cases} \tag{2}$$

with transaction costs defined by *t* and *s* respectively.

Let us denote the three cases for Δ_{2t} as 1, 2 and 3, and similarly denote the possible cases for Δ_{3t} as A, B, and C.

Under the conditions described above one would test $\Delta_{1t} = \Delta_{2t} - \Delta_{3t}$ for stationarity.

It is clear that Δ_1 would be stationary when both Δ_2 and Δ_3 are stationary, i.e. in cases 1A, 1C, 3A and 3C. Similarly when Δ_2 is stationary, but Δ_3 is not and vice versa, i.e. in the cases 2A, 2C,1B and 3B, then Δ_1 would be nonstationary.

The remaining case is 2B, when both Δ_2 and Δ_3 contain unit root.

In this case we can write

$$\Delta_{2t} = y_t - z_t = \Delta_{2,t-1} + u_t = y_{t-1} - z_{t-1} + u_t \text{ and}$$

$$\Delta_{3t} = x_t - z_t = \Delta_{3,t-1} + v_t = x_{t-1} - z_{t-1} + v_t$$

from where

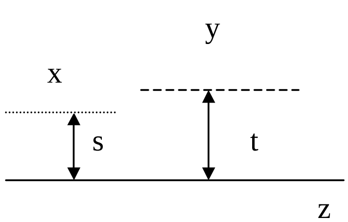
$$\Delta_{1t} = \Delta_{2t} - \Delta_{3t} = y_{t-1} - z_{t-1} + u_t - x_{t-1} + z_{t-1} - v_t = y_{t-1} - x_{t-1} + u_t - v_t = \Delta_{1,t-1} + u_t - v_t$$

Since both u_t and v_t are stationary it follows that Δ_1 contains unit root.

If we assume that t>s, we can present the above cases graphically. For this illustrative purpose without any loss of generality we can fix *z*.

Stationary cases

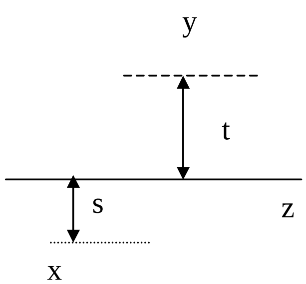
Case 1A



In this case *y* and *x* essentially share the same are above *z+t* (and *x* can also be in the interval (*z+s*,*z+t*)). The difference Δ_1 can therefore range from zero to very high values.

Note that case 3C is essentially the same as 1A, only that *y* and *x* lie below *z*. For this reason we will not draw any of the type 3 cases.

Case 1C

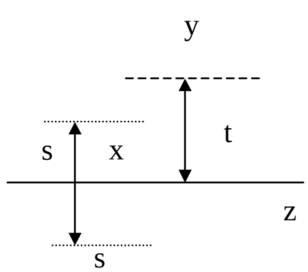


Here $\Delta_1 > t + s$.

Case 3A is similar.

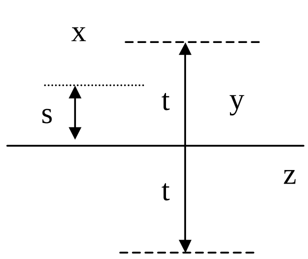
Non-stationary cases

Case 1B



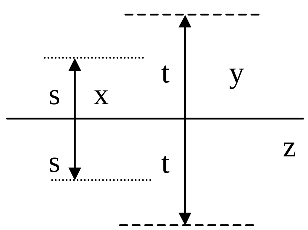
Here $\Delta_1 > t - s$. Similar same situation arises in case 3B.

Case 2A.



In this case, similarly to the stationary cases 1A and 3C, one cannot determine the magnitude of Δ_1, which can take arbitrary (in absolute terms) values. Case 2C is similar.

Case 2B.



In this case $\Delta_1 < t + s$.

Overall although cases 1C, 3A, 1B, 3B seem consistent with a ‘transaction cost’ band of *t+s,* case 2B is somehow out of line, being consistent with an inner ‘transaction cost’ band of *t-s*. More worryingly however the stationary cases 1A and 3C, as well as the non-stationary cases 2A and 2C have nothing to do with a ‘transaction cost’ band representation. Note furthermore that in all the above cases which can to some extent fit a ‘transaction cost’ band explanation, *y* and *x* are ‘synchronised with regard to the ‘unobserved’ *z*, in the sense that they are both in an inner or outer band, when these are defined with regard to the common market price *z.*

The general conclusion from the above analysis is that no threshold type of unit roots test is appropriate for testing the LOP if the two markets under consideration are not directly linked. It does not matter whether these are the conventional threshold unit root tests, their generalised smooth transition versions or indeed a non-linear unit root tests. The issue is that the stationarity of the price differentials is not function of their magnitude.

What are the general conclusions from this analysis? The LOP can be tested in the conventional way, only if the two markets under consideration are directly linked through an arbitrage type of relation. If this is not the case one may try to identify the appropriate reference markets and test the LOP with regard to the reference market(s).

Note that even if one cannot identify in advance the appropriate reference markets, formal tests of the LOP can still be carried out. For example in the unit root tests setup, we have identified five different regimes for the price differential dynamics. One may estimate a general regime-switching model[1] for the price differential and identify whether the latter fits with the same type of dynamics as identified above. One may actually expect to find smaller number of regimes, since for example cases 1C and 3A may not be very realistic in principle, unless there are considerable additional transaction costs between these two markets[2]. Also some of the identified regimes may not be present in particular instance of data used for testing the LOP.

Note however that the model we use here is still a rather simplified one. For example we assume that there is a single reference market, which does not change. One may envisage situations where for example both market under consideration have different reference markets and only the latter are linked together through a common reference market. Furthermore, relaxing the assumption that the reference market does not change introduces additional complications. Therefore even the regime switching approach suggested above may fall short of properly capturing the underlying price dynamics.

The implications for the PPP tests are much more serious. Since the LOP is a pre-requisite for the PPP, there does not seem to be any way to circumvent the reference market problem in a PPP setting. Even if the extremely restrictive and dubious assumption that there is a single reference market for all goods is implemented, the resulting pattern of 'violations' to the LOP for different goods is so complex that no reliable inference about what type of PPP test could be feasible seems possible.

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[1] I.e. where the regimes do not depend on the magnitude of the price differential. Markov-Switching models seem to be a natural candidate for this purpose.

[2] Since the prices in these cases fall on the opposite sites from the reference market price, one may ask why there can be no direct arbitrage between the two market in consideration. This can probably only be the case whet considerable trade barriers for entry onto the higher priced market exist.

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Appendix

In a more general setup one may use threshold co-integration, instead of threshold unit root tests to account for tastes, preferences and quality differences between the concerned markets.

In this case we simply need to redefine
$\Delta_{1t} = y_t - ax_t$; $\Delta_{2t} = y_t - bz_t$; $\Delta_{3t} = x_t - cz_t$. With *a, b* and *c* taken from the cointegrating relationships (ignoring the constants), so that **(1)** and **(2)** still hold with regard to the so redefined Δ_2 and Δ_3, if there are no intercepts. If there are intercepts they will modify the thresholds, which does not impact substantially on our argument. For example, if b_0 is the intercept for Δ_2, meaning that the co-integration relationship is $0 = y_t - bz_t - b_0$, one simply needs to replace *t* with $t + b_0$ and –t with $-t + b_0$. This has the effect of shifting the thresholds, so that they are no longer symmetrical. Therefore without any loss of generality, unless we want to draw the separate cases, we can safely ignore the intercepts.
Since $\Delta_2 - a\Delta_3 = y - bz - a(x - cz) = y - ax + (ac - b)z$
We can write

$$\Delta_1 = \Delta_2 - a\Delta_3 + (b - ac)z \tag{3}$$

As before we are interested in whether Δ_1 contains unit root.
Lets us firsts consider the case when *b*=*ac*. Then the last term in (3) vanishes. It is easy to see that then all our previous conclusions hold. The only difference is case 2B, which may also turn to be stationary if the variables Δ_2 and Δ_3 happened to be cointegrated with a vector (1, -*a*).
If on the other hand $b \neq ac$, since the last term in (3) is non-stationary, all the stationary cases in our previous analysis will now turn non-stationary. The situation with the non-stationary cases is slightly more involved, since the relevant expressions contain two non-stationary variables.
Bearing in mind that

$$\Delta_2 + (b - ac)z = y - acz \tag{4}$$

and

$$-a\Delta_3 + (b - ac)z = bz - ax \tag{5}$$

If the expression in (4) for example is a local co-integration relationship for case 2 (and similarly for (5) for case B), the corresponding cases may revert to stationarity.
Overall however, most cases will be non-stationary. What is important for the argument presented here is that similarly to the threshold unit roots, threshold cointegration cannot provide valid representation to the LOP in the presence of transaction costs, unless the proper reference markets are identified in advance.