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A Different Approach of Tax Progressivity Measurement

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1. Introduction

According to economical, political and social circumstances, fiscal authorities take appropriate measures to adjust fiscal policy according to economic and social trends. Such fiscal modifications frequently include changing the income tax rate. In some cases, the reason for modifying the income tax rate is an attempt to redistribute income in a society. In other cases, the increase or decrease of the income tax rate will have a direct effect in the purchasing power of taxpayers, thus maintaining the existing state of vertical equity.

When it comes to the a more equal redistribution of income, fiscal authorities apply lower tax rates for low income groups and higher tax rates for high income groups. When measures are related to changes in purchasing power, the tax rate changes (decrease/increase) occurs proportionally among all groups of taxpayers. The proportional change does not affect the redistributive aspect of taxation.

In order to understand whether such measures are in line with a desired goal, should be used an indicator (coefficient), one that clearly reflects the changes. When fiscal measures are related to redistribution income, the disproportional change of the tax liability will change the coefficient in a direction that decreases for low-income taxpayers and increases for taxpayers with high incomes. If the government's aim is to change the purchasing power, proportional changes in tax liability will result in an unchanged coefficient.

Structuring and setting an optimal income tax system is a difficult task for policymakers. On one hand, policymakers try to avoid distortions in the system and aim for a fair distribution of income – vertical equity in other words, which directly affects social justice and economic equality. However, in most cases governments tend to prioritize efficiency and economic growth, which affects horizontal equity in the process of tax liability distribution between individuals with different incomes.¹

2. Measuring methods for income tax progression

An income tax system may be proportional, regressive or progressive. A proportional income tax means that as income increases, so does tax liability. An income tax system is considered regressive when the tax liability rate increase is lower than the income increase. A regressive tax system may be: proportional – when there is proportional rate of regression, progressive – when there is a progressive rate of regression, and regressive – when the regression rate is regressive. A progressive tax system means that the rate of tax liability increase is higher than the rate of income increase. A progressive tax system may be: proportional - the rate of progression is proportional in all income scales, regressive – the rate of progression falls when entering into higher income brackets, and progressive – the rate of progression increases as income increases.

Regarding measuring the degree of income tax progression, various measures are proposed that express the ratio of change in the variables that are used for calculation.² It is not difficult to

¹ Most collected taxes come from individuals with low incomes as they constitute the majority of taxpayers.

² There are disagreements about which variables to use when measuring tax progressivity. Regardless of the disagreements, all these theories accept the definition of progressivity to mean: an income tax system is proportional when the average tax rate equals the marginal tax rate;

measure these changes - it is simply an arithmetic calculation. The question, however, is which variables to use for the calculation and how to interpret the results? Interpreting these results correctly is important for taxpayers and policymakers. First, the coefficient is the price taxpayers have to pay for a given income level. Second, fiscal authorities can use this coefficient as an indicator for measurement during the course of fiscal policy changes in accordance with other economic and social policy. Third, the coefficient which represents the degree of progression should be comparable between time frames and countries.

Authors with special interest in the field of income tax progression have proposed different models for measuring income tax progression. The coefficients of progression that result from these measures differ not only in terms of the amplitude, but also the degree of progression. Furthermore, the coefficient which shows income tax progressivity, may decrease or increase for a given income tax system depending on the model that is applied.

Average rate progression (AP) measures the change rate of the average tax rate (Pigou, 1960). It may be written as:

$$AP = \Delta a_{(y)} / \Delta y$$

It may be also calculated from the following formula:

$$AP = (m_{(y)} - a_{(y)}) / y - \Delta y$$

where y – income, Δy – change in income, $a_{(y)}$ - average rate, $\Delta a_{(y)}$ – change in average rate, $m_{(y)}$ - marginal rate.

For $AP=0$ proportional tax structure, $AP<0$ regressive tax structure, $AP>0$ progressive tax structure.

Marginal rate progression (MP) is defined as the ratio of change in the marginal tax rate to the change in income (Pigou, 1960). The coefficient may be calculated from the following formula:

$$MP = \Delta m_{(y)} / \Delta y$$

where $\Delta m_{(y)}$ is a change of the marginal rate in a given level of income and Δy is a change in income.

The coefficient will equal zero when the tax is proportional, exceed zero when the tax is progressive and fall short of zero when the tax is regressive.

Liability progression (LP) measures the ratio of percentage change in tax liability to the percentage change in income (Musgrave and Thin, 1948). Authors proposed two models for measuring liability progression that differ in whether the changes in income are small or major.

In case of small changes in income, the coefficient is defined as:

$$LP = (\Delta t_{(y)} / \Delta y) * (y / t_{(y)})$$

It may be also written as:

$$LP = m_{(y)} / a_{(y)}$$

progressive – when the average tax rate is lower than the marginal tax rate; and regressive - when the average tax rate exceeds the marginal tax rate.

If the coefficient equals 1, then tax is proportional, if it exceeds 1, then tax is progressive and if it falls short of 1, then tax is regressive.

For major changes in income, liability progression may be calculated from the following formula:

$$LP = [\Delta t_{(y)} / (t_{(y)} - \Delta t_{(y)})] * [(y - \Delta y) / \Delta y]$$

or

$$LP = \Delta m / a$$

The coefficient which is recommended to be used in this case, measures the rate of tax liability increases to the rate of income increases. In other words, this ratio shows how many times taxation has increased as income has increased at a given level.

Residual income progression (RP) measures the ratio of the percentage change in income after tax, to the percentage change in income before tax (Musgrave and Thin, 1948).

For major income changes, the coefficient is defined as:

$$RP = \{[\Delta(y - t_{(y)}) / [(y - \Delta y) - (t - \Delta t)]] * [(y - \Delta y) / \Delta y]$$

When the changes in income are small, residual income progression is expressed as:

$$RP = [\Delta(y - t_{(y)}) / \Delta y] * [y / (y - t_{(y)})]$$

Or

$$RP = (1 - m_{(y)}) / (1 - a_{(y)})$$

If the coefficient equals 1, taxation is proportional, if exceeds 1, taxation is regressive, and if it falls short of 1, then taxation is progressive.

The models mentioned above all take into account the income change or average tax rate in the denominator. However, the results obtained by measuring the rate of progression in a given level of income from these methods do not show any connection with the previous income level. With the exception of residual income progression, all methods of measurement have different range of the values that exceed the magnitude of ± 1 , even if the average rates are low.

The divergences between the different methods of approach of defining progression determines that, within the methods of marginal rate progression, liability progression and residual income progression, the marginal rate in different forms ($m_{(y)}$, $1 - m_{(y)}$ and $\Delta m_{(y)}$) is always in the numerator. The average rate progression is defined by changes in the average tax rate $\Delta a_{(y)}$, so the average tax rate is in the numerator.

3. An alternative measure of tax progressivity

The model of tax progression (TP) measurement introduced here expresses the rate of tax progression at a given level of income, which keeps into account the tax liability in the prior level of incomes. The coefficient measures the difference between the rate of tax liability change and the rate of income change in a given level.

The rate of tax liability change in a given level of income may be written as $\Delta t_{(y)} / t_{(y)}$. Similarly, the rate of income change may be written as $\Delta y / y$. In order to maintain the link

between tax liabilities in different income levels for purposes of comparing tax progression, the calculation would be as follows:

$$T_p = (\Delta t_{(y)} / t_{(y)}) - (\Delta y / y) \quad \langle 1 \rangle$$

Thus, the proportion of income changes is subtracted from the proportion of tax liability changes. The coefficient calculated in this way signifies how much the tax liability proportion exceeds the income proportion in a given level of income. The amplitude of the coefficient does not exceed ± 1 .

The difference is equal to zero ($T_p=0$) when taxation is proportional ($m=a$), negative ($T_p<0$) when taxation is regressive ($m<a$), and positive ($T_p>0$) when taxation is progressive ($m>a$). The main difference between the tax liability rate change and rate of income change, depends on whether taxation is more progressive. If the coefficient remains the same at all income levels, then tax progression is proportional. If changes in the degree of progression increase when income increases, then tax progression is progressive. And conversely, if changes in the degree of progression decrease as income increase, the tax progression will be regressive.

When tax liability in a given level of income is multiplied with a progression coefficient, it will result in a number (amount) which, when added to the tax liability in the prior level of income, equals the average tax rates in both levels of income (see Appendix). Mathematically, this may be expressed as:

$$a_{(y1)} = a_{(y2)} \Rightarrow [(T_p (y2) t_{(y2)}) + t_{(y1)}] / y_1 \quad \langle 2 \rangle$$

Since $a_{(y)} = t_{(y)} / y$, than $a_{(y1)}$ will equal $a_{(y2)}$ also as:

$$a_{(y1)} = a_{(y2)} \Rightarrow \{[(t_{(y2)} / y_2) - (t_{(y1)} / y_1)] y_1 + t_{(y1)}\} / y_1 \quad \langle 3 \rangle$$

The formulation $[(t_{(y2)} / y_2) - (t_{(y1)} / y_1)]$ may be written as $\Delta a_{(y2)}$, so we have:

$$a_{(y1)} = a_{(y2)} \Rightarrow [(\Delta a_{(y2)} y_1) + t_{(y1)}] / y_1 \quad \langle 4 \rangle$$

From this, we may derive:

$$T_p = [\Delta a_{(y)} (y - \Delta y)] / t_{(y)} \quad \langle 5 \rangle$$

Since:

$$\Delta a_{(y)} (y - \Delta y) = \Delta y (m_{(y)} - a_{(y)}) \quad \langle 6 \rangle$$

the coefficient of tax progression may be defined also as:

$$T_p = [\Delta y (m_{(y)} - a_{(y)})] / t_{(y)} \quad \langle 7 \rangle$$

Since equations (1), (5) and (7) provide identical results, then it may be written as:

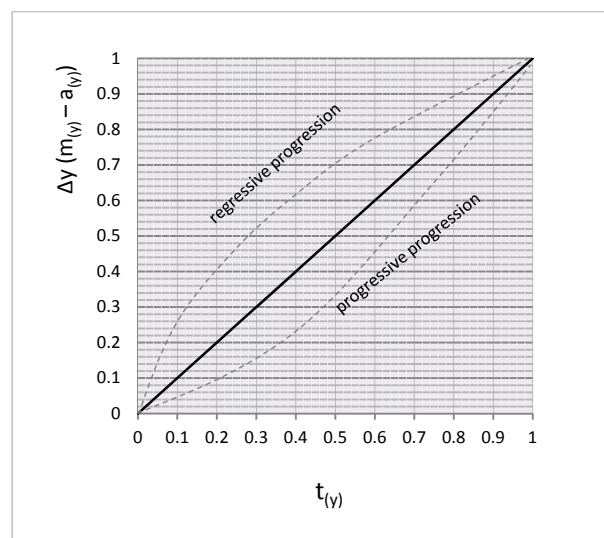
$$[(\Delta t_{(y)} / t_{(y)}) - (\Delta y / y)] = [\Delta a_{(y)} (y - \Delta y)] / t_{(y)} = [\Delta y (m_{(y)} - a_{(y)})] / t_{(y)} \quad \langle 8 \rangle$$

The coefficient fulfills the two axioms of progressivity (Kakwani, 1986):

1. The degree of progressivity is unaffected if the share of tax liability of every individual remains the same.
2. If the total tax liabilities share of a person with income x is increased (decreased) and that of a person with a lower income is decreased (increased), then progressivity must increase (decrease).

The coefficients of tax progressivity suggested in this paper, can be graphed by plotting denominator values on the horizontal axis and nominator values on the vertical axis. This is shown in Figure 1. The diagonal line shows the proportional distribution of tax liability. In other words, if the value of the coefficient in all income levels equals the value in the diagonal line, then the tax system is proportionally progressive. If the coefficient increases when moving up to on the income scale, the line of progression will fall under the proportionality line. Conversely, if the coefficient decreases when moving up the income level, the line of progression will fall above the proportionality line.

Figure 1 Distribution of tax liability



The degree of (progressive) progression is high, if the distance between the progression line and the diagonal line increases when moving from lower to higher income ranges. Conversely, the degree of (progressive) progression is low, if the difference between the progression line and the diagonal line decreases when moving from lower to higher income ranges. This interpretation is valid if the line of progression lies below the diagonal line.

The degree of (regressive) progression is low, when the line of progression which lies above diagonal line, goes near to the diagonal line when moving from lower to higher income ranges. The wider the distance between two lines when moving up the income scale, the higher the degree of (regressive) progression.

4. Conclusion

In this paper is introduced an alternative measuring method for income tax progression. This method is consistent with general definition of progressivity, and fulfills criterions for evolving alternative measure of tax progression. The specific of this measurement methodology is that, on the one hand, resulted coefficient shows the difference between proportion of tax liability changes and proportion of income changes in a given income level. On the other hand, the coefficient shows the percentage of tax liability that a group with higher income pays more for an income to the previous (lower income) bracket, than tax liability that a group falling into that previous (lower income) bracket has to pay. So, changes (increase/decrease) in coefficient resulted in a taxable income bracket may be compared with a coefficient resulted in previous taxable income bracket.

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